

# Nonlinear wave phenomena at optical boundaries: spatial soliton refraction

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## Abstract

The behaviour of light at the interface between different materials underpins the entire field of Optics. Arguably the simplest manifestation of this phenomenon – the reflection and refraction characteristics of (infinitely-wide) *plane waves* at the boundary between two linear dielectric materials – can be found in many standard textbooks on electromagnetism (for instance, see Ref. 1). However, laser sources tend to produce collimated output in the form of a *beam* (whose transverse cross-section is typically “bell-shaped” and, hence, finite). When describing light beyond the plane-wave limit, a more involved and sophisticated treatment is generally required.

If a beam of light is propagating in a nonlinear planar waveguide, its innate tendency to diffract (spread out) may be compensated by the self-lensing properties of the host medium (whose refractive index is intensity-dependent). Such nonlinear photonic systems are driven and dominated by complex light-medium feedback loops. Under the right conditions (e.g., where the amplitude and phase of the input beam have the correct transverse distribution), the light may evolve with a stationary (invariant) intensity profile. Such self-localizing and self-stabilizing nonlinear waves are known as *spatial optical solitons*.

The seminal analyses of spatial soliton refraction were performed by Aceves, Moloney, and Newell [2,3] more than two decades ago. They reduced the full electromagnetic complexity of the interface problem by adopting the scalar approximation, and describing the light field within an intuitive (paraxial) nonlinear Schrödinger-type framework. In this way, they provided the first description of how light beams behave when impinging on the boundary between two dissimilar Kerr-type materials. These early, ground-breaking works yielded a great deal of physical insight. However, the assumption of beam paraxiality necessarily restricts the angles of incidence, reflection, and refraction (relative to the interface) to negligibly (or near-negligibly) small values. Ideally, one would like to find a way of lifting this inherent angular limitation without forfeiting a straightforward and analytically-tractable governing equation.

Recently, the seminal works of Aceves *et al.* [2,3] have been built upon by our Group. A new modelling formalism has been developed, based on an inhomogeneous nonlinear Helmholtz equation, whose flexibility accommodates both bright [4,5] and dark [6,7] spatial solitons at arbitrary angles of incidence, reflection, and refraction with respect to the interface:

$$\kappa \frac{\partial^2 u}{\partial \zeta^2} + i \frac{\partial u}{\partial \zeta} + \frac{1}{2} \frac{\partial^2 u}{\partial \xi^2} \pm |u|^2 u = \left[ \frac{\Delta}{4\kappa} \pm (1-\alpha)|u|^2 \right] H(\xi, \zeta) u. \quad (1)$$

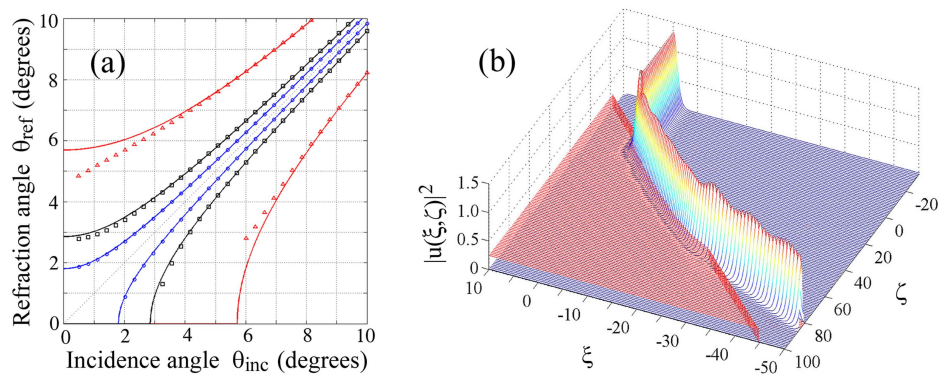
Here,  $u$  is the dimensionless electric field envelope, while  $(\xi, \zeta)$  are the transverse and longitudinal coordinates (normalized with respect to the waist and diffraction length of a reference Gaussian beam, respectively). The small parameter  $\kappa \ll O(1)$  quantifies beam waist compared to the optical wavelength, and the  $\pm$  sign flags focusing/defocusing materials. Mismatches between the linear and nonlinear refractive index are quantified by the parameters  $\Delta$  and  $\alpha$ , respectively, while the Heaviside unit function  $H$  defines the location and orientation of the interface in the  $(\xi, \zeta)$  coordinate frame.

The use of Helmholtz-type nonparaxial models allows for a complete angular characterization of spatial solitons, which is crucial in studies of this type. Most recently, we have been investigating systematic generalizations of Eq. (1) to capture higher-order medium effects. For instance, the classic power-law model [8] replaces terms at  $|u|^2$  with the generic form  $|u|^q$ , where the exponent  $q$  assumed a continuum of values  $0 < q < 4$ ; exact analytical power-law Helmholtz solitons [9] can then be deployed as a nonlinear basis for analyzing refraction phenomena. This model plays a fundamental role in photonics, and can be used to capture, for instance, leading-order effects due to saturation of the nonlinear-optical response.

In this presentation we detail our latest research, which has investigated arbitrary-angle scattering of spatial solitons at the planar boundary between two dissimilar cubic-quintic optical materials [10,11]. This is, to the best of our knowledge, the first time that refraction effects have been considered within this type of material context. The derivation of our novel Helmholtz refraction law will be discussed in detail:

$$\gamma n_{01} \cos \theta_{\text{inc}} = n_{02} \cos \theta_{\text{ref}}, \quad (2)$$

where  $n_{01}$  and  $n_{02}$  are the refractive indexes of the material before and after the interface respectively,  $\theta_{\text{inc}}$  and  $\theta_{\text{ref}}$  are the angles of incidence and refraction of the soliton, respectively, and  $\gamma$  describes the interplay between beam nonparaxiality and mismatches in medium properties. An illustrative selection of results will be presented, which compare theoretical predictions to full numerical computations. The level of agreement between the predictions of Eq. (2) and full numerical computations with Eq. (1) has been found to be generally excellent [see Fig. 1(a)]. Examples of the Goos-Hänchen shift (a phenomenon occurring close to the critical angle, where the trajectory of the reflected beam is displaced laterally relative to the path predicted by geometrical optics [12,13]) have also been uncovered [see Fig. 1(b)].



**Figure 1.** (a) Comparison of Snell Law predictions (solid lines) against full numerical simulations (points) for a range of linear interface parameters. (b) Non-trivial Goos-Hänchen shift for an incident Helmholtz soliton close to the critical angle.

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