Relativistic & pseudo-relativistic connections in optical pulse propagation

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We explore a more complete model for describing the evolution of scalar optical pulses in generic nonlinear waveguides. The electromagnetic wave envelope u satisfies a dimensionless equation that is of the Helmholtz type, namely

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i\left(\frac{\partial u}{\partial \zeta} + \alpha \frac{\partial u}{\partial \tau} \right) + \frac{s}{2} \frac{\partial^2 u}{\partial \tau^2} + f\left(|u|^2 \right) u = 0 \,. \tag{1}$$

The space/time coordinates are ζ/τ, while (α, s, κ) are system parameters and the function $f(|u|^2)$ describes the light-induced properties of the host material. Our approach is to retain the first term in Eq. (1), which is routinely neglected – with few exceptions [1] – throughout nearly 50 years of literature. Pulse propagation problems are firmly rooted in frame-of-reference considerations, and as such the mathematical structure of Eq. (1) allows us to draw intriguing parallels with Einstein's special theory of relativity (e.g., the velocity combination rule for pulses is akin to that for particles in relativistic kinematics). Exact analytical bright solitons have been derived for a range of classic nonlinearities when $s = +1$, and their robustness has been tested through exhaustive computations (see figure 1).

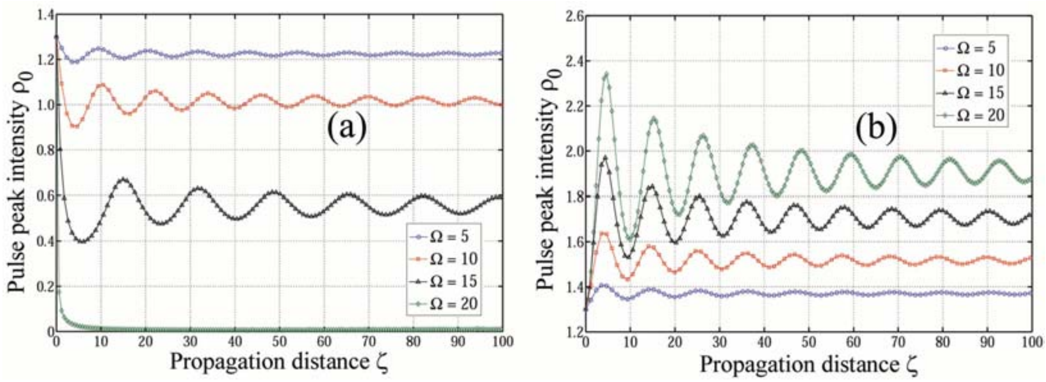

Figure 1. Simulations testing the robustness of perturbed bright solitons of Eq. (1) for a cubic-quintic nonlinearity when the net spatial dispersion parameter [2] κ is (a) positive, and (b) negative. Solutions are more robust in this latter regime, and they radiate less strongly during the self-reshaping process.

References

[1] Kh. I. Pushkarov, D. I. Pushkarov, and I. V. Tomov, Opt. Quantum Electron. **11**, 471–478 (1979).

[2] F. Biancalana and C. Creatore, Opt. Exp. **16**, 14882–14893 (2008).