The role of turbulent diffusion on thermal comfort in naturally ventilated buildings

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ABSTRACT: In recent years there has been considerable effort to develop simple models for wind and buoyancy induced natural ventilation in buildings that can be used to improve existing building thermal simulation tools. These models have been successful at predicting the bulk flow rate through a ventilated room, but less successful at predicting the vertical temperature distribution within a room. This is due to turbulent mixing within the room caused by people walking, convection, and ventilation inflow momentum. Prediction of the vertical temperature profile is important because it directly influences people's perception of thermal comfort. We examine the role of mixing in terms of diffusivity, whether molecular or turbulent, on the steady-state stratification in a ventilated filling box. The buoyancy driven displacement ventilation model of Linden, Lane-Serff & Smeed (1990) predicts that, when a single plume is introduced into an enclosure with vents at the top and bottom, a vertical stratification with two well mixed layers will form. The model assumes that mixing and diffusion play no roles in the development of the ambient temperature stratification of the room; diffusion is a relatively slow process and the entrainment of ambient fluid into the plume will act to sharpen the interface at the temperature step between the two layers. The prediction of a sharp interface has been confirmed by small scale salt bath experiments. However, full scale measurements in ventilated rooms, and complementary CFD simulations, indicate that the interface between the two layers is not sharp but rather smeared over a finite thickness. We show that as the cross-sectional area of the enclosure increases and for a given plume buoyancy flux, the volume of fluid that must be entrained by the plume in order to maintain a sharp interface increases. Therefore the balance between the diffusive thickening of the interface and plume-driven sharpening favors a thicker interface. This paper describes a diffuse layer thickness model and discusses the significance of the layer thickness for occupants' thermal comfort.

1 INTRODUCTION

Increasing energy costs and a desire to reduce greenhouse gas emissions have led to renewed interest in energy efficient building design. To that end, significant research has addressed simple models that describe the physics of wind and buoyancy driven building natural ventilation flows. Basic models consider a single room with floor and ceiling level vents, and have since been extended to consider more complex building geometry. The models are built on a number of simplifying assumptions, including the assumption that diffusion plays no significant role in these ventilation flows.

1.1 *Natural ventilation models*

The two primary models for buoyancy driven ventilation of a single room are based on two extreme assumptions about the distribution of the heat load in the room. The mixed ventilation model of Gladstone and Woods (2001) assumes that the heat load is distributed evenly across the entire room floor and that turbulent thermal convection heats the room uniformly so that the temperature is the same at all heights and the warm air in the room produces a stack driven ventilation flow. The other extreme is the assumption made in the displacement ventilation model of Linden et al. (1990). In their work the heat load is assumed to be concentrated as a given point producing a turbulent plume that rises upward and spreads out at the top of the room. As the plume rises it entrains ambient fluid and its volume flow rate Q increases with height as

$$Q = 0.16F^{1/3}z^{5/3} \qquad (1)$$

(see Morton et al. (1956)). Q is the volume flow rate, z is the vertical distance above the source and F is the buoyancy flux, which is related to the source heat release, or power output, by

$$F = \frac{Pg}{C_p \rho_0 T_0} \qquad (2)$$

(see Batchelor (1954)), where P is the power output by the source, g is acceleration due to gravity, C_p is the specific heat of air, ρ_0 is the reference density, and T_0 is the reference temperature in Kelvin.

After an initial transient, a steady flow develops with a two layer thermal stratification in the room. The upper warm layer drives a stack flow through the vents (A_U and A_L) and the sharp density step at the interface (height h) between the upper and lower layers suppresses vertical mixing. See figure 1 for a schematic diagram of the steady flow. The only air crossing the interface is through the plume. Balancing the stack driven flow with the plume flow rate at the interface gives

$$Q = 0.16F^{1/3}h^{5/3} = A^*\sqrt{g'h} \qquad (3)$$

where $g' = g(\rho - \rho_0)/\rho_0 = F^{2/3}/(0.16h^{5/3})$ is the reduced gravity of the upper layer and A^* is the effective vent area given by

$$\frac{1}{A^{*2}} = \frac{1}{2c^2A_L^2} + \frac{1}{2c^2A_U^2} \qquad (4)$$

in which c is a loss coefficient for the vents. Scaling the interface height on the room height, $\zeta = h/H$, and simplifying equation 3 leads to an expression for the interface height in terms of the room height and the vent effective area:

$$\frac{A^*}{H^2} = 0.16^{3/2} \left(\frac{\zeta^5}{1-\zeta}\right)^{1/2}. \qquad (5)$$

The model assumes a sharp interface, and therefore no diffusive effects. The buoyancy of the upper layer is equal to the buoyancy in the plume at the interface height

$$g' = \frac{F}{Q} = \frac{F^{2/3}}{0.16h^{5/3}} \tag{6}$$

which can be related back to a temperature difference between the floor and ceiling ΔT_r by

$$\Delta T_r = \frac{g'T}{g} \tag{7}$$

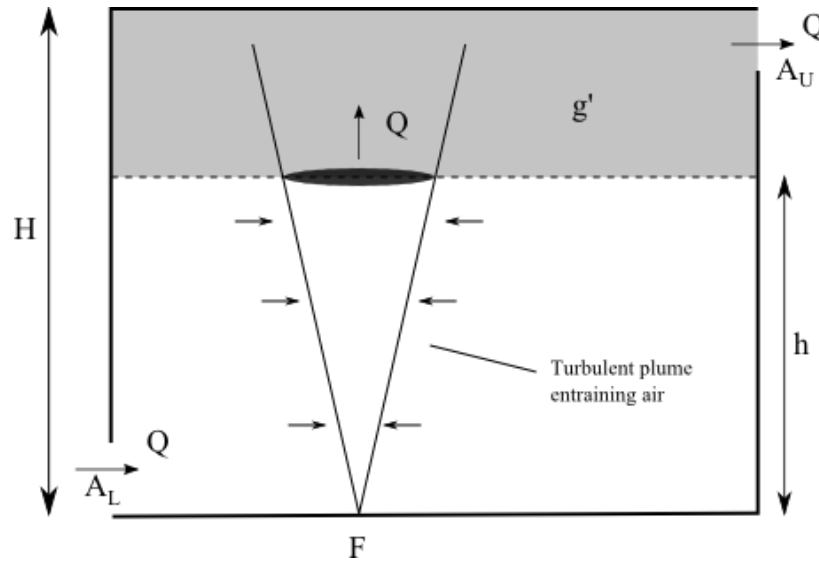where T is the ambient temperature in Kelvin.

Figure 1 Schematic diagram of the Linden et al. (1990) model for buoyancy driven displacement ventilation.

1.2 *Thermal comfort*

One of the goals of any ventilation system is to ensure adequate thermal comfort and indoor air quality for room occupants. The advantage of displacement ventilation is that warm stale polluted air is displaced upward into the warm upper layer. Provided there is sufficient vent area, the room can be designed so that the interface height is above the breathing zone of the room's occupants. For a full review of thermal comfort conditions and indoor air quality, see Awbi (2003).

One potential concern with the Linden et al. (1990) model is that turbulent or molecular diffusion will cause the interface to smear drawing warm stale air down to occupant level. This paper considers when such diffusive smearing is likely to pose a thermal comfort problem, particularly the problem of thermal discomfort caused by vertical variations in temperature over a person's body. Olsen et al. (1979) showed that, for seated occupants, up to 10% will experience thermal discomfort if the temperature difference between their feet and head was 3°K or greater. ASHRAE 55-1992 further requires that this 3°K difference be enforced for standing occupants up to a height of 1.7m. This paper considers the circumstances under which an otherwise comfortable design will lead to thermal discomfort due to diffusion. The remainder of the paper is as follows. Section 2 reviews the role of diffusion on displacement ventilation and extends the existing model to consider thermal comfort. Section 3 presents results for the limits on thermal comfort. Discussion of the significance of these results, and conclusions, are drawn in section 4.

2 MODEL DEVELOPMENT

Our thermal comfort model is based on the model of Kaye et al. (accepted) on the role of diffusive processes on the displacement ventilation model of Linden et al. (1990).
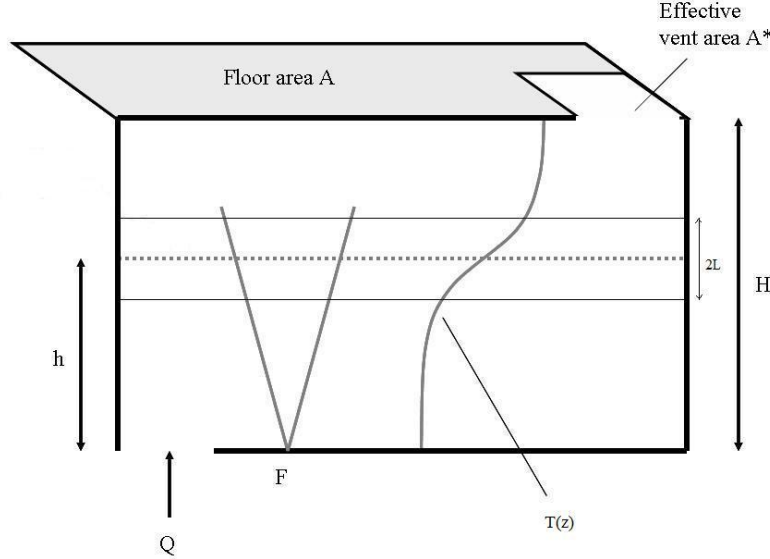

Figure 2 Schematic diagram showing the diffuse layer of thickness 2L along with the previous Linden et al. (1990) parameters and floor area, A, needed to analyze the role of diffusion.

2.1 *Diffuse layer thickness*

While Linden et al. (1990) assume that the interface between two layers is sharp, diffusive processes will act to smear it. The appropriate length scale for the extent of one dimensional diffusion over time is given by $L = \sqrt{4\kappa t}$ where κ is a diffusion coefficient, whether molecular or turbulent. See figure 2 for a schematic diagram of the diffuse layer. Eventually a balance develops within the diffuse layer between the growth of the layer due to diffusion, and the thinning of the layer due to entrainment of layer fluid into the plume. This balance leads to an approximate half thickness of the diffuse layer of

$$L = \left[\frac{6\kappa A}{0.8F^{1/3}h^{2/3}}\right]^{1/2} \qquad \Longleftrightarrow \qquad \frac{L}{H} = \left(\frac{2}{R}\right)^{1/2}\left[\frac{2\sqrt{5}}{3\pi\zeta}\right]^{1/3} \qquad (8)$$

where R quantifies the balance of convective interface thinning and diffusive spreading.

$$R = \frac{(2\alpha)^{4/3}F^{1/3}H^{8/3}}{\kappa A} \qquad (9)$$

where A is the room floor area and α is the plume entrainment coefficient ($\alpha = 0.11$, see Morton et al. (1956)). See Kaye et al. (accepted) for a full description of the model development. Note that equation 8 includes the Linden et al. (1990) predicted interface height, ζ, so the interface height is therefore a function of both R and A^*/H^2. A plot of this relationship is shown in figure 3.
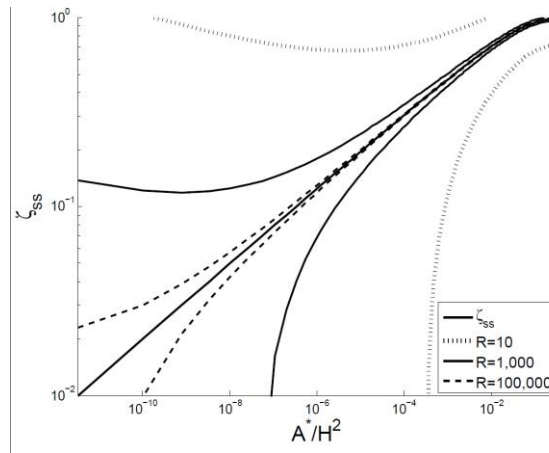
Figure 3 Steady state interface height and diffuse layer thickness as a function of A^*/H^2, for a range of R.

For a given interface height and plume buoyancy flux, the interface will get thicker as the room floor area increases due to the greater volume of air that needs to be removed from the diffuse layer by the plume. Conversely, for a given floor area, the interface will get thinner as the plume buoyancy flux increases, since this increases the rate of entrainment into the plume. Typical values of R for a room are strongly dependent on the room geometry and plume buoyancy flux. To give a sense of scale using molecular diffusivity, a 100m^2 room of height 3m and heat load of 1kW has $R = 300$. However, this is a very conservative estimate as the actual turbulent diffusivity is likely to be at least an order of magnitude larger leading to much smaller values of $R \sim O(10^0 - 10^1)$.
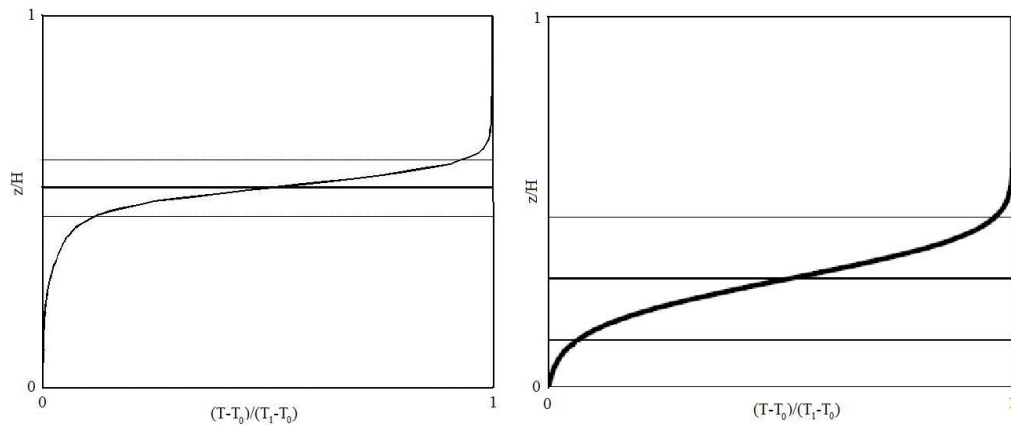

Figure 4 Comparison between approximate layer thickness model (equation 8) and (a) Filling box diffusion model for $R = 100$, and (b) a CFD simulation for $\frac{A^*}{H^2} = 0.0208$. Central line is interface height; outer lines represent the extent of the diffuse layer based on equation 8. The horizontal axis is the relative temperature difference.

2.2 *Model verification*

The simple phenomenological model presented above was verified theoretically through two sets of numerical simulations. First the full set of differential equations that describe the development of the plume and the room stratification were solved numerically for a range of R and A^*/H^2. See Baines and Turner (1969) and Linden et al. (1990) for the full model description. Second, a series of 3D CFD simulations were conducted, see Kaye et al. (2009), to model the development

of the stratification. The steady state temperature profiles from these two sets of simulations were compared to the approximate model for the interface thickness. Examples of vertical profiles based on each of the simulations is plotted in figure 4 along with the interface bound, based on equation 8. Agreement between both sets of simulations and equation 8 is very good.

2.3 *Three layer model for thermal comfort*

The primary goal of this paper is to examine the impact of the diffuse layer described above on the thermal comfort of room occupants. As stated in the introduction, up to 10% of room occupants will experience thermal discomfort if the temperature difference between their feet and head exceeds 3^oK. In order to estimate when this is likely to be a concern, the temperature gradient is approximated by a three layer model. The upper and lower layers are well mixed and given by the model of Linden et al. (1990). The diffuse middle layer is described by a straight line; see figure 5, for an example. In terms of the layer thickness (equation 8), the interface height ζ (equation 5) and the floor to ceiling temperature difference ΔT_r, the temperature difference between the floor and any height, z, in the diffuse layer is given by

$$\Delta T = \left(\frac{z+L}{H} - \zeta\right)\frac{H}{2L}\Delta T_r.$$
(10)

Recalling that $\zeta = \zeta(A^*/H^2)$ (equation 5), $L/H = L/H(A^*/H^2, R)$ (equation 8), and that $\Delta T_r = \Delta T_r(\zeta, P)$ (equations 2, 6 and 7) we can express the temperature at any height in the diffuse layer as a function of A^*/H^2, R, and P. Due to the complexity in inverting equation 5, an explicit form of this relationship is not given; instead individual cases are examined numerically.
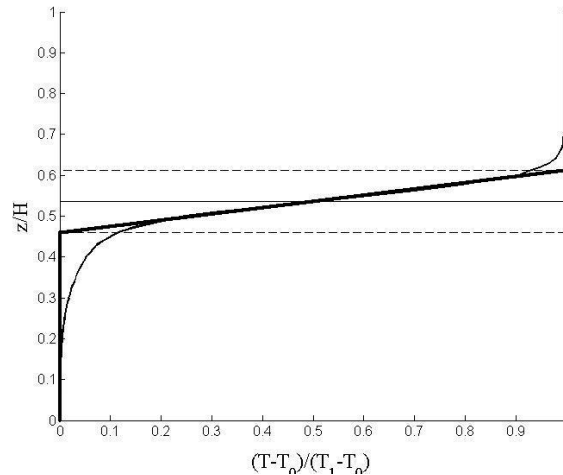

Figure 5 Three layer model with linear temperature variation in the diffuse layer drawn over the results of a CFD simulation temperature profile. The dark line is the three layer model; thin line, the Linden et al. (1990) interface height prediction; dashed lines, the diffuse layer edge (equation 8); light solid line, the CFD simulation profile.

3 MODEL RESULTS

In order to demonstrate the possibility of thermal discomfort caused by interface diffusion, calculations were done for a 3m high room to find the value of R at which a person with a height of 1.7m ($z/H = 0.57$) feels a temperature difference of 3^oK from their feet to their head. The first set of calculations show the value of R as a function of ζ (figure 6 (a)) and A^*/H^2 (figure 6

(b)) for different floor to ceiling temperature differences. Data below each line represents uncomfortable conditions. For example, a room with a design interface height of 2.1m ($\zeta = 0.7$) and a floor to ceiling temperature difference of 10°K will have uncomfortable conditions for $R < 2$. Therefore, although the design interface height would suggest that the person occupies only space below the warm upper layer, diffusion has the potential to create uncomfortable conditions, particularly in larger floor area rooms or rooms with high turbulent diffusivity due to drafts, people walking, or excessive ventilation inlet velocities.
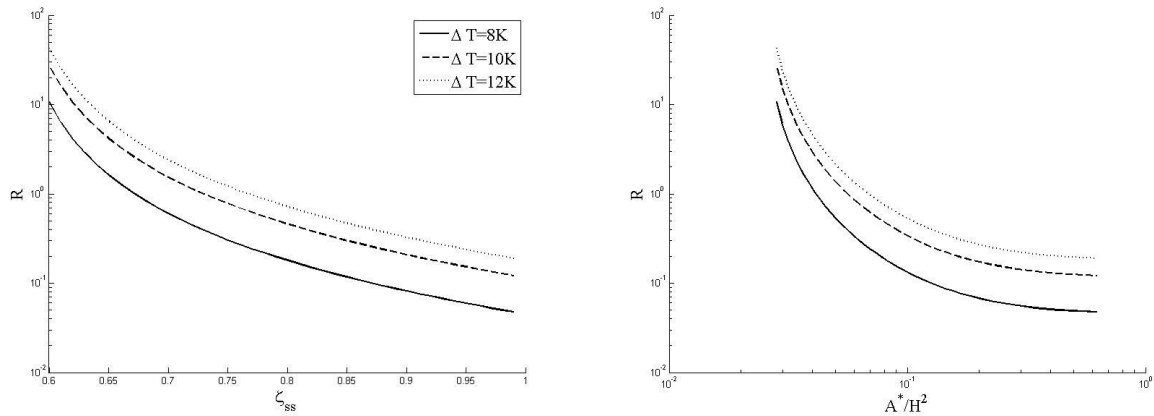

Figure 6 Log-log plots of the transition from comfortable and uncomfortable conditions as a function of (a) R and ζ and (b) **R** and A*/H^2 , in a 3m room for different floor to ceiling temperature differences. Uncomfortable conditions are below each line.
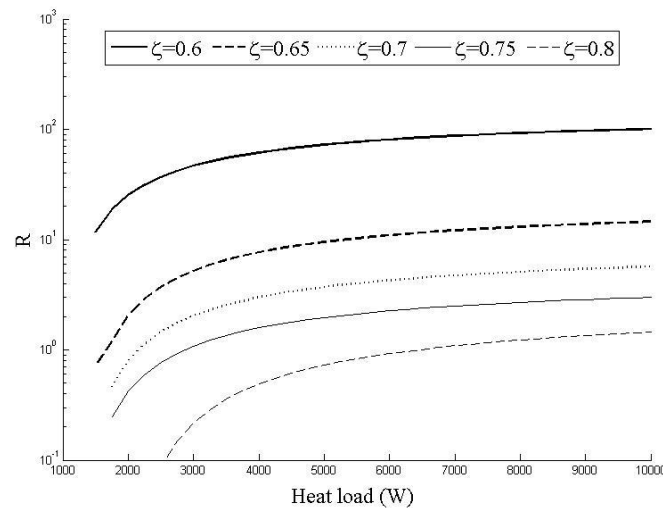

Figure 7 Transition from comfortable to uncomfortable conditions as a function of heat load in a 3m high room for different design interface heights. Uncomfortable conditions are below the lines.

The second set of calculations (figure 7) shows the value of R for the transition from comfortable to uncomfortable conditions as a function of heat load for different design interface heights. Again, taking data from the plot, a room with a design interface height of 1.95m ($\zeta = 0.65$) and a heat load of 5.5kW, conditions will become uncomfortable for $R < 10$. Note that the values of R plotted in the figures are independent of the value of thermal diffusivity as the calculations are

done based on the second part of equation 8, to find the diffusive layer thickness, and equation 10, to determine the 1.7m height temperature difference.

From a design perspective, for a given room area, height and heat load, the value of R can be calculated given estimates of the design interface height and the room's thermal diffusivity. From this, figures 5 and 6 can be used to determine whether or not the diffusion of the interface will lead to uncomfortable conditions. If it does, then the design interface height needs to be raised and the calculation repeated.

4 DISCUSSION AND CONCLUSIONS

The current trend toward energy efficient building design is leading to increased demand for simple design tools for analyzing the performance of energy efficient ventilation systems. In order to be useful, these ventilation models and design tools attempt to capture the key physical processes that drive ventilation flows and control thermal comfort. A classic example of this is the model of Linden et al. (1990) which demonstrated that non-uniformity in horizontal distribution of heat load throughout a room will lead to vertical variations in temperature through the action of localized convective plumes. However, as a simplifying assumption, their model ignores the effect of thermal diffusion, arguing that diffusion is a slow process compared to turbulent plume convection. This paper demonstrates that, under certain circumstances, diffusion can lead to thermal discomfort resulting from vertical temperature differences across an occupant's body.

While the results presented in this paper can be used as a guide to the role of diffusivity, more work is required to accurately parameterize turbulent mixing in the indoor environment. Work is ongoing to establish the most appropriate parameterization of mixing in ambient air for use in CFD simulations and other models.

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