# PETRI-NET-BASED SUPERVISORY CONTROL OF DISCRETE EVENT SYSTEMS AND THEIR LADDER LOGIC DIAGRAM IMPLEMENTATIONS

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*To my parents,*

*Mehmet and Zeynep Uzam,*

*and*

*To my wife,*

*Aynur Uzam*

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# ABSTRACT

The last decade has witnessed rapid developments in computer technology, which in return, has found widespread applications in manufacturing systems, communication networks, robots etc. Such systems are called *Discrete Event Systems* (DESs), in which properties such as non-determinism, conflict and parallelism are exhibited. As DESs become more complex, the need for an effective design tool and its implementation becomes more important. *Supervisory control theory*, based on *finite state machines* (FSM) and formal languages, is a well established framework for the study of DESs. In supervisory control, given a model of the system and the desired system behaviour specifications, the objective is to find a supervisor (controller) such that the controlled behaviour of the system does not contradict the specifications given and does not unnecessarily constrain the behaviour of the system. In general, the classes of specifications that have been considered within the supervisory control fall into two categories: the forbidden state problem, in which the control specifications are expressed as forbidden conditions that must be avoided, and the desired string problem, in which the control specifications are expressed as sequence of activities that must be provided.

In supervisory control, there are some problems when using FSMs as an underlying modelling tool. Firstly, the number of states grows exponentially as the system becomes bigger. Secondly, FMSs lack from graphical visiualisation. To overcome these problems *Petri nets* have been considered as an alternative modelling tool for the analysis, design and implementation of such DESs, because of their easily understood graphical representation in addition to their well formed mathematical formalism.

The thesis investigates the use of Petri nets in *supervisory control*. Both the forbidden state problem and the desired string problem are solved. In other words, this work presents systematic approaches to the synthesis of Petri-nets-based supervisors (controllers) for both the forbidden state problem and the desired string problem and introduces the details of supervisory design procedures. The supervisors obtained are the form of a net structure as oppose to supervisors given as a feedback function. This means that a controlled model of the system can be constructed and analysed using the techniques regarding to Petri net models.

In particular the thesis considers discrete manufacturing systems. The results obtained can be applied to high level control of manufacturing systems, where the role of the supervisor is to coordinate the control of machines, robots, etc. and to low-level control of manufacturing systems, where the role of the supervisor is to arrange low-level interactions between the control devices, such as motors, actuators, etc.

An approach to the conversion from the supervisors to *ladder logic diagrams* (LLDs) for implementation on a *programmable logic controller* (PLC) is proposed. A discrete manufacturing system example is then considered. The aim of this is to illustrate the applicability, strengths and drawbacks of the design techniques proposed.

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| | |
|---|---|
| APN(s) | Automation Petri Net(s) |
| APN-SM | APN-State Machine |
| C | Counter |
| CAP | CAPacity |
| CD | Count Down |
| C-TPM rule method | A synthesis technique involving the construction of the RG of the Controlled model and the use of TPM rules |
| CU | Count Up |
| DEC | Discrete Event Controller |
| DECS | Discrete Event Control System |
| DES(s) | Discrete Event System(s) |
| F | Flag |
| FIFO | First-In-First-Out |
| FRRG | Final Reduced Reachability Graph |
| FSM(s) | Finite State Machine(s) |
| I | Input |
| IEC | International Electrotechnical Commission |
| LLD(s) | Ladder Logic Diagram(s) |
| M | Marking |
| $M_1 [\chi > M_2$ | Marking $M_2$ is reachable from marking $M_1$ if $\chi$ occurs. |
| p | place |
| PLC(s) | Programmable Logic Controller(s) |
| PN(s) | Petri Net(s) |
| Q | Output |
| R | Reset |
| RG | Reachability Graph |
| S | Set |
| SM | State Machine |
| SR | On delay timer |
| t | transition |
| T | Timer (also time delay) |
| TPL | Token Passing Logic |
| TPLC | Token Passing Logic Controller |
| TPM | Token Passing Marking |
| TTPN | Timed Transition Petri Net |
| U-TPM rule method | A synthesis technique involving the construction of the RG of the Uncontrolled model and the use of TPM rules |
| $\chi$ | firing condition of a transition |

# CHAPTER 1

# INTRODUCTION

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## 1.1. INTRODUCTION

The last decade has witnessed rapid developments in computer technology, which in return, has found widespread applications in manufacturing systems, communication networks, robots, etc. Such systems fall into the category of Discrete Event Dynamic Systems (DEDS) or simply Discrete Event Systems (DES), in which properties such as non-determinism, conflict and parallelism are exhibited. These characteristics are very difficult to describe using traditional control theory, which deals with systems of continuous or synchronous discrete variables modeled by differential or difference equations. DESs have emerged as a new discipline to cope with the control problems of modern industrial systems. Before the emergence of this discipline, the problems faced were not so complicated that is was not difficult to solve them by heuristic methods. This fashion still exists such that the design of the control systems for DES problems is often made by trial and error, based on the experience and ingenuity of the control engineer. As DESs become more complex, the need for an effective formal design tool and its implementation becomes more important.

## 1.2. DESIGN OF DISCRETE EVENT CONTROLLERS

For the formal study of DESs, there are mainly four techniques: automata, Petri nets, minimax and other algebras, and queuing networks (Koussoulas, 1994). The *automata approach*, which is also known Finite State Machine (FSM) approach, represents the most serious effort to extend control theory concepts for continuous systems to the discrete event environment. FSMs provide a logical model for DESs. The objective of this theory has been to examine concepts such as controllability, observability, decentralized and hierarchical control for DESs. (Ramadge and Wonham, 1989; Lin and Wonham, 1988a; Lin and Wonham, 1988b). There are mainly two obstacles when using this technique: the computational complexity of the resulting algorithms and the high initial effort that one has to expend to get familiar with the necessary mathematical tools.

*Petri nets* were first proposed by a German mathematician (Petri, 1962) and have become one of the most popular models for DESs, both in the fields of computing and manufacturing (Koussoulas, 1994). Petri nets are a superset of Finite State Machines. They are a suitable model in various contexts, such as parallel processing computer software, flexible manufacturing etc.

The *algebraic approach* to DES modelling allows for greater compactness than the other methods since a large complicated model can be built through the combination of simpler ones in a way guided by the structure of the original system (Koussoulas, 1994). There have been a number of algebraic techniques proposed suitable for modelling DESs (Cuninghame-Green, 1979; Inan and Varaiya, 1989). However, they have been mainly used for performance evaluation of Discrete Event Systems (Cohen et al, 1985; Cohen et al, 1989).

Finally, *queuing networks* have also been proposed. A queuing network is a collection of queues with interdependent operation. (Kleinrock, 1975; Gross and Harris, 1974; Walrand, 1988). Queuing networks have been a very successful modelling tool for computer networks and similar communications systems. They have the drawback that the necessary mathematical analysis and computations rapidly become heavy or impossible as the complexity of the system increases (Koussoulas, 1994).

As stated above *automata* or *FSM* method represents the most serious effort to develop a formal way for designing control systems for DESs. Within this context, the theory of *supervisory control* of DESs was introduced by Ramadge and Wonham (Ramadge and Wonham, 1986; Ramadge and Wonham, Sept. 1987; Ramadge and Wonham, Jan. 1987; Wonham and Ramadge, May 1987). The supervisory control is a unifying framework for the control of DESs. It is based on formal languages, that allow the designer to model specifications and solve the given DES control problem with standard algorithms. The framework involves a discrete state plant (system) and a discrete state supervisor (controller) modelled by finite state machines (FSM). The plant and supervisor have an identical alphabet set that is partitioned into controllable and uncontrollable symbols. The plant automaton accepts the language generated by the plant. The state of the supervisor is used to decide the controllable symbols that will not be permitted to occur in the plant. The supervisor is assumed to have an inhibiting action only on the controllable symbols. Given a plant automaton, it is of interest to synthesise a supervisor that prevents the occurrence of controllable symbols of the plant to enforce specifications in the closed-loop system. In general, the classes of specifications that have been considered in the supervisory control literature fall into two categories: The *forbidden state problem* (Ramadge and Wonham, Sept. 1987), in which the control specifications are expressed as forbidden conditions that must be avoided, and the forbidden string problem (Ramadge and Wonham, Jan. 1987), also called the *desired string problem,* in which the control specifications are expressed as sequence of activities that must be provided, while not allowing the undesired sequence of activities to occur. The supervisor to be synthesised is expected to be both *nonblocking,* i.e., the forbidden states are avoided and *maximally permissive,* i.e., all events which do not contradict the specifications are allowed to happen.

FSMs provide a general framework for establishing fundamental properties of DES control problems. Nevertheless, there are some disadvantages in using FSMs. Firstly, for practical systems the number of states, which are used to model the system, increases exponentially as the system gets bigger. This means that FSMs are computationally inefficient. Secondly, graphical representation is almost impossible, i.e., when using FSMs, graphical visualisation can not be realised easily.

To overcome these problems Petri nets have been considered as an alternative modelling tool for the analysis, design and implementation of such DESs, because of their easily understood graphical representation in addition to their well formed mathematical formalism. Petri Nets have several advantages over FSMs (Giua, 1996). Firstly, the states of a Petri net are represented by the possible markings and not by the places: thus they give a compact description, i.e., the structure of the net may be maintained small even if the number of the markings grow. Secondly, instead of using ambiguous textual descriptions or mathematical notations, which can be difficult to understand, the plant and the specifications can be represented graphically using Petri nets. Finally, using Petri net models, the same model can be used for the analysis of behavioural properties and performance evaluation as well as for systematic construction of the discrete event controllers (Zurawski and Zhou, 1994). There are three main design approaches for the control of DES using Petri net models (Holloway et al, 1998): Controller behaviour approach, logic controller approach and control theoretic approach.

In the *controlled behaviour approach,* which is commonly used for modelling manufacturing systems, the Petri net model includes both the behaviour of the plant as well as the controller. When the desired controlled behaviour is obtained, it is necessary to extract the controller logic for implementation. This approach is preferable when a declarative model, rather than procedural model, is used. Bottom-up, top-down or hybrid, i.e., both bottom-up and top-down, design rules may be used to make sure that the final model will have the properties of interests such as liveness, boundedness, reversibility, etc. Examples of this approach can be found in (Jeng and DiCesare, 1993; Zhou et al, June 1992; Zhou et al, Nov. 1992; Zhou and DiCesare, 1993).

The *logical controller approach* focuses on the direct design and implementation of a controller for the DES. The objective is to define the input-output behaviour of the controller to achieve the desired controller behaviour for the system. Generally, the controller receives commands from an external agent and then translates them into a sequence of operations to be performed by the system. In this approach, it is necessary to validate the controlled behaviour through simulation. This approach leads naturally to the physical implementation of the control program. Examples of this approach can be found in (Valette, 1983; Courvoiser et al, 1983; Nketsa and Courvoiser, 1990; Bruno and Marchetto, 1986). The relationship between Petri nets and the programming language GRAFCET for specification of controller logic was discussed in (David and Alla, 1992; David, 1993).

The *control theoretic approach* is mainly based on the classical supervisory control framework proposed by Ramadge and Wonham. Given an uncontrolled model of the system and a specification for the desired controlled behaviour, the objective is to synthesise a controller to achieve the specifications. In this approach, there is a clear distinction between the system and the controller, and the information flow between the system and controller is modelled explicitly.

Because of the advantages of Petri nets over FSMs, Petri nets have emerged as a strong alternative formalism for the study of DES control. Petri net models are generally more compact and more powerful than FSMs and they provide structured models which can be exploited in developing more efficient algorithms for controller synthesis. Recent research on the application of Petri net models to the analysis and synthesis of controllers for discrete event systems has been reviewed in (Holloway et al, 1998). Several issues related to the use of Petri nets in the supervisory control of discrete event systems are discussed in (Giua, 1996). There are mainly two groups of Petri-net-based supervisors proposed: *mapping supervisor*, whose control policy is efficiently computed by an on-line controller as a feedback function of the marking of the system, and *compiled supervisor*, whose control policy is represented as a net structure. There are several advantages in fully compiling the supervisor action into a net structure (Giua, 1996). Firstly, the computation of the control action is faster, since it does not require separate on-line computation. Secondly, the same Petri net system execution algorithms may be used for both the original system and the supervisor. Finally, a closed-loop model of the system under control can be built with standard net composition constructions.

In addition to the forbidden state problems and the desired string problems, a class of specification so called *generalised mutual exclusion constraints (GMEC)* has also been considered in the literature. A classic approach to discrete event modelling and control considers complex systems as interacting subsystems. Depending on the particular tasks demanded from the system, and on the way the subsystems are interconnected some specific constraints must be imposed on the systems behavior. A GMEC limits a weighted sum of tokens contained in a subset of places in a Petri net. Several solutions have been proposed for this problem. Several control structures capable of enforcing GMECs on marked graphs with control safe places have been discussed in (Giua et al, 1993). In this work, how a constraint may be enforced by a place, called *monitor*, has been shown. A maximally permissible control law for a set of constraints may always be implemented by a set of monitors. The use of monitors as control structure to be added to the net structure for enforcing GMEC's, also called *place invariants*, has been discussed in (Moody et al, 1994; Moody et al, 1995; Moody and Antsaklis, 1995; Yamalidou et al, 1996). In this work, an algorithm has been given to compute a monitor such that a given place invariant will not be violated. In this case, very simple controllers are obtained in the form of monitor places, which only constrain controllable transitions. In this technique, when there are uncontrollable transitions, monitor based solutions are still in use. However, in this case, the solution may not be maximally permissive. Note that in the presence of uncontrollable transitions, a problem of mutual exclusion, or place invariant problem, is transformed into a forbidden state problem.

In the case of the *forbidden state problem*, an important step forward has been the introduction of so called *controlled Petri nets (CtlPN)* (Krogh, 1987; Holloway and Krogh, 1990). The basic restriction of this method is that the net is a marked graph, i.e., each place has exactly one input arc and one output arc. Also it was assumed that there is no conflict in the net. This technique has involved the computation of the control law in two steps: off-line computation and on-line computation. Both these computations are very simple. Therefore, this approach is very efficient. However, because the controller is given as a feedback law, it is not possible to design a net model of the controlled system. In other words, the supervisor obtained is a *mapping supervisor*. This approach has received a lot of attention in the literature and has also been extended to classes of nets other than marked graphs: controlled state machines (Boel et al, 1995), forward and backward conflict-free nets (Chen, 1994), coloured Petri nets (Boel et al, 1993; Makungu et al, 1994). In (Holloway et al, 1996), the technique has been extended to be applicable to a very general class of controlled Petri nets which can include both marked graph structures and state graph structures. These extensions also permit the control of Petri nets with markings which are not safe or live and may even be unbounded.

Recently an interesting approach has been proposed in (Godon and Ferrier, 1997) to solve the forbidden state problems for coloured Petri nets. In this work, the compiled supervisor is obtained in two main steps: In the first step, the primary supervisor is obtained through the coverability tree analysis. In the second step, the final supervisor is obtained by applying algebraic or algorithmic methods to the primary supervisor, taking into account the required properties such as liveness, reversibility, etc. Sreenivas (Sreenivas, 1993; Sreenivas, 1994; Sreenivas, 1996) has addressed both the forbidden state and the *desired string problems* using Petri nets. In the case of the forbidden state problem, through the analysis of reachability tree of the system, the control law is obtained as a table that lists the controllable events to be disabled for every reachable state of the system. Then, the supervisor is heuristically designed such that the control law is met. In the case of the desired string problem, by using so called Deterministic Sequential Petri Net Languages, the supervisor is constructed such that the supervised system will only accept the desired sequences of events. The results obtained in this case are based on formal Petri net languages concepts. In (Sreenivas and Krogh, 1992), the *desired string problem* has been considered. In this work, a class of supervisory control problems that require infinite state supervisors have been considered and Petri nets with inhibitor arcs have been introduced to model the supervisors. In (Giua and DiCesare, 1991), how a *compiled supervisor* can be designed using Petri nets has been shown. In fact in this case, the desired string problem is converted into a forbidden state problem and then it is solved. In this method, the design requires two steps. In the first step, a coarse structure of a supervisor is synthesised by means of so called *concurrent composition* of different modules. In the second step, the structure is refined by *ad hoc* methods to avoid reaching forbidden markings. This work has then been extended in (Kumar and Holloway, 1996), where an algorithm has been obtained for computing a minimally restrictive control when the system behaviour is a deterministic Petri net language and the desired behaviour is regular language.

## 1.3. IMPLEMENTATION OF DISCRETE EVENT CONTROLLERS

The control of discrete event systems is referred to as 'logic control' (Ferrani and Maffezzoni, 1991), 'sequential control' (Zhou and Twiss, 1995; Venkatesh et al, 1994; Greene, 1990) or 'discrete event control' (Ventakesh et al, 1995; Bigou et al, 1987). In today's automated modern factories the majority of the discrete event control systems (DECS) are implemented by Programmable Logic Controllers (PLC). A PLC is a replacement for the hard-wired relay and timer logic to be found in traditional control panels. PLCs provide ease and flexibility of control based on programming and executing simple logic instructions. They are designed through Ladder Logic Diagrams (LLD), which are known to be very difficult to debug and modify when written in a heuristic manner. In general, the LLD involved is small enough to be very easily understood in terms of representation and operation. However, when larger and more complex control operations have to be performed it quickly becomes apparent that an informal and unstructured approach to LLD design will only result in programs which are difficult to understand, modify, troubleshoot and document (Lloyd, 1985). The matter of fact is that even with these shortfalls, LLDs dominate industrial discrete event control (Cook and Gardner, 1991; Pollard, 1994).

To provide some degree of structured programming for implementation of DES control systems, there are mainly three approaches. The first approach is called state machine or state based methods (Lorenz and Eberlein, 1988; Jones, 1991; Morihara, 1994; Ready, 1991; Mandado et al, 1996). The fundamentals of state machine logic are quite straight forward. For a sequence of steps in a process, each step defines a set of outputs which control the action occurring (or expected to occur) at that time. Each unique set of outputs in a sequence is then defined as a logical 'machine state'. Appropriate transitions in the status for the 'current state machine' will define when the current state must be changed to the next machine state. The techniques involve representing the state by 'flags' and using the flags to control the flow of the discrete event control system.

The second approach is called GRAFCET, which is also known as Sequential Function Chart (SFC). GRAFCET was specifically developed for describing sequential control systems (Fisher, 1989; Llyod, 1985; David and Alla, 1992). GRAFCET is a European standard, established in 1977 by the French AFCET committee. It is based on Petri nets (Desrochers and Al-Jaar, 1995). It is closely related to a sub-set of Petri nets called condition/event nets. A condition/event net is a Petri net where each place has maximum of one token and the transitions are called events. Therefore, a transition can not fire if one of its output places has a token, even if it is enabled. If it does, that output place will have two tokens which is not allowed. This is required since the places represent a condition that could be either true (token exists) or false (no token).

The basic elements of GRAFCET are steps, actions, transitions, and receptivities. Macro steps can also be defined. Actions are associated with the steps to represent the desired control to be executed. Steps are represented as squares, and the associate actions are written next to them. steps are similar to conditions in condition/event nets, which are places with capacity of one. The transitions are drawn as black bars, and are equivalent to the events (transitions) in the condition/event nets. The receptivities are logical conditions associated with the transitions. They describe a true/false condition that must be satisfied before the transitions can occur (fire). A black dot inside the step represents an active step, just as a token in a Petri net marks a place and indicates the state of the system. GRAFCET evolves by clearing the enabled transitions if the associated receptivities are true.

Petri nets, as graphical and mathematical tools, are another powerful tool for modelling, formal analysis and design of discrete event systems. Petri nets were named after Carl A. Petri, who invented a net-like mathematical tool for the study of communications with automata in 1963. Petri nets enable a discrete event system of any kind whatsoever to be modelled (David and Alla, 1994). Petri nets can be used to model properties such as process synchronisation, asynchronous events, concurrent operations and conflicts or resource sharing. Petri nets describe a discrete event system graphically and this contributes to a better understanding of the complex interactions within the system. A Petri net consists of places and transitions, which are linked to each other by directed arcs. Graphically places are represented by circles. Places represent passive system components, which store 'items' (called tokens), and take particular states. Transitions are represented by bars, which are the active system components. They may produce, transport and change the tokens. Places may contain tokens, while arcs indicate the flow of tokens. According to the classical Petri net theory, a transition is enabled if there is at least one token in each of its input places. When enabled, a transition 'fires' by removing a token from each input place and by adding a token to each output place. Comparisons between Petri nets and LLDs have been reported (Silva and Veilla, 1982; Venkatesh et al, 1994; Zhou and Twiss, 1995; Venkatesh et al, 1995). Petri net based PLCs have been proposed (Valette et al, 1983; Courvoiser et al, 1983; Nketsa and Courvoiser, 1990). Some attempts have also been made at producing a technique to convert Petri nets into ladder logic diagrams (Greene, 1990; Satoh et al, 1992; Rattigan, 1992; Jafari and Boucher, 1994; Burns and Bidanda, 1994; Taholakian and Hales, 1995; Q. Zhou et al, 1995). However, none of these, to-date, have produced a general technique for conversion of Petri nets into LLDs in the sense that it can deal with flags, timers, counters, timed Petri nets and Coloured Petri nets.

State machine method can only be applied to very simple systems. When state machines are used to model and control DESs in a straightforward manner the exponential increase in the number of states makes it very difficult to implement complex DESs. Graphical representation is almost impossible and thus graphical visualisation can not be easily realised (Zhou and DiCesare, 1993). GRAFCET is closely related to a sub-set of Petri nets. It has two advantages over Petri nets. Firstly, GRAFCET is an applied model that is defined with its interpretations as it relates to an actual system. Secondly, The GRAFCET standard is strict. Developers of GRAFCET models must adhere to the rules of drawing, labelling and inscription. This facilitates the exchange of documents and controllers among various companies and different products. Nevertheless, there are some disadvantages in using GRAFCET. Specifically: The powerful and important notion of conflict can not be accommodated. A transition can fire even if one of the output steps has a token. These disadvantages reduce the modelling power and applicability of GRAFCET in many manufacturing systems, where conflict, concurrency and asynchronous operations are exhibited. Another drawback in using GRAFCET is that it can only be implemented on GRAFCET PLCs (Bowman, 1989). Also, no analysis can be done using GRAFCET. On the other hand, Petri nets, as mathematical and graphical tools, are widely used for modelling, analysis and control of discrete event systems. They are superior to the previously defined methods. They have the ability to tackle conflict, concurrency, and asynchronous operations. However, it has been reported that the use of Petri nets is still restricted to research laboratories and academic institutions because of the lack of widely available inexpensive software tools suitable for the development of industrial type systems (Zurawski and Zhou, 1994). In fact, PLCs can offer a great deal of flexibility for programming and execution of Petri net based controllers, but as mentioned before there is no general technique that will allow the conversion of such controllers into a PLC code.

## 1.4. OBJECTIVES OF THE THESIS

Petri-net-based approaches to the supervisory control design, as an alternative to the original FSM framework, has been at the heart of recent research into discrete event control system design. This is because Petri nets provide a very compact description of the systems, and they represent the systems by means of easily understood graphical representation as opposed to difficult to understand textual descriptions and mathematical notations. In addition, the same model can easily be used for analysis and the systematic construction of supervisory controllers. In general, both the forbidden state specification and the desired string specification problems have been considered. As explained there are two types of Petri net based supervisors proposed namely mapping supervisors, whose control policy is a feedback function, and compiled supervisors, whose control policy is represented as a net structure.

For the reasons given, compiled supervisors are preferable to mapping supervisors. However, to date the design of compiled supervisors has only been done by heuristic methods. Therefore, it is very important to design compiled supervisors using a formal design technique. An important issue in designing complied supervisors in the case of the forbidden state specification is that the supervisor should have the following properties; it must be nonblocking, i.e., the forbidden states are avoided, and maximally permissive, i.e., the supervisor does not unnecessarily constrain the behaviour of the system. In the case of the desired string problem, the construction of supervisors is generally based on formal languages concepts. However, the results obtained are either difficult to apply to real systems or difficult to understand in most cases. Therefore, it is also crucial to introduce some simple design techniques to facilitate the design of compiled supervisors in the case of the desired string problem as well as making sure that the results obtained can readily be used for real problems. Supervisory control problems occur at all level of the manufacturing system control hierarchy, ranging from the low-level interaction between equipment controllers and devices through the coordination of workcells, to the factory-wide coordination of workstation controllers. Therefore, in this thesis manufacturing systems are considered as an example of DESs. It is desirable to obtain some techniques for the design of supervisors, which can be applied to both high-level and low-level manufacturing control problems.

The design phase is only the first step towards the control of DESs. After designing a controller (supervisor), it is necessary to have an automatic means for the generation of control code from the controller. However, the results obtained in the supervisory control literature are mostly related to the theoretic studies as opposed to practical (implementation) studies. It is crucial to come up with a technique to convert the controllers into ladder logic diagram (LLD) code since LLDs are the most popular implementation language used on programmable logic controllers (PLCs). In the light of this discussion the main objectives of this thesis may thus be stated as follows:

i) the extension of existing Petri net based control design techniques, to allow the formal design of compiled supervisors for both the forbidden state specifications and the desired string specifications.

ii) the development of a conversion technique from the Petri net based supervisors into ladder logic diagrams (LLDs) for the implementation of the corresponding supervisors on programmable logic controllers (PLCs).

## 1.5. OUTLINE OF THE THESIS

This chapter has introduced the literature relevant to the research carried out, together with the objectives of the research.

Chapter 2, provides a brief introduction to Petri nets and modelling of discrete event systems. The chapter starts by defining simple Petri nets. Then, some important properties of Petri nets and analysis tools for Petri nets are considered. This is followed by the definition of extended Petri nets such as inhibitor arc Petri nets and timed Petri nets. After that, some Petri net modules, which can be used as building blocks when modelling a system with Petri nets, are described. Finally, an extended Petri net formalism, called Automation Petri net (APN), which allows sensor readings and actuator operations to be included into the Petri net framework, is described.

In the chapter 3, four design techniques, called *inhibitor arc method*, *enabling arc method*, *intermediate place method* and *APN-SM method*, are proposed for the design of compiled supervisors for the control of DESs in the case of the forbidden state problem. In these methods, the uncontrolled model of the system is obtained using APNs. In the first three methods, the supervisor is a controlled model of the system, which contains the uncontrolled model, so called model supervisor, and the control policy. The model supervisor and the control policy are determined by constructing the reachability graph and by reducing it according to the forbidden state specifications. In the inhibitor arc method the model supervisor is connected to the uncontrolled model through the use of inhibitor arcs such that the control policy is met. In the enabling arc method the model supervisor is connected to the uncontrolled model through the use of enabling arcs such that the control policy is satisfied. Similarly, in the intermediate place method a set of places called intermediate places are connected between the uncontrolled model and the model supervisor according to the control policy. In contrast to the first three methods, in the APN-SM method the supervisor contains only one net structure. In this case the incomplete supervisor, called the model supervisor in the previous methods, is obtained as defined in the previous methods. The control policy defines a set of actions to be assigned to some of the places within the incomplete supervisor. After this process, the supervisor becomes the (complete) supervisor. Note that the supervisors obtained are maximally permissive, nonblocking, and correct by construction. To show how these methods can be used to obtain a compiled supervisor, a manufacturing system is considered. The comparison between these methods is also provided.

In the chapter 4, two design techniques are proposed as alternative methods to the previous four methods, for the design of compiled supervisors for the control of DESs in the case of the forbidden state problem. The first method represents a top-down synthesis technique, involving the construction of the reachability graph (RG) of the uncontrolled model of the system and involving the use of token passing marking (TPM) rules. Therefore, it is called U-TPM rule method. The TPM rules are obtained through the RG analysis. The TPM rules are implemented on the uncontrolled model by enabling arcs. This process produces the controlled model, i.e., the supervisor. In this case, the supervisor obtained is correct by construction, maximally permissive and nonblocking. On the other hand, the second method represents a bottom-up synthesis technique, involving the construction of the reachability graph of the controlled model (i.e. the supervisor) of the system and involving the use of TPM rules. Therefore, it is called C-TPM rule method. In this case the TPM rules are obtained directly from the forbidden state specifications and then the controlled model, i.e., the supervisor is obtained by implementing the TPM rules on the uncontrolled model through the use of enabling arcs. However, the correctness of the controlled model must be checked by reachability graph analysis. The supervisor in this case may not be maximally permissive. The manufacturing system example introduced in the previous chapter is used to show how these two methods can be used to obtain a compiled supervisor for a DES. The results obtained for the manufacturing system are also compared for these two methods.

In the chapter 5, a methodology is proposed for the purpose of designing compiled supervisors for the control of DESs in the case of the desired string problem. In this case it is assumed that the problem is only related to the desired string problem. That is to say if there is any forbidden state problem related to a system it is assumed to be solved previously. The model of the system, called the untreated model, is represented as an APN. A simple design technique is used as an alternative to the use of formal language concepts. In this case the desired specification is represented by an APN, called specification APN. Then, the untreated model is combined with the specification APN through the use of concurrent composition. The technique proposed can be used when the desired string require a deterministic specification APN as well as a nondeterministic specification APN.

In the chapter 6, a general methodology for converting Automation Petri Nets into LLDs is proposed. Ladder Logic Diagrams (LLDs) are the most popular programming language for programming PLCs. Because of this, a general methodology, called Token Passing Logic (TPL), is proposed to convert APNs into LLDs. The TPL method is conceptually simple, and permits a direct conversion of Automation Petri Nets into LLDs. It also provides a straight forward mapping between the basic sequencing information and the programming steps. The method accommodates timers and counters and timed APNs.

In the chapter 7, a discrete manufacturing system is considered to illustrate the applicability, strengths and drawbacks of the design techniques proposed. It is important to point out that this chapter shows how low level manufacturing control problems can be solved with the methods proposed. Both the forbidden state and the desired string problems are considered. The details of the design and implementation issues are provided. Finally, the results obtained are compared in terms of the number of places and the transitions used in different methods as well as the number of LLD rungs produced from the supervisors.

Finally, in the chapter 8, conclusions are provided together with a discussion of the original contributions and possible further directions of research.

# CHAPTER 2

# INTRODUCTION TO PETRI NETS AND MODELLING OF DISCRETE EVENT SYSTEMS

# CHAPTER 2

# INTRODUCTION TO PETRI NETS AND MODELLING OF DISCRETE EVENT SYSTEMS

## 2.1. INTRODUCTION

The rapid development in science and technology has brought about a lot of man-made systems, which cannot be described with traditional differential or difference equations. The examples of these systems include flexible manufacturing systems, computer network systems, various transportation systems and others. The behaviour of these systems is determined mostly by discrete events functioning in them. Such systems are called discrete event systems (DES) or discrete event dynamic systems (DEDS), whose characteristics can be identified as follows:

*Concurrency:* In a discrete event system many operations may take place at the same time, i.e., simultaneously.

*Asynchronous operations:* Unlike the systems, in which each change or step is synchronised by a global clock, in discrete event systems, the events often occur asynchronously.

*Event-driven:* Discrete event systems can be characterised by a discrete state space, in which changes in state are caused by event occurrences. In this case, any event may be dependent on the occurrence of other events, i.e., the completion of one operation may initiate another operation.

*Non-determinism:* Non-determinism results from uncertain event occurrences, i.e., different evolutions may be possible from a given state.

Petri nets, as a graphical and mathematical tool, are being increasingly used in the modelling, analysis, design and control of discrete event systems (Zhou and DiCesare, 1993). Petri nets were named after Carl A. Petri, a contemporary German mathematician, introduced a net-like mathematical tool for the study of communication with automata (Petri, 1962). Ever since, there has been a great deal of research in different disciplines, including manufacturing systems, computer science, communication systems, etc. (Zurawski and Zhou, 1994). Petri nets enable a discrete event system of any kind to be modelled. They present two interesting characteristics. Firstly, they make it possible to model and visualise behaviours comprising concurrency, synchronisation and resource sharing. Secondly, the theoretical results concerning them are plentiful (Alla and David, 1994). Petri nets have proven to be very useful in the modelling, analysis, simulation, and control of manufacturing systems. They provide very useful models for the following reasons (Desrochers and Al-Jaar, 1995):

- Petri nets capture the precedence relations and structural interactions of stochastic, concurrent, and asynchronous events. In addition, their graphical nature helps to visualise such complex systems.
- Conflicts and buffer sizes can be modelled easily and efficiently.
- Deadlocks in the system can be detected.
- Petri net models represent a hierarchical modelling tool with a well-developed mathematical and practical foundation.
- Various extensions of Petri nets, such as timed Petri nets, stochastic (timed) Petri nets, coloured Petri nets, and predicate transition nets, allow for both qualitative and quantitative analysis of resource utilisation, effect of failures, and throughput rate, and so on.
- Petri net models can be used for both carrying out a systematic analysis of complex systems and systematic construction (i.e., synthesis) of the discrete event controllers
- Finally, Petri net models can also be used to implement real-time control systems for flexible and agile manufacturing systems.

Ordinary Petri nets are not always sufficient to represent and analyse complex industrial systems. This has prompted the development of new classes of Petri nets. For example, when modelling complex systems, consisting of many similar interacting activities ordinary Petri nets increase the graphical complexity of the model. In order to address this issue, Petri nets, which allow tokens to have distinct identity, were proposed. These nets, referred to as high-level Petri nets, include predicate-transition nets (Genrish and Lautenbach, 1981), coloured nets (Jensen, 1981), and nets with individual tokens (Reisig, 1985). An important development in the area of high-level Petri nets was the introduction of object oriented Petri nets (Sibentin-Blanc, 1985). Due to the need for representing approximate and uncertain information has led to the various types of fuzzy Petri nets (Chen *et al*, 1990; Garg *et al*, 1991; Loony, 1988; Valette *et al*, 1989). The need for the temporal analysis of the systems resulted in the introduction of temporal Petri nets (Timed Petri nets) (Suzuki and Lu, 1989).

The purpose of this chapter is to provide a brief introduction to Petri nets. The remainder of this chapter is arranged as follows: Firstly, some Petri net basics are introduced. This is followed by some important Petri net extensions, such as inhibitor arc Petri nets, weighted arc Petri nets, etc. After that, some Petri net modules are considered for modelling of manufacturing systems. Finally, an extended Petri net formalism, called Automation Petri nets (APN) is proposed.

## 2.2. SIMPLE PETRI NETS

*An ordinary Petri net* is a *directed graph* represented by a quadruple;

**PN = *(P, T, Pre, Post)*.......................................................................................(1)**

Where,

- $P = \{ p_1, \ldots\ldots\ldots, p_n\}$ is a finite set of places,

- $T = \{ t_1, .........., t_m \}$ is a finite set of transitions,
- Pre is an input mapping $P \times T \rightarrow \{0, 1\}$ corresponding to the set of *directed arcs* from P to T.
- Post is an output mapping $P \times T \rightarrow \{0, 1\}$ corresponding to the set of *directed arcs* from T to P.

Note that P and T are disjoint sets and that any element of $P \cup T$ is called *a node.* Petri nets are assumed to be *connected.* This means that there exists at least one path between any two nodes. Generally, places are used to express the states of the systems, while transitions correspond to control evolutions from one state to another.

Petri nets can be represented graphically, which is helpful in both describing how they work and gaining an understanding of a particular model. A Petri net graph uses circles and bars to represent places and transitions, respectively. The input and output functions are represented by directed arcs between the two types of nodes. An arc directed from a place to a transition defines the place to be an input place of the transition. Similarly, an arc directed from a transition to a place defines the place to be an output place of the transition.

*A marked Petri net* contains *tokens* in addition to the elements described above. Tokens reside in places, travel along arcs, and their flow through the net is controlled by transitions. They are represented graphically by dots. The *marking M(p)* of a Petri net is a mapping of each place to a non-negative integer representing the number of tokens in that place. A marked Petri net is defined by the quintuple:

**PN = *(P, T, Pre, Post, M)*..........................................................................................(2)**

The marking M is an n-dimensional vector whose ith component *M(Pi)* represents the number of tokens in the ith place *Pi*. The *initial marking* is denoted by *Mo*. A simple Petri net, showing places, transitions, directed arcs and a token, is given in Fig. 2.1.

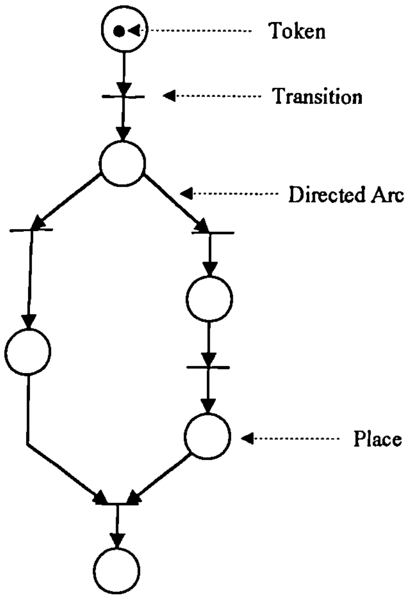


Figure 2.1. A simple Petri net.

The *execution* of an ordinary Petri net is controlled by the number and distribution of tokens in the net and causes the token to flow in the net. Execution is performed by firing enabled transitions. A transition is *enabled* when each of its input places is marked with at least one token. A transition fires by removing a token from each of its input places and by placing a token in each of its output places. The firing of transitions causes tokens to flow through the net.

### 2.2.1. Firing of a Simple Petri Net

The firing of a simple Petri net is shown in Fig. 2.2, where there are four places P = { $p_1$, $p_2$, $p_3$, $p_4$ } and two transitions T = { $t_1$, $t_2$ }. Initially, as shown in Fig. 2.2.(a), transition $t_1$ is enabled, because $M(p_1) = 1$, $Pre(p_1, t_1) = 1$ and $M(p_2) = 1$, $Pre(p_2, t_1) = 1$, and transition $t_2$ is not enabled, because $M(p_3) = 0$ and $Pre(p_3, t_2) = 1$. When transition $t_1$ fires, it removes one token each from places $p_1$ and $p_2$ and deposits one token in place $p_3$, as shown in Fig. 2.2(b). In this case, transition $t_2$ becomes, because $M(p_3) = 1$ and $Pre(p_3, t_2) = 1$. When transition $t_2$ fires, it removes one token from places $p_3$ and deposits one token in place $p_4$, as shown in Fig. 2.2(c).

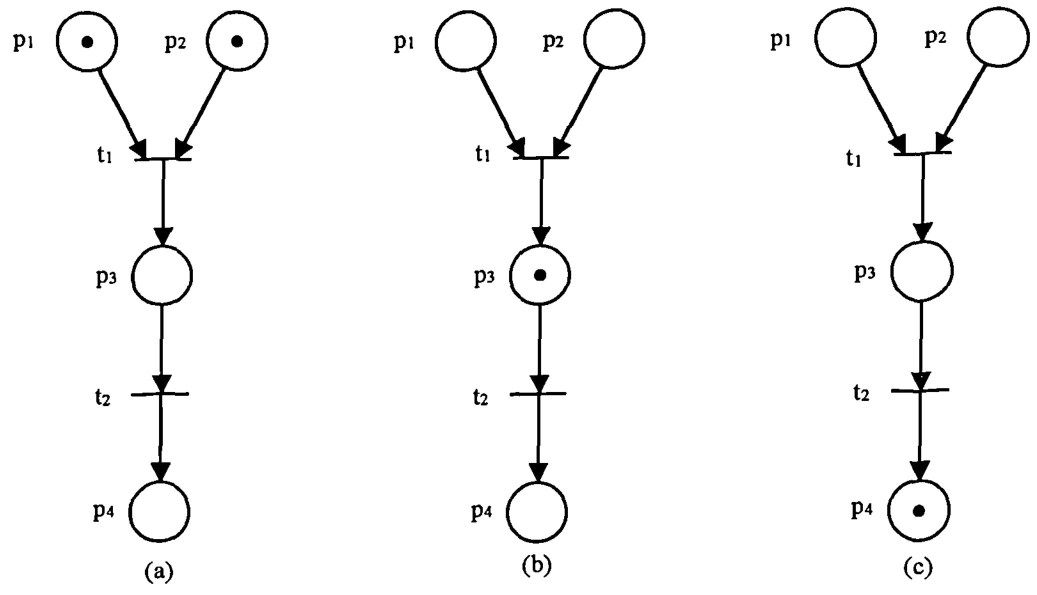


Figure 2.2. A simple Petri net with : (a) initial marking. (b) marking after $t_1$ fires. (c) marking after $t_2$ fires.

The tokens, places and transitions must be assigned a meaning for proper integration of the model. In general, they are interpreted in the following way: Places represent resources or possible states of the system. The existence of one or more tokens in a place represents the availability of a particular resource or presence of a condition being met. A transition represents changings in the system states. A firing transition may be interpreted as an activity happening. Places and transitions together represent conditions and precedence relations in the system's operation. For example, a token in a place can imply that the condition is true, and no token, that it is false.

### 2.2.2. Properties of Petri nets

Petri nets as graphical and mathematical tools have a lot of properties. Such properties, when interpreted in terms of the modelled system, make it possible to identify the presence or absence of functional properties of the system under design (Zurawski and Zhou, 1994). There are two types of properties, namely, behavioural and structural. The former depends on the initial marking of the Petri net, while the latter does not depend on the initial marking. The structural properties are related to the net structure of a given Petri net. In this section, some of the most important behavioural properties, from the practical point of view, are provided. These properties are reachability, boundedness, safeness, conservativeness, liveness and reversibility. Detailed information about the other behavioural properties and the structural properties of a Petri net can be found in (Murata, 1989).

**Reachability**: The firing of an enabled transition changes the marking, i.e., token distribution of a Petri net. A marking $M_i$ is said to be *reachable* from an initial marking $M_0$ if there exist a sequence of firings that can transform $M_0$ to $M_i$. A firing sequence is represented by $\sigma = t_1, t_2, t_3......t_n$. To show $M_i$ is reachable from $M_o$ by $\sigma$ the following representation is used: $M_0 [\sigma > M_i$.

**Boundedness**: A Petri net is said to be *k-bounded* or *bounded* if the number of tokens in each place does not exceed a finite number 'k' for every marking reachable from the initial marking $M_0$.

**Safeness**: A Petri net is said to be *safe* if all its places are safe. A place 'p' is safe if it contains no more than one token. In other words, a Petri net is called *safe* if it is 1-bounded.

**Conservativeness**: A Petri net is said to be *conservative* if the total number of tokens in all its places for all reachable markings is constant.

**Liveness**: A transition is said to be *live* if for all markings of the Petri net there is a firing sequence, which takes the net to a marking, in which the transition is enabled. A Petri net is *live* if all its transitions are live. If a Petri net is live and the model is correct, it indicates the absence of *deadlocks* in the operation of the system.

**Reversibility**: A Petri net is said to be *reversible* if the initial marking $M_0$ is reachable from each marking.

### 2.2.3. Analysis of Petri nets

In general, there are two techniques for the analysis of Petri nets: linear algebraic method and graph-based method. The linear algebraic method is based on matrix equations. In this case matrix equations represent the dynamic behaviour of Petri nets. The fundamental to this approach is the incidence matrix, which defines all possible interconnections between places and transitions in a Petri net. The use of the incidence matrix representation results in a homogeneous system of linear algebraic equations. This immediately poses some problems, since the solutions will not be unique (Koussoulas, 1994). Additionally, this method does not provide the firing sequences necessary to reach a certain marking. Finally, the linear algebraic analysis technique can not be applied on all Petri nets; they have to be free of self-loops. The advantages of this technique over the graph-based analysis technique is the existence of simple linear-algebraic properties (Desrochers and Al-Jaar, 1995).

The graph-based analysis method can be split into two parts for bounded systems: the reachability tree analysis and the reachability graph analysis. Both methods involve essentially the enumeration of all reachable markings and it should be able to apply to all different types of Petri nets. However, they are limited to not very big systems, because of the computational complexity, and the so called the *state explosion problem*: the number of markings can be exponential with respect to the size of the Petri net. For a bounded Petri net, the reachability tree contains all possible markings. The analysis problems (i.e., properties of Petri nets), as discussed in the previous section, can be solved by the reachability tree (Murata, 1989). For bounded systems, the reachability tree provides all valid firing sequences together with all reachable markings, but the reachability graph provides only all the reachable markings and firing of transitions among them.

Given a Petri net, from the initial marking $M_0$, as many "new" markings as the number of the enabled transitions can be obtained. From each new marking, more markings can be reached. This process results in a tree representation of the markings. Nodes represent the markings generated from the initial marking $M_0$ and its successors, and each arc represents a transition firing, which transforms one marking to another. Consider the Petri net shown in Fig. 2.3.(a), where there are three places, $P = \{ p_1, p_2, p_3 \}$ and three transitions $T = \{ t_1, t_2, t_3 \}$. The reachability tree of this Petri net is shown in Fig. 2.3.(b). Note that the valid firing sequences of the transitions are as follows: $t_1t_3$, $t_1t_2t_1t_3$, $t_1t_2t_1t_2t_1t_3$, ......

The reachability graph associated with a system is a graph, in which each node represents a marking reachable from the initial marking $M_0$ and each arc represents the firing of a transition. If the marked Petri net is bounded the graph construction process finishes when all possible firing from the reachable markings have been explored. For the Petri net, shown in Fig. 2.3.(a), the reachability graph is shown in Fig. 2.3.(c). When the reachability tree and the reachability graph are considered the difference between these two techniques can be seen easily. The former simply provides all the valid firing sequences of a Petri net together with all reachable markings, while the latter only provides all possible markings and the firing of transitions, which go from one marking to another. It is important to note that when carrying out reachability tree/graph analysis only one transition is assumed to fire at a time.

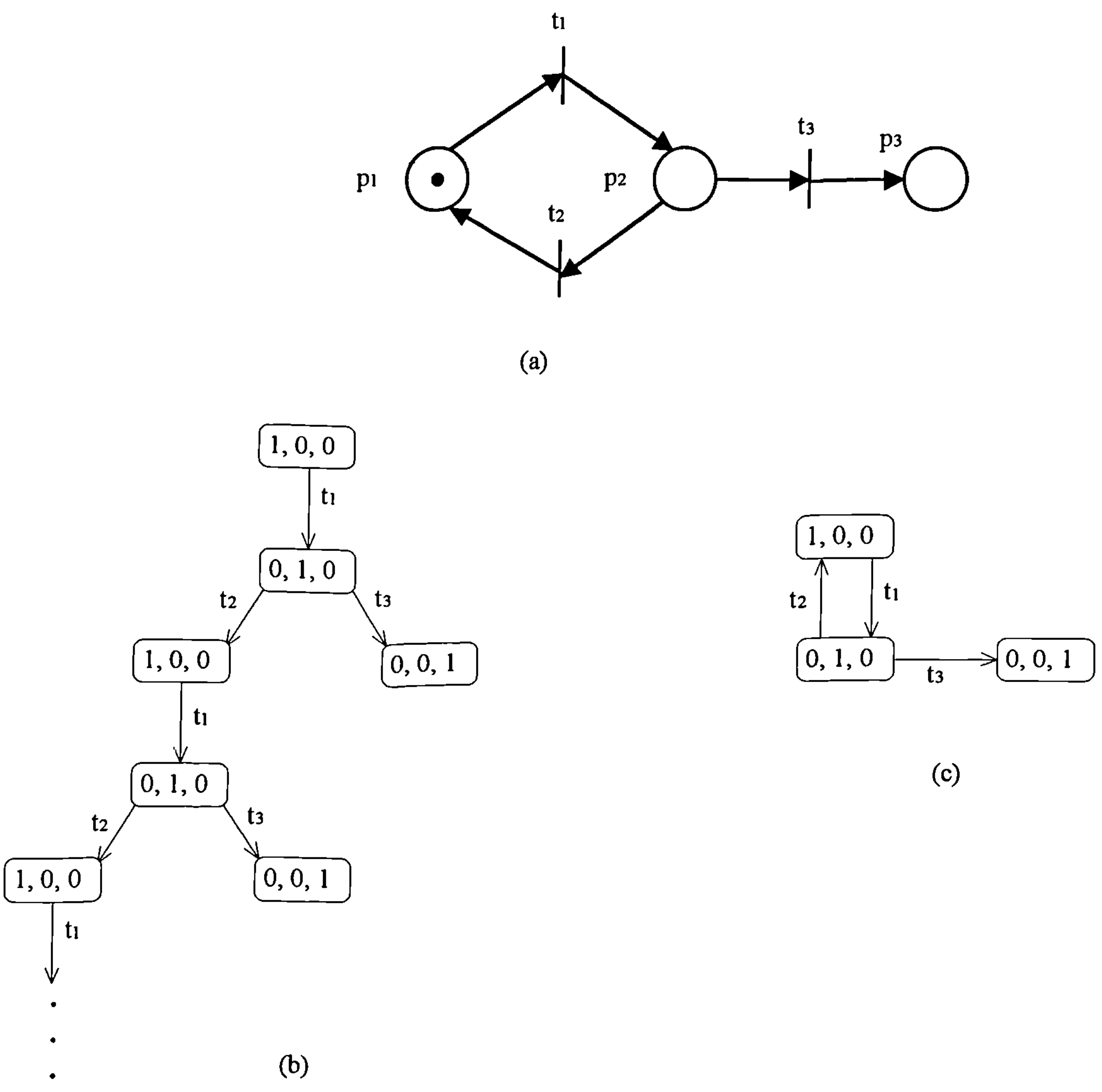


Figure 2.3. (a) A Petri net. (b) Its reachability tree. (c) Its reachability graph.

## 2.3. EXTENDED PETRI NETS

Several extensions have been made to ordinary Petri net framework in order to be able to represent complex systems easily. In this section some of these extensions are considered. The extensions considered in this section involves the following:

- Weighted arc Petri net
- Inhibitor arc Petri net
- Enabling arc Petri net
- Finite capacity Petri net
- Timed Petri net

## 2.3.1. Weighted arc Petri net

A weighted arc Petri net is one in which weights are associated with arcs. The Pre and Post mappings may take values over the set of all non-negative integers. In this case, each arc is said to have *multiplicity k,* where k represents the weight of arcs. Ordinary Petri nets have a multiplicity of 1. The weight of an arc is indicated by a non-negative integer assigned to the arc. A transition is enabled, if each of its input places is marked with at least the number of the tokens equal to the weight of the related arc, which connects the input place to the transition. The transition fires by removing necessary number of tokens from input places, according to the weights of the input arcs, and by putting sufficient number of tokens to the output places, according to the weights of the output arcs. Such a weighted arc Petri net is shown in Fig. 2.4.(a), in which the input arc $p_1 \rightarrow t_1$ has the weight of 'n', i.e., $Pre(p_1, t_1) = n$, and the output arc $t_1 \rightarrow p_2$ has the weight of 'm', i.e., $Post(t_1, p_2) = m$. In this case, if the number of tokens in the input place $p_1$ is at least equal to the number 'n', then the transition $t_1$ is enabled. When the transition $t_1$ fires, it removes 'n' tokens from input place $p_1$ and deposits 'm' tokens to the output place $p_2$. Instead of using weighted arcs Peterson used the concept of '*bag of arcs*' (Peterson, 1981). In this case, Peterson would use 'n' number of arcs directed from place $p_1$ to transition $t_1$ and 'm' number of arcs directed from transition $t_1$ to place $p_2$. This is shown in Fig. 2.4.(b). When the transition is fired, every arc, directed from place $p_1$ to transition $t_1$, will remove one token from place $p_1$ -'n' tokens in total- and every arc, directed from transition $t_1$ to place $p_2$, will deposit one token to the place $p_2$ -'m' tokens in total. However, it is possible to represent the weighted arcs by using the representation shown in Fig. 2.4.(c), where 2n+1 places are used to represent place $p_1$ and the weighted arc Pre($p_1$, $t_1$), and m+1 places are used to represent place $p_2$ and the weighted arc Post($t_1$, $p_2$). In this case, number of the tokens in place $p_1$ is equal to the sum of the tokens in places $p_1^i$, $p_1^1$, $p_1^2$, ...., $p_1^n$, i.e., $M(p_1) = M(p_1^i) + M(p_1^1) + M(p_1^2) + .....+ M(p_1^n)$, and the number of the tokens in place $p_2$ is equal to sum of the tokens in places $p_2^o$, $p_2^1$, $p_2^2$, ...., $p_2^m$, i.e., $M(p_2) = M(p_2^o) + M(p_2^1) + M(p_2^2) + .....+ M(p_2^m)$.

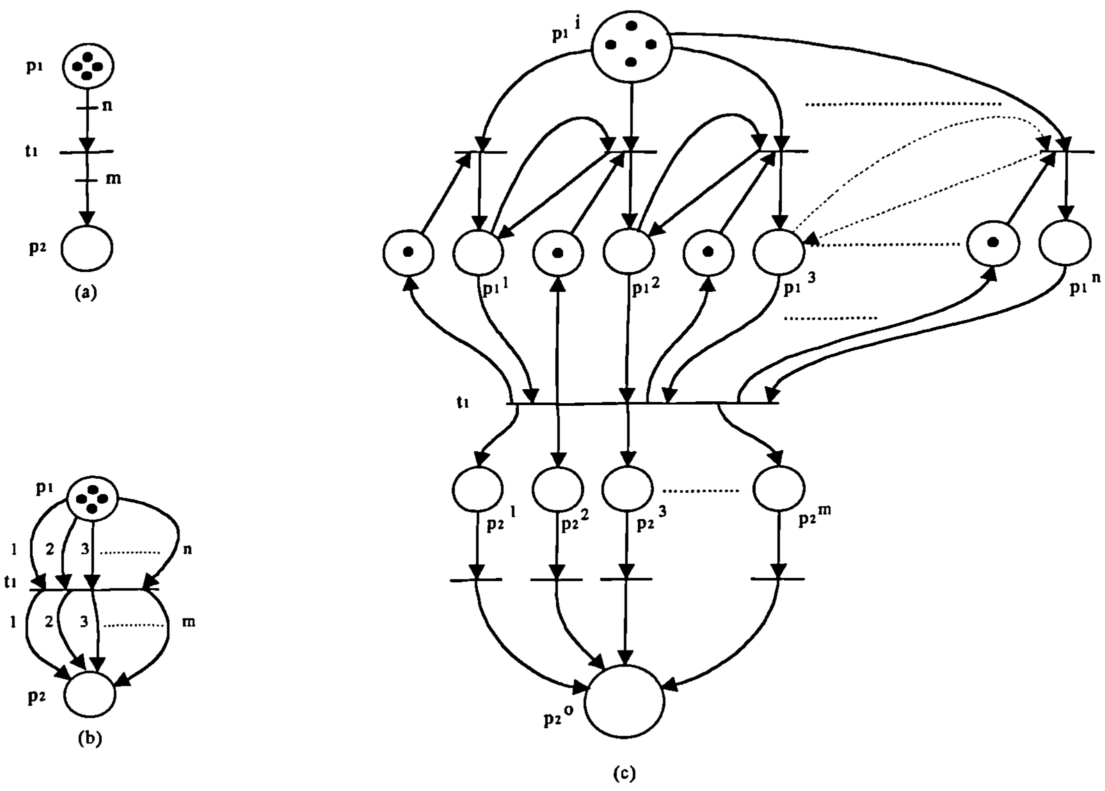


Figure 2.4. (a). A weighted arc Petri net (b). Equivalent Petri nets with 'bag of arcs' representing the weights. (c). Equivalent ordinary Petri net representing the weighted arcs.

Now consider the firing of a weighted arc Petri net, shown in Fig. 2.5, where there are four places P = { $p_1$, $p_2$, $p_3$, $p_4$ } and one transition T = { $t_1$ }. In this Petri net, the input arc $p_2 \rightarrow t_1$ has the weight of 2, i.e., Pre($p_2$, $t_1$) = 2, and the output arc $t_1 \rightarrow p_3$ has the weight of 3, i.e., Post($t_1$, $p_3$) = 3. The other arcs, whose weights are not explicitly specified, have a weight of 1. In Fig. 2.5.(a), transition $t_1$ is not enabled, because Pre($p_2$, $t_1$) = 2 and $M(p_2) = 1$, although Pre($p_1$, $t_1$) = 1 and $M(p_1) = 3$ and similarly, in Fig 2.5.(b), transition $t_1$ is not enabled, because $Pre(p_1, t_1) = 1$ and $M(p_1) = 0$, although $Pre(p_2, t_1) = 2$ and $M(p_2) = 2$. However, the Petri net, shown in Fig. 2.5.(c), is enabled, because $Pre(p_1, t_1) = 1$ and $M(p_1) = 2$, and, $Pre(p_2, t_1) = 2$ and $M(p_2) = 3$. When transition $t_1$ fires, it removes one token from place $p_1$ and two tokens from place $p_2$ and at the same time it deposits three tokens into place $p_3$ and one token into place $p_4$, as shown in Fig. 2.5.(d).

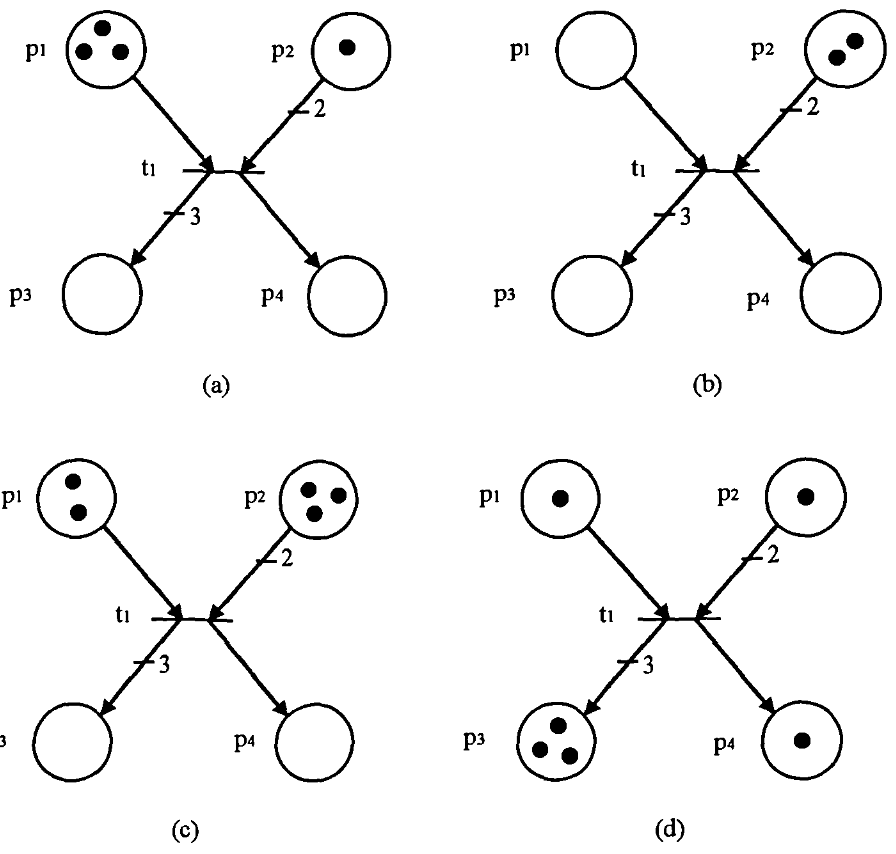


Figure 2.5. A weighted arc Petri net. (a). Not enabled. (b). Not enabled.
(c). (Enabled) before firing. (d). After firing.

## 2.3.2. Inhibitor arc Petri net

The modelling power of Petri nets can be increased by adding the '*zero testing*' ability, i.e., the ability to test whether a place has no token. This is achieved by introducing an *inhibitor arc*. The inhibitor arc connects an input place to a transition and is represented by an arc whose end is marked by a small circle. The presence of an inhibitor arc connecting an input place to a transition means that the transition is only enabled if the input place does not have any tokens. Firing of a transition does not change the marking of a place, which is connected to the transition with an inhibitor arc. In the general case, an inhibitor arc Petri net can not be transformed into an ordinary Petri net (David and Alla, 1992). An inhibitor arc Petri net is shown in Fig. 2.6, where there are three places P = { $p_1$, $p_2$, $p_3$ } and one transition T = { $t_1$ }. In the Petri net, the arc $p_2 \rightarrow t_1$ is an inhibitor arc, i.e., In($p_2$, $t_1$). The Petri net is not enabled in Fig. 2.6.(a), because Pre($p_1$, $t_1$) = 1 and M($p_1$) = 0, although In($p_2$, $t_1$) = 1 and M($p_2$) = 0 and similarly, in Fig 2.6.(b), transition $t_1$ is not enabled, because In($p_2$, $t_1$) = 1 and M($p_2$) = 1, although Pre($p_1$, $t_1$) = 1 and M($p_1$) = 1. However, the inhibitor arc Petri net in Fig. 2.6.(c) is enabled, because Pre($p_1$, $t_1$) = 1 and M($p_1$) = 1, and, In($p_2$, $t_1$) = 1 and M($p_2$) = 0. When transition $t_1$ fires, it removes one token from place $p_1$ and deposits one token into place $p_3$, as shown in Fig. 2.6.(d). Note that after the firing of transition $t_1$, the marking of the place $p_2$ remains the same.

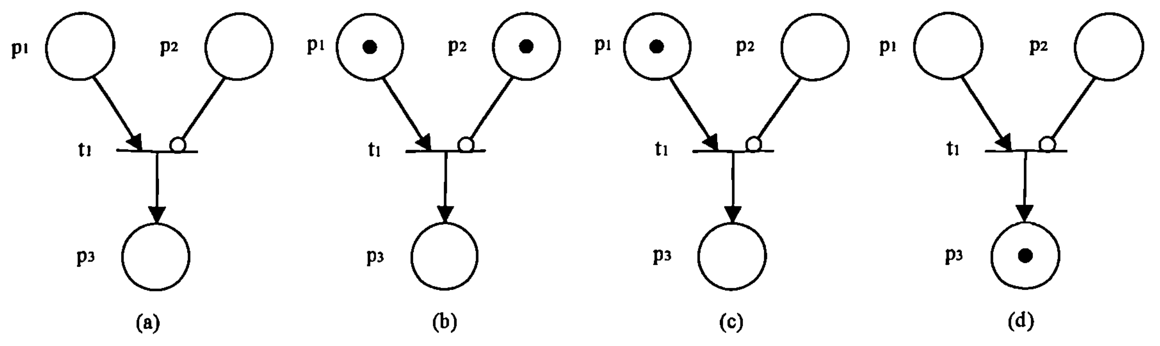


Figure 2.6. A inhibitor arc Petri net : (a). Not enabled. (b). Not enabled. (c). (Enabled) before firing. (d). After firing.

It is possible to associate weights with inhibitor arcs. In this case, an inhibitor arc is called *weighted inhibitor arc*, which has the ability to test the number of tokens in a place. If the number of tokens in an input place, connected to a transition with a weighted inhibitor arc whose weight is 'k', is less than the weight value, then the transition is enabled. If it is equal to or bigger than 'k' then the transition is not enabled. However, in this case it is assumed that all the other input places have sufficient tokens to enable the transition. The firing does not change the marking in the inhibitor arc connected places. A weighted inhibitor arc Petri net is shown in Fig. 2.7, where there are three places $P = \{ p_1, p_2, p_3 \}$ and one transition $T = \{ t_1 \}$. In the Petri net, the arc $p_2 \rightarrow t_1$ is a weighted inhibitor arc with the weight of 3, i.e., $In(p_2, t_1) = 3$. The Petri net is not enabled in Fig. 2.7.(a), because $Pre(p_1, t_1) = 1$ and $M(p_1) = 0$, although $In(p_2, t_1) = 3$ and $M(p_2) = 1$ and similarly, in Fig 2.7.(b), transition $t_1$ is not enabled, because $In(p_2, t_1) = 3$ and $M(p_2) = 4$, although $Pre(p_1, t_1) = 1$ and $M(p_1) = 1$. However, the Petri net in Fig. 2.7.(c) is enabled, because $Pre(p_1, t_1) = 1$ and $M(p_1) = 1$, and, $In(p_2, t_1) = 3$ and $M(p_2) = 2$. When transition $t_1$ fires, it removes one token from place $p_1$ and deposits one token into place $p_3$, as shown in Fig. 2.7.(d).

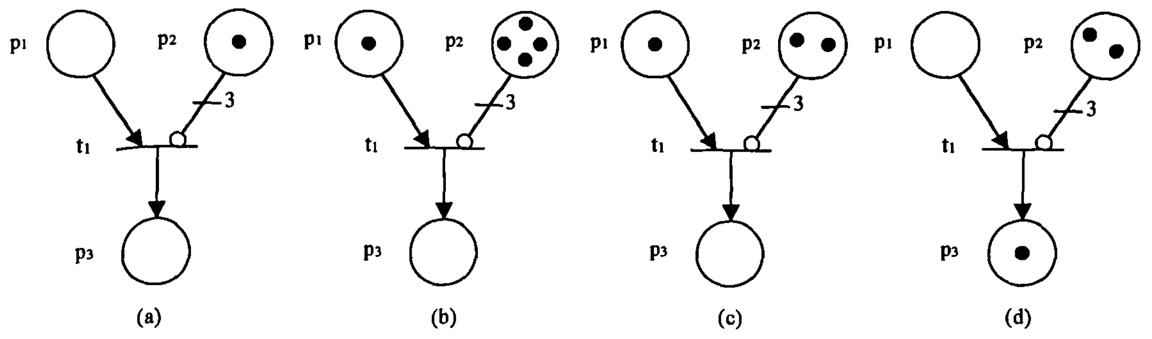


Figure 2.7. A weighted inhibitor arc Petri net : (a). Not enabled. (b). Not enabled. (c). (Enabled) before firing. (d). After firing.

## 2.3.3. Enabling arc Petri net

The modelling power of Petri nets can be increased by adding the '*one testing*' ability, i.e., the ability to test whether a place has a token(s). This is achieved by introducing an 'enabling arc'. The enabling arc connects an input place to a transition and is represented by an arc, whose end is marked by an empty arrow. The presence of an enabling arc connecting an input place to a transition means that the transition is only enabled if the input place has a token(s). The firing does not change the marking in the enabling arc connected places. An enabling arc Petri net is shown in Fig. 2.8, where there are three places P = { $p_1$, $p_2$, $p_3$ } and one transition T = { $t_1$ }. In the Petri net, the arc $p_2 \rightarrow t_1$ is an enabling arc, i.e., En($p_2$, $t_1$). The Petri net is not enabled in Fig. 2.8.(a), because En($p_2$, $t_1$) = 1 and M($p_2$) = 0, although Pre($p_1$, $t_1$) = 1 and M($p_1$) = 1, and similarly, in Fig 2.8.(b), transition $t_1$ is not enabled, because Pre($p_1$, $t_1$) = 1 and M($p_1$) = 0, although En($p_2$, $t_1$) = 1 and M($p_2$) = 1. However, the enabling arc Petri net in Fig. 2.8.(c) is enabled, because Pre($p_1$, $t_1$) = 1 and M($p_1$) = 1, and, En($p_2$, $t_1$) = 1 and M($p_2$) = 1. When transition $t_1$ fires, it removes one token from place $p_1$ and deposits one token into place $p_3$, as shown in Fig. 2.8.(d). Note that after the firing of transition $t_1$, the marking of the place $p_2$ remains the same.

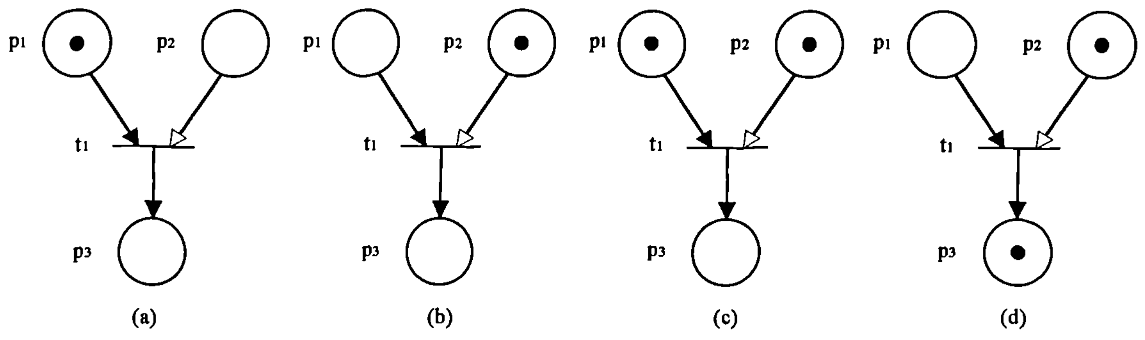


Figure 2.8. An enabling arc Petri net : (a). Not enabled. (b). Not enabled. (c). (Enabled) before firing. (d). After firing.

Although an enabling arc can be represented by two ordinary arcs, as shown in Fig. 2.9. (a) and (b), enabling arcs are distinctively different from ordinary arcs in the sense that they do not lead to conflicts in a Petri net. This is shown in Fig. 2.9.(c), where transition $t_1$ and $t_2$ can fire at any time without any conflict. However, if the enabling arcs En($p_2$, $t_1$)

and En ($p_2$, $t_2$) are replaced with normal arcs as shown in Fig. 2.9.(d), then it is obvious that this is a potential conflict situation.

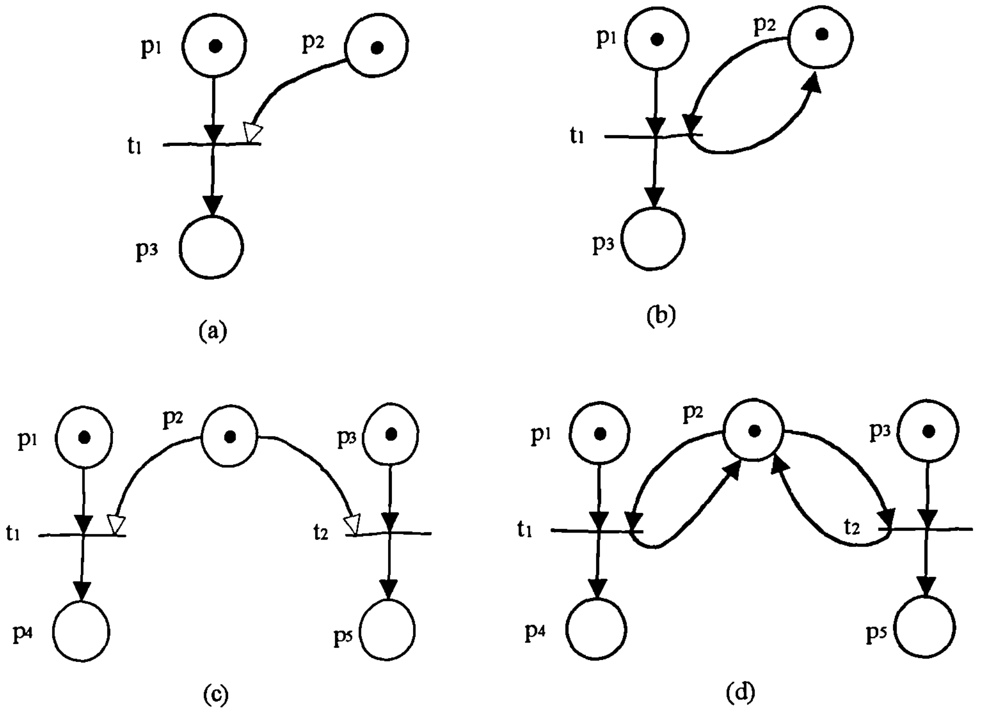


Figure 2.9. (a) An enabling arc Petri net. (b) Its equivalent.
(c). An enabling arc, where there is no conflict. (d) An ordinary Petri net, where there is a conflict.

It is also possible to associate weights with enabling arcs. In this case an enabling arc is called *weighted enabling arc*, which has the ability to test the number of tokens in a place. If the number of tokens in an input place, connected to a transition with a weighted enabling arc, whose weight is 'k', is at least equal to the weight value then the transition is enabled. If it is less than 'k' then the transition is blocked, i.e., it is not enabled. However, in this case it is assumed that all the other input places have sufficient tokens to enable the transition. The firing does not change the marking in the weighted enabling arc connected places. A weighted enabling arc Petri net is shown in Fig. 2.10, where there are three places $P = \{ p_1, p_2, p_3 \}$ and one transition $T = \{ t_1 \}$. In the Petri net, the arc $p_2 \rightarrow t_1$ is a weighted enabling arc with the weight of 3, i.e., $En(p_2, t_1) = 3$. The Petri net is not enabled in Fig. 2.10.(a), because $En(p_2, t_1) = 3$ and $M(p_2) = 2$, although $Pre(p_1, t_1) = 1$ and $M(p_1) = 1$, and similarly, in Fig 2.10.(b), transition $t_1$ is not enabled, because $Pre(p_1, t_1) = 1$ and $M(p_1) = 0$, although $En(p_2, t_1) = 3$ and $M(p_2) = 3$. However, the Petri net in Fig. 2.10.(c) is enabled, because $Pre(p_1, t_1) = 1$ and $M(p_1) = 1$, and, $En(p_2, t_1) = 3$ and $M(p_2) = 3$. When transition $t_1$ fires, it removes one token from place $p_1$ and deposits one token into place $p_3$, as shown in Fig. 2.10.(d).

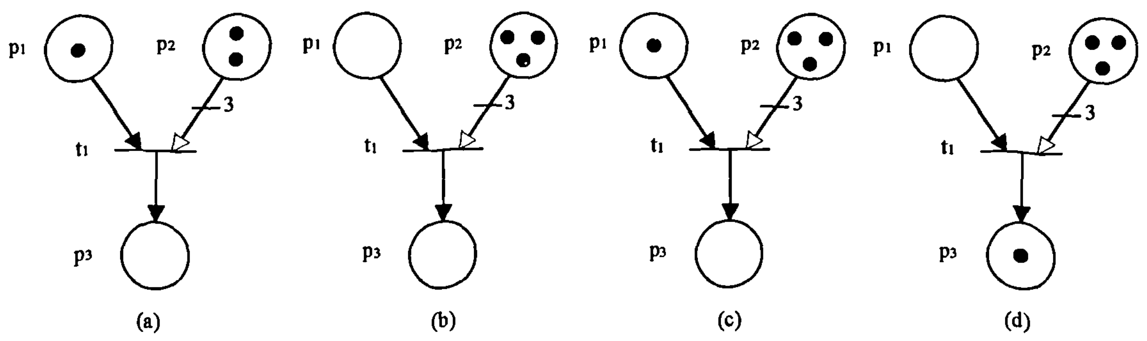


Figure 2.10. A weighted enabling arc Petri net : (a). Not enabled. (b). Not enabled. (c). (Enabled) before firing. (d). After firing.

## 2.3.4. Finite capacity Petri net

A finite capacity Petri net is one in which capacities (positive integers) are associated with places. Firing of an input transition of a place $p_i$, whose capacity is $CAP(p_i)$, is only possible, if firing of this transition does not result in a number of tokens in $p_i$ that exceeds the capacity (David and Alla, 1992). Place $p_2$ in Fig. 2.11 is a finite capacity place with the capacity of 2, i.e., $CAP(p_2) = 2$. Firing of $t_1$ in Fig. 2.11.(a) results in the marking shown in Fig. 2.11.(b) and similarly firing of $t_1$ in Fig. 2.11.(b) results in the marking shown in Fig. 2.11.(c). However, transition $t_1$ in Fig. 2.11.(c) can not fire anymore, because the marking of place $p_2$ has reached its maximum capacity. It is possible to represent the finite capacity place with two places ($p_2$ and $p_2$'). In this case, first place ($p_2$) represents the place itself and the marking of second place, i.e., $M(p_2')$, represents the capacity of the place. In other words, the marking invariant $M(p_2) + M(p_2') = 2$ is hold. This is shown in Fig. 2.12. Note that Fig. 2.12 (a), (b) and (c) is equivalent to Fig. 2.11 (a), (b) and (c), respectively. Another representation of a finite capacity place can be done by using weighted inhibitor arc, whose weight '*k*' equals to the capacity of the place. This is shown in Fig. 2.13. Note that Fig. 2.13 (a), (b) and (c) is equivalent to Fig. 2.11 (a), (b) and (c), respectively.

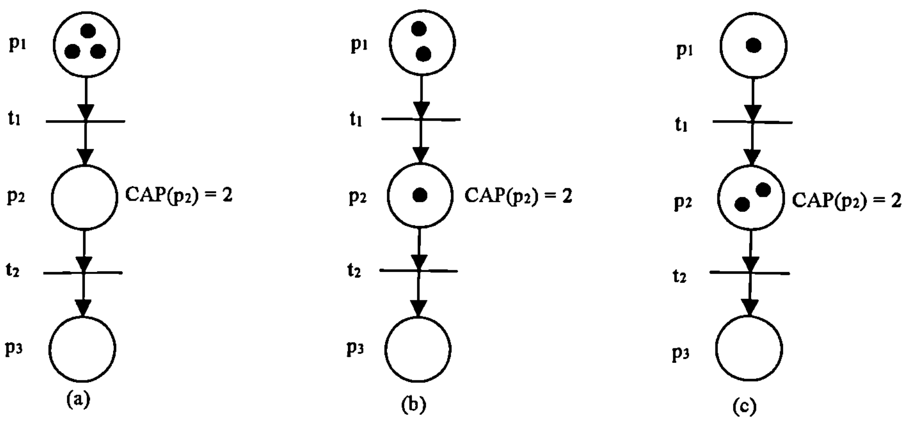


Figure 2.11. A finite capacity Petri net : (a). Initial marking ($t_1$ is enabled). (b). Marking after $t_1$ fires ($t_1$ and $t_2$ are enabled). (c). Marking after $t_1$ fires (only $t_2$ is enabled).

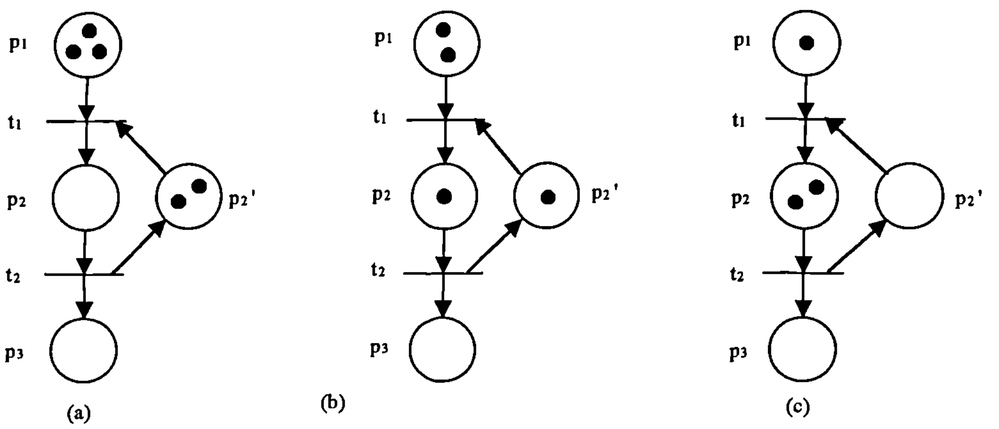


Figure 2.12. A finite capacity place, represented by two places. (a). Initial marking ($t_1$ is enabled). (b). Marking after $t_1$ fires ($t_1$ and $t_2$ are enabled). (c). Marking after $t_1$ fires (only $t_2$ is enabled).

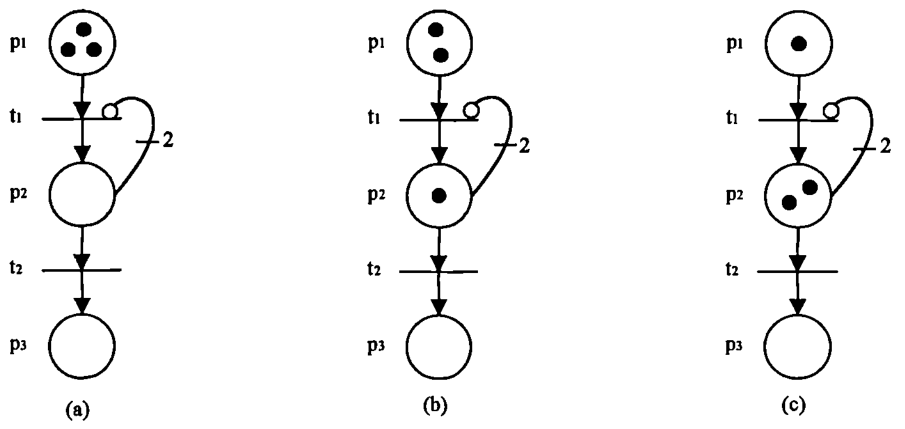


Figure 2.13. A finite capacity place, represented by weighted inhibitor arc. (a). Initial marking ($t_1$ is enabled). (b). Marking after $t_1$ fires ($t_1$ and $t_2$ are enabled). (c). Marking after $t_1$ fires (only $t_2$ is enabled).

## 2.3.5. Timed Petri net

Ordinary Petri nets do not include any concept of time. With this class of nets, it is possible only to describe the logical structure of the modelled system, but not its time evolution. Due to the need for the temporal analysis of discrete event systems, time has been introduced into Petri nets in variety of ways. In general, there are two types of timed Petri nets, namely timed-place Petri nets and timed-transition Petri nets. If the timings are associated with the places, then the Petri net is called timed-place Petri net. If the timings are associated with the transitions, then the Petri net is called timed-transition Petri net. In this thesis only the timed-transition Petri net is considered.

A timed-transition Petri net (TTPN) is a tuple as defined in (David and Alla, 1992);

**TTPN = ( PN , $\tau$ )**.........................................................................................**(3)**

In (4), PN is a marked Petri net and $\tau$ is a function from the set of transitions to the set of positive or zero rational numbers. $\tau$ ($t_i$) = $T_i$ = timing associated with transition $t_i$. In this case, a token can have two states: it can be *reserved* for the firing of a timed-transition $t_i$ or it can be *unreserved*. If a timed transition is enabled, then it is ready to be fired. When the firing condition for the transition occurs, the token of input place to this transition is said to be reserved for a specified amount of time($T_i$). When the time $T_i$ has elapsed, the transition is effectively fired: the reserved token is removed from the input place and an unreserved token is put into the output place(s). This is illustrated in Fig. 2.14, where the transition $t_2$ is a timed-transition with the time delay $T_2$. At the beginning, there is a token in place $p_1$, as shown in Fig. 2.14.(a). When transition $t_1$ is fired a token is deposited in place $p_2$, thereby resulting in the enabling of timed-transition $t_2$, as shown in Fig. 2.14.(b). Then, the firing condition for transition $t_2$ may occur at any moment after this. When the firing condition occurs, the token required for this firing is reserved, as shown in Fig. 2.14.(c). When time delay $T_2$, has elapsed, the transition is effectively fired. The token reserved for firing is then removed from place $p_2$ and an unreserved token is deposited in place $p_3$. This is shown in Fig. 2.14.(d). Note that timed-place Petri nets and timed-transition Petri nets are equivalent and it is possible to move from one to another (David and Alla, 1992).

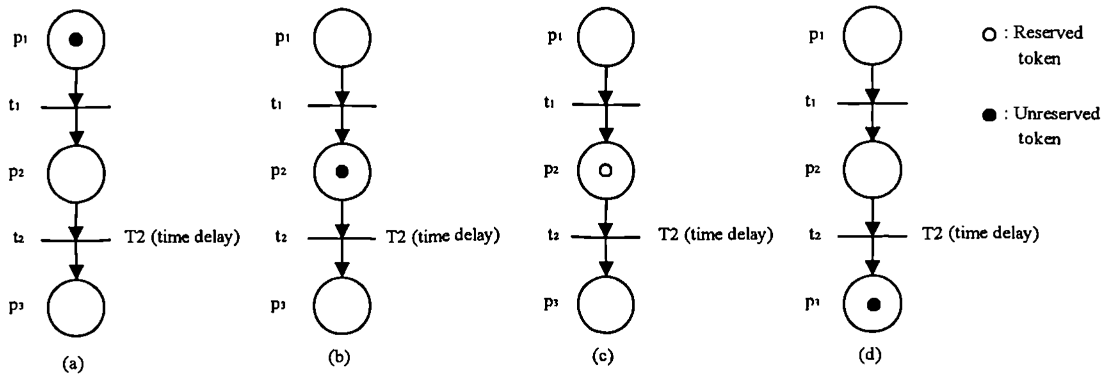


Figure 2.14. A timed-transition Petri net (TTPN) : (a). Initial marking ($t_1$ is enabled). (b). After $t_1$ is fired, an unreserved token is deposited in place $p_2$ for a time $T_2$ ($t_2$ is enabled). (c). Firing condition occurs for $t_2$ and then the unreserved token becomes reserved for firing transition $t_2$, ($0<t<T_2$). (d). After time $T_2$ has elapsed, transition $t_2$ is effectively fired and a token is deposited in place $p_3$.

## 2.4. BASIC DESIGN MODULES

In this section, some basic design modules are considered. These modules are useful in the construction of models for both high level and low level system operations. The advantage of these modules is obvious: they are specific and therefore easily accepted and applied when a practical problem is encountered. In this section the following design modules are considered:

- sequence
- concurrency and synchronisation
- conflict
- buffer
- FIFO queue
- machine
- motor and actuator

### 2.4.1. Sequence

A sequence in a Petri net represents a series of successive operations. It is possible to model a sequence Petri net as shown in Fig. 2.15, where there are $p_{n+1}$ places and $t_n$ transitions. In this case, activities or operations are represented by places. Transitions represent the end of one activity and at the same time starting of another one. Note that each activity depends on the completion of the previous one, except for the first activity.

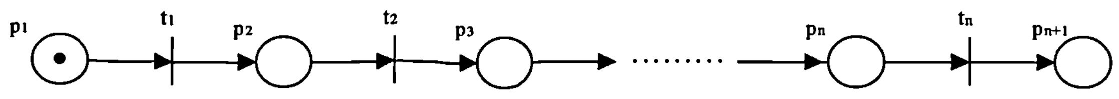


Figure 2.15. A sequence Petri net.

### 2.4.2. Concurrency and Synchronisation

In a system some activities or operations may be happening concurrently. For example, two machines can be running concurrently producing two different part-types. There is no need to synchronise events unless it is required by the underlying system, which is being modelled. When synchronisation is needed, it is easy to do so. For example, if two part types, produced by two machines mentioned above, are required to be assembled, then it can be done when each machine operation is complete. These examples exhibit the characteristic of concurrency and synchronisation.

Concurrency and synchronisation are shown in Fig. 2.16. As can be seen from the Petri net when the system starts ($t_1$ fires) two machines, i.e., machine 1 and machine 2, start operating concurrently in order to produce parts, part 1 and part 2, respectively. When machine 1 finishes its operation ($t_2$ fires), it produces a part 1. When machine 2 finishes its operation ($t_3$ fires), it produces a part 2. One machine could finish its operation before the other one. However, in order to make an assembly both parts are required (synchronisation).

In terms of Petri nets, concurrency means that two or more events are occurring at the same time. That is, concurrency will be present when more than one transition is enabled and firable at the same time. Synchronisation is present, when there is more than one input place, each of which representing a different activity, to a transition.

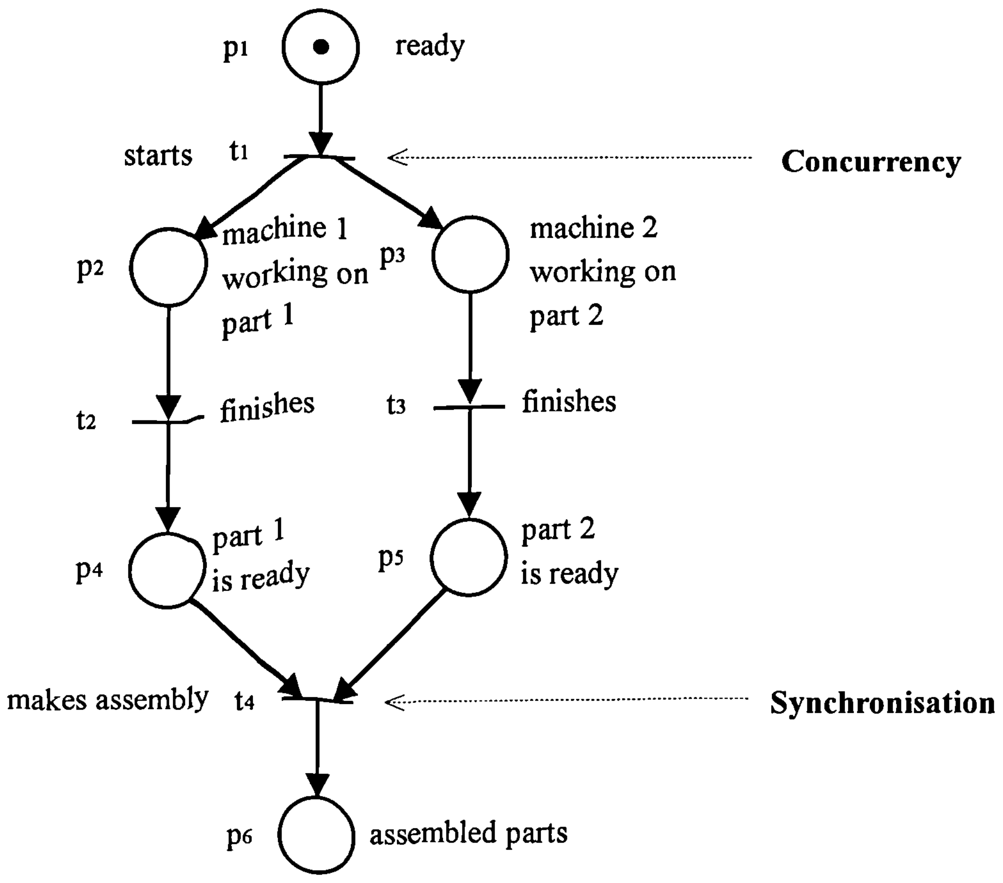
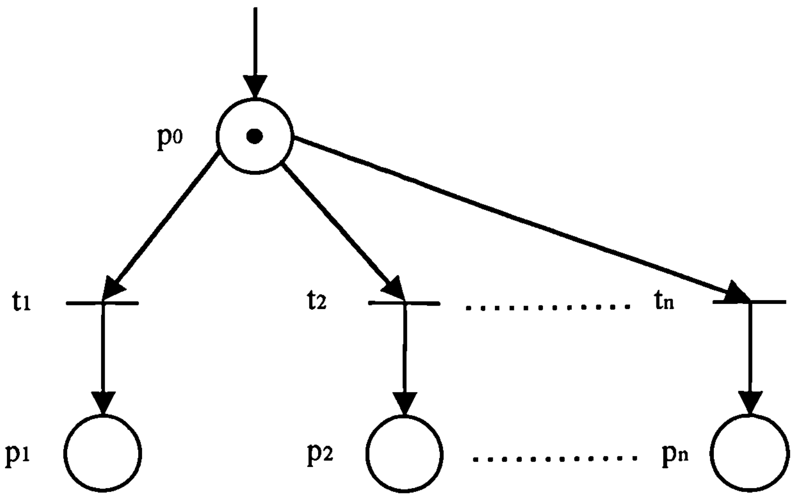


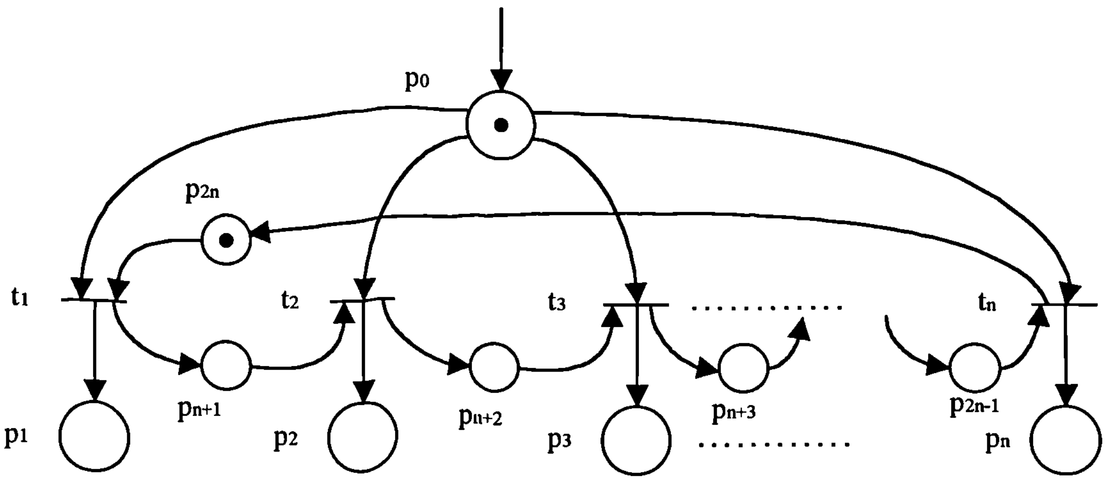
Figure 2.16. Concurrency and synchronisation

### 2.4.3. Conflict

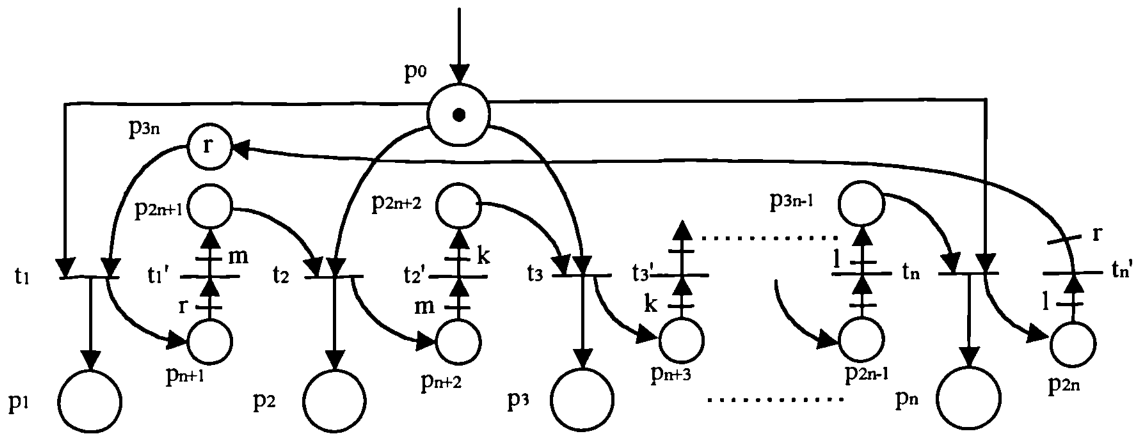
In a manufacturing system, when two machines share the same resource and both try to access it at the same time, this situation leads to *conflict*. In a Petri net, a conflict situation occurs when a place enables more than one transition at the same time. In conflict, only one transition can fire. A conflict in Petri nets is shown in Fig. 2.17.(a). As can be seen from Fig. 2.17.(a), when there is a token in place $p_0$ all transitions are enabled. Since only one transition can fire in the case of conflict, any conflict, arising in a Petri net, must be solved. The conflict can be solved by assigning a priority between the conflicting transitions. Such a priority for resolving conflict has been proposed by (Zhou and Dicesare, 1993) as shown in Fig. 2.17.(b), where there are 2n+1 places and 'n' transitions. As can be seen from the structure of the Petri net, shown in Fig. 2.17.(b), each token deposited into place $p_0$ is equally shared by places $p_1$, $p_2$, $p_3$......$p_n$, one by one. It is also possible to introduce a priority scheme, in which every place, i.e., $p_1$, $p_2$, ....., $p_n$, will receive different number of tokens. Such priority can be represented as shown in Fig. 2.17.(c), where there are 3n+1 places and 2n transitions. Initially, there are 'r' tokens in place $p_{3n}$ and *r*, *m*, *k*, *l* are non-negative integers. Note that in this case weighted arcs used to represent different number of tokens. Upon reaching the specified number of tokens for each output transition the Petri net structure enables the next output transition, and so on.



(a)



(b)



(c)

Figure 2.17. (a) Conflict. (b) Conflict free Petri net. (c) Conflict free Petri net in general case.

Fig. 2. 18 is considered to explain these two conflict resolution techniques in detail. In Fig. 2.18.(a), there is a conflict between transitions $t_1$ and $t_2$. If this conflict is to be resolved such that $t_1$ and $t_2$ fire one after another, then this can be done as shown in Fig. 2.18.(b) by using the Zhou and Dicesare's approach. As can be seen from the Petri net, firstly, transition $t_1$ fires, then if there is a token in $p_1$, transition $t_2$ fires and so on. If this conflict is to be solved such that first $t_1$ fires twice and then $t_2$ fires three times and so on. This can be done this time, as shown in Fig. 2.18.(c), by using the method proposed in this thesis. Note that initially there are two tokens in place $p_6$. These tokens enable transition $t_1$. After $t_1$ fires twice, there will be two tokens in place $p_3$ and transition $t_1$' will fire immediately by removing these two transitions from place $p_3$ and by depositing three tokens in place $p_5$. When there is a token(s) in place $p_5$, this will enable transition $t_2$. After $t_2$ fires three times, there will be three tokens in place $p_4$ and transition $t_2$' will fire immediately by removing these three transitions from place $p_4$ and by depositing two tokens in place $p_6$. This process carries on in a repeating fashion.

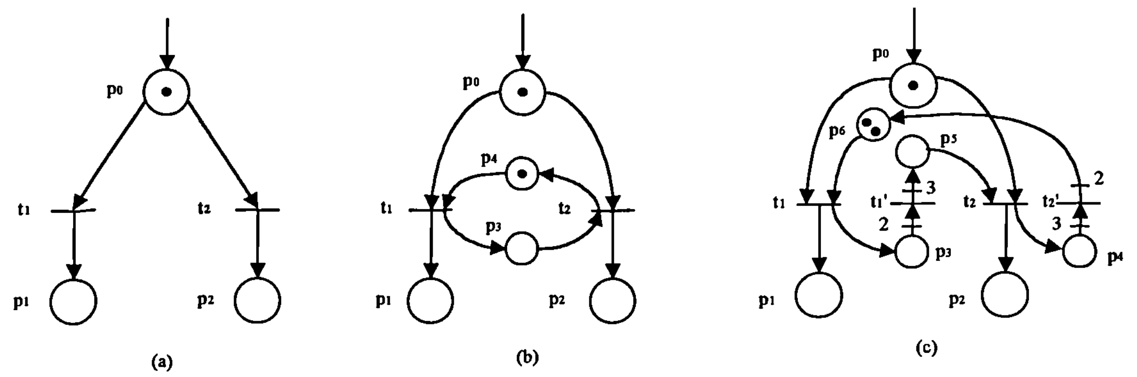


Figure 2.18. (a) Conflict. (b) Conflict free Petri net. (c) Conflict free Petri net in general case.

### 2.4.4. Buffer

Buffers, in a manufacturing context, are used to provide temporary storage of workpieces between operations. For example, it is assumed that the buffer between machine 1 and machine 2, shown in Fig. 2.19.(a), can hold *k* parts. One possible Petri net buffer model is shown in Fig. 2. 19.(b), where the number of tokens in place $p_1$ represents the number of available spaces in the buffer, while the number of tokens in place $p_2$ represents the available parts in the buffer. Note that transition $t_1$ represents a part entering into the buffer and transition $t_2$ represents a part leaving the buffer. Initially there are *k* tokens in place $p_1$, indicating that the buffer has capacity k and it is presently empty. When a part enters the buffer (transition $t_1$ fires) one token is removed from place $p_1$ and one token is deposited in place $p_2$, indicating that the number of parts in the buffer is incremented by one, while the number available spaces in the buffer is decremented by one. The parts can be put into the buffer as long as there is enough space, i.e., there are tokens in place $p_1$. If there is no token in place $p_1$ this means that the buffer is full. When a part leaves the buffer (transition $t_2$ fires) one token is removed from place $p_2$ and one token is deposited in place $p_1$, indicating that the number of parts in the buffer is decremented by one, while the number of available spaces in the buffer is incremented by one. It is also possible to model a buffer with a place and a weighted inhibitor arc as shown in Fig. 2.19.(c), where the number of tokens in place $p_1$ represents the number of parts in the buffer and the weight $k$ of the inhibitor arc represents the available spaces. In this case, transition $t_1$ can fire as long as the number of tokens in place $p_1$ is less than $k$. When the number of tokens in place $p_1$ equals to k, transition $t_1$ can not fire. This means that the buffer is full.

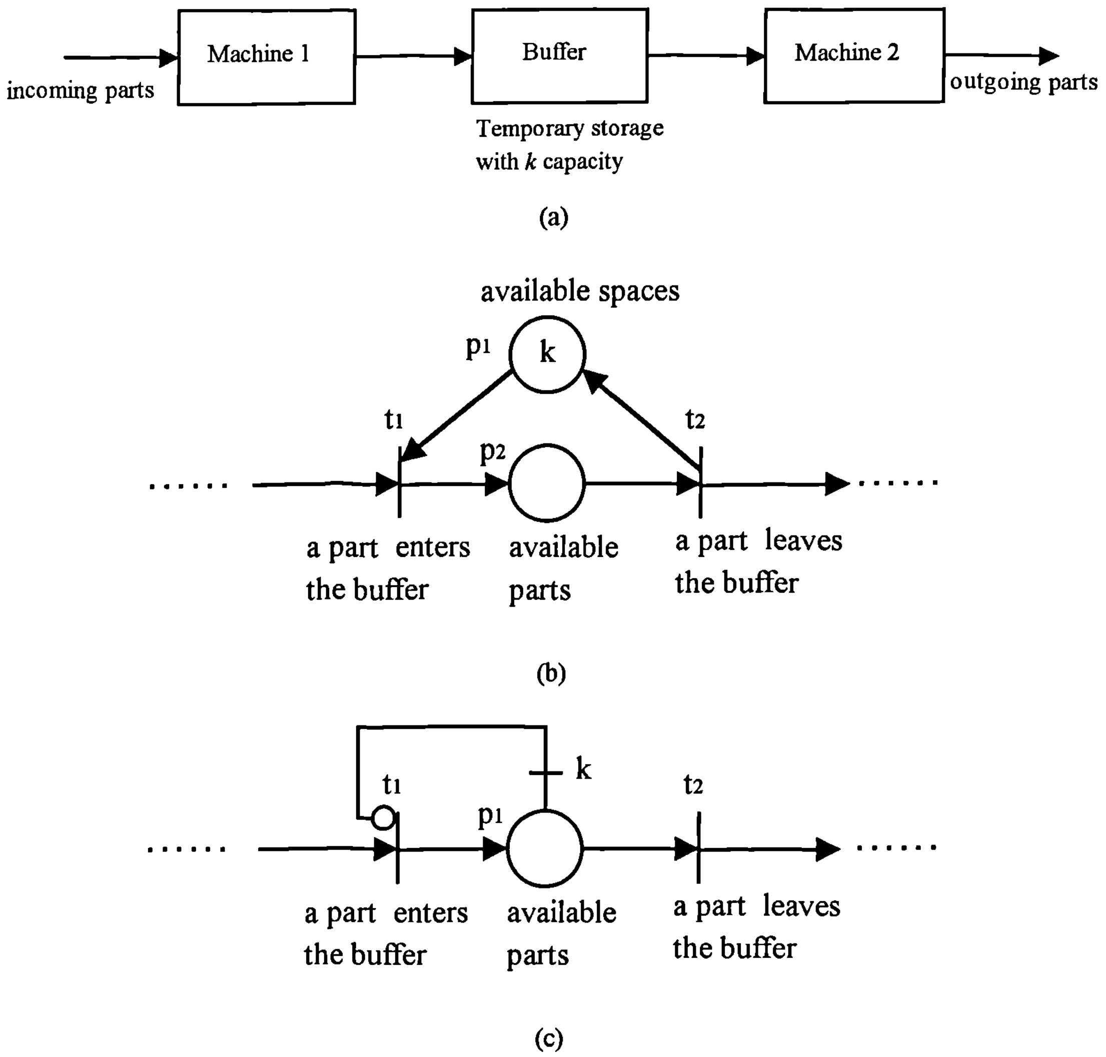
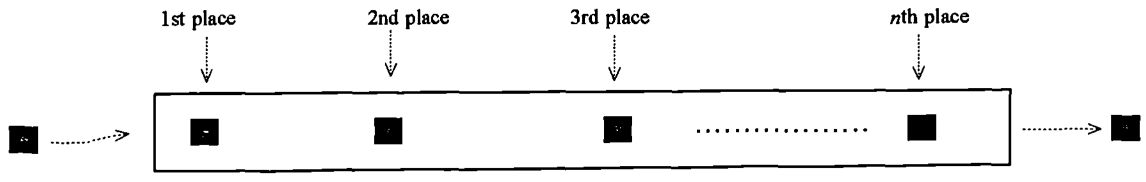


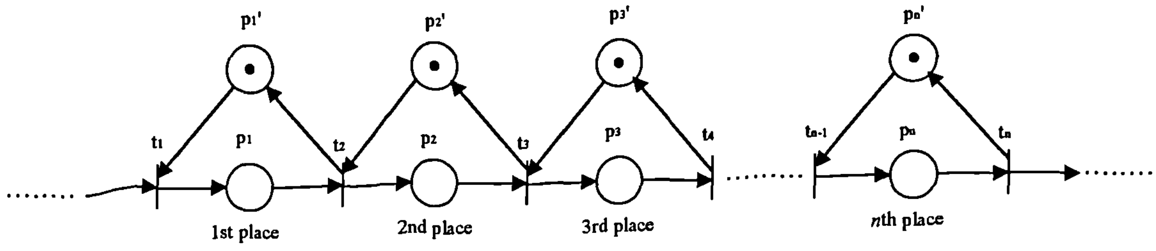
Figure 2.19. (a). Buffer in a manufacturing system. (b). Buffer model, implemented by two places. (c). Buffer model, implemented by a place and a weighted inhibitor arc.

### 2.4.5. FIFO queue

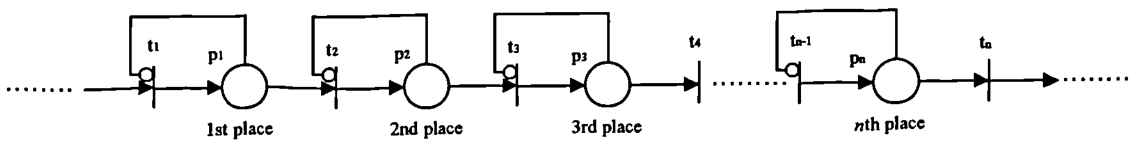
A First-In-First-Out (FIFO) Queue is also called First-Come-First-Serve (FCFS) Queue in manufacturing context. A conveyor belt is an example of a FIFO part queue, where the first part put onto the belt is the first part to come off the other end. The structure of the FIFO queue depends on whether the queue is supposed to store only one part-type or multiple part-types. When the single part-type case is considered, it is possible to use the models proposed for modelling the buffer. It is also possible to introduce a 'safe' Petri net model for a FIFO queue. For example, consider the conveyor belt shown in Fig. 2.20.(a). Assume that the conveyor belt can carry *n* parts. To model this FIFO queue structure, it is possible to use the Petri net as shown in Fig. 2.20.(b), where tokens in places $p_1$, $p_2$......$p_n$ represent the presence of parts on the 1st place, on the 2nd place, ......., on the nth place of the conveyor belt respectively, while tokens in places $p_1'$, $p_2'$......$p_n'$ represent the presence of available places on the 1st place, 2nd place......., nth place of the conveyor belt, respectively. An alternative safe FIFO queue model is shown in Fig. 2.20.(c), where inhibitor arcs are used to specify available spaces on the conveyor belt.



(a)



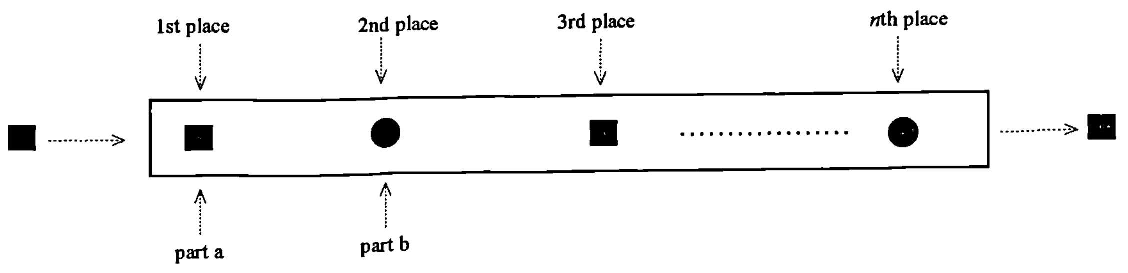
(b)



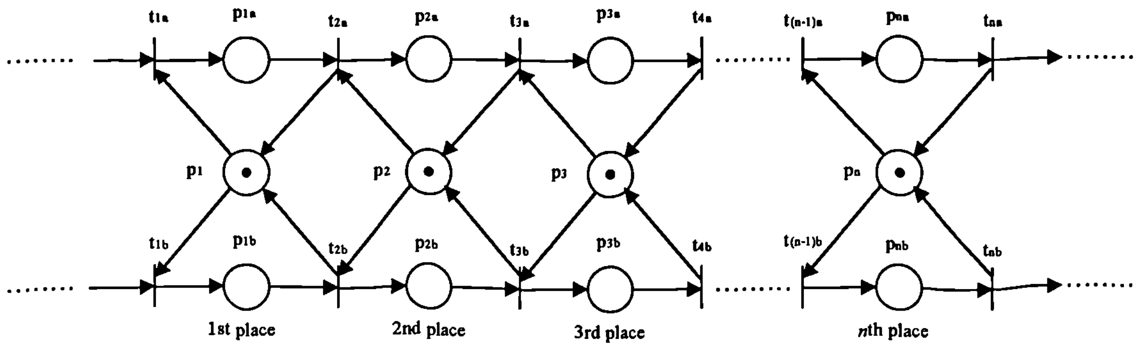
(c)

Figure 2.20. (a) A conveyor belt. (b). Its FIFO queue model.
(c) FIFO queue model implemented with inhibitor arcs.

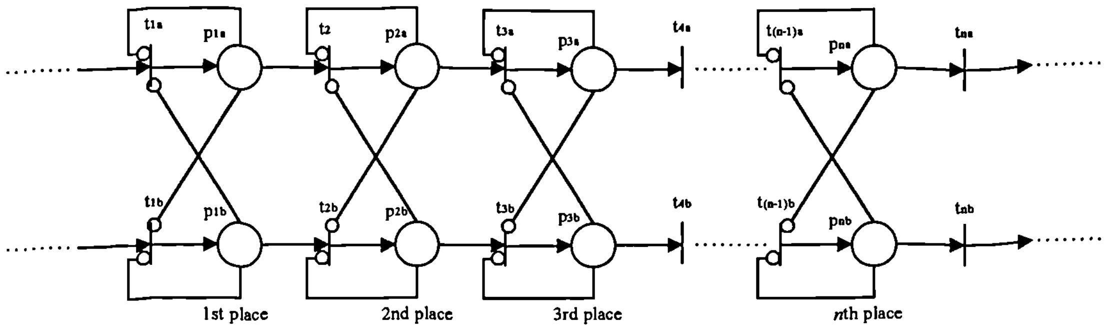
The FIFO queue models considered above are simple, because it was assumed that all tokens (parts) were the same. However, in many practical situations the FIFO queues have more than one part-type. For example, in Fig. 2.21.(a) there are two part-types, namely part a and part b, on a conveyor. This is an example of a FIFO queue with two parts. It is possible to model this system with a Petri net as shown in Fig. 2.21(b), where tokens in places $p_{1a}$, $p_{2a}$, $p_{3a}$, ....., $p_{na}$ represent the presence of part a's on the 1st place, 2nd place, 3rd place......., on the nth place of the conveyor belt respectively, while tokens in places $p_{1b}$, $p_{2b}$, $p_{3b}$, ....., $p_{nb}$ represent the presence of part b's on the 1st place, 2nd place 3rd place, ......., on the nth place of the conveyor belt respectively. Tokens in places $p_1$, $p_2$, $p_3$, ......, $p_n$ represent the absence of part a's or b's on the 1st place, 2nd place, 3rd place, ......., on the nth place of the conveyor belt. An alternative safe FIFO queue for two part types is shown in Fig. 2.21.(c), where inhibitor arcs are used to specify available spaces on the conveyor.



(a)



(b)



(c)

Figure 2.21. (a) A conveyor belt with two part-types. (b). Its FIFO queue model.
(c) FIFO queue model for two part-types, implemented with inhibitor arcs.

Note that $p_1'$, $p_2'$, $p_3'$.....$p_n'$ of the Fig. 2.20.(b) as well as $p_1$, $p_2$, $p_3$.....$p_n$ of the Fig. 2.21.(b) are called *monitor places*. In Fig. 2.20.(b) the place invariant $M(p_1) + M(p_1') = 1$ is hold for places $p_1$ and $p_1'$. This means that the number of tokens that can be present in places $p_1$ and $p_1'$ can not be more than one. Similarly, in Fig. 2.21.(b), the place invariant $M(p_{1a}) + M(p_{1b}) + M(p_1) = 1$ is hold for places $p_{1a}$, $p_{1b}$ and $p_1$. This means that the number of tokens that can be present in places $p_{1a}$, $p_{1b}$ and $p_1$ can not be more than one. How the place invariant method is used to represent the maximum number of tokens, which can be present in a group of places, is considered in detail in (Moody et al 1994, Yamalidou et al, 1996).

### 2.4.6. Machine

There are two types of machine operations, that can be considered. In the first case, a machine is assumed to work without any breakdown. The machine in this case is called a *reliable machine*. In the second case, any possible breakdown is also taken into account. The machine in this case is called an *unreliable machine*. A reliable machine can be modelled as shown in Fig. 2.22.(a), where a token in place $p_1$ represents the machine being idle and a token in place $p_2$ represents the machine working. Initially, the machine is idle. When it is started to its operation ($t_1$ fires), it is working. When it is stopped, it is idle again. An unreliable machine can be modelled as shown in Fig. 2.22.(b), where the machine has three different states: idle, working and down. When the machine is working it may either finish its operation (transition $t_2$) or it may breakdown (transition $t_3$). When the machine is down it needs to be repaired (transition $t_4$) before returning back to its working state.

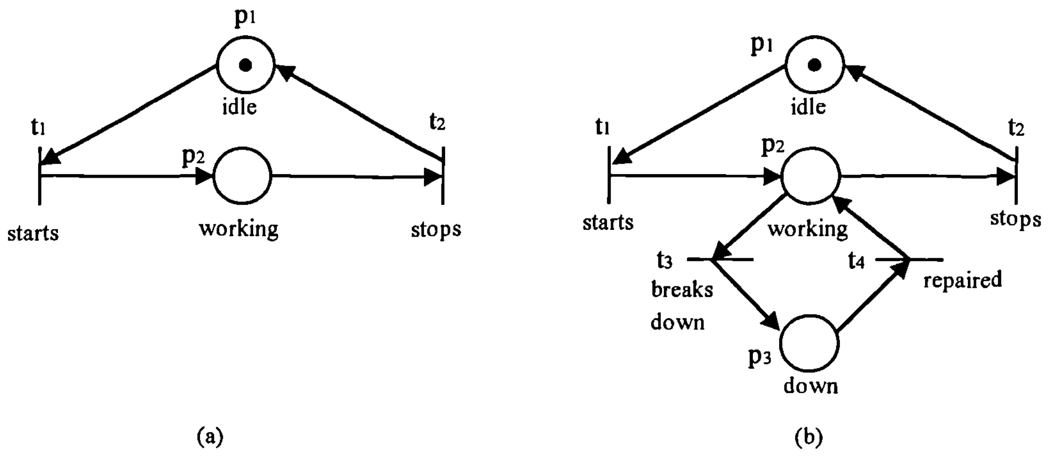


Figure 2.22. (a) Reliable machine. (b) Unreliable machine.

### 2.4.7. Motor and Actuator

Normally, if a motor's operation involves turning in the same direction, it is possible to describe its operation with *on* and *off* states, as shown in Fig. 2.23.(a). In this case, initially the motor is *off*. When it is switched on (transition $t_1$ fires), it is *on*. When it is switched *off* (transition $t_2$ fires), it is *off*. If a motor's operation involves turning both forwards and backwards, then it can be modelled as shown in Fig. 2.23.(b). In this case, initially the motor is *off*. It can be switched-on-forwards (transition $t_4$) or switched-on-backwards (transition $t_3$). In addition an actuator's operation can also be modelled as shown in Fig. 2.23.(a). This means that an actuator is either *off* or *on*. When the actuator's state is shown as being *on* in the model, it is assumed that in the real system it is in operation and vice versa.

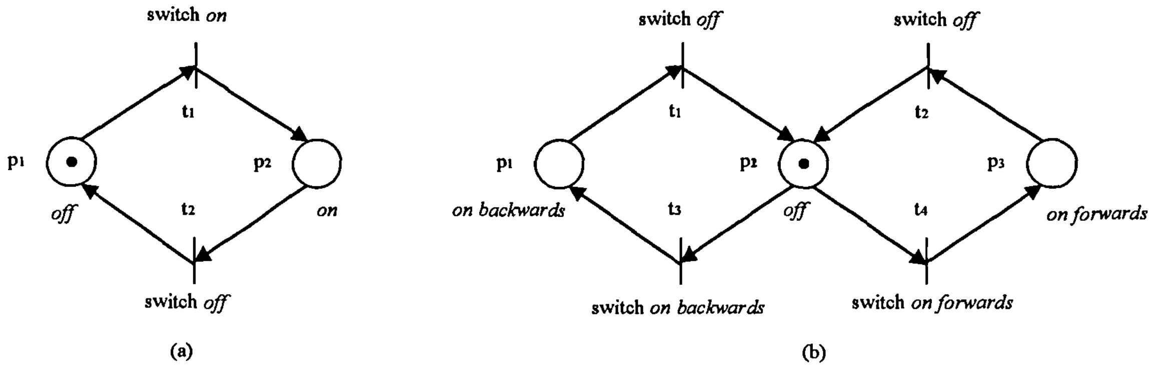


Figure 2.23. (a). A motor model, with on and off states.
(b). A motor model, with on forwards, off and on backwards states.

## 2.5. AUTOMATION PETRI NETS

As manufacturing systems become more complex, the need for an effective automation tool to produce Discrete Event Control System (DECS) becomes increasingly more important. Petri nets have appeared as the most promising tool to facilitate such design work. In this section, Automation Petri nets (APN) are proposed as a new method for the design of DECSs. Since ordinary Petri nets do not deal with sensors and actuators, the Petri net concepts are extended, by including actions and sensor readings as formal structures within the APN. These extensions involve extending the Petri nets to accommodate sensor signals at transitions and to assign action to the places. A typical discrete event control system (DECS) is shown in Fig. 2.24.(a). It consists of a discrete event system (DES), to be controlled and a discrete event controller (DEC). Sensor readings are regarded as inputs from the DES to the DEC, and control actions are considered as outputs from the DEC to the DES. The main function of the DEC is to supervise the desired DES operation and to avoid forbidden operations. To do this, the DEC processes the sensor readings and then it forces the DES to conform to the desired specifications through control actions. Nowadays, PLCs are the most popular implementation tools for this type of DEC. Petri nets can be used to design such DECs. However, ordinary Petri nets do not deal with actuators or sensors. Because of this, it is necessary to define a Petri net-based controller (Automation Petri net, APN) which can embrace both actuators and sensors within an extended Petri net framework. An APN is shown in Fig. 2.24.(b). In the APN, sensor readings can be used as firing conditions at transitions. The presence or absence of sensor readings can be used in conjunction with the extended Petri net pre-conditions to fire transitions. In the APN, two types of actuation can be considered, namely impulse actions and level actions. Actions are associated with places. With these additional features, it is possible to design Discrete Event Control Systems. Fig. 2.24.(c) shows how an APN can be used as a DEC in a DECS.

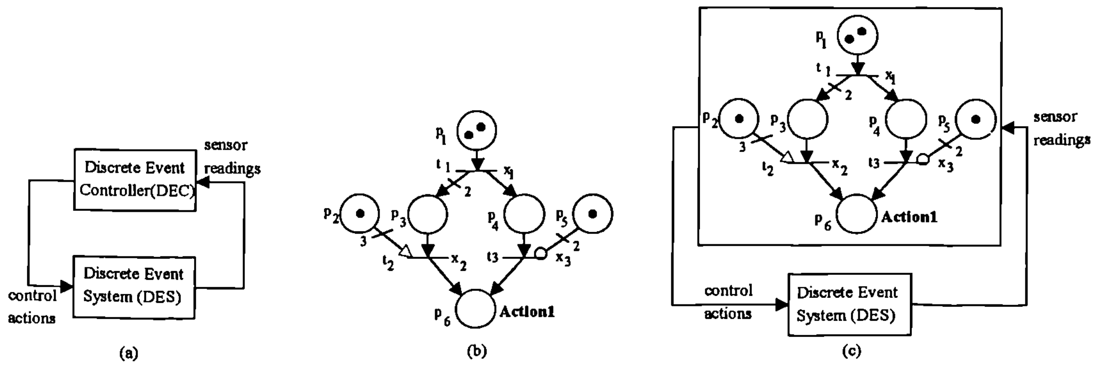


Figure 2.24. (a). A typical discrete event control system.
(b). Automation Petri Net (APN). (c). APN as a controller in a DECS.

Formally an APN can be defined as follows:

**APN = (P, T, Pre, Post, In, En, $\chi$, Q, M$_0$)**......................................................................**(4)**

Where,

- P = { $p_1$, $p_2$, ......,$p_n$ } is a finite, nonempty set of places,
- T = { $t_1$, $t_2$, ......., $t_n$ } is a finite, nonempty set of transitions, $P \cup T \neq \varnothing$ and $P \cap T = \varnothing$,
- Pre: (P×T) → N is an input function that defines directed ordinary arcs from places to transitions, where N is a set of nonnegative integers,
- Post: (P×T) → N is an output function that defines directed ordinary arcs from transitions to places,
- In: (P×T) → N is an inhibitor input function that defines inhibitor arcs from places to transitions,
- En: (P×T) → N is an enabling input function that defines enabling arcs from places to transitions,
- $\chi$ = { $\chi_1$, $\chi_2$, ........, $\chi_m$ } is a finite, nonempty set of firing conditions associated with the transitions,
- Q = { $q_1$, $q_2$, ......., $q_n$ } is a finite set of actions that might be assigned to the places,
- $M_0$ : P → N is the initial marking.

The APN consists of two types of nodes called places, represented by circles ( O ), and transitions, represented by bars ( — ). There are three types of arcs used in the APN, namely, ordinary arcs, represented by a directed arrow ( ⟶ ), inhibitor arcs, represented by an arrow, whose end is a circle ( —o ), and finally enabling arcs, represented by a directed arrow, whose end is empty ( —▹ ). Weighted and directed ordinary arcs connect places to transitions and vice versa, while weighted enabling arcs and inhibitor arcs connect only places to transitions. Places represent the status of the system and transitions represent events. Each transition has a set of input and output places, which represent the pre-condition and post-condition of the transition. The actions (Q), assigned to the places, can be either impulse actions or level actions. Impulse actions are enabled at the instant when a token is deposited into the place and level actions are enabled when there is a token(s) at the place. More than one action may be assigned to a place. Firing conditions in the APN are recognised as external events such as sensor readings. A firing condition, $\chi$, associated with a transition *t*, is a Boolean variable that can be 0, in which case related transition *t* is not allowed to fire, or it can be 1, in which case related transition *t* is allowed to fire if it is enabled. The marking of the APN is represented by the number of tokens in each place. Tokens are represented by black dots (•). Movement of tokens between places describes the evolution of the APN and is accomplished by the firing of the enabled transitions. The following rules are used to govern the flow of tokens:

**Enabling Rules**: In the APN, there are mainly three rules which define whether a transition is enabled to fire.

1. If the input place of a transition *t* is connected to the transition with a directed ordinary arc, then transition *t* is said to be enabled when the input place *p* contains at least the number of tokens equal to the weight of the directed ordinary arc connecting *p* to *t*.

2. If the input place of a transition *t* is connected to the transition with an enabling arc, then transition *t* is said to be enabled when the input place *p* contains at least the number of tokens equal to the weight of the enabling arc connecting *p* to *t*.

3. If the input place of a transition *t* is connected to the transition with an inhibitor arc, then transition *t* is said to be enabled when the input place *p* contains less tokens than the weight of the inhibitor arc connecting *p* to *t*.

In the case, where a transition may have the mixture of these input arcs, enabling rule for the transition must be analysed accordingly.

Firing Rules: In the APN, an enabled transition *t* can or can not fire depending on the external firing condition $\chi$ of *t*. These firing conditions can be, positive level or zero level of a sensor reading. Broadly speaking, a firing condition $\chi$ may include more than one sensor reading with 'AND', 'OR' and 'NOT' logical operators. When dealing with more than one sensor readings as firing conditions, the logical operators of firing conditions must be taken into account accordingly. In the special case, where $\chi = 1$, transition *t* is always allowed to fire when it is enabled. When an enabled transition t fires, it removes from each input place *p* the number of tokens equal to the weight of the directed ordinary arc connecting *p* to *t*. It deposits, at the same time, in each output place *p* the number of tokens equal to the weight of the directed arc connecting *t* to *p*. It should be noted that, the firing of an enabled transition t does not change the marking of the input places, which are connected to the transition t only by enabling or inhibitor arcs. It is also possible to consider timed APNs, as in normal Petri nets.

## 2.6. DISCUSSION

In this chapter, an introduction to Petri nets has been given. This has included firstly, the definition of ordinary Petri nets and firing of a simple Petri net. After that some of the most important behavioural properties such as reachability, boundedness, liveness, of Petri nets have been considered. Analysis techniques for Petri nets have also been discussed. It is necessary to note that Petri net models considered in this thesis are bounded, live, and also reversible, safe and conservative unless otherwise stated. Since ordinary Petri nets are not always sufficient to represent and analyse complex systems, some new classes of Petri nets have been proposed in the literature. Therefore, some important extended Petri nets, such as weighted arc Petri net and timed Petri nets, have been introduced by the research community. Then, some basic design modules have been considered. It is important point out that the basic Petri net modules provided can be used as ***building blocks*** when modelling a system with Petri nets. Finally, in this thesis an extended Petri net formalism, called Automation Petri nets, has been proposed in order to include sensor readings and actuator operations within the Petri net formalism. APNs make it possible to design a controller for a discrete event system.

# CHAPTER 3

# PETRI-NET-BASED STATE MACHINE SUPERVISORS FOR THE FORBIDDEN STATE PROBLEM

# CHAPTER 3

# PETRI-NET-BASED STATE MACHINE SUPERVISORS FOR THE FORBIDDEN STATE PROBLEM

## 3.1. INTRODUCTION

The *supervisory control theory* was introduced just more than 10 years ago as a conceptual framework for studying the supervision (i.e., control) of discrete event systems (DES) (Ramadge and Wonham, 1986; Ramadge and Wonham, Jan. 1987; Ramadge and Wonham, Sept. 1987; Wonham and Ramadge, May 1987). The key concepts in the supervisory control of DESs are as follows:

- There are two types of events that may occur in the DES, namely controllable events, that may be controlled by control action, and uncontrollable events, that may not be controlled by control action.
- Given a model of a DES and a specified desired behaviour of the controlled system, the objective is to synthesise a supervisor and a supervisory control policy to realise the specified controlled behaviour.
- Controlled behaviour of the DES must be nonblocking, i.e., it must not contradict the specifications given.
- Controlled behaviour of the DES must be maximally permissive within the specifications given.

The supervisory control theory is based on finite state machines (FSM) and formal language concepts. Although, FSMs provide a general framework for establishing fundamental properties of DES control problems, there are some disadvantages in using FSMs (Giua, 1996). Firstly, for practical systems the number of states, which are used to model the system, increases exponentially as the system becomes bigger. This means that FSMs are computationally inefficient. Secondly, a meaningful graphical representation is impossible for all but modest problems.

Petri-nets-based solutions, have several advantages over FSMs (Giua, 1996). Firstly, the states of a Petri net are represented by the possible markings and not by the places: thus Petri nets give a more compact description, i.e., the structure of the net may be maintained small even if the number of the markings grow. Secondly, instead of using ambiguous textual descriptions or mathematical notations, which are difficult to understand, the plant and the specifications can be represented graphically in an easily understood format using Petri nets. Finally, by using Petri net models, the model can be used for the analysis of their properties, performance evaluation and the systematic construction of the discrete event supervisors. Because of these advantages over FSM models, Petri nets have gained in popularity as an alternative framework for the design of supervisory controllers for discrete event systems (Holloway and Krogh, May 1990; Krogh and Holloway, 1991; Giua and DiCesare, Dec. 1991, Sreenivas, 1993; Sreenivas, 1994; Sreenivas, 1996). In general, there are two types of supervisors considered, namely mapping supervisors, whose control policy is a function computed after each new event generated by the system, and compiled supervisors, whose control policy is represented as a net structure (Giua, 1996). There are several advantages in fully compiling the supervisor into a net structure (Giua, 1996). Firstly, the computation of the control action is faster, since it does not require separate on-line computation. Secondly, the same Petri net system execution algorithms may be used for both the original system and the supervisor. Finally, a controlled model of the system under control can be built with standard net composition constructions. It is obvious that compiled supervisors are preferred to mapping supervisors. However, to-date the construction of such supervisors has been based on heuristic methods. Therefore, an important issue within the synthesis of supervisor for a DES is to develop a formal methodology for the design of such a supervisor.

The classes of specifications that have been considered within the supervisory control problem fall into two categories: the *forbidden state problem* (Ramadge and Wonham, Sept. 1987) and the *desired* (also called *forbidden) string problem* (Ramadge and Wonham, Jan. 1987). Note that in this chapter only the forbidden state problem is considered.

In the forbidden state problem, the control specifications are expressed as forbidden conditions. Forbidden conditions are a compact way of defining classes of undesirable markings which should be avoided (Holloway et al., 1996). In a discrete manufacturing context, the forbidden state problem can be specified as undesirable operating conditions, for which the production goals can not be satisfied, or catastrophic situations, in which data or equipment can be damaged (Krogh and Holloway, 1991). In this case, the supervisor implements a state feedback. That is, the control input is a function of the present state of the system and the objective is to synthesise a supervisor and a feedback policy which guarantees that the system will not enter a forbidden state. Supervisory control and forbidden state problems occur at all level of the manufacturing system control hierarchy, ranging from the low-level interaction between equipment controllers and devices through the coordination of workcells, to the factory-wide coordination of workstation controllers (Krogh and Holloway, 1991).

In this chapter four techniques are proposed for the purpose of designing compiled supervisors for the control of DESs in the case of the forbidden state problem. Automation Petri Nets (APN) are used as an underlying formalism for the design of such compiled supervisors for the control of DESs. The approach used in these techniques is based on information feedback on the occurrence of events and Petri net concepts. In particular, discrete event manufacturing systems are considered. The control synthesis procedures proposed in this thesis can be applied to high level or low level manufacturing problems. The methodologies proposed in this chapter offer the following advantages:

- The compiled supervisor and the control policy obtained are correct by construction, i.e., controlled behaviour of the system is nonblocking and does not contradict the forbidden state specifications.

- All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications.

Note that, in this thesis supervisory control means the following:

- Monitoring of the system behavior via sensory feedback.
- Control evaluation in accordance with a compiled supervisor and the corresponding supervisory control policy that maps the behaviour of the system to corresponding controls.
- Control enforcement via ladder logic diagram (LLD) implementation of the supervisory control system on a Programmable Logic Controller (PLC).

To explain the *supervisory control* and the *forbidden state problem* in a simple example, let us make up a scenario, in which there is a father and a child in a room, together with a box of matches, some food to eat, some toys to play with, a TV to watch cartoons, and finally a knife. In this case, the father represents a supervisor, the child represents the system to be controlled. The forbidden state specification is as follows: Do not let the child hurt himself or cause any damage, but at the same time let him do as many things as he wishes to. In the case, where the father is not in the room, the child can play with the matches and cause a fire, can play with the knife and hurt himself, can eat some food, can play with the toys and finally can watch TV. This represents the uncontrolled system behaviour, i.e., unsupervised system behaviour. Consider the case in which the model supervisor has an inhibitory effect over the model, then in this example, the father is expected to say to the child "don't play with the matches" and "don't play with the knife". This represents the control policy of the supervisory control in this case. This implies that the child can 'eat some food', 'play with toys' and 'watch TV'. Since the father does not forbid the things the child can do, in a logical sense, it can be said that this supervisory control is maximally permissive. This represents the controlled system behaviour, i.e., the supervised system behaviour in this case. In the unlikely case, if the father were to add 'don't play with the toys' into the control policy, the child would be left with only two things to do: 'eat some food' and 'watch TV', then this would not be a maximally permissive supervisor.

Consider the other case in which the model supervisor has an enabling effect over the model, then this represents the father saying to the child the following: "you can 'play with the toys', 'eat some food' and 'watch TV' ". This represents the control policy of the supervisory control in this case. This means that the child can do these things if he wishes to. This also implies that he can not 'play with the matches' and can not 'play with the knife'. This represents the controlled system behaviour, i.e., supervised system behaviour in this case. Again, since the father does not reduce the number of things the child can do, it can be said that this supervisory control is maximally permissive. In the unlikely case, if the father were to restrict the things the child can do by saying the following: "you can 'eat some food' and 'watch TV' ", then this would not be a maximally permissive supervisor.

If there were, say, 98 things that the child could do and 2 things that he could not do, then it would be easier to use the first case where the supervisory control policy includes only the things that the child can not do. In contrast, if there were 98 things that the child could not do and 2 things that he could do, then it would be easier to use the second case where the supervisory control policy includes only the things that the child can do.

In fact, in this example the child can do five different things all together and these are all controllable events in the supervisory control sense. Some uncontrollable events can be introduced in the system. For example, this would be the need for going to the toilet.

That is, if the child wants to go to the toilet the father would not stop him from doing so. Therefore, this could be an example for uncontrollable events within the supervisory control context.

In this chapter there are four techniques proposed, namely, the *inhibitor arc method*, the *enabling arc method*, the *intermediate place method* and the *APN-SM* (APN state machine) *method*. In the first three methods, the supervisor, which is used to control DES, consists of the *uncontrolled APN model*, which represents the uncontrolled behaviour of the DES, the *APN model supervisor*, which represents the maximally permissible system behaviour according to the forbidden state specifications, and the *control policy*, which defines a set of actions to take in order to force the uncontrolled model to behave within the maximally permissible state space. In the *inhibitor arc method*, the APN model supervisor has an inhibiting effect over the APN model and the control policy is a static table that provides a list of related controllable transitions of the APN model to be disabled for each reachable state of the maximally permissible state space. Note that these controllable transitions are related to the forbidden state specifications. The control policy is then enforced by connecting inhibitor arcs from places of the APN model supervisor to the related controllable transitions of the APN model such that the control policy is satisfied. In the *enabling arc method*, the APN model supervisor has an enabling effect over the APN model and the control policy is a static table that provides a list of related controllable transitions to be enabled for each reachable state of the maximally permissible state space. Note that these controllable transitions are related to the forbidden state specifications. The control policy is then enforced by connecting enabling arcs from places of the APN model supervisor to the related controllable transitions of the APN model such that the control policy is satisfied. In the *intermediate place method*, as an alternative to the use of inhibitor and enabling arcs in the controlled model, intermediate places are used. These places are connected to the related controllable transitions of the APN model as input places with normal arcs. Note that these controllable transitions are related to the forbidden state specifications. The control policy is a table that specifies a set of input transitions and output transitions from the APN model supervisor for the intermediate places. In contrast to the first three methods, in the *APN-SM method* the supervisor consists of only one net structure, which is referred to the APN model supervisor in the previous methods. In this case, the model supervisor is called the *incomplete supervisor* and it becomes a *complete supervisor* (or simply the supervisor) when some actions are assigned to some of its places such that the control policy is met. The supervisor in this case has the following characteristics: every transitions has only one input and one output place and in the entire net there is only one token. All places have the capacity of 1. These type of nets are called Petri net state machines (Peterson, 1981). Therefore this method is called APN state machine method (APN-SM). In these four techniques, the uncontrolled model that represents the uncontrolled behaviour of the DES is constructed by using APNs. The APN model supervisors (or the incomplete supervisor in the case of APN-SM method) are synthesised by using reachability graph analysis, which shows all the possible markings of a system. These are common steps in the synthesis procedures of these four techniques. However, the determination of the control policy and its implementation is different for each method.

## 3.2. SUPERVISORY CONTROL OF DESs

A typical supervisory control of a DES is shown in Fig. 3.1. This architecture is used in the first three methods, namely, the inhibitor arc method, the enabling arc method, the intermediate place method. Fig. 3.1 consists of four parts; i) the discrete event system (DES), to be controlled, ii) the supervisor, iii) sensor readings as outputs from the DES, and iv) control actions as inputs to the DES. The objective of the supervisor is to make sure that no forbidden state will be reached, and the controlled system operation is maximally permissive, i.e., the supervisor does not unnecessarily constrain the system operation.

The APN model supervisor is a special APN, in which every transition has only one input place and one output place, and in the entire net there is only one token. It is a safe Petri net, i.e., all places have the capacity of 1. Every place in the APN model supervisor, represents an admissible marking of the system. There are no actions assigned to places of the model supervisor. The role of the model supervisor is to represent the current state of the plant. Therefore, the transitions are assigned either controllable or uncontrollable events, but their role is only to monitor the behaviour of the plant. It, therefore, acts as a monitor showing the current state of the plant. When an event occurs in the plant, this causes the model supervisor to change its state.

From some states of the system, if not supervised, the system can get into a forbidden state through the firing of controllable transitions. To prevent this, the supervisory control policy simply defines a set of controllable transitions and corresponding markings, such that if that marking is reached then the corresponding transition is stopped from firing. This blocking process prevents the system from reaching the forbidden state, but ensures that every admissible state of the system can still be reached, i.e., the supervisor is maximally permissive.

In brief, the events occurring in the plant are realised by the APN model supervisor as a sensory feedback. Then, the model supervisor changes its state accordingly. If there are any controllable transition to be stopped from firing in the APN model, this is carried out according to the control policy. Next, the APN model fires its transitions, according to the sensory feedback and the supervision of the model supervisor through the control policy. When there is a token in the places, to which an action(s) is assigned, this is used as a control action to tell the plant what to do, i.e., to start or stop motors, machines, actuators etc.

The control policy as defined above is based on stopping the controllable transitions from firing in order to supervise the plant not to get into a forbidden state. However, sometimes the system has a very limited admissible state space compared with the whole state space. This may result in a lot of "don't"s in the control policy. Therefore instead of telling the system "*what not to do*" the control policy may as well prefer to say "*what to do*". So, the control policy in this case becomes a set of controllable transitions of the APN model to be enabled to fire at each reachable state within the maximally permissible state space, represented by the APN model supervisor. In simple terms, the control policy in this case, provides a set of "do"s for each admissible marking to make sure the legal behaviour of the system.

## 3.3. THE INHIBITOR ARC METHOD

In this method, *the supervisor* consists of a controlled model of the DES. The supervisory control policy is a static table that provides a list of controllable transitions to be disabled for each reachable state of the maximally permissible state space so that the forbidden state specifications are met. This table is then enforced by using inhibitor arcs from the corresponding places of the APN model supervisor to the controllable transitions of the APN model. This is shown in Fig. 3.2. In other words, the model supervisor has an inhibiting effect over the APN model. The *inhibitor arc method* for the supervisory control of the DESs is divided into four main steps:

Step 1 - Design the uncontrolled model of the system using APNs

Step 2 - Synthesise the APN model supervisor and determine the control policy

Step 3 - Construct the controlled model of the system

Step 4 - Implement the supervisor (the controlled model) on a programmable logic controller (PLC) as ladder logic diagrams (LLDs)

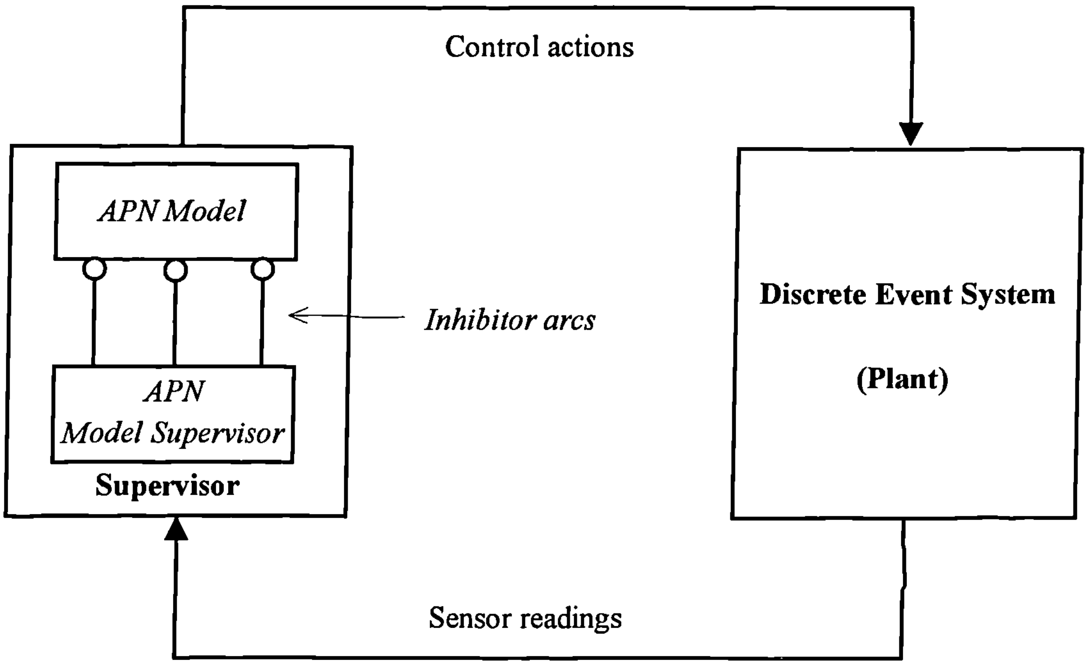


Figure 3.2. The use of inhibitor arcs in the controlled model.

### 3.3.1. Step 1 - Design the Uncontrolled Model of the System Using APNs

In this thesis, APNs are used for designing the uncontrolled models of the DESs in order to capture the uncontrolled behaviour of the system. In practical modelling, the firing of an enabled transition is generally associated with an external event, such as sensor readings. This means that a transition is fired when it is enabled and a related external event occurs. The external events are subdivided into *controllable events*, i.e., the events which may be disabled through control, and *uncontrollable events*, i.e., the events which may not be disabled through control. However, as explained in the chapter 1, ordinary PNs do not deal with actuators and sensors. Therefore, APNs are proposed to embrace both actuators and sensor readings as an extension to the PN framework. In an APN, sensor readings can be used as firing conditions at transitions. The presence or absence of sensor readings can be used in conjunction with the normal Petri net pre-conditions to enable or fire transitions. In an APN, actuators can be associated with places. With these additional features, it is possible to design uncontrolled models of the DESs.

Models of the uncontrolled behavior of the DESs can be designed efficiently by means of a modular modelling concept, which is based on a set of predefined, standard modules for typical devices of the DESs, such as actuators, drives, valves, pushers, stopper, FIFO queues, etc. The *concurrent composition* (Giua and DiCesare, 1991), can then be used to merge common transitions to form the uncontrolled model. The *place invariant technique* (Moody *et al*, 1994; Yamalidou *et al*, 1996) can also be used to enforce any physical constraints on the uncontrolled model.

The final uncontrolled APN model consist of a set of places and a set of transitions connected to each other. The number of tokens in each place represents the state of the APN. When a transition is enabled it may fire with an external event, realised by a sensor reading. When a transition fires, tokens are moved from one place to another. Actions, which are associated with places, assume to have an enabling effect on the actuators in the real system. Some transitions in the uncontrolled model are controllable by the model supervisor, i.e., they may be disabled by the model supervisor, and some transitions uncontrolled model are uncontrollable by the model supervisor, i.e., they may not be disabled by the model supervisor.

### 3.3.2. Step 2 - Synthesise the Automation Petri Net Model Supervisor and Determine the Control Policy

The objective in this step is to synthesise an APN model supervisor and a control policy, using the APN model constructed in the previous step, so that controlled behaviour of the system will be maximally permissive and will conform to the forbidden state specifications given. In this step the following sub-steps are considered:

Step 2.1. Generate the reachability graph of the APN model

Step 2.2. Identify and remove the "bad states" from the reachability graph

Step 2.3. Design the APN model supervisor and determine the control policy

### 3.3.2.1. *Step2.1. Generate the reachability graph of the APN model*

The reachability graph (RG), in which each node represents a marking reachable from the initial marking $M_0$ and each arc represents the firing of a transition, of the APN model is generated. Note that the RG represents the uncontrolled behaviour of the DES considered. In other words it represents all the possible markings, i.e., the whole state space, of the system.

### 3.3.2.2. *Step2.2. Identify and remove the "bad states" from the reachability graph*

In this step, forbidden state specifications, generally given as abstract explanations about the system considered, are represented in terms of the RG states, and defined as "bad states". After identifying these "bad states", the next step is to remove these "bad states" from the RG together with any related arcs connecting them to the rest of the RG. In some cases, the forbidden state problem can map to a "bad transition", in which case the related "bad transition(s)" is removed from the RG. Then, "unreachable states", i.e., states to which there are no arcs coming from the other states and "blocking states", i.e., states from which there is no arcs going to the other states, are also removed from the RG. This process yields the final reduced reachability graph (FRRG), which represents the maximally permissible behaviour of the system.

### 3.3.2.3. *Step2.3. Design the APN model supervisor and determine the control policy*

The APN model supervisor is designed by converting the FRRG into an APN structure such that each state of the FRRG is represented by an APN place, the arcs of the FRRG are represented by the APN transitions and the initial marking is also represented by a token in the APN place representing the initial state of the FRRG. The APN model supervisor, designed using this methodology, has the following characteristics: It is a safe PN, i.e., all places have the capacity of 1. There is only one token in the entire APN representing the current marking of the system. Since each place in the APN model supervisor represents an admissible marking within the legal behaviour, the APN model supervisor acts as a monitor showing the current state of the system.

The control policy for the system according to the forbidden state specifications is also determined by using the FRRG. This is done as follows: The "good markings" of the FRRG are considered together with their arcs. If there is an arc which can lead to a "bad marking", this means that an event may take place at "good markings" that can result in a "bad marking". Therefore, the supervisory control policy in the *inhibitor arc method* involves inhibiting the controllable events, which result in the "bad markings". As a result, a static table of the transitions (events) to be disabled for each "good marking" of the FRRG, is produced so as to avoid the "bad markings". This static table constitutes the control policy.

### 3.3.3. Step3 - Construct the controlled model of the system

In this final step, the controlled model (the supervisor) of the system is obtained. To do this, the APN model supervisor is connected to the APN model with inhibitor arcs, such that the control policy is maintained. Note that, the APN model supervisor is assumed to have an inhibitory effect over the APN model and also note that each place in the APN model supervisor represents an admissible marking of the APN model. Therefore, the control policy is simply enforced by inhibitor arcs, which are directed from the places of the APN model supervisor, representing the markings for which there are some events to stop, to the corresponding controllable transitions of the APN model, representing the events to be stopped, in order to force the system to behave within the specification. This yields the supervised model of the system, which is maximally permissive and behaves according to the specifications. It is also important to note that the behaviour of the controlled model (the supervisor) of the system is correct by construction and therefore there is no need for verification.

### 3.3.4. Step 4- Implement the supervisor on a PLC as LLDs

Note that the supervisory control can be enforced by implementing the supervisor on an industrial computer. The implementation can be done by using high-level languages, such as C, C++ or low-level languages, such as machine language, ladder logic diagrams. In order to convert the controlled model (i.e., the supervisor) into an LLD code for implementation on a PLC, Token Passing Logic Methodology (TPL), details of which can be found in (Jones *et al*, May 1996; Uzam and Jones, July 1996), can be used. In brief, to convert an APN into an LLD code, *counters* are assigned to the places, whose token capacity is bigger than or equal to 1, and *flags* are assigned to the places, whose token capacity equals to 1. The simulated movement of tokens is achieved by incrementing and decrementing the counters (or setting and resetting the flags).

### 3.3.5. Example for the Inhibitor Arc Method

#### 3.3.5.1. *Problem description*

As an example consider a manufacturing system, which consists of two machines and a buffer of size 1. Machine 1 and Machine 2 are connected by the buffer as shown in Fig. 3.3. The machines are either idle, working, or down. The buffer is either full or empty. Initially the buffer is assumed to be empty and the machines are assumed to be idle. The transfer of workpieces is assumed to be part of the machines' workcycle, during which the machines pick up workpieces upstream, and transfer workpieces downstream. The machines operation and repair must be coordinated according to the followings production and repair specifications:

1. The buffer must not overflow or underflow: Machine 1 may not start operating while a workpiece is present in the buffer.

2. Machine 2 has repair and return to service priority over Machine 1: in case both machines are down, Machine 2 must be repaired and returned to service first.

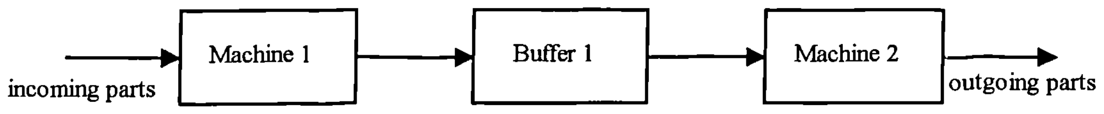


Figure 3.3. A small manufacturing system.

### 3.3.5.2. *Synthesis of Supervisory Controller*

Recall that the synthesis of supervisory controller is divided into three main steps:

#### 3.3.5.2.1. *Step1. Design the Uncontrolled Model of the System Using APNs*

As a first step in the modelling, consider the standard APN modules for the system, shown in Fig. 3.4. Firstly, Let us consider Machine 1. Initially it is idle ($M_1^i$), since place $p_1$ has a token, $M(p_1) = 1$. If the event $s_1$ occurs, then transition $t_1$ fires, resulting in the marking $M(p_2) = 1$, i.e., Machine 1 starts operating ($M_1^w$). Consequently, either it may finish its workcycle, in which case transition $t_4$ is fired by the event $f_1$ and it becomes idle, or it may break down, in which case transition $t_2$ is fired by the event $b_1$ and it becomes down ($M_1^d$), i.e., $M(p_3) = 1$. If the Machine 1 is down, it is repaired and returned to service, in which case transition $t_3$ is fired by the event $r_1$. When the Machine is returned to service it is brought to its working position. The same applies to the Machine 2, in which events $s_2$, $b_2$, $r_2$ and $f_2$ play similar roles. Now, consider the buffer 1. Initially, it is empty ($B_1^e$), i.e., $M(p_4) = 1$. When Machine 1 finishes its workcycle, realised by event $f_1$ assigned to transition $t_4$, the workpiece, that is being processed by the Machine 1 is deposited into the Buffer 1, which is realised by event $f_1$, which is assigned to transition $t_{4'}$, in which case the buffer becomes full ($B_1^f$), i.e., $M(p_5) = 1$. When the Machine 2 is idle and the Buffer 1 is full, the Machine 2 may start operating, realised by event $s_2$ assigned to transition $t_{5'}$, resulting in the buffer becoming empty.

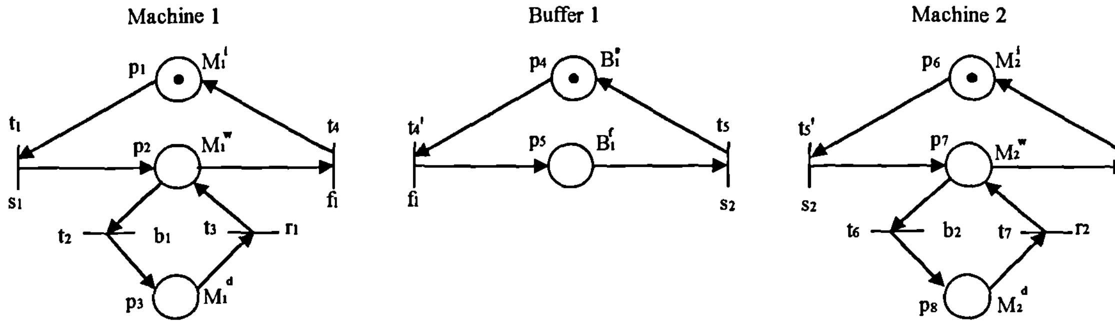


Figure 3.4. Standard APN modules for the manufacturing system.

Secondly, by using concurrent composition, i.e., by merging the transitions with the same events, the uncontrolled model of the manufacturing system is obtained as an APN, shown in Fig. 3.5, where there are eight places, namely, $P = \{ p_1, p_2, \dots , p_8 \}$ and eight transitions, namely, $T = \{ t_1, t_2, \dots , t_8 \}$. The initial marking of the APN model is $M_0 = (1,0,0,1,0,1,0,0)^T$ or simply $M_0 = (1,4,6)$, i.e., Machine 1 and Machine 2 are idle, and the buffer is empty. Detailed information about the APN model is given in Table 3.1. Places $p_1$, $p_2$ and $p_3$ represent the Machine 1, being idle ($M_1^i$), working ($M_1^w$), and down ($M_1^d$) respectively, and similarly places $p_6$, $p_7$ and $p_8$ represent the Machine 2, being idle ($M_2^i$), working ($M_2^w$), and down ($M_2^d$) respectively. Also places $p_4$ and $p_5$ denote the buffer, being empty ($B_1^e$) and being full ($B_1^f$) respectively. Events, $e = \{ s_1, b_1, r_1, f_1, s_2, b_2, r_2, f_2 \}$ are associated with transitions $T = \{ t_1, t_2, \dots , t_8 \}$ respectively. Note that the uncontrolled APN model, shown in Fig. 3.5 is safe, i.e., 1-bounded, live, reversible, and conservative.

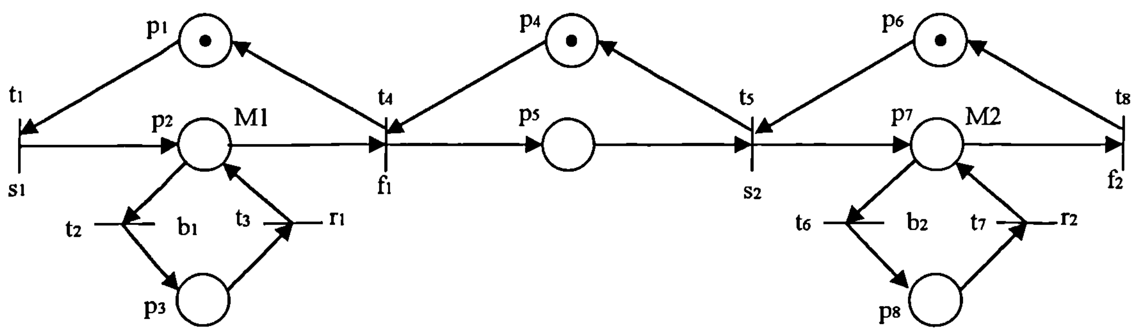


Figure 3.5. The uncontrolled model of the manufacturing system as an APN.

The transfer of workpieces is assumed to be part of the machines' workcycle, so that when transitions $t_1$ and $t_5$ are fired, the machines are assumed to have picked up a workpiece upstream, and when transitions $t_4$ and $t_8$ are fired, the machines are assumed to have transferred a workpiece downstream. The transitions $t_1$, $t_3$, $t_5$ and $t_7$ are controllable, in other words, events $s_1$, $r_1$, $s_2$, and $r_2$ are controllable: they can be disabled and enabled by control action. The transitions $t_2$, $t_4$, $t_6$ and $t_8$ are uncontrollable, in other words, events $b_1$, $f_1$, $b_2$, and $f_2$ are uncontrollable: they can not be disabled and may occur spontaneously. Places $p_2$ and $p_7$ are assigned actions M1 and M2 respectively. It is to show that Machine 1 (Machine 2) is ON when there is a token in place $p_2$ ($p_7$).

| places | | | transitions | | |
|---|---|---|---|---|---|
| $p_1$ | $M_1^i$ | Machine 1 is idle | $t_1$ | $s_1$ | Machine 1 starts operating |
| $p_2$ | $M_1^w$ | Machine 1 is working | $t_2$ | $b_1$ | Machine 1 breaks down |
| $p_3$ | $M_1^d$ | Machine 1 is down | $t_3$ | $r_1$ | repair & return to service of Machine 1 |
| $p_4$ | $B_1^e$ | Buffer 1 is empty | $t_4$ | $f_1$ | Machine 1 finishes operating |
| $p_5$ | $B_1^f$ | Buffer 1 is full | $t_5$ | $s_2$ | Machine 2 starts operating |
| $p_6$ | $M_2^i$ | Machine 2 is idle | $t_6$ | $b_2$ | Machine 2 breaks down |
| $p_7$ | $M_2^w$ | Machine 2 is working | $t_7$ | $r_2$ | repair & return to service of Machine 2 |
| $p_8$ | $M_2^d$ | Machine 2 is down | $t_8$ | $f_2$ | Machine 2 finishes operating |

Table 3.1. Places and transitions of the APN model.

### 3.3.5.2.2. *Step2. Synthesise the APN Model Supervisor and the Control Policy*

Remember that in this step there are three sub-steps:

#### 3.3.5.2.2.1. Step2.1. Generate the reachability graph of the APN model

The reachability graph (RG) of the APN model is shown in Fig. 3.6. In the RG there are eighteen nodes M = { $M_0$, $M_1$, $M_2$, ... , $M_{17}$ }, representing all the possible markings reachable from the initial marking $M_0$. Table 3.2 provides the information on the meaning of the RG nodes. Note that there are two notations used for the markings as shown in the column 1 and 2 of the Table 3.2 and the notation given in the 2nd column will be used in the text for the explanations as it is easy to follow. There exist forty-two arcs, representing the firing of a transition in the APN model. For simplicity, only events associated with the transitions are shown in the RG.

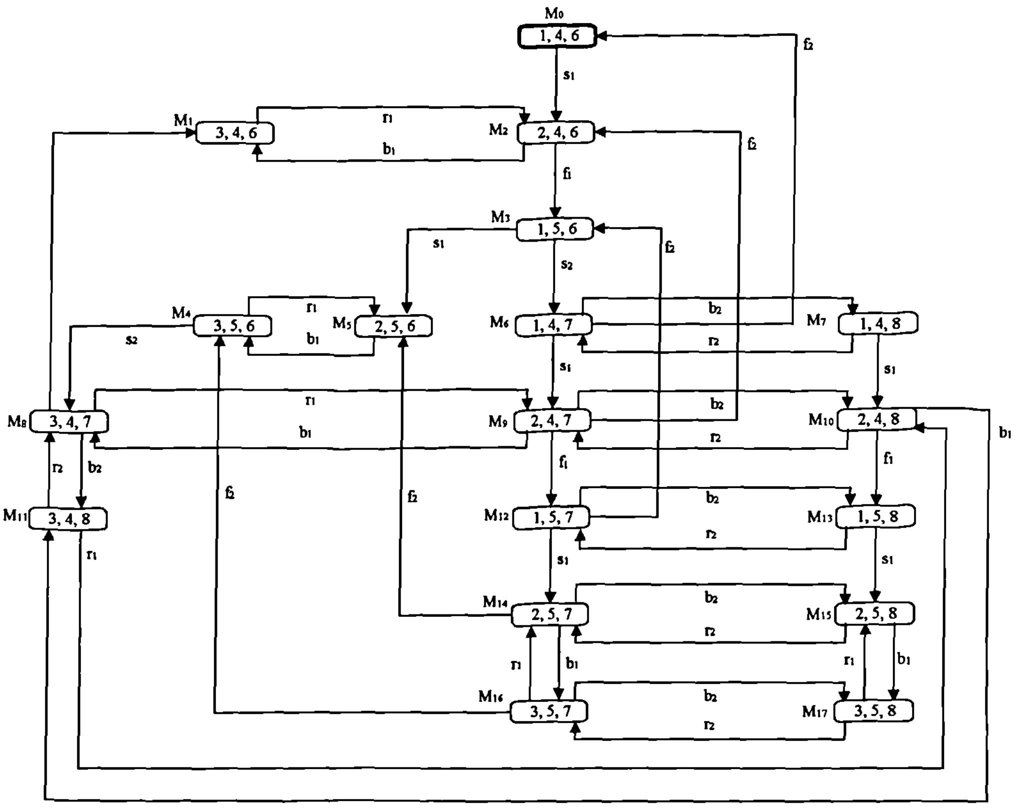


Figure 3.6. The reachability graph of the APN model of the manufacturing system.

| Marking | | Machine 1 | Buffer 1 | Machine 2 |
|---|---|---|---|---|
| $M_0 = (1,0,0,1,0,1,0,0)^T$ | $M_0 = (1, 4, 6)$ | idle | empty | idle |
| $M_1 = (0,0,1,1,0,1,0,0)^T$ | $M_1 = (3, 4, 6)$ | down | empty | idle |
| $M_2 = (0,1,0,1,0,1,0,0)^T$ | $M_2 = (2, 4, 6)$ | working | empty | idle |
| $M_3 = (1,0,0,0,1,1,0,0)^T$ | $M_3 = (1, 5, 6)$ | idle | full | idle |
| $M_4 = (0,0,1,0,1,1,0,0)^T$ | $M_4 = (3, 5, 6)$ | down | full | idle |
| $M_5 = (0,1,0,0,1,1,0,0)^T$ | $M_5 = (2, 5, 6)$ | working | full | idle |
| $M_6 = (1,0,0,1,0,0,1,0)^T$ | $M_6 = (1, 4, 7)$ | idle | empty | working |
| $M_7 = (1,0,0,1,0,0,0,1)^T$ | $M_7 = (1, 4, ,8)$ | idle | empty | down |
| $M_8 = (0,0,1,1,0,0,1,0)^T$ | $M_8 = (3, 4, 7)$ | down | empty | working |
| $M_9 = (0,1,0,1,0,0,1,0)^T$ | $M_9 = (2, 4, 7)$ | working | empty | working |
| $M_{10} = (0,1,0,1,0,0,0,1)^T$ | $M_{10} = (2, 4, 8)$ | working | empty | down |
| $M_{11} = (0,0,1,1,0,0,0,1)^T$ | $M_{11} = (3, 4, 8)$ | down | empty | down |
| $M_{12} = (1,0,0,0,1,0,1,0)^T$ | $M_{12} = (1, 5, 7)$ | idle | full | working |
| $M_{13} = (1,0,0,0,1,0,0,1)^T$ | $M_{13} = (1, 5, 8)$ | idle | full | down |
| $M_{14} = (0,1,0,0,1,0,1,0)^T$ | $M_{14} = (2, 5, 7)$ | working | full | working |
| $M_{15} = (0,1,0,0,1,0,0,1)^T$ | $M_{15} = (2, 5, 8)$ | working | full | down |
| $M_{16} = (0,0,1,0,1,0,1,0)^T$ | $M_{16} = (3, 5, 7)$ | down | full | working |
| $M_{17} = (0,0,1,0,1,0,0,1)^T$ | $M_{17} = (3, 5, 8)$ | down | full | down |

Table 3.2. The meaning of the markings, in terms of the machines and the buffer.

#### 3.3.5.2.2.2. Step2.2. Identify and remove the "bad states" from the reachability graph.

In this step, the "bad states" of the RG according to the specifications given are identified and removed from the RG. They are also called "bad markings". To achieve this the two forbidden state specifications, also known as constraints, are considered.

*Specification 1.* The first specification says that Machine 1 may not start operating while a workpiece is present in the buffer. On considering the RG, it is evident that markings $M_5 = (2, 5, 6)$, $M_{14} = (2, 5, 7)$ and $M_{15} = (2, 5, 8)$ are "bad markings", because these markings represent the Machine 1 working while there is a workpiece in the buffer. Therefore, these "bad markings" must be removed from the RG together with their arcs coming from other states to these states or going from these states to the other states in the RG. These bad markings and their arcs are shown in Fig. 3.7.

*Specification 2*. The second specification says that in case both machines are down, Machine 2 must be repaired and returned to service first. In other words, if both machines are down, then do not let the Machine 1 be repaired and returned to service first, but let the Machine 2 be repaired and returned to service. This specification does not produce any "bad marking" to remove from the RG. In this case, the marking $M_{11}$ = (3, 4, 8) is in the focus, because it represents the situation where, both machines are down. The marking $M_{11}$ can either be reached from the marking $M_8$ through the event $b_2$, $M_8[b_2>M_{11}$, or be reached from the marking $M_{10}$ through the event $b_1$, $M_{10}[b_1>M_{11}$. In other words, in the case where the Machine 1 is down, the buffer 1 is empty and the Machine 2 is working, the Machine 2 may break down and likewise in the case where the Machine 1 is working, the buffer 1 is empty and the Machine 2 is down, the Machine 1 may break down. Therefore, $M_{11}$, which is reachable from markings, $M_8$ and $M_{10}$, is the only marking representing the both machines being down. From $M_{11}$, marking $M_8$ can be reached through the event $r_2$, $M_{11}[r_2>M_8$, and marking $M_{10}$ can be reached through the event $r_1$, $M_{11}[r_1>M_{10}$. In other words, either the Machine 2 ($r_2$) or the Machine 1 ($r_1$) is repaired and returned to service. In this case, because of the specification Machine 1 must not be repaired and returned to service. Therefore, the arc pointing from $M_{11}$ to $M_{10}$, represents a "bad transition" and must be removed from the RG.

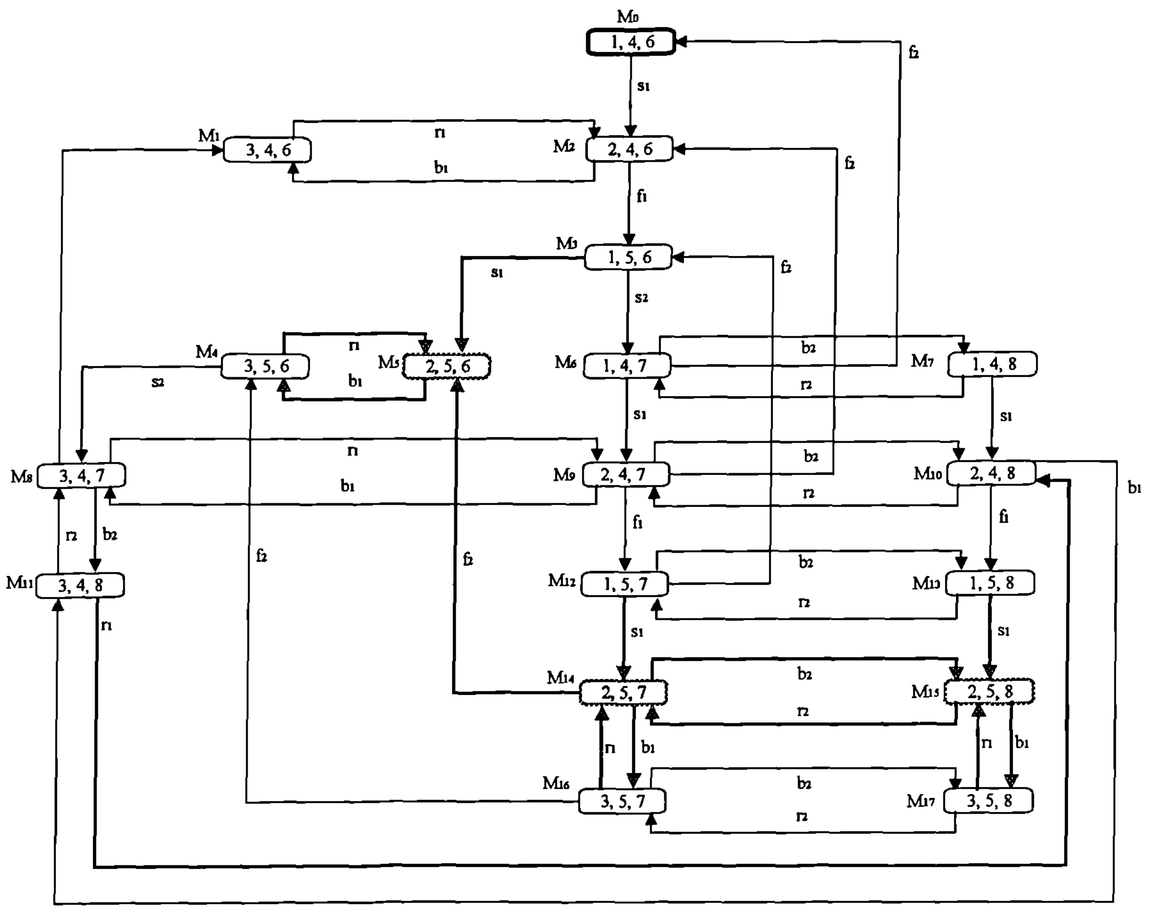


Figure 3.7. The reachability graph (RG), with the "bad markings" $M_5$, $M_{14}$ and $M_{15}$ and the "bad transition" $r_1$, in $M_{11}$ $[r_1{>}M_{10}$.

After removing the "bad markings" $M_5 = (2, 5, 6)$, $M_{14} = (2, 5, 7)$ and $M_{15} = (2, 5, 8)$ with their related arcs and the "bad transition" $r_1$, $M_{11}[r_1{>}M_{10}$ from the RG, the 1st reduced reachability graph (RRG) is obtained, as shown in Fig. 3.8.

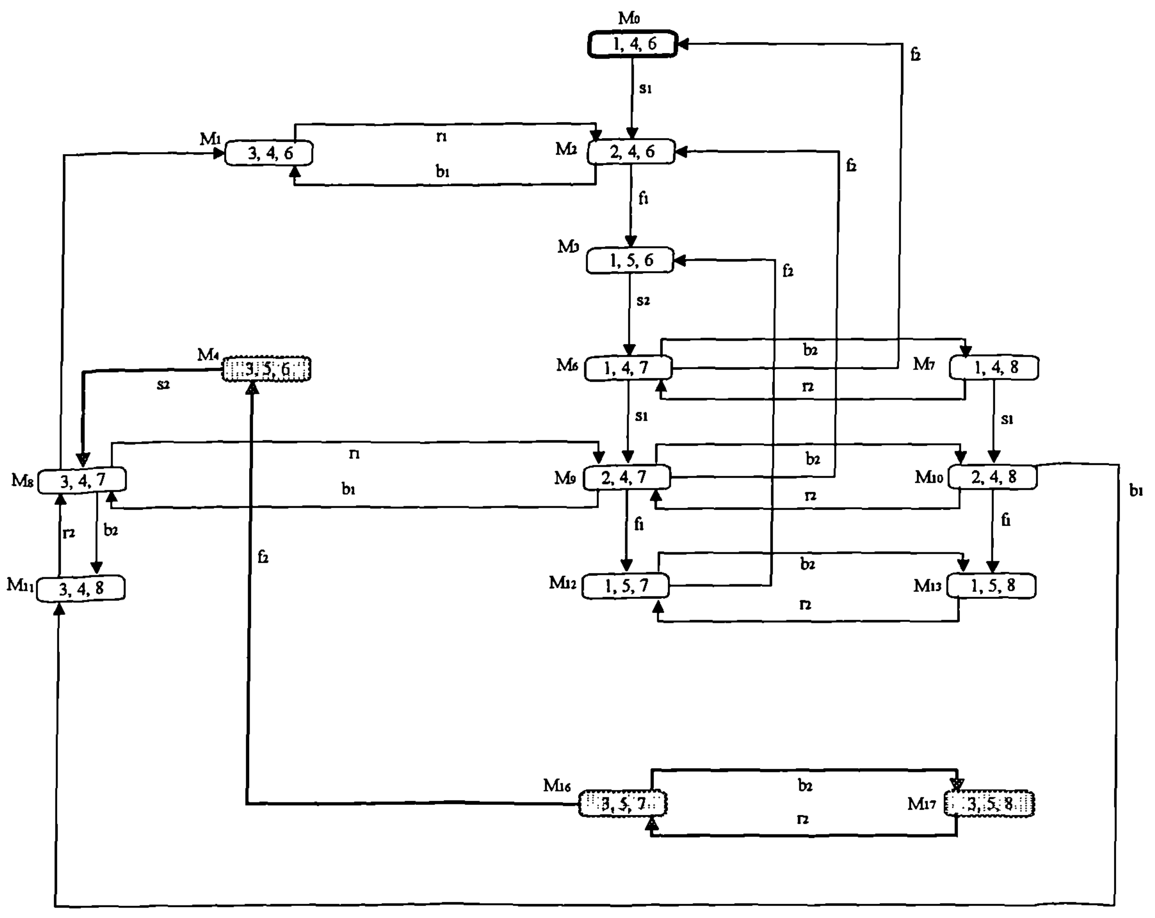


Figure 3.8. The 1st reduced reachability graph (RRG), after removing the "bad markings" $M_5$, $M_{14}$ and $M_{15}$ and the "bad transition" $r_1$, in $M_{11}$ $[r_1>M_{10}$ from the RG.

*"unreachable" and "blocking" states*. Next, it is necessary to consider the "unreachable states" and "blocking states" that have emerged after removing "bad markings" $M_5$, $M_{14}$ and $M_{15}$ and the "bad transition" $r_1$, in $M_{11}$ $[r_1>M_{10}$. As can be seen from the 1st RRG, there is no "blocking states". However, the markings $M_4 = (3, 5, 6)$, $M_{16} =(3, 5, 7)$ and $M_{17} = (3, 5, 8)$, shown in Fig. 3.8, are "unreachable markings", because there aren't any arcs coming from the other states and pointing to them. Thus, they must be removed from the RG. After removing the unreachable markings $M_4$, $M_{16}$ and $M_{17}$ with their related arcs from the 1st RRG, the final reduced reachability graph (FRRG) is obtained, as shown in Fig. 3.9.

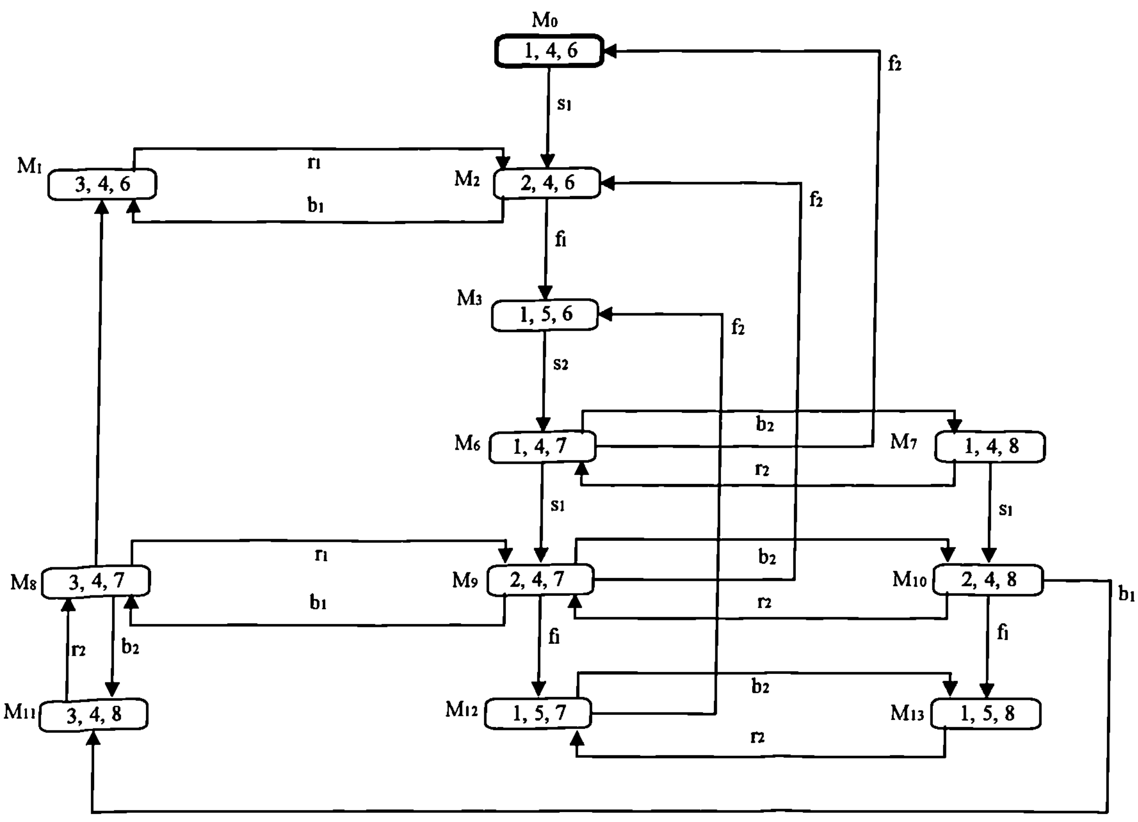


Figure 3.9. The final reduced reachability graph (FRRG), after removing the "unreachable markings" $M_4$, $M_{16}$ and $M_{17}$.

**3.3.5.2.2.3.** Step2.3. Design the APN model supervisor and determine the control policy

The FRRG represents the maximally permissible system behaviour and therefore the objective of the APN model supervisor, to be designed, is to make sure that the DES behaves within this legal behaviour and does not behave in any undesirable way. In order to achieve this objective in this step the APN model supervisor is designed and the control policy related to the APN model supervisor is determined by using the FRRG. Firstly, the APN model supervisor is designed. To do this, the FRRG is converted into a related APN such that every state (or marking) of the FRRG is represented by an APN place and the arcs of the FRRG are represented by the APN transitions. Note that in this special APN, there is no actions assigned to the places, because the APN model supervisor designed in this way behaves as a monitor that represents the current state of the system. The initial marking is also represented by a token in the APN place representing the initial state. When this technique is applied to the manufacturing example, the FRRG, shown in Fig. 3.9, is converted into the APN model supervisor as shown in Fig. 3.10. The APN model supervisor has twelve places $P = \{ p_9, p_{10}, p_{11}, \dots , p_{20} \}$ and twenty-four transitions $T = \{ t_9, t_{10}, t_{11}, \dots , t_{32} \}$. The initial marking for the APN model supervisor is $M_0 = (1,0,0,0,0,0,0,0,0,0,0,0)^T$, i.e., $M_0 = (9)$. Note that each place within the APN model supervisor represents an admissible marking of the APN model of the manufacturing problem, i.e., places $p_9, p_{10}, \dots , p_{20}$ represent the markings $M_0 = (1, 4, 6)$, $M_1 = (3, 4, 6)$, $M_2 = (2, 4, 6)$, $M_3 = (1, 5, 6)$, $M_6 = (1, 4, 7)$, $M_7 = (1, 4, 8)$, $M_8 = (3, 4, 7)$, $M_9 = (2, 4, 7)$, $M_{10} = (2, 4, 8)$, $M_{11} = (3, 4, 8)$, $M_{12} = (1, 5, 7)$ and $M_{13} = (1, 5, 8)$ of the FRRG respectively. It is also necessary to determine the control policy, with which the controlled model of the system can be obtained.

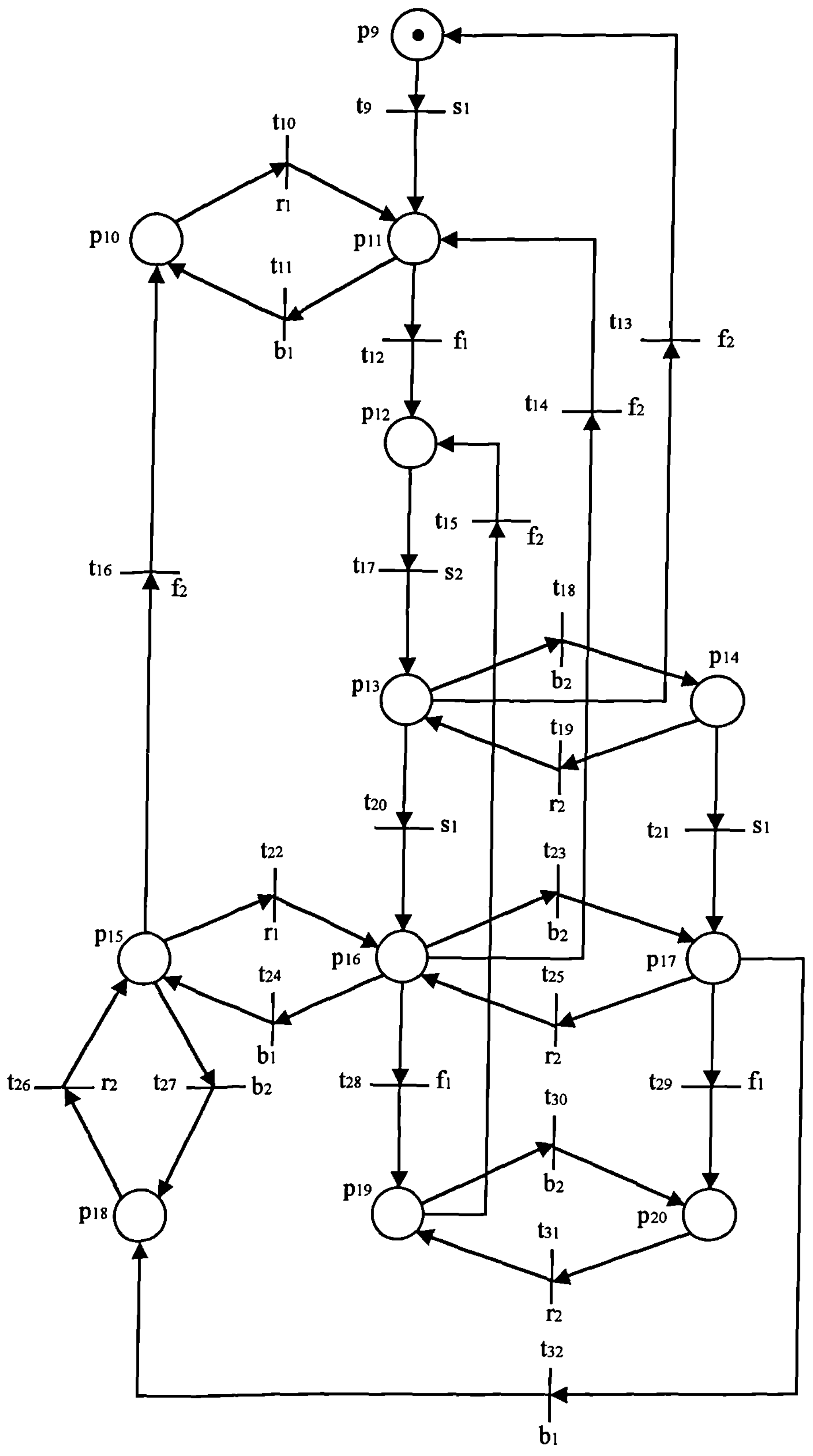


Figure 3.10. The APN model supervisor of the manufacturing system.

Secondly, it is necessary to determine the control policy for the manufacturing system by using the FRRG. In order to determine the control policy, which consists of the set of control actions to be taken for each admissible marking, the FRRG together with its arcs, which are leading to "bad markings", is considered. When the FRRG, shown in Fig. 3.9, and the RG given in Fig. 3.6, are taken into account, it is obvious that from the admissible markings $M_3$, $M_{12}$ and $M_{13}$, the "bad markings" $M_5$, $M_{14}$, and $M_{15}$ are reachable as follows $M_3[s_1>M_5$, $M_{12}[s_1>M_{14}$, and $M_{13}[s_1>M_{15}$. Also the marking $M_{10}$ is reachable from $M_{11}$ through the event $r_1$. This is shown in Fig. 3.11. In order to make the system behave within the admissible state space and not to get into "bad markings" each event leading from a "good state" to a "bad state" must be blocked. This represents the control policy used. For example, the "bad marking" $M_5$ is reachable from the "good marking" $M_3$ through the event $s_1$, i.e., $M_3[s_1>M_5$, therefore the blocking action of the APN model supervisor, when reaching the "good marking" $M_3$, must be "stop $s_1$" so that "bad marking" $M_5$ will not be reached. The "bad marking" $M_{14}$ is reachable from the "good marking" $M_{12}$ through the event $s_1$, i.e., $M_{12}[s_1>M_{14}$, therefore the blocking action of the APN model supervisor, when reaching the "good marking" $M_{12}$, must be "stop $s_1$" so that "bad marking" $M_{14}$ will not be reached. The "bad marking" $M_{15}$ is reachable from the "good marking" $M_{13}$ through the event $s_1$, i.e., $M_{13}[s_1>M_{15}$, therefore the blocking action of the APN model supervisor, when reaching to the "good marking" $M_{13}$, must be "stop $s_1$" so that "bad marking" $M_{15}$ will not be reached. Finally, the "bad transition" $r_1$ can occur at the marking $M_{11}$ which results in the marking $M_{10}$, i.e., $M_{11}[r_1>M_{10}$, therefore control action for the APN model supervisor when reaching to the "good marking" $M_{11}$ must be "stop $r_1$" so that "bad transition" $r_1$ will not take place. The final control policy for the *inhibitor arc method* is shown in Table 3.3.

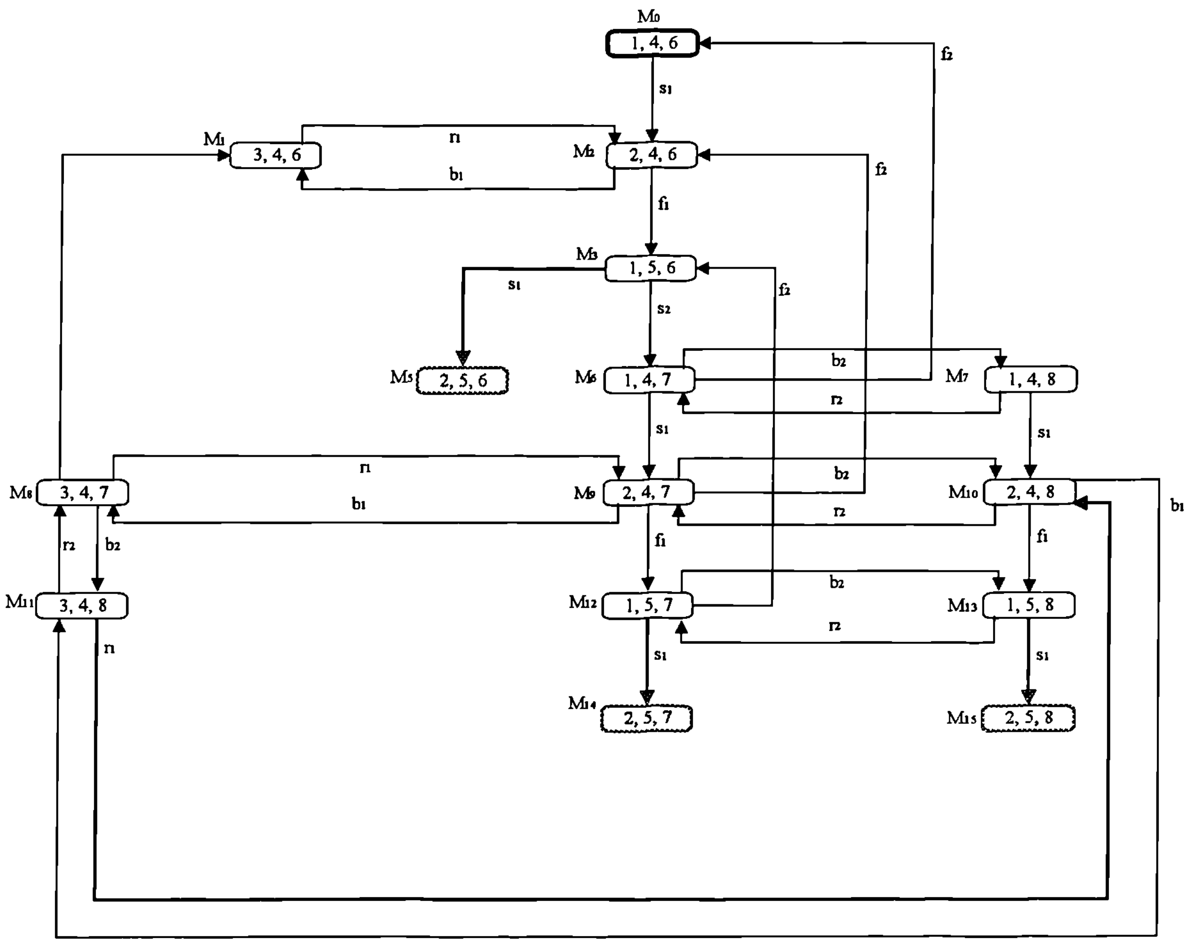


Figure 3.11. The final reduced reachability graph (FRRG), and "bad markings" reachable from it.

| Marking | Places of the model supervisor | Control action |
|---|---|---|
| $M_0 = (1, 4, 6)$ | $p_9$ | - ( = Don't care ) |
| $M_1 = (3, 4, 6)$ | $p_{10}$ | - |
| $M_2 = (2, 4, 6)$ | $p_{11}$ | - |
| $M_3 = (1, 5, 6)$ | $p_{12}$ | Stop $s_1$ |
| $M_6 = (1, 4, 7)$ | $p_{13}$ | - |
| $M_7 = (1, 4, ,8)$ | $p_{14}$ | - |
| $M_8 = (3, 4, 7)$ | $p_{15}$ | - |
| $M_9 = (2, 4, 7)$ | $p_{16}$ | - |
| $M_{10} = (2, 4, 8)$ | $p_{17}$ | - |
| $M_{11} = (3, 4, 8)$ | $p_{18}$ | Stop $r_1$ |
| $M_{12} = (1, 5, 7)$ | $p_{19}$ | Stop $s_1$ |
| $M_{13} = (1, 5, 8)$ | $p_{20}$ | Stop $s_1$ |

Table 3.3. The control policy for the manufacturing system in the *inhibitor arc method.*

**3.3.5.2.3.** Step3. *Construct the controlled model of the system.*

After designing the APN model supervisor and determining the control policy, it is possible to easily establish the supervisory control of the manufacturing system. This is done as follows. In brief, there are three things to consider, namely the APN model, i.e., the uncontrolled model of the system, the APN model supervisor and the control policy. The APN model represents the uncontrolled behaviour of the system, and the APN model supervisor represents the maximally permissible supervised behaviour. The control policy represents transitions to be blocked in the APN model, when the APN model supervisor is in certain blocking states. Then, to implement the control policy, inhibitor arcs are taken from the appropriate blocking states of the APN model supervisor to the APN model. Through the blocking of controllable events, the "bad markings", which are not allowed by the forbidden state specifications, are never reached by the machines and the buffer. In the light of this, *the supervisor*, which consists of the APN model, the APN model supervisor and the control policy, can be obtained as shown in Fig. 3.12. To explain how the control policy is enforced, consider the situation where the Machine 1 is idle, the buffer 1 is full and the Machine 2 is idle, which corresponds to the markings $M_3$ = (1, 5, 6) of the APN model. In this case the marking of the APN model supervisor is M = $(0,0,0,1,0,0,0,0,0,0,0,0)^T$ , i.e., M = (12) and the control action is "stop $s_1$". Therefore, by using an inhibitor arc, $In(p_{12}, t_1)$, connected from place $p_{12}$ to transition $t_1$, this blocking action is enforced. In the second situation, both machines are down, and the buffer 1 is empty, which corresponds to the marking $M_{11}$ = (3, 4, 8) of the APN model. In this case the marking of the APN model supervisor is M = $(0,0,0,0,0,0,0,0,0,1,0,0)^T$, i.e., M = (18) and the control action is "stop $r_1$". Therefore, by using an inhibitor arc, $In(p_{18}, t_3)$, connected from place $p_{18}$ to transition $t_3$, this blocking action is enforced. In the third situation, the Machine 1 is idle, the buffer 1 is full and the Machine 2 is working, which corresponds to the markings $M_{12}$ = (1, 5, 7) of the APN model. In this case the marking of the APN model supervisor is M = $(0,0,0,0,0,0,0,0,0,0,1,0)^T$, i.e., M = (19) and the control action is "stop $s_1$". Therefore, by using an inhibitor arc, $In(p_{19}, t_1)$, connected from place $p_{19}$ to transition $t_1$, this blocking action is enforced. In the fourth and final situation, the Machine 1 is idle, the buffer 1 is full and the Machine 2 is down, which corresponds to the markings $M_{13}$ = (1, 5, 8) of the APN model. In this case the marking of the APN model supervisor is M = $(0,0,0,0,0,0,0,0,0,0,0,1)^T$, i.e., M = (20) and the control action is "stop $s_1$". Therefore, by using an inhibitor arc, $In(p_{20}, t_1)$, connected from place $p_{20}$ to transition $t_1$, this blocking action is enforced. In the other markings of the model there is no action to be enforced.

Note that the APN model and the APN model supervisor have the same set of events associated with their transitions. This means that, the APN model supervisor is synchronised with the APN model. Therefore, they both run concurrently, i.e., in the controlled model when two transitions with the same event, one from the model and the other from the model supervisor, are enabled they are assumed to fire concurrently.

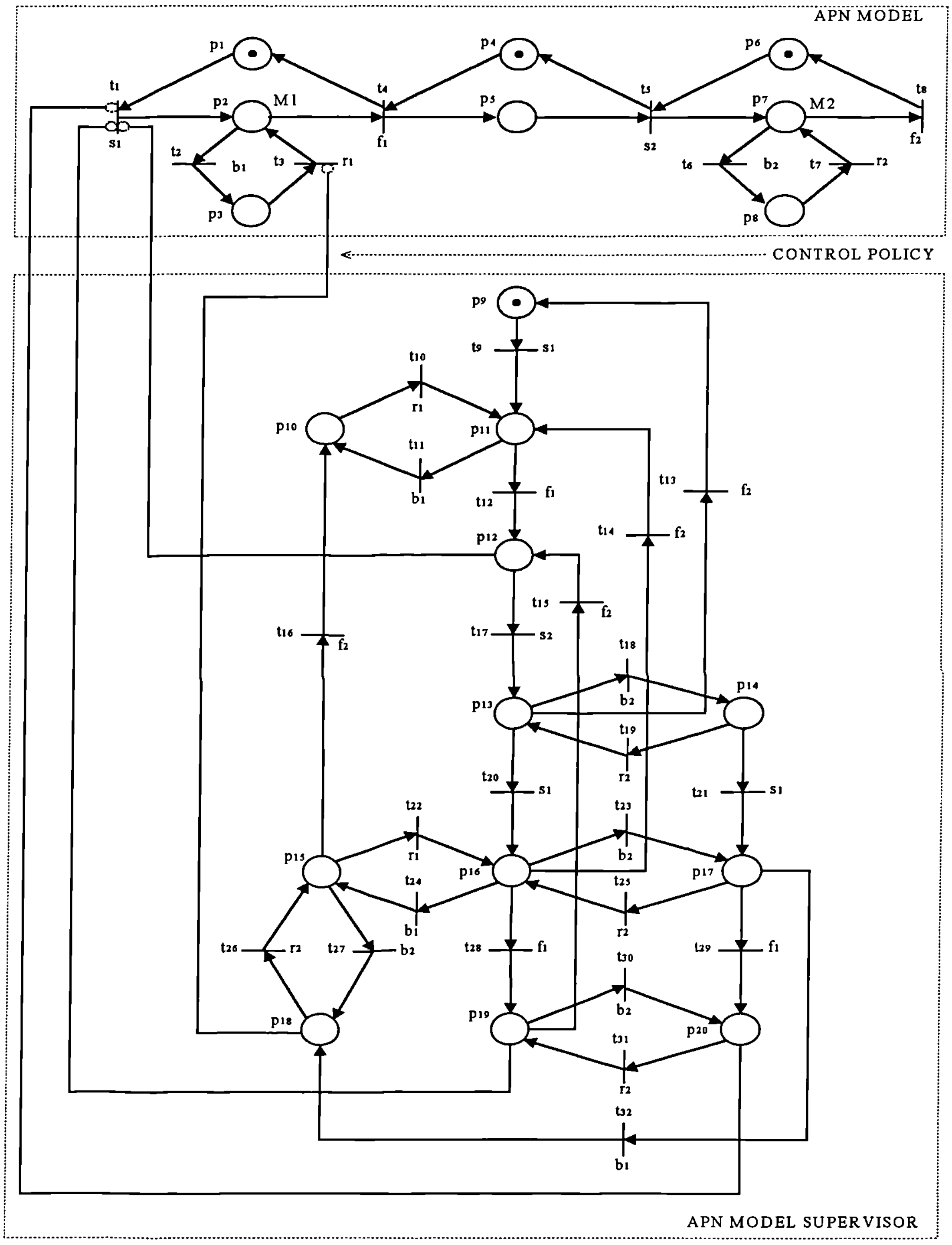


Figure 3.12. The supervisor for the manufacturing system in the *inhibitor arc method.*

## 3.4. THE ENABLING ARC METHOD

In this method, *the supervisor* consists of a controlled model of the DES as shown in Fig. 3.13. However, in this case the controlled model is obtained by connecting the APN model supervisor to the APN model through enabling arcs such that the control policy is met. This means that the model supervisor has an enabling effect over the model. In order to obtain the control policy, first of all it is necessary to determine the controllable transitions of the uncontrolled APN model that are related to the forbidden state specifications. The supervisory control policy is a static table that provides a list of places of the model supervisor from which the related controllable transitions of the APN model are to be enabled such that in the controlled model the forbidden state specifications are met. This table is enforced by enabling arcs. The *enabling arc method* for the supervisory control of the DESs is divided into the following steps:

Step 1 - Design the uncontrolled model of the system using APNs

Step 2 - Synthesise the APN model supervisor and determine the control policy

- Step 2.1. Generate the reachability graph of the APN model
- Step 2.2. Identify and remove the "bad states" from the reachability graph
- Step 2.3. Design the APN model supervisor and determine the control policy

Step 3 - Construct the controlled model of the system

Step 4 - Implement the supervisor (the controlled model) on a programmable logic controller (PLC) as ladder logic diagrams (LLDs)

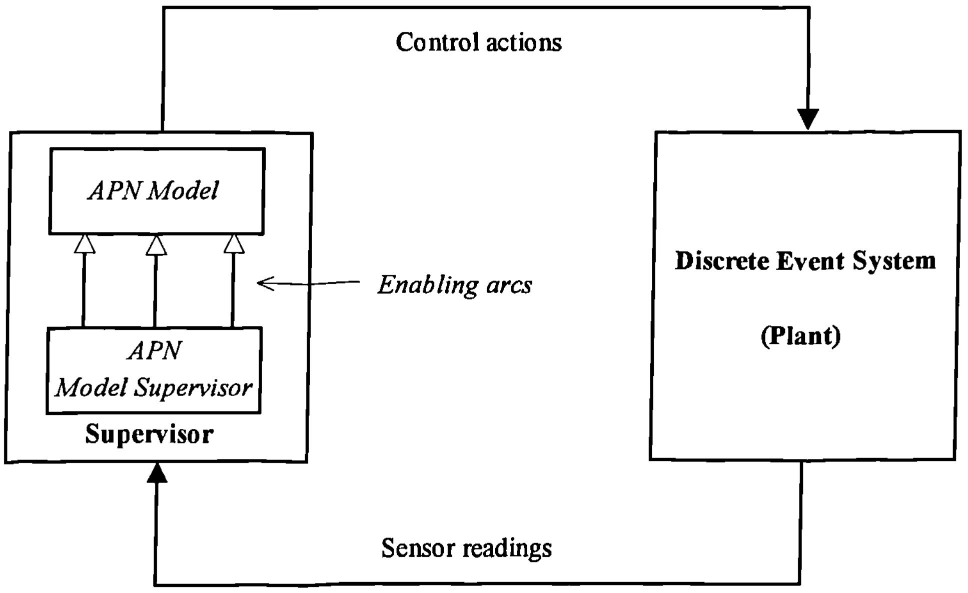


Figure 3.13. The use of enabling arcs in the controlled model.

The *enabling arc method* have common steps up to the step 2.3 with the *inhibitor arc method*, therefore in this section the step 2.3 and the step 3 will be considered. Note that the implementation of the supervisor in this method is carried out by using the token passing logic (TPL) methodology as described in the inhibitor arc method. In the step 2.3 the design of APN model supervisor is carried out again the same way as in the *inhibitor arc method*. In the *enabling arc method*, the control policy is determined in a different manner. In order to determine the control policy, firstly, it is necessary to determine the controllable transitions of the APN model that are related to the forbidden state specifications. Then, each related controllable transition within the APN model is taken into account and its' associated controllable events are identified from the APN model supervisor. In one column of a table, the list of the related controllable transitions is provided. In the next column, the model supervisor places, that are to be used to enable these transitions, are provided. This represents the control policy of the *enabling arc method*.

In this method in order to obtain the controlled model of the system, APN model supervisor is connected to the APN model with enabling arcs such that the control policy is satisfied. In this case, the APN model supervisor has an enabling effect over the APN model. As explained, the control policy shows from which places of the model supervisor which related controllable transitions of the APN model must be enabled in order to make sure the correct system behaviour. This yields the controlled model (i.e., the supervisor) of the system, that is maximally permissive and behaves according to the specifications.

When connecting the APN model supervisor to the APN model with enabling arcs, if there is more than one place to enable a related controllable transition, then the related controllable transition is duplicated as many as the number of these places. This is done simply to accommodate the *or* operation within the Petri net formalism.

### 3.4.1. Example for the Enabling Arc Method

To compare the four methods proposed in this chapter, the same manufacturing example is considered for each method. By doing this in this section only the control policy is defined and the controlled model is obtained for the manufacturing example. Note that the APN model of the manufacturing system is shown in Fig. 3.5 and the APN model supervisor is also shown in Fig. 3.10 for the forbidden state specifications given in section 3.3.5.1. These results are obtained by following the design steps given in the section 3.4.

Now it is necessary to determine the control policy for the *enabling arc method.* To do this, firstly the controllable transitions of the uncontrolled APN model that are related to the forbidden state specifications are determined. Recall that the forbidden state specifications are as follows:

1. Machine 1 may not start operating while a workpiece is present in the buffer.
2. In case both machines are down, Machine 2 must be repaired and returned to service first.

As can be seen from Fig. 3.5, when there is a token in place $p_2$, Machine 1 is in operation. The controllable transition $t_1$ is responsible for depositing a token into place $p_2$. Therefore, for the first forbidden state specification, transition $t_1$ is identified as the controllable transition that is related to this forbidden state specification. Similarly, consider the second forbidden state specification, which also means that when Machine 1 is down, it can only be repaired and returned to service if Machine 2 is not down. It is obvious from Fig. 3.5 that the controllable transition $t_3$ is responsible for repairing and returning Machine 1 to service. Therefore, for the second forbidden state specification, transition $t_3$ is identified as the controllable transition that is related to this forbidden state specification. As a result the controllable transitions $t_1$ with the event $s_1$ and $t_3$ with the event $r_1$ are related to the forbidden state specifications. In other words, the objective of the control policy is to decide when to let the controllable transitions $t_1$ and $t_3$ fire such that the forbidden state specifications are met.

Secondly, consider the APN model supervisor as shown in Fig. 3.14. Note that, the event $s_1$ means the 'machine 1 starts operating'. The event $s_1$ is associated with the transitions $t_9$, $t_{20}$, and $t_{21}$ in the APN model supervisor. These transitions are called the *identical transitions* of the related controllable transition $t_1$. The input places of the identical transitions are places $p_9$, $p_{13}$, and $p_{14}$ respectively. These places of the APN model supervisor are called the *base places* of the transition $t_1$. Therefore in the control policy, base places $p_9$, $p_{13}$, and $p_{14}$ are identified as places from which the related controllable transition $t_1$ is to be enabled by enabling arcs. This is the control policy for the transition $t_1$. Similarly, the related controllable transition $t_3$ with event $r_1$ has the identical transitions $t_{10}$ and $t_{22}$ and therefore it has the base places $p_{10}$ and $p_{15}$ from the model supervisor. Thus, in the control policy, base places $p_{10}$ and $p_{15}$ are identified as places from which the related controllable transition $t_3$ is to be enabled by enabling arcs. This is the control policy for the transition $t_3$. The resulting control policy for the manufacturing system in the *enabling arc method* is given in Table 3.4

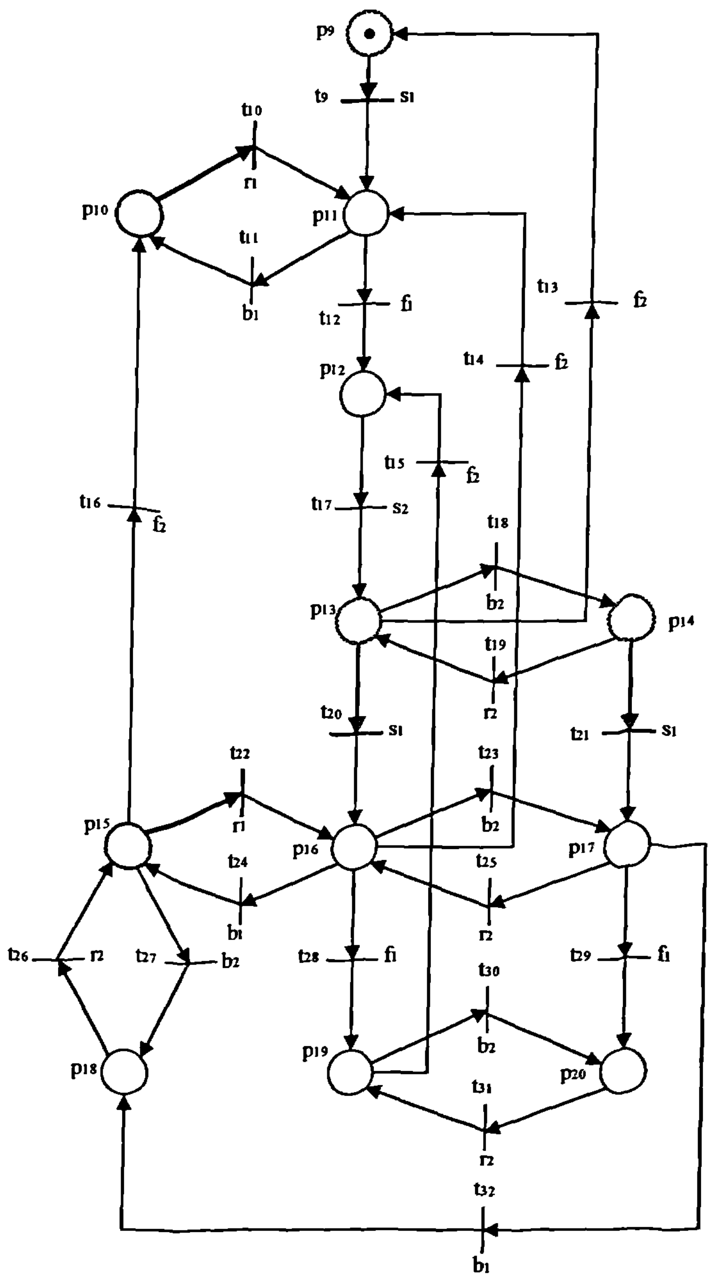


Figure 3.14. The APN model supervisor of the manufacturing system, used in determining the control policy in the *enabling arc method.*

| Related controllable transitions of the APN model | Places from which an enabling arc is to be connected |
|---|---|
| $t_1$ | $p_9$ *or* $p_{13}$ *or* $p_{14}$ |
| $t_3$ | $p_{10}$ *or* $p_{15}$ |

Table 3.4. The control policy for the manufacturing system in the *enabling arc method.*

After designing the APN model supervisor and determining the control policy, the controlled model of the system can be obtained. To do this the APN model supervisor is connected to the APN model by enabling arcs such that the control policy is satisfied. In a way, the model supervisor guides the model by enabling the related controllable transitions at certain markings. Through enabling only the related controllable transitions, the controlled model allows only the "good states" to take place, i.e., the controlled model does not allow the "bad states" to take place. In fact, the *enabling arc method* represents the complement of the *inhibitor arc method*, where the role of the model supervisor is to dictate the model *what not to do*. In contrast, in the *enabling arc method* the model supervisor dictates the model *what to do*.

The controlled model (i.e., the supervisor), shown in Fig. 3.15, for the *enabling arc method* is obtained by using the APN model, shown in Fig. 3.5, the APN model supervisor, shown in Fig. 3.10, and the control policy given in Table 3.4. Note that since the related controllable transition $t_1$ is to be enabled by places $p_9$ or $p_{13}$ or $p_{14}$, in the controlled model it is replaced by three identical transitions namely $t_1^1$, $t_1^2$, $t_1^3$ and to implement the control policy enabling arcs En($p_9$, $t_1^3$), En($p_{13}$, $t_1^2$) and En($p_{14}$, $t_1^1$) are connected from places $p_9$, $p_{13}$ and $p_{14}$ to transitions $t_1^3$, $t_1^2$ and $t_1^1$, respectively. The same applies to related controllable transition $t_3$ where it is replaced by transitions $t_3^1$, $t_3^2$. To implement the control policy enabling arcs En($p_{10}$, $t_3^1$) and En($p_{15}$, $t_3^2$) are connected from places $p_{10}$ and $p_{15}$ to transitions $t_3^1$ and $t_3^2$, respectively.

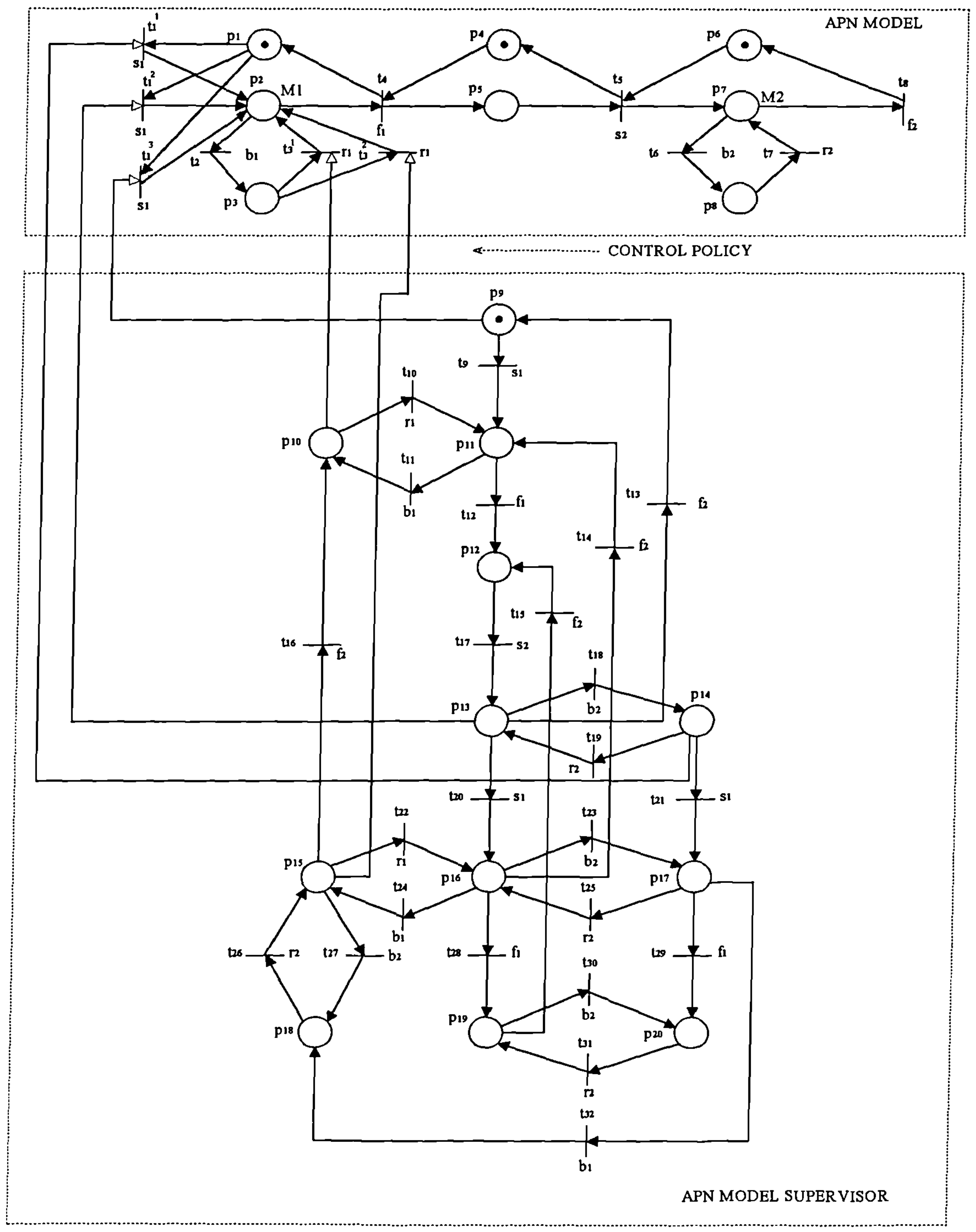


Figure 3.15. The supervisor for the manufacturing system in the *enabling arc method.*

## 3.5. THE INTERMEDIATE PLACE METHOD

In this method, in order to obtain the controlled model of the system, firstly the controllable transitions of the uncontrolled APN model that are related to the forbidden state specifications are determined. Then, a set of places called *intermediate places* are connected to these related controllable transitions of the APN model through ordinary arcs. The role of the control policy, in this case, is to provide a set of input and output transitions for the intermediate places from the APN model supervisor. Fig. 3.16 shows how the intermediate places are used to obtain the controlled model. The *enabling arc method* for the supervisory control of the DESs is divided into the following steps:

Step 1 - Design the uncontrolled model of the system using APNs

Step 2 - Synthesise the APN model supervisor and determine the control policy

- Step 2.1. Generate the reachability graph of the APN model
- Step 2.2. Identify and remove the "bad states" from the reachability graph
- Step 2.3. Design the APN model supervisor and determine the control policy

Step 3 - Construct the controlled model of the system

Step 4 - Implement the supervisor (the controlled model) on a programmable logic controller (PLC) as ladder logic diagrams (LLDs)

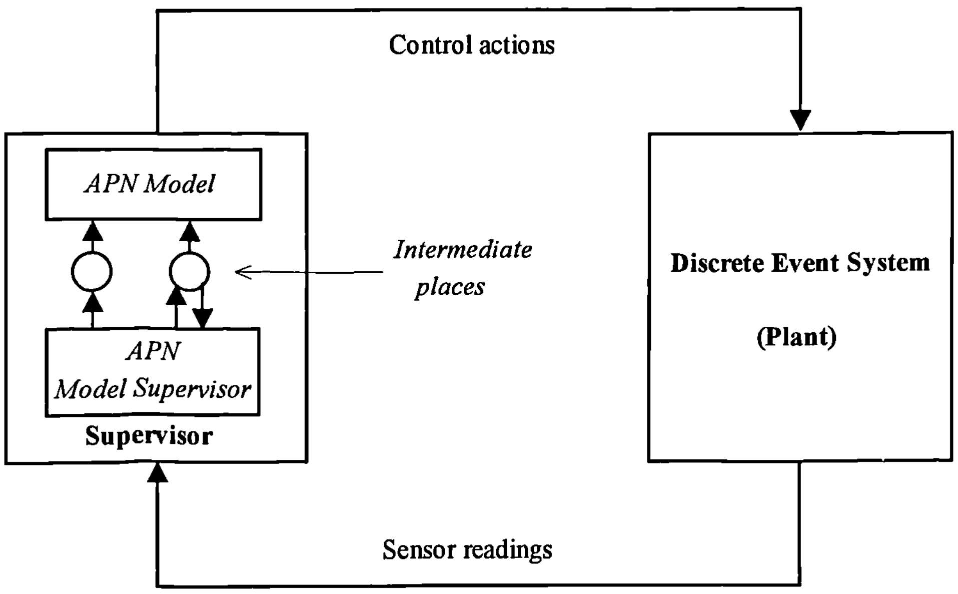


Figure 3.16. The use of intermediate places in the controlled model.

The *intermediate place method* have common steps up to the step 2.3 with the *inhibitor arc method*, therefore in this section the step 2.3 and the step 3 will be considered. Note that the implementation of the supervisor in this method is carried out by using the token passing logic (TPL) methodology as described in the inhibitor arc method. In the step 2.3 the design of the APN model supervisor is carried out again the same way as in the *inhibitor arc method*. In the *intermediate place method*, in order to determine the control policy firstly, the controllable transitions of the uncontrolled APN model that are related to the forbidden state specifications are determined and an intermediate place is connected to each related controllable transition of the APN model by an ordinary arc. Then, the control policy defines when to put and remove a token into and from these intermediate places. By depositing and removing a token to and from the intermediate places, the model supervisor guides the model so that it behaves within the given set of forbidden state specifications. The control policy simply defines for each intermediate place a set of input transitions from the model supervisor that deposit a token into the intermediate place and likewise a set of output transitions, if any, from the model supervisor, that remove a token from the intermediate place. To find which model supervisor transitions are the input/output transitions of an intermediate place, firstly the related controllable event associated with the output transition of the intermediate place is defined. Secondly, the identical controllable events and of course their related transitions, called *identical transitions*, are identified from the model supervisor. Then, the input places, called *base place*s, of the identical transitions are identified. The control policy for depositing a token into an intermediate place is then obtained such that the input transitions of the base places become the input transitions of the intermediate place and likewise the output transitions of the base places become the output transitions of the intermediate place. However, if there is more than one base place for an intermediate place and if these base places have a common transition, then this common transition between the base places is not taken into account when defining the control policy. This means that the token movements between the base places does not affect the movement of the token in the related intermediate place. The identical controllable transitions with the same events within the model supervisor are not also considered when determining the control policy, because the intermediate place will consume its token through the related controllable transitions within the APN model. It should be noted that if a base place initially has a token within the model supervisor then a token must be put into the corresponding intermediate place.

In the *intermediate place method*, in order to obtain the controlled model, the APN model, together with the intermediate places, connected to the related controllable transitions via ordinary arcs, is connected to the APN model supervisor such that the control policy is satisfied. Note that the control policy is a set of input and output arcs to be connected from the model supervisor for depositing and removing a token to and from the intermediate places. This yields the controlled model of the system that is maximally permissive and behaves within the forbidden state specifications.

### 3.5.1. Example for the Intermediate Place Method

Consider the manufacturing system introduced in the section 3.3.5. Note that the APN model of the manufacturing system is shown in Fig. 3.5 and the APN model supervisor is also shown in Fig. 3.10 for the forbidden state specifications, given in the section 3.3.5.1. These results are obtained by following the design steps given in the section 3.5.

Now it is necessary to determine the control policy for the *intermediate place method.* To do this, firstly the controllable transitions that are related to the forbidden state specifications are determined. As explained in the enabling arc method, the controllable transitions $t_1$ with the event $s_1$ and $t_3$ with the event $r_1$ are related to the forbidden state specifications. In other words the objective of the control policy is to decide when to let transitions $t_1$ and $t_3$ fire such that the forbidden state specifications are met. Then one intermediate place is connected to the related controllable transitions with ordinary arcs. Therefore, intermediate places $p_{21}$ and $p_{22}$ are connected to the related controllable transitions $t_1$ and $t_3$, respectively, by ordinary arcs $Pre(p_{21}, t_1)$ and $Pre(p_{22}, t_3)$. This is shown in Fig. 3.17. Secondly, the base places for these related controllable transitions are identified from the APN model supervisor. To do this, the identical controllable transitions with the same controllable events are identified. The controllable transition $t_1$ with the event $s_1$ has identical transitions $t_9$, $t_{20}$, and $t_{21}$ with the same event in the APN model supervisor, as shown in Fig. 3.18. Therefore, the input places $p_9$, $p_{13}$, and $p_{14}$ of these transitions are the base places for transition $t_1$. In the control policy, the input transitions of places $p_9$, $p_{13}$, and $p_{14}$ are identified as the input transitions of the intermediate place $p_{21}$ and likewise the output transitions of places $p_9$, $p_{13}$, and $p_{14}$ are identified as the output transitions of the intermediate place $p_{21}$. When doing this, the identical transitions $t_9$, $t_{20}$, and $t_{21}$ and also the transitions $t_{13}$, $t_{18}$ and $t_{19}$, that connect one base place to another, are not included. This is the control policy for the transition $t_1$. The controllable transition $t_3$ with the event $r_1$ has identical transitions $t_{10}$ and $t_{22}$. Therefore, the input places $p_{10}$ and $p_{15}$ of these transitions are the base places for transition $t_3$. In the control policy, the input transitions of these places are identified as the input transitions of the intermediate place $p_{22}$ and likewise the output transitions of these places are identified as the output transitions of the intermediate place $p_{22}$. When doing this, the identical transition $t_{10}$ and $t_{22}$ and also the transition $t_{16}$ that connect one base place to another, are not included. The resulting control policy for the manufacturing system in the *intermediate place method* is given in Table 3.5

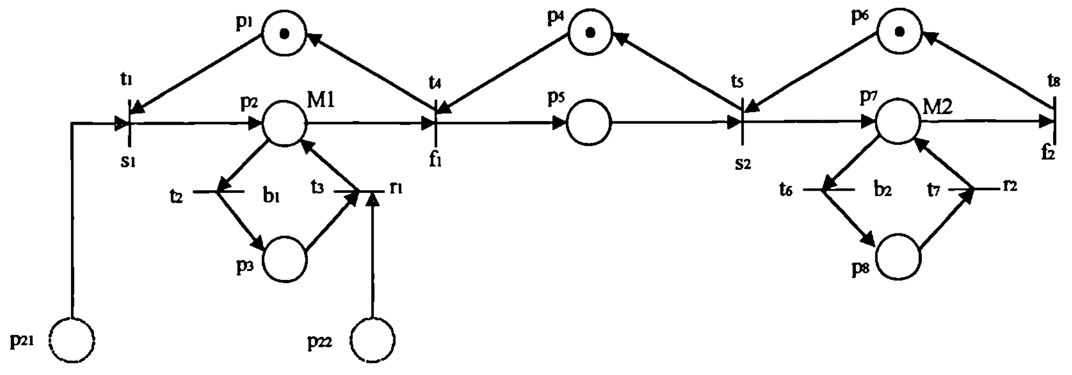


Figure 3.17. The intermediate places connected to the related controllable transitions of the uncontrolled model.

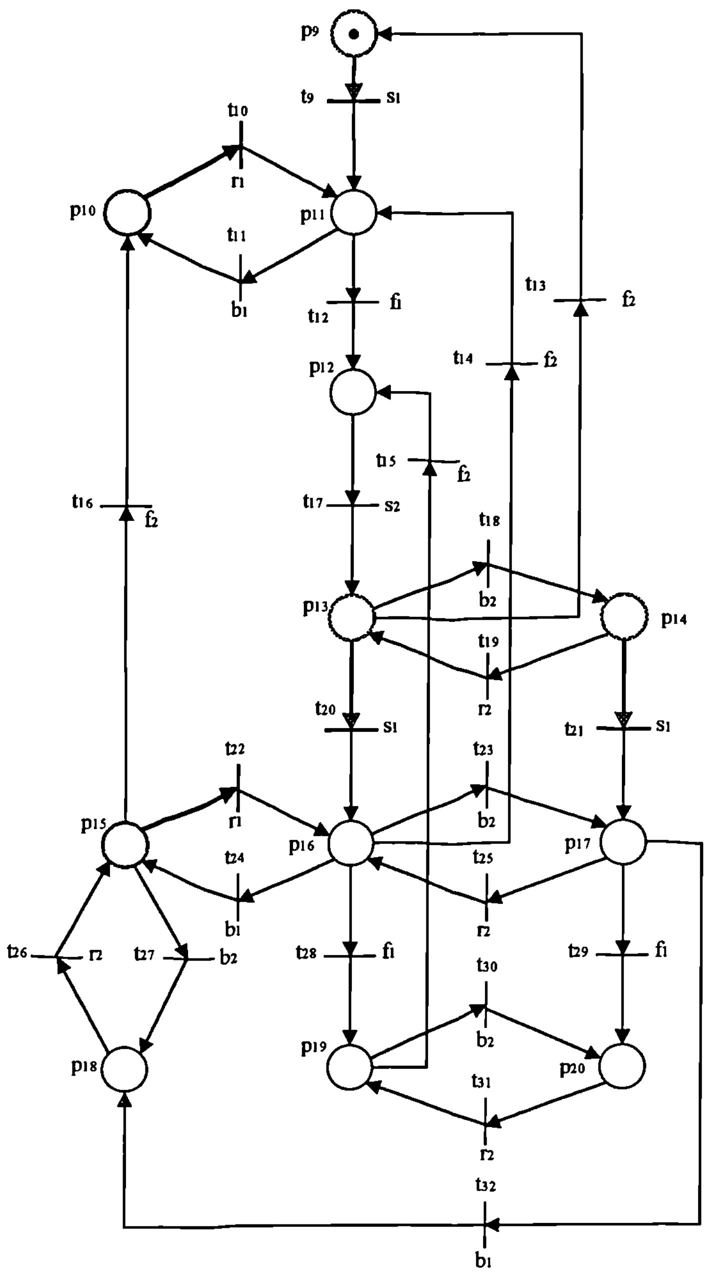


Figure 3.18. The APN model supervisor of the manufacturing system used in determining the control policy in the *intermediate place method.*

| Intermediate place | Input transition(s) | Output transition(s) |
|---|---|---|
| $p_{21}$ | $t_{17}$ | - |
| $p_{22}$ | $t_{11}$, $t_{24}$, $t_{26}$ | $t_{27}$ |

Table 3.5. The control policy for the manufacturing system in the *intermediate place method.*

After designing the APN model supervisor and determining the control policy the controlled model of the system can be obtained for the *intermediate place method*. To do this, ordinary arcs are connected from the APN model supervisor transitions to the intermediate places such that the control policy is satisfied. In a way, the model supervisor guides the model by producing a token in the intermediate places. Through enabling some of the controllable transitions the controlled model allows only the "good states" to take place, i.e., the controlled model does not allow the "bad states" to take place.

The controlled model (i.e., the supervisor), shown in Fig. 3.19, for the *intermediate place method* is obtained by using the APN model, shown in Fig. 3.5, the APN model supervisor, shown in Fig. 3.10, and the control policy given in Table 3.5. The control policy is implemented as follows: since place $p_9$, with $M_0(p_9) = 1$, is a base place for the controllable transition $t_1$, one token as its initial marking is deposited into the intermediate place $p_{21}$. According to the control policy, the arc $Post(t_{17}, p_{21})$ is connected from transition $t_{17}$ to the intermediate place $p_{21}$. For the intermediate place $p_{22}$, input arcs $Post(t_{11}, p_{22})$, $Post(t_{24}, p_{22})$ and $Post(t_{26}, p_{22})$ and the output arc $Pre(p_{22}, t_{27})$ are connected.

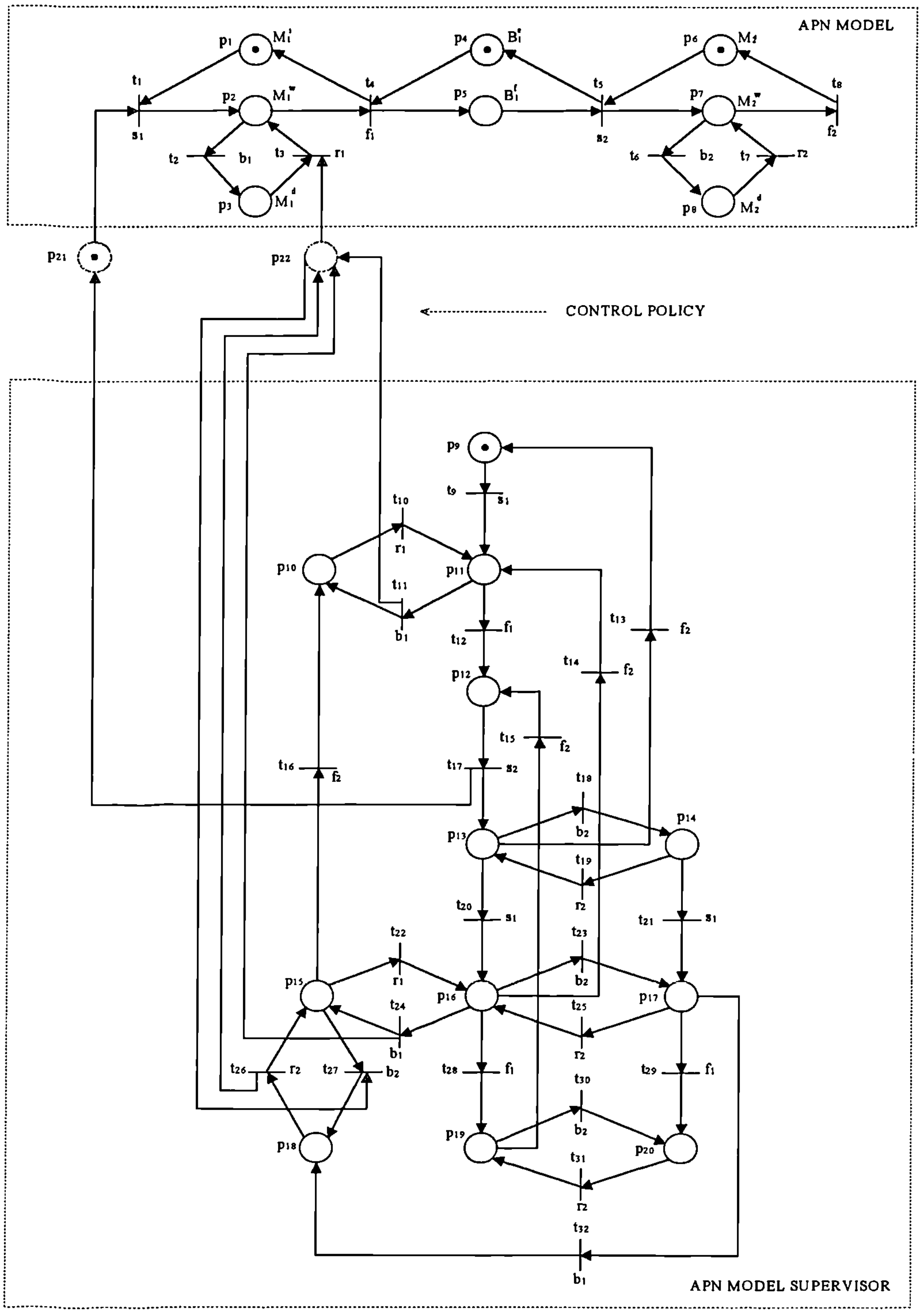


Figure 3.19. The supervisor of the manufacturing system for the *intermediate place method.*

## 3.6. THE APN-SM METHOD

In contrast to the first three methods, in this method the supervisor, as shown in Fig. 3.20, consists of only one net structure, which is referred to the APN model supervisor in the previous methods. In this case, the model supervisor is called the *incomplete supervisor*. The supervisory control policy is a static table that provides a list of actions to be assigned to the places within the supervisor such that the forbidden state specifications are met. This table is then enforced by assigning the related actions to the places of the supervisor. In other words, the supervisor represents the maximally permissible system behaviour as an APN and enforces it by actions assigned to its places. The incomplete supervisor becomes a *complete supervisor* (or only the supervisor) when some actions are assigned to the places according to the control policy. The supervisor in this case has the following characteristics: every transitions has only one input and one output place and in the entire net there is only one token. All places have the capacity of 1. This type of nets are called Petri net state machines (Peterson, 1981). Therefore this method is called APN state machine method (APN-SM). The APN-SM method for the supervisory control of the DESs is divided into the following steps:

Step 1 - Design the uncontrolled model of the system using APNs

Step 2 - Synthesise the incomplete supervisor and determine the control policy

- Step 2.1. Generate the reachability graph of the APN model
- Step 2.2. Identify and remove the "bad states" from the reachability graph
- Step 2.3. Design the incomplete supervisor and determine the control policy

Step 3 - Construct the complete supervisor

Step 4 - Implement the supervisor (the controlled model) on a programmable logic controller (PLC) as ladder logic diagrams (LLDs)

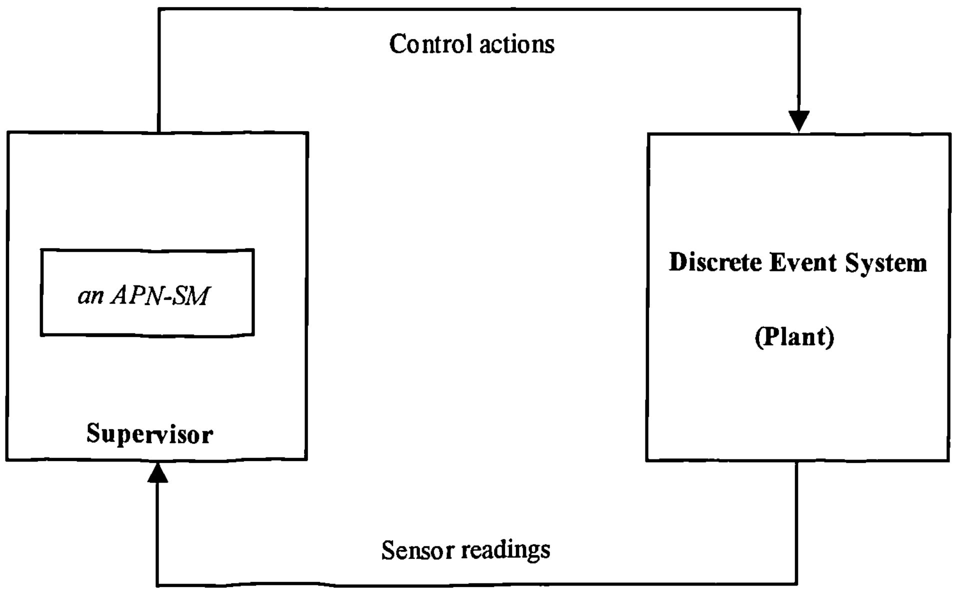


Figure 3.20. The use of an APN-SM as the supervisor in supervisory control.

In the APN-SM method, uncontrolled model of the system to be controlled is obtained using APNs, as explained in the inhibitor arc method. Similarly, the reachability graph of the APN model, that shows the whole state space of the system, is generated and the "bad states", "bad transitions", "blocking states", and "unreachable states" are identified and removed from the reachability graph (RG), yielding the FRRG. The incomplete supervisor, which is referred to the APN model supervisor in the previous methods, is obtained by converting the FRRG into an APN, as explained before. However, the control policy in this case is a static table that lists a set of actions, if any, to be assigned to each place within the supervisor. After assigning the related actions to the places, the supervisor is *complete* and it can directly be used for the control of the system. In order to obtain the control policy, firstly, the APN model places with actions, called *action places*, are considered. Then, the FRRG is checked to see if it contains a marking, in which the action places is shown to have a token. Finally, if a marking within the FRRG represents an action place having a token, then in the control policy, the supervisor place, representing this marking, is to be assigned the related action within the complete supervisor. In this case the supervisor allows only the actions that are allowed to happen at the current marking, represented by a place within the supervisor. The resulting supervisor, therefore, is maximally permissive and behaves according to the specifications. The behaviour of the supervisor is correct by construction and therefore there is no need for verification.

### 3.6.1. Example for the APN-SM Method

Consider the manufacturing system example introduced in the section 3.3.5. The APN model (uncontrolled model) of the manufacturing system is shown in Fig. 3.5. For the forbidden state specifications, given in section 3.3.5.1, the FRRG is shown in Fig. 3.9, and the incomplete supervisor, which is obtained by converting the FRRG into a related net, is shown in Fig. 3.10. These results are obtained by following the design steps given in the section 3.6. Note that the implementation of the supervisor in this method is carried out by using the token passing logic (TPL) methodology as described in the inhibitor arc method.

Now it is necessary to determine the control policy for the APN-SM method. In this case, the supervisory control policy is a static table that provides a list of actions to be assigned to the places within the supervisor. It is obvious from Fig. 3.5 that places $p_2$ and $p_7$ are action places in the APN model, because the *actions* M1 *and* M2 *are assigned to* them, respectively. This means that when there is a token in place $p_2$ ($p_7$), the Machine 1 (the Machine 2) is switched *on*. Now, consider the FRRG given in Fig. 3.21. The markings that represent the action place $p_2$ having a token are $M_2 = (2, 4, 6)$, $M_9 = (2, 4, 7)$ and $M_{10} = (2, 4, 8)$. Therefore, places $p_{11}$, $p_{16}$, and $p_{17}$, that represent these markings respectively, are to be assigned the action M1 in the complete supervisor. Similarly, when considering the action M2, assigned to the action place $p_7$ in the APN model, it is obvious that at the markings $M_6 = (1, 4, 7)$, $M_8 = (3, 4, 7)$, $M_9 = (2, 4, 7)$ and $M_{12} = (1, 5, 7)$, place $p_7$ has a token. Therefore, places $p_{13}$, $p_{15}$, $p_{16}$, and $p_{19}$, that represent these markings respectively, are to be assigned the action M2 in the complete supervisor. The resulting control policy for the manufacturing system is given in Table 3.6. After assigning the actions shown in Table 3.6 to the related places within the incomplete supervisor, the complete supervisor is obtained as shown in Fig. 3.22.

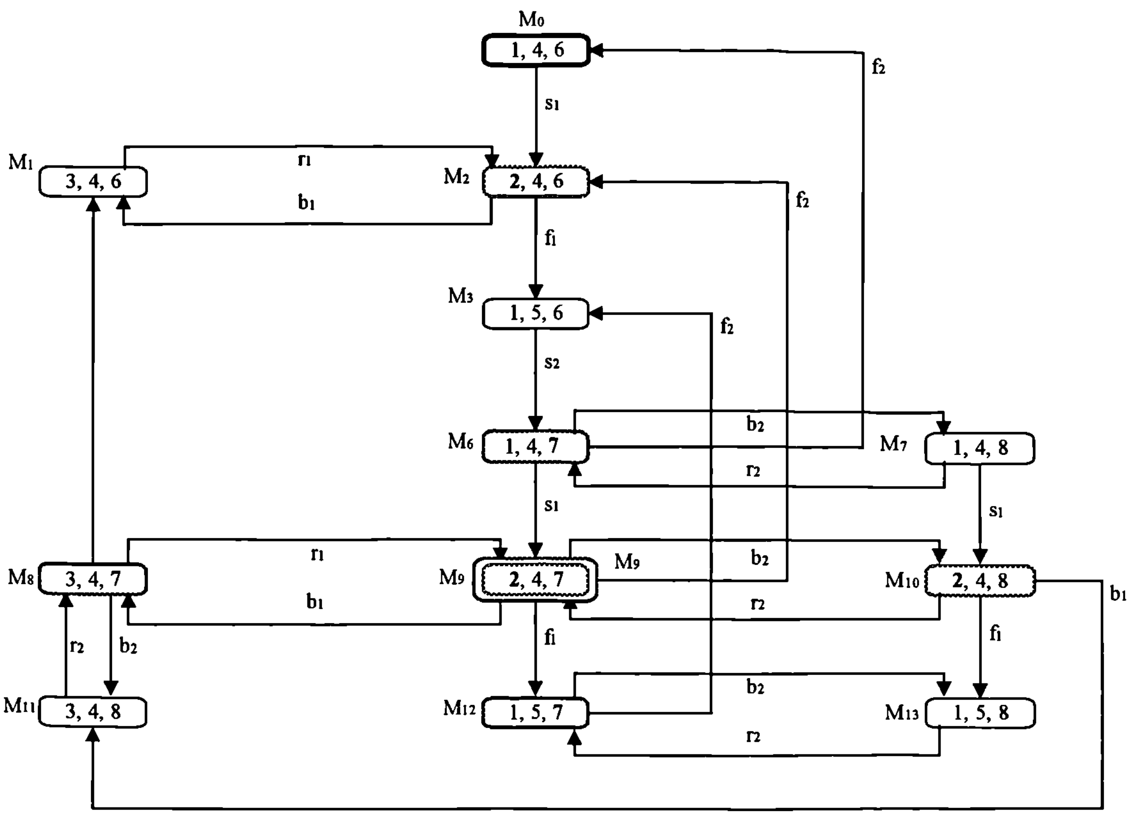


Figure 3.21. The final reduced reachability graph (FRRG), used in determining the control policy in the *APN-SM method.*

| Marking | Supervisor place | Action |
|---|---|---|
| $M_0 = (1, 4, 6)$ | $p_9$ | - |
| $M_1 = (3, 4, 6)$ | $p_{10}$ | - |
| $M_2 = (2, 4, 6)$ | $p_{11}$ | M1 |
| $M_3 = (1, 5, 6)$ | $p_{12}$ | - |
| $M_6 = (1, 4, 7)$ | $p_{13}$ | M2 |
| $M_7 = (1, 4, 8)$ | $p_{14}$ | - |
| $M_8 = (3, 4, 7)$ | $p_{15}$ | M2 |
| $M_9 = (2, 4, 7)$ | $p_{16}$ | M1 & M2 |
| $M_{10} = (2, 4, 8)$ | $p_{17}$ | M1 |
| $M_{11} = (3, 4, 8)$ | $p_{18}$ | - |
| $M_{12} = (1, 5, 7)$ | $p_{19}$ | M2 |
| $M_{13} = (1, 5, 8)$ | $p_{20}$ | - |

Table 3.6. The control policy for the manufacturing system in the *APN-SM method.*

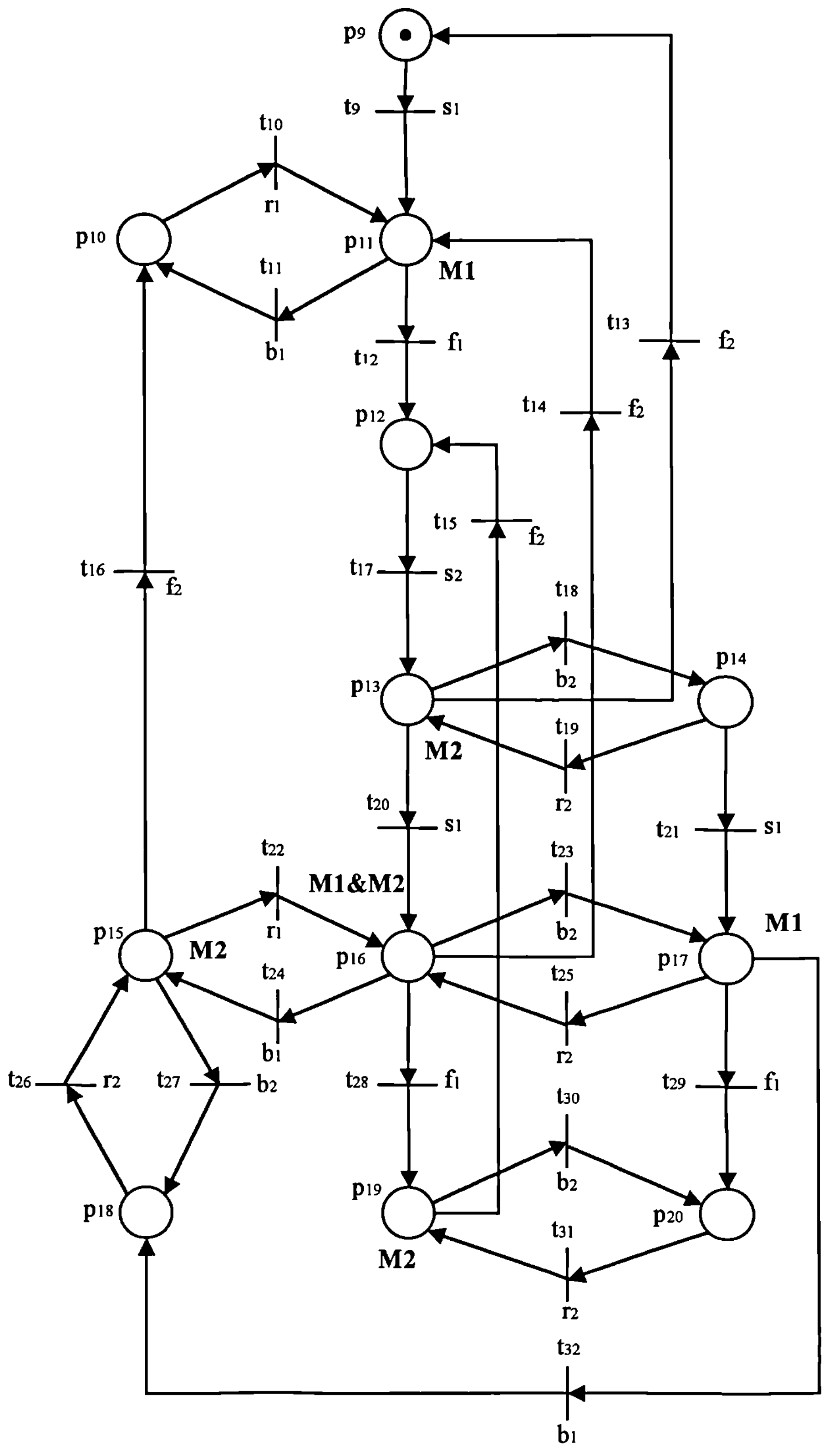


Figure 3.22. The (complete) supervisor for the manufacturing system in the *APN-SM method.*

## 3.7. DISCUSSION

In this chapter, four design techniques, called the *inhibitor arc method*, the *enabling arc method*, the *intermediate place method* and the *APN-SM method*, have been proposed for the design of compiled supervisors for the control of DESs in the case of the forbidden state problem. In the first three methods, the supervisor is a controlled model of the system that behaves according to the given forbidden state specifications. The supervisor, that guides the DES by control action, consists of the APN model of the DES, the APN model supervisor and the control policy. The design of the APN model and the APN model supervisor have been done in the same manner in all these four methods. However, the determination of control policy and the construction of the controlled model is different for each method. In the *inhibitor arc method*, the model supervisor is assumed to have an inhibitory effect over the model and therefore the role of the APN model supervisor is to stop some controllable events at certain markings so that the control policy is satisfied. As a result the controlled model of the system, in the *inhibitor arc method*, has been obtained by connecting the model supervisor to the model through the use of inhibitor arcs. In the enabling arc method, the model supervisor is assumed to have an enabling effect over the model and therefore the role of the APN model supervisor is to enable only the controllable events which are allowed to happen at certain markings so that the control policy is satisfied. As a result, the controlled model of the system, in the *enabling arc method*, has been obtained by connecting the model supervisor to the model through the use of enabling arcs. In the *intermediate place method*, the controlled model has been obtained by using a set of intermediate places, whose role is to enable or disable the controllable transitions of the model according to the current marking of the system represented by a place within the model supervisor. In contrast to the first three methods, in the *APN-SM method*, the supervisor consists of only one net structure, which is referred to the APN model supervisor in the previous methods. In this case, the model supervisor is called the *incomplete supervisor*. The supervisory control policy is a static table that provides a list of actions to be assigned to the places within the supervisor such that the forbidden state specifications are met. The incomplete supervisor becomes a *complete supervisor* (or only the supervisor) when the actions are assigned to the places according to the control policy.

The APN-SM method is a Petri net equivalent of Wonham's FSM supervisors. The other three techniques show how the FSM supervisory technique can be deployed within a context of supervising a Petri net model of the system. These three techniques provide a bridge between FSM and supervised Petri net models.

The results obtained in these methods can be applied to high level manufacturing control, where the role of the supervisor is to coordinate factory-wide control of machines, and to low-level manufacturing control, where the role of the supervisor is to arrange low-level interaction between the control devices, such as motors, actuators, etc.

Although the example considered in this chapter required an untimed and safe APN, i.e., a net in which a place can have only one token at most, for the design, these methods can also deal with timed models and the systems that require models in which more than one token can be present in a place within the net. In all of these four methods the whole state space of an uncontrolled model has to be considered as a reachability graph (RG). The computation of the whole state space poses the following problem: the whole state space of the system grows exponentially in the size of the model, i.e., we are faced with the *state explosion problem*. Similar problems are faced by any techniques which deploy a finite state machine type solution path (Wonham and Ramadge, 1987). To show the effect of the *state explosion problem* in this case, consider the same manufacturing system example in which the buffer has the capacity of three. With the same forbidden state specifications the RG of the uncontrolled system model has 36 reachable states and 91 arcs. Within this RG, 3 states are "bad states", 3 states are "unreachable states" and 3 arcs represent "bad transitions". As shown in Table 3.7, the supervisor for the manufacturing system example, in which the buffer has the capacity of three, would have

38 places, in the case of the inhibitor arc method, 38 places, in the case of the enabling arc method, 40 places, in the case of the intermediate place method and 30 places, in the case of the APN-SM method. On the other hand, the supervisor that has been designed for the same manufacturing system, when buffer has the capacity of one, has 20 places, in the case of the inhibitor arc method, 20 places, in the case of the enabling arc method, 22 places, in the case of the intermediate place method and 12 places, in the case of the APN-SM method. This shows the exponential complexity of these four methods.

| | number of places used in the supervisor for the manufacturing system (buffer capacity = 1) | number of places used in the supervisor for the manufacturing system (buffer capacity = 3) |
|---|---|---|
| inhibitor arc method | 20 | 38 |
| enabling arc method | 20 | 38 |
| intermediate place method | 22 | 40 |
| APN-SM method | 12 | 30 |

Table 3.7. The number of places used in the supervisor for the manufacturing system.

Nevertheless, these methods represent a basic framework that shows how Petri-net-based compiled supervisors can be constructed by using the models of the systems in a systematic way as opposed to heuristic methods. The methodologies proposed in this chapter offer the following advantages: The compiled supervisor and control policy obtained are correct by construction, i.e., controlled behaviour of the system is nonblocking and does not contradict the forbidden state specifications. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications.

Note that these results are based on the assumption that there is a sufficient number of discrete event actuators, motors, etc. and discrete event sensors available within the system in order to be able to control the system.

# CHAPTER 4

# PETRI-NET-BASED TOKEN PASSING MARKING RULE SUPERVISORS FOR THE FORBIDDEN STATE PROBLEM

# CHAPTER 4

# PETRI-NET-BASED TOKEN PASSING MARKING RULE SUPERVISORS FOR THE FORBIDDEN STATE PROBLEM

## 4.1. INTRODUCTION

In this chapter two techniques are proposed for the purpose of designing compiled supervisors for the control of DESs in the case of the forbidden state problem. Automation Petri Nets (APN) are used as an underlying formalism for the design of such compiled supervisors for the control of DESs. The approach is based on information feedback on the occurrence of events and Petri net concepts. In particular, discrete event manufacturing systems are considered. The control synthesis procedures proposed in this chapter can be applied to both high-level and low-level manufacturing control.

The *first method* proposed in this chapter is a top-down synthesis technique involving the construction of the reachability graph (RG) of the uncontrolled APN model of the system. In this case, the supervisor to be synthesised is a controlled model of the system, which is obtained by using the uncontrolled APN model and the Token Passing Marking (TPM) rules. The uncontrolled APN model represents the uncontrolled system behaviour. The uncontrolled TPM rules are obtained through a top-down synthesis technique as follows: In order to obtain the TPM rules firstly, the RG of the uncontrolled APN model is generated. Next, the final reduced RG (FRRG) is obtained by identifying the bad markings according to the forbidden state specifications and then by removing these bad markings from the RG. Then, by considering the FRRG together with the controllable transitions, which are related to forbidden state specifications, the control policy is obtained. The control policy simply defines the related controllable transitions and markings of the uncontrolled APN model at which these transitions are to be enabled. This table is then converted into associated TPM rules, which are of the form:

> *if* <markings(s)>
> *then* <a controllable transition is to be enabled>

In the *if* part of a TPM rule a particular marking of the uncontrolled APN model is given and in the *then* part of the TPM rule a controllable transition of the uncontrolled APN model is provided. In this arrangement, a specified controllable transition, given in the *then* part of a TPM rule, is allowed to fire only at the marking(s), given in the *if* part of the rule. This also means that if the marking of the system is different from the one specified in the *if* part of the TPM rule, then the specified controllable transition is not allowed to fire. In order to obtain the controlled model of the system the TPM rules are implemented on the uncontrolled APN model by using enabling arcs which are connected from the corresponding places to the related controllable transitions. Because the synthesis technique in this case involves the construction of the RG of the Uncontrolled model and the use of TPM rules, it is called the *U-TPM rule method.*

The <u>*second method*</u> proposed in this chapter is a bottom-up synthesis technique involving the construction of the RG of the controlled APN model of the system. In this case, the supervisor is a controlled model of the system, which is obtained by using the uncontrolled APN model and the TPM rules, related to the forbidden state specifications. In contrast to the U-TPM rule method, the TPM rules are obtained directly from the forbidden state specifications and they are implemented through the use of a mixture of enabling arcs and inhibitor arcs, which are connected from the corresponding places to the related controllable transitions. However, in this case it is necessary to verify the correctness of the controlled model obtained, through the RG analysis. Because the synthesis technique in this case involves the construction of the RG of the Controlled model and the use of TPM rules, it is called the *C-TPM rule method.*

In the remainder of this chapter, firstly how the TPM rules are implemented and what sort of effect they have on the uncontrolled model are explained. Then, the U-TPM rule method and subsequently the C-TPM rule method are considered.

## 4.2. TOKEN PASSING MARKING RULES

The Token Passing Marking rules are used to construct the controlled model of a system. They can be obtained through RG analysis as in the U-TPM rule method or directly from the forbidden state specifications given. The TPM rules are of the form

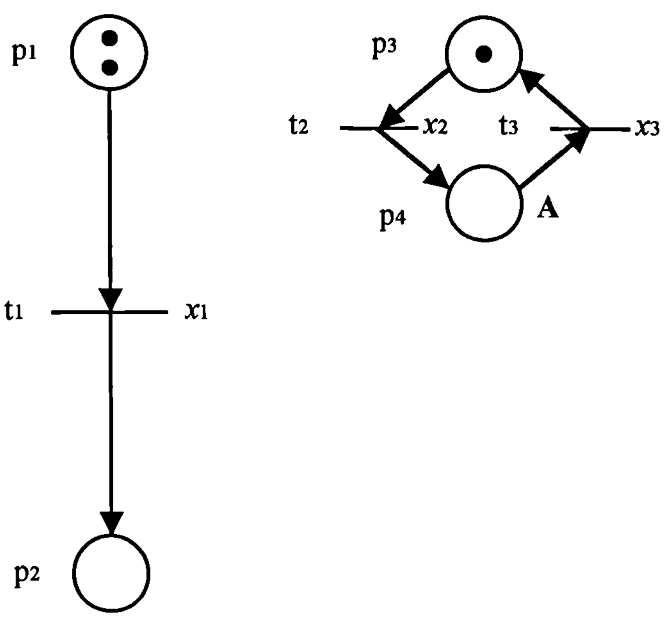
*if* <marking(s)>
*then* <a controllable transition is to be enabled>

The *if* part of the TPM rule is the marking(s) at which a controllable transition can be allowed to fire. The *if* part can contain the logical AND and OR operations. If the OR operation is used in the *if* part, then the controllable transition is duplicated as many as $2^n$ - 1 times, where *n* is the number of places appearing in the marking. The *then* part of the TPM rule represents the controllable transition, which is to be enabled at the specific marking, which is given in the *if* part of the TPM rule. Note that when the marking of the system is other than the one specified in the *if* part of the TPM rule, then the controllable transition is blocked, i.e., it is not enabled to fire. The *if* part can involve checking the presence of a token in a place. For instance *if* <$M(p_1) = 1$> means 'if there is a token in place $p_1$'. In this case, an enabling arc is used to implement this TPM rule. Assuming that the *then* part is '*then* <$t_1$ is to be enabled>', an enabling arc $En(p_1, t_1)$, connected from place $p_1$ to transition $t_1$, is used to implement this rule. The *if* part can also involve checking the absence of a token in a place. For instance *if* <$M(p_1) = 0$> means 'if there is no token in place $p_1$'. In this case, an inhibitor arc is used to implement this TPM rule. Assuming that the *then* part is '*then* <$t_1$ is to be enabled>', an inhibitor arc $In(p_1, t_1)$, connected from place $p_1$ to transition $t_1$ is used to implement this rule. The use of the

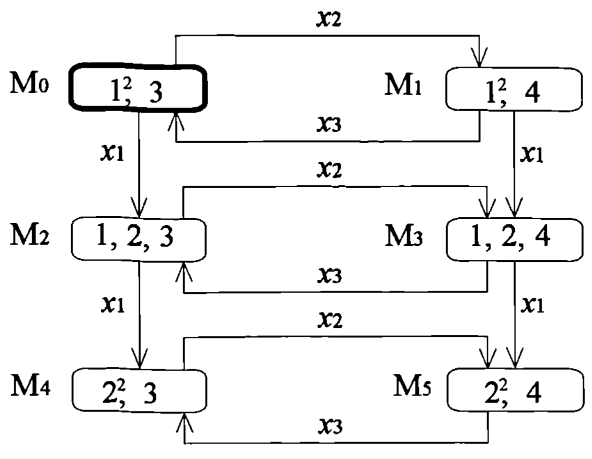
TPM rules, implemented either by an enabling arc or by an inhibitor arc reduces the number of arcs and/or the reachable states (markings) of the uncontrolled model of a system. In the *if* part of the TPM rules the number of markings is either one or more. In this case when the markings of the *if* part is more than one, logical OR or logical AND function can be used.

### 4.2.1. TPM Rules With One Marking

In the case, where the *if* part of the TPM rule contains only one place marking, the place marking is used to check either the presence or the absence of a token in a particular place. To show these two cases, firstly an uncontrolled system model, shown in Fig. 4.1.(a), where there are four places $P = \{ p_1, p_2, p_3, p_4 \}$ and three transitions $T = \{ t_1, t_2, t_3 \}$, is considered. It is assumed that transition $t_1$ is a controllable transition. The action A is assigned to place $p_4$. The initial marking of the APN model is $M_0 = (2, 0,1, 0)$ or simply $M = (1^2, 3)$, which means that there are two tokens in place $p_1$ and there is a token in place $p_3$. Initially, transitions $t_1$ and $t_2$ are enabled since $M(p_1) \geq 1$ and $M(p_3) = 1$. When transition $t_1$ is enabled, i.e., $M(p_1) \geq 1$, if the firing condition $\chi_1$ of transition $t_1$ occurs, then transition $t_1$ fires by removing a token from place $p_1$ and by depositing a token in place $p_2$. When transition $t_2$ is enabled, i.e., $M(p_3) = 1$, if the firing condition $\chi_2$ of transition $t_2$ occurs, then transition $t_2$ fires by removing a token from place $p_3$ and by depositing a token in place $p_4$. When transition $t_3$ is enabled, i.e., $M(p_4) = 1$, if the firing conditions $\chi_3$ occurs, then transition $t_3$ fires by removing a token from place $p_4$ and by depositing a token in place $p_3$. All possible reachable markings (states) of this system is shown as a reachability graph in Fig. 4.1.(b). Note that in this case the concurrent firing of transition is not considered.



(a)



(b)

Figure 4.1. (a) An uncontrolled APN model of a system. (b) Its reachability graph.

Now, to show the case where the TPM rule contains only one place marking and is used to check the presence of a token in a particular place assume that the TPM rule is as follows:

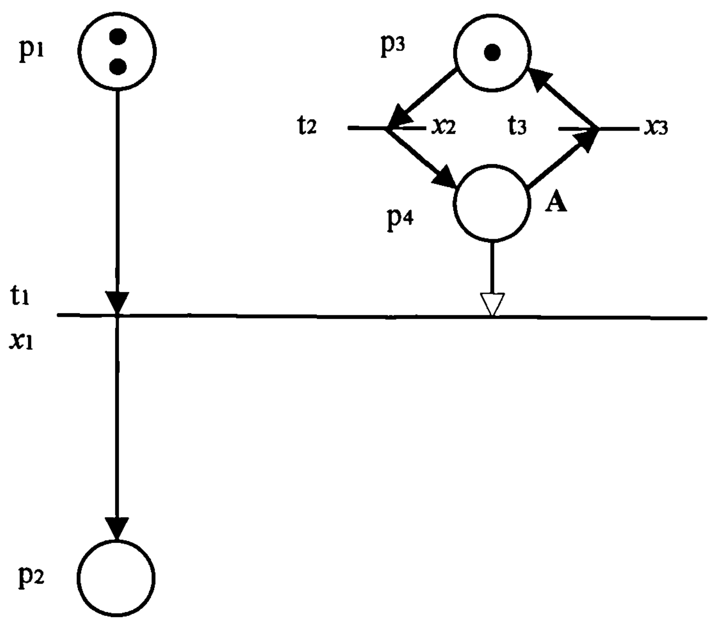
*if* $<M(p_4) = 1>$

*then* $<t_1$ is to be enabled$>$

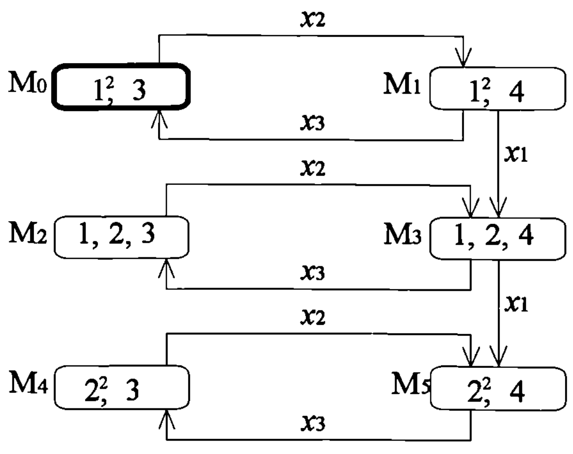
In order to obtain the controlled model from the uncontrolled model, shown in Fig. 4.1.(a), and the TPM rule, given above, in this case an enabling arc *En(p₄, t₁)* is connected from place $p_4$ to the controllable transition $t_1$, as shown in Fig. 4.2.(a). The RG of this controlled model is shown in Fig. 4.2.(b), in which arcs $M_0$ $[\chi_1>$ $M_2$ and $M_2$ $[\chi_1>$ $M_4$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$ and a token in place $p_4$, and if the firing condition $\chi_1$ occurs. This also implies that if there is no token in place $p_4$, then transition $t_1$ is blocked, i.e., it is not enabled. Therefore the complement of the TPM rule given above is as follows:

*if* $<M(p_4) = 0>$

*then* $<t_1$ is to be blocked$>$



(a)



(b)

Figure 4.2. (a) The controlled APN model of the system, shown in Fig. 4.1. (b) Its reachability graph.

Now, to show the case where the TPM rule contains only one place marking and is used to check the absence of a token in a particular place assume that the TPM rule is as follows:

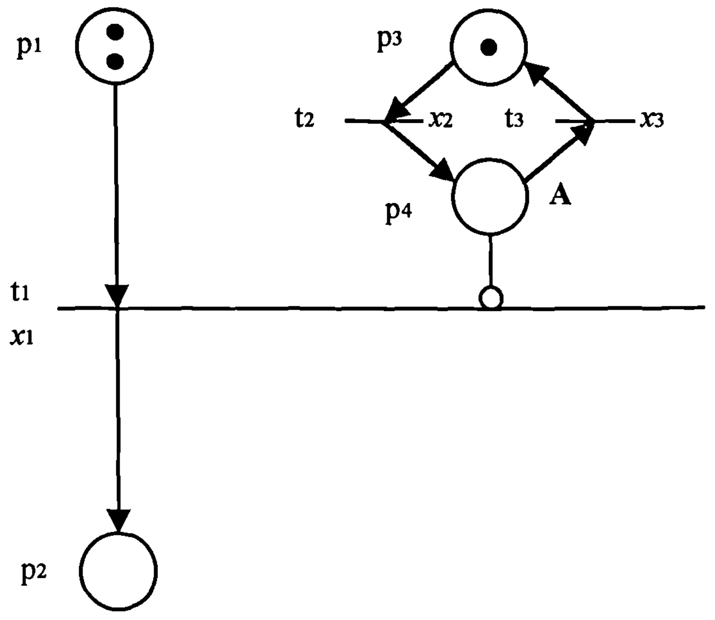
*if* &lt;$M(p_4) = 0$&gt;

*then* &lt;$t_1$ is to be enabled&gt;

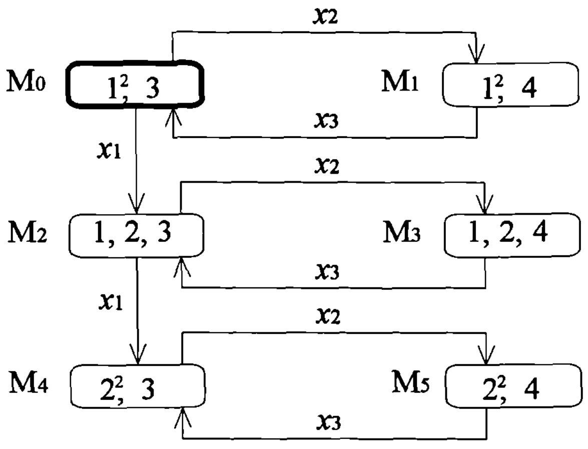
In order to obtain the controlled model from the uncontrolled model, shown in Fig. 4.1.(a), and the TPM rule, given above, in this case an inhibitor arc $In(p_4, t_1)$ is connected from place $p_4$ to controllable transition $t_1$ as shown in Fig. 4.3.(a). The RG of this controlled model is shown in Fig. 4.3.(b), in which arcs $M_1$ [$\chi_1$> $M_3$ and $M_3$ [$\chi_1$> $M_5$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token in place $p_1$ and a token in place $p_4$, and if the firing condition $\chi_1$ is occurs. This also implies that if there is a token in place $p_4$, then transition $t_1$ is blocked, i.e., it is not enabled. Therefore the complement of the TPM rule given above is as follows:

*if* &lt;$M(p_4) = 1$&gt;

*then* &lt;$t_1$ is to be blocked&gt;



(a)



(b)

Figure 4.3. (a) The controlled APN model of the system, shown in Fig. 4.1. (b) Its reachability graph.

This section shows that the same rule can be represented in two ways, one complementing the other. It also shows how the enabling / blocking process works to obtain the controlled model of the system.

### 4.2.2. TPM Rules With More Than One Marking

In the case, where the *if* part of the TPM rule contains more than one marking, these markings are used to check either the presence or the absence of a token in a group of places. When doing this either logical AND or logical OR function is used. These logical functions can be used for checking only the absence of markings in the places, for checking only the presence of markings in the places, or can be used for checking both the presence and the absence of markings in the places. In this section, these cases are explained. In addition it is also shown how the TPM rules can reduce the number of reachable states.

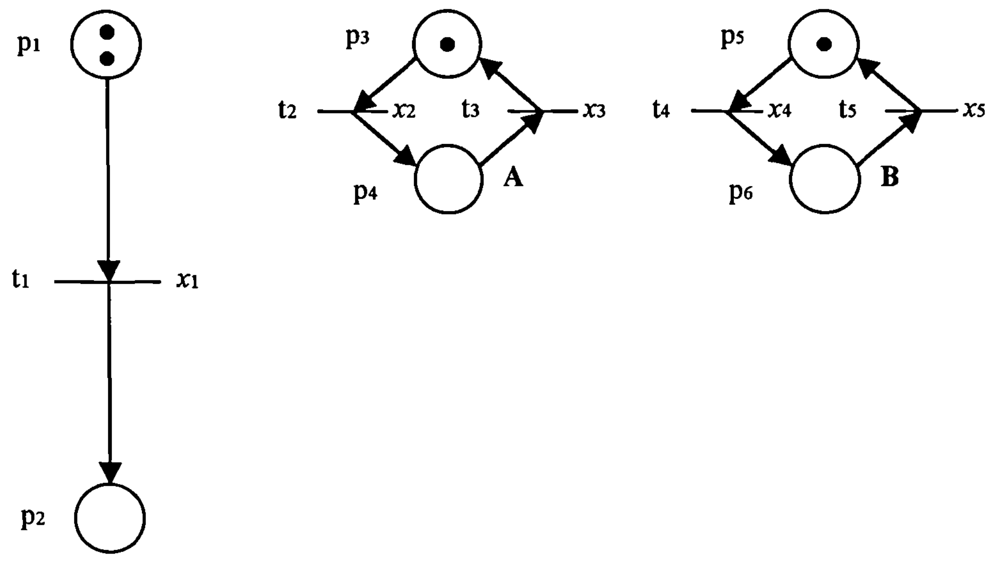
#### 4.2.2.1. The AND Function

##### 4.2.2.1.1. The AND function and checking the absence of markings

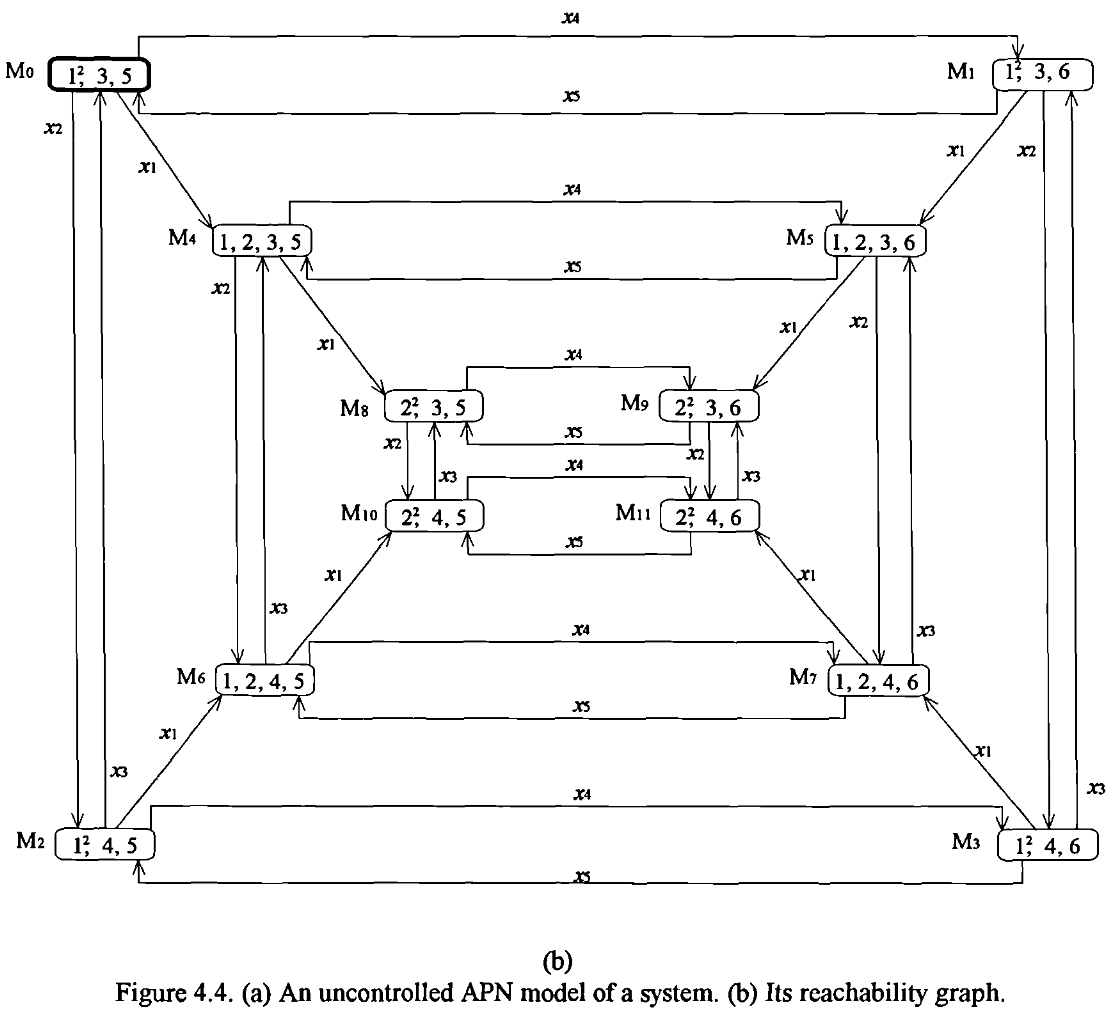
In this case the structure of the TPM rule is as follows:

*if* $<M(p_1) = 0>$ AND $<M(p_2) = 0>$ AND $< M(p_3) = 0>$ AND.......
*then* <a controllable transition is to be enabled>

To explain this case, the uncontrolled model of the APN model of a system, shown in Fig. 4.4.(a), where there are six places $P = \{ p_1, p_2, .... p_6 \}$ and five transitions $T = \{ t_1, t_2, ......, t_5 \}$, is considered. It is assumed that transition $t_1$ is a controllable transition. Note that the actions A and B are assigned to places $p_4$ and $p_6$, respectively. The initial marking of the uncontrolled APN model is $M_0 = (2, 0,1, 0, 1, 0)^T$ or simply $M = (1^2, 3, 5)$, which means that there are two tokens in place $p_1$, one token in place $p_3$ and one token in place $p_5$. The whole state space, i.e., all possible reachable markings, of the system is shown as a reachability graph (RG) in Fig. 4.4.(b).



(a)



(b)

Figure 4.4. (a) An uncontrolled APN model of a system. (b) Its reachability graph.

Now, assume that the TPM rule is as follows:

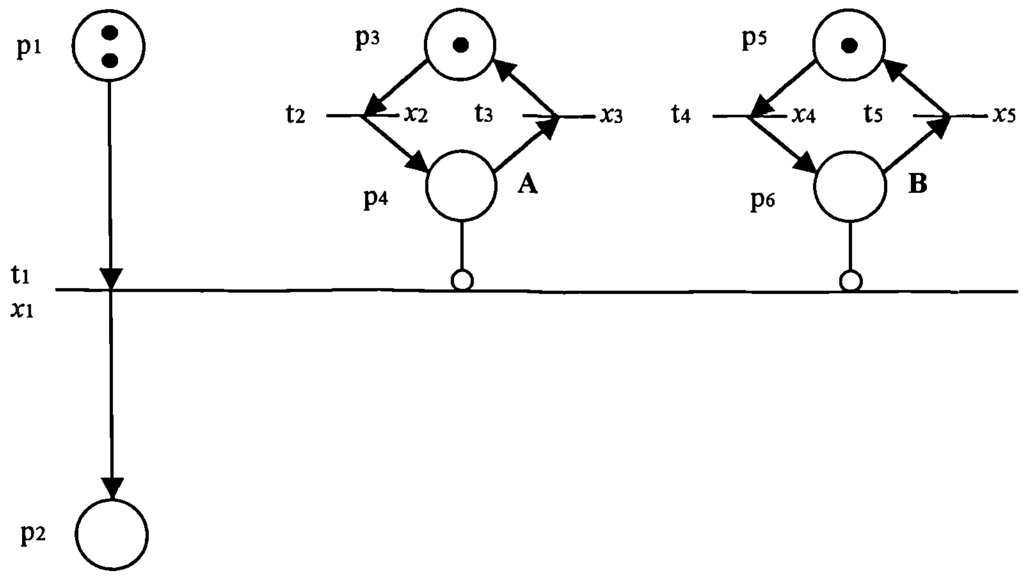
*if* &lt;M($p_4$) = 0&gt; AND &lt;M($p_6$) = 0&gt;

*then* &lt;$t_1$ is to be enabled&gt;

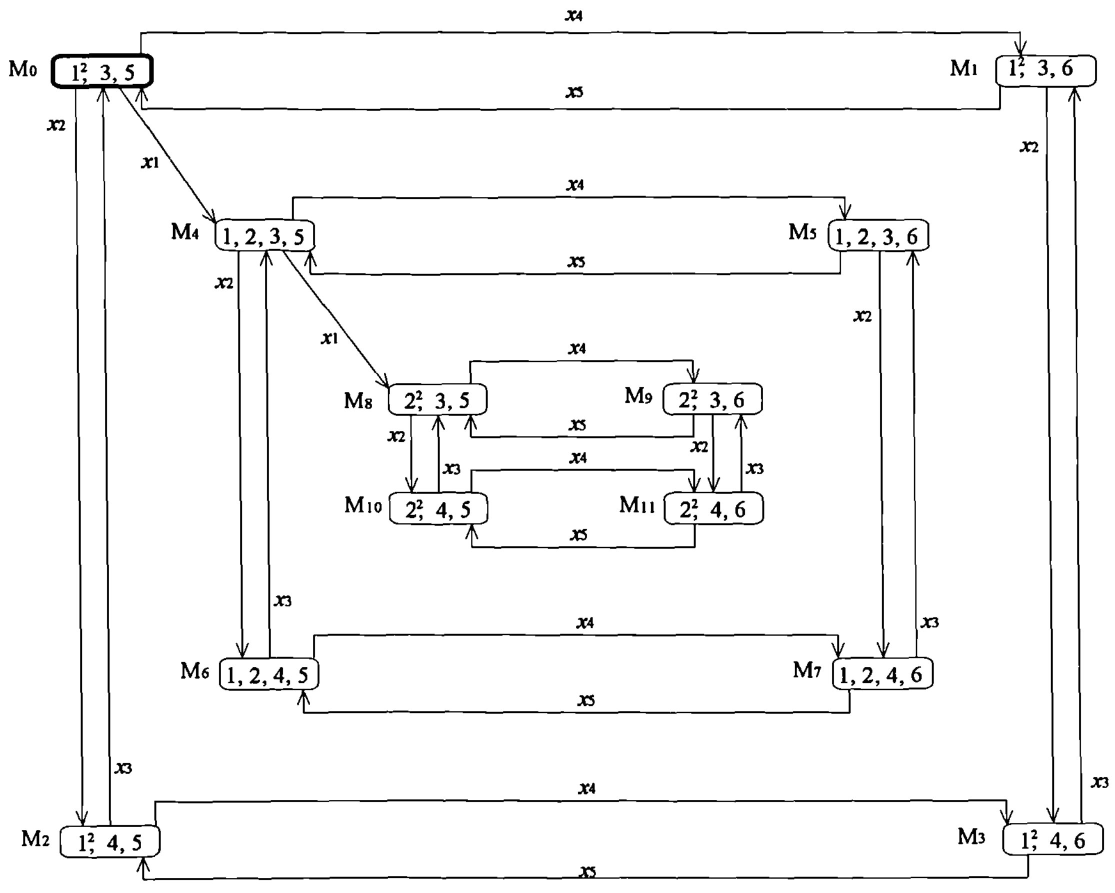
In order to obtain the controlled model from the uncontrolled model, shown in Fig. 4.4.(a), and the TPM rule, given above, in this case two inhibitor arcs *In($p_4$, $t_1$)* and *In ($p_6$, $t_1$)* are connected from place $p_4$ and $p_6$ to the controllable transition $t_1$, as shown in Fig. 4.5.(a). The RG of this controlled model is shown in Fig. 4.5.(b), in which arcs $M_1$ [$\chi_1$> $M_5$, $M_3$ [$\chi_1$> $M_7$, $M_2$ [$\chi_1$> $M_6$, $M_5$[$\chi_1$> $M_9$, $M_7$ [$\chi_1$> $M_{11}$ and $M_6$ [$\chi_1$> $M_{10}$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$, no token in place $p_4$, and no token in place $p_6$, and if the firing condition $\chi_1$ occurs. This also implies that if there is a token in either place $p_4$ or in place $p_6$, then transition $t_1$ is blocked, i.e., it is not enabled. Therefore, the complement of the TPM rule given above is as follows:

*if* &lt;M($p_4$) = 1&gt; OR &lt;M($p_6$) = 1&gt;

*then* &lt;$t_1$ is to be blocked&gt;



(a)



(b)

Figure 4.5. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

### 4.2.2.1.2. The AND function and checking the presence of markings

In this case the structure of the TPM rule is as follows:

*if* <M($p_1$) = 1> AND <M($p_2$) = 1 > AND <M($p_3$) = 1> AND......

*then* <a controllable transition is to be enabled>

To explain this case, consider the uncontrolled APN model, shown in Fig. 4.4.(a), and its RG, shown in Fig. 4.4.(b). Now, assume that the TPM rule is as follows:

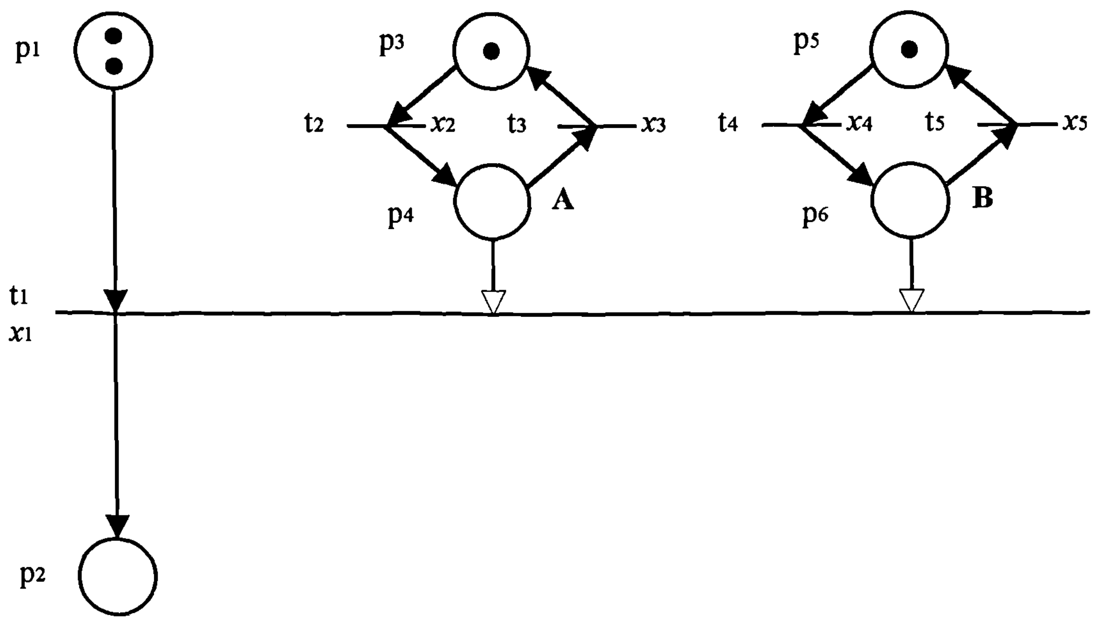
*if* <M($p_4$) = 1> AND <M($p_6$) = 1>

*then* <$t_1$ is to be enabled>

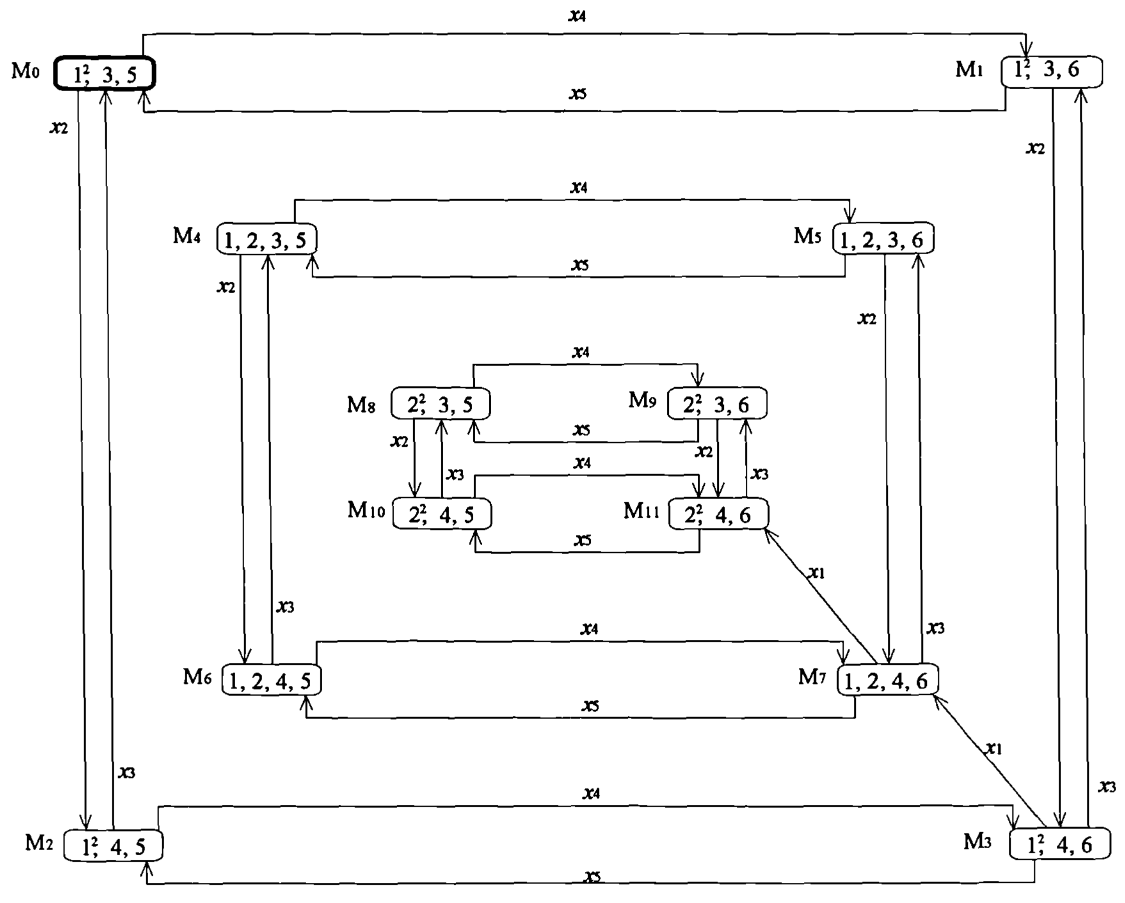
In order to obtain the controlled model from the uncontrolled model, shown in Fig. 4.4.(a) and the TPM rule as shown above, in this case the two enabling arcs *En($p_4$, $t_1$)* and *En($p_6$, $t_1$)* are connected from place $p_4$ and $p_6$ to transition $t_1$, as shown in Fig. 4.6.(a). The RG of this controlled model is shown in Fig. 4.6.(b), in which arcs $M_0$ [$\chi_1$> $M_4$, $M_1$ [$\chi_1$> $M_5$, $M_2$ [$\chi_1$> $M_6$, $M_4$[$\chi_1$> $M_8$, $M_5$ [$\chi_1$> $M_9$ and $M_6$ [$\chi_1$> $M_{10}$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$ and a token each in places $p_4$ and $p_6$, and if the firing condition $\chi_1$ is occurs. This also implies that if there is no token either in place $p_4$ or in place $p_6$, then transition $t_1$ is blocked, i.e., it is not enabled. Therefore the complement of the TPM rule given above is as follows:

*if* <M($p_4$) = 0> OR <M($p_6$) = 0>

*then* <$t_1$ is to be blocked>



(a)



(b)

Figure 4.6. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

### 4.2.2.1.3. The AND function and checking both the presence and absence of markings

In this case the structure of the TPM rule is as follows:

*if* <M($p_1$) = 1 (or 0)> AND <M($p_2$) = 0 (or 1) > AND ......

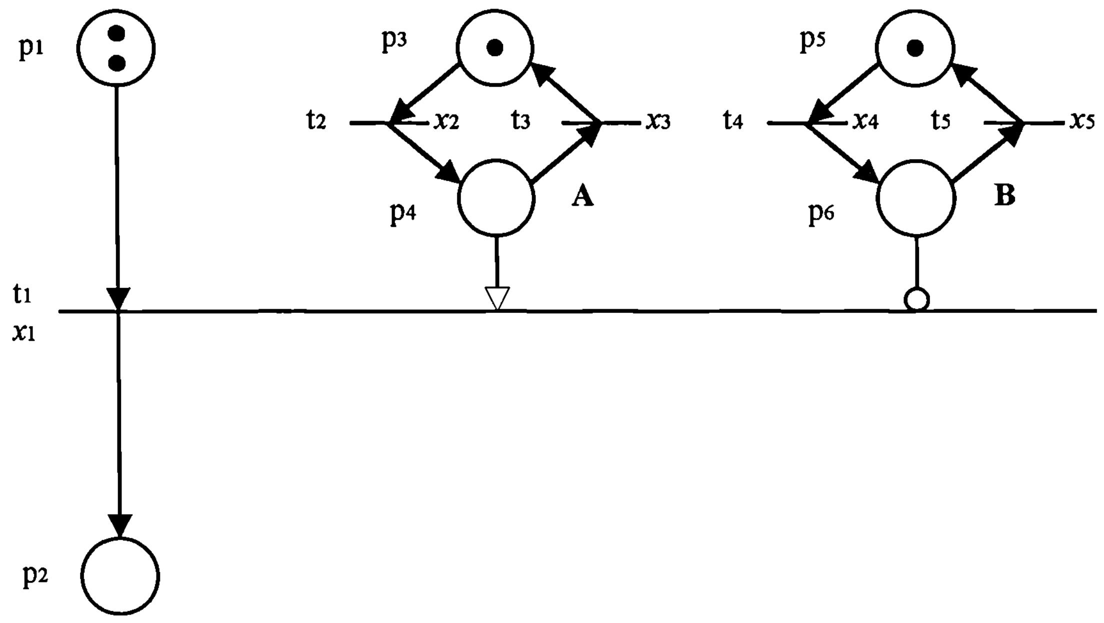
*then* <a controllable transition is to be enabled>

To explain this case, consider the uncontrolled APN model, shown in Fig. 4.4.(a), and its RG, shown in Fig. 4.4.(b). Now, assume that the TPM rule is as follows:

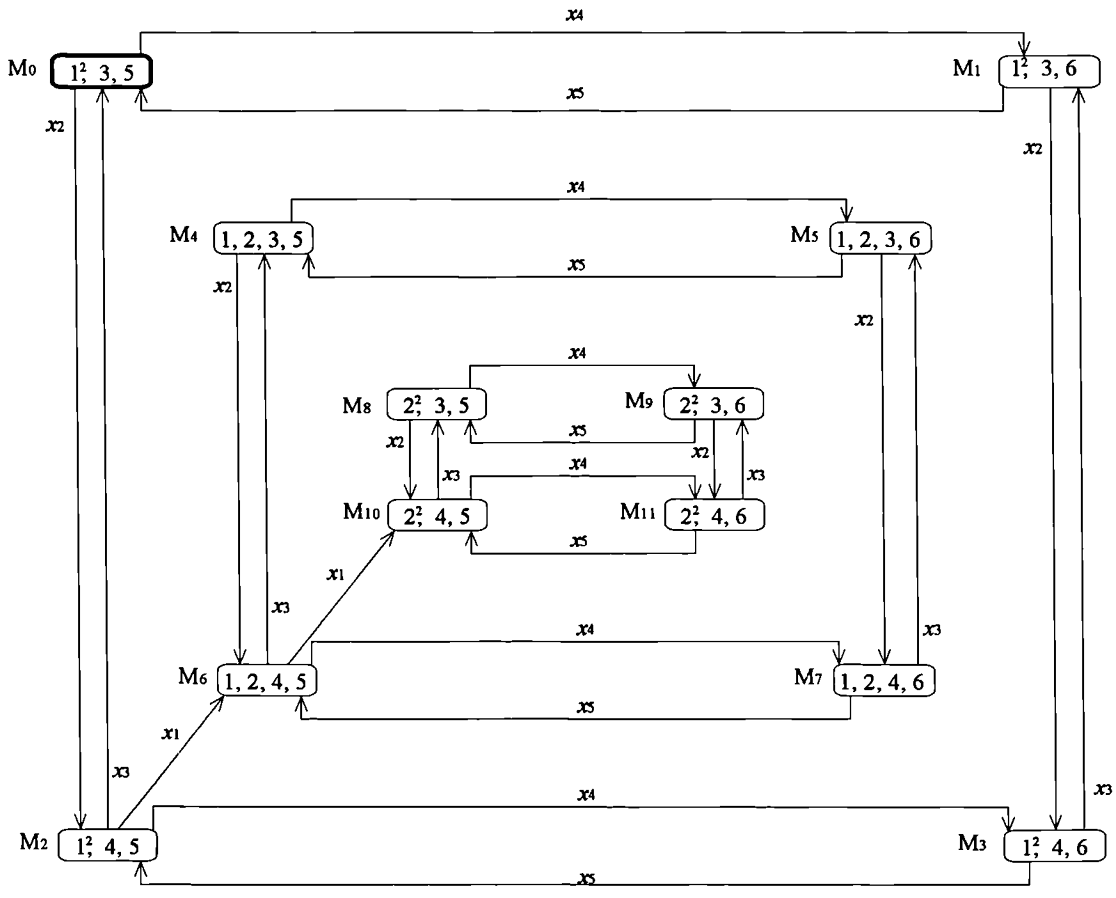
*if* &lt;$M(p_4) = 1$&gt; AND &lt;$M(p_6) = 0$&gt;
*then* &lt;$t_1$ is to be enabled&gt;

In order to obtain the controlled model from the uncontrolled model, shown in Fig. 4.4.(a) and the TPM rule as shown above, in this case an enabling arc $En(p_4, t_1)$ is connected from place $p_4$ to transition $t_1$, and an inhibitor arc $In(p_6, t_1)$ is connected from place $p_6$ to transition $t_1$, as shown in Fig. 4.7.(a). The RG of this controlled model is shown in Fig. 4.7.(b), in which arcs $M_0$ [$\chi_1$> $M_4$, $M_1$ [$\chi_1$> $M_5$, $M_3$ [$\chi_1$> $M_7$, $M_4$[$\chi_1$> $M_8$, $M_5$ [$\chi_1$> $M_9$ and $M_7$ [$\chi_1$> $M_{11}$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$ and a token in place $p_4$ and no token in place $p_6$, and if the firing condition $\chi_1$ occurs. This also implies that if there is no token in place $p_4$ or there is a token in place $p_6$, then transition $t_1$ is blocked, i.e., it is not enabled. Therefore the complement of the TPM rule given above is as follows:

*if* &lt;$M(p_4) = 0$&gt; OR &lt;$M(p_6) = 1$&gt;
*then* &lt;$t_1$ is to be blocked&gt;



(a)



(b)

Figure 4.7. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

### 4.2.2.2. The OR Function

Note that in this case, in order to obtain the controlled model of the system the controllable transition, which appears in the *then* part of the TPM rule, is duplicated as many as $2^n$ -1 times, where *n* is the number of places appearing in the marking(s). This is simply done to accommodate the logical OR operation within the Petri net formalism.

#### 4.2.2.2.1. The OR function and checking the absence of markings

In this case the structure of the TPM rule is as follows:

*if* <M($p_1$) = 0 > OR <M($p_2$) = 0 > OR <M($p_3$) = 0 > OR..........
*then* <a controllable transition is to be enabled>

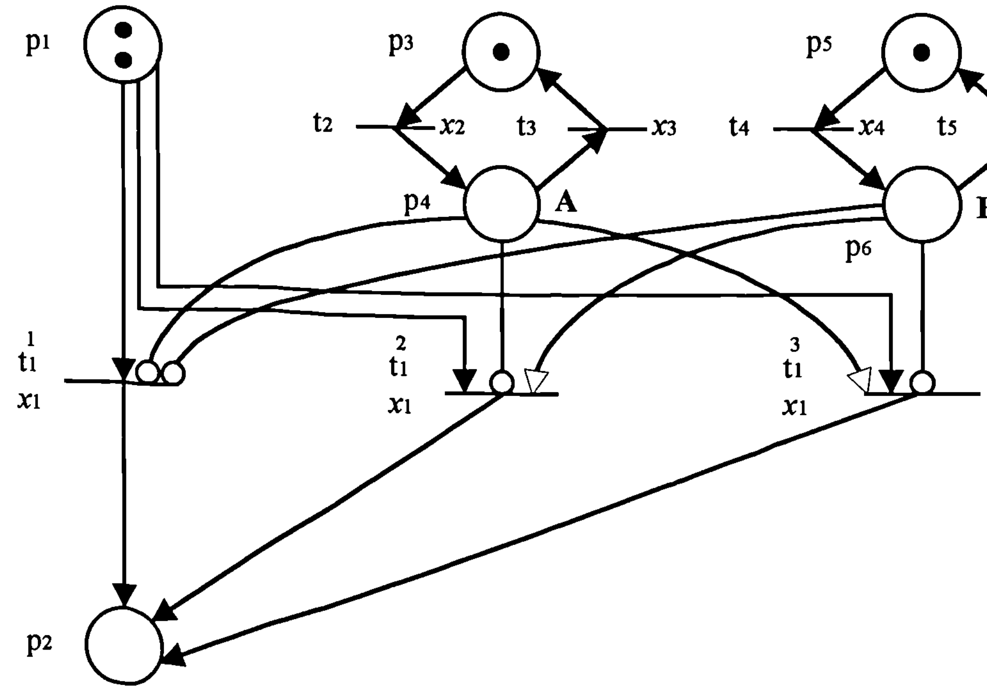
To explain this case, consider the uncontrolled APN model, shown in Fig. 4.4.(a), and its RG, shown in Fig. 4.4.(b). Now, assume that the TPM rule is as follows:

*if* <M($p_4$) = 0> OR <M($p_6$) = 0>
*then* <$t_1$ is to be enabled>

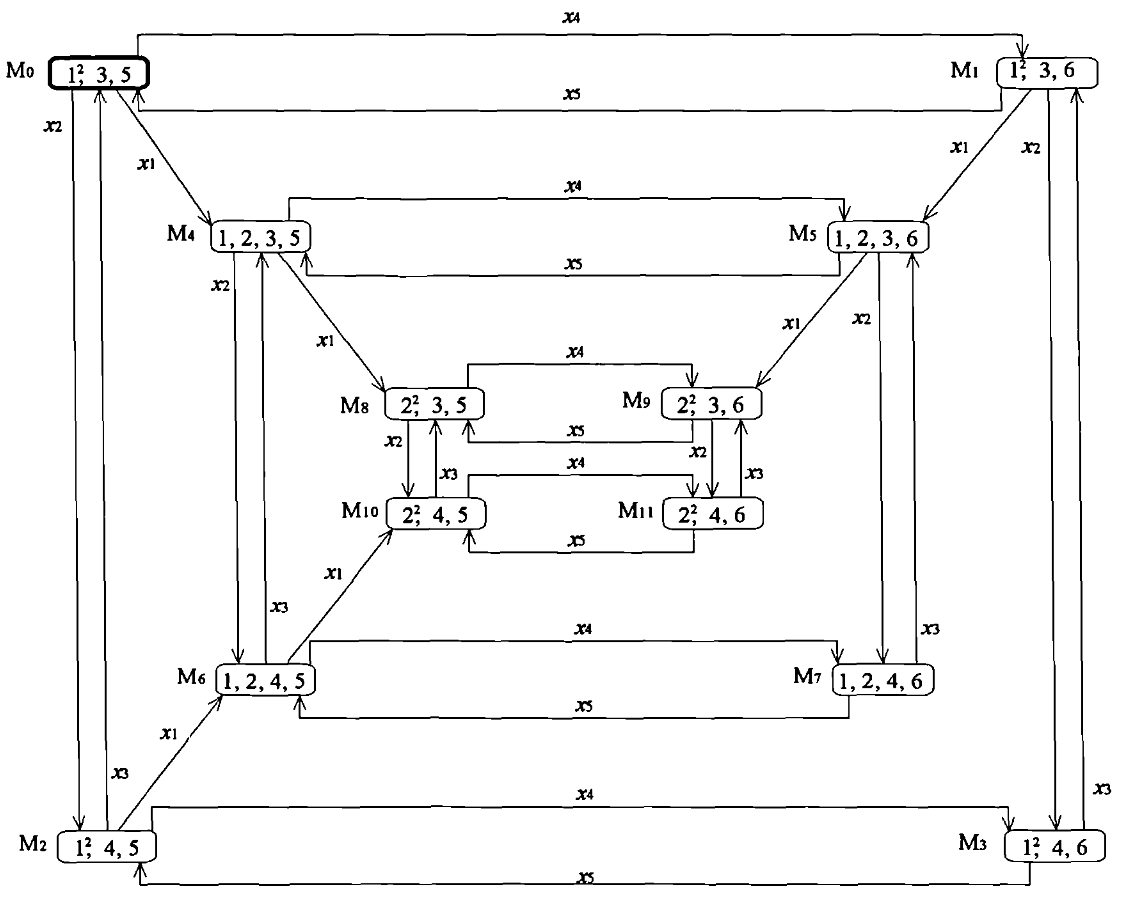
In order to obtain the controlled model of the system, the uncontrolled model, shown in Fig. 4.4.(a) and the TPM rule, given above, are used. To accommodate the logical OR function transition $t_1$ is duplicated as $t_1^1$, $t_1^2$ and $t_1^3$ in the controlled model. Then, enabling and inhibitor arcs are connected from places $p_4$ and $p_6$ to transitions $t_1^1$, $t_1^2$ and $t_1^3$ as shown in Fig. 4.8.(a) such that the TPM rule is satisfied. The controlled model is shown in Fig. 4.8.(a). The RG of this controlled model is shown in Fig. 4.8.(b), in which arcs $M_3$ [$\chi_1$> $M_7$, and $M_7$ [$\chi_1$> $M_{11}$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$ and no token either in place $p_4$ 'or' in place $p_6$, and if the firing condition $\chi_1$ is occurs. This also means that if there is a token each in places $p_4$ and $p_6$, then transition $t_1$ is blocked. Therefore, the complement of the TPM rule given above is as follows:

*if* <M($p_4$) = 1> AND <M($p_6$) = 1>

*then* <$t_1$ is to be blocked>



(a)



(b)

Figure 4.8. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

### 4.2.2.2.2. The OR function and checking the presence of markings

In this case the structure of the TPM rule is as follows:

*if* <M($p_1$) = 1> OR <M($p_2$) = 1> OR <M($p_3$) = 1> OR..........

*then* <a controllable transition is to be enabled>

To explain this case, consider the uncontrolled APN model, shown in Fig. 4.4.(a), and its RG, shown in Fig. 4.4.(b). Now, assume that the TPM rule is as follows:

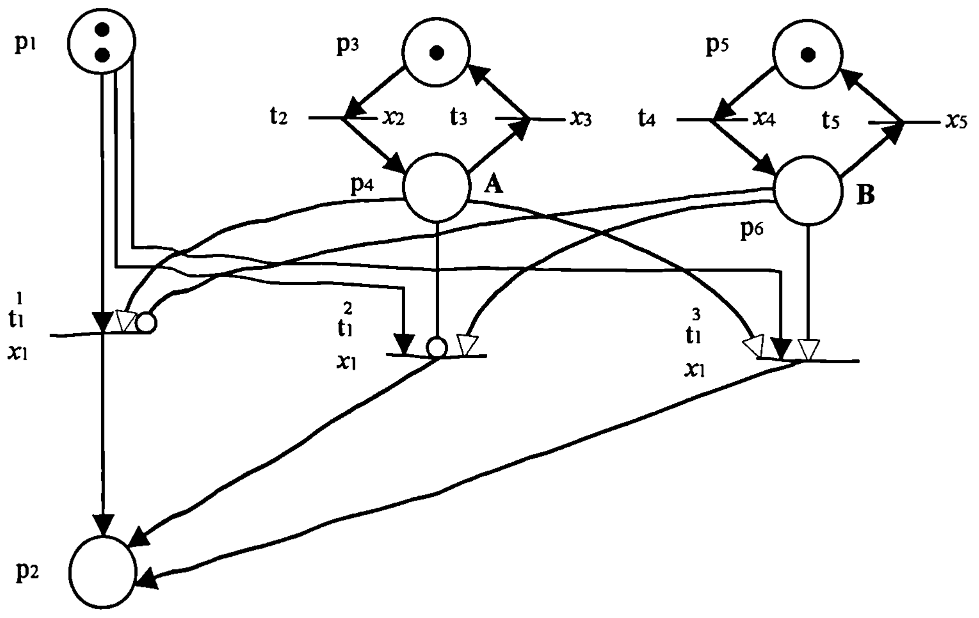
*if* $<M(p_4) = 1>$ OR $<M(p_6) = 1>$

*then* $<t_1$ is to be enabled$>$

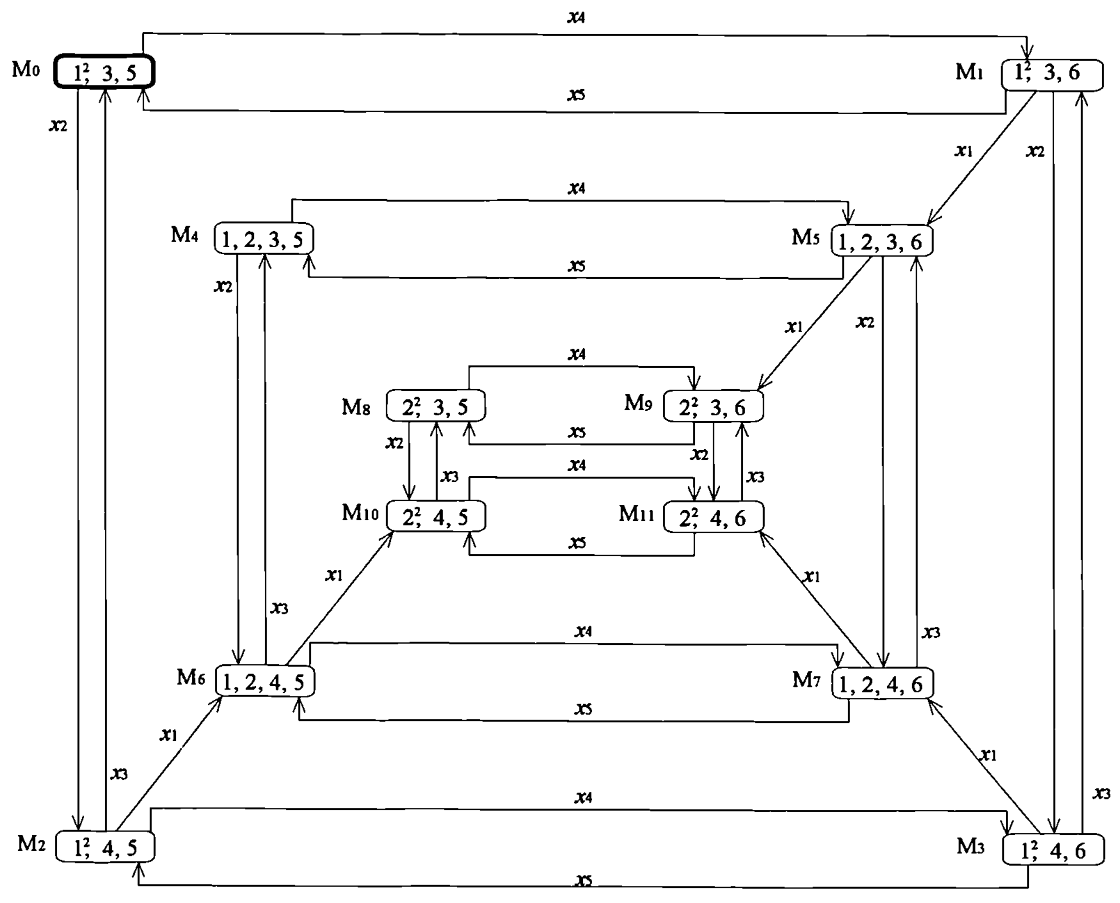
In order to obtain the controlled model of the system, the uncontrolled model, shown in Fig. 4.4.(a) and the TPM rule, given above, are used. To accommodate the logical OR function transition $t_1$ is duplicated as $t_1^1$, $t_1^2$ and $t_1^3$ in the controlled model. Then enabling and inhibitor arcs are connected from places $p_4$ and $p_6$ to transitions $t_1^1$, $t_1^2$ and $t_1^3$ as shown in Fig. 4.9.(a) such that the TPM rule is satisfied. The controlled model is shown in Fig. 4.9.(a). The RG of this controlled model is shown in Fig. 4.9.(b), in which arcs $M_0$ $[\chi_1> M_4$, and $M_4$ $[\chi_1> M_8$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$ and a token either in place $p_4$ 'or' in place $p_6$, and if the firing condition $\chi_1$ is occurs. This also means that if there is no token in place $p_4$ and place $p_6$, then transition $t_1$ is blocked. Therefore, the complement of the TPM rule given above is as follows:

*if* $<M(p_4) = 0>$ AND $<M(p_6) = 0>$

*then* $<t_1$ is to be blocked$>$



(a)



(b)

Figure 4.9. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

### 4.2.2.2.3. The OR function and checking both the presence and absence of markings

In this case the structure of the TPM rule is as follows:

*if* <M($p_1$) = 1 (or 0) > OR <M($p_2$) = 0 (or 1) > OR..........

*then* <a controllable transition is to be enabled>

To explain this case, consider the uncontrolled APN model, shown in Fig. 4.4.(a), and its RG, shown in Fig. 4.4.(b). Now, assume that the TPM rule is as follows:

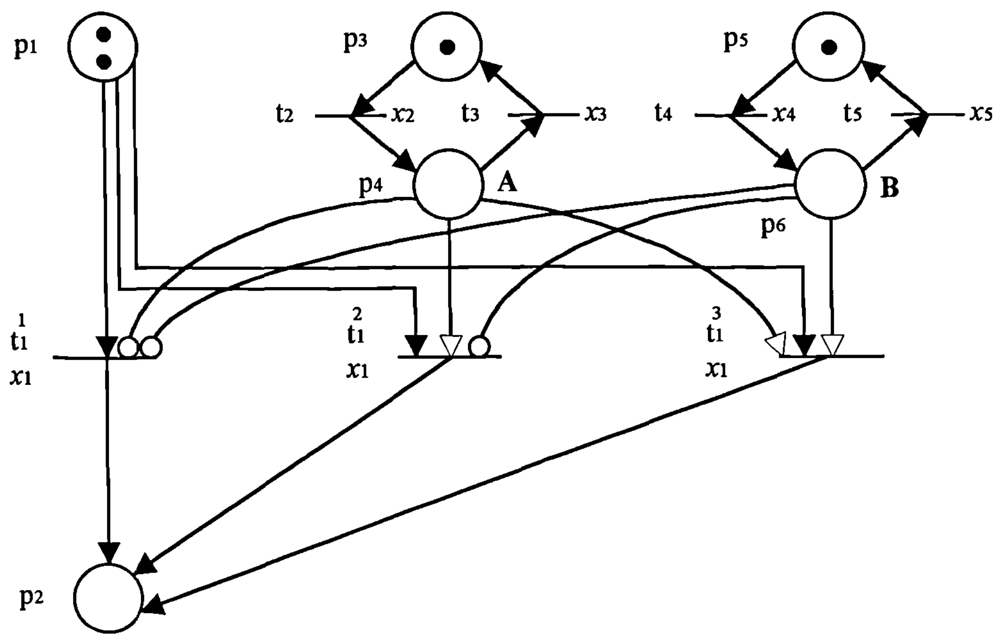
*if* <$M(p_4) = 1$> OR <$M(p_6) = 0$>

*then* <$t_1$ is to be enabled>

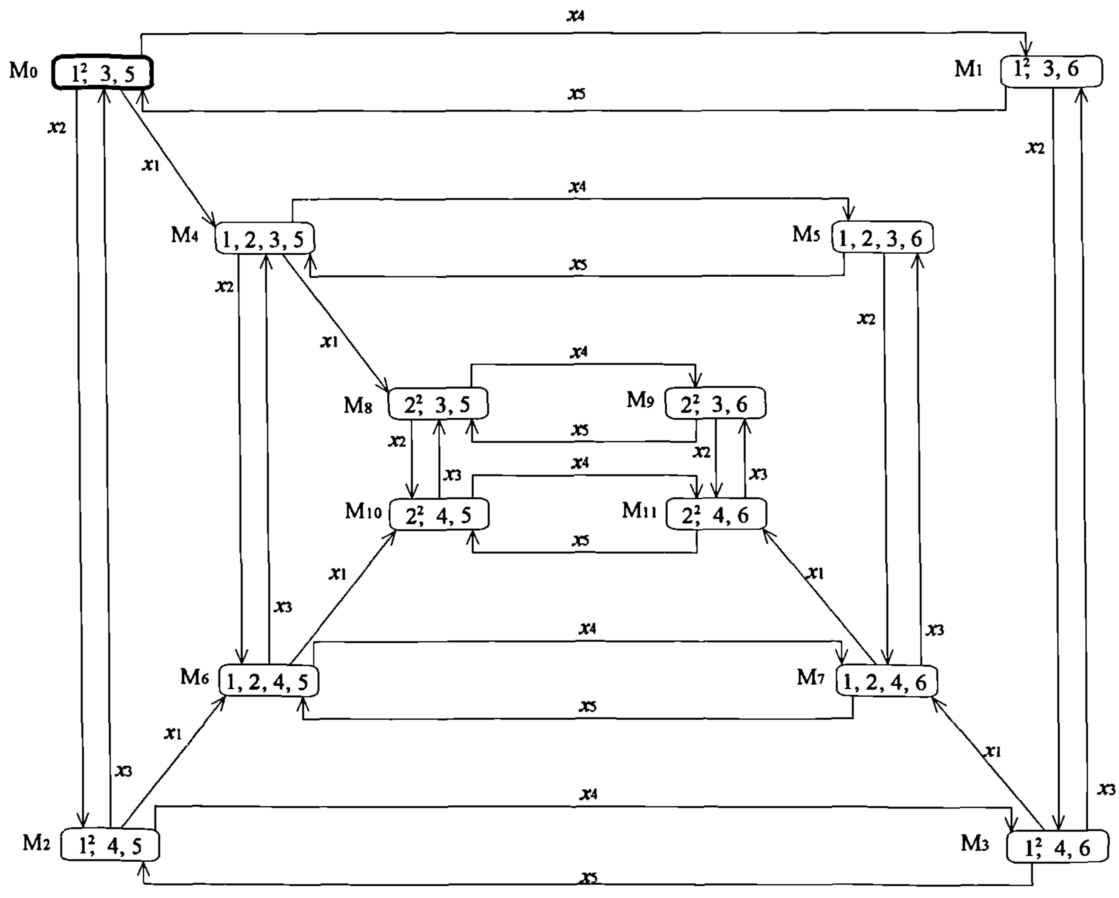
In order to obtain the controlled model of the system, the uncontrolled model, shown in Fig. 4.4.(a) and the TPM rule, given above, are used. To accommodate the logical OR function transition $t_1$ is duplicated as $t_1^1$, $t_1^2$ and $t_1^3$ in the controlled model. Then enabling and inhibitor arcs are connected from places $p_4$ and $p_6$ to transitions $t_1^1$, $t_1^2$ and $t_1^3$ as shown in Fig. 4.10.(a) such that the TPM rule is satisfied. The controlled model is shown in Fig. 4.10.(a). The RG of this controlled model is shown in Fig. 4.10.(b), in which arcs $M_1$ [$\chi_1$> $M_5$, and $M_5$ [$\chi_1$> $M_9$ of the uncontrolled model do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$ and a token in place $p_4$ 'or' no token in place $p_6$, and if the firing condition $\chi_1$ is occurs. This also means that if there is no token in place $p_4$ and a token in place $p_6$, then transition $t_1$ is blocked. Therefore, the complement of the TPM rule given above is as follows:

*if* <$M(p_4) = 0$> AND <$M(p_6) = 1$>

*then* <$t_1$ is to be blocked>



(a)



(b)

Figure 4.10. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

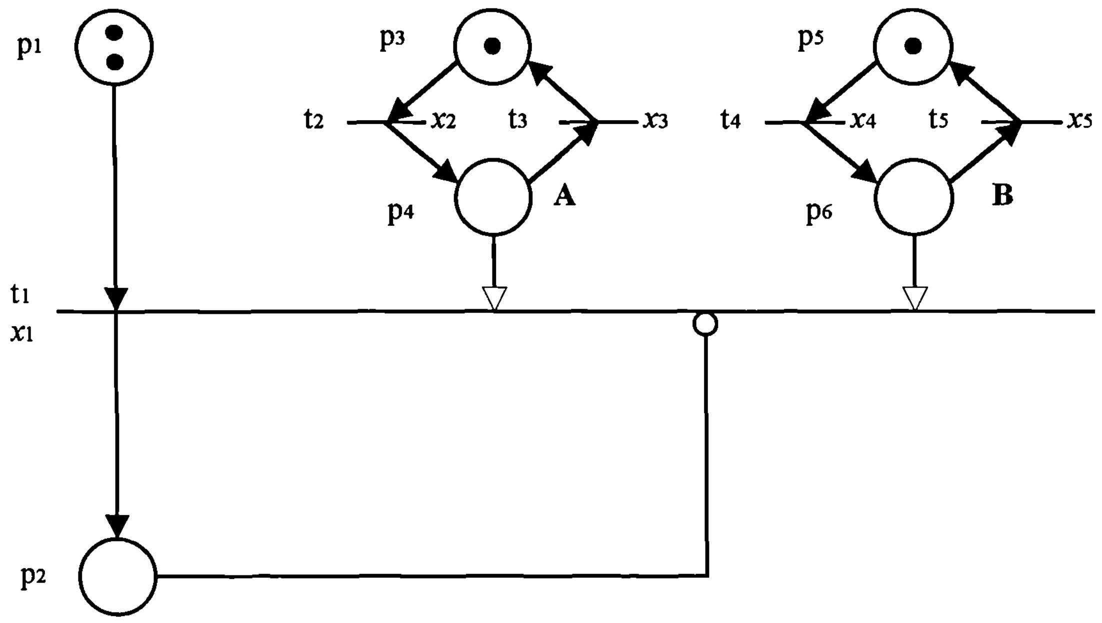
### 4.2.3. Reduction of the Reachable Markings by means of the TPM rules

When obtaining the controlled model of a system by means of the TPM rules, the number of arcs and markings (states), which appear in the reachability graph (RG) of the controlled model is less than the ones that appear in the RG of the uncontrolled model. The examples considered in the previous sections show the reduction of the arcs in the controlled model. To show how the number of markings is reduced, consider the uncontrolled model of the system, shown in Fig. 4.4.(a), and its RG, shown in Fig. 4.4.(b). Now, assume that the TPM rule is as follows:

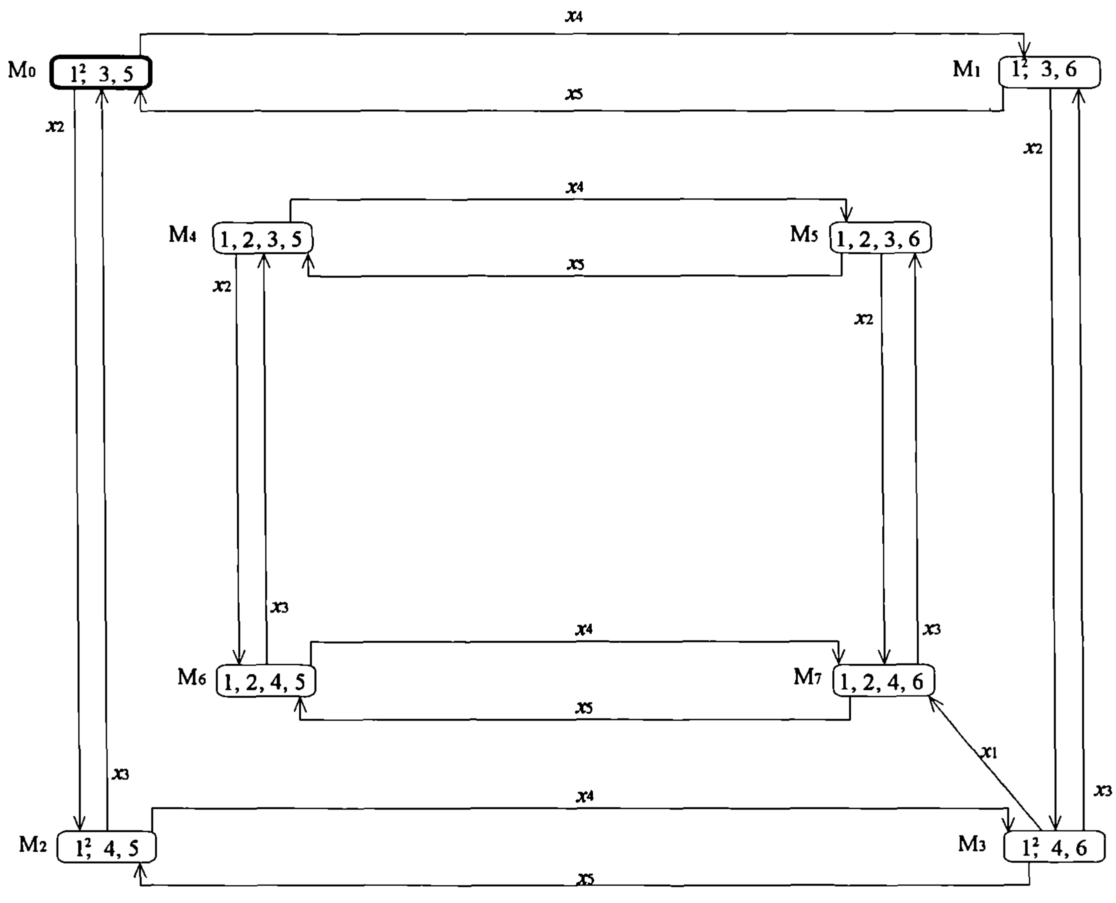
*if* <$M(p_2) = 0$> AND <$M(p_4) = 1$> AND <$M(p_6) = 1$>
*then* <$t_1$ is to be enabled>

In order to obtain the controlled model of the system from the uncontrolled model, shown in Fig. 4.4.(a), and the TPM rules, given above, an inhibitor arc $In(p_2, t_1)$ is connected from place $p_2$ to transition $t_1$ and two enabling arcs $En(p_4, t_1)$ and $En(p_6, t_1)$ are connected from places $p_4$ and $p_6$ to transition $t_1$ as shown in Fig. 4.11.(a), which shows the controlled model of the system. The RG of this controlled model is shown in Fig. 4.11.(b), in which markings $M_8$, $M_9$, $M_{10}$ and $M_{11}$ of the uncontrolled model together with fifteen arcs do not exist. This means that transition $t_1$ can only fire, when there is a token(s) in place $p_1$, a token in place $p_4$, a token in place $p_6$ and no token in place $p_2$ and, if the firing condition $\chi_1$ is occurs. This also implies that if there is a token in place $p_2$ or no token in place $p_4$, or no token in place $p_6$, then transition $t_1$ is blocked, i.e., it is not enabled. Therefore the complement of the TPM rule given above is as follows:

*if* <$M(p_2) = 1$> OR <$M(p_4) = 0$> OR <$M(p_6) = 0$>
*then* <$t_1$ is to be blocked>



(a)



(b)

Figure 4.11. (a) The controlled APN model of the system, shown in Fig. 4.4. (b) Its reachability graph.

## 4.3. THE U-TPM RULE METHOD

In this method, as shown in Fig. 4.12, *the supervisor* consists of a controlled APN model of the DES. However, in this case the controlled model is obtained by connecting enabling arcs from places within the model to its controllable transitions, such that the control policy is satisfied. This means that the model has an enabling action over itself. The supervisory control policy is a static table that provides a list of places of the model from which controllable transitions of the model will be enabled. The control policy is represented by the corresponding TPM rules. To obtain the controlled model, the TPM rules are implemented by enabling arcs such that in the controlled model the forbidden state specifications are met. The U-TPM rule method for the supervisory control of the DESs is divided into the following steps:

Step 1- Design the uncontrolled model of the system using APNs.

Step 2- Determine the control policy

  Step 2.1. Generate the reachability graph of the APN model

  Step 2.2. Identify and remove the "bad states" from the reachability graph

  Step 2.3. Determine the control policy

Step 3- Construct the controlled model of the system

Step 4- Implement the supervisor (the controlled model) on a programmable logic controller (PLC) as ladder logic diagrams (LLDs)

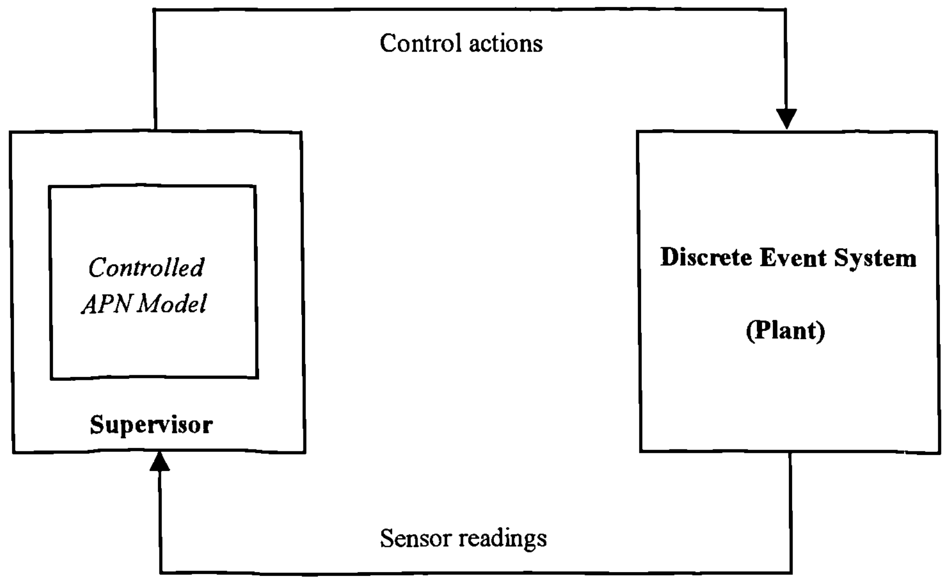


Figure 4.12. The use of controlled APN model as a supervisor in supervisory control in the U-TPM rule method.

In the U-TPM rule method, the uncontrolled model of the system is obtained using APNs, as explained in the inhibitor arc method. Note that the implementation of the supervisor in this method is carried out by using the token passing logic (TPL) methodology as described in the inhibitor arc method. The reachability graph of the APN model (uncontrolled model), that shows the whole state space of the system, is generated and the "bad states", "bad transitions", "blocking states", and "unreachable states" are identified and removed from the reachability graph (RG), yielding the FRRG. Then the FRRG is used in determining the control policy, which is a static table that lists some places of the model from which the controllable transitions will be enabled by enabling arcs. Then the control policy is converted into the corresponding TPM rules. However, it is important to note that when implementing the TPM rules, i.e., connecting enabling arcs from a place to a controllable transition, if there is already an ordinary arc connected from the same place to the same transition, then the enabling arc is simply omitted, because the ordinary arc will do the same job and therefore there is no need for another arc. In order to determine the control policy, firstly, the uncontrolled APN model is considered and the controllable transitions, which are related to the forbidden state specifications, are determined. Each related controllable transition within the APN model is taken into account and the arcs, representing the firing of these controllable events, are identified from the FRRG. In one column of this static table the list of the controllable transitions is provided. In the next column, places of the uncontrolled model, that are to be used to enable these transitions, are provided. This represents the control policy of the U-TPM rule method.

In this case, the APN model has an enabling action over itself. In order to obtain the controlled APN model of the system, places of the uncontrolled model are connected to the controllable transitions with enabling arcs, according to the TPM rules. This process yields the controlled model of the system, that is maximally permissive and behaves according to the specifications.

### 4.3.1. Example for the U-TPM rule method

Consider the manufacturing system example introduced in the section 3.3.5 in the Chapter 3. The APN model (uncontrolled model) of the manufacturing system is shown in Fig. 3.5, and for the forbidden state specifications, given in the section 3.3.5.1, the FRRG is shown in Fig. 3.9. These results are obtained by following the design steps given in the section 4.3.

Now, it is necessary to determine the control policy for the U-TPM rule method. As is known the controllable transitions $t_1$ and $t_3$, with events $s_1$ and $r_1$, of the APN model, are related with the forbidden state specifications. From the FRRG, it can be seen that firing of transition $t_1$ is represented only by arcs $M_0[s_1>M_2$, $M_6[s_1>M_9$, and $M_7[s_1>M_{10}$. These arcs are called the *identical arcs* for the controllable transition $t_1$. The input markings of these arcs are markings $M_0 = (1, 4, 6)$, $M_6 = (1, 4, 7)$, and $M_7 = (1, 4, 8)$ respectively. These markings of the FRRG are called the *base markings* for the transition $t_1$. Therefore in the control policy, these base markings $M_0$, $M_6$, and $M_7$ are identified as markings at which the controllable transition $t_1$ is to be enabled by enabling arcs. This is the control policy for the controllable transition $t_1$. The controllable transition $t_3$ with event $r_1$ has the identical arcs $M_1[r_1>M_2$, and $M_8[r_1>M_9$ and therefore it has the base markings $M_1 = (3, 4, 6)$ and $M_8 = (3, 4, 7)$ from the FRRG. Thus, in the control policy, these base markings $M_1$ and $M_8$ are identified as markings at which the controllable transition $t_3$ is to be enabled by enabling arcs. This is the control policy for the transition $t_3$. The resulting control policy for the manufacturing system in U-TPM rule method is given in Table 4.1.

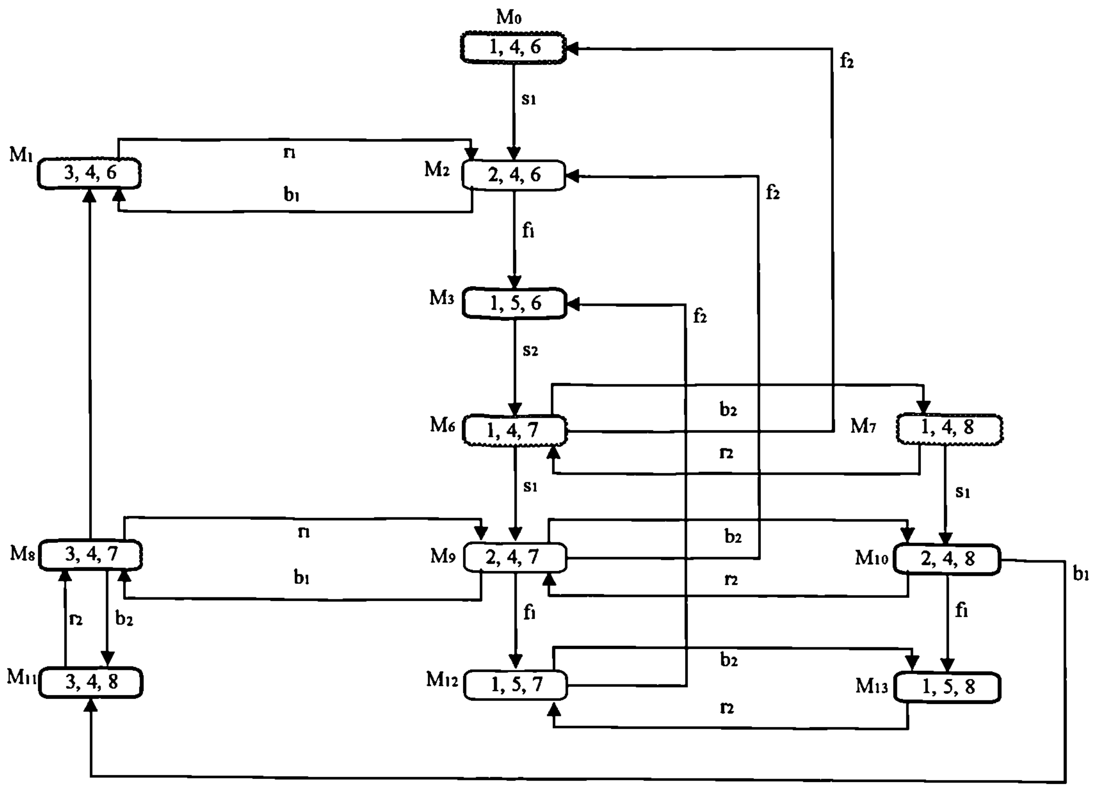


Figure 4.13. The final reduced reachability graph (FRRG) used in determining the control policy in the U-TPM rule method.

| Related controllable transition | Markings at which the transition is to be enabled |
|---|---|
| $t_1$ | $M_0 = (1, 4, 6)$ *or* $M_6 = (1, 4, 7)$ *or* $M_7 = (1, 4, 8)$ |
| $t_3$ | $M_1 = (3, 4, 6)$ *or* $M_8 = (3, 4, 7)$ |

Table 4.1. The control policy for the manufacturing system in the U-TPM rule method.

Note that the control policy can be written as TPM rules as follows:

1. *if* $< M_0 >$ *or* $< M_6 >$ *or* $< M_7 >$
   *then* <transition $t_1$ is to be enabled>

2. *if* $< M_1 >$ or $< M_8 >$
   *then* <transition $t_3$ is to be enabled>

Note that these TPM rules can be re-written by separating the *or* operation for each marking as follows:

1.i. *if* $< M_0 >$
 *then* <transition $t_1$ is to be enabled>
*or*
ii. *if* $< M_6 >$
 *then* <transition $t_1$ is to be enabled>
*or*
iii. *if* $< M_7 >$
 *then* <transition $t_1$ is to be enabled>

2.i. *if* $< M_1 >$
 *then* <transition $t_3$ is to be enabled>
*or*
ii. *if* $< M_8 >$
 *then* <transition $t_3$ is to be enabled>

These rules can be represented by putting the individual markings in the *if* part of the rules as follows:

1.i. *if* $<M(p_1) = 1>$ AND $<M(p_4) = 1>$ AND $<M(p_6) = 1>$
 *then* <transition $t_1$ is to be enabled>
*or*
ii. *if* $<M(p_1) = 1>$ AND $<M(p_4) = 1>$ AND $<M(p_7) = 1>$
 *then* <transition $t_1$ is to be enabled>
*or*
ii. *if* $<M(p_1) = 1>$ AND $<M(p_4) = 1>$ AND $<M(p_8) = 1>$
 *then* <transition $t_1$ is to be enabled>

2.i. *if* $<M(p_3) = 1>$ AND $<M(p_4) = 1>$ AND $<M(p_6) = 1>$
 *then* <transition $t_3$ is to be enabled>
*or*
ii. *if* $<M(p_3) = 1>$ AND $<M(p_4) = 1>$ AND $<M(p_7) = 1>$
 *then* <transition $t_3$ is to be enabled>

The controlled model (i.e., the supervisor), shown in Fig. 4.14, for the *U-TPM rule method* is obtained by using the uncontrolled APN model, shown in Fig. 3.5, and the TPM rules given below. Since the TPM rule 1 is split into three parts and contains an *or* operation, transition $t_1$ of the uncontrolled APN model is replaced with the transitions $t_1^1$, $t_1^2$ and $t_1^3$ within the controlled APN model. Similarly, since the TPM rule 2 is split into two parts and contains an *or* operation, transition $t_3$ of the uncontrolled APN model is duplicated to accommodate the *or* operation within the Petri net formalism and replaced with transitions $t_3^1$ and $t_3^2$ within the controlled APN model. Therefore, the TPM rules are modified as follows:

1.i. *if* <M($p_1$) = 1> AND <M($p_4$) = 1> AND <M($p_6$) = 1>
*then* <transition $t_1^1$ is to be enabled>
*or*
ii. *if* <M($p_1$) = 1> AND <M($p_4$) = 1> AND <M($p_7$) = 1>
*then* <transition $t_1^2$ is to be enabled>
*or*
ii. *if* <M($p_1$) = 1> AND <M($p_4$) = 1> AND <M($p_8$) = 1>
*then* <transition $t_1^3$ is to be enabled>

2.i. *if* <M($p_3$) = 1> AND <M($p_4$) = 1> AND <M($p_6$) = 1>
*then* <transition $t_3^1$ is to be enabled>
*or*
ii. *if* <M($p_3$) = 1> AND <M($p_4$) = 1> AND <M($p_7$) = 1>
*then* <transition $t_3^2$ is to be enabled>

In order to implement the TPM rule 1.i, the enabling arcs $En(p_4, t_1^1)$ and $En(p_6, t_1^1)$ are connected from places $p_4$ and $p_6$ to the controllable transition $t_1^1$. However, since there is an ordinary arc connecting place $p_1$ to the controllable transition $t_1^1$, it is not necessary to connect an enabling arc from $p_1$ to $t_1^1$. To implement TPM rule 1.ii, the enabling arcs $En(p_4, t_1^2)$ and $En(p_7, t_1^2)$ are connected from places $p_4$ and $p_7$ to the controllable transition $t_1^2$. However, since there is an ordinary arc connecting place $p_1$ to controllable transition $t_1^2$, it is not necessary to connect an enabling arc from $p_1$ to $t_1^2$. To implement TPM rule 1.iii, the enabling arcs $En(p_4, t_1^3)$ and $En(p_8, t_1^3)$ are connected from places $p_4$ and $p_8$ to the controllable transition $t_1^3$. However, since there is an ordinary arc connecting place $p_1$ to controllable transition $t_1^3$, it is not necessary to connect an enabling arc from $p_1$ to $t_1^3$. Similarly, to implement the TPM rule 2.i, the enabling arcs *En*($p_4$, $t_3^1$), and *En*($p_6$, $t_3^1$) are connected from places $p_4$ and $p_6$ to the controllable transition $t_3^1$. However, since there is an ordinary arc connecting place $p_3$ to the controllable transition $t_3^1$, it is not necessary to connect an enabling arc from $p_3$ to $t_3^1$. To implement TPM rule 2.ii, the enabling arcs *En*($p_4$, $t_3^2$), and *En*($p_7$, $t_3^2$) are connected from places $p_4$ and $p_7$ to the controllable transition $t_3^2$. However, since there is an ordinary arc connecting place $p_3$ to the controllable transition $t_3^2$, it is not necessary to connect an enabling arc from $p_3$ to $t_3^2$. This process yields the controlled model of the system.

Note that the controlled model (the supervisor) obtained does not contradict the forbidden state specifications, i.e., the controlled behaviour of the system is nonblocking. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications. It is also important to point out that the controlled model (the supervisor) obtained is correct by construction.

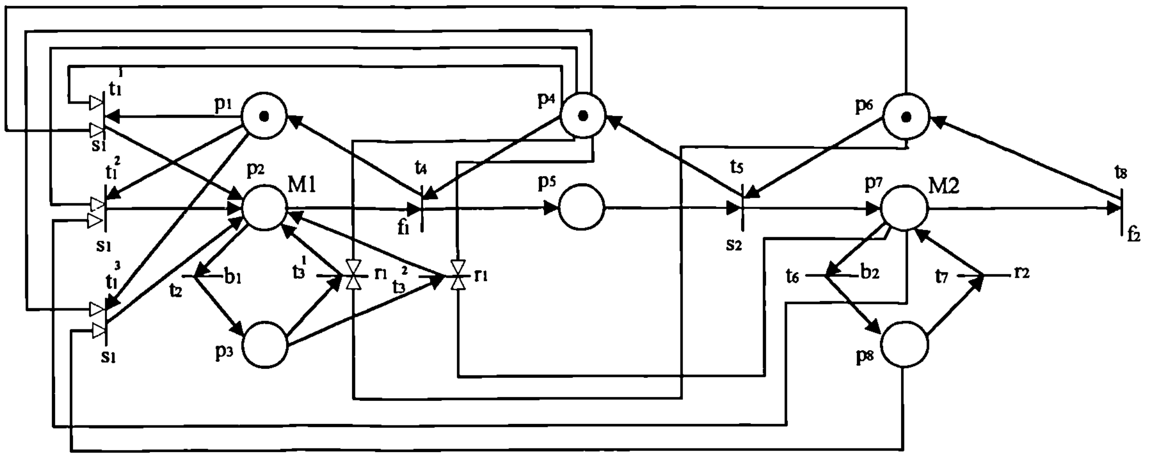


Figure 4.14. The controlled APN model (supervisor) of the manufacturing system in the U-TPM rule method.

## 4.4. THE C-TPM RULE METHOD

In this method, as shown in Fig. 4.15, the supervisor consists of a controlled APN model of the DES. The TPM rules are used to obtain the controlled model from the uncontrolled model. Note that in this case, the TPM rules are assumed to be given. In fact, the forbidden state specifications are converted into related TPM rules. In these rules some markings of the uncontrolled model are identified for restricting the firing of some of the controllable transitions, which are related to the forbidden state specifications. However, in this case when the controlled model of the system is constructed it is necessary to verify its correctness by using reachability graph (RG) analysis. In this method, it is not clear whether the controlled model obtained is maximally permissive. However, it may be proved by comparing the uncontrolled behaviour with the controlled behaviour, but this process requires further RG analysis for the uncontrolled model which may be computationally prohibitive for complex systems. The C-TPM rule method is divided into the following steps:

Step 1- Design the uncontrolled model of the system using APNs

Step 2- Convert the forbidden state specifications, given, into related TPM rules

Step 3- Construct the controlled model of the system by combining the uncontrolled model and the TPM rules

Step 4- Generate the reachability graph (RG) of the controlled model

Step 5- Check whether the controlled model behaves according to the specifications: If it does not behave according to the specifications, go to the step 2 and make necessary corrections

Step 6- Implement the supervisor (the controlled model) on a PLC as LLDs

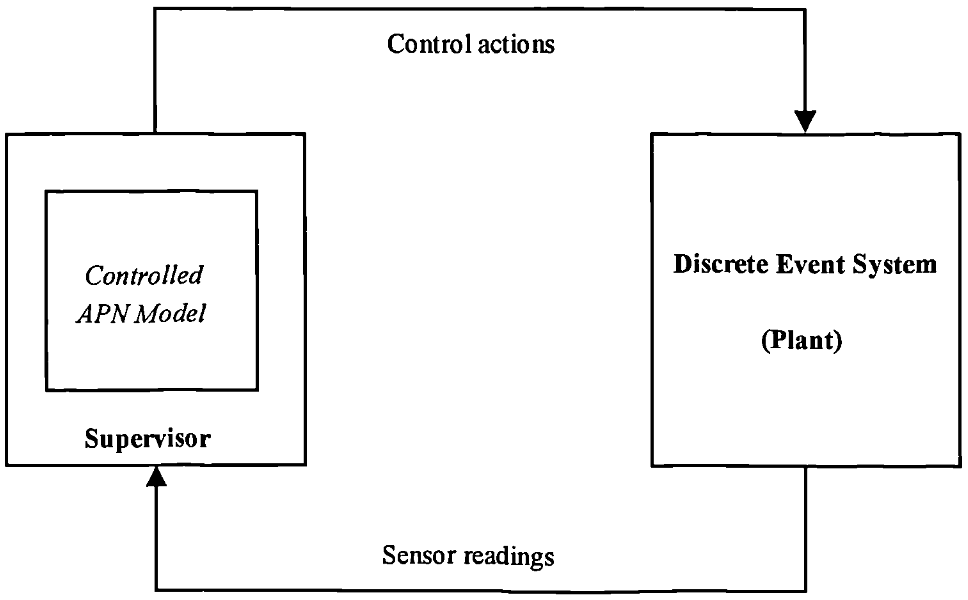


Figure 4.15. The use of the controlled APN model as the supervisor in supervisory control in the C-TPM rule method.

The uncontrolled behaviour of the system is captured by using APN models. A modular modelling concept can be used to obtain structured models. To describe concurrent systems the *concurrent composition* can be used. After obtaining uncontrolled system behaviour as an APN model, the forbidden state specifications are considered. These specifications are converted into a set of TPM rules such that these rules specifies the desired behaviour of the controlled system.

After obtaining the controlled model of the system it is necessary to prove that the controlled model behaves according to the specifications given. To do this the RG of the controlled model is generated and then the markings within the RG is checked to see if they all represent the desirable system behaviour. If all the markings within the RG do not contradict the specifications given then this means that the controlled model obtained is correct. If some of the markings within the RG represent undesirable system behaviour then this means that the controlled model obtained is not correct. Therefore it must be re-constructed to overcome the problems faced. In this method, it is necessary to point out that the behaviour of the controlled model of the system may not be maximally permissive. In order to check whether the behaviour of the controlled model of the system is maximally permissive, it is necessary to generate the RG of the uncontrolled model and to compare it with the RG obtained for the controlled model. This process might be computationally prohibitive for complex systems. Note that the implementation of the supervisor in this method is carried out by using the token passing logic (TPL) methodology as described in the inhibitor arc method.

### 4.4.1. Example 1 for the C-TPM rule method

Let us consider the manufacturing system introduced in section 3.3.5. Note that the APN model of the manufacturing system representing the uncontrolled system behaviour is shown in Fig. 3.5. This represents the first design step of the method 6, given in the section 4.4. The forbidden state specifications are as follows:

Specification 1. The buffer must not overflow or underflow: Machine 1 may not start operating while a workpiece is present in the buffer.

Specification 2. Machine 2 has repair and return to service priority over Machine 1: in case both machines are down, Machine 2 must be repaired and returned to service first.

Consider the first specification, it implies that if Machine 1 is already idle and the buffer is empty then Machine 1 can start its operation. Therefore the first specification can be re-written as follows:

|  |  |  |
|---|---|---|
| Specification 1 : | *if* | <Machine 1 is idle> AND <the buffer is empty> |
|  | *then* | <Machine 1 can be started> |

Similarly, consider the second specification, which implies that Machine 1 can be repaired and returned to service provided that it is down and at that instant Machine 2 is not down, therefore the second specification can be re-written as follows:

Specification 2 : *if* <Machine 1 is down> AND <Machine 2 is NOT down>
*then* <Machine 1 can be repaired>

Now, in order to obtain the TPM rules for these two specifications, the *if* part of the specifications can be represented with the related markings and the *then* part of the specifications can be represented with the related controllable transitions of the uncontrolled model of the manufacturing system. Now, firstly the first specification is considered. In the *if* part of the first TPM rule, <Machine 1 is idle> can be represented with <$M(p_1) = 1$>, because when there is a token in place $p_1$ this means that Machine 1 is idle. Similarly, <$M(p_4) = 1$> can be put instead of <the buffer is empty> because when there is a token in place $p_4$ this means that the buffer is empty. Now, consider the *then* part of the first TPM rule. As can be seen from the uncontrolled model of the manufacturing system when there is a token in place $p_2$, the action M1 is active, i.e., Machine 1 is switched *on*. To control this process the controllable transition $t_1$ with firing condition (event) $s_1$ is used. That is, when $t_1$ fires, Machine 1 is switched *on*. Therefore, the *then* part of the specification 1, i.e., <Machine 1 can be started>, is represented with <transition $t_1$ is to be enabled> in the *then* part of the TPM rule 1. As a result, the TPM rule 1 for the specification 1 is as shown below. Now, secondly, the second specification is considered. In the *if* part of the second TPM rule <Machine 1 is down> can be represented with <$M(p_3) = 1$>, because when there is a token in place $p_3$ this means that Machine 1 is down. Similarly, <$M(p_8) = 0$> can be put instead of <Machine 2 is NOT down>, because when there is no token in place $p_8$ this means that Machine 2 is not down. Now, consider the *then* part of the second TPM rule. Using the similar approach, the *then* part of the specification 2, i.e., <Machine 1 can be repaired>, is represented with <transition $t_3$ is to be enabled> in the *then* part of the TPM rule 2. As a result, the TPM rule 2 for the specification 1 is as shown below.

TPM rule 1 : *if* <$M(p_1) = 1$> AND <$M(p_4) = 1$>
*then* <transition $t_1$ is to be enabled>

TPM rule 2 : *if* $<M(p_3) = 1>$ AND $< M(p_8) = 0>$
*then* <transition $t_3$ is to be enabled>

After obtaining the TPM rules, the controlled model of the system can be obtained from the uncontrolled model (APN model). To implement the TPM rule 1, an enabling arc, *En* $(p_4, t_1)$, is connected from place $p_4$ to transition $t_1$. Similarly, to realise the TPM rule 2, an inhibitor arc, *In* $(p_8, t_3)$, is connected from place $p_8$ to transition $t_3$. Note that since there are ordinary arcs connecting place $p_1$ to controllable transition $t_1$ and connecting place $p_3$ to controllable transition $t_3$, superfluous enabling arcs are not connected from the same places to the same transitions. Finally, The controlled model (controlled APN model) of the system is constructed as shown in Fig. 4.16.

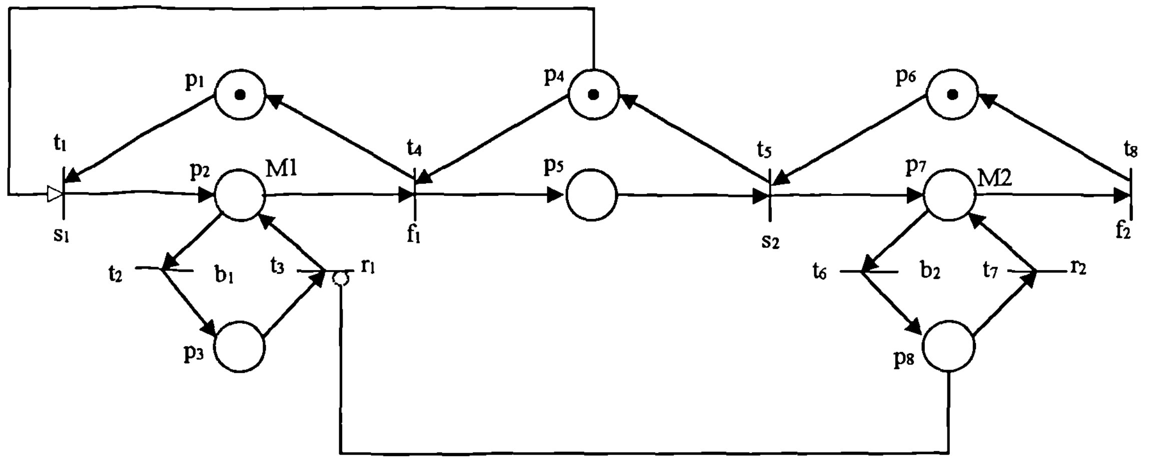


Figure 4.16. The controlled APN model (supervisor) of the manufacturing system in the C-TPM rule method.

In order to make sure about the correctness of the controlled model behaviour, the RG, shown in Fig. 4.17, is generated. Its markings are also shown in Table 4.2. When the RG is checked, it can be seen that there is no undesirable system operation, i.e., the behaviour of the controlled model does not contradict the specifications given. Therefore, the controlled model is correct. As a matter of fact the RG shown in Fig. 4.10

is identical with the FRRGs obtained in the first five methods. This means that we have not only obtained the correct controlled model, but also obtained the maximally permissive system operation, in this particular example. However, this may not be the case for every problem. In other words, in order to prove the maximally permissiveness of a controlled model, it is necessary to generate the RG of the uncontrolled model and compare it with the RG obtained for the controlled model.

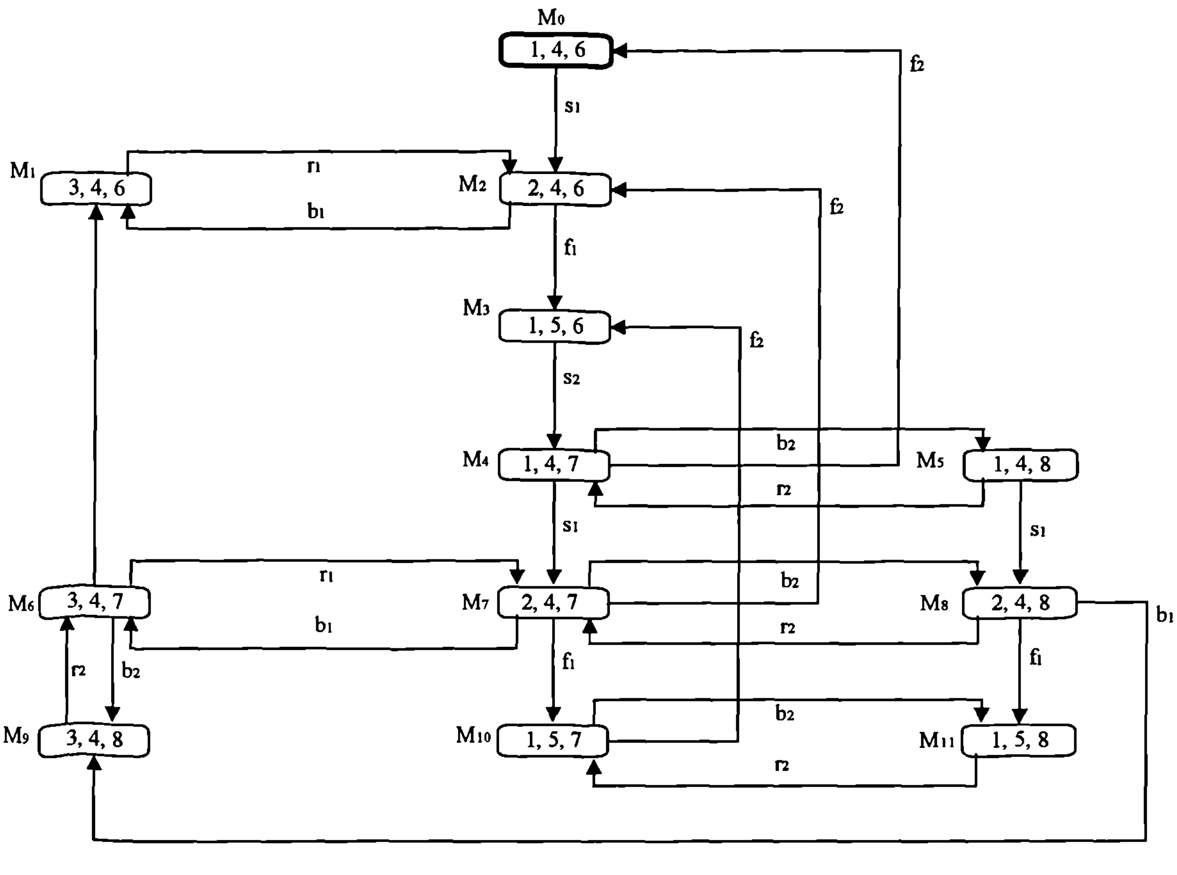


Figure 4.17. The reachability graph of the controlled APN model in the C-TPM rule method.

| Marking |
|---|
| $M_0 = (1, 4, 6)$ |
| $M_1 = (3, 4, 6)$ |
| $M_2 = (2, 4, 6)$ |
| $M_3 = (1, 5, 6)$ |
| $M_4 = (1, 4, 7)$ |
| $M_5 = (1, 4, ,8)$ |
| $M_6 = (3, 4, 7)$ |
| $M_7 =(2, 4, 7)$ |
| $M_8 =(2, 4, 8)$ |
| $M_9 =(3, 4, 8)$ |
| $M_{10} =(1, 5, 7)$ |
| $M_{11} =(1, 5, 8)$ |

Table 4.2. The markings appearing in the RG.

### 4.4.2. Example 2 for the C-TPM rule method

The C-TPM rule method provides the simplest solution compared with the previous five methods and it does not suffer from the state explosion problem. To show this the same manufacturing system is considered, in which the buffer has the capacity of three. For this case, the uncontrolled model of the manufacturing system is as shown in Fig. 4.11. Note that the only difference in this case is the capacity of the buffer, i.e., $M_0$ $(p_4) = 3$.

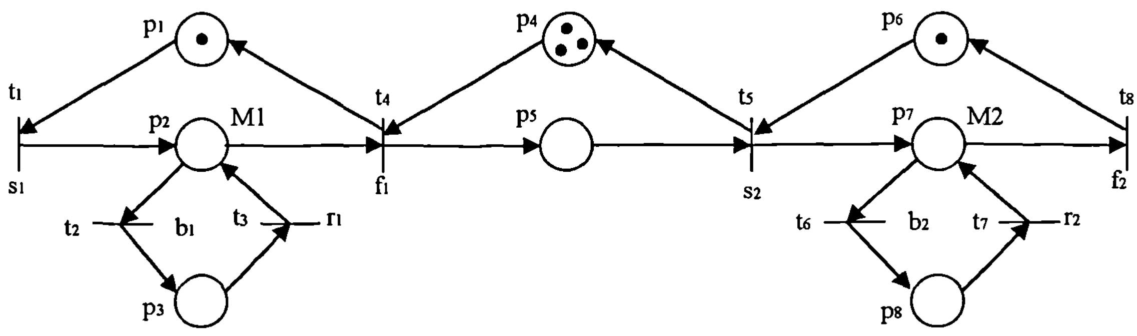


Figure 4.18. The uncontrolled APN model of the manufacturing system, where the buffer has the capacity of three.

Assume that the same forbidden state specifications are given. This means that we have the same TPM rules for the system. Therefore, the controlled model of the system is constructed as shown in Fig. 4.19. Note that the structure of the controlled model, i.e., places, transitions and arcs, is exactly the same as the controlled model constructed in the previous case, where the buffer had the capacity of one. This simply shows that Petri-nets-based supervisors can keep the structure of the net simple even if the marking of the net gets bigger. In this example, if the capacity of the buffer is to be described with tens and hundreds, then the structure of the resulting controlled model will be the same and the only difference will be the number of tokens to be put into place $p_4$. The RG, which reflects the behaviour of the controlled model in this case, has 36 states (markings) reachable and 90 arcs and when it is checked it can be seen that there is no undesirable system operation, i.e., the behaviour of the controlled model, given in Fig. 4.19, does not contradict the specifications given. Therefore, the controlled model obtained is correct.

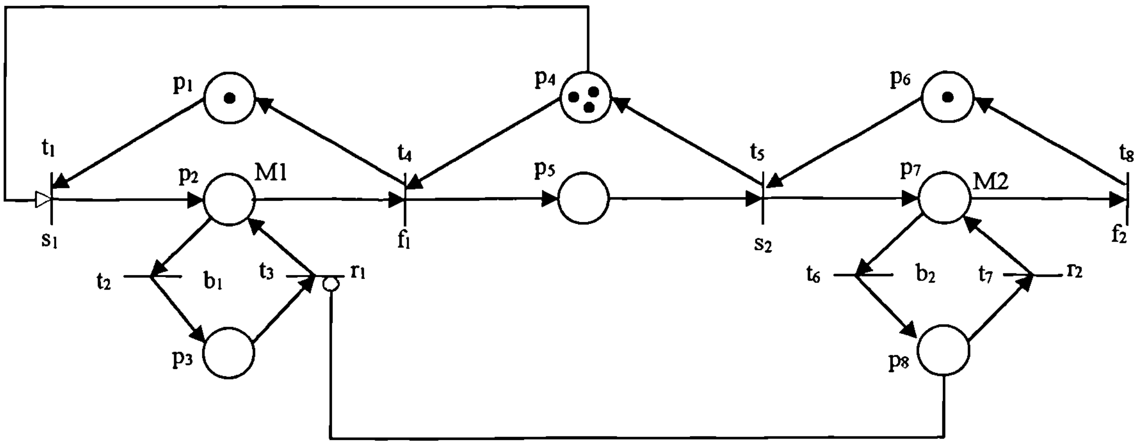


Figure 4.19. The controlled APN model (supervisor) of the manufacturing system, where the buffer has the capacity of three.

## 3.5. DISCUSSION

In this chapter, two new design techniques, called the U-TPM rule method and the C-TPM rule method, have been proposed for the design of compiled supervisors for the control of DESs in the case of the forbidden state problem. These two techniques do not involve any Petri net state machines within the supervisor as were deployed in chapter 3. For this reason, the size of supervisors do not suffer from the state explosion problem. The U-TPM rule method represents a top-down synthesis technique, involving the construction of the reachability graph (RG) of the uncontrolled APN model of the system. In contrast, the C-TPM rule method represents a bottom-up synthesis technique, involving the construction of the RG of the controlled APN model of the system. In the case of the U-TPM rule method the state explosion problem has an effect only on the computation of the supervisor. That is, the computation of the supervisor becomes very difficult as the system becomes bigger. However, the number of places and transitions used in the supervisor does not increase exponentially in the size of the model. In this case, the supervisor is *maximally permissive*, i.e., it does not unnecessarily constrain the system behaviour and *nonblocking*, i.e., it does not contradict the specifications given. In addition, the supervisors obtained are correct by construction. On the other hand, the C-TPM rule method does not suffer from the state explosion problem. However, the correctness of the supervisor obtained must be verified by using reachability graph (RG) analysis. In return this could still pose a problem, because for very big systems the RG of the system could still be very big. In this case, the supervisor is not necessarily maximally permissive. Nevertheless, it may be proved by comparing the uncontrolled behaviour with the controlled behaviour, but this process requires further RG analysis for the uncontrolled model which may be computationally prohibitive for complex systems.

The results obtained in these two methods can be applied to high level manufacturing control, where the role of the supervisor is to coordinate factory-wide control of machines, and to low-level manufacturing control, where the role of the supervisor is to arrange low-level interaction between the control devices, such as motors, actuators, etc. The results obtained in these three methods can be applied to systems that require untimed or timed, safe APNs, i.e., an APN model in which a place can have only one token at most, as well as APN models that can accommodate more than one token a place.

Note that these results are based on the assumption that there is a sufficient number of discrete event actuators, motors, etc. and discrete event sensors available within the system in order to be able to control the system.

# CHAPTER 5

# PETRI-NET-BASED SUPERVISORS FOR THE DESIRED STRING PROBLEM

# CHAPTER 5

# PETRI-NET-BASED SUPERVISORS FOR THE DESIRED STRING PROBLEM

## 5.1. INTRODUCTION

In this chapter, a methodology is proposed for the purpose of designing supervisors for the control of DESs in the case of the *desired string problem*. As addressed by Ramadge and Wonham, (Ramadge and Wonham, Jan. 1987) in the desired string problem a typical objective is to force some desirable event sequences to occur, this might be understood as preventing some undesirable event sequences from occurring. The supervisory control theory is based on finite state machines (FSMs) and formal language concepts. In the original framework, a DES (a plant) and its supervisor are modelled by FSMs. The plant and its supervisor have an identical alphabet set. The plant generates a language and the supervisor accepts the language generated by the plant. These languages, that are representable as FSM models, are called *regular languages* (Kumar and Holloway, 1996). Recently, Petri net models have received attention as an alternative model for investigating discrete event control theory (Sreenivas, 1996; Sreenivas, 1993; Giua and DiCesare, 1991; Giua, 1996; Kumar and Holloway, 1996; Sreenivas and Krogh, 1992). Petri nets have more descriptive power than FSMs in the sense that the set of Petri net languages is a superset of regular languages and they allow a more concise model description (Kumar and Holloway, 1996). In supervisory control the events that are generated in the discrete event system (the plant) are partitioned into *controllable events*, which can be disabled if desired, and *uncontrollable events*, which can not be disabled by control action. The control specification is given as a specification language. Given a discrete event system and a specification language, representing the desired behaviour, also called controller, which dynamically disables some of the controllable events while never trying to prevent any uncontrollable event from occurring, such that the controlled plant behaviour equals to the desired behaviour (Kumar and Holloway, 1996). In this chapter, this type of supervisory control problem is called the *desired string problem*. Note that in the literature the same problem is also called the *forbidden string problem* (Sreenivas, 1996), the *string avoidance problem* (Sreenivas and Krogh, 1992) and the *language control problem* (Kumar and Holloway, 1996). In the desired string problem by dictating only the acceptable string of events, undesired or forbidden strings are eliminated.

The theoretical results obtained in this area of research are very difficult to apply to practical problems and they involve ambiguous textual descriptions or mathematical notations, which are difficult to understand. To overcome these problems, a practical approach is proposed in this chapter. In brief, the desired behaviour of the system given as a string of events is represented as an APN and then it is combined with the uncontrolled model, using the *concurrent composition* (Giua and DiCesare, 1991). This yields the supervised model of the system, which is used as the supervisor to force the system to behave according to the specifications given.

In this chapter, the desired string problem is solved as follows: if the control of a DES includes both the forbidden state problem and the desired string problem, then the forbidden state problem is solved firstly and consequently the desired string problem is solved. If the control of a DES includes only the desired string problem then it is directly solved and in this case no forbidden states are expected to come into existence since the solution simply organises a sequence of events to occur one after another.

To explain the desired string problem within the supervisory control context, now the father and the child example, introduced in the chapter 3, is reconsidered. Note that in this scenario, there is a father and a child in a room, together with a box of matches, some food to eat, some toys to play with, a TV to watch cartoons and finally a knife. In this case, the father plays the role of a supervisor, while the child acts as a system (plant).

If the forbidden state problem is as follows: ‘do not let the child hurt himself or cause any damage, but at the same time let him do as many things as he wishes to’, then the supervisory control simply forbids the child ‘to play with the knife’ and ‘to play with the matches’ while allowing him ‘to play with the toys’, ‘to eat some food’ and ‘to watch TV’. After solving the forbidden state problem now a desired string problem is established as follows: Assume that the father removed the matches and the knife from the room. However, it is lunch time now and first of all the father wishes the child to eat some food. After doing this since there is some time before children’s programs start, the father wishes the child to play with toys. Finally, when it is time the father would like to let the child watch TV. This gives us the following sequences of events for the child to follow: eat some food - play with toys - watch TV. This also implies that the child can not play with the toys and can not watch TV before eating some food. In other words, the following sequence of events are not acceptable according to the desired string specification given:

1- eat some food - watch TV - play with toys
2- watch TV - eat some food - play with toys
3- watch TV- play with toys - eat some food
4- play with toys - eat some food - watch TV
5- play with toys - watch TV- eat some food

Therefore, by declaring the sequence of events ‘eat some food - play with toys - watch TV’ as the legal behaviour, the five sequences as shown above become illegal system behaviour and are not allowed to happen.

A typical supervisory control of a DES, as used in this chapter, is shown in Fig. 5.1. It consists of four parts; i) the discrete event system (DES), to be controlled, ii) the supervisor, iii) sensor readings as outputs from the DES, and iv) control actions as inputs to the DES. In this case, the objective of the supervisor is to make sure that a sequence of events will take place within the plant such that the desired string specification is satisfied.

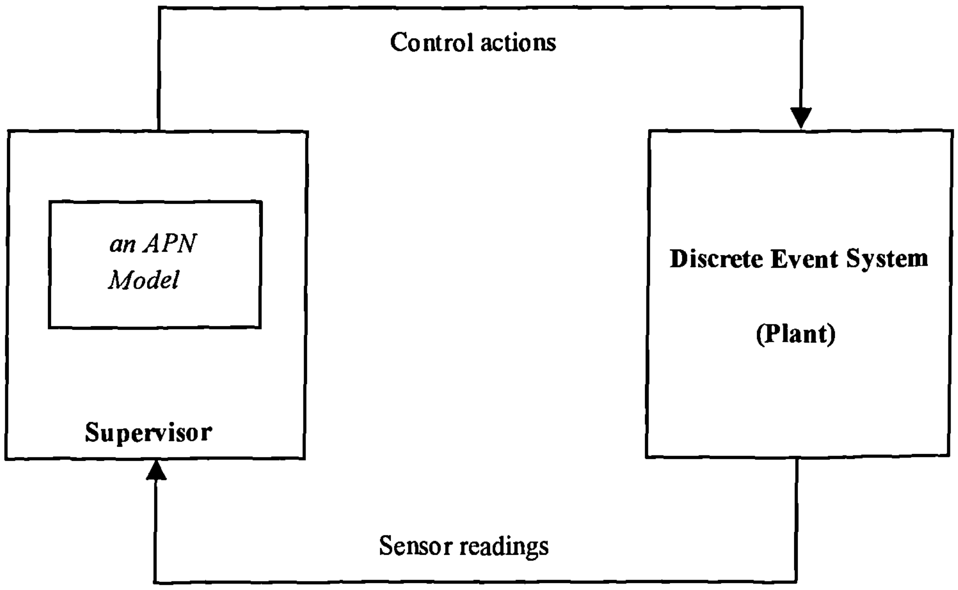


Figure 5.1. The use of an APN model as the supervisor in supervisory control, in the case of the desired string problem.

The plant and the supervisor are assumed to run in parallel in the following fashion. Transitions within the supervisor are synchronous with identically labelled events in the plant. When an event (controllable or uncontrollable) occurs in the plant, this is realised by the supervisor through sensory feedback. This results in the state change within the supervisor. Since the supervisor is a dynamic-feedback controller, its controlling actions depend on the previous states of the plant. Therefore, at the current state, depending on the previous state, the supervisor provides a set of actions to force the plant to behave according to the desired string specifications given.

The supervisor in this case is an APN model, which consists of an *untreated model* of the system, as well as the sequencing information to accommodate the desired string specification. The untreated model is defined in the following text as a model without any forbidden state problems. If a supervisory control assignment includes both the forbidden state problem and the desired string problem, then first of all the forbidden state problem is solved. The resulting supervisor obtained for the forbidden state problem, then becomes the *untreated model* for the desired string problem. If a supervisory control assignment includes only a desired string problem, then the untreated model is just the uncontrolled model of the system. The untreated model of the system has some of its places with actions assigned, in order to control the motors, actuators, etc. of the plant. The occurrence of events in the plant causes the token flow in the untreated model. In this case, no forbidden state is assumed to have come to existence in the APN model (supervisor). This may be checked by constructing the reachability tree of the APN model (supervisor).

The desired string of events is represented by an APN, called a *specification APN*, which represents the sequencing information as a Petri net structure to accommodate the desired string specification. In other words, a specification APN simply represents a desired string as a net structure. Such nets are called a language generator (Sreenivas, 1996; Sreenivas, 1993; Sreenivas and Krogh, 1992). The *untreated model* and the specification APN are combined by using the concurrent composition, an operator that requires the transitions with the same events to be merged. The concurrent composition is also termed as the synchronous composition (Kumar and Holloway, 1996).

The type of languages considered in the Petri net literature falls into two categories: i) deterministic Petri net languages, also called regular languages, and ii) nondeterministic Petri net languages, also called nonregular languages. Note that although the methodology proposed in this chapter does not use the language concepts, the concepts used are closely related to the languages as defined. For example, when a deterministic sequence of events is specified by a specification APN, it is called deterministic specification APN and it is closely related to the regular languages. Similarly, when a nondeterministic sequence of events is specified by a specification APN, it is called nondeterministic specification APN and it is closely related to the nonregular languages. Therefore, the specification APNs considered in this chapter, are falls into two categories: i) deterministic specification APNs and ii) nondeterministic specification APNs.

## 5.2. DETERMINISTIC SPECIFICATION APNs

The specification APN, considered in this section, is deterministic, i.e., the exact firing sequence of the specification is known beforehand. A specification APN is a special Automation Petri net, in which a desired string specification is represented by a net structure. In this particular APN there are no actions assigned to the places. A specification APN can be either irreversible or reversible. In the former case, a string of transitions $t_1t_2t_3....t_n$, where $t_i \in T$ ($i = 1, 2, ....., n$), is said to be a *valid firing sequence* starting from the marking $M_o(p_1) = 1$. For $i = \{1, 2, ....., n\text{-}1\}$ the firing of the transition $t_i$ produces a marking under which the transition $t_{i+1}$ is enabled. The string of firing conditions (events) $\chi_1$, $\chi_2$, $\chi_3$,....$\chi_n$ are associated with the transitions $t_1$, $t_2$,....$t_n$ respectively. This means that if a desired string specification is given as $\chi_1\chi_2\chi_3....\chi_n$, i.e., a system is required to carry out events in this given sequence one after another, then it can be represented as a specification APN as shown in Fig. 5.2.(a). In this case, the sequence is executed just once if all the events appearing in the sequence occur. Therefore, this is called an *irreversible specification APN.* This may be used for systems in which a sequence of events wanted to occur just once and then system is to be stopped. In order to obtain a reversible specification APN, an arc Post($t_n$, $p_1$), connecting transition $t_n$ to place $p_1$, can be used. By doing this once the firing sequence of transitions $t_1$, $t_2$,.....$t_n$ is completed, another firing sequence can be started again and again. This may well be used for systems in which there is a sequence of activities carried out repetitively. The specification APN, in this case, is called a *reversible specification APN* and it is shown in Fig. 5.2.(b). Note that when establishing these specification APNs only controllable transitions are allowed to be put into the firing sequence, because uncontrollable transitions can not be stopped from firing. It is assumed that the plant, which is subject to the desired string problem, is capable of producing the string given by a deterministic specification APN. If a desired string can be generated by the system considered, then this string is called a *valid string.* If a desired string can not be generated by the system, then this string is called an *invalid string.* It is possible to obtain the set of valid firing sequences, i.e., valid strings, of an APN model by generating the reachability tree of the model. It is important to point out that the technique explained in this section, is a very crude way of representing a deterministic desired string problem. For example, if there is an event repeating a lot of times one after another, in this way a lot of places and transitions must be used to accommodate this situation. In order to overcome this problem, it is possible to construct more efficient specification APNs, in the sense that less places and transitions are used in the APNs.

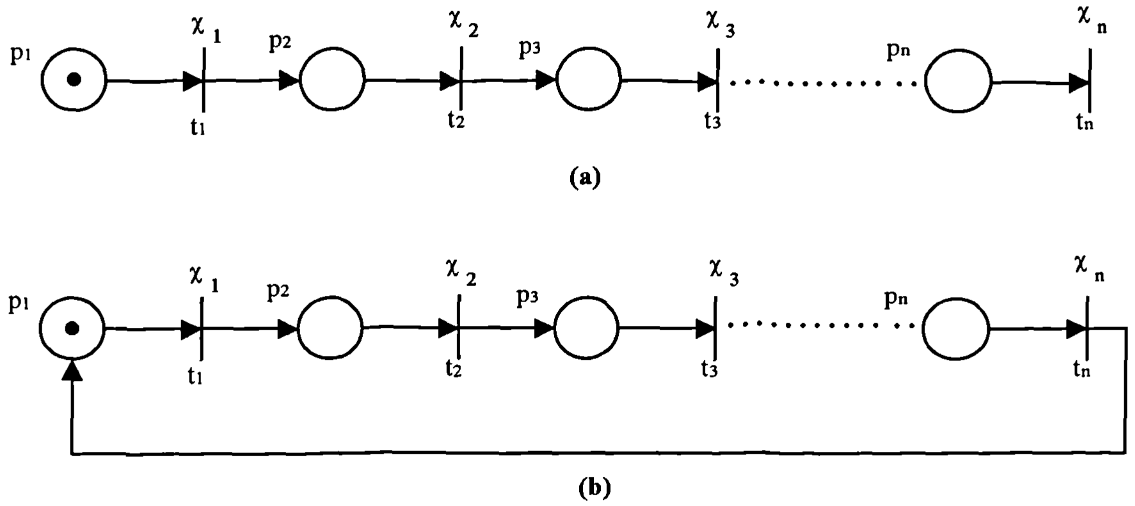
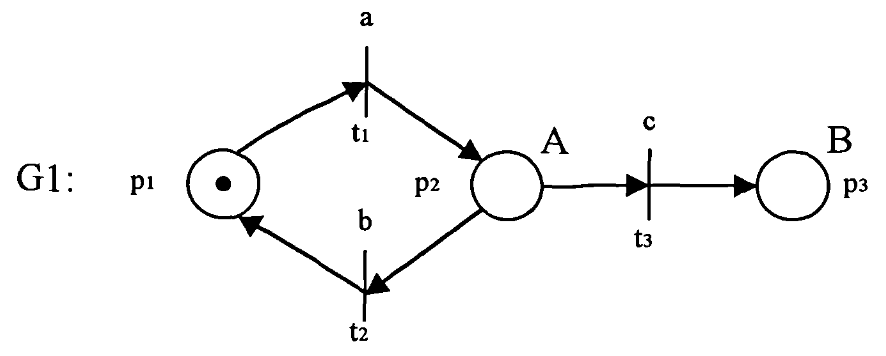


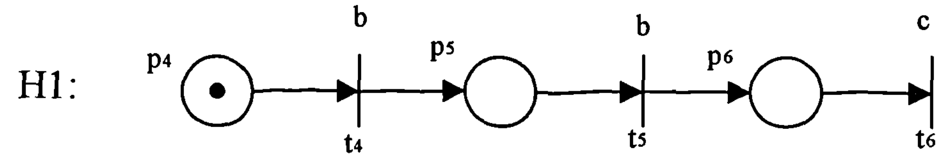
Figure 5.2. Deterministic Specification APNs. a) Irreversible. b) Reversible.

## 5.2.1. Example 1

To explain how irreversible specification APNs can be used for the supervisory control of DESs in the case of the desired string problem the model of a DES, ($G_1$), as shown in Fig. 5.3.(a)., is considered. In $G_1$, there are three places, $P_{G1} = \{p_1, p_2, p_3\}$ and three transitions $T_{G1} = \{t_1, t_2, t_3\}$, with firing conditions (events) a, b, c, assigned to these transitions, respectively. Events b and c are controllable and the event a is not controllable. There are two actions A and B, assigned to places $p_2$ and $p_3$, respectively. Assume that the desired string is “bbc”. This means that the controlled (supervised) model of the system should allow ‘b’ to happen exactly twice before letting ‘c’ to take place. The irreversible specification APN (H1), which represents this desirable string specification, is shown in Fig. 5.3.(b), where there are three places, $P_{H1} = \{p_4, p_5, p_6\}$ and three transitions $T_{H1} = \{t_4, t_5, t_6\}$, with firing conditions (events) b, b, c, assigned to these transitions, respectively.



(a)

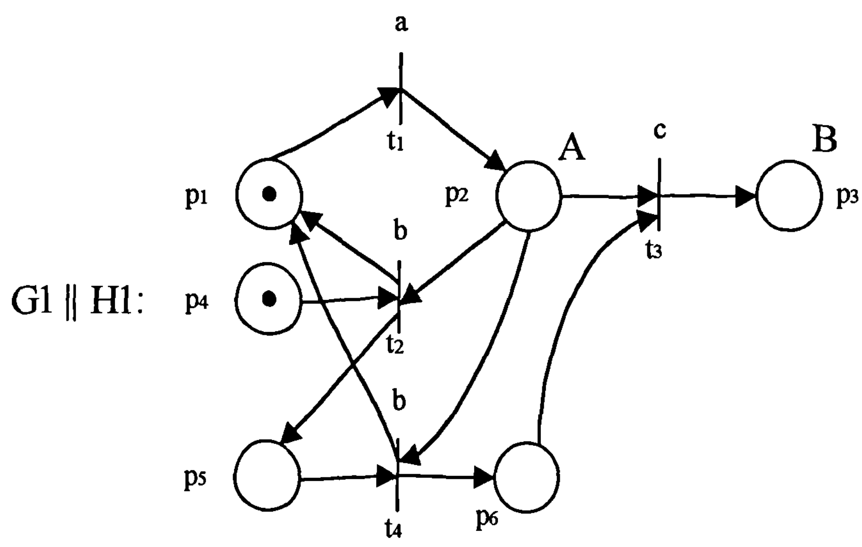


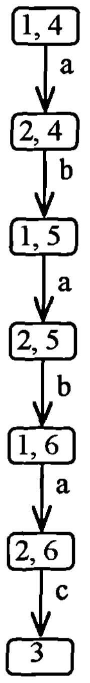
(b)

Figure 5.3. (a) The model of a system G1, i.e., untreated model.
(b) An irreversible deterministic specification APN, representing the desired string 'bbc'.

When the concurrent composition is used, by merging transitions with the same events, the controlled model of the system is obtained, as shown in Fig. 5.4.(a), which behaves according to the desired string specification given. As can be seen from the reachability tree of the controlled model (the supervisor), given in Fig. 5.4.(b), the behaviour of the controlled model produces the string 'bbc' as desired.

Note that when merging transitions with the same events, if there is more than one transition with the same event in the specification APN then the transition, having the same event from the untreated model, is duplicated as many as the number of transitions with the same event within the specification APN. For example, in this case, transition $t_2$ of the untreated model with the event b becomes doubled as $t_2$ and $t_4$ in the controlled model with the event b, because of the specification APN.





(a) (b)

Figure 5.4. (a) The controlled model. (b) Its reachability tree.

Note that the desired string ‘bbc’ is a *valid string*, i.e., the plant can produce such a string. However, if the string ‘bbcb’ is considered, then it is obvious that the plant can not produce such a string. In other words, the string ‘bbcb’ is an *invalid string* for the system G1. Therefore, it is necessary to make sure that the desired string specification represents a valid string for the system considered. In order to find out the valid firing sequences, i.e., valid strings of a system, it is possible to use the *reachability tree* of a model, because the reachability tree of the model represents all possible firing sequences of the system. In this example, the reachability tree shown in Fig. 5.5 represents the firing sequences, i.e., valid strings, of the model G1. As can be seen from Fig. 5.5, without considering the uncontrollable event ‘a’, the valid firing sequences, i.e., the valid strings, for this system are as follows: c, bc, bbc, bbbc, bbbbc.....

Note that nodes of the reachability tree represent markings of the model G1 and arcs represent firing of transitions within the model. In the nodes; 1, 2 and 3 mean that places $p_1$, $p_2$ and $p_3$ have one token each, respectively. Instead, (1,0,0), (0,1,0) and (0,0,1,) can be used as an alternative notation to show the same markings.

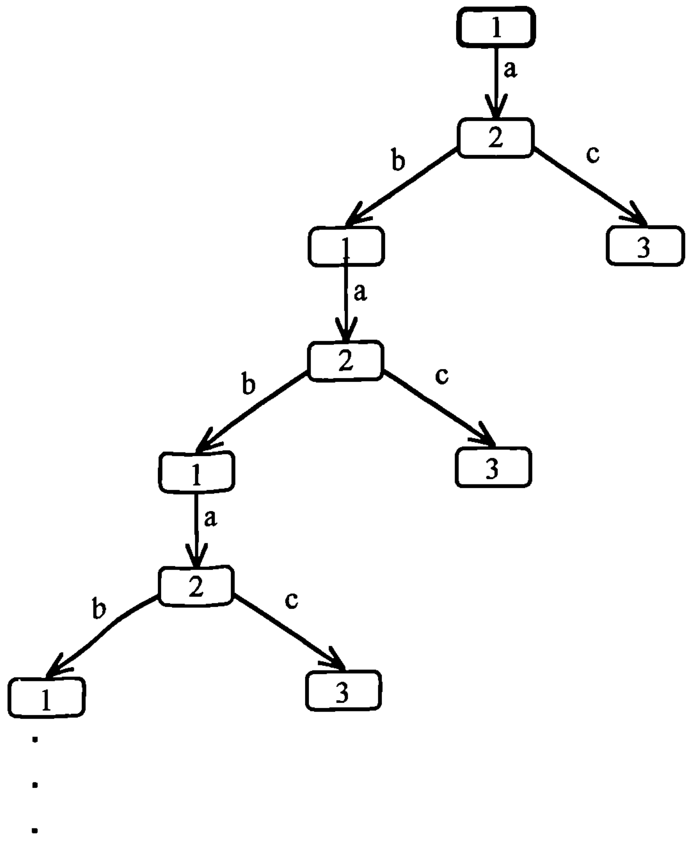


Figure 5.5. The reachability tree of the model G1.

### 5.2.2. Example 2

Now consider a plant capable of producing reversible events. In this example, a robot and a machine are considered. The models of the robot and the machine are shown in Fig. 5.6. The model of the robot, G1, is shown in Fig. 5.6.(a) and the model of the machine, G2, is shown in Fig. 5.6.(b). Actions R and M are assigned to places $p_2$ and $p_4$, respectively. The action R represents that the robot is working, while the action M means the machine is *on*. In this case, events a, c and d are controllable and the event b is uncontrollable. Initially, both the robot and the machine are idle. When the robot picks up one part (event a), it starts working its way to load the part on the machine (action R). When it loads the part on the machine (event b), it ceases working and becomes idle. When the robot loads a part on the machine (event b), the machine starts working on the part (action M). When the machine finishes working on the part, it outputs the produced part either on conveyor A (event c) or on conveyor B (event d). When the machine outputs the produced part (event c or d) it becomes idle. If this system is required to produce one part on conveyor A and then one part on conveyor B in a repeating fashion, then this maps to the desired string 'acad' repeated all along, i.e., acadacadacad...... This desired string specification can be represented as a reversible deterministic specification APN, as shown in Fig. 5.6.(c). Note that the desired string 'acad' is a valid string, i.e., the system can generate such a firing sequence.

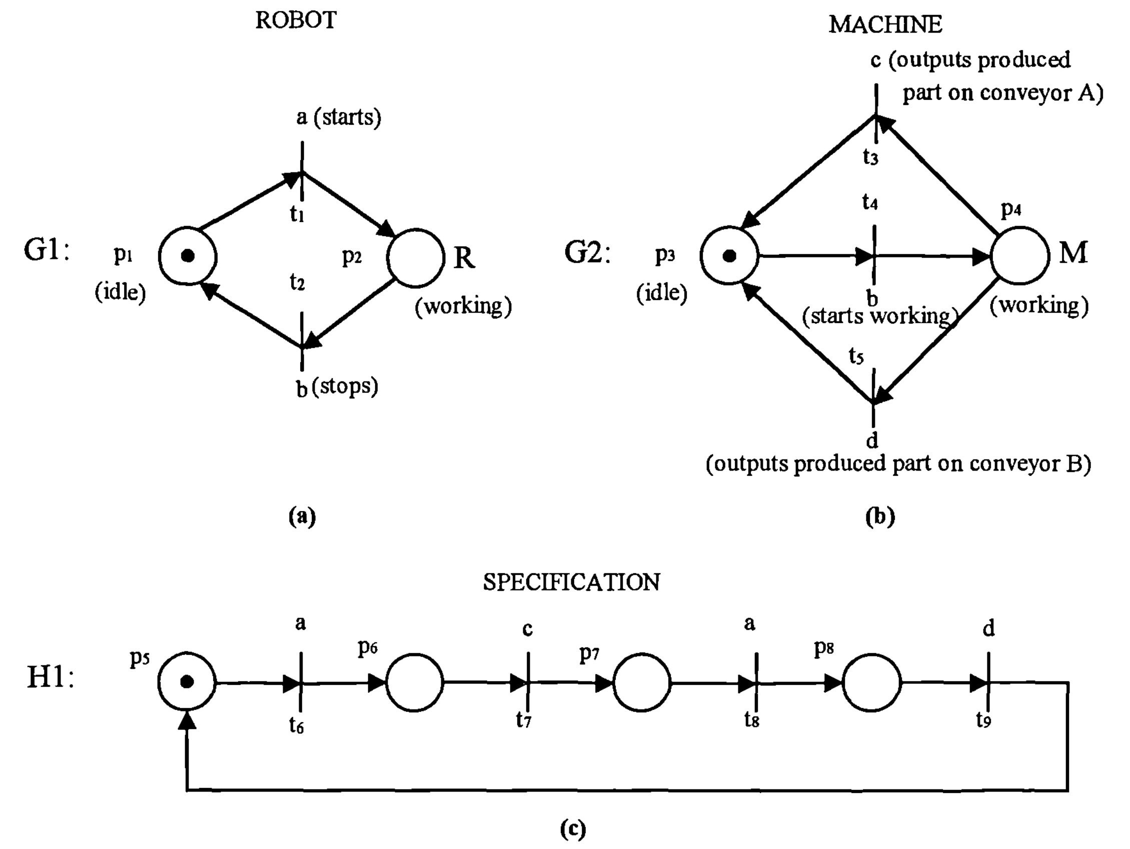


Figure 5.6. (a) The model of the robot, G1. (b) The model of the machine, G2.
(c) The reversible deterministic specification APN, representing the desired string 'acad'

When the concurrent composition is used, by merging transitions with the same events, the controlled model (i.e., the supervisor) of the system is obtained, as shown in Fig. 5.7.(a). Note that since the event 'a' appears twice in the specification APN, transition $t_1$ of the untreated model (uncontrolled model), with the event 'a', is duplicated in the controlled model and represented by transitions $t_1$ and $t_2$, having the event a assigned to them.

In this case, the uncontrollable event 'b' is assigned to transitions $t_2$ and $t_4$ of the untreated model. Therefore, they are merged, in the controlled model and they are both represented by transition $t_3$, with the event 'b'. The controlled model (the supervisor) behaves according to the desired string specification given, i.e., it produces the desired string 'acad' in a repeating fashion. This can also be seen from the reachability tree given in Fig. 5.7.(b). Now, the behaviour of the controlled model is considered in detail. At the beginning, i.e., at the marking (1, 3, 5), the only transition enabled is $t_1$ and when it fires with the event a, the robots starts working (action R). In this case, the new marking is (2, 3, 6) and the robot works its way to load the part on the machine. Now, the only transition enabled is $t_3$ and when it fires with the event b, the robot loads the part on the machine and ceases working and at the same time the machine starts working (action M). In this case, the new marking is (1, 4, 6) and the machine tries to finish its work on the part. Now, the only transition enabled is $t_4$ and when it fires with the event c, the machine outputs the produced part on the conveyor A and stops working. In this case, the new marking is (1, 3, 7) and both the robot and the machine are idle once again. Now, the only transition enabled is $t_1$ and when it fires with the event a, the robot starts working (action R). In this case, the new marking is (2, 3, 8) and the robot tries to load the part on the machine. Now, the only transition enabled is $t_3$ and when it fires with the event b, the robot loads the part on the machine and stops, and at the same time the machine starts working (action M). In this case, the new marking is (1, 4, 8) and the machine works on the part. Now, the only transition enabled is $t_5$ and when it fires with the event d, the machine outputs the produced part on the conveyor B and stops working. After finishing this sequence of events, the system will have finished the string 'acad' and it will start the same sequence from the beginning.

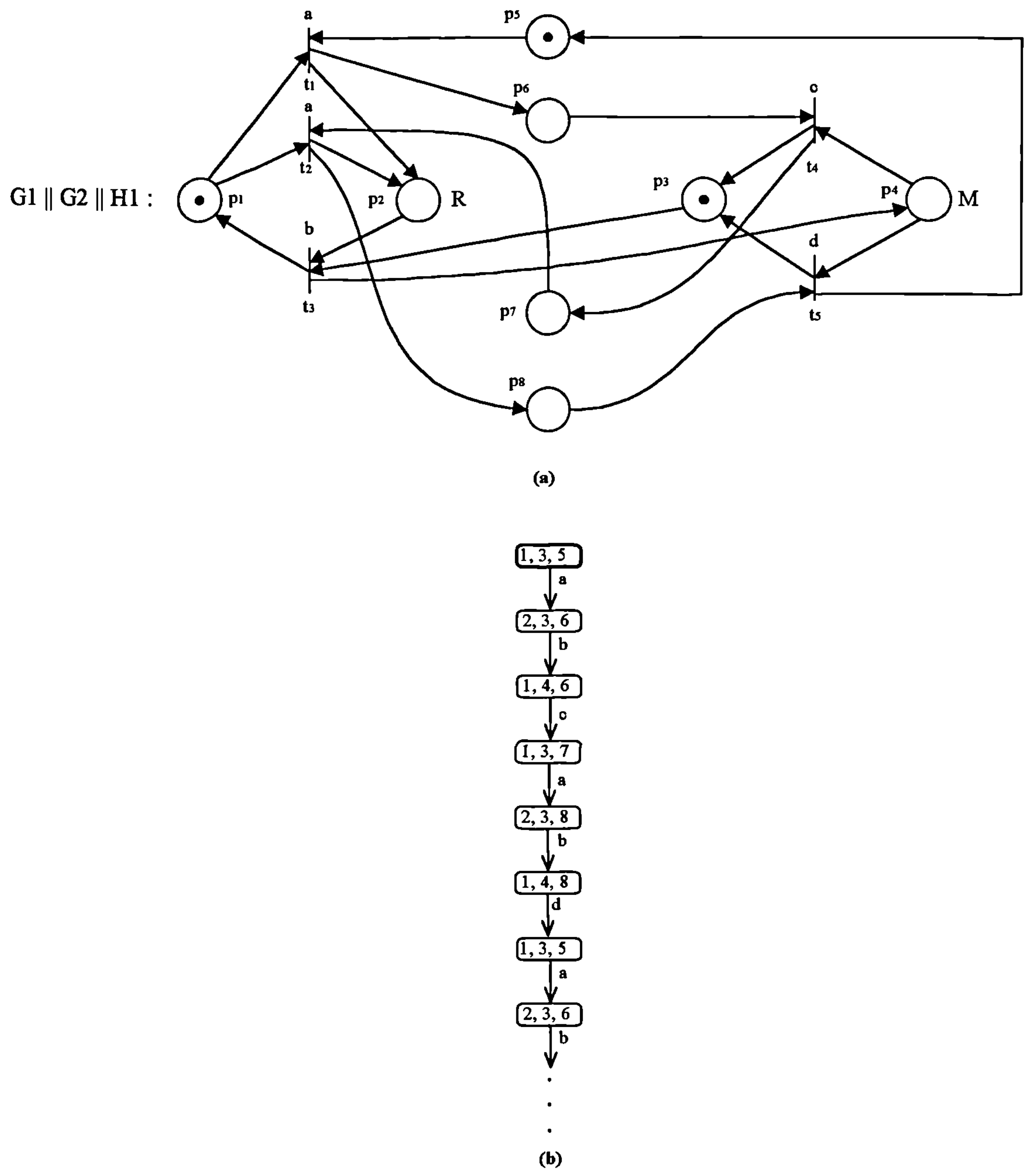
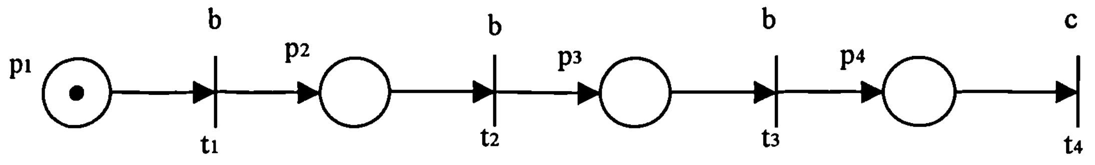


Figure 5.7. (a) The controlled model (the supervisor) of the system.
(b) The reachability tree of the controlled model.

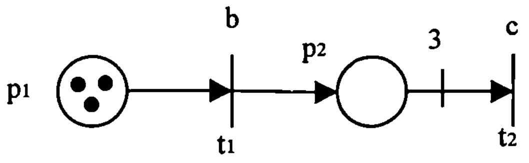
### 5.2.3. Simplifications For Deterministic Specification APNs

Some deterministic specification APNs can be simplified when the same events repeat a lot of times in a desired string specification. In general, three different cases can be considered. In the first case, repeating events can be at the beginning of the specification APN. In the second case, repeating events can be in the middle of the specification APN. Finally, in the third case, a set of repeating events can be at the beginning and another set of repeating events at the other parts of the specification APN.

Now, firstly consider a deterministic specification APN in which repeating events are at the beginning of the APN. For example, Fig. 5.8.(a) shows such a specification APN, where there are four places $P = \{p_1, p_2, p_3, p_4\}$ and four transitions $T = \{t_1, t_2, t_3, t_4\}$, with events b, b, b, c assigned respectively. This irreversible deterministic specification APN represents the desired string 'bbbc'. In the APN, when $t_1$ fires with the event b, the token moves from place $p_1$ to place $p_2$. In the next step, when $t_2$ fires then the token moves from place $p_2$ to place $p_3$ and so on. Instead of using this arrangement it is possible to represent the same desired string 'bbbc' with the irreversible deterministic specification APN shown in Fig. 5.8.(b). In this APN, initially place $p_1$ has three tokens and the event b is associated with transition $t_1$. In this case, the event b occurs three times, i.e., as many as the number of tokens in place $p_1$. After that the number of tokens in place $p_2$ becomes three. This means that since $M(p_2) = 3$ and $Pre(p_2, t_2) = 3$, transition $t_2$ is enabled and the event c is allowed to occur. It is obvious that in this case, the number of places and transitions is less than the APN shown in Fig. 5.8.(a).



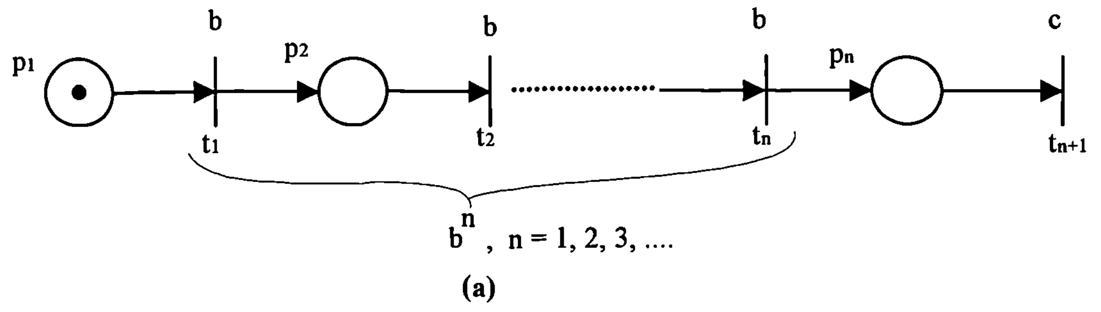
**(a)**



**(b)**

Figure 5.8. (a) A deterministic specification APN representing the desired string 'bbbc'.
(b) An alternative deterministic specification APN to represent the same desired string 'bbbc'.

It is possible to generalise this simplification technique. The general case for this simplification technique is shown in Fig. 5.9. In Fig. 5.9.(a), a string of events $\{b^n \mid n = 1, 2, 3.....\}$ appears at the beginning of a specification APN. Instead of repeating the string of places and transitions as many as $n$ times as in Fig. 5.9.(a), a simplified specification APN can be simply used, as shown in Fig. 5.9. (b), where initially there are $n$ tokens in place $p_1$, i.e., $M_0(p_1) = n$, and the weight of the arc is $Pre(p_2, t_2) = n$. This means that the event b is allowed to occur $n$ times and then the event c can occur.



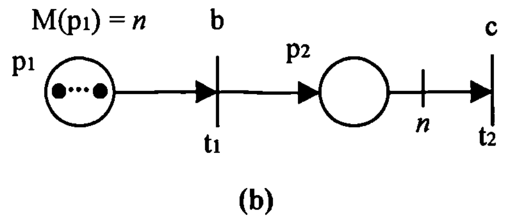
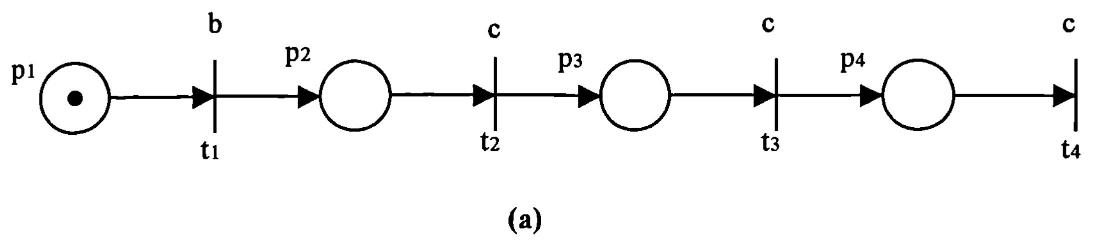


Figure 5.9. (a) A deterministic specification APN representing the desired string $\{b^nc \mid n = 1, 2, 3, \ldots\}$
(b) An alternative deterministic specification APN to represent the same desired string.

Now, secondly consider a deterministic specification APN in which repeating events are in the middle of the APN. For example, Fig. 5.10.(a) shows such a specification APN, where there are four places $P = \{p_1, p_2, p_3, p_4\}$ and four transitions $T = \{t_1, t_2, t_3, t_4\}$, with events b, c, c, c assigned respectively. This irreversible deterministic specification APN, represents the desired string 'bccc'. In other words, in the specification APN, if the event b occurs, then the event 'c' may occur three times. Instead of representing this string with the APN shown in Fig. 5.10.(a). It is possible to represent the same string with the specification APN shown in Fig. 5.10. (b). In this APN, when the event 'b' occurs three tokens are put into place $p_2$ via $Post(t_1, p_2) = 3$ and then by means of these three tokens in place $p_2$ the event c is allowed to occur three times.



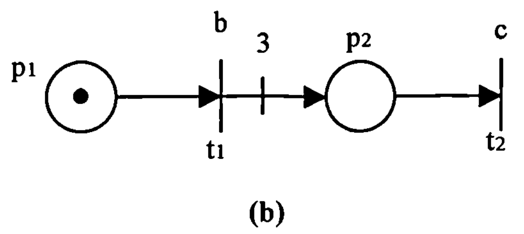


Figure 5.10. (a) A deterministic specification APN representing the desired string 'bccc'.
(b) An alternative deterministic specification APN to represent the same desired string.

This technique can also be generalised, as shown in Fig. 5.11. In Fig. 5.11.(a), when the event b occurs a string of events $\{c^m \mid m = 1, 2, 3....\}$ is allowed to happen. This is done by using *m* places and *m* transitions. Instead the specification APN shown in Fig. 5.11.(b) represents the same desirable string in a compact way. In this APN, if the event 'b' occurs, then *m* tokens are deposited into place $p_2$ via Post($t_1$, $p_2$) = *m*. After that the event 'c' is allowed to occur up to *m* times by means of *m* tokens in place $p_2$.

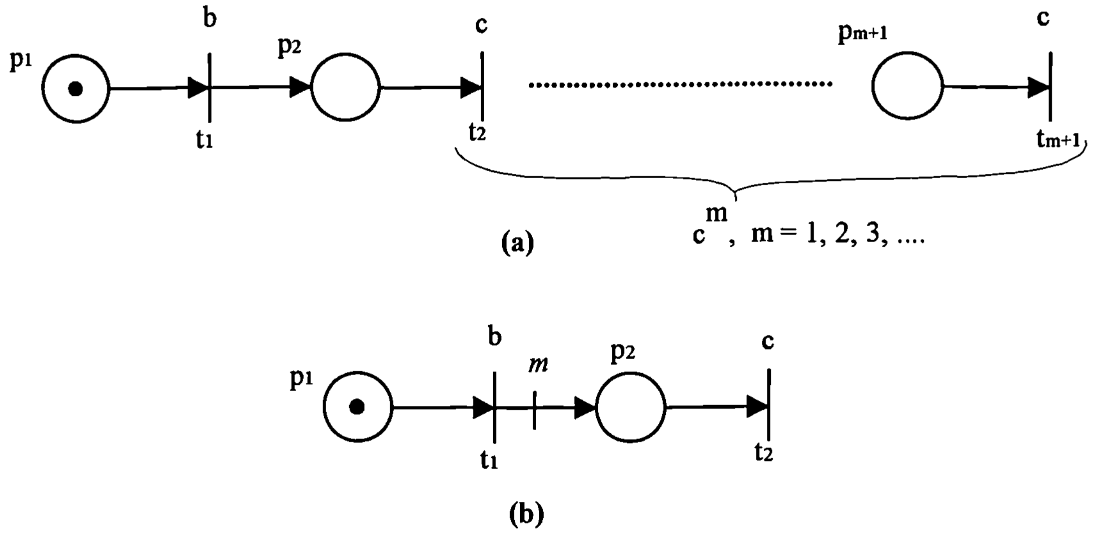


Figure 5.11. (a) A deterministic specification APN representing the desired string {bc$^{m}$' | m = 1, 2, 3,...}
(b) An alternative deterministic specification APN to represent the same desired string.

Now, finally consider a deterministic specification APN, in which repeating events are all over the APN, i.e., at the beginning as well as in the middle. For instance such a specification APN is shown in Fig. 5.12.(a), where there are nine places P = {$p_1$, $p_2$, ...., $p_9$} and nine transitions T = {$t_1$, $t_2$,.... $t_9$} with events b, b, c, c, c, c, d, d, d assigned respectively. This irreversible deterministic specification APN, represents the desired string 'bbccccddd'. Instead, the same specification APN can be represented as shown in Fig. 5.12.(b). In this case, simplification techniques introduced in the previous two cases are simply used together to obtained the specification APN shown in Fig. 5.12.(b).

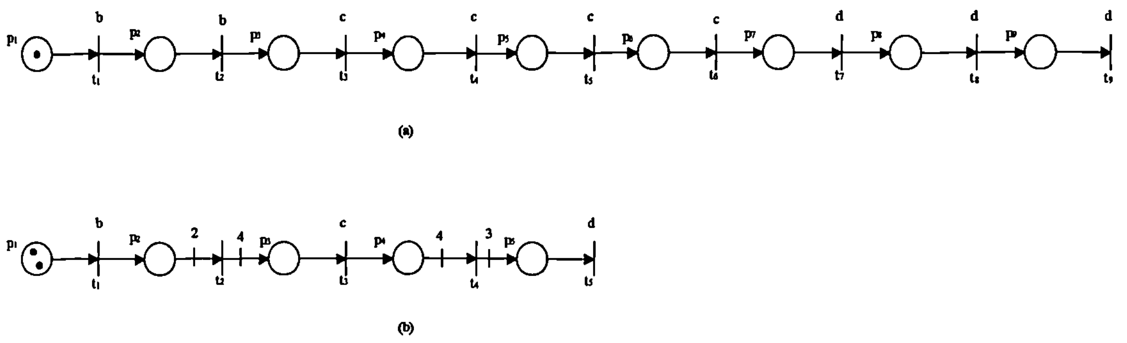


Figure 5.12. (a) A deterministic specification APN representing the desired string 'bbccccddd'.
(b) An alternative deterministic specification APN to represent the same desired string.

In general, a string of events can appear in a specification APN, as shown in Fig. 5.13.(a), where the desired string $\{b^n c^m d^k \mid n = 1, 2, 3...., m = 1, 2, 3...., k = 1, 2, 3.... \}$ is represented. To do this in an inefficient way, total of *n+m+k* places and *n+m+k* transitions must be used. Instead, the specification APN shown in Fig. 5.13.(b) can represent the same string of events by using only four places and four transitions. Note that initially *n* tokens are put in place $p_1$ and the arcs must be arranged as shown in the Fig. 5.13.(b). Although in these cases, only irreversible deterministic specification APNs are considered, the same techniques can be used with the reversible ones as well.

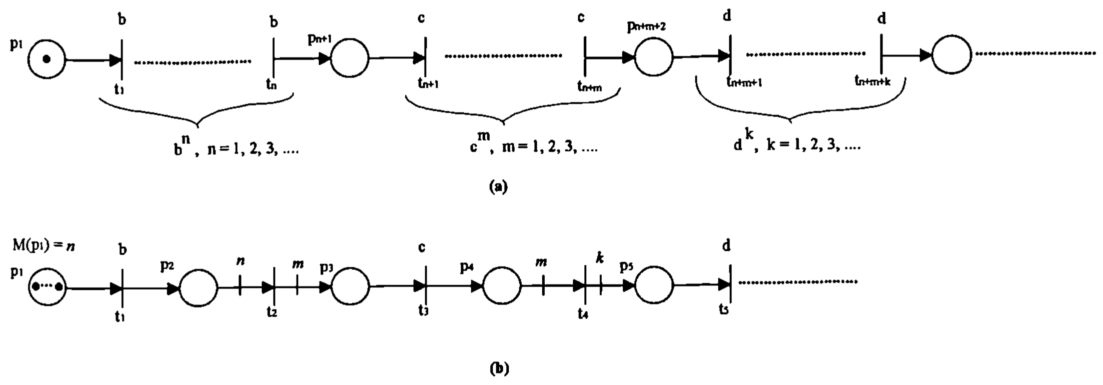
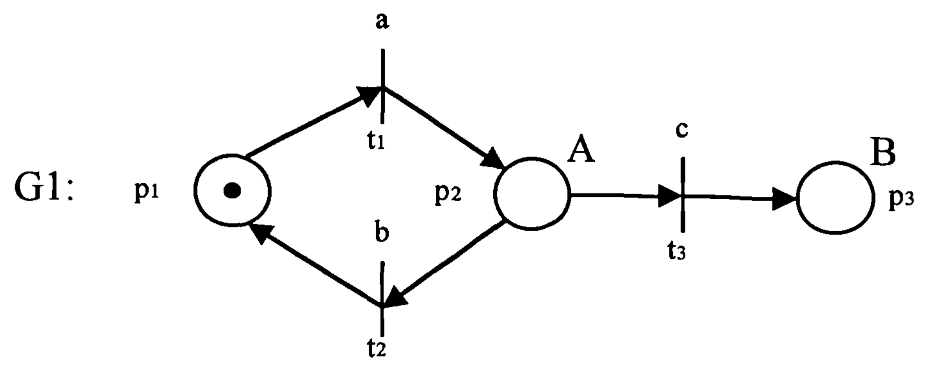


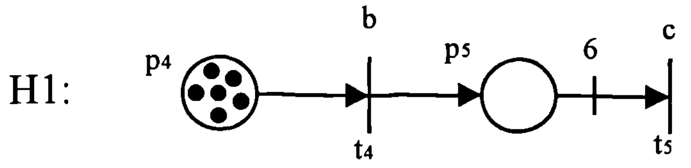
Figure 5.13. (a) A deterministic specification APN representing the desired string $\{b^n c^m d^k \mid n = 1, 2, 3...., m = 1, 2, 3...., k = 1, 2, 3.... \}$
(b) An alternative deterministic specification APN to represent the same desired string.

### 5.2.4. Example 3

Now an example is considered to show how useful the simplifications are for the supervisory control of DESs in the case of the desired string problem. $G_1$, the model of a DES, is shown in Fig. 5.14.(a), where there are three places and three transitions with events a, b, c. The events 'b' and 'c' are controllable, while 'a' is not. There are two actions A and B assigned to the places $p_2$ and $p_3$ respectively. In this case, assume that the desired string is 'bbbbbbc'. This means that the controlled model of the system should allow 'b' to happen exactly six times before letting 'c' take place. The desired string can be represented as an irreversible deterministic specification APN, in which there are only two places and two transitions as shown in Fig. 5.14. (b). If normal specification APNs are used to solve this problem, it is necessary to use seven places and seven transitions.



(a)



(b)

Figure 5.14. (a) The model of a system G1, i.e., untreated model.
(b) An irreversible deterministic specification APN, representing the desired string 'bbbbbbc'.

When the concurrent composition is used, the controlled model of the system is obtained as shown in Fig. 5.15. The controlled model behaves according to the desired string specification given, i.e., 'bbbbbbc' is generated by the system.

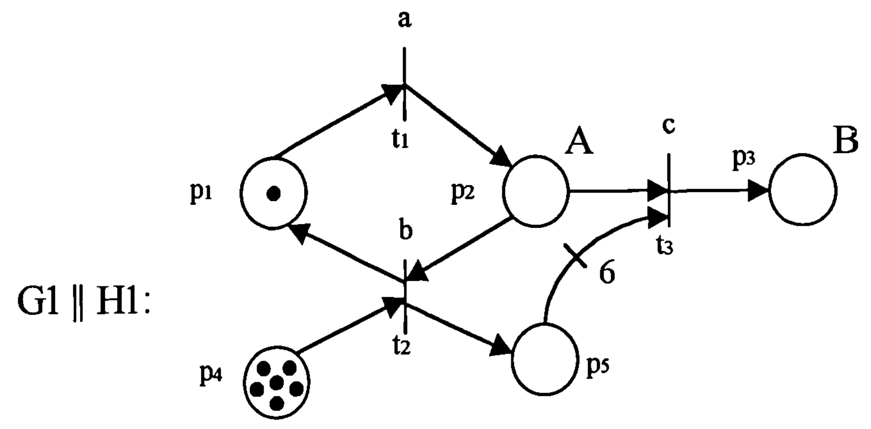


Figure 5.15. (a) The controlled model of the system .

## 5.3. NONDETERMINISTIC SPECIFICATION APNs

The specification APN, considered in this section, is nondeterministic, i.e., the exact firing sequence of the specification is not known beforehand. Instead, a family of firing sequences is considered. The supervisory control problems considered require infinite state supervisors. The state of a Petri net is given by the marking of the net, which represents the distribution of tokens in each place. When the value of the marking is unbounded, finite Petri nets can represent infinite state systems. Therefore, in this section the desired string problem will be solved for the systems that require nondeterministic infinite state supervisors. To solve this problem, firstly the desired string specification represented as a nondeterministic specification APN and then the concurrent composition is used to obtain the controlled model (the supervisor) of the system. It is assumed that the desired string represents a valid string, which can be generated by the system considered. Note once again that in the specification APN there are no actions assigned to the places. The nondeterministic specification APNs can be reversible or irreversible. The reachability tree analysis can be used to check the valid firing sequences of the controlled model or the uncontrolled model (untreated model).

### 5.3.1. Irreversible Nondeterministic Specification APN

This type of APNs represent a series of activities that takes place once. However, the exact sequence of events is not known and it is defined by events themselves as they occur. The length of the desired string, therefore, may be infinite. In this case, the desired string specification is represented by an irreversible nondeterministic APN. It is assumed that the desired string can be produced by the system considered. To explain this case, the system, shown in Fig. 5.16.(a), is considered, where there are two places $P = \{p_1, p_2\}$ and three transitions $T = \{t_1, t_2, t_3\}$ with events a, b, b assigned to them respectively. Initially, there is only one token in place $p_1$. The string of events that can be generated by this system is $w = \{a^n b^m \mid 0 \le n \le p, 0 \le m \le p \mid p = 0, 1, 2, 3\ldots\}$. That is,

$p = 0 \quad \Rightarrow \quad w_0 = \{a^n b^m \mid n = 0, m = 0\} = \{\lambda\}$

$p = 1 \quad \Rightarrow \quad w_1 = \{a^n b^m \mid 0 \le n \le 1, 0 \le m \le 1\} = \{\lambda, a, b, ab\}$

$p = 2 \quad \Rightarrow \quad w_2 = \{a^n b^m \mid 0 \le n \le 2, 0 \le m \le 2\} = \{\lambda, a, aa, b, bb, ab, abb, aab, aabb\}$

$p = 3 \quad \Rightarrow \quad w_3 = \{a^n b^m \mid 0 \le n \le 3, 0 \le m \le 3\} = \{\lambda, a, aa, b, bb, ab, abb, aab, aabb, aaa, bbb, abbb, aabbb, aaab, aaabb, aaabbb\}$

.. .. ....

Where $w$ represents the strings generated by the system and $\lambda$ means there are no events taking place. The $w = \{a^n b^m \mid 0 \le n \le p, 0 \le m \le p \mid p = 0, 1, 2, 3\ldots\}$ represents a system, where the first occurrence of the event 'b' terminates the occurrence of the event 'a'. The number of events that can occur in the plant is infinite.

Assume that the desired string specification is $w_d = \{a^n b^m \mid 0 \le m \le n \le p \mid p = 0, 1, 2, 3\ldots\}$. That is, p = 0 ⇒ $w_d = \{a^n b^m \mid n = 0, m = 0\} = \{\lambda\}$

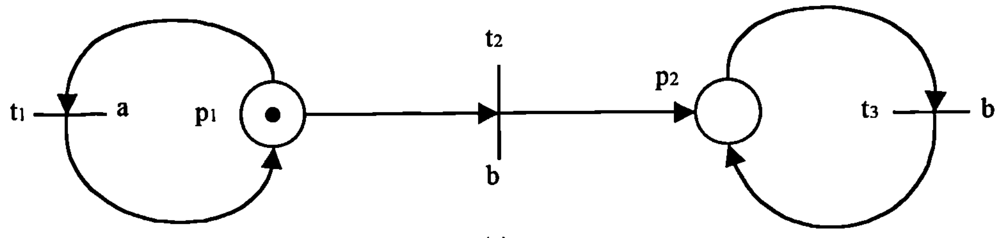
p = 1 ⇒ $w_d = \{a^n b^m \mid 0 \le m \le n \le 1\} = \{\lambda, a, ab\}$

p = 2 ⇒ $w_d = \{a^n b^m \mid 0 \le m \le n \le 2\} = \{\lambda, a, aa, ab, aab, aabb\}$

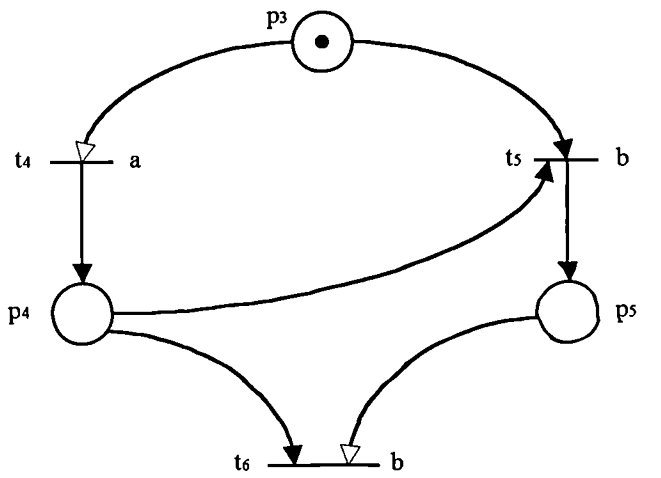
p = 3 ⇒ $w_d = \{a^n b^m \mid 0 \le m \le n \le 3\} = \{\lambda, a, aa, ab, aab, aaa, aabb, aaab, aaabb, aaabbb\}$

.. .. ....

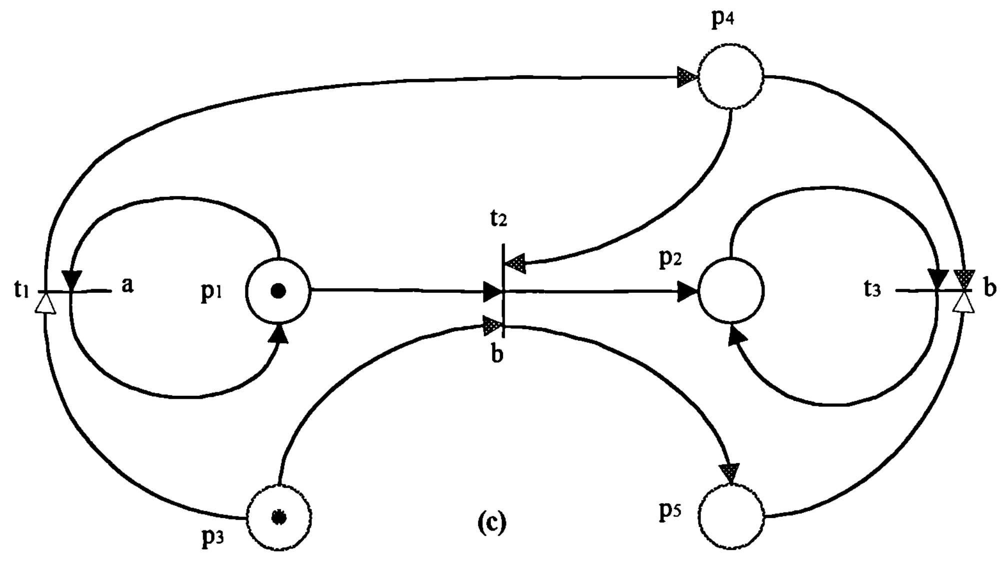
Note that according to the desired string specification before the event 'b' takes place, the event 'a' must occur and the maximum number of event 'b's that can occur is confined to the number of the event 'a's that have occurred previously. The desired string $w_d$ can be represented by an irreversible nondeterministic APN as shown in Fig. 5.16.(b), where there are three places P = {$p_3$, $p_4$, $p_5$} and three transitions T = {$t_4$, $t_5$, $t_6$} with events a, b, b, assigned to them. Initially only place $p_3$ has a token. This means that transition $t_4$ is enabled (i.e., the event 'a' can occur). Every firing of $t_4$ (event a) puts a token in place $p_4$ and re-enables $t_4$ and $t_5$. Upon the first firing of $t_5$ (event b) $t_4$ is disabled permanently and $t_6$ (event 'b') can fire repeatedly until the tokens in place $p_4$ are depleted. The controlled (supervised) model of the system is shown in Fig. 5.16.(c). The controlled model is obtained by combining the system and the specification APN through the concurrent composition. Note that the controlled model of the system produces the strings $w_d = \{a^n b^m \mid 0 \le m \le n \le p \mid p = 0, 1, 2, 3.... \}$. This can also be seen from Fig. 5.17, which shows the reachability tree of the controlled model, representing the strings generated by the controlled model.



(a)



(b)



(c)

Figure 5.16.(a). The model of a system, that generates the nondeterministic strings $w = \{a^n b^m \mid 0 \le n \le p, 0 \le m \le p \mid p = 0, 1, 2, 3..... \}$. (b).The irreversible nondeterministic specification APN, that represents the desired strings $w_d = \{a^n b^m \mid 0 \le m \le n \le p \mid p = 0, 1, 2, 3..... \}$.
(c). The supervised model of the system.

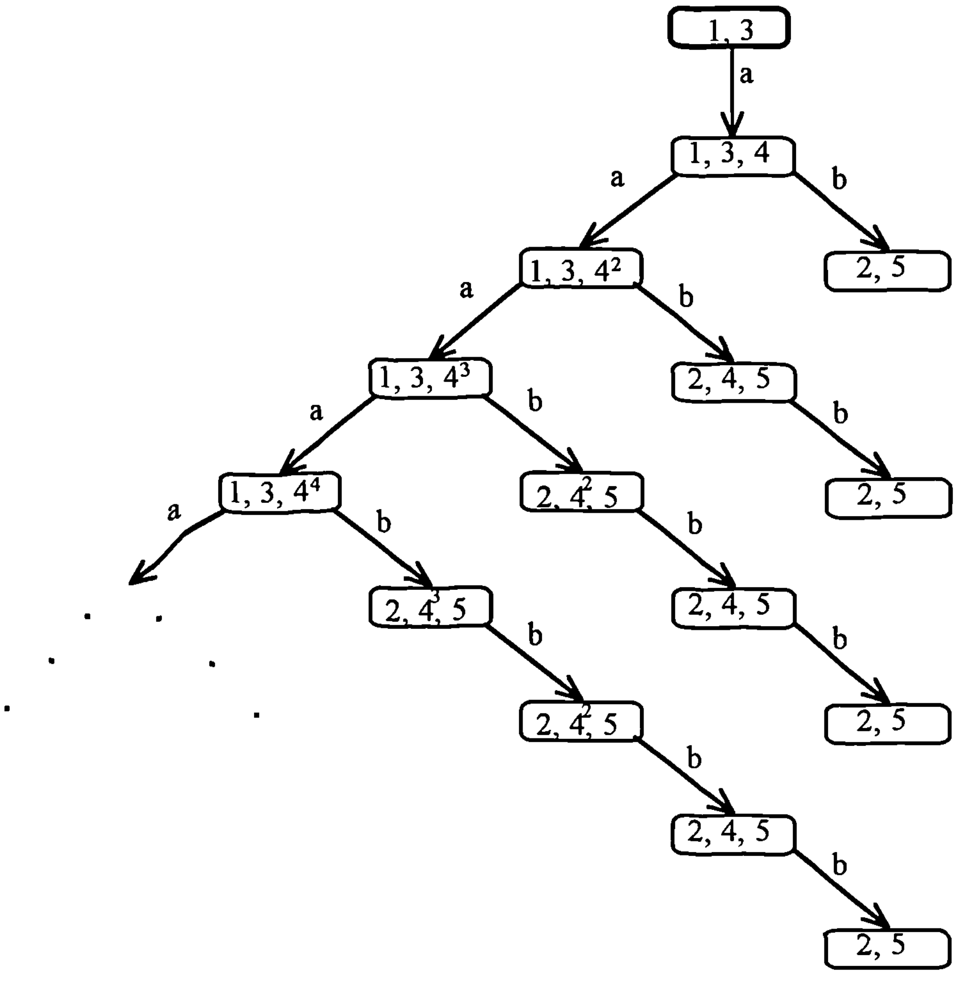


Figure 5.17. The reachability tree of the supervised model.

## 5.3.2. Reversible Nondeterministic Specification APN

This type of APNs represent series of activities that take place in a repeating fashion. However, each time the sequence of events may differ one from another, because of the nondeterminism. The length of the desired string may be infinite. In this case, the desired string is represented by a reversible nondeterministic APN. It is assumed that the desired string can be generated by the system considered. To explain this case, the system, shown in Fig. 5.18.(a), is considered, where there are two places P = $\{p_1, p_2\}$ and two transitions T = $\{t_1, t_2\}$, with events a and b assigned to them respectively. Initially, there is one token in place $p_1$ and in place $p_2$ respectively. The strings of events that can be generated by this system is $w = \{a^{n_1}b^{m_1}a^{n_2}b^{m_2}a^{n_3}b^{m_3} \ldots\ldots a^{n_p}b^{m_p} \cup b^{n_1}a^{m_1}b^{n_2}a^{m_2}b^{n_3}a^{m_3} \ldots\ldots a^{n_p}b^{m_p} \mid 0 \le n_1 + n_2 + n_3 +\ldots+ n_p \le p, 0 \le m_1 + m_2 + m_3 +\ldots+ m_p \le p \mid p = 0, 1, 2, 3\ldots\ldots \}$. That is,

p = 0 $\Rightarrow$ $w_0 = \{\lambda\}$

p = 1 $\Rightarrow$ $w_1 = \{ a^{n_1}b^{m_1} \cup b^{n_1}a^{m_1} \mid 0 \le n_1 \le 1, 0 \le m_1 \le 1 \} = \{\lambda$, a, b, ab, ba$\}$

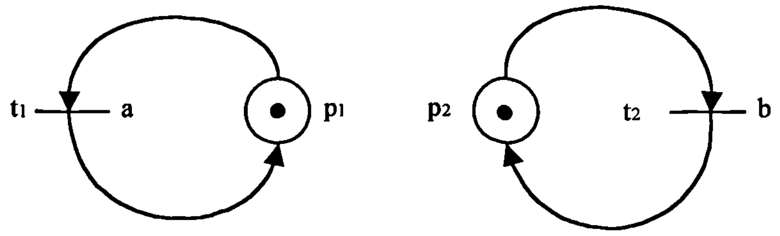
p = 2 $\Rightarrow$ $w_2 = \{ a^{n_1}b^{m_1}a^{n_2}b^{m_2} \cup b^{n_1}a^{m_1}b^{n_2}a^{m_2} \mid 0 \le n_1 + n_2 \le 2, 0 \le m_1 + m_2 \le 2 \} = \{\lambda$, a, b, aa, ab, bb, ba, abb, aab, aba, baa, bba, bab, aabb, abab, abba, bbaa, baba, baab$\}$

p = 3 $\Rightarrow$ $w_3 = \{ a^{n_1}b^{m_1}a^{n_2}b^{m_2}a^{n_3}b^{m_3} \cup b^{n_1}a^{m_1}b^{n_2}a^{m_2}b^{n_3}a^{m_3} \mid 0 \le n_1 + n_2 + n_3 \le 3, 0 \le m_1 + m_2 + m_3 \le 3 \} = \{\lambda$, a, b, aa, ab, bb, ba, aaa, aab, aba, abb, bbb, bba, bab, baa, aaab, abab, abba, abaa, aabb, abbb, aaba, bbba, baba, baab, babb, bbaa, baaa, bbab, aaabb, aabbb, aabab, abbba, aabba, ababa, abbab, ababb, abaab, abbaa, bbbaa, bbaaa, bbaba, baaab, bbaab, babab, baaba, babaa, babba, baabb, aaabbb, aababb, aabbab, ababba, aabbba, abbaba, abbaab, abbbaa, abaabb, ababab, bbbaaa, bbabaa, bbaaba, babaab, bbaaab, baabab, baabba, baaabb, babbaa, bababa$\}$

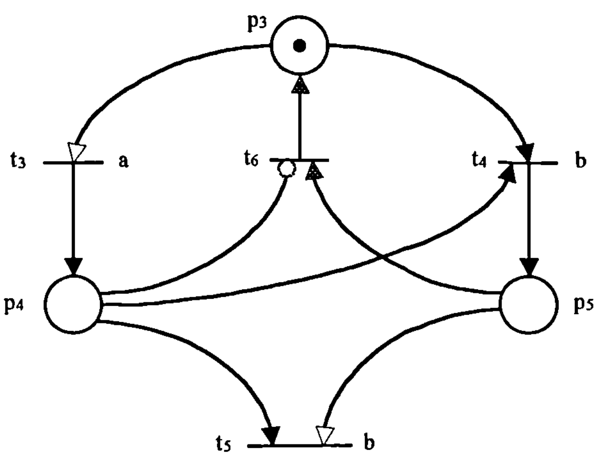
.. .. ....

This represents a system, in which two events (a and b) occur independently one from another. Note that simultaneous firing of these transitions is not considered. The number of events that can occur in the plant is infinite and the firing sequences are nondeterministic. Assume that the reversible desired string specification is $w_d = \{\text{rev}\ (a^n b^m) \mid 0 \le m \le n \le p \mid p = 0, 1, 2, 3\ldots. \}$. This means that once a desired string of events takes place another ones are also allowed to occur repeatedly.

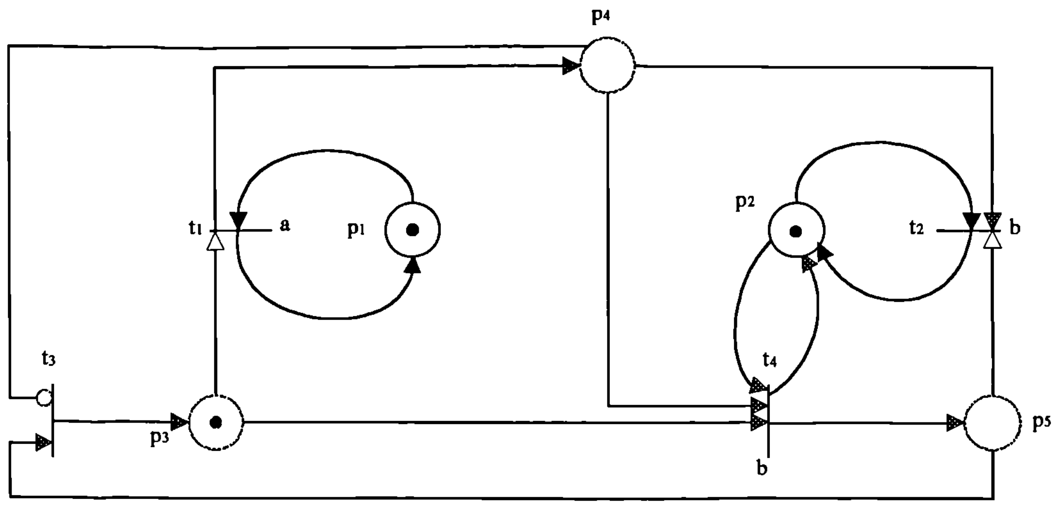
Note that according to the desired string specification before the event 'b' takes place, the event 'a' must occur and the maximum number of event 'b's that can occur is confined to the number of the event 'a's that have occurred previously. The reversible desired string $w_d$ can be represented by a reversible nondeterministic APN as shown in Fig. 5.18.(b), where there are three places P = {$p_3$, $p_4$, $p_5$} and four transitions T = {$t_3$, $t_4$, $t_5$, $t_6$}. Note that events a, b, and b are assigned to transitions $t_3$, $t_4$, $t_5$ respectively and there is no firing condition (event) associated with transition $t_6$, i.e., it fires as soon as it is enabled. Initially, only place $p_3$ has a token. This means that transitions $t_3$ is enabled (i.e., the event 'a' can occur). Every firing of $t_3$ (event 'a') puts a token in place $p_4$ and re-enables $t_3$ and $t_4$. Upon the first firing of $t_4$ (event 'b') $t_3$ is disabled and $t_5$ (event 'b') can fire repeatedly until the tokens in place $p_4$ are consumed. When there is no token in place $p_4$ and there is a token in place $p_5$, this represents the end of a string. In this case transition $t_6$ is enabled and it fires by removing the token from place $p_5$ and by depositing a token in place $p_3$. This means that the specification APN goes back to its initial marking and another nondeterministic string can be generated again. The supervised model of the system is shown in Fig. 5.18.(c), which is obtained by combining the model and the nondeterministic reversible nondeterministic specification APN through the concurrent composition. Note that the supervised model of the system produces the strings $w_d$ = {rev($a^n b^m$) | $0 \leq m \leq n \leq p$ | p = 0, 1, 2, 3.... }. This can also be seen from Fig. 5.19, which shows the reachability tree of the supervised model, representing the reversible strings generated by the supervised model.



(a)



(b)



(c)

Figure 5.18.(a). The model of a system, that generates the nondeterministic strings $w = \{a^{n_1}b^{m_1}a^{n_2}b^{m_2}a^{n_3}b^{m_3} \ldots\ldots a^{n_p}b^{m_p} \cup b^{n_1}a^{m_1}b^{n_2}a^{m_2}b^{n_3}a^{m_3} \ldots\ldots a^{n_p}b^{m_p} \mid 0 \leq n_1 + n_2 + n_3 + \ldots + n_p \leq p, 0 \leq m_1 + m_2 + m_3 + \ldots + m_p \leq p \mid p = 0, 1, 2, 3 \ldots\ldots \}$. (b).The reversible nondeterministic specification APN, that represents the desired strings $w_d = \{rev(a^n b^m) \mid 0 \leq m \leq n \leq p \mid p = 0, 1, 2, 3 \ldots\ldots \}$.
(c). The supervised model of the system.

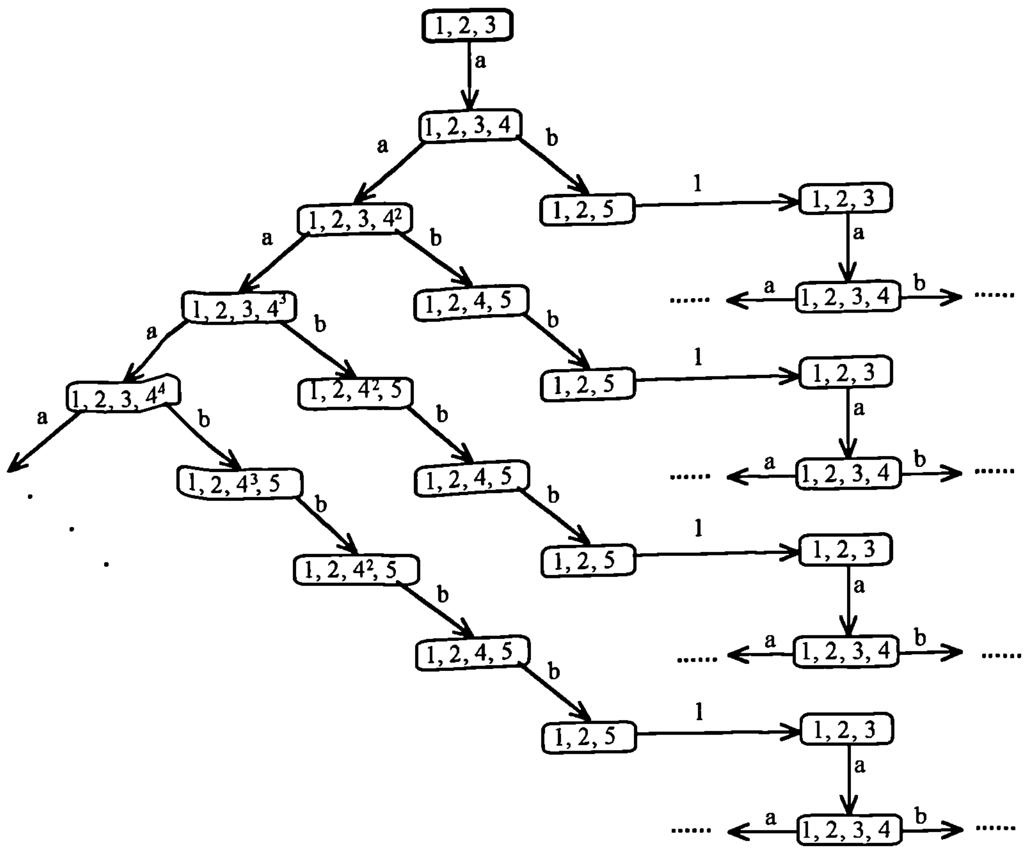


Figure 5.19. The reachability tree of the supervised model.

## 5.4. DISCUSSION

In this chapter, the *concurrent composition* concept, proposed by Giua and DiCesare (Giua and DiCesare, 1991) and the *language generator* concept, proposed by Sreenivas (Sreenivas, 1993 & 1994), have been brought together in a novel manner to solve the desired string problem. Note that a language generator is termed as a specification APN in this chapter. The methodology proposed can cope with deterministic and nondeterministic desired string problems. In the former the exact desired string is known beforehand. In contrast, in the latter case the exact desired string is not known beforehand. The strings considered (deterministic or nondeterministic) can be either reversible or irreversible. The irreversible strings can be used for systems, in which a sequence of events wanted to occur just once. The reversible strings can be used for systems, in which a sequence of events wanted to occur in a repeating fashion. In order to solve the desired string problem, first of all the desired string is represented by a specification APN and then it is combined with the uncontrolled model (in this case also called untreated model) by using the concurrent composition. This yields the controlled (supervised) model, which becomes the supervisor to supervise the system considered. It is important to note that, the desired string considered represents a valid desired string, i.e., it can be generated by the system. Note also that the reachability tree analysis can readily be used to check whether the controlled model behaviour conforms to the desired string given.

The results obtained in this chapter can be applied to high level manufacturing control, where the role of the supervisor is to coordinate factory-wide control of machines, and to low-level manufacturing control, where the role of the supervisor is to arrange low-level interaction between the control devices, such as motors, actuators, etc.

Note that these results are based on the assumption that there is a sufficient number of discrete event actuators, motors, etc. and discrete event sensors available within the system in order to be able to control the system.

# CHAPTER 6

# CONVERSION OF AUTOMATION PETRI NETS INTO LADDER LOGIC DIAGRAMS

# CHAPTER 6

# CONVERSION OF AUTOMATION PETRI NETS INTO LADDER LOGIC DIAGRAMS

## 6.1. INTRODUCTION

In today's modern factory Programmable Logic Controllers (PLC) have emerged as the mainstay in the execution of automation tasks. Their selection for discrete event control tasks is due to their low-cost, ruggedness and ease of programming. Indeed, the majority of PLCs can be programmed in a graphical symbolic language called *ladder logic diagrams* (LLD). The very simplicity of the LLDs which makes them so transparent is also their greatest downfall. This is because when developing complex control systems involving parallel tasks, which interact periodically, the ladder logic programming language offers little in the way of structural constructs to deal with the problem. However, this problem has been recognised and a structured approach to the design of discrete event control systems, which makes use of interpreted Petri nets, has emerged called Grafcet (David and Alla, 1992). Grafcet is a graphical programming language, which is made up of steps and transitions joined by directed links. The technique facilitates the design of concurrent interacting tasks and has become an international standard. Grafcet techniques are very powerful, but they do not contain all of the power and flexibility of the originating interpreted Petri net analysis. Moreover, many industrial users of PLCs still prefer to program PLCs in LLDs using heuristic approaches (Pollard, 1994).

For simple systems it is easy to write down PLC programs by heuristic methods. However, as systems get more complex it becomes very difficult to handle the problem effectively. The difficulty is compounded when multi-product systems are considered. In systems) by using heuristic approaches. The complexity problem of heuristic LLDs has long been recognised (Venkatesh *et al,* 1994). The most successful solutions to the problem have involved the use of Petri net for the conceptual design. Because of the success of Petri net designs there have been some attempts to produce methods to convert Petri nets into LLDs, (Greene, 1989; Rattigan, 1992; Satoh *et al,* 1992; Jafari and Boucher, 1994; Burns and Bidanda, 1994; Taholakian and Hales, 1995). However, until the advent of the Token Passing Logic (TPL) methodology (Jones *et al,* May 1996), none of these methods to-date have produced a technique that is general, in the sense that it can deal with timers, counters, coloured Petri nets and timed Petri nets. The TPL methodology bridges the gap between Petri net analysis and LLDs. The technique is powerful and yet simple to both understand and implement. Moreover, the technique has been extended to deal with timed-place Petri nets (Jones *et al,* May 1996), (Uzam and Jones, July 1996), timed-transition Petri nets (Jones *et al,* Sept. 1996), and Coloured Petri nets (Uzam and Jones, August 1996; Jones and Uzam, August 1996). The TPL methodology has also been developed to embrace statement lists (Jones and Uzam, Sept. 1996; Uzam and Jones, Sept. 1996), and knowledge-based systems (Jones *et al,* 28-30 May 1996; Jones *et al,* June 1996; Jones and Uzam, Dec. 1996; Uzam and Jones, Dec. 1996; Jones and Uzam, 1998 ). An attempt to introduce a Petri net based formal controllers is made in (Uzam and Jones, November 1996). This is followed by (Uzam and Jones, 1997), in which the IEC 1131-3 standard, which is a standard of International Electrotechnical Commission, dealing with five programming languages for programmable controllers, is considered for possible implementations of Automation Petri Net Controllers using the IEC 1131-3 instruction list (IL) code, and likewise by (Uzam and Jones, 1998), in which the IEC1131-3 standard is considered for possible implementations of Automation Petri Nets using the IEC 1131-3 Ladder Diagram (LD) code.

The (TPL) methodology makes use of the fact that the prime control mechanism within the Petri net is the token. This concept of using a token within a ladder logic program goes against industrial practice, where in general the program control is achieved through the use of flags or auxiliary relays. Indeed, it is this fundamental conceptual departure from conventional practice which is the key to the new proposed method.

The purpose of this chapter is to introduce a general methodology for converting Automation Petri Nets into LLDs. In order to implement Petri net designs directly as controllers some additional features to ordinary Petri nets have to be defined. This is because Petri nets do not have constructs to adequately deal with actuators and sensors. This deficiency has prompted the advent of Automation Petri nets (APNs) (Uzam and Jones, 1998), which extends the ordinary Petri nets to deal with discrete event control applications. These extensions involve interfacing the Petri net to actuators and sensors. The TPL method is conceptually simple, and permits a direct conversion of Automation Petri Nets into LLDs. It also provides a straight forward mapping between the basic sequencing information and the programming steps. Moreover, the method accommodates timers and counters. The method can also deal with coloured and/or timed APNs. Furthermore, because of the structure of the method it is very easy to modify or extend the program if the control requirements change. Finally, it is believed that this method is a candidate to become a world-wide standard method for programming PLCs.

## 6.2. CONVERSION OF AUTOMATION PETRI NETS INTO LADDER LOGIC DIAGRAMS

Recently introduced Token Passing Logic (TPL) methodology can be easily used to convert Automation Petri Nets (APNs) into ladder logic diagrams (LLD). The TPL method is conceptually simple, and permits the conversion of APNs into LLDs. It also provides a straight forward mapping between the basic sequencing information and the programming steps. Moreover, the method accommodates timers and counters. The method can also deal with coloured and/or timed APNs.

### 6.2.1. Token Passing Logic Methodology

The prime feature of the TPL technique is that it facilitates the direct conversion of any Automation Petri Net (APN) into a Token Passing Logic Controller (TPLC). The TPLC is a generic form of control logic which may be implemented with low level languages such as machine language, STatement Lists(STL), Ladder Logic Diagrams (LLDs), etc. or with high level languages such as C, C++, etc. This is achieved by adopting the Petri net concept of using tokens as the main mechanism for controlling the flow of the control logic. Hence, each place within the APN corresponds to a place within the TPLC. The simulated movement of tokens is achieved by deploying counters at each place in APN, whose capacity is greater than 1. These counters are then incremented and decremented to simulate token flow. Thus, each place within the APN has at least an associated counter in the TPLC, the current count of which represents the number of tokens in the place. There are different types of counters with different specifications, depending on the manufacturers. In TPL methodology counters have the following characteristics: If the count value of the counter is greater than zero, then the status of the counter becomes 'one', and if the count value of the counter is zero then the status of the counter is 'zero'. The assignment of a counter to an APN place through the TPL is shown in Fig. 6.1.(a), where C stands for counter. Finally, to complete the Petri net synergy, if the count associated with a place in the APN is non-zero and the firing condition of a Petri net-like transition associated with that place becomes true, then the counter at the place is decremented by one, and the subsequent place linked by the transition is incremented by one. In the case of single capacity places the counters can be replaced by flags. The assignment of a flag to an APN place through the TPL is also shown in Fig. 6.1.(b), where F stands for flag.

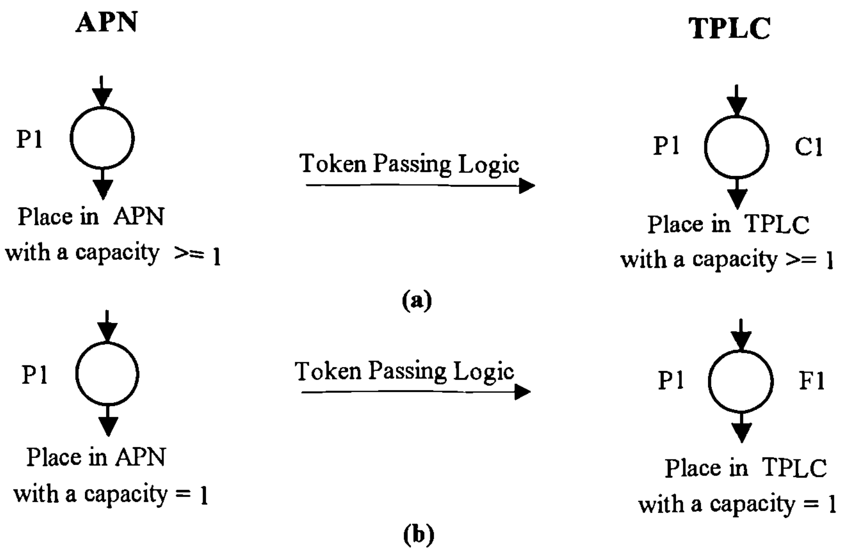


Figure 6.1. APN places and their equivalent TPLC places.

In essence, the APN places are represented by places in TPLC, and the APN tokens are represented by the counts in separate counters at each logic place. Moreover, the flow of Petri net tokens is simulated by counting down and counting up the counters or similarly by setting and resetting the appropriate flags at the appropriate places. In APNs, actions are assigned to places. Places for which actions are assigned are called action places. Transition firing conditions in APNs are logical functions of sensor states. In theory, the TPL methodology can cope with any number of tokens at an APN place. The TPLC provides a visual description of the control program which has all the advantages of a full Petri net analysis. Timed Automation Petri Nets, namely timed-place APN and timed-transition APN can be converted into TPLC by using an *on delay timer*. Furthermore, coloured APNs can also be converted into control logic using this methodology, simply by assigning more than one counter or flag to each place. It is believed that this new technique provides a tool which is a simple, but sophisticated way of developing complex Discrete Event Control Systems. It is these very features which will be vital to the success of agile manufacturing systems.

It should be noted that the LLDs generated from TPLCs rely on the updating of outputs only at the end of a ladder scan for proper functioning. The TPL methodology is illustrated by considering the following structures:

1. Initial marking
2. APN without action
3. APN with action
4. Inhibitor arc APN
5. Enabling arc APN
6. And transition APN
7. Or transition APN
8. Weighted arc APN
9. Conflict in APN
10. Timed-transition APN

#### 6.2.1.1. Initial marking

The initial markings have to be put into the controller before the system is started. When implementing the TPLC structure as LLDs initial markings has to be taken into account at the beginning of the ladder logic code to ensure correct operation. In order to put initial marking into LLDs at the first rung of the LLD a flag, called *initialisation flag*, is used. One normally closed contact of the initialisation flag is used to set the corresponding flags and the corresponding counters to the correct numbers in order to put initial markings into LLD. After this is done, the initialisation flag resets itself. For the next scans initialisation flag will be set and as a result initial marking will be put in the LLD once. This is shown in Fig. 6.2, where F0 is an initialisation flag.

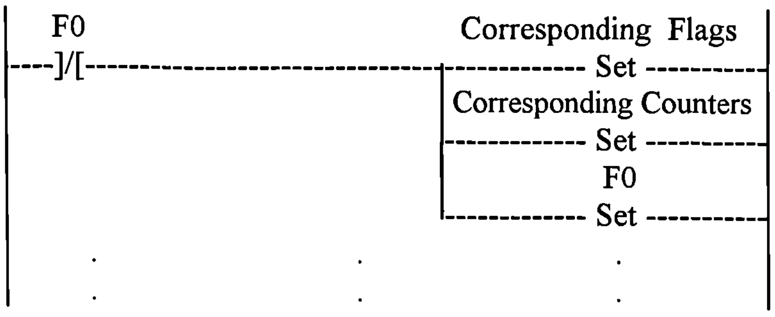


Figure 6.2. The LLD for the initial marking.

#### 6.2.1.2. APN without action

An APN with no actions assigned to its places is shown in Fig. 6.3.(a). In an APN, a transition can only be fired if the number of tokens in the input place(s) is non-zero, that is, transition is enabled, and the firing condition $\chi$ of the transition occurs. When the transition is fired it removes a token from the input place(s) and puts a token to the output place(s). To convert an APN into a TPLC a counter (or a flag) is assigned to the places. In TPLC, each transition withdraws a token from the current logic place and adds a token to the next logic place. This is achieved by using a counter (or a flag) at each place to represent the tokens. When a transition is fired, to simulate the passing of a token the input counter is decremented and the output counter is incremented by one (or similarly the input flag is reset and the output flag is set). The equivalent TPLC for the APN, given in Fig. 6.3.(a), is shown in Fig. 6.3.(b). The Ladder logic diagram for the TPLCs shown in Fig. 6.3.(b), is given in Fig. 6.4. In this case, the initial marking is also included in LLD.

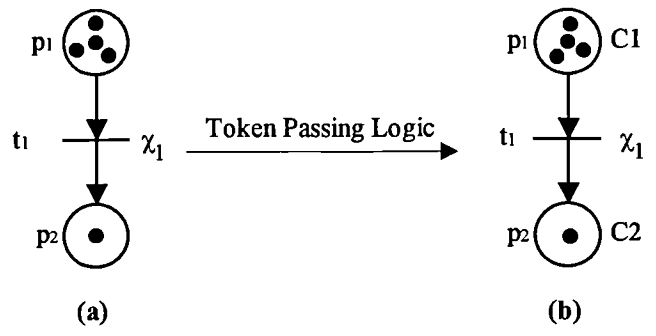


Figure 6.3. (a) An APN with no actions assigned to its places. (b) The equivalent TPLC.

```
|  F0                                        C1 : 4          |
|----]/[-----------------------------+--------- Set ---------|
|                                    |       C2 : 1          |
|                                    +--------- Set ---------|
|                                    |         F0            |
|                                    +--------- Set ---------|
|  C1    χ1                                    C1            |
|-----] [-----] [--------------------+------ Count Down -----|
|                                    |         C2            |
|                                    +----- Count Up --------|
|                                                            |
```

Figure 6.4. The LLD for the TPLC, given in Fig. 6.3.(b).

### 6.2.1.3. APN with action

Fig. 6.5.(a) shows an APN, where an action is assigned to place $p_2$. An action at a given place within a Petri net occurs only if the number of token at the place is non-zero. Note that if the firing condition for transition $t_2$ is 1, i.e., it fires as soon as there is a token in place $p_2$, then the action assigned to place $p_2$ is called an *impulse action*. Similarly, if the firing condition for transition $t_2$ is not 1, i.e., it fires when there is a token in place $p_2$ and firing condition of transition $t_2$ occurs, then the action assigned to place $p_2$ is called a *level action*. To convert an APN into a TPLC, a counter or a flag is assigned to the places. In a TPLC, a level action is controlled by the counter or flag at the place. If the count value of a counter at the control place is greater than zero or the related flag is set then any actions associated with the place are enabled. Fig. 6.5.(b) shows the equivalent

TPLC for the APN, given in Fig. 6.5.(a). The LLD for the TPLC, shown in Fig. 6.5.(b), is given in Fig. 6.6. Note that in this case the initial marking is not shown in the LLD. Also note that if the same action is assigned to more than one place, then the flags associated with the places have to be 'OR'ed together to activate the action. This implies that each action only appears once in the LLD code.

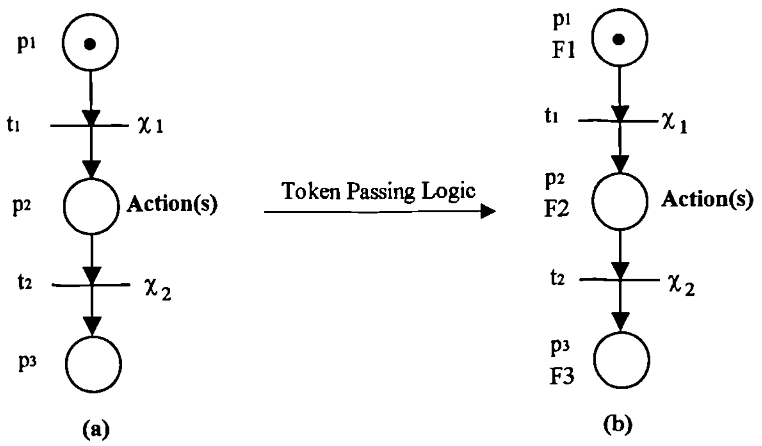


Figure 6.5. (a). An APN in which a level action(s) assigned on a place. (b) The equivalent TPLC.

```
|  F1   χ1                              F1         |
|-----] [----] [---------------------------------- Reset ---------|
|                                     |      F2         |
|                                     L--------- Set ---------|
|  F2                                     Action(s)  |
|-----] [----------------------------------------( )---------|
|  F2   χ2                              F2         |
|-----] [----] [---------------------------------- Reset ---------|
|                                     |      F3         |
|                                     L--------- Set ---------|
```

Figure 6.6. The LLD for the TPLC, shown in Fig. 6.5.(b).

### 6.2.1.4. Inhibitor arc APN

An inhibitor arc APN is shown in Fig. 6.7.(a). The transition $t_1$ has two input places $p_1$ and $p_2$, where $p_2$ has an inhibitor arc, *In*$(p_2,t_1)$. The transition $t_1$ is fired, when place $p_1$

has at least one token, place $p_2$ has no token and the firing condition $\chi_1$ occurs. When it is fired a token is removed from place $p_1$ and a token is deposited into the output place $p_3$, but the marking of inhibitor arc connected place $p_2$ does not change. There can be more than one output place in which case a single token would be passed to every output places. The transition $t_1$ is inhibited from firing if there is a token in place $p_2$. To convert an APN into a TPLC a counter or a flag is assigned to the places. Fig. 6.7.(b) shows the equivalent inhibitor arc TPLC for the inhibitor arc APN. In this case transition $t_1$ is fired if the count value of C1 is greater than zero and the count value of C2 is zero and firing condition $\chi_1$ occurs. When transition $t_1$ is fired, the count value of the counter C1, associated with place $p_1$, is decremented, thus a token is withdrawn from place $p_1$, and the count value of the counter C3, associated with place $p_3$, is incremented thus a token is added into place $p_3$. If the count value of the counter C2, associated with place $p_2$, is non-zero, then it will inhibit transition $t_1$ from firing. The LLD for the TPLC, shown in Fig. 6.7.(b), is given in Fig. 6.8. Note that in this case the initial marking is not shown in the LLD.

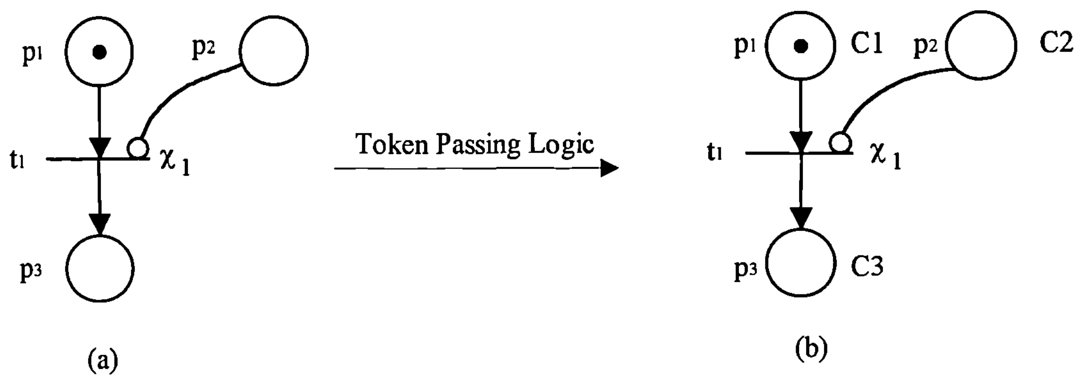


Figure 6.7. (a). An inhibitor arc APN. (b). The equivalent inhibitor arc TPLC.

```
|   C1    C2    χ1                            C1           |
|----] [-----]/[-----] [------------------+---- Count Down -----|
|                                         |       C3           |
|                                         +----- Count Up -------|
|                                                              |
```

Figure 6.8. The LLD for the inhibitor arc TPLC shown in Fig. 6.7.(b).

### 6.2.1.5. Enabling arc APN

An enabling arc APN is shown in Fig. 6.9. The transition $t_1$ has two input places $p_1$ and $p_2$, where $p_2$ has an enabling arc, *En(p₂,t₁)*. The transition $t_1$ is fired if both place $p_1$ and $p_2$ have at least one token each and firing condition $\chi_1$ occurs. When it is fired a token is removed from place $p_1$ and a token is deposited into the output place $p_3$, but the marking of enabling arc connected place $p_2$ does not change. There can be more than one output place in which case a single token would be passed to every output place. The transition $t_1$ is not enabled to fire if there is no token in place $p_2$. To convert an APN into a TPLC a counter or a flag is assigned to the places. Fig. 6.9.(b) shows the equivalent inhibitor arc TPLC for the enabling arc APN. In this case transition $t_1$ is fired if the count value of C1 and C2 are greater than zero and firing condition $\chi_1$ occurs. When transition $t_1$ is fired, the count value of the counter C1, associated with place $p_1$, is decremented, thus a token is withdrawn from place $p_1$, and the count value of C3, associated with place $p_3$, is incremented, thus a token is added into place $p_3$. After transition $t_1$ is fired, the count value of C2 will remain the same. If the count value of C2, associated with place $p_2$, is zero, then it will not enable the transition $t_1$ to fire. The LLD for the TPLC shown in Fig. 6.9.(b) is given in Fig. 6.10. Note that in this case the initial marking is not shown in the LLD.

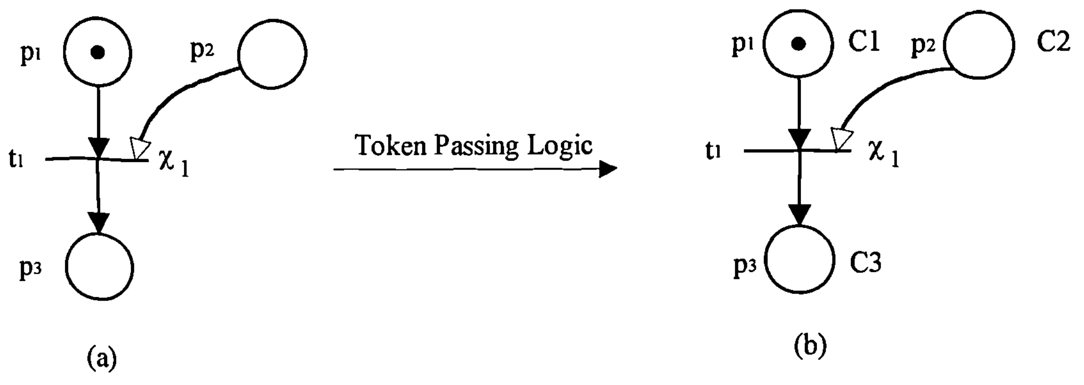


Figure 6.9. (a). An enabling arc APN. (b). The equivalent enabling arc TPLC.

```
|   C1     C2    χ1                          C1              |
|-----] [-----] [-----] [-----------------+---- Count Down ------|
|                                         |      C3              |
|                                         +---- Count Up --------|
|                                                                |
```

Figure 6.10. The LLD for the enabling arc TPLC shown in Fig. 6.9.(b).

### 6.2.1.6. And transition APN

An *and transition* in an APN is shown in Fig. 6.11.(a). Transition $t_1$ can only be fired when all the input places have at least one token and the firing condition $\chi_1$ occurs. When it is fired a token is removed from place $p_1$ and $p_2$ and a token is deposited to the output place $p_3$. To convert an APN into a TPLC a counter or a flag is assigned to the places. The equivalent *and transition* in a TPLC for the and transition APN is shown in Fig. 6.11.(b). If a transition is fired it withdraws a token from each of the input places of that transition and adds a token to the output place(s) of that transition. This goal can be achieved by decrementing the counters at each input place by one and incrementing the counter of every output place by one. The LLD for the and transition APN, shown in Fig. 6.11.(b), is given in Fig. 6.12. Note that in this case the initial marking is not shown in the LLD.

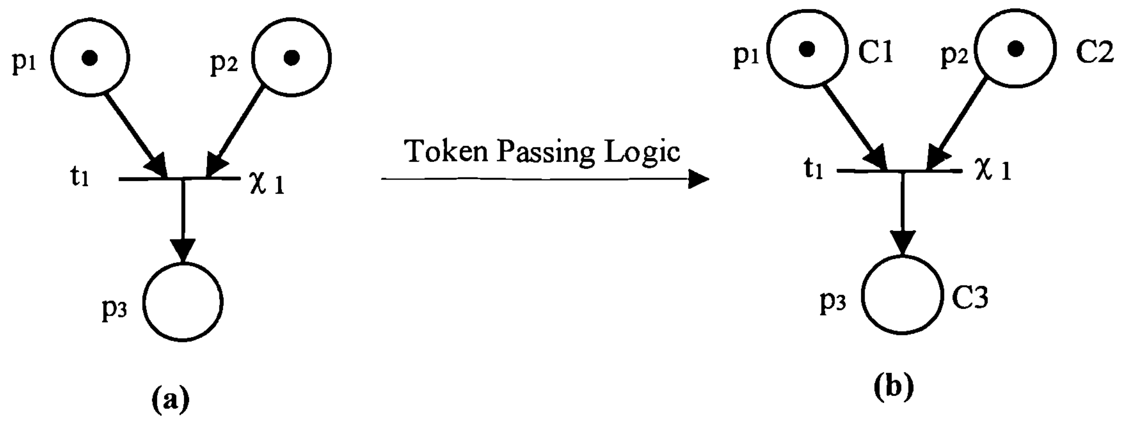


Figure 6.11. (a). An and transition in an APN. (b). The equivalent and transition in a TPLC.

```
|    C1    C2    χ1                          C1                  |
|-----] [-----] [-----] [-------------------┬---- Count Down ------|
|                                           |     C2              |
|                                           ├-- Count Down -------|
|                                           |     C3              |
|                                           └---- Count Up --------|
|                                                                 |
```

Figure 6.12. The LLD for the TPLC, shown in Fig. 6.11.(b).

## 6.2.1.7. Or transition APN

In general, the *or transition* is not formally defined in terms of a Petri net. However, in LLDs the 'Boolean or' and 'Exclusive or' are used widely. An event driven Boolean or can be implemented in APNs by using $(2^n - 1)$ transitions and $n.(2^{(n-1)} - 1)$ inhibitor arcs, where *n* is the number of input places. An 'or transition APN' with three input transitions is shown in Fig. 6.13.(a). To convert an APN into a TPLC a counter or a flag is assigned to the places. The equivalent 'or transition TPLC' for the or transition APN is shown in Fig. 6.13.(b). The LLD for the TPLC shown in Fig. 6.13.(b) is given in Fig. 6.14. Note that in this case the initial marking is not shown in the LLD.

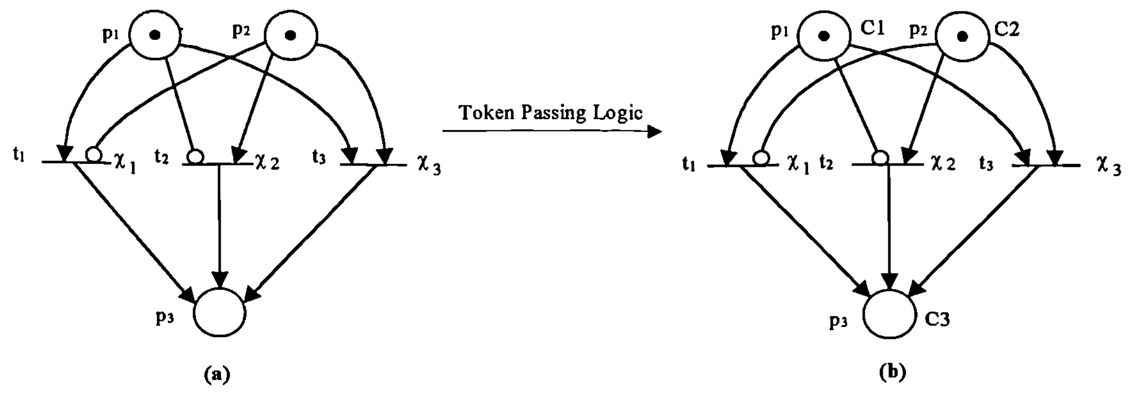


Figure 6.13. (a). An or transition in an APN. (b). The equivalent or transition in a TPLC.

```
|  C1    C2    χ1                            C1              |
|-----] [-----]/[-----] [------------------+----- Count Down -----|
|                                          |     C3              |
|                                          L----- Count Up --------|
|  C1    C2    χ2                            C2              |
|-----]/[-----] [-----] [------------------+----- Count Down -----|
|                                          |     C3              |
|                                          L----- Count Up --------|
|  C1    C2    χ3                            C1              |
|-----] [-----] [-----] [------------------+---- Count Down -----|
|                                          |     C2              |
|                                          |-- Count Down ------|
|                                          |     C3              |
|                                          L---- Count Up -------|
|                                                                |
```

Figure 6.14. The LLD for the TPLC, shown in Fig. 6.13.(b).

### 6.2.1.8. Weighted arc APN

A weighted arc APN is shown in Fig. 6.15.(a). The transition $t_1$ can only be fired if the number of tokens at the input place $p_1$ is either equal to or greater than $n$ and the firing condition $\chi_1$ occurs. When the transition $t_1$ is fired, it will remove $n$ tokens from place $p_1$ and will add $m$ tokens to place $p_2$. To convert an APN into a TPLC a counter or a flag is assigned to the places. The equivalent weighted arc TPLC for the weighted arc APN is shown in Fig. 6.15.(b). In the weighted arc TPLC, transition $t_1$ will be enabled if the count value of C1 is greater than or equal to $n$ and the firing condition $\chi_1$ occurs. When it is fired it will decrement the count value of C1 by $n$ and increment the count value of C2 by $m$. The LLD for the *weighted arc* TPLC is given in Fig. 6.16, where CMP is a compare instruction, ADD is an addition instruction, SUB is a subtraction instruction, $n$ is the weight of the input arc, and $m$ is the weight of the output arc. Note that in this case the initial marking is not shown in the LLD.

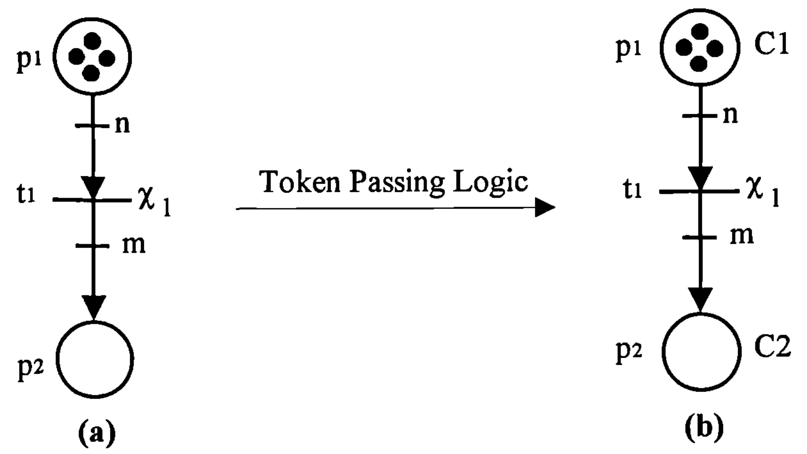


Figure 6.15. (a). A weighted arc APN (b). The equivalent weighted arc TPLC.

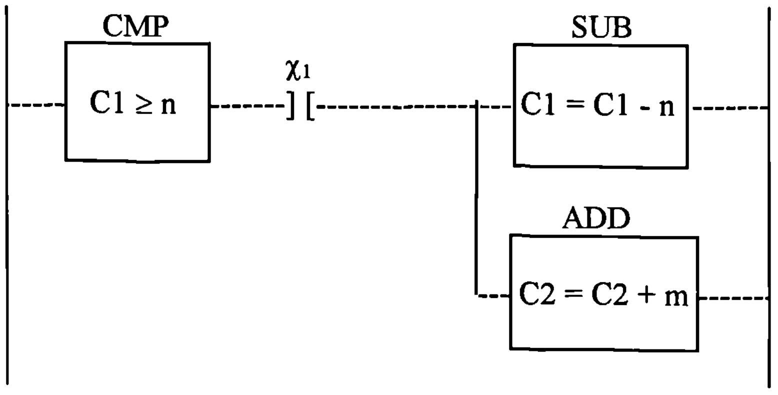


Figure 6.16 LLD for the TPLC shown in Fig. 6.15.(b).

Nevertheless, the instructions CMP, SUB and ADD are not available in some PLCs. In this case, the weighted arc APN can be replaced with its equivalent, shown in Fig. 6.17.(a), so as to convert it into LLDs. TPLC equivalent of the APN, given in Fig. 6.17.(a), is shown in Fig. 6.17.(b). The input and output places, $p_1$ and $p_2$ are represented by counters, $C1^i$ and $C2^o$ respectively. Places $p_1^1$, $p_1^2$, $p_1^3$,..., $p_1^n$ and $p_2^1$, $p_2^2$, $p_2^3$,..., $p_2^m$ are represented by input flags $F1^1$, $F1^2$, $F1^3$,..., $F1^n$ and by output flags $F2^1$, $F2^2$, $F2^3$,..., $F2^m$, respectively. Note that firing condition for the transitions $t_1^{i1}$, $t_1^{i2}$, $t_1^{i3}$,..., $t_1^{in}$ and $t_1^{o1}$, $t_1^{o2}$, $t_1^{o3}$,..., $t_1^{om}$ is 1 and so, to keep the APN simple, firing conditions for these transitions are not shown in Fig. 6.17. When a new token enters the input place $p_1$, the counter $C1^i$ is incremented. This token is then routed automatically to an empty place ($p1^1$, $p1^2$, $p1^3$,..., $p1^n$ ) by the APN structure. Transition $t_1$ is fired, when all the input flags $F1^1$, $F1^2$, $F1^3$,..., $F1^n$ are set and the firing condition $\chi_1$ occurs. When transition $t_1$ is fired the input flags are reset and the output flags $F2^1$, $F2^2$, $F2^3$,..., $F2^m$ are set. Each one of the output flags will increment the counter $C2^o$ by one. The LLD for the TPLC shown in Fig. 6.17.(b) is given in Fig. 6.18. Note that in this case the initial marking is not shown in the LLD.

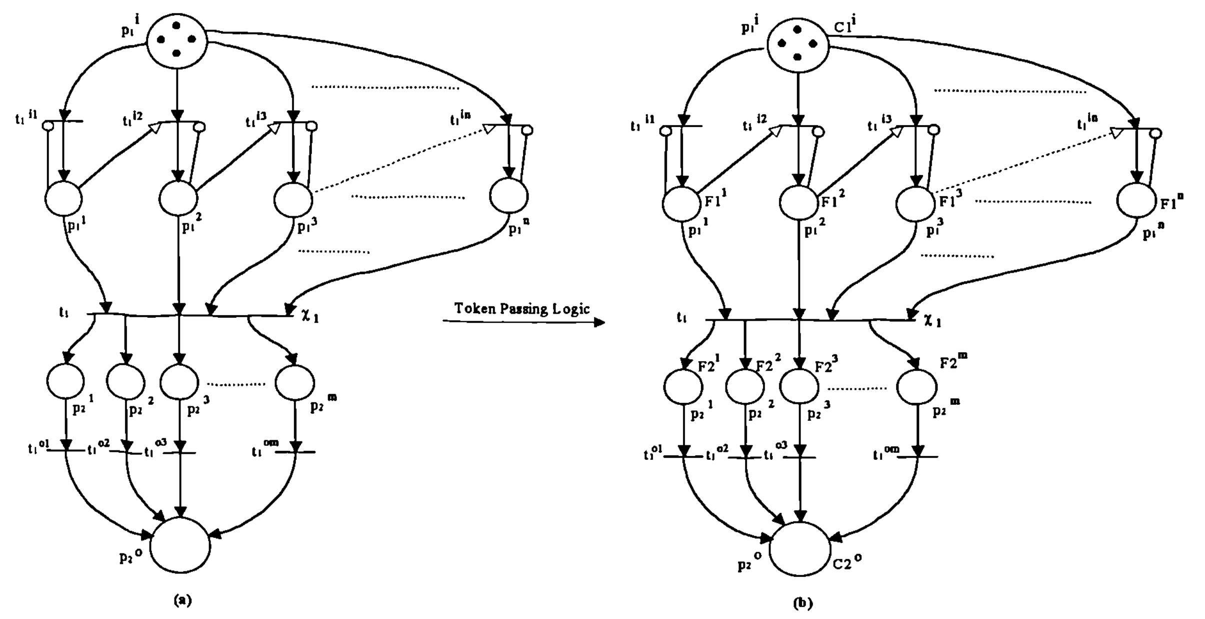


Figure 6.17. (a). The equivalent of the weighted arc APN shown in Fig 6.15.(a). (b). The equivalent TPLC for the weighted arc APN, shown in Fig 6.17.(a).

```
|  C1^i  F1^n    F^n-1                          F^n
|-----] [-----]/[-----] [-------------------┬--------- Set -----------|
|                                           |          C1^i           |
|                                           └---- Count Down ---------|
|   .            .          .                                         |
|   .            .          .                                         |
|   .            .          .                                         |
|  C1^i   F1^3    F1^2                          F1^3                  |
|----] [-----]/[-----] [--------------------┬---------- Set ----------|
|                                           |          C1^i           |
|                                           └----- Count Down --------|
|  C1^i   F1^2    F1^1                          F1^2                  |
|----] [-----]/[-----] [--------------------┬---------- Set ----------|
|                                           |          C1^i           |
|                                           └----- Count Down --------|
|                                                                     |
```

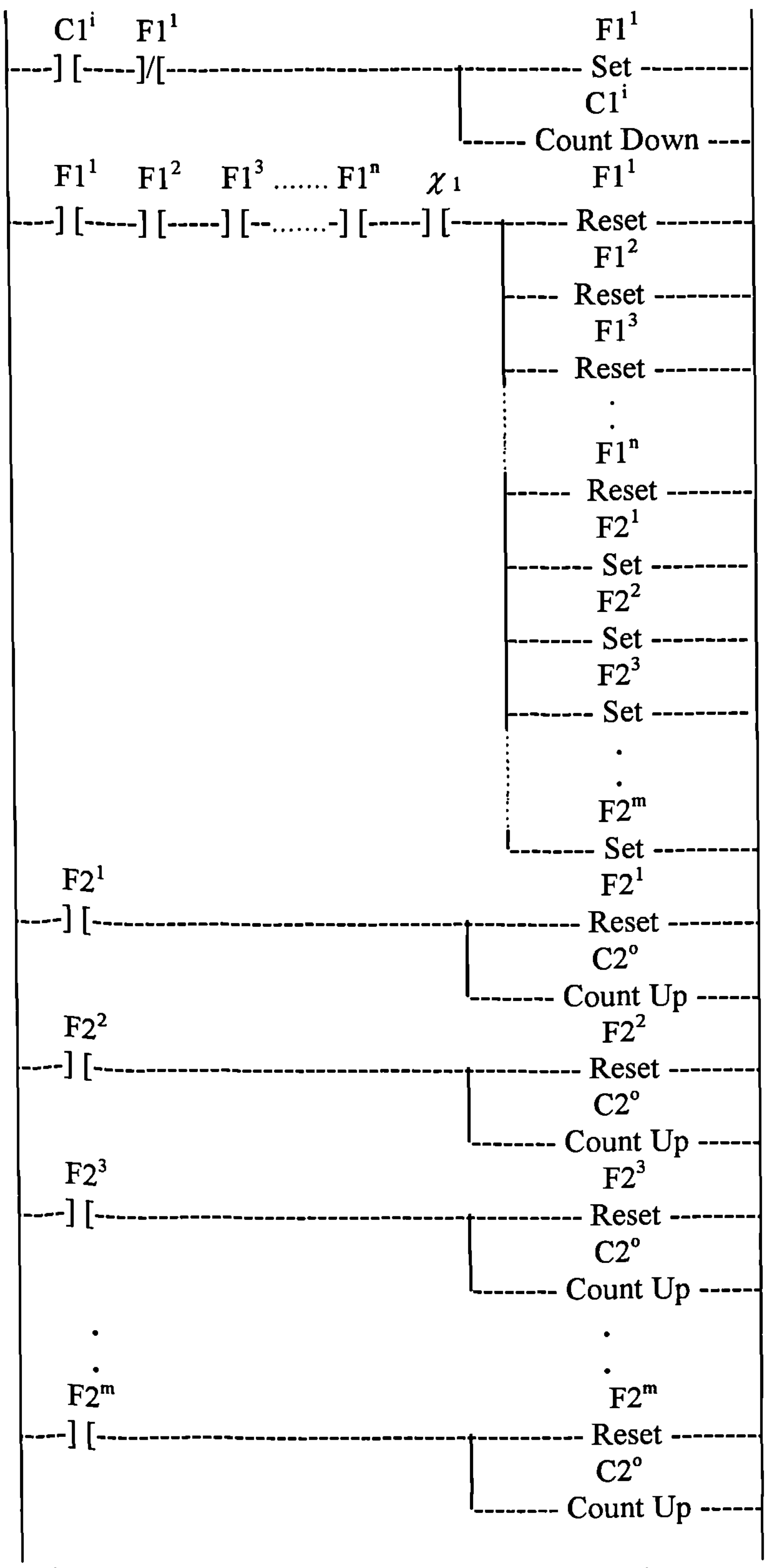


Figure 6.18. The LLD for the TPLC shown in Fig. 6.17.(b).

### 6.2.1.9. Conflict in APN

A *conflict* corresponds to the existence of an input place which has at least two output transitions. According to the definition of Petri nets, only one output place can receive a token in the case of conflict. One simple way to resolve the conflict is to assign a priority to each of the transitions, i.e., a technique is used to resolve the conflict by choosing which transition is to be allowed to fire (Desrochers and Al-Jaar, 1995). This choice is often based on a priority scheme. Conflict in an APN may occur as shown in Fig. 6.19.(a), when there is a token in place $p_1$ and firing condition $\chi_1$ and $\chi_2$ occur at the same time. Alternatively, if the firing condition $\chi_1$ of transition $t_1$ equals to the firing condition $\chi_2$ of transition $t_2$, and then this represents a conflict situation in APNs. Conflict in a TPLC may occur in the same manner as shown in Fig. 6.19.(b). The conflict in Fig. 6.19.(b) can be resolved by assigning a priority between this two transitions. To convert an APN into a TPLC, a counter or a flag is assigned to the places. Fig. 6.20 shows a LLD for the TPLC, given in Fig. 6.19.(b). In LLDs, conflict resolution is achieved by firstly deciding the order of priorities for the conflicting transitions. Once this has been done, each rung of ladder logic for each transition is then written in the same order. Because of the nature of LLD this process will automatically resolve any conflict such that the chosen priorities are met. Therefore, as can be seen from Fig. 6.20 transition $t_1$ of TPLC has the priority over transition $t_2$. If the LLD program was written the other way around, transition $t_2$ would have the priority over $t_1$. Note that in this case the initial marking is not shown in the LLD.

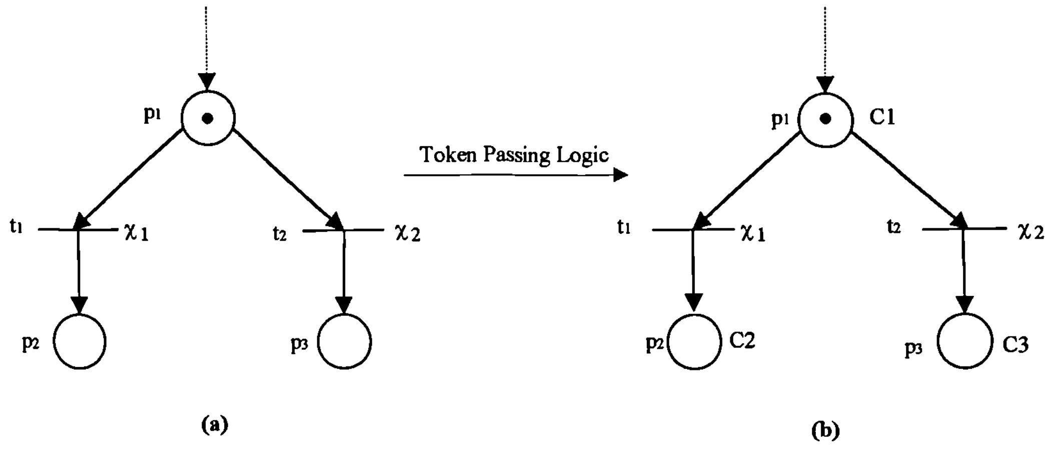


Figure 6.19. (a). Conflict in an APN. (b). Conflict in a TPLC.

```
|  C1   χ1                               C1              |
|-----] [-----] [--------------------------- Count Down ---|
|                                  |         C2            |
|                                  L------ Count Up -------|
|  C1   χ2                               C1              |
|-----] [-----] [--------------------------- Count Down ---|
|                                  |         C3            |
|                                  L------ Count Up -------|
|                                                          |
```

Figure 6.20. The LLD for the TPLC shown in Fig. 6.19.(b).

The APN in Fig. 6.21.(a) has two additional places $p_4$ and $p_5$, which are used to have a conflict free APN. With this arrangement, there is no conflict and transitions $t_1$ and $t_2$ will be fired one after another. To convert an APN into a TPLC a counter or a flag is assigned to the places. Fig. 6.22 shows a LLD for the TPLC given in Fig. 6.21.(b). In this case one token has to be put in either place $p_4$ or place $p_5$, indicating first priority of the conflict resolution. In Fig. 6.22, at the beginning the priority has been given to transition $t_1$, by putting a token in place $p_4$.

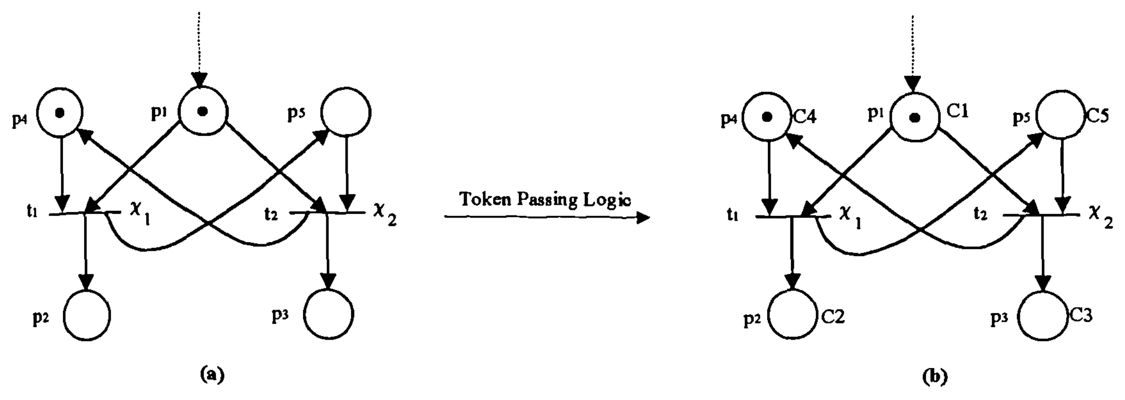


Figure 6.21 (a). Conflict resolution in APN. (b). The equivalent TPLC.

```
|  C1     C4    χ1                                 C1                |
|----] [-----] [-----] [------------------+----- Count Down ----|
|                                         |        C4                |
|                                         +---- Count Down -----|
|                                         |        C2                |
|                                         +------ Count Up ------|
|                                         |        C5                |
|                                         +------ Count Up ------|
|  C1     C5    χ2                                 C1                |
|----] [-----] [-----] [------------------+----- Count Down ----|
|                                         |        C5                |
|                                         +---- Count Down -----|
|                                         |        C3                |
|                                         +------ Count Up ------|
|                                         |        C4                |
|                                         +------ Count Up ------|
|                                                                    |
```

Figure 6.22. The LLD for the TPLC shown in Fig. 6.21.(b).

### 6.2.1.10. Timed-transition APN

Note that in this thesis only the timed-transition APN is explained, but timed-place APNs can also be converted into LLDs using the TPL method. From the definition of timed-transition APNs, a token can have two states: it can be *reserved* for the firing of a timed-transition $t_i$ or it can be *unreserved*. If a timed transition is enabled then it is ready to be fired. When the firing condition $\chi$ for the transition occurs, the token of the input place to this transition is said to be reserved for a specified amount of time($\alpha_i$). When the time $\alpha_i$ has elapsed, the transition is effectively fired : the reserved token is removed from the input place and a token is put into the output place(s). Fig. 6.23.(a) shows a timed-transition APN, where $t_1$ is a timed-transition. When transition $t_i$ is fired, an unreserved token is deposited in place $p_1$. When firing condition $\chi_1$ occurs, firing of transition $t_1$ is started and the unreserved token in place $p_1$ becomes reserved for firing of transition $t_1$. During the time $\alpha_1$, transition $t_1$ is being fired. After the time has elapsed, the transition is effectively fired: the reserved token is removed from the input place $p_1$ and a token is put into the output place $p_2$. In the equivalent timed-transition TPLC for the timed-transition APN is shown in Fig. 6.23.(b). An *on delay timer*, whose timing diagram is shown in Fig. 6.24, is used to represent time delay of the timed-transition. If the flag F1 is set and the firing condition $\chi_1$ has not yet occurred then this represents the unreserved token. If the flag F1 is set and the firing condition $\chi_1$ occurs then this represents the reserved token. In Fig. 6.23.(b), when transition $t_i$ is fired, the flag F1 is set. When flag F1 is set, transition $t_1$ is enabled. If F1 is set and firing condition $\chi_1$ occurs, then firing of timed-transition $t_1$ is started, i.e., the on delay timer T1 is started its operation for a time delay '$\alpha_1$'. When the time has elapsed, transition $t_1$ is effectively fired: F1 is reset and counter C2 is incremented by one. The LLD for the timed-transition TPLC, shown in Fig. 6.23.(b), is obtained by direct mapping from the TPLC to ladder logic, and it is given in Fig. 6.25. Note that in this case the initial marking is not shown in the LLD.

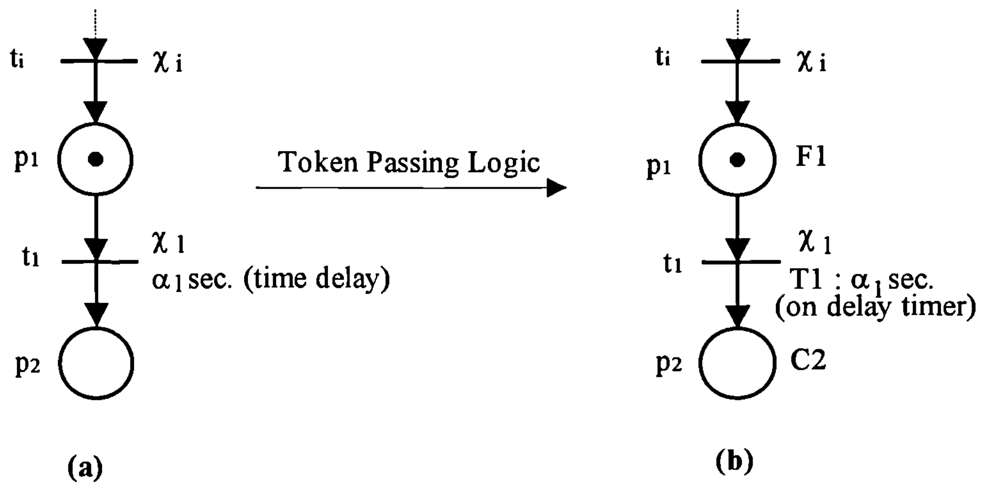


Figure 6.23. (a). A timed-transition APN. (b). The equivalent timed-transition TPLC.

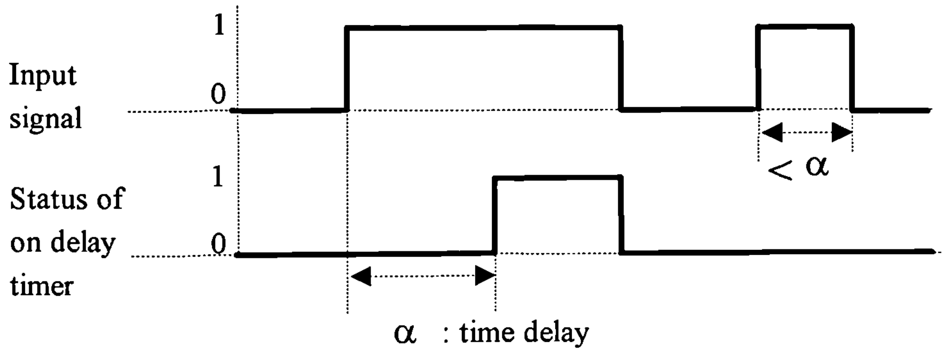


Figure 6.24. The timing diagram for an *on delay timer.*

```
|  χi                                           F1                |
|----] [------------------------------------------ Set -------------|
|  F1    χ1                                  T1 : α sec.            |
|----] [-----] [-------------------------- On Delay Timer ------|
|  F1    χ1      T1                            F1                 |
|----] [-----] [-----] [------------------┬-------- Reset ---------|
|                                         |         C2             |
|                                         └---- Count Up ---------|
|                                                                   |
```

Figure 6.25. The LLD for the TPLC, shown in Fig. 6.23.(b).

## 6.3. DISCUSSION

In this chapter, a general methodology for converting Automation Petri Nets into LLDs has been proposed. Ladder Logic Diagrams (LLDs) are the most popular programming language for programming such PLCs. Because of this, a general methodology, called Token Passing Logic (TPL), has been proposed to convert APNs into LLDs. The TPL method is conceptually simple, and permits a direct conversion of Automation Petri Nets into LLDs. It also provides a straight forward mapping between the basic sequencing information and the programming steps. It has been shown that, the method accommodates timers and counters and timed APNs. Furthermore, because of the structure of the method it is very easy to modify or extend the LLD program if the control requirements change. Finally, it is believed that this technique provides for the first time a general way of converting Petri nets into LLDs that can include all of the automation requirements of a modern factory.

# CHAPTER 7

# APPLICATION EXAMPLES

# CHAPTER 7

# APPLICATION EXAMPLES

## 7.1. INTRODUCTION

In this chapter, a discrete manufacturing system is considered to show how the methodologies proposed in this thesis can be applied to real supervisory control problems. To do this, the Bytronic Associates Industrial Control Trainer (ICT) is used as the discrete manufacturing system. For implementation purposes a Siemens programmable logic controller (PLC) (S5-100U) is used. Note that the manufacturing system used in this chapter is an example of a low-level control problem. The forbidden state problem regarding the manufacturing system is considered in the section A, while the desired string problem is considered in the section B of this chapter. In the forbidden state problem, the results obtained are compared in terms of the number of places and transitions used in the supervisors for each method as well as the ladder logic diagram code (LLD) generated from these supervisors. In the case of the desired string problem, two examples regarding the manufacturing system are used to show how the desired string problems can be solved in the case of low-level control problems.

The conversion from the supervisors (the controlled models) into LLDs for implementation on S5-100 PLC is considered in detail by using the Token Passing Logic (TPL) concept.

## SECTION A

## THE FORBIDDEN STATE PROBLEM

## 7.2. THE FORBIDDEN STATE PROBLEM

In this section, an example manufacturing system is explained and example forbidden state specifications are provided. After that the inhibitor arc method, the enabling arc method, the intermediate place method, the APN-SM method, the U-TPM rule method and the C-TPM rule method, are considered (in the sections A1, A2, A3, A4, A5, and A6 respectively) to solve the forbidden state problem. The corresponding LLD code for each method is also provided.

### 7.2.1. Problem Description

The Manufacturing System, shown in Fig. 7.1, represents a component sorting and assembly processes that can be controlled by virtually any PLC. The upper conveyor and the lower conveyor are driven by the upper conveyor motor (Actuator 1) and the lower conveyor motor (Actuator 2) respectively. A random selection of metallic pegs and plastic rings are placed on the upper conveyor. The rings and pegs need to be identified and separated. This is done by two sensors, a proximity sensor (Sensor 1) and an infra-red reflective sensor (Sensor 2). By using these two sensors a distinction can be made between the peg and the ring. By means of the sort solenoid (Actuator 3), plastic rings can be ejected down the assembly chute, which can have up to five plastic rings. Metallic pegs, meanwhile, continue on the upper conveyor and are deflected down the feeder chute. The feeder chute automatically feeds pegs onto the lower conveyor. An infra-red emitter/detector (Sensor 3) is used to determine whether or not the assembly area is empty. If it is, the assembly solenoid (Actuator 4) is used to dispense a ring from the assembly chute into the assembly area. The assembly area is positioned just above the lower conveyor and, when a metallic peg passes, the peg engages with the hole in the ring and the two components are assembled. The lower conveyor is used to carry completed components into the collection tray.

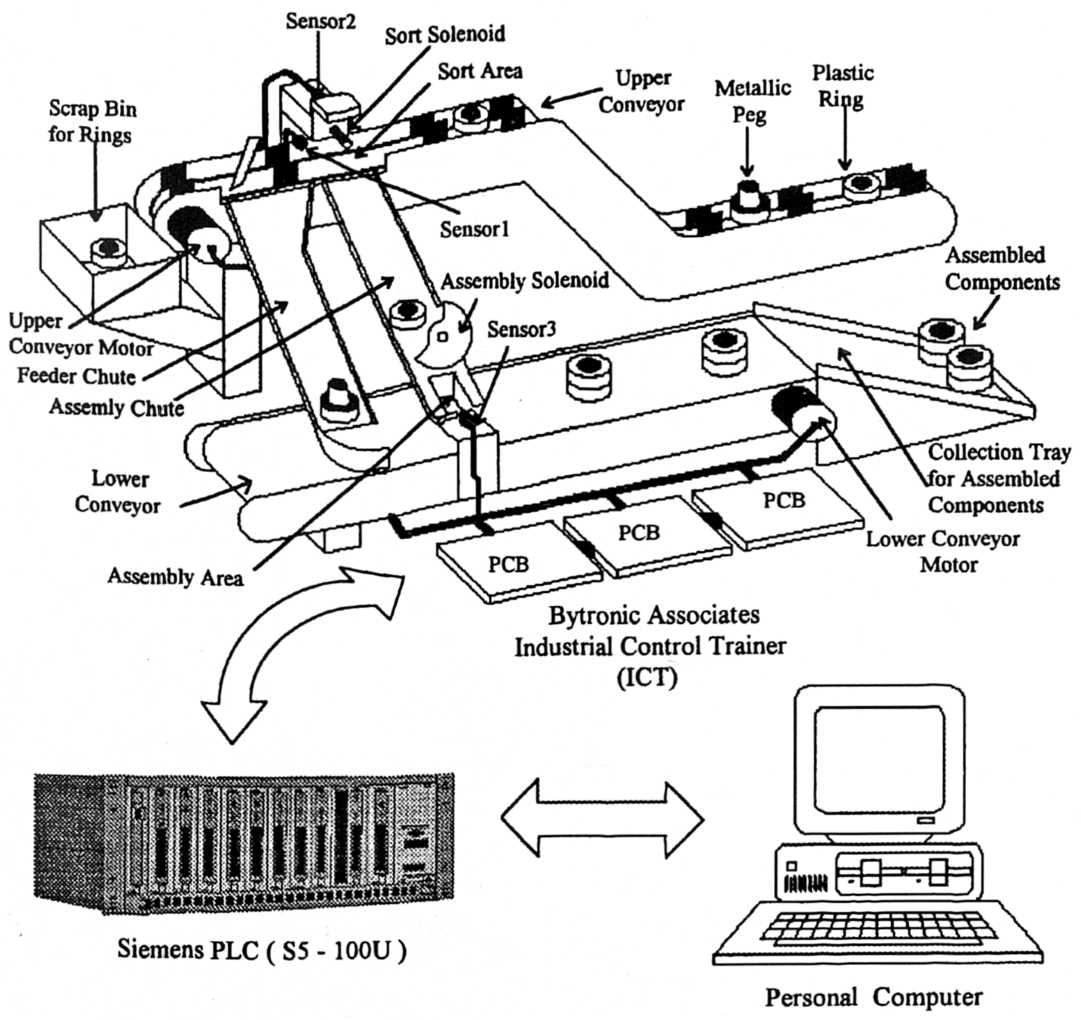


Figure 7.1. Discrete manufacturing system.

A Siemens PLC (S5-100U) is used to control the process, and a PC-based package called 'Quadriga' is used to program the PLC. PLC inputs and outputs are given in Table 7.1 and in Table 7.2 respectively.

| PLC Inputs | Sensor No. | Definition |
|---|---|---|
| I0.0 | Sensor 1 | Detects a ring or a peg at the sort area |
| I0.1 | Sensor 2 | Detects a peg at the sort area |
| I0.2 | Sensor 3 | Detects a ring in the assembly area |

Table 7.1. PLC inputs.

| PLC Outputs | Actuator No. | Definition |
|---|---|---|
| Q2.0 | Actuator 1 | Upper conveyor motor |
| Q2.1 | Actuator 2 | Lower conveyor motor |
| Q2.2 | Actuator 3 | Sort solenoid |
| Q2.3 | Actuator 4 | Assembly solenoid |

Table 7.2. PLC outputs.

For simplification purposes it is assumed that the assembly chute can have only one ring at a time. It is also assumed that when the system is switched on, both the upper conveyor motor and the lower conveyor motor are switched on automatically. The forbidden state specifications are as follows:

1. Operate the sort solenoid only when there is space in the assembly chute and there is a ring at the sort area.

2. Operate the assembly solenoid only when there is space at the assembly area and there is a ring in the assembly chute.

## SECTION A1

## THE FORBIDDEN STATE PROBLEM

## THE INHIBITOR ARC METHOD

### 7.2.1.1. The Inhibitor Arc Method

#### 7.2.1.1.1. Synthesis of Supervisory Controller

Recall that the synthesis of supervisory controller in the inhibitor arc method is divided into four main steps:

Step 1 - Design the uncontrolled model of the system using APNs.

Step 2- Synthesise the APN model supervisor and determine the control policy

Step 3- Construct the controlled model of the system

Step 4- Implement the supervisor (the controlled model) on a PLC as LLDs

##### 7.2.1.1.1.1. Step 1 - Design the Uncontrolled Model of System Using APNs

As a first step in capturing the uncontrolled behaviour of the manufacturing system, consider the standard APN modules and structures given in Fig. 7.2, where there are ten places, $P = \{ p_1, p_2, ...., p_{10} \}$ and nine transitions, $T = \{ t_1, t_2, t_3, t_3', t_4, t_4', t_5, t_6, t_7 \}$, with which firing conditions $\chi_1 = \overline{I0.0}$, $\chi_2 = I0.0 \ \& \ \overline{I0.1}$, $\chi_3 = \chi_3' = \overline{I0.0}$, $\chi_4 = \chi_4' = I0.2$, $\chi_5 = \overline{I0.2}$, $\chi_6 = 1$, $\chi_7 = 1$, are associated respectively. Note that transitions $t_3$, $t_3'$ and $t_5$ are timed transitions with time delays 0.7 sec., 0.7 sec. and 1.5 sec. respectively. Places $p_7$ and $p_8$ represent the *off* and *on* states of the sort solenoid respectively. Likewise, places $p_9$ and $p_{10}$ represent the *off* and *on* states of the assembly solenoid. A token in places $p_1$, $p_3$ and $p_5$ represent the available spaces in the sort area, in the assembly chute and in the assembly area respectively. A token in places $p_2$, $p_4$ and $p_6$ depicts the presence of a plastic ring in the sort area, in the assembly chute and in the assembly area respectively. Initially, both solenoids are *off* and there are no plastic rings in the manufacturing system.

When there is no ring at the sort area, i.e., M $(p_1)$ = 1, and the presence of a ring is detected, i.e., $\chi_2 = I0.0 \ \& \ \overline{I0.1}$, transition $t_2$ fires by removing the token from place $p_1$ and by depositing a token into place $p_2$. This means that there is a ring at the sort area, i.e., M $(p_2)$ = 1. When there is a ring at the sort area either it clears the sort area through transition $t_1$ or it is put into the assembly chute through transition $t_3$. If there is a ring at the sort area, the sort solenoid is *off*, i.e., M $(p_7)$ = 1, and the absence of a ring is detected, i.e., $\chi_1 = \overline{I0.0}$ then transition $t_1$ fires by removing the token from place $p_2$ and by depositing a token in place $p_1$. This means that the ring cleared the sort area. If there is a ring at the sort area, i.e., M $(p_2)$ = 1, the sort solenoid is *on*, i.e., M $(p_8)$ = 1, there is space in the assembly chute, i.e., M $(p_3)$ = 1, and the absence of a ring is detected, i.e., $\chi_3 = \overline{I0.0}$, then timed-transition $t_3$ is being fired for 0.7 sec., after which the token at the sort area is removed, i.e., M $(p_2)$ = 0, and a token is deposited into the assembly chute, i.e., M $(p_4)$ = 1, by using the empty space in the assembly chute, i.e., M $(p_3)$ = 0. This means that the ring at the sort area is put into the assembly chute by means of the sort solenoid and this process takes 0.7 sec.

If there is a ring in the assembly chute, i.e., M $(p_4)$ = 1, there is space at the assembly area, i.e., M $(p_5)$ = 1, the assembly solenoid is *on*, i.e., M $(p_{10})$ = 1, then the ring is dispensed from the assembly chute to the assembly area, i.e., the tokens are removed from places $p_4$ and $p_5$ and a token is deposited into place $p_6$, by means of transition $t_4$ with $\chi_4 = I0.2$. This also means that there is space in the assembly chute, i.e., M $(p_3)$ = 1. If there is a ring at the assembly area, i.e., M $(p_6)$ = 1, and a peg engages with the hole in the ring, i.e., $\chi_5 = \overline{I0.2}$, then it takes 1.5 sec. for the ring and the peg to be assembled and to clear the assembly area. After this, there is space at the assembly area, i.e., M $(p_5)$ = 1.

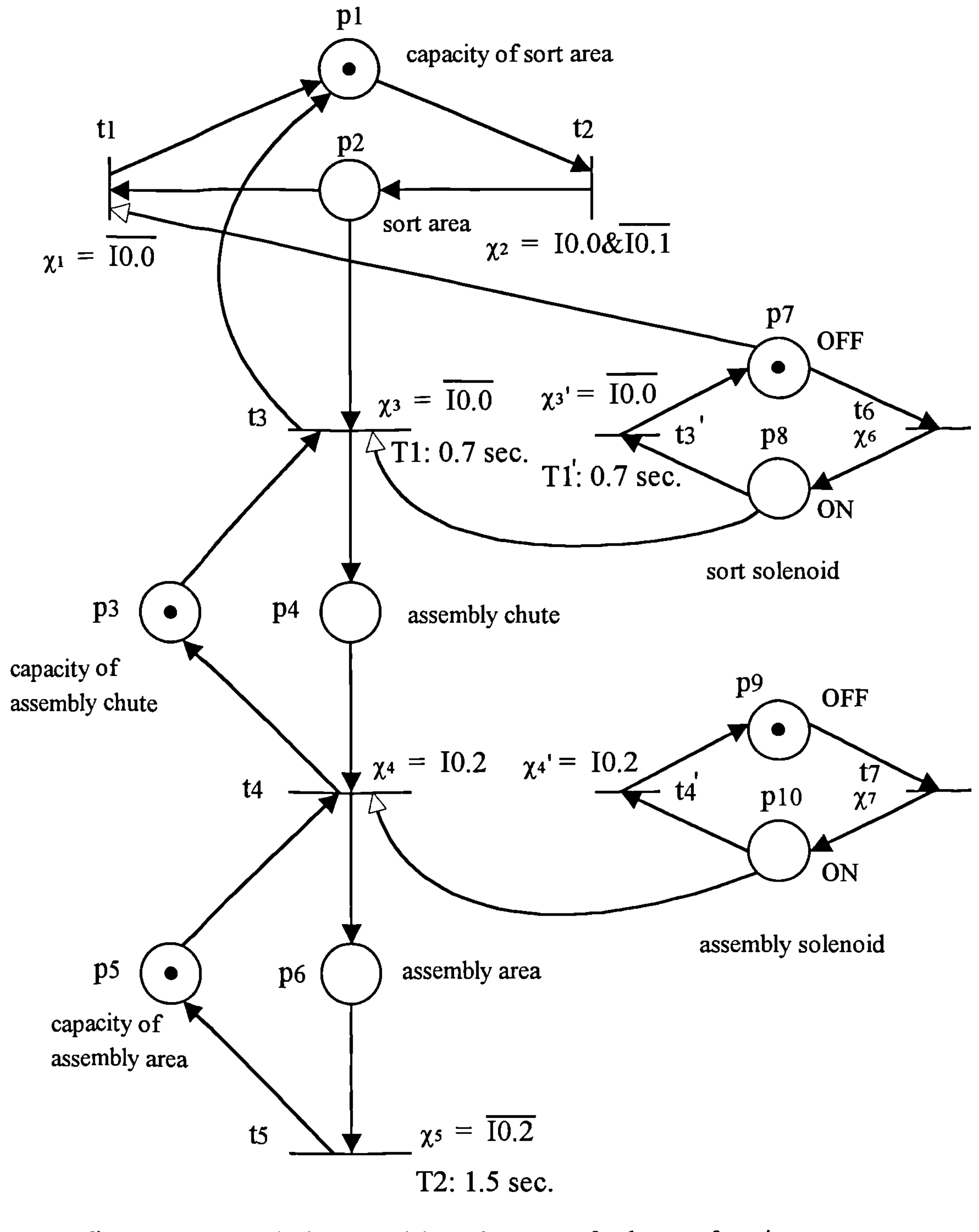


Figure 7.2. The standard APN modules and structures for the manufacturing system.

Secondly, by using the concurrent composition, i.e., by merging the transitions with the same events, the uncontrolled model is obtained. It is obvious from Fig. 7.2 that timed-transitions $t_3$ and $t_3$' have the same time delay as well as the same firing condition (event). Therefore, they are merged as $t_3$ in the uncontrolled model. Similarly transitions $t_4$ and $t_4$' have the same firing condition (event). Therefore they are merged as $t_4$ in the uncontrolled model. The uncontrolled model of the manufacturing system is obtained as an APN as shown in Fig. 7.3, where there are ten places, $P = \{ p_1, p_2, ..., p_{10} \}$ and seven transitions $T = \{ t_1, t_2, ...., t_7 \}$, with which the firing conditions, $\chi = \{ \chi_1, \chi_2, ....., \chi_7 \}$ are associated respectively. In the uncontrolled APN model transitions $t_3$ and $t_5$, are timed-transitions with time delays 0.7 sec and 1.5 sec. respectively. Note that actions Q2.2 and Q2.3 are assigned to places $p_8$ and $p_{10}$ respectively. They represent the sort solenoid and the assembly solenoid operations respectively. It is important to point out that after merging transitions $t_3$ and $t_3$' of the Fig. 7.2, as $t_3$ in the uncontrolled model, the enabling arc *En($p_8$, $t_3$), connecting place $p_8$ to transition $t_3$, is omitted, because there is already a* normal arc *Pre($p_8$, $t_3$)*, connecting the same place to the same transition. The same applies to the enabling arc *En($p_{10}$, $t_4$)*, connecting place $p_{10}$ to transition $t_4$. The initial marking of the uncontrolled model is $M_0 = ( 1, 0, 1, 0, 1, 0, 1, 0, 1, 0 )^T$ or simply $M_0 = ( 1, 3, 5, 7, 9 )$. This means that initially, there is no ring in the manufacturing system and both the sort solenoid and the assembly solenoid are *off*. Note that the events $\chi_1$, $\chi_2$, and $\chi_5$ are uncontrollable events, while the events $\chi_3$, $\chi_4$, $\chi_6$ and $\chi_7$ are controllable events. In fact the objective in this case is to come up with a supervisor to decide when to fire transitions $t_6$ and $t_7$ such that the forbidden state specifications are met. Note that the uncontrolled APN model, shown in Fig. 7.3 is safe, i.e., 1-bounded, live, reversible, and conservative.

$\chi_1 = \overline{I0.0}$

$\chi_2 = I0.0\&\overline{I0.1}$

$\chi_3 = \overline{I0.0}$

$\chi_4 = I0.2$

$\chi_5 = \overline{I0.2}$

$\chi_6 = 1$

$\chi_7 = 1$

T1: 0.7 sec.

T2: 1.5 sec.

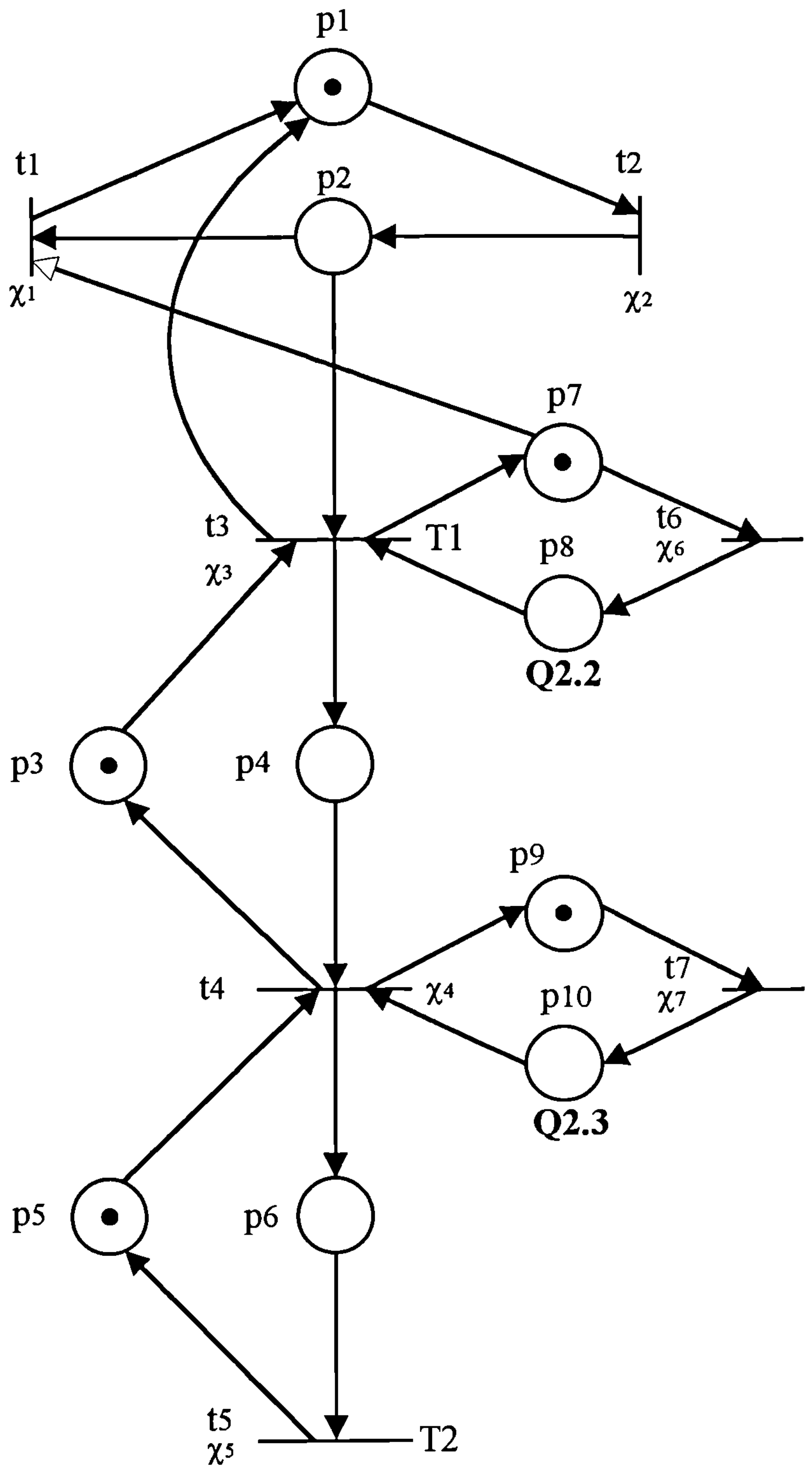


Figure 7.3. The uncontrolled model of the manufacturing system as an APN.

#### 7.2.1.1.1.2. Step 2 - Synthesise the APN model supervisor and determine the control policy

Remember that in this step there are three sub-steps:

Step 2.1. Generate the reachability graph of the APN model

Step 2.2. Identify and remove the "bad states" from the reachability graph

Step 2.3. Design the APN model supervisor and determine the control policy

##### 7.2.1.1.1.2.1. *Step 2.1. Generate the reachability graph of the APN model*

The reachability graph (RG) of the uncontrolled APN model is shown in Fig. 7.4, where there are seventy-nine arcs, representing the firing of transitions in the uncontrolled model, and there are thirty-two nodes $M = \{ M_0, M_1, M_2, ..., M_{31} \}$, representing the all possible markings reachable from the initial marking $M_0$. Table 7.3 provides detailed information about the RG nodes. Note that for simplicity reasons only the events, which are associated with the transitions, are shown in the RG. Therefore the events (firing conditions) $\chi = \{ \chi_1, \chi_2, ....., \chi_7 \}$ in the RG represent the firing of corresponding transitions $T = \{ t_1, t_2, ....., t_7 \}$respectively. It is also important to note that although it is not explicitly written in the RG, time delays 0.7 sec. and 1.5 sec. are associated with the firing of transitions $t_3$ and $t_5$ respectively.

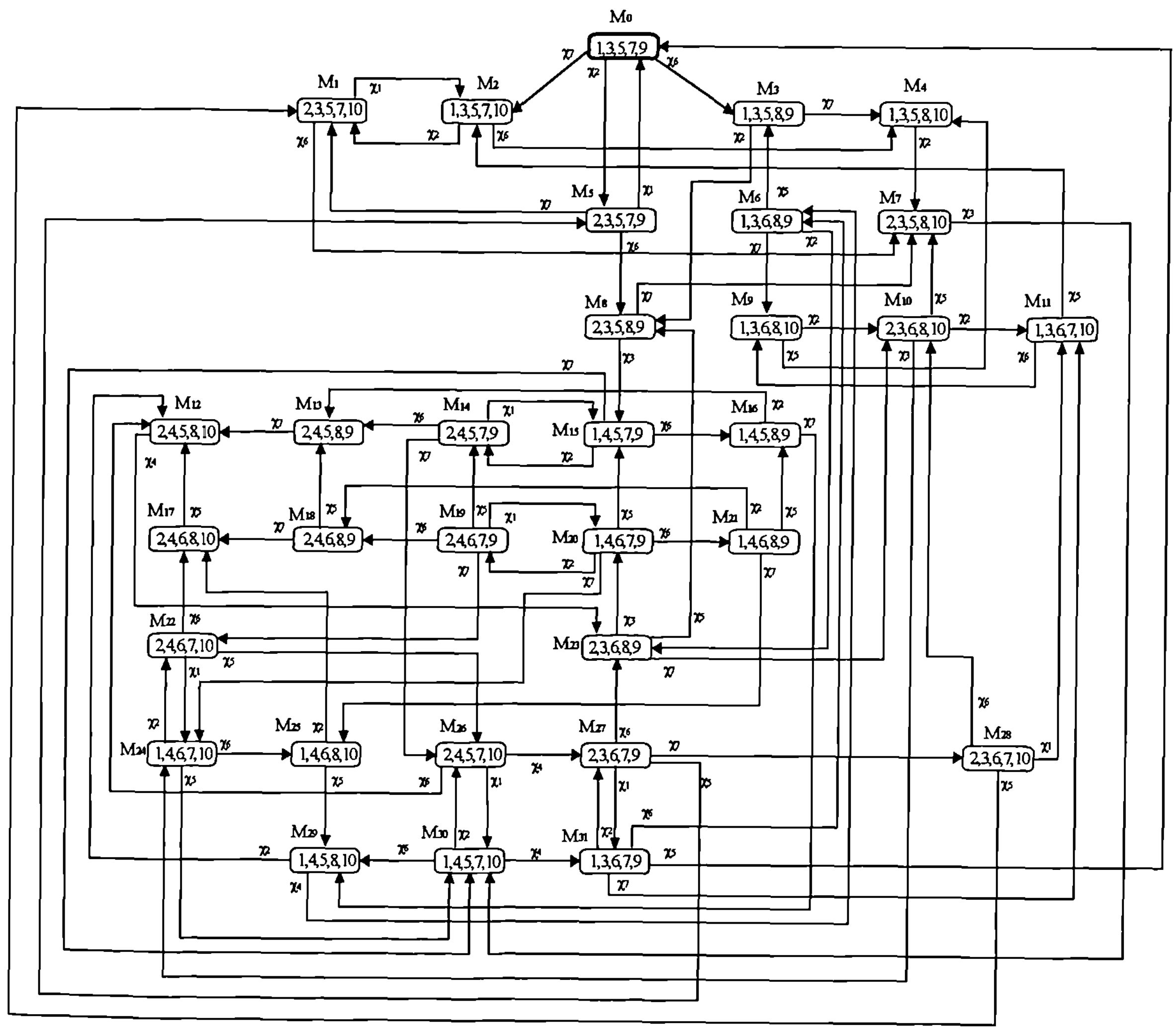


Figure 7.4. The reachability graph (RG) of the uncontrolled APN model.

| Marking | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $M_0$ = (1,3,5,7,9) | 1 | | 1 | | 1 | | 1 | | 1 | |
| $M_1$ = (2,3,5,7,10) | | 1 | 1 | | 1 | | 1 | | | 1 |
| $M_2$ = (1,3,5,7,10) | 1 | | 1 | | 1 | | 1 | | | 1 |
| $M_3$ = (1,3,5,8,9) | 1 | | 1 | | 1 | | | 1 | 1 | |
| $M_4$ = (1,3,5,8,10) | 1 | | 1 | | 1 | | | 1 | | 1 |
| $M_5$ = (2,3,5,7,9) | | 1 | 1 | | 1 | | 1 | | 1 | |
| $M_6$ = (1,3,6,8,9) | 1 | | 1 | | | 1 | | 1 | 1 | |
| $M_7$ = (2,3,5,8,10) | | 1 | 1 | | 1 | | | 1 | | 1 |
| $M_8$ = (2,3,5,8,9) | | 1 | 1 | | 1 | | | 1 | 1 | |
| $M_9$ = (1,3,6,8,10) | 1 | | 1 | | | 1 | | 1 | | 1 |
| $M_{10}$ = (2,3,6,8,10) | | 1 | 1 | | | 1 | | 1 | | 1 |
| $M_{11}$ = (1,3,6,7,10) | 1 | | 1 | | | 1 | 1 | | | 1 |
| $M_{12}$ = (2,4,5,8,10) | | 1 | | 1 | 1 | | | 1 | | 1 |
| $M_{13}$ = (2,4,5,8,9) | | 1 | | 1 | 1 | | | 1 | 1 | |
| $M_{14}$ = (2,4,5,7,9) | | 1 | | 1 | 1 | | 1 | | 1 | |
| $M_{15}$ = (1,4,5,7,9) | 1 | | | 1 | 1 | | 1 | | 1 | |
| $M_{16}$ = (1,4,5,8,9) | 1 | | | 1 | 1 | | | 1 | 1 | |
| $M_{17}$ = (2,4,6,8,10) | | 1 | | 1 | | 1 | | 1 | | 1 |
| $M_{18}$ = (2,4,6,8,9) | | 1 | | 1 | | 1 | | 1 | 1 | |
| $M_{19}$ = (2,4,6,7,9) | | 1 | | 1 | | 1 | 1 | | 1 | |
| $M_{20}$ = (1,4,6,7,9) | 1 | | | 1 | | 1 | 1 | | 1 | |
| $M_{21}$ = (1,4,6,8,9) | 1 | | | 1 | | 1 | | 1 | 1 | |
| $M_{22}$ = (2,4,6,7,10) | | 1 | | 1 | | 1 | 1 | | | 1 |
| $M_{23}$ = (2,3,6,8,9) | | 1 | 1 | | | 1 | | 1 | 1 | |
| $M_{24}$ = (1,4,6,7,10) | 1 | | | 1 | | 1 | 1 | | | 1 |
| $M_{25}$ = (1,4,6,8,10) | 1 | | | 1 | | 1 | | 1 | | 1 |
| $M_{26}$ = (2,4,5,7,10) | | 1 | | 1 | 1 | | 1 | | | 1 |
| $M_{27}$ = (2,3,6,7,9) | | 1 | 1 | | | 1 | 1 | | 1 | |
| $M_{28}$ = (2,3,6,7,10) | | 1 | 1 | | | 1 | 1 | | | 1 |
| $M_{29}$ = (1,4,5,8,10) | 1 | | | 1 | 1 | | | 1 | | 1 |
| $M_{30}$ = (1,4,5,7,10) | 1 | | | 1 | 1 | | 1 | | | 1 |
| $M_{31}$ = (1,3,6,7,9) | 1 | | 1 | | | 1 | 1 | | 1 | |

Table 7.3. The markings of the reachability graph (RG).

### 7.2.1.1.1.2.2. *Step 2.2. Identify and remove the "bad states" from the reachability graph*

Consider the forbidden state specifications:

*Specification 1:* Operate the sort solenoid only when there is space in the assembly chute and there is a ring at the sort area. This also implies that when there is no space in the assembly chute and/or there is no ring at the sort area do not operate the sort solenoid. Therefore it is obvious from Fig. 7.4 that markings $M_3$, $M_4$, $M_5$ and $M_9$ are bad markings (i.e., bad states), because they represent the states, where there is no ring at the sort area and the sort solenoid is *on*. Then, markings $M_{16}$, $M_{21}$, $M_{25}$ and $M_{29}$ are bad markings (i.e., bad states), because they represent the states, where there is no ring at the sort area, there is a ring in the assembly chute and the sort solenoid is *on*. Finally, markings $M_{12}$, $M_{13}$, $M_{17}$ and $M_{18}$ are bad markings (i.e., bad states), because they represent the states, where there is a ring at the sort area, there is a ring in the assembly chute and the sort solenoid is *on*.

*Specification 2:* Operate the assembly solenoid only when there is space at the assembly area and there is a ring in the assembly chute. This also implies that when there is no ring in the assembly chute and/or there is no space at the assembly area do not operate the assembly solenoid. As can be seen from Fig. 7.4 markings $M_1$, $M_2$, $M_4$ and $M_7$ are bad markings (i.e., bad states), because they represent the states, where there is no ring in the assembly chute, there is no ring at the assembly area and the assembly solenoid is *on*. Then, markings $M_{17}$, $M_{22}$, $M_{24}$ and $M_{25}$ are also bad markings (i.e., bad states), because they represent the states, where there is a ring in the assembly chute, there is a ring at the assembly area and the assembly solenoid is *on*. Finally, markings $M_{10}$, $M_{11}$, and $M_{28}$ are bad markings (i.e., bad states), because they represent the states, where there is a ring at the assembly area, there is no ring in the assembly chute and the assembly solenoid is *on*.

As a result, according to the forbidden state specifications there are twenty bad markings (states), namely, $M_1$, $M_2$, $M_3$, $M_4$, $M_6$, $M_7$, $M_9$, $M_{10}$, $M_{11}$, $M_{12}$, $M_{13}$, $M_{16}$, $M_{17}$, $M_{18}$, $M_{21}$, $M_{22}$, $M_{24}$, $M_{25}$, $M_{28}$ and $M_{29}$, as shown in Fig 7.5.

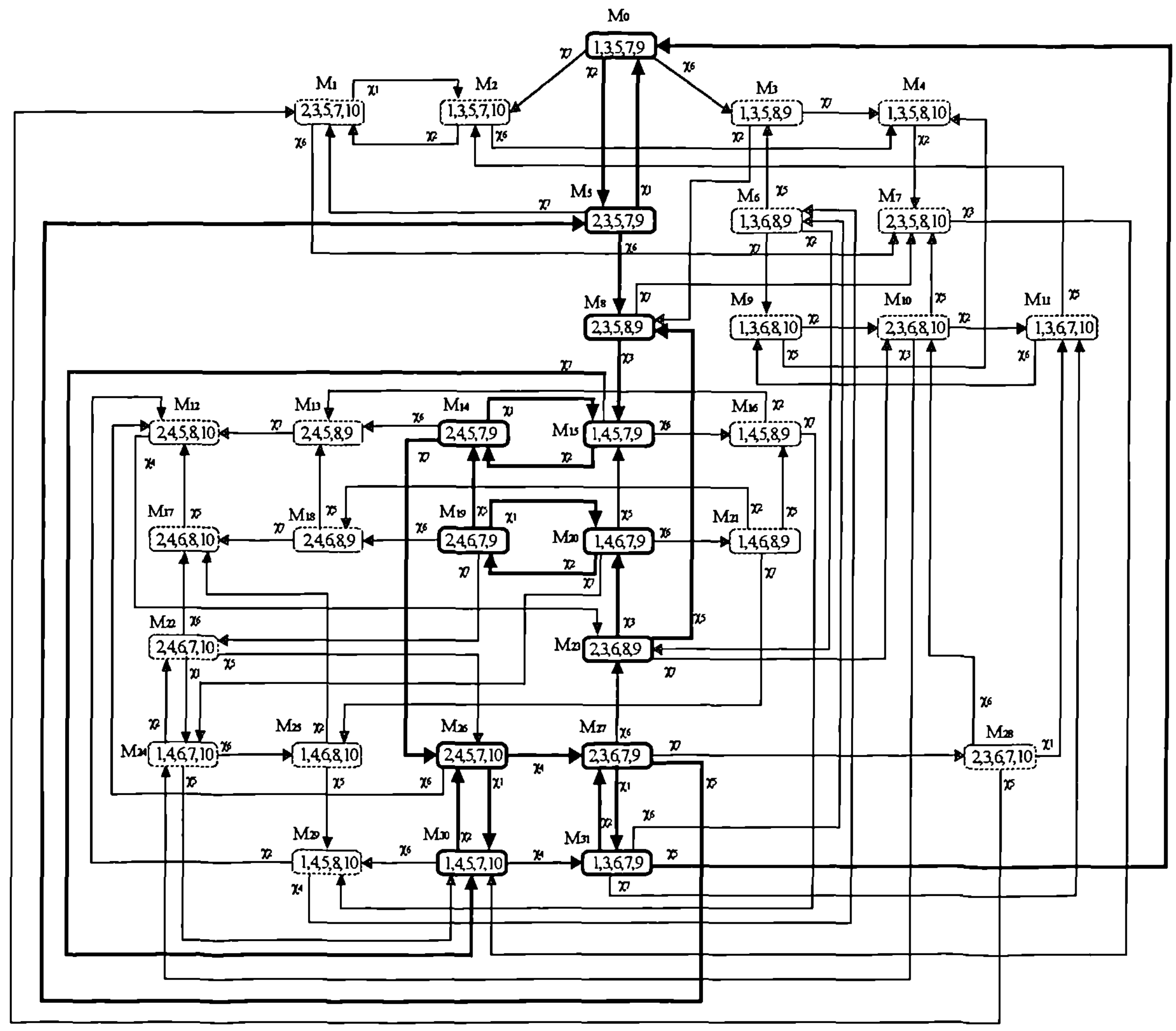


Figure 7.5. The 'bad markings' and the '**good markings**' of the reachability graph (RG).

These bad markings must be removed from the RG together with their arcs connecting them to the rest of the RG. After removing these bad markings and their arcs from the RG, the final reduced reachability graph (FRRG) is obtained as shown in Fig. 7.6. Note that the FRRG represents the maximally permissible state space for the forbidden state specifications given.

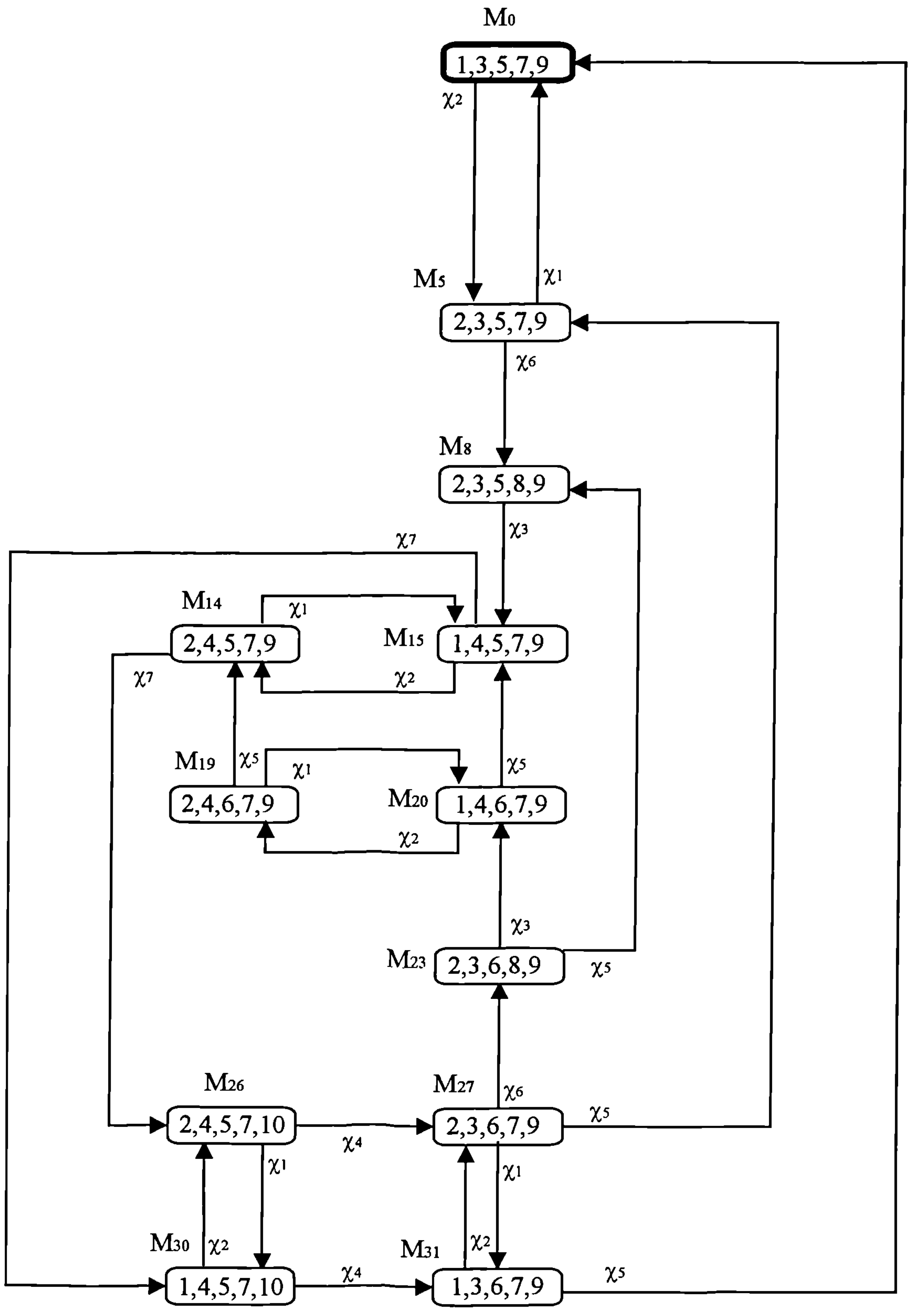


Figure 7.6. The final reduced reachability graph (FRRG), according to the forbidden state specifications.

### 7.2.1.1.1.2.3. *Step 2.3. Design the APN model supervisor and determine the control policy*

Firstly, the APN model supervisor is designed. To do this, the FRRG is converted into a related APN such that every state (or marking) of the FRRG is represented by an APN place and the arcs of the FRRG are represented by the APN transitions. Note that in this special APN, there are no actions assigned to the places, because the APN model supervisor designed in this way behaves as a monitor that represents the current state of the system. The initial marking is also represented by a token in the APN place representing the initial state. When this technique is applied to the manufacturing system, the FRRG is converted into the APN model supervisor as shown in Fig. 7.7. The APN model supervisor has twelve places $P = \{ p_{11}, p_{12}, p_{13}, \dots , p_{22} \}$ and twenty-three transitions $T = \{ t_8, t_9, t_{10}, \dots , t_{30} \}$. The initial marking of the APN model supervisor is $M_0 = (11)$, i.e., initially, there is a token in place $p_{11}$. Note that each place within the APN model supervisor represents an admissible marking of the APN model of the manufacturing problem, i.e., places $p_{11}, p_{12}, \dots , p_{22}$ represent the markings $M_0$, $M_5$, $M_8$, $M_{14}$, $M_{15}$, $M_{19}$, $M_{20}$, $M_{23}$, $M_{26}$, $M_{27}$, $M_{30}$ and $M_{31}$ of the FRRG respectively.

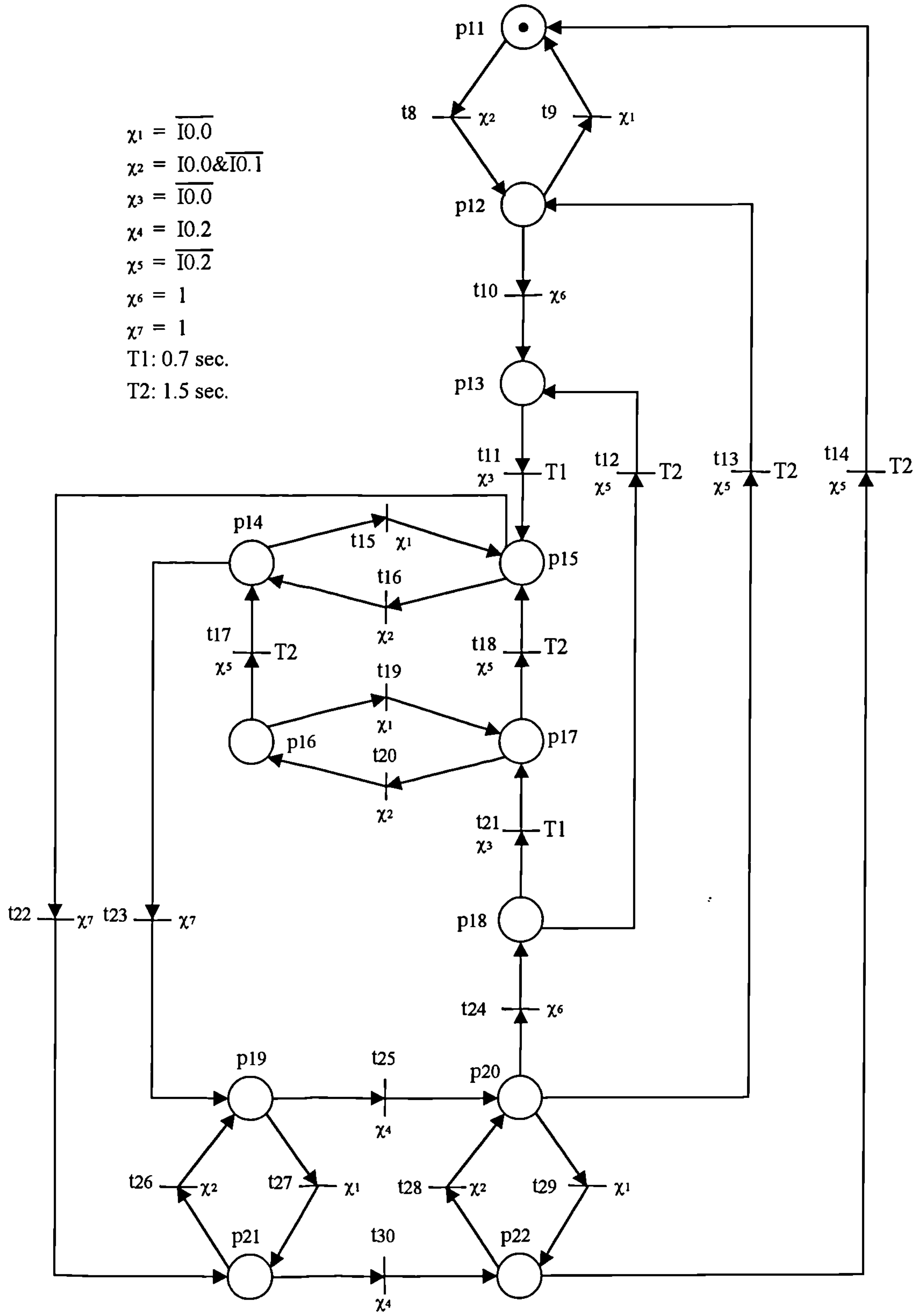


Figure 7.7. The APN model supervisor for the manufacturing system.

Secondly, the control policy is determined. The FRRG is considered together with its arcs, which are leading from the "good markings" to the "bad markings". It is obvious that from Fig. 7.8 that the "bad markings" $M_1$, $M_2$, $M_3$, $M_6$, $M_7$, $M_{10}$, $M_{11}$, $M_{12}$, $M_{13}$, $M_{16}$, $M_{18}$, $M_{21}$, $M_{22}$, $M_{24}$, $M_{28}$ and $M_{29}$ can be reached from the "good markings" $M_0$, $M_5$, $M_8$, $M_{14}$, $M_{15}$, $M_{19}$, $M_{20}$, $M_{23}$, $M_{26}$, $M_{27}$, $M_{30}$ and $M_{31}$ as follows: $M_0[\chi_7>M_2$, $M_0[\chi_6>M_3$, $M_5[\chi_7>M_1$, $M_8[\chi_6>M_7$, $M_{14}[\chi_6>M_{13}$, $M_{15}[\chi_6>M_{16}$, $M_{19}[\chi_6>M_{18}$, $M_{19}[\chi_7>M_{22}$, $M_{20}[\chi_6>M_{21}$, $M_{20}[\chi_7>M_{24}$, $M_{23}[\chi_7>M_{10}$, $M_{26}[\chi_6>M_{12}$, $M_{27}[\chi_7>M_{28}$, $M_{30}[\chi_6>M_{29}$, $M_{31}[\chi_6>M_6$ and $M_{31}[\chi_7>M_{11}$. This can be seen from Fig. 7.8. In order to make sure the correct system operation each event leading from a "good state" to a "bad state" must be stopped. This constitutes the control policy. For example, the "bad marking" $M_1$ can be reached from the "good marking" $M_5$ through the controllable event $\chi_7$, i.e., $M_5[\chi_7>M_1$. Therefore, the control policy when reaching the marking $M_5$ must be 'stop $\chi_7$' so that the bad marking $M_1$ will not be reached. The final control policy is shown in Table 7.4.

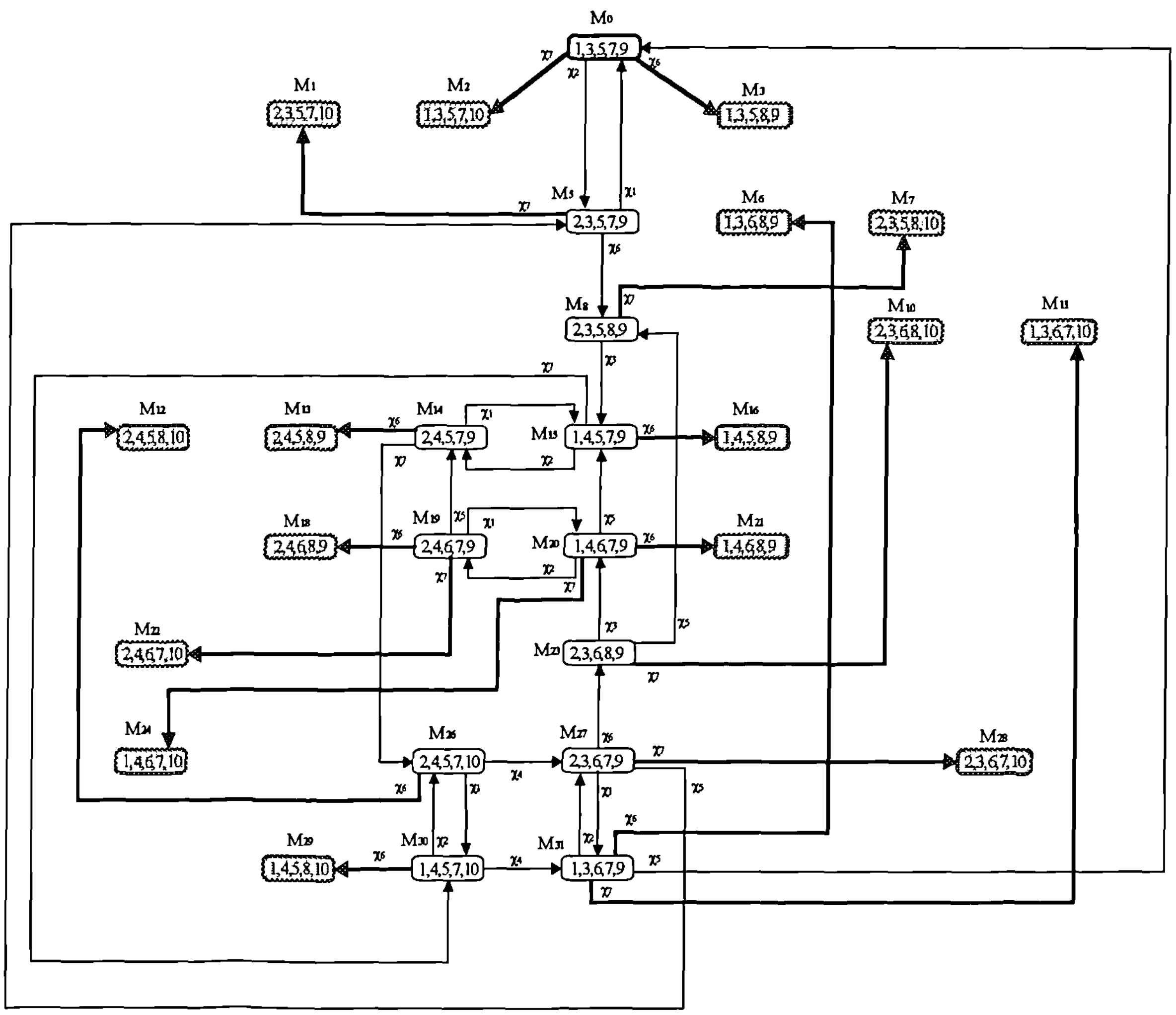


Figure 7.8. The FRRG and the “bad markings” reachable from it.

| Marking | Supervisor place | Control action |
|---|---|---|
| $M_0 = (1,3,5,7,9)$ | $p_{11}$ | stop $\chi_6$ & $\chi_7$ |
| $M_5 = (2,3,5,7,9)$ | $p_{12}$ | stop $\chi_7$ |
| $M_8 = (2,3,5,8,9)$ | $p_{13}$ | stop $\chi_7$ |
| $M_{14} = (2,4,5,7,9)$ | $p_{14}$ | stop $\chi_6$ |
| $M_{15} = (1,4,5,7,9)$ | $p_{15}$ | stop $\chi_6$ |
| $M_{19} = (2,4,6,7,9)$ | $p_{16}$ | stop $\chi_6$ & $\chi_7$ |
| $M_{20} = (1,4,6,7,9)$ | $p_{17}$ | stop $\chi_6$ & $\chi_7$ |
| $M_{23} = (2,3,6,8,9)$ | $p_{18}$ | stop $\chi_7$ |
| $M_{26} = (2,4,5,7,10)$ | $p_{19}$ | stop $\chi_6$ |
| $M_{27} = (2,3,6,7,9)$ | $p_{20}$ | stop $\chi_7$ |
| $M_{30} = (1,4,5,7,10)$ | $p_{21}$ | stop $\chi_6$ |
| $M_{31} = (1,3,6,7,9)$ | $p_{22}$ | stop $\chi_6$ & $\chi_7$ |

Table 7.4. The control policy for the inhibitor arc method.

### 7.2.1.1.1.3. Step 3 - Construct the controlled model of the system

The controlled model (i.e., the supervisor) of the system is obtained as shown in Fig. 7.9. The controlled model consists of the uncontrolled model, i.e., the APN model, the APN model supervisor and the control policy, which is implemented as inhibitor arcs. The inhibitor arcs are connected from the places of the APN model supervisor to the controllable transitions of the APN model such that the control policy is satisfied. This is simply done by connecting the inhibitor arcs $In(p_{11}, t_6)$, $In(p_{11}, t_7)$, $In(p_{12}, t_7)$, $In(p_{13}, t_7)$, $In(p_{14}, t_6)$, $In(p_{15}, t_6)$, $In(p_{16}, t_6)$, $In(p_{16}, t_7)$, $In(p_{17}, t_6)$, $In(p_{17}, t_7)$, $In(p_{18}, t_7)$, $In(p_{19}, t_6)$, $In(p_{20}, t_7)$, $In(p_{21}, t_6)$, $In(p_{22}, t_6)$, and $In(p_{22}, t_7)$ from places $P = \{ p_{11}, p_{12}, ..., p_{22} \}$ to the controllable transitions $t_6$ and $t_7$ as shown in Fig. 7.9.

Note that the controlled model (the supervisor) obtained does not contradict the forbidden state specifications, i.e., controlled behaviour of the system is nonblocking. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications. It is also important to point out that the controlled model (the supervisor) obtained is correct by construction.

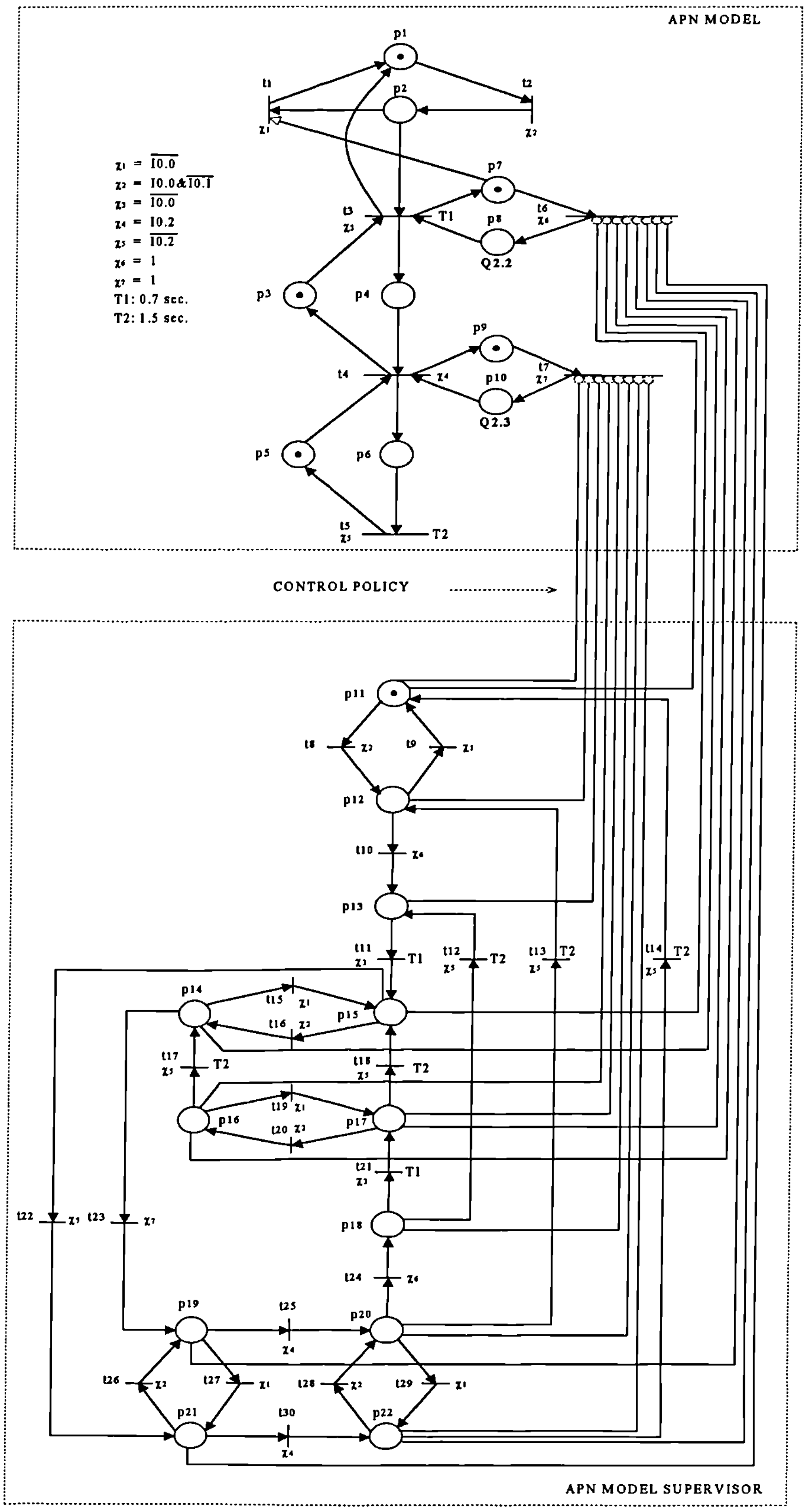


Figure 7.9. The controlled model (supervisor) of the manufacturing system for the inhibitor arc method.

### 7.2.1.1.1.1.4. Step 4 - Implement the supervisor (the controlled model) on a PLC as LLDs

In order to convert the supervisor (the controlled model) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions. Note that in the controlled model since the APN model supervisor acts as a monitor there is no actions or on delay timers associated with its places and transitions. On the other hand, in the APN model there are actions assigned to places and on delay timers are also associated with timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code. However, it should be noted that for proper functioning the order of the LLD code, must be arranged as follows: first, the initial marking is written; next, the LLD code related to the APN model supervisor is written; and finally, the LLD code for the APN model is written. This is because after the initial marking is represented as LLD, the APN model supervisor monitors the system behaviour and changes its state, and then according to the current state and the control policy, the behaviour of the APN model is restricted if necessary. Note that, while on delay timers are only associated with the timed-transitions in the APN model, the time evolution of these timers are followed by the timed-transitions within the APN model supervisor.

As a result to convert the supervisor, given in Fig. 7.9, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2 are assigned to the places $P = \{ p_1, p_2, \ldots.., p_{10} \}$ of the APN model respectively. Similarly, flags F2.0, F2.1, F2.2, F2.3, F2.4, F2.5, F2.6, F2.7, F3.0, F3.1, F3.2, F3.3 are assigned to the places $P = \{p_{11}, p_{12}\ldots\ldots p_{22}\}$ of the APN model supervisor respectively. On delay timers T1 with 0.7 sec. time delay and T2 with 1.5 sec. time delay are assigned only to the timed-transitions $t_3$ and $t_5$ of the APN model. After the TPLC is obtained as shown in Fig. 7.10, it is then converted into the LLD code, as shown in Fig. 7.11, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The APN model supervisor is converted into LLD at the rungs from 1 to 23, where the rungs 1, 2, 3, ....., 23 represent the transitions $T = \{ t_8, t_9, t_{10}, ......, t_{30} \}$. The APN model is converted into LLD at the rungs from 24 to 32, where rungs 24, 25, ......., 32 represent the transitions $T = \{ t_1, t_2, ...., t_7 \}$ of the APN model. Then, action places $p_8$ and $p_{10}$ are represented by rungs 33 and 34 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 35. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

| LLD Symbol | Definition |
|---|---|
| S | Set |
| R | Reset |
| T | Timer |
| I | Input |
| Q | Output |
| F | Flag |
| SR | On delay timer |
| CD | Count Down |
| CU | Count Up |
| ----] [---- | Normally open contact |
| ----]/[---- | Normally closed contact |

Table 7.5. The LLD symbols for Siemens S5-100U PLC.

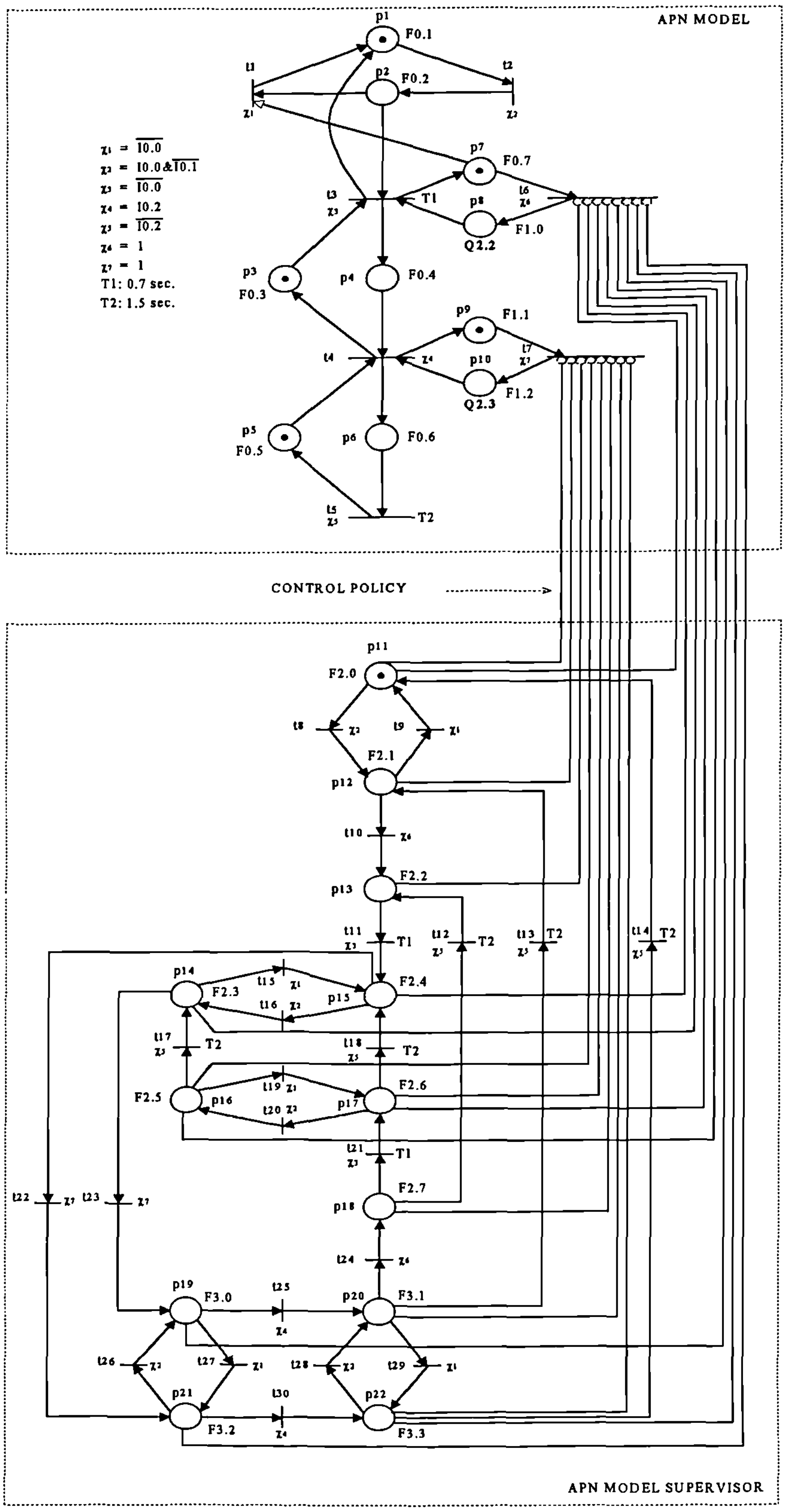


Figure 7.10. The TPLC for the supervisor (controlled model), shown in Fig. 7.9.

```
               | 0  F0.0                                          F0.1     |
initialisation | ------]/[-----------------------------------------+----------- S ---------|
               |                                                  |           F0.3     |
               |                                                  |----------- S ---------|
               |                                                  |           F0.5     |
               |                                                  |----------- S ---------|
               |                                                  |           F0.7     |
               |                                                  |----------- S ---------|
               |                                                  |           F1.1     |
               |                                                  |----------- S ---------|
               |                                                  |           F2.0     |
               |                                                  |----------- S ---------|
               |                                                  |           F0.0     |
               |                                                  |----------- S ---------|
               | 1  F2.0  I0.0   I0.1                                          F2.0     |
t8             | ------] [-----] [-----]/[--------------------------+----------- R --------|
               |                                                  |           F2.1     |
               |                                                  |----------- S ---------|
               | 2  F2.1  I0.0                                                 F2.1     |
t9             | ------] [-----]/[----------------------------------+----------- R --------|
               |                                                  |           F2.0     |
               |                                                  |----------- S ---------|
               | 3  F2.1                                                       F2.1     |
t10            | ------] [------------------------------------------+----------- R ---------|
               |                                                  |           F2.2     |
               |                                                  |----------- S ---------|
               | 4  F2.2  I0.0   T1                                            F2.2     |
t11            | ------] [-----]/[-----] [--------------------------+----------- R ---------|
               |                                                  |           F2.4     |
               |                                                  |----------- S ---------|
               | 5  F2.7  I0.2   T2                                            F2.7     |
t12            | ------] [-----]/[-----] [--------------------------+----------- R ---------|
               |                                                  |           F2.2     |
               |                                                  |----------- S ---------|
               | 6  F3.1  I0.2   T2                                            F3.1     |
t13            | ------] [-----]/[-----] [--------------------------+----------- R ---------|
               |                                                  |           F2.1     |
               |                                                  |----------- S ---------|
               | 7  F3.3  I0.2   T2                                            F3.3     |
t14            | ------] [-----]/[-----] [--------------------------+----------- R ---------|
               |                                                  |           F2.0     |
               |                                                  |----------- S ---------|
               | 8  F2.3  I0.0                                                 F2.3     |
t15            | ------] [-----]/[----------------------------------+----------- R ---------|
               |                                                  |           F2.4     |
               |                                                  |----------- S ---------|
               | 9  F2.4  I0.0   I0.1                                          F2.4     |
t16            | ------] [-----] [-----]/[--------------------------+----------- R ---------|
               |                                                  |           F2.3     |
               |                                                  |----------- S ---------|
       10  F2.5  I0.2   T2                                      F2.5
t17  ------] [-----]/[-----] [----------------------------┬---------- R --------
                                                          |          F2.3
                                                          └---------- S --------
       11  F2.6  I0.2   T2                                      F2.6
t18  ------] [-----]/[-----] [----------------------------┬---------- R --------
                                                          |          F2.4
                                                          └---------- S --------
       12  F2.5  I0.0                                           F2.5
t19  ------] [-----]/[------------------------------------┬---------- R --------
                                                          |          F2.6
                                                          └---------- S --------
       13 F2.6  I0.0   I0.1                                     F2.6
t20  ------] [-----] [-----]/[----------------------------┬---------- R --------
                                                          |          F2.5
                                                          └---------- S --------
       14  F2.7  I0.0   T1                                      F2.7
t21  ------] [-----]/[-----] [----------------------------┬---------- R --------
                                                          |          F2.6
                                                          └---------- S --------
       15  F2.4                                                 F2.4
t22  ------] [--------------------------------------------┬---------- R --------
                                                          |          F3.2
                                                          └---------- S --------
       16  F2.3                                                 F2.3
t23  ------] [--------------------------------------------┬---------- R --------
                                                          |          F3.0
                                                          └---------- S --------
       17  F3.1                                                 F3.1
t24  ------] [--------------------------------------------┬---------- R --------
                                                          |          F2.7
                                                          └---------- S --------
       18  F3.0  I0.2                                           F3.0
t25  ------] [-----] [------------------------------------┬---------- R --------
                                                          |          F3.1
                                                          └---------- S --------
       19  F3.2  I0.0   I0.1                                    F3.2
t26  ------] [-----] [-----]/[----------------------------┬---------- R --------
                                                          |          F3.0
                                                          └---------- S --------
       20  F3.0  I0.0                                           F3.0
t27  ------] [-----]/[------------------------------------┬---------- R --------
                                                          |          F3.2
                                                          └---------- S --------
       21  F3.3  I0.0   I0.1                                    F3.3
t28  ------] [-----] [-----]/[----------------------------┬---------- R --------
                                                          |          F3.1
                                                          └---------- S --------
       22  F3.1  I0.0                                           F3.1
t29  ------] [-----]/[------------------------------------┬---------- R --------
                                                          |          F3.3
                                                          └---------- S --------
```

```
     | 23  F3.2  I0.2                                          F3.2         |
t30  | ------] [-----] [-----------------------------------+------- R -------|
     |                                                     |       F3.3     |
     |                                                     +------- S -------|
     | 24  F0.2  F0.7  I0.0                                    F0.2         |
t1   | ------] [-----] [-----]/[---------------------------+------- R -------|
     |                                                     |       F0.1     |
     |                                                     +------- S -------|
     | 25  F0.1  I0.0   I0.1                                   F0.1         |
t2   | ------] [-----] [-----]/[---------------------------+------- R -------|
     |                                                     |       F0.2     |
     |                                                     +------- S -------|
     | 26  F0.2  F0.3  F1.0  I0.0                          T1: 0.7 sec.     |
t3   | ------] [-----] [-----] [-----]/[------------------------- SR -------|
     | 27 F0.2  F0.3  F1.0  I0.0   T1                          F0.2         |
t3   | ------] [-----] [-----] [-----]/[-----] [-----------+------- R -------|
     |                                                     |       F0.3     |
     |                                                     +------- R -------|
     |                                                     |       F1.0     |
     |                                                     +------- R -------|
     |                                                     |       F0.1     |
     |                                                     +------- S -------|
     |                                                     |       F0.4     |
     |                                                     +------- S -------|
     |                                                     |       F0.7     |
     |                                                     +------- S -------|
     | 28  F0.4  F0.5  F1.2  I0.2                              F0.4         |
t4   | ------] [-----] [-----] [-----] [-------------------+------- R -------|
     |                                                     |       F0.5     |
     |                                                     +------- R -------|
     |                                                     |       F1.2     |
     |                                                     +------- R -------|
     |                                                     |       F0.3     |
     |                                                     +------- S -------|
     |                                                     |       F0.6     |
     |                                                     +------- S -------|
     |                                                     |       F1.1     |
     |                                                     +------- S -------|
     | 29  F0.6  I0.2                                      T2: 1.5 sec.     |
t5   | ------] [-----]/[---------------------------------------- SR -------|
     | 30   F0.6  I0.2   T2                                    F0.6         |
t5   | ------] [-----]/[-----] [---------------------------+------- R -------|
     |                                                     |       F0.5     |
     |                                                     +------- S -------|
     |31 F0.7 F2.0  F2.3 F2.4  F2.5 F2.6  F3.0  F3.2  F3.3     F0.7         |
t6   | ----] [---]/[----]/[----]/[----]/[----]/[----]/[----]/[+------ R -------|
     |                                                     |       F1.0     |
     |                                                     +------- S -------|
     |32 F1.1 F2.0  F2.1 F2.2  F2.5 F2.6  F2.7  F3.1  F3.3     F1.1         |
t7   | ----] [---]/[----]/[----]/[----]/[----]/[----]/[----]/[+------ R -------|
     |                                                     |       F1.2     |
     |                                                     +------- S -------|
     |                                                                      |
```

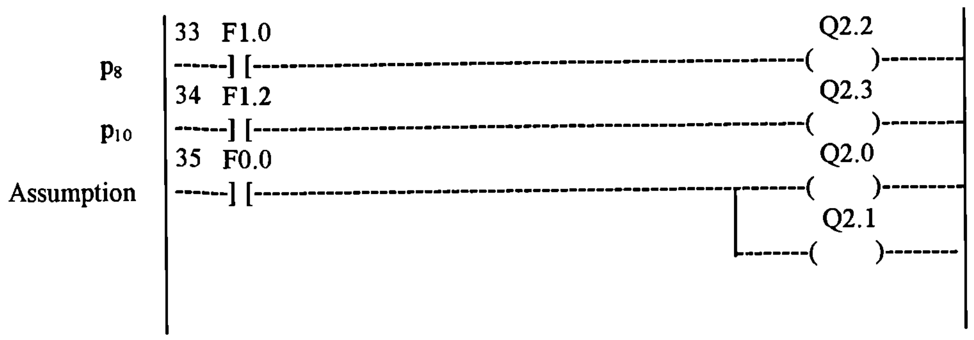


Figure 7.11. The LLD for the TPLC, shown in Fig. 7.10.

## SECTION A2

## THE FORBIDDEN STATE PROBLEM

## THE ENABLING ARC METHOD

### 7.2.1.2. The Enabling Arc Method

#### 7.2.1.2.1. Synthesis of Supervisory Controller

Recall that the synthesis of supervisory controller in the enabling arc method is divided into four main steps:

Step 1 - Design the uncontrolled model of the system using APNs
Step 2- Synthesise the APN model supervisor and determine the control policy
    Step 2.1. Generate the reachability graph of the APN model
    Step 2.2. Identify and remove the "bad states" from the reachability graph
    Step 2.3. Design the APN model supervisor and determine the control policy
Step 3- Construct the controlled model of the system
Step 4- Implement the supervisor (the controlled model) on a PLC as LLDs

Note that the enabling arc method has common design steps up to the step 2.3 with the inhibitor arc method. Therefore in this section only the steps 2.3 , 3 and 4 are considered.

##### 7.2.1.2.1.1. Step 2.3. *Design the APN model supervisor and determine the control policy*

Consider the manufacturing system introduced in the section 7.2.1. Note that the uncontrolled APN model of the manufacturing system is shown in Fig. 7.3 and the APN model supervisor is also shown in Fig. 7.7 for the forbidden state specifications given in the section 7.2.1. These results are obtained by following the design steps given above.

Note that since the APN model supervisor is already designed in the step 2.3, in this section only the control policy is determined for the enabling arc method. To do this, first of all it is necessary to determine the controllable transitions that are related to the forbidden state specifications. Recall that the forbidden state specifications are as follows:

1. Operate the sort solenoid only when there is space in the assembly chute and there is a ring at the sort area.
2. Operate the assembly solenoid only when there is space at the assembly area and there is a ring in the assembly chute.

As can be seen from Fig. 7.2 and Fig. 7.3 when there is a token in place $p_8$ the sort solenoid is in operation. The controllable transition $t_6$ is responsible for putting a token into place $p_8$. Similarly, when there is a token in place $p_{10}$ the assembly solenoid is in operation. The controllable transition $t_7$ is responsible for putting a token into place $p_{10}$. Therefore in this case the controllable transitions $t_6$ with the event $\chi_6$ and $t_7$ with the event $\chi_7$ are related to the forbidden state specifications. In other words the objective of the control policy is to decide when to let transitions $t_6$ and $t_7$ fire such that the forbidden state specifications are met. Remember that in this case the supervisory control policy is a static table that provides a list of places of the APN model supervisor from which controllable transitions of the model will be enabled such that in the controlled model the forbidden state specifications are met. This table is enforced by enabling arcs. Now, consider the APN model supervisor, shown in Fig. 7.12. Note that transitions $t_{10}$ and $t_{24}$ are identical transitions of the controllable transition $t_6$, because they have the same event $\chi_6$ assigned to them. The input places, i.e., the base places, of these identical transitions are $p_{12}$ and $p_{20}$. Therefore in the control policy, the base places $p_{12}$ and $p_{20}$ are identified as places from which the controllable transitions $t_6$ is to be enabled by enabling arcs. This is the control policy for the controllable transition $t_6$. The controllable transition $t_7$ with event $\chi_7$ has the identical transitions $t_{22}$ and $t_{27}$ and therefore it has the base places $p_{14}$ and $p_{15}$ from the APN model supervisor. Thus, in the control policy, base places $p_{14}$ and $p_{15}$ are identified as places from which the controllable transition $t_3$ is to be enabled by enabling arcs. This is the control policy for the transition $t_3$. The resulting control policy for the manufacturing system in the *enabling arc method* is given in Table 7.6.

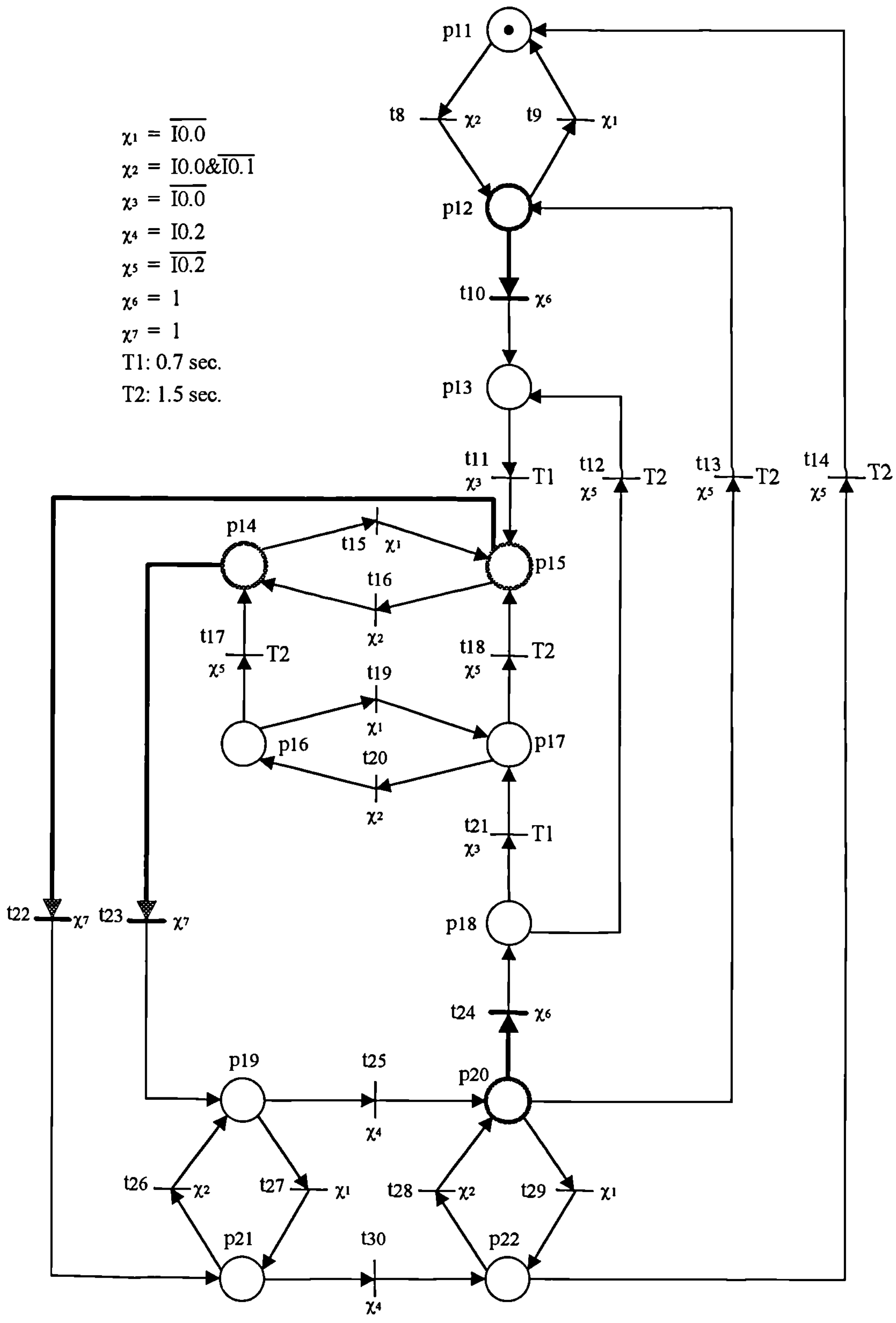


Figure 7.12. The APN model supervisor of the manufacturing system, used in determining the control policy in the *enabling arc method.*

| Transition | Base places from which an enabling arc is to be connected |
|---|---|
| $t_6$ | $p_{12}$ *or* $p_{20}$ |
| $t_7$ | $p_{14}$ *or* $p_{15}$ |

Table 7.6. The control policy for the manufacturing system in the enabling arc method.

#### 7.2.1.2.1.2. Step 3 - Construct the controlled model of the system

The controlled model (i.e., the supervisor), shown in Fig. 7.13, for the *enabling arc method* is obtained by using the APN model, shown in Fig. 7.3, the APN model supervisor, shown in Fig. 7.7, and the control policy given in Table 7.5. Note that since the controllable transition $t_6$ is to be enabled by places $p_{12}$ or $p_{20}$, in the controlled model it is replaced with two transitions, namely $t_6$ and $t_6$'. The same applies to the controllable transition $t_7$, where *it is replaced by transition $t_7$ and $t_7$'. Then the control policy is* implemented by simply connecting enabling arcs *$En(p_{12}, t_6)$, $En(p_{20}, t_6')$, $En(p_{14}, t_7)$* and *$En(p_{15}, t_7')$* from places $p_{12}$, $p_{20}$, $p_{14}$ and $p_{15}$ to the controllable transitions $t_6$, $t_6$', $t_7$ and $t_7$' respectively.

Note that the controlled model (the supervisor) obtained does not contradict the forbidden state specifications, i.e., controlled behaviour of the system is nonblocking. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications. It is also important to point out that the controlled model (the supervisor) obtained is correct by construction.

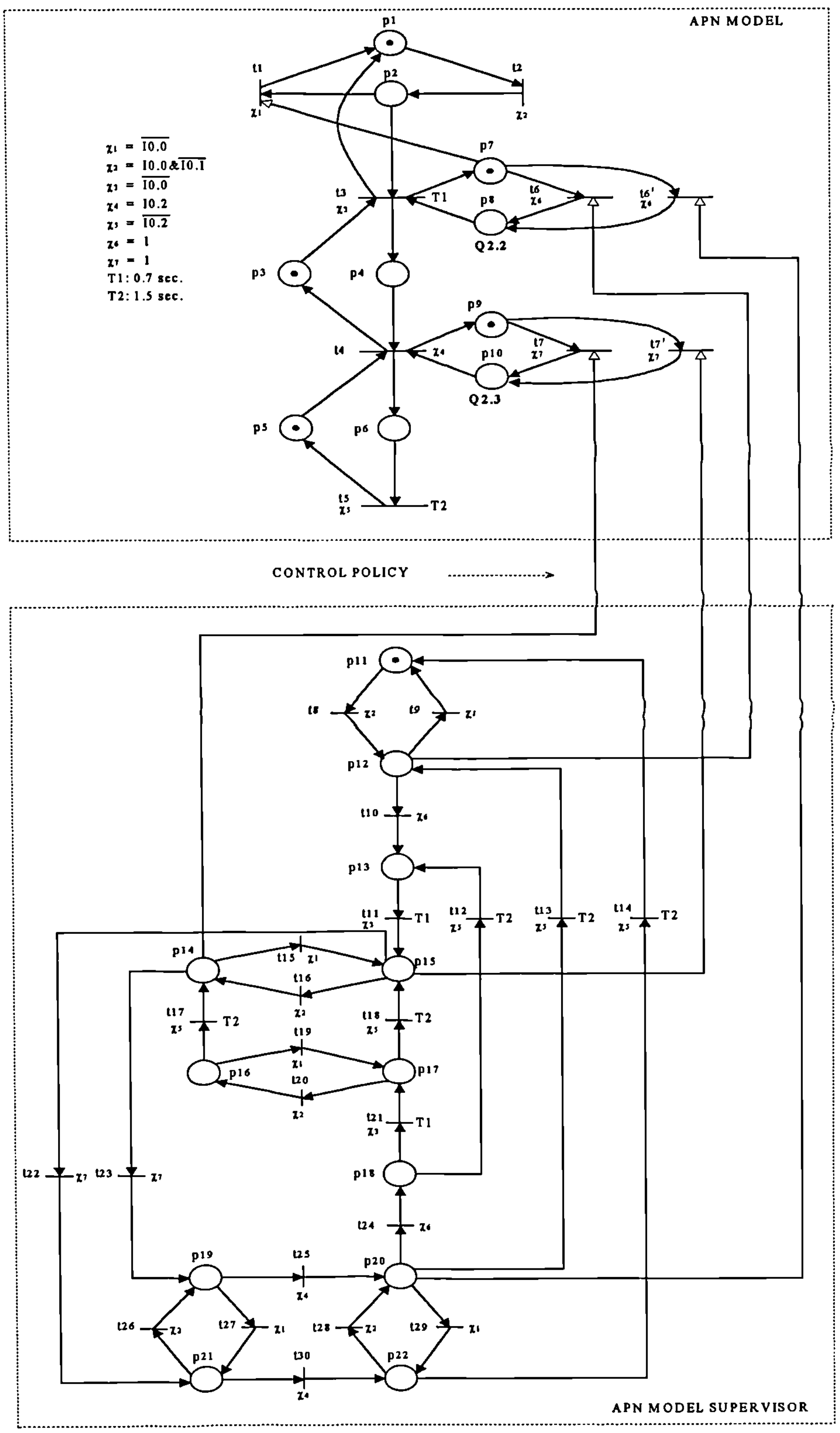


Figure 7.13. The controlled model (supervisor) of the manufacturing system for the enabling arc method.

### 7.2.1.2.1.3. Step 4 - Implement the supervisor (the controlled model) on a PLC as LLDs

In order to convert the supervisor (the controlled model) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions. Note that in the controlled model since the APN model supervisor acts as a monitor there is no actions or on delay timers associated with its places and transitions. On the other hand, in the APN model there are actions assigned to places and on delay timers are also associated with timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code. However, it should be noted that for proper functioning the order of the LLD code, must be arranged as follows: first, the initial marking is written; next, the LLD code related to the APN model supervisor is written; and finally, the LLD code for the APN model is written. In addition, in this case the LLD code for each controllable transition, to which an enabling arc is connected from a base place because of the control policy, is relocated between the LLD codes for the input transitions and the output transitions of the base place. This is because after the initial marking is represented as LLD, the APN model supervisor monitors the system behaviour and changes its state, and then according to the current state and the control policy, the behaviour of the APN model is restricted if necessary. Note that, while on delay timers are only associated with the timed-transitions in the APN model, the time evolution of these timers are followed by the timed-transitions within the APN model supervisor.

As a result to convert the supervisor, given in Fig. 7.13, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2 are assigned to the places $P = \{ p_1, p_2, .....,$

$p_{10}$ } of the APN model respectively. Similarly, flags F2.0, F2.1, F2.2, F2.3, F2.4, F2.5, F2.6, F2.7, F3.0, F3.1, F3.2, F3.3 are assigned to the places P = {$p_{11}$, $p_{12}$......$p_{22}$} of the APN model supervisor respectively. On delay timers T1 with 0.7 sec. time delay and T2 with 1.5 sec. time delay are assigned only to the timed-transitions $t_3$ and $t_5$ of the APN model. After the TPLC is obtained as shown in Fig. 7.14, it is then converted into the LLD code, as shown in Fig. 7.15, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. Then the APN model supervisor, represented by the transitions T = { $t_8$, $t_9$, $t_{10}$, ......, $t_{30}$ }, is converted into LLD. Finally, the APN model, represented by the transitions T = { $t_1$, $t_2$, ...., $t_7$ }, is converted into LLD. After this is done, the LLD code for each controllable transition, to which an enabling arc is connected from a base place because of the control policy, is relocated between the LLD codes for the input transitions and the output transitions of the base place. For example, the LLD code for the controllable transition $t_6$, to which an enabling arc *En*($p_{12}$, $t_6$) is connected from place $p_{12}$, is relocated between the LLD code for the input transition $t_8$ and the LLD code for the output transitions $t_9$ and $t_{10}$ of the base place $p_{12}$. The same applies to the controllable transitions $t_6$', $t_7$ and $t_7$'. The action places $p_8$ and $p_{10}$ are represented by rungs 35 and 36 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the last rung 37. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

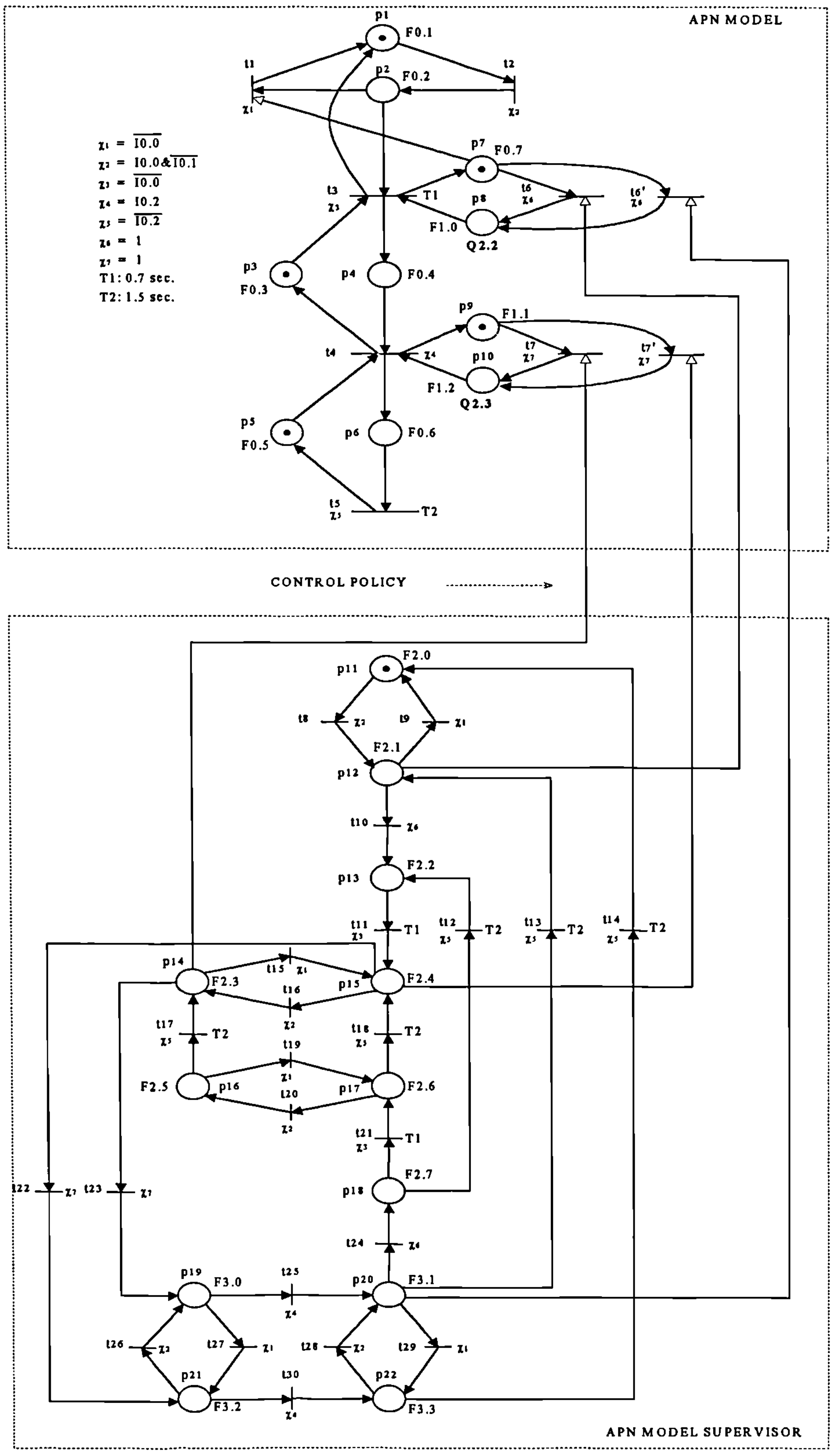
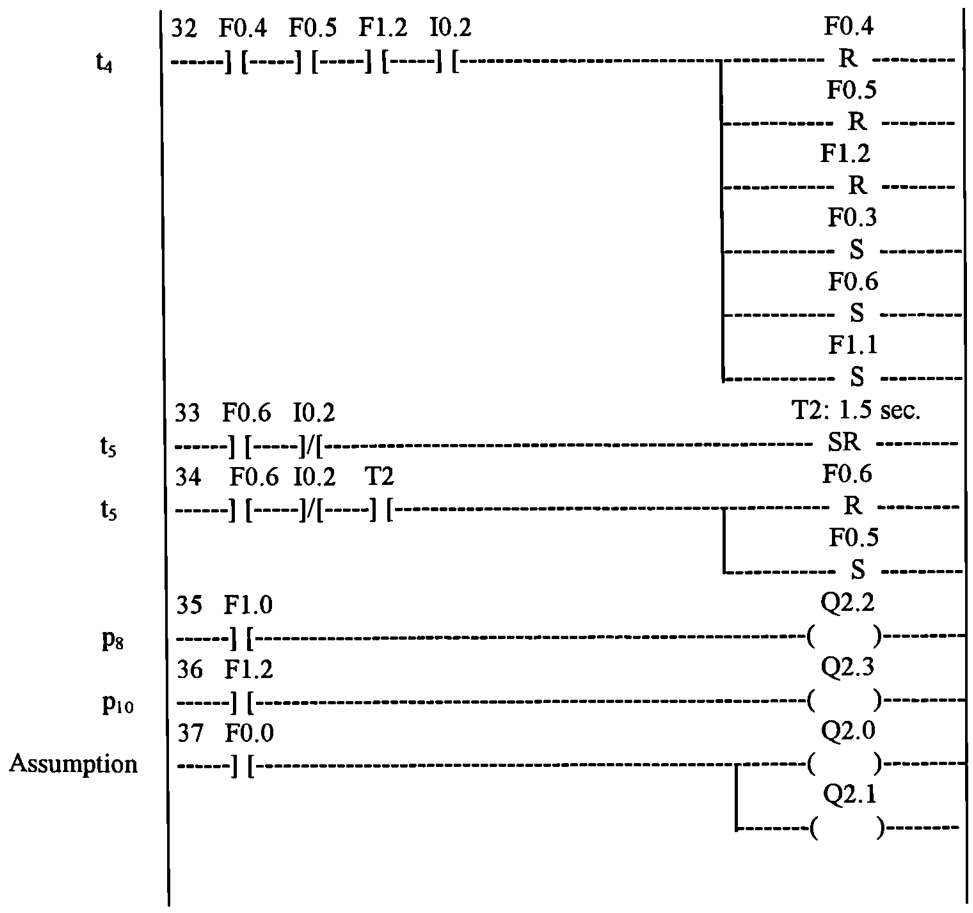


Figure 7.14. The TPLC for the supervisor (controlled model), shown in Fig. 7.13.

```
                | 0  F0.0                                            F0.1
initialisation  | ------]/[----------------------------------------------- S --------|
                |                                            |        F0.3
                |                                            |------------ S --------|
                |                                            |        F0.5
                |                                            |------------ S --------|
                |                                            |        F0.7
                |                                            |------------ S --------|
                |                                            |        F1.1
                |                                            |------------ S --------|
                |                                            |        F2.0
                |                                            |------------ S --------|
                |                                            |        F0.0
                |                                            |------------ S --------|
                | 1  F2.0  I0.0   I0.1                               F2.0
t8              | ------] [-----] [-----]/[----------------------------- R --------|
                |                                            |        F2.1
                |                                            |------------ S --------|
                | 2  F0.7 F2.1                                       F0.7
t6              | ------] [-----] [------------------------------------- R --------|
                |                                            |        F1.0
                |                                            |------------ S --------|
                | 3  F2.1  I0.0                                      F2.1
t9              | ------] [-----]/[------------------------------------- R --------|
                |                                            |        F2.0
                |                                            |------------ S --------|
                | 4  F2.1                                            F2.1
t10             | ------] [--------------------------------------------- R --------|
                |                                            |        F2.2
                |                                            |------------ S --------|
                | 5  F2.2  I0.0   T1                                 F2.2
t11             | ------] [-----]/[-----] [----------------------------- R --------|
                |                                            |        F2.4
                |                                            |------------ S --------|
                | 6  F2.7  I0.2   T2                                 F2.7
t12             | ------] [-----]/[-----] [----------------------------- R --------|
                |                                            |        F2.2
                |                                            |------------ S --------|
                | 7  F3.1  I0.2   T2                                 F3.1
t13             | ------] [-----]/[-----] [----------------------------- R --------|
                |                                            |        F2.1
                |                                            |------------ S --------|
                | 8  F3.3  I0.2   T2                                 F3.3
t14             | ------] [-----]/[-----] [----------------------------- R --------|
                |                                            |        F2.0
                |                                            |------------ S --------|
                | 9  F2.3  I0.0                                      F2.3
t15             | ------] [-----]/[------------------------------------- R --------|
                |                                            |        F2.4
                |                                            |------------ S --------|
                |                                                                   |
      | 10 F2.6  I0.2   T2                                   F2.6
t18   |------] [-----]/[-----] [---------------------------┬---------- R --------|
      |                                                    |          F2.4
      |                                                    └---------- S --------|
      | 11  F1.1 F2.4                                         F1.1
t7'   |------] [-----] [-----------------------------------┬---------- R --------|
      |                                                    |          F1.2
      |                                                    └---------- S --------|
      | 12  F2.4  I0.0   I0.1                                 F2.4
t16   |------] [-----] [-----]/[---------------------------┬---------- R --------|
      |                                                    |          F2.3
      |                                                    └---------- S --------|
      | 13  F2.5  I0.2   T2                                   F2.5
t17   |------] [-----]/[-----] [---------------------------┬---------- R --------|
      |                                                    |          F2.3
      |                                                    └---------- S --------|
      | 14  F1.1  F2.3                                        F1.1
t7    |------] [-----] [-----------------------------------┬---------- R --------|
      |                                                    |          F1.2
      |                                                    └---------- S --------|
      | 15  F2.5  I0.0                                        F2.5
t19   |------] [-----]/[-----------------------------------┬---------- R --------|
      |                                                    |          F2.6
      |                                                    └---------- S --------|
      | 16 F2.6  I0.0   I0.1                                  F2.6
t20   |------] [-----] [-----]/[---------------------------┬---------- R --------|
      |                                                    |          F2.5
      |                                                    └---------- S --------|
      | 17  F2.7  I0.0   T1                                   F2.7
t21   |------] [-----]/[-----] [---------------------------┬---------- R --------|
      |                                                    |          F2.6
      |                                                    └---------- S --------|
      | 18  F2.4                                              F2.4
t22   |------] [-------------------------------------------┬---------- R --------|
      |                                                    |          F3.2
      |                                                    └---------- S --------|
      | 19  F2.3                                              F2.3
t23   |------] [-------------------------------------------┬---------- R --------|
      |                                                    |          F3.0
      |                                                    └---------- S --------|
      | 20  F3.0  I0.2                                        F3.0
t25   |------] [-----] [-----------------------------------┬---------- R --------|
      |                                                    |          F3.1
      |                                                    └---------- S --------|
      | 21  F3.2  I0.0   I0.1                                 F3.2
t26   |------] [-----] [-----]/[---------------------------┬---------- R --------|
      |                                                    |          F3.0
      |                                                    └---------- S --------|
      | 22 F3.0  I0.0                                         F3.0
t27   |------] [-----]/[-----------------------------------┬---------- R --------|
      |                                                    |          F3.2
      |                                                    └---------- S --------|
      |                                                                          |
     | 23  F3.3  I0.0   I0.1                                    F3.3       |
t28  | ------] [-----] [-----]/[------------------------------------------- R ---------|
     |                                                  |           F3.1      |
     |                                                  L----------- S ---------|
     | 24  F0.7  F3.1                                          F0.7      |
t6'  | ------] [-----] [------------------------------------------------- R ---------|
     |                                                  |           F1.0      |
     |                                                  L----------- S ---------|
     | 25  F3.1                                                F3.1      |
t24  | ------] [--------------------------------------------------------- R ---------|
     |                                                  |           F2.7      |
     |                                                  L----------- S ---------|
     |                                                                         |
     | 26  F3.1  I0.0                                          F3.1      |
t29  | ------] [-----]/[------------------------------------------------- R ---------|
     |                                                  |           F3.3      |
     |                                                  L----------- S ---------|
     | 27  F3.2  I0.2                                          F3.2      |
t30  | ------] [-----] [------------------------------------------------- R ---------|
     |                                                  |           F3.3      |
     |                                                  L----------- S ---------|
     | 28  F0.2  F0.7  I0.0                                    F0.2      |
t1   | ------] [-----] [-----] [------------------------------------------ R ---------|
     |                                                  |           F0.1      |
     |                                                  L----------- S ---------|
     | 29  F0.1  I0.0   I0.1                                    F0.1      |
t2   | ------] [-----] [-----]/[------------------------------------------ R ---------|
     |                                                  |           F0.2      |
     |                                                  L----------- S ---------|
     | 30  F0.2  F0.3  F1.0  I0.0                         T1: 0.7 sec.  |
t3   | ------] [-----] [-----] [-----]/[--------------------------------- SR --------|
     | 31  F0.2  F0.3  F1.0  I0.0   T1                          F0.2      |
t3   | ------] [-----] [-----] [-----]/[-----] [-------------------------- R ---------|
     |                                                  |           F0.3      |
     |                                                  |----------- R ---------|
     |                                                  |           F1.0      |
     |                                                  |----------- R ---------|
     |                                                  |           F0.1      |
     |                                                  |----------- S ---------|
     |                                                  |           F0.4      |
     |                                                  |----------- S ---------|
     |                                                  |           F0.7      |
     |                                                  L----------- S ---------|
     |                                                                         |
```



```
           | 32  F0.4  F0.5  F1.2  I0.2                         F0.4      |
  t4       | ------] [-----] [-----] [-----] [---------------┬---------- R ---------|
           |                                                 |          F0.5      |
           |                                                 ├---------- R --------|
           |                                                 |          F1.2      |
           |                                                 ├---------- R --------|
           |                                                 |          F0.3      |
           |                                                 ├---------- S ---------|
           |                                                 |          F0.6      |
           |                                                 ├---------- S ---------|
           |                                                 |          F1.1      |
           |                                                 └---------- S ---------|
           | 33  F0.6  I0.2                                     T2: 1.5 sec.      |
  t5       | ------] [-----]/[------------------------------------------ SR --------|
           | 34  F0.6  I0.2   T2                                F0.6      |
  t5       | ------] [-----]/[-----] [-------------------------┬---------- R ---------|
           |                                                 |          F0.5      |
           |                                                 └---------- S ---------|
           | 35  F1.0                                           Q2.2      |
  p8       | ------] [-----------------------------------------------(    )--------|
           | 36  F1.2                                           Q2.3      |
  p10      | ------] [-----------------------------------------------(    )--------|
           | 37  F0.0                                           Q2.0      |
Assumption | ------] [----------------------------------------┬--------(    )--------|
           |                                                 |        Q2.1      |
           |                                                 └--------(    )--------|
           |                                                                      |
```

Figure 7.15. The LLD for the TPLC, shown in Fig. 7.14.

## SECTION A3

## THE FORBIDDEN STATE PROBLEM

## THE INTERMEDIATE PLACE METHOD

### 7.2.1.3. The Intermediate Place Method

#### 7.2.1.3.1. Synthesis of Supervisory Controller

Recall that the synthesis of supervisory controller in the intermediate place method is divided into four main steps:

Step 1 - Design the uncontrolled model of the system using APNs
Step 2- Synthesise the APN model supervisor and determine the control policy
    Step 2.1. Generate the reachability graph of the APN model
    Step 2.2. Identify and remove the "bad states" from the reachability graph
    Step 2.3. Design the APN model supervisor and determine the control policy
Step 3- Construct the controlled model of the system
Step 4- Implement the supervisor (the controlled model) on a PLC as LLDs

Note that the intermediate place method has common design steps up to the step 2.3 with the inhibitor arc method. Therefore in this section only the steps 2.3 , 3 and 4 are considered.

##### 7.2.1.3.1.1. Step 2.3. *Design the APN model supervisor and determine the control policy*

Consider the manufacturing system introduced in the section 7.2.1. Note that the uncontrolled APN model of the manufacturing system is shown in Fig. 7.3 and the APN model supervisor is also shown in Fig. 7.7 for the forbidden state specifications given in the section 7.2.1. These results are obtained by following the design steps given above.

Note that since the APN model supervisor is already designed in the step 2.3, in this section only the control policy is determined for the intermediate place method. To do this, first of all it is necessary to determine the controllable transitions that are related to the forbidden state specifications. As explained in the enabling arc method, the controllable transitions $t_6$ with the event $\chi_6$ and $t_7$ with the event $\chi_7$ are related to the forbidden state specifications. In other words the objective of the control policy is to decide when to let transitions $t_6$ and $t_7$ fire such that the forbidden state specifications are met. Remember that in this method one intermediate place is connected to the related controllable transitions with ordinary arcs. Therefore, intermediate places $p_{23}$ and $p_{24}$ are connected to the related controllable transitions $t_7$ and $t_6$ respectively, by ordinary arcs $Pre(p_{23}, t_7)$ and $Pre(t_{24}, t_6)$. This is shown in Fig. 7.16. The role of the control policy is to provide a set of input and output transitions for the intermediate places $p_{23}$ and $p_{24}$ from the APN model supervisor. Now consider the APN model supervisor, shown in Fig. 7.17. The controllable transition $t_7$ with the event $\chi_7$ of the APN model has identical transitions $t_{22}$ and $t_{23}$ within the APN model supervisor and therefore it has the base places $p_{14}$ and $p_{15}$. In the control policy, the input transitions of base places $p_{14}$ and $p_{15}$ are identified as the input transitions of the intermediate place $p_{23}$. When doing this, the identical transitions $t_{22}$ and $t_{23}$ and also the transitions $t_{15}$ and $t_{16}$, that connect one base place to another, are not included in the control policy. As a result the transitions $t_{11}$, $t_{17}$ and $t_{18}$ are identified as the input transitions of the intermediate place $p_{23}$. Note that in this case there are no output transitions for the intermediate place $p_{23}$. This is the control policy for the transition $t_6$. Similarly, the controllable transition $t_6$ with the event $\chi_6$ of the APN model has identical transitions $t_{10}$ and $t_{24}$ within the APN model supervisor and therefore it has the base places $p_{12}$ and $p_{20}$. In the control policy, the input transitions of places $p_{12}$ and $p_{20}$ are identified as the input transitions of the intermediate place $p_{24}$ and likewise the output transitions of places $p_{12}$ and $p_{20}$ are identified as the output transitions of the intermediate place $p_{24}$. When doing this, the identical transitions $t_{10}$ and $t_{24}$ and also the transition $t_{13}$, that connects one base place to another, are not included in the control policy. As a result the transitions $t_8$, $t_{25}$ and $t_{28}$ are identified as the input transitions of the intermediate place $p_{24}$, and likewise the transitions $t_9$ and $t_{29}$ are identified as the output transitions of the intermediate place $p_{24}$. This is the control policy for the transition $t_6$. The resulting control policy for the intermediate place method is shown in Table 7.7.

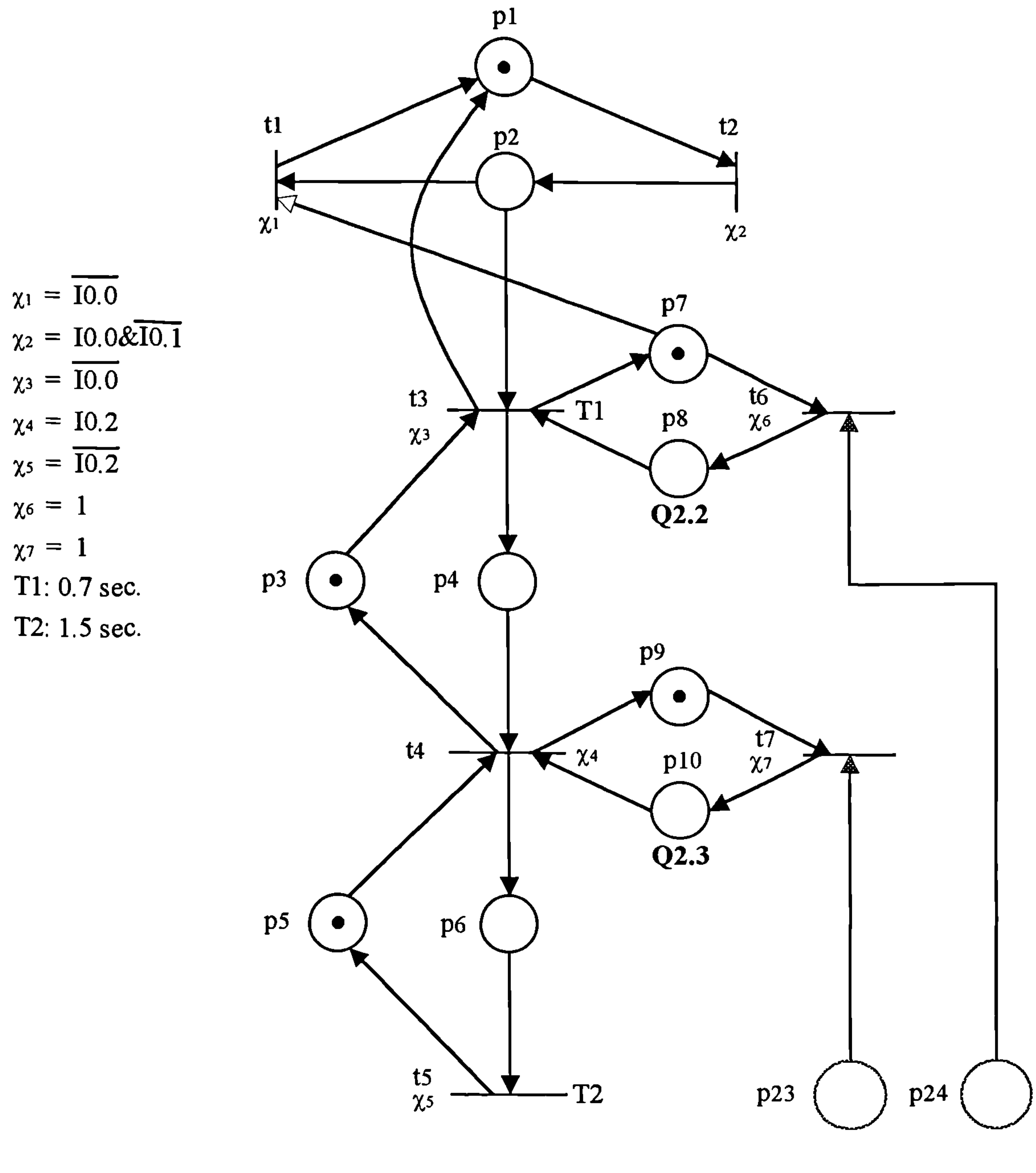


Figure 7.16. The intermediate places, connected to the uncontrolled APN model.

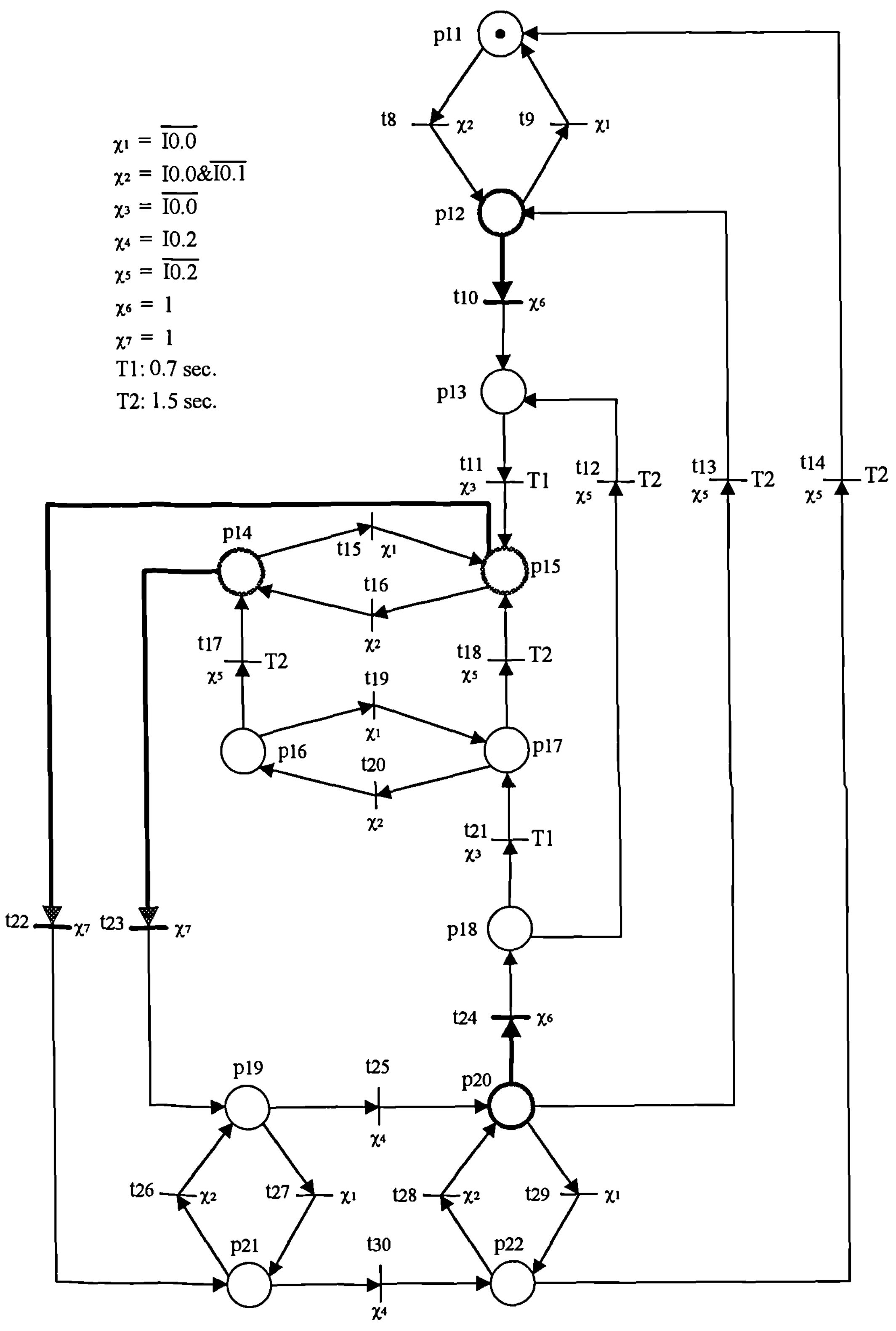


Figure 7.17. The APN model supervisor of the manufacturing system, used in determining the control policy in the *intermediate place method.*

| Intermediate place | Input transitions | Output transitions |
|---|---|---|
| $p_{23}$ | $t_{11}$, $t_{17}$, $t_{18}$ | - |
| $p_{24}$ | $t_8$, $t_{25}$, $t_{28}$ | $t_9$, $t_{29}$ |

Table 7.7. The control policy for the manufacturing system in the intermediate place method.

### 7.2.1.3.1.2. Step 3 - Construct the controlled model of the system

The controlled model (i.e., the supervisor), shown in Fig. 7.18, for the *intermediate place method* is obtained by using the APN model, shown in Fig. 7.16, the APN model supervisor, shown in Fig. 7.7, and the control policy given in Table 7.6. The control policy is implemented as follows. For the intermediate place $p_{24}$ the arcs $Post(t_8, p_{24})$, $Post(t_{25}, p_{24})$ and $Post(t_{28}, p_{24})$ are connected from transitions $t_8$, $t_{25}$ and $t_{28}$ to the intermediate place $p_{24}$ and the arcs $Pre(p_{24}, t_9)$ and $Pre(p_{24}, t_{29})$ are connected from the intermediate place $p_{24}$ to the transitions $t_9$ and $t_{18}$ respectively.

Note that the controlled model (the supervisor) obtained does not contradict the forbidden state specifications, i.e., controlled behaviour of the system is nonblocking. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications. It is also important to point out that the controlled model (the supervisor) obtained is correct by construction.

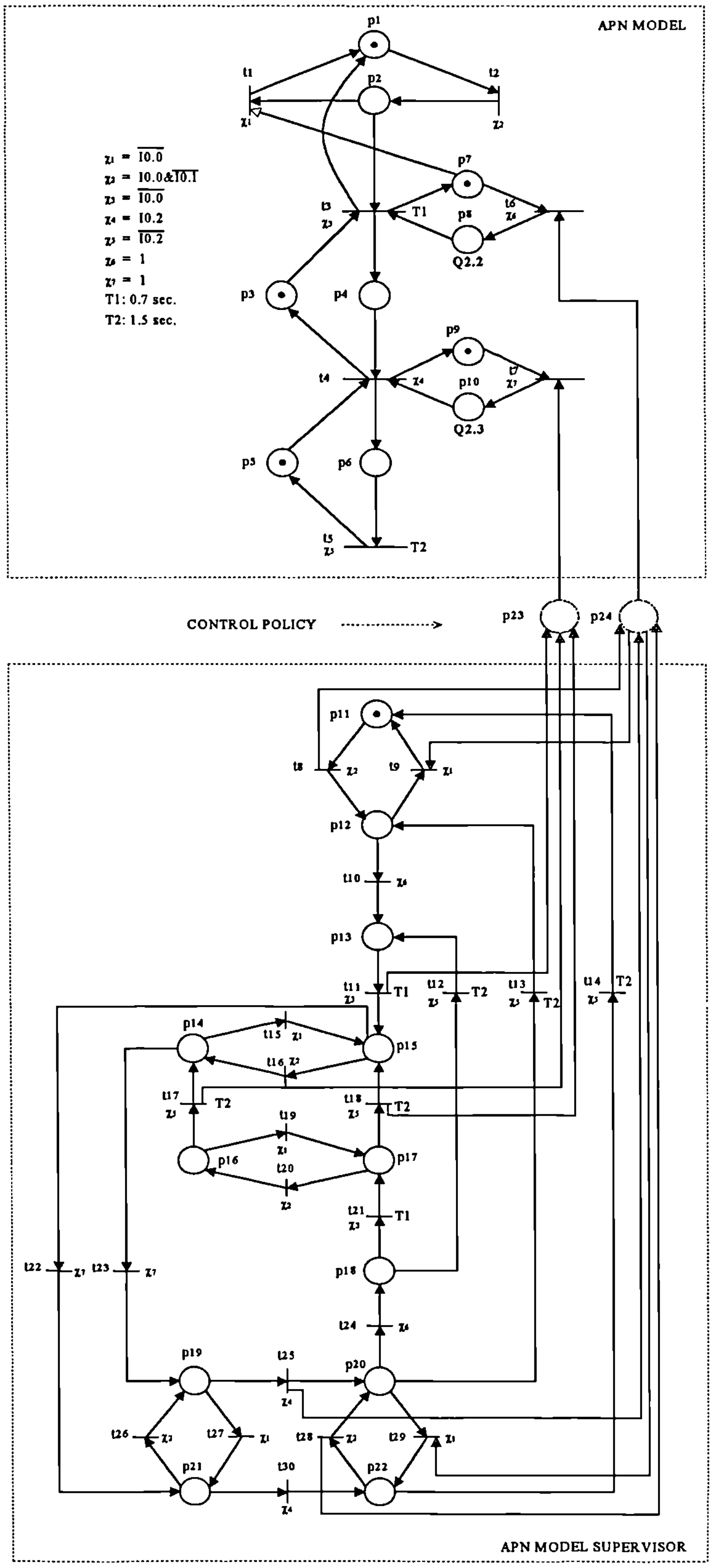


Figure 7.18. The controlled model (supervisor) of the manufacturing system for the intermediate place method.

### 7.2.1.3.1.3. Step 4 - Implement the supervisor (the controlled model) on a PLC as LLDs

In order to convert the supervisor (the controlled model) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions. Note that in the controlled model since the APN model supervisor acts as a monitor there is no actions or on delay timers associated with its places and transitions. On the other hand, in the APN model there are actions assigned to places and on delay timers are also associated with timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code. However, it should be noted that for proper functioning the order of the LLD code, must be arranged as follows: first, the initial marking is written; next, the LLD code related to the APN model supervisor is written; and finally, the LLD code for the APN model is written. This is because after the initial marking is represented as LLD, the APN model supervisor monitors the system behaviour and changes its state, and then according to the current state and the control policy, the behaviour of the APN model is restricted if necessary. Note that, while on delay timers are only associated with the timed-transitions in the APN model, the time evolution of these timers are followed by the timed-transitions within the APN model supervisor.

As a result to convert the supervisor, given in Fig. 7.18, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2 are assigned to the places P = { $p_1$, $p_2$, ....., $p_{10}$ } of the APN model respectively. Similarly, flags F2.0, F2.1, F2.2, F2.3, F2.4, F2.5, F2.6, F2.7, F3.0, F3.1, F3.2, F3.3 are assigned to the places P = {$p_{11}$, $p_{12}$......$p_{22}$} of the

APN model supervisor respectively. On delay timers T1 with 0.7 sec. time delay and T2 with 1.5 sec. time delay are assigned only to the timed-transitions $t_3$ and $t_5$ of the APN model. After the TPLC is obtained as shown in Fig. 7.19, it is then converted into the LLD code, as shown in Fig. 7.20, by using a *direct mapping from TPLC to LLD*. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The APN model supervisor is converted into LLD at the rungs from 1 to 23, where the rungs 1, 2, 3, ....., 23 represent the transitions $T = \{ t_8, t_9, t_{10}, ......, t_{30} \}$. The APN model is converted into LLD at the rungs from 24 to 32, where rungs 24, 25, ......., 32 represent the transitions $T = \{ t_1, t_2, ...., t_7 \}$ of the APN model. Then, action places $p_8$ and $p_{10}$ are represented by rungs 33 and 34 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 35. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

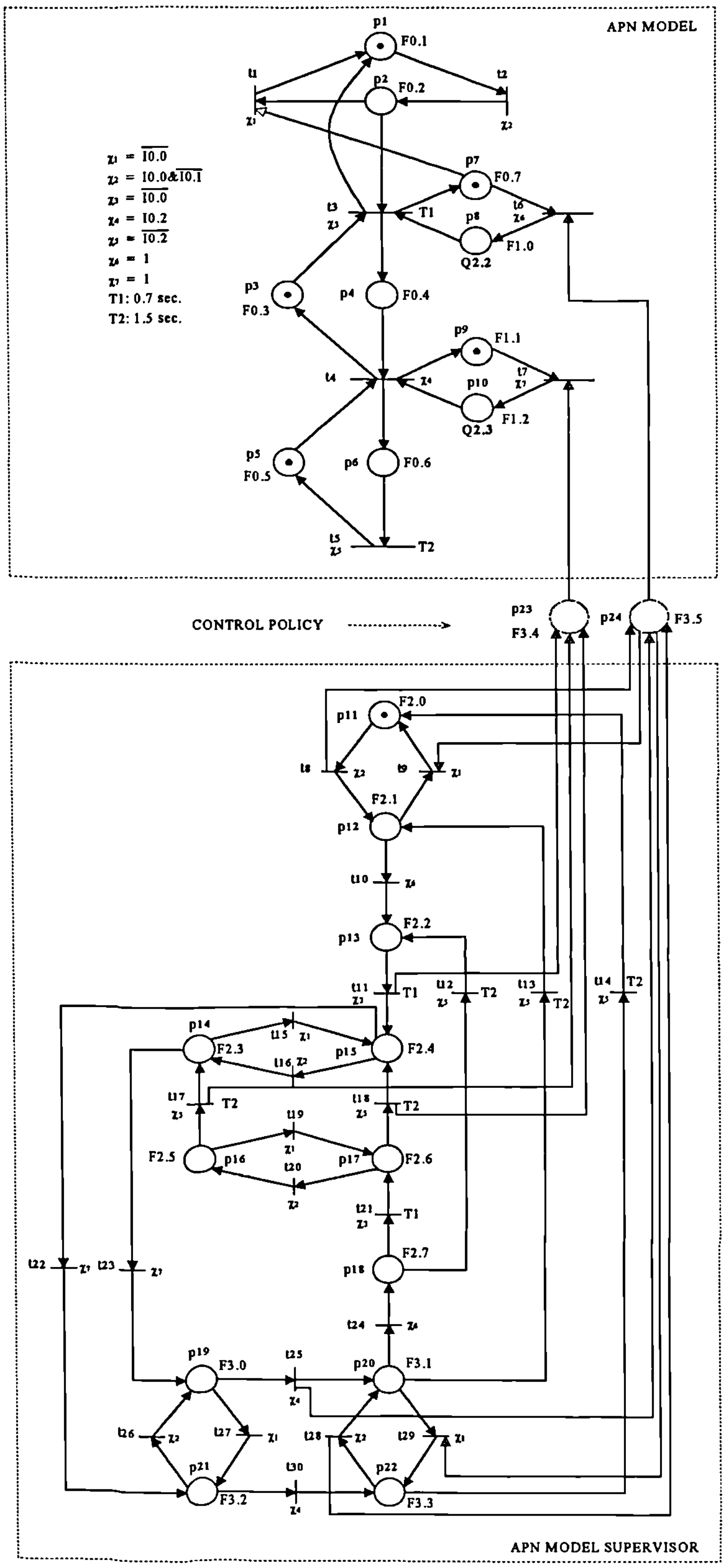


Figure 7.19. The TPLC for the supervisor (controlled model), shown in Fig. 7.18.

```
                 | 0  F0.0                                          F0.1     |
initialisation   | ------]/[--------------------------------------T--------- S ---------|
                 |                                                |     F0.3      |
                 |                                                |--------- S ---------|
                 |                                                |     F0.5      |
                 |                                                |--------- S ---------|
                 |                                                |     F0.7      |
                 |                                                |--------- S ---------|
                 |                                                |     F1.1      |
                 |                                                |--------- S ---------|
                 |                                                |     F2.0      |
                 |                                                |--------- S ---------|
                 |                                                |     F0.0      |
                 |                                                L--------- S ---------|
                 | 1   F2.0  I0.0   I0.1                               F2.0      |
t8               | ------] [-----] [-----]/[----------------------T--------- R --------|
                 |                                                |     F2.1      |
                 |                                                |--------- S ---------|
                 |                                                |     F3.5      |
                 |                                                L--------- S ---------|
                 | 2   F2.1  F3.5  I0.0                                F2.1      |
t9               | ------] [-----] [-----]/[----------------------T--------- R --------|
                 |                                                |     F3.5      |
                 |                                                |--------- R --------|
                 |                                                |     F2.0      |
                 |                                                L--------- S ---------|
                 | 3   F2.1                                            F2.1      |
t10              | ------] [--------------------------------------T--------- R ---------|
                 |                                                |     F2.2      |
                 |                                                L--------- S ---------|
                 | 4   F2.2  I0.0    T1                                F2.2      |
t11              | ------] [-----]/[-----] [----------------------T--------- R ---------|
                 |                                                |     F2.4      |
                 |                                                |--------- S ---------|
                 |                                                |     F3.4      |
                 |                                                L--------- S ---------|
                 | 5   F2.7  I0.2    T2                                F2.7      |
t12              | ------] [-----]/[-----] [----------------------T--------- R ---------|
                 |                                                |     F2.2      |
                 |                                                L--------- S ---------|
                 | 6   F3.1  I0.2    T2                                F3.1      |
t13              | ------] [-----]/[-----] [----------------------T--------- R ---------|
                 |                                                |     F2.1      |
                 |                                                L--------- S ---------|
                 | 7   F3.3  I0.2    T2                                F3.3      |
t14              | ------] [-----]/[-----] [----------------------T--------- R ---------|
                 |                                                |     F2.0      |
                 |                                                L--------- S ---------|
                 | 8   F2.3  I0.0                                      F2.3      |
t15              | ------] [-----]/[------------------------------T--------- R ---------|
                 |                                                |     F2.4      |
                 |                                                L--------- S ---------|
     | 9   F2.4  I0.0   I0.1                                        F2.4      |
t16  | ------] [-----] [-----]/[------------------------------------------ R ---------|
     |                                                      |         F2.3      |
     |                                                      L---------- S ---------|
     | 10  F2.5  I0.2   T2                                          F2.5      |
t17  | ------] [-----]/[-----] [------------------------------------------ R ---------|
     |                                                      |         F2.3      |
     |                                                      |---------- S ---------|
     |                                                      |         F3.4      |
     |                                                      L---------- S ---------|
     | 11  F2.6  I0.2   T2                                          F2.6      |
t18  | ------] [-----]/[-----] [------------------------------------------ R ---------|
     |                                                      |         F2.4      |
     |                                                      |---------- S ---------|
     |                                                      |         F3.4      |
     |                                                      L---------- S ---------|
     | 12  F2.5  I0.0                                               F2.5      |
t19  | ------] [-----]/[------------------------------------------------ R ---------|
     |                                                      |         F2.6      |
     |                                                      L---------- S ---------|
     | 13 F2.6  I0.0   I0.1                                         F2.6      |
t20  | ------] [-----] [-----]/[------------------------------------------ R ---------|
     |                                                      |         F2.5      |
     |                                                      L---------- S ---------|
     | 14  F2.7  I0.0   T1                                          F2.7      |
t21  | ------] [-----]/[-----] [------------------------------------------ R ---------|
     |                                                      |         F2.6      |
     |                                                      L---------- S ---------|
     | 15  F2.4                                                     F2.4      |
t22  | ------] [------------------------------------------------------- R ---------|
     |                                                      |         F3.2      |
     |                                                      L---------- S ---------|
     | 16  F2.3                                                     F2.3      |
t23  | ------] [------------------------------------------------------- R ---------|
     |                                                      |         F3.0      |
     |                                                      L---------- S ---------|
     | 17  F3.1                                                     F3.1      |
t24  | ------] [------------------------------------------------------- R ---------|
     |                                                      |         F2.7      |
     |                                                      L---------- S ---------|
     | 18  F3.0  I0.2                                               F3.0      |
t25  | ------] [-----] [------------------------------------------------ R ---------|
     |                                                      |         F3.1      |
     |                                                      |---------- S ---------|
     |                                                      |         F3.5      |
     |                                                      L---------- S ---------|
     | 19  F3.2  I0.0   I0.1                                        F3.2      |
t26  | ------] [-----] [-----]/[------------------------------------------ R ---------|
     |                                                      |         F3.0      |
     |                                                      L---------- S ---------|
     |                                                                          |
      | 20  F3.0  I0.0                                              F3.0
t27   | ------] [-----]/[-------------------------------------------┬---------- R --------|
      |                                                             |          F3.2
      |                                                             └---------- S --------|
      | 21  F3.3  I0.0   I0.1                                       F3.3
t28   | ------] [-----] [-----]/[-----------------------------------┬---------- R --------|
      |                                                             |          F3.1
      |                                                             ├---------- S --------|
      |                                                             |          F3.5
      |                                                             └---------- S --------|
      |
      | 22  F3.1  F3.5  I0.0                                        F3.1
t29   | ------] [-----] [-----]/[-----------------------------------┬---------- R --------|
      |                                                             |          F3.5
      |                                                             ├---------- R --------|
      |                                                             |          F3.3
      |                                                             └---------- S --------|
      | 23  F3.2  I0.2                                              F3.2
t30   | ------] [-----] [-------------------------------------------┬---------- R --------|
      |                                                             |          F3.3
      |                                                             └---------- S --------|
      | 24  F0.2  F0.7  I0.0                                        F0.2
t1    | ------] [-----] [-----] [-----------------------------------┬---------- R --------|
      |                                                             |          F0.1
      |                                                             └---------- S --------|
      | 25  F0.1  I0.0   I0.1                                       F0.1
t2    | ------] [-----] [-----]/[-----------------------------------┬---------- R --------|
      |                                                             |          F0.2
      |                                                             └---------- S --------|
      | 26  F0.2  F0.3  F1.0  I0.0                                  T1: 0.7 sec.
t3    | ------] [-----] [-----] [-----]/[------------------------------------- SR --------|
      | 27  F0.2  F0.3  F1.0  I0.0    T1                            F0.2
t3    | ------] [-----] [-----] [-----]/[-----] [-------------------┬---------- R --------|
      |                                                             |          F0.3
      |                                                             ├---------- R --------|
      |                                                             |          F1.0
      |                                                             ├---------- R --------|
      |                                                             |          F0.1
      |                                                             ├---------- S --------|
      |                                                             |          F0.4
      |                                                             ├---------- S --------|
      |                                                             |          F0.7
      |                                                             └---------- S --------|
           | 28 F0.4  F0.5  F1.2  I0.2                       F0.4   |
t4         | ------] [-----] [-----] [-----] [---------------------- R --------|
           |                                              |        F0.5   |
           |                                              |----------- R -------|
           |                                              |        F1.2   |
           |                                              |----------- R -------|
           |                                              |        F0.3   |
           |                                              |----------- S --------|
           |                                              |        F0.6   |
           |                                              |----------- S --------|
           |                                              |        F1.1   |
           |                                              |----------- S --------|
           | 29  F0.6  I0.2                              T2: 1.5 sec.  |
t5         | ------] [-----]/[---------------------------------- SR --------|
           | 30  F0.6 I0.2  T2                               F0.6   |
t5         | ------] [-----]/[-----] [---------------------------- R --------|
           |                                              |        F0.5   |
           |                                              |----------- S --------|
           | 31  F0.7  F3.5                                   F0.7   |
t6         | ------] [-----] [------------------------------------ R --------|
           |                                              |        F3.5   |
           |                                              |----------- R -------|
           |                                              |        F1.0   |
           |                                              |----------- S --------|
           |                                                              |
           | 32  F1.1  F3.4                                   F1.1   |
t7         | ------] [-----] [------------------------------------ R --------|
           |                                              |        F3.4   |
           |                                              |----------- R -------|
           |                                              |        F1.2   |
           |                                              |----------- S --------|
           | 33  F1.0                                         Q2.2   |
p8         | ------] [----------------------------------------------(    )--------|
           | 34  F1.2                                         Q2.3   |
p10        | ------] [----------------------------------------------(    )--------|
           | 35  F0.0                                         Q2.0   |
Assumption | ------] [----------------------------------------------(    )--------|
           |                                              |        Q2.1   |
           |                                              |--------(    )--------|
           |                                                              |
```

Figure 7.20. The LLD for the TPLC, shown in Fig. 7.19.

## SECTION A4

## THE FORBIDDEN STATE PROBLEM

## THE APN-SM METHOD

### 7.2.1.4. The APN-SM Method

#### 7.2.1.4.1. Synthesis of Supervisory Controller

Recall that the synthesis of supervisory controller in the APN-SM method is divided into four main steps:

Step 1 - Design the uncontrolled model of the system using APNs
Step 2 - Synthesise the APN model supervisor and determine the control policy
    Step 2.1. Generate the reachability graph of the APN model
    Step 2.2. Identify and remove the "bad states" from the reachability graph
    Step 2.3. Design the incomplete supervisor and determine the control policy
Step 3 - Construct the complete supervisor
Step 4 - Implement the supervisor (the complete supervisor) on a PLC as LLDs

Note that the APN-SM method has common design steps up to the step 2.3 with the inhibitor arc method. Therefore in this section only the steps 2.3 , 3 and 4 are considered.

##### 7.2.1.4.1.1. Step 2.3. *Design the incomplete supervisor and determine the control policy*

Consider the manufacturing system introduced in the section 7.2.1. Note that the uncontrolled APN model of the manufacturing system is shown in Fig. 7.3 for the forbidden state specifications given in the section 7.2.1. The final reduced reachability graph (FRRG) is shown in Fig. 7.6 and the incomplete APN supervisor, called APN model supervisor in the previous methods, is obtained by converting the FRRG into an APN as shown in Fig. 7.7. These results are obtained by following the design steps given above.

Now it is necessary to determine the control policy, which simply provides a list of actions to be assigned to the places within the incomplete supervisor. After these actions are assigned to the related places, the incomplete supervisor becomes a complete supervisor. The control policy is determined as follows. Firstly, the action places, on which actions are assigned, are identified from the uncontrolled model. Then, the FRRG is checked to see if it contains a marking, in which an action place is shown to have a token. Finally, if a marking within the FRRG represents an action place having a token, then in the control policy, the incomplete supervisor place, representing this marking, is to be assigned the related action within the complete supervisor. It is obvious from Fig. 7.3 that places $p_8$ and $p_{10}$ are action places of the uncontrolled APN model, because the actions Q2.2 and Q2.3 are assigned to them respectively. This means that when there is a token in place $p_8$, the sort solenoid is switched *on*. Similarly, when there is a token in place $p_{10}$, the assembly solenoid is switched *on*. Now, consider the FRRG given in Fig. 7.21. The markings that represent the action place $p_8$ having a token are $M_8$ = (2, 3, 5, **8**, 9) and $M_{23}$ = (2, 3, 6, **8**, 9). Therefore, places $p_{13}$ and $p_{18}$, that represent these markings within the incomplete supervisor respectively, are to be assigned the action Q2.2 in the complete supervisor. Similarly, consider the action Q2.3, assigned to the action place $p_{10}$ in the uncontrolled APN model. It is obvious that at the markings $M_{26}$ = (2, 4, 5, 7, **10**) and $M_{30}$ = (1, 4, 5, 7, **10**), place $p_{10}$ has a token. Therefore, places $p_{19}$ and $p_{21}$, that represent these markings within the incomplete supervisor respectively, are to be assigned the action Q2.3 in the complete supervisor. The resulting control policy for the manufacturing system is given in Table 7.8.

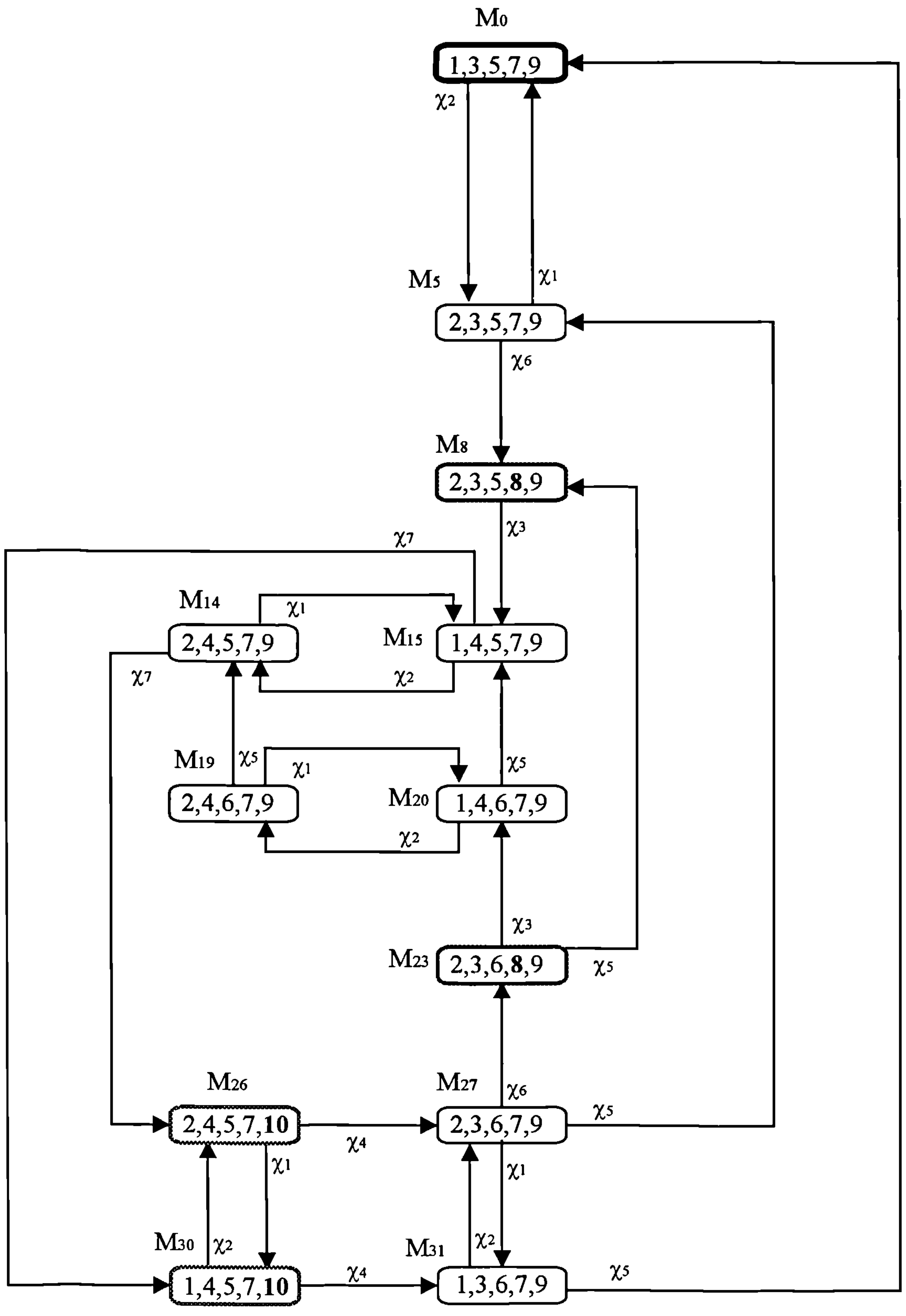


Figure 7.21. The FRRG, used in determining the control policy in the *APN-SM method*.

| Marking | Supervisor place | Action |
|---|---|---|
| $M_5 = (2,3,5,7,9)$ | $p_{13}$ | Q2.2 |
| $M_{23} = (2,3,6,8,9)$ | $p_{18}$ | Q2.2 |
| $M_{26} = (2,4,5,7,10)$ | $p_{19}$ | Q2.3 |
| $M_{30} = (1,4,5,7,10)$ | $p_{21}$ | Q2.3 |

Table 7.8. The control policy for the manufacturing system in the APN-SM method .

### 7.2.1.4.1.2. Step 3 - Construct the complete supervisor

The supervisor (i.e., the complete supervisor), shown in Fig. 7.22, for the *APN-SM method* is obtained by using the incomplete supervisor, shown in Fig. 7.7 and the control policy given in Table 7.7. In order to implement the control policy action Q2.2 is assigned to places $p_{13}$ and $p_{18}$ and action Q2.3 is assigned to places $p_{19}$ and $p_{21}$.

Note that the supervisor obtained does not contradict the forbidden state specifications, i.e., controlled behaviour of the system is nonblocking. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications. It is also important to point out that the supervisor obtained is correct by construction.

$\chi_1 = \overline{\mathrm{I0.0}}$

$\chi_2 = \mathrm{I0.0}\&\overline{\mathrm{I0.1}}$

$\chi_3 = \overline{\mathrm{I0.0}}$

$\chi_4 = \mathrm{I0.2}$

$\chi_5 = \overline{\mathrm{I0.2}}$

$\chi_6 = 1$

$\chi_7 = 1$

T1: 0.7 sec.

T2: 1.5 sec.

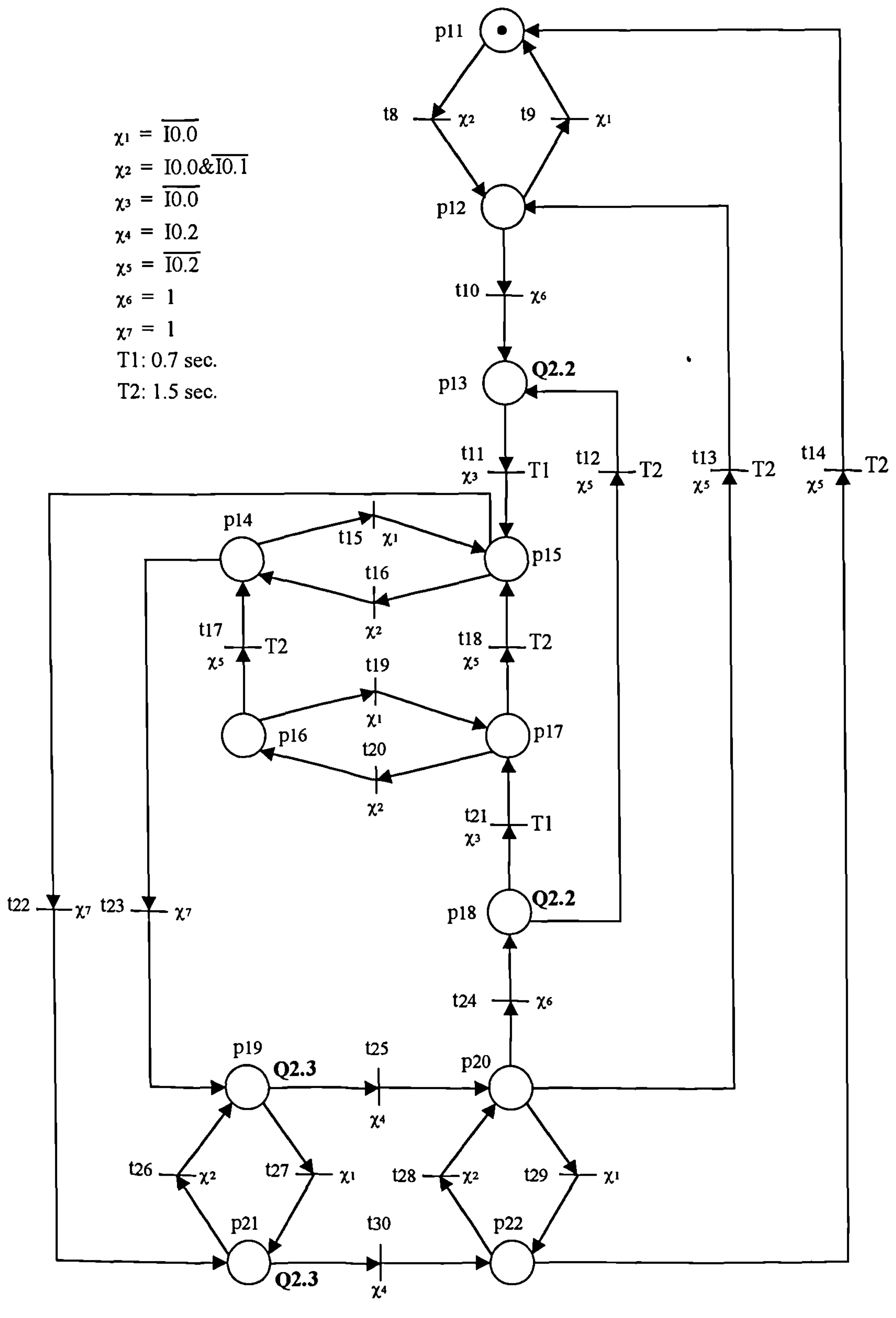


Figure 7.22. The supervisor for the manufacturing system in the APN-SM method .

### 7.2.1.4.1.3. Step 4 - Implement the supervisor (the complete supervisor) on a PLC as LLDs

In order to convert the supervisor (the complete supervisor) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code.

As a result to convert the supervisor, given in Fig. 7.22, into a TPLC, flags F2.0, F2.1, F2.2, F2.3, F2.4, F2.5, F2.6, F2.7, F3.0, F3.1, F3.2, F3.3 are assigned to the places P = $\{p_{11}, p_{12}......p_{22}\}$ of the supervisor respectively. The on delay timer T1 with 0.7 sec. time delay is assigned to the timed-transitions $t_{11}$ and $t_{21}$. Similarly, the on delay timer T2 with 1.5 sec. time delay is assigned to the timed-transitions $t_{12}$, $t_{13}$, $t_{14}$, $t_{17}$ and $t_{18}$. After the TPLC is obtained as shown in Fig. 7.23, it is then converted into the LLD code, as shown in Fig. 7.24, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The rungs 1, 2, 3, ....., 23 represent the transitions T = $\{ t_8, t_9, t_{10}, ......, t_{30} \}$ respectively. The timing requirements for the timed-transitions $t_{11}$ and $t_{21}$ are represented by the rung 24. Similarly, the timing requirements for the timed-transitions $t_{12}$, $t_{13}$, $t_{14}$, $t_{17}$ and $t_{18}$ are represented by the rung 25. The action places $p_{13}$ and $p_{18}$ are represented by the rung 26 and similarly the action places $p_{19}$ and $p_{21}$ are represented by the rung 27. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 28. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

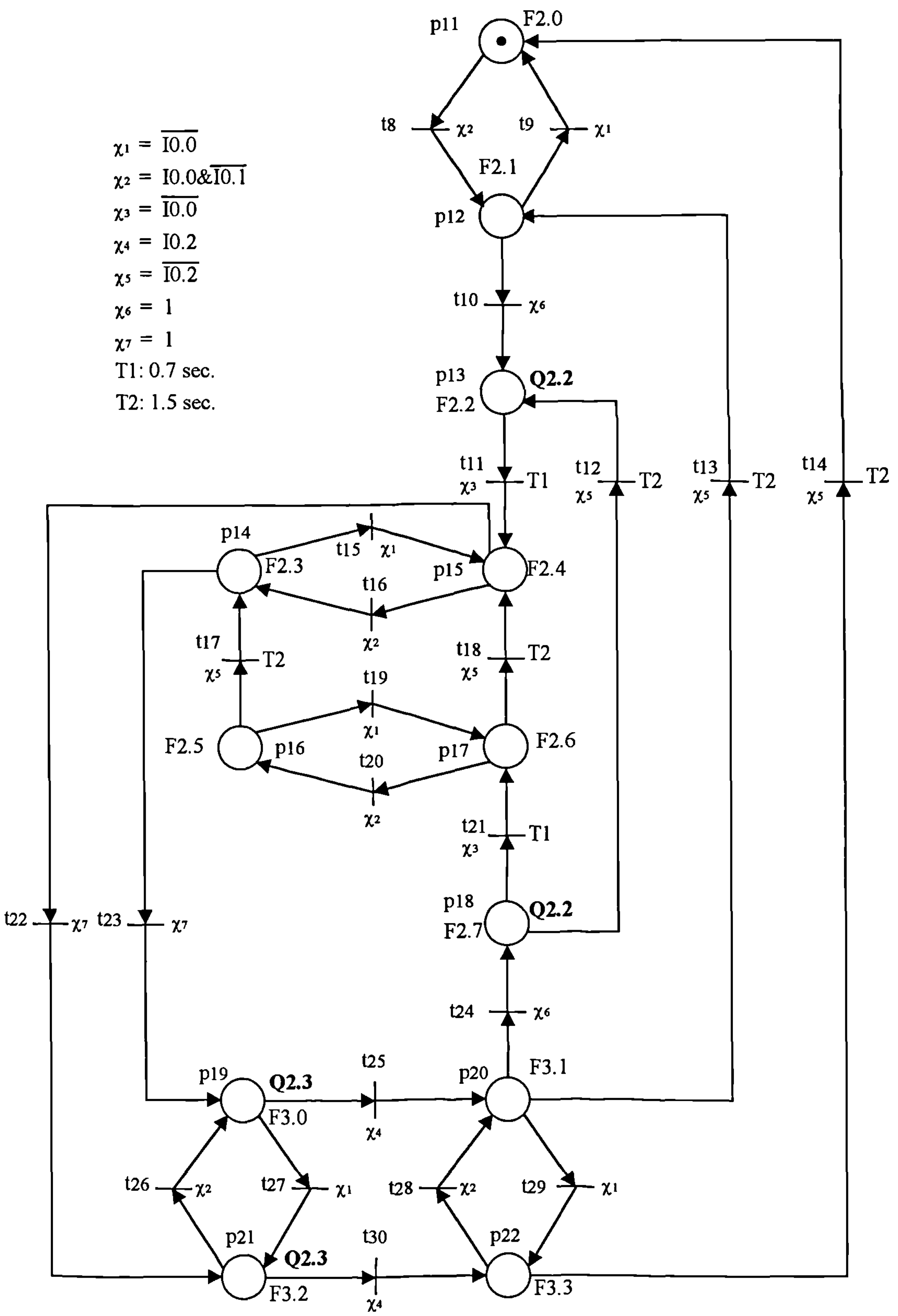


Figure 7.23. The TPLC for the supervisor shown in Fig. 7.22.

```
               | 0  F0.0                                              F2.0     |
initialisation | ------]/[------------------------------------------┬---------- S --------|
               |                                                    |     F0.0            |
               |                                                    └---------- S --------|
               | 1  F2.0  I0.0   I0.1                                 F2.0     |
      t8       | ------] [-----] [-----]/[--------------------------┬---------- R --------|
               |                                                    |     F2.1            |
               |                                                    └---------- S --------|
               | 2  F2.1  I0.0                                        F2.1     |
      t9       | ------] [-----]/[----------------------------------┬---------- R --------|
               |                                                    |     F2.0            |
               |                                                    └---------- S --------|
               | 3  F2.1                                              F2.1     |
      t10      | ------] [------------------------------------------┬---------- R --------|
               |                                                    |     F2.2            |
               |                                                    └---------- S --------|
               | 4  F2.2  I0.0   T1                                   F2.2     |
      t11      | ------] [-----]/[-----] [--------------------------┬---------- R --------|
               |                                                    |     F2.4            |
               |                                                    └---------- S --------|
               | 5  F2.7  I0.2   T2                                   F2.7     |
      t12      | ------] [-----]/[-----] [--------------------------┬---------- R --------|
               |                                                    |     F2.2            |
               |                                                    └---------- S --------|
               | 6  F3.1  I0.2   T2                                   F3.1     |
      t13      | ------] [-----]/[-----] [--------------------------┬---------- R --------|
               |                                                    |     F2.1            |
               |                                                    └---------- S --------|
               | 7  F3.3  I0.2   T2                                   F3.3     |
      t14      | ------] [-----]/[-----] [--------------------------┬---------- R --------|
               |                                                    |     F2.0            |
               |                                                    └---------- S --------|
               | 8  F2.3  I0.0                                        F2.3     |
      t15      | ------] [-----]/[----------------------------------┬---------- R --------|
               |                                                    |     F2.4            |
               |                                                    └---------- S --------|
               | 9  F2.4  I0.0   I0.1                                 F2.4     |
      t16      | ------] [-----] [-----]/[--------------------------┬---------- R --------|
               |                                                    |     F2.3            |
               |                                                    └---------- S --------|
               | 10 F2.5  I0.2   T2                                   F2.5     |
      t17      | ------] [-----]/[-----] [--------------------------┬---------- R --------|
               |                                                    |     F2.3            |
               |                                                    └---------- S --------|
               | 11 F2.6  I0.2   T2                                   F2.6     |
      t18      | ------] [-----]/[-----] [--------------------------┬---------- R --------|
               |                                                    |     F2.4            |
               |                                                    └---------- S --------|
               | 12 F2.5  I0.0                                        F2.5     |
      t19      | ------] [-----]/[----------------------------------┬---------- R --------|
               |                                                    |     F2.6            |
               |                                                    └---------- S --------|
               |                                                                          |
     | 13 F2.6   I0.0  I0.1                                    F2.6      |
t20  | -----] [-----] [-----]/[------------------------------┬---------- R --------|
     |                                                       |         F2.5        |
     |                                                       └---------- S --------|
     |                                                                             |
     | 14 F2.7  I0.0    T1                                     F2.7                |
t21  | -----] [-----]/[-----] [------------------------------┬---------- R --------|
     |                                                       |         F2.6        |
     |                                                       └---------- S --------|
     | 15  F2.4                                                F2.4                |
t22  | -----] [----------------------------------------------┬---------- R --------|
     |                                                       |         F3.2        |
     |                                                       └---------- S --------|
     | 16  F2.3                                                F2.3                |
t23  | -----] [----------------------------------------------┬---------- R --------|
     |                                                       |         F3.0        |
     |                                                       └---------- S --------|
     | 17  F3.1                                                F3.1                |
t24  | -----] [----------------------------------------------┬---------- R --------|
     |                                                       |         F2.7        |
     |                                                       └---------- S --------|
     | 18  F3.0  I0.2                                          F3.0                |
t25  | -----] [-----] [--------------------------------------┬---------- R --------|
     |                                                       |         F3.1        |
     |                                                       └---------- S --------|
     | 19  F3.2  I0.0    I0.1                                  F3.2                |
t26  | -----] [-----] [-----]/[------------------------------┬---------- R --------|
     |                                                       |         F3.0        |
     |                                                       └---------- S --------|
     | 20  F3.0  I0.0                                          F3.0                |
t27  | -----] [-----]/[--------------------------------------┬---------- R --------|
     |                                                       |         F3.2        |
     |                                                       └---------- S --------|
     | 21  F3.3  I0.0    I0.1                                  F3.3                |
t28  | -----] [-----] [-----]/[------------------------------┬---------- R --------|
     |                                                       |         F3.1        |
     |                                                       └---------- S --------|
     | 22  F3.1  I0.0                                          F3.1                |
t29  | -----] [-----]/[--------------------------------------┬---------- R --------|
     |                                                       |         F3.3        |
     |                                                       └---------- S --------|
     | 23  F3.2  I0.2                                          F3.2                |
t30  | -----] [-----] [--------------------------------------┬---------- R --------|
     |                                                       |         F3.3        |
     |                                                       └---------- S --------|
     | 24  F2.2      I0.0                                   T1: 0.7 sec.           |
t11  | -----] [-----┬--]/[------------------------------------------- SR ------|
     |   F2.7       |                                                              |
t21  | -----] [-----┘                                                              |
     |                                                                             |
           | 25  F2.5    I0.2                                      T2: 1.5 sec. |
t17        | ------] [-----+--]/[-------------------------------------- SR ------ |
                F2.6     |                                                      |
t18        | ------] [-----+                                                     |
                F2.7     |                                                      |
t12        | ------] [-----+                                                     |
                F3.1     |                                                      |
t13        | ------] [-----+                                                     |
                F3.3     |                                                      |
t14        | ------] [-----+                                                     |
           | 26  F2.2                                              Q2.2         |
p13        | ------] [-----+-------------------------------------------(     )---|
                F2.7     |                                                      |
p18        | ------] [-----+                                                     |
           | 27  F3.0                                              Q2.3         |
p19        | ------] [-----+-------------------------------------------(     )---|
                F3.2     |                                                      |
p21        | ------] [-----+                                                     |
           | 28  F0.0                                              Q2.0         |
Assumption | ------] [---------------------------------------+-------(     )---|
           |                                              |    Q2.1            |
           |                                              +-------(     )---   |
           |                                                                    |
```

Figure 7.24. The LLD for the TPLC, shown in Fig. 7.23.

## SECTION A5

## THE FORBIDDEN STATE PROBLEM

## THE U-TPM RULE METHOD

#### 7.2.1.5. The U-TPM Rule Method

##### 7.2.1.5.1. Synthesis of Supervisory Controller

Note that in this case the closed model is obtained by connecting enabling arcs from places within the uncontrolled model to its controllable transitions, such that the control policy is satisfied. This means that the model has an enabling action over itself. The supervisory control policy is a static table that provides a list of places of the uncontrolled model from which controllable transitions of the model will be enabled such that in the controlled model the forbidden state specifications are met. This table is enforced by enabling arcs. Recall that the synthesis of supervisory controller in the U-TPM rule method is divided into four main steps:

Step 1 - Design the uncontrolled model of the system using APNs
Step 2- Determine the control policy
    Step 2.1. Generate the reachability graph of the APN model
    Step 2.2. Identify and remove the "bad states" from the reachability graph
    Step 2.3. Determine the control policy
Step 3- Construct the controlled model of the system
Step 4- Implement the supervisor (the controlled model) on a PLC as LLDs

Note that the U-TPM rule method has common design steps up to the step 2.3 with the inhibitor arc method. Therefore in this section only the steps 2.3 , 3 and 4 are considered.

###### 7.2.1.5.1.1. Step 2.3. *Determine the control policy*

Consider the manufacturing system introduced in the section 7.2.1. Note that the uncontrolled APN model of the manufacturing system is shown in Fig. 7.3 and the final reduced reachability graph (FRRG), which is obtained by removing the "bad states" from the reachability graph, is shown in Fig. 7.6 for the forbidden state specifications, given in the section 7.2.1. These results are obtained by following the design steps given above.

In order to determine control policy, firstly it is necessary to determine the controllable transitions that are related to the forbidden state specifications. Then, the uncontrolled APN model is considered. Each related controllable transition within the uncontrolled model is taken into account and the arcs, representing the firing of these controllable transitions, are identified from the FRRG. In one column of the control policy the list of the related controllable transitions is provided. In the next column, places of the uncontrolled model, that are to be used to enable these transitions, are provided. This represents the control policy of the U-TPM rule method. As explained in the enabling arc method, the controllable transitions $t_6$ with the event $\chi_6$ and $t_7$ with the event $\chi_7$ are related to the forbidden state specifications. In other words, the objective of the control policy is to provide markings at which transitions $t_6$ and $t_7$ are to be enabled such that the forbidden state specifications are met. It can be seen from the FRRG, shown in Fig. 7.25, that the firing of transition $t_6$ is represented by identical arcs $M_5[\chi_6>M_8$ and $M_{27}[\chi_6>M_{23}$. The input markings of these arcs are markings $M_5 = (2, 3, 5, 7, 9)$ and $M_{27} = (2, 3, 6, 7, 9)$ respectively. These arcs are called the *identical arcs* for the controllable transition $t_6$. Therefore in the control policy, these base markings $M_5$ and $M_{27}$ are identified as markings at which the controllable transition $t_6$ is to be enabled by enabling arcs. This is the control policy for the transition $t_6$. The controllable transition $t_7$ with event $\chi_7$ has the identical arcs $M_{14}[\chi_7>M_{26}$ and $M_{15}[\chi_7>M_{30}$ and therefore it has the base markings $M_{14} = (2, 4, 5, 7, 9)$ and $M_{15} = (1, 4, 5, 7, 9)$ from the FRRG. Thus, in the control policy, the base markings $M_{14}$ and $M_{15}$ are identified as markings at which the controllable transition $t_7$ is to be enabled by enabling arcs. This is the control policy for the transition $t_7$. The resulting control policy for the intermediate place method is shown in Table 7.9.

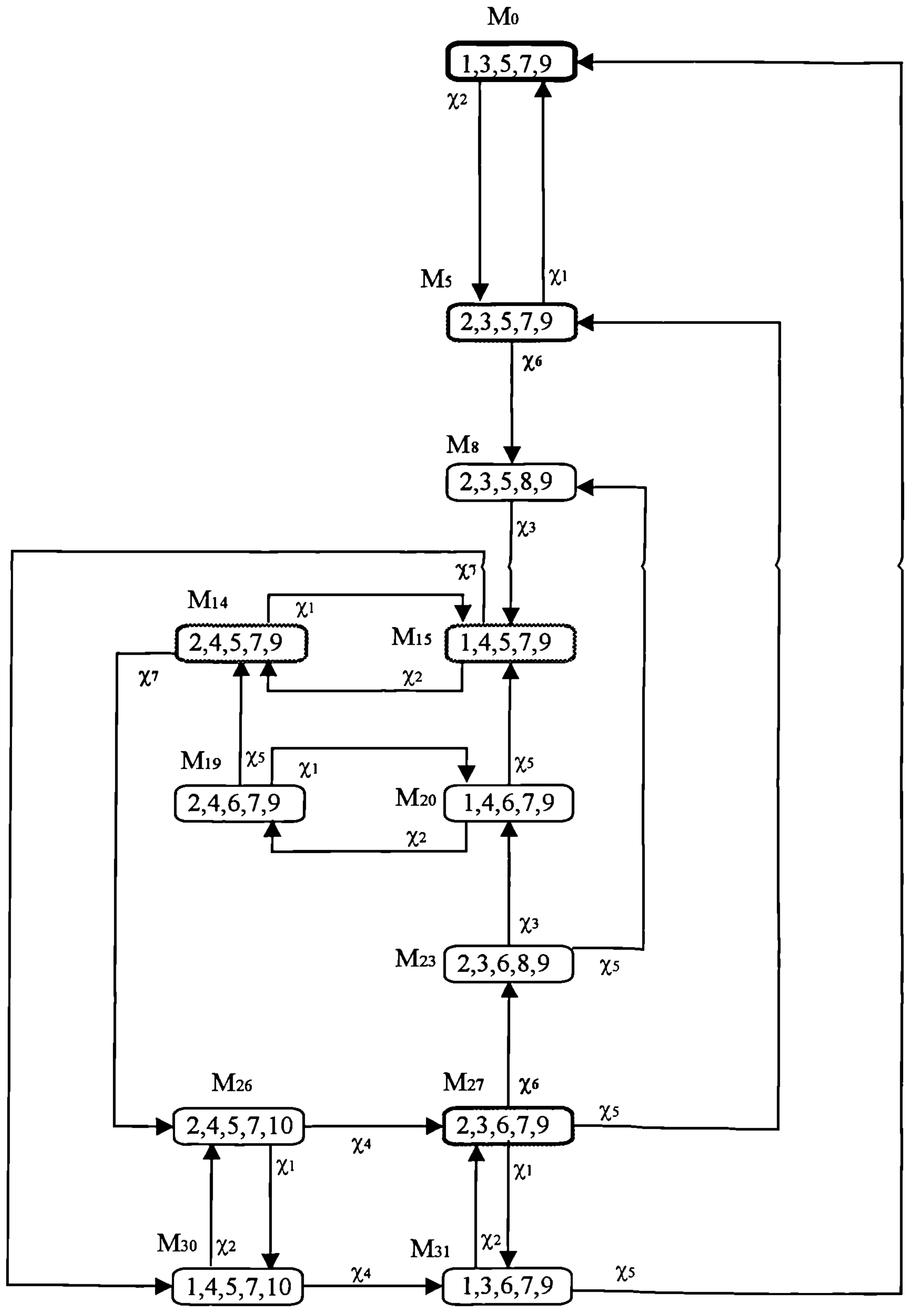


Figure 7.25. The FRRG, used in determining the control policy in the *U-TPM rule method.*

| Transition | Markings at which the transition is to be enabled |
|---|---|
| $t_6$ | $M_5 = (2,3,5,7,9)$ *or* $M_{27} = (2,3,6,7,9)$ |
| $t_7$ | $M_{14} = (2,4,5,7,9)$ *or* $M_{15} = (1,4,5,7,9)$ |

Table 7.9. The control policy for the manufacturing system in the U-TPM rule method.

Note that the control policy can be written as TPM rules as follows:

1. *if* $< M_5>$ OR $< M_{27}>$
   *then* <transition $t_6$ is to be enabled>

2. *if* $< M_{14}>$ OR $< M_{15}>$
   *then* <transition $t_7$ is to be enabled>

Note that these TPM rules can be re-written by separating the OR operation for each marking as follows:

1.i. *if* $< M_5>$
*then* <transition $t_6$ is to be enabled>
*OR*
ii. *if* $< M_{27}>$
*then* <transition $t_6$ is to be enabled>

2.i. *if* $< M_{14}>$
*then* <transition $t_7$ is to be enabled>
*OR*
ii. *if* $< M_{15}>$
*then* <transition $t_7$ is to be enabled>

These rules can be represented by putting the individual markings in the *if* part of the rules as follows:

1.i. *if* $<M(p_2) = 1>$ AND $<M(p_3) = 1>$ AND $<M(p_5) = 1>$ AND $<M(p_7) = 1>$ AND $<M(p_9) = 1>$
*then* <transition $t_6$ is to be enabled>
*OR*
ii. *if* $<M(p_2) = 1>$ AND $<M(p_3) = 1>$ AND $<M(p_6) = 1>$ AND $<M(p_7) = 1>$ AND $<M(p_9) = 1>$
*then* <transition $t_6$ is to be enabled>

2.i. *if* <$M(p_2) = 1$> AND <$M(p_4) = 1$> AND <$M(p_5) = 1$> AND <$M(p_7) = 1$> AND <$M(p_9) = 1$>
*then* <transition $t_7$ is to be enabled>
*OR*
ii. *if* <$M(p_1) = 1$> AND <$M(p_4) = 1$> AND <$M(p_5) = 1$> AND <$M(p_7) = 1$> AND <$M(p_9) = 1$>
*then* <transition $t_7$ is to be enabled>

### 7.2.1.5.1.2. Step 3 - Construct the controlled model of the system

The controlled model (i.e., the supervisor), shown in Fig. 7.26, for the *U-TPM rule method* is obtained by using the uncontrolled APN model, shown in Fig. 7.3, and the control policy given in Table 7.8. The control policy is implemented in the form of TPM rules shown above. Since the TPM rule 1 is split into two parts and contains an *or* operation, transition $t_6$ of the uncontrolled APN model is duplicated to accommodate the *or* operation within the Petri net formalism and replaced with transitions $t_6$ and $t_6$' within the controlled APN model. Similarly, since the TPM rule 2 is split into two parts and contains an *or* operation, transition $t_7$ of the uncontrolled APN model is duplicated to accommodate the *or* operation within the Petri net formalism and replaced with transitions $t_7$ and $t_7$' within the controlled APN model. Therefore the APM rules are modified as follows:

1.i. *if* <$M(p_2) = 1$> AND <$M(p_3) = 1$> AND <$M(p_5) = 1$> AND <$M(p_7) = 1$> AND <$M(p_9) = 1$>
*then* <transition $t_6$ is to be enabled>
ii. *if* <$M(p_2) = 1$> AND <$M(p_3) = 1$> AND <$M(p_6) = 1$> AND <$M(p_7) = 1$> AND <$M(p_9) = 1$>
*then* <transition $t_6$' is to be enabled>

2.i. *if* <$M(p_2) = 1$> AND <$M(p_4) = 1$> AND <$M(p_5) = 1$> AND <$M(p_7) = 1$> AND <$M(p_9) = 1$>
*then* <transition $t_7$ is to be enabled>
ii. *if* <$M(p_1) = 1$> AND <$M(p_4) = 1$> AND <$M(p_5) = 1$> AND <$M(p_7) = 1$> AND <$M(p_9) = 1$>
*then* <transition $t_7$' is to be enabled>

In order to implement the TPM rule 1.i, the enabling arcs $En(p_2, t_6)$, $En(p_3, t_6)$, $En(p_5, t_6)$, and $En(p_9, t_6)$ are connected from places $p_2$, $p_3$, $p_5$ and $p_9$ to controllable transition $t_6$. However, since there is an ordinary arc connecting place $p_7$ to controllable transition $t_6$, it is not necessary to connect an enabling arc from $p_7$ to $t_6$. To implement TPM rule 1.ii, the enabling arcs $En(p_2, t_6')$, $En(p_3, t_6')$, $En(p_6, t_6')$ and $En(p_9, t_6')$ are connected from places $p_2$, $p_3$, $p_6$ and $p_9$ to transition $t_6$'. However, since there is an ordinary arc connecting place $p_7$ to transition $t_6$', it is not necessary to connect an enabling arc from $p_7$ to $t_6$'. Similarly, to implement the TPM rule 2.i, the enabling arcs $En(p_2, t_7)$, $En(p_4, t_7)$, $En(p_5, t_7)$, and $En(p_7, t_7)$ are connected from places $p_2$, $p_4$, $p_5$ and $p_7$ to controllable transition $t_6$. However, since there is an ordinary arc connecting place $p_9$ to controllable transition $t_7$, it is not necessary to connect an enabling arc from $p_9$ to $t_7$. To implement TPM rule 2.ii, the enabling arcs $En(p_1, t_7')$, $En(p_4, t_7')$, $En(p_5, t_7')$ and $En(p_7, t_7')$ are connected from places $p_1$, $p_4$, $p_5$ and $p_7$ to controllable transition $t_7$'. However, since there is an ordinary arc connecting place $p_9$ to transition $t_7$', it is not necessary to connect an enabling arc from $p_9$ to $t_7$'. This process yields the controlled model of the system.

Note that the controlled model (the supervisor) obtained does not contradict the forbidden state specifications, i.e., controlled behaviour of the system is nonblocking. All events that do not contradict the forbidden state specifications are allowed to happen, i.e., the controlled behavior of the system is maximally permissive within the specifications. It is also important to point out that the controlled model (the supervisor) obtained is correct by construction.

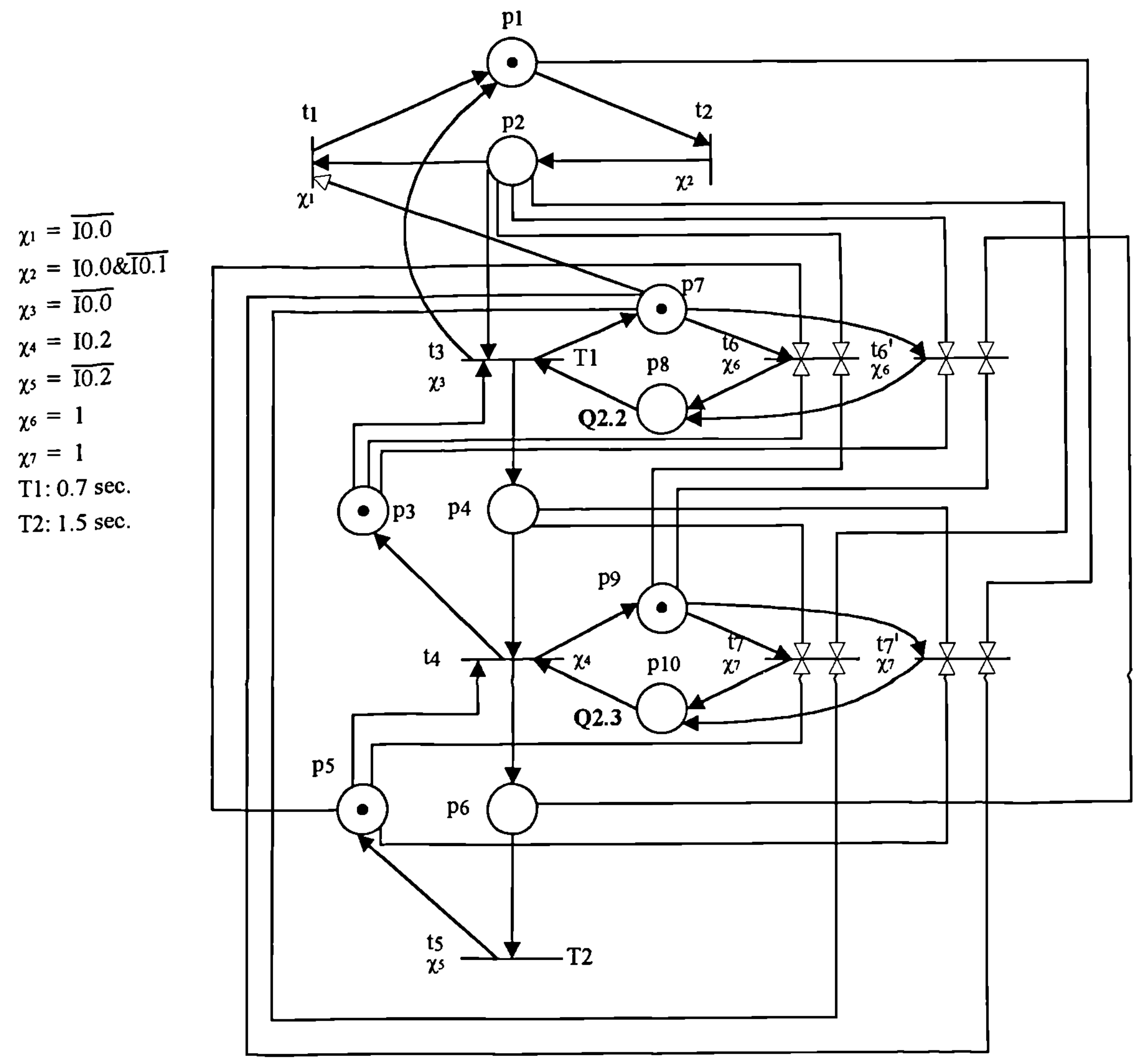


Figure 7.26. The supervisor (controlled model) for the manufacturing system in the U-TPM rule method.

### 7.2.1.5.1.3. Step 4 - Implement the supervisor (the controlled model) on a PLC as LLDs

In order to convert the supervisor (the controlled model) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code.

As a result to convert the supervisor, given in Fig. 7.26, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2 are assigned to the places P = { $p_1$, $p_2$, ....., $p_{10}$ } of the supervisor respectively. The on delay timers T1 with 0.7 sec. time delay and T2 with 1.5 sec. time delay are assigned to the timed-transitions $t_3$ and $t_5$. After the TPLC is obtained as shown in Fig. 7.27, it is then converted into the LLD code, as shown in Fig. 7.28, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The rungs 1, 2, 3, ....., 11 represent the transitions T = { $t_1$, $t_2$, $t_3$, $t_4$, $t_5$, $t_6$, $t_6$', $t_7$, $t_7$' }. Then, action places $p_8$ and $p_{10}$ are represented by rungs 12 and 13 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 14. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

$\chi_1 = \overline{I0.0}$

$\chi_2 = I0.0\&\overline{I0.1}$

$\chi_3 = \overline{I0.0}$

$\chi_4 = I0.2$

$\chi_5 = \overline{I0.2}$

$\chi_6 = 1$

$\chi_7 = 1$

T1: 0.7 sec.

T2: 1.5 sec.

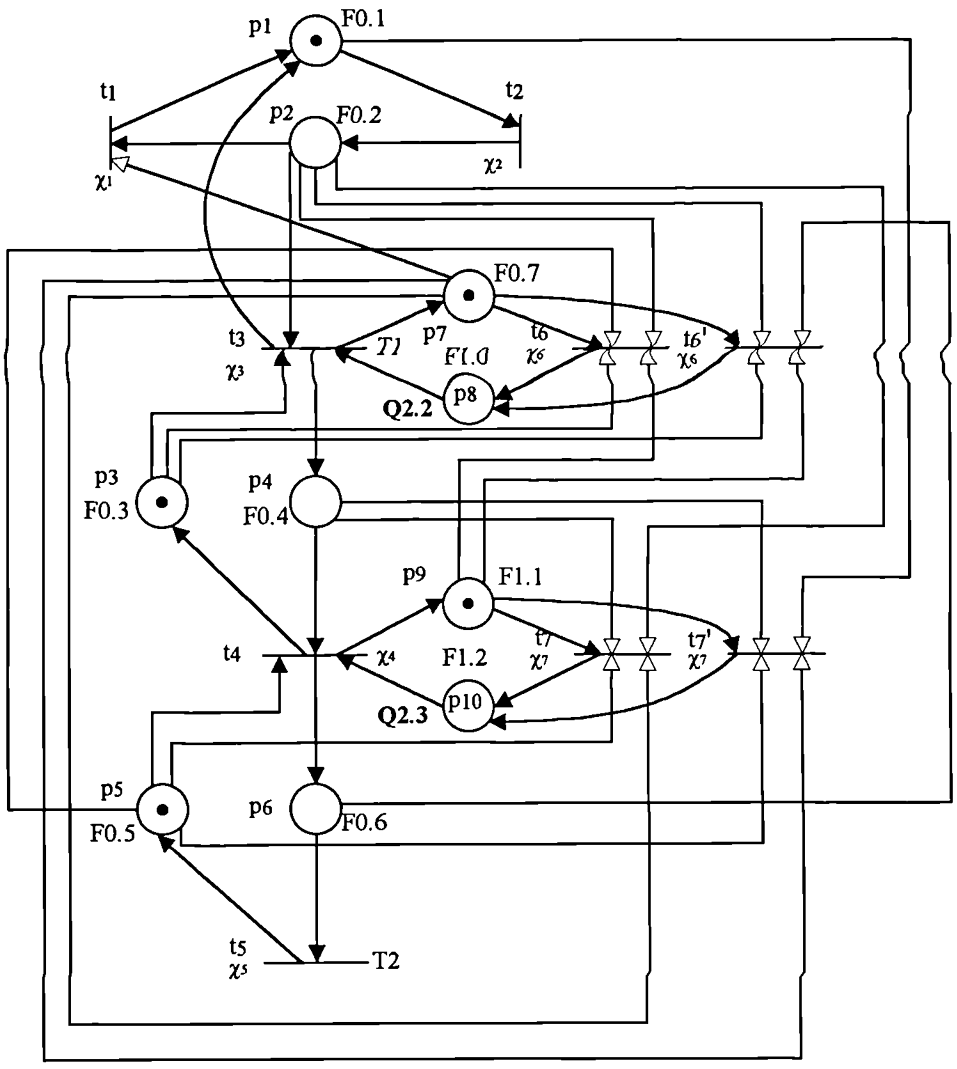


Figure 7.27. The TPLC for the supervisor shown in Fig. 7.26.

```
               | 0  F0.0                                       F0.1   |
initialisation | ------]/[--------------------------------+------- S -------|
               |                                          |     F0.3  |
               |                                          |------- S -------|
               |                                          |     F0.5  |
               |                                          |------- S -------|
               |                                          |     F0.7  |
               |                                          |------- S -------|
               |                                          |     F1.1  |
               |                                          |------- S -------|
               |                                          |     F0.0  |
               |                                          |------- S -------|
               | 1  F0.2  F0.7   I0.0                           F0.2  |
t1             | ------] [-----] [------]/[---------------+------- R -------|
               |                                          |     F0.1  |
               |                                          |------- S -------|
               | 2  F0.1  I0.0    I0.1                          F0.1  |
t2             | ------] [-----] [-----]/[----------------+------- R -------|
               |                                          |     F0.2  |
               |                                          |------- S -------|
               | 3  F0.2  F0.3  F1.0  I0.0                  T1: 0.7 sec. |
t3             | ------] [-----] [-----] [-----]/[------------------ SR -------|
               | 4   F0.2  F0.3  F1.0  I0.0   T1                F0.2  |
t3             | ------] [-----] [-----] [-----]/[-----] [----+------- R -------|
               |                                          |     F0.3  |
               |                                          |------- R -------|
               |                                          |     F1.0  |
               |                                          |------- R -------|
               |                                          |     F0.1  |
               |                                          |------- S -------|
               |                                          |     F0.4  |
               |                                          |------- S -------|
               |                                          |     F0.7  |
               |                                          |------- S -------|
               | 5  F0.4  F0.5  F1.2  I0.2                      F0.4  |
t4             | ------] [-----] [-----] [-----] [--------+------- R -------|
               |                                          |     F0.5  |
               |                                          |------- R -------|
               |                                          |     F1.2  |
               |                                          |------- R -------|
               |                                          |     F0.3  |
               |                                          |------- S -------|
               |                                          |     F0.6  |
               |                                          |------- S -------|
               |                                          |     F1.1  |
               |                                          |------- S -------|
               | 6  F0.6  I0.2                              T2: 1.5 sec. |
t5             | ------] [-----]/[-------------------------------- SR -------|
               |                                                      |
           |7   F0.6  I0.2   T2                                          F0.6 |
t5         |------] [-----]/[-----] [-------------------------------┬---------- R --------|
           |                                                        |         F0.5 |
           |                                                        └---------- S --------|
           |8  F0.2   F0.3  F0.5  F0.7  F1.1                              F0.7 |
t6         |------] [-----] [-----] [-----] [-----] [---------------┬---------- R --------|
           |                                                        |         F1.0 |
           |                                                        └---------- S --------|
           |9  F0.2   F0.3  F0.6  F0.7  F1.1                              F0.7 |
t6'        |------] [-----] [-----] [-----] [-----] [---------------┬---------- R --------|
           |                                                        |         F1.0 |
           |                                                        └---------- S --------|
           |10 F0.2   F0.4  F0.5  F0.7  F1.1                              F1.1 |
t7         |------] [-----] [-----] [-----] [-----] [---------------┬---------- R --------|
           |                                                        |         F1.2 |
           |                                                        └---------- S --------|
           |11 F0.1   F0.4  F0.5  F0.7  F1.1                              F1.1 |
t7'        |------] [-----] [-----] [-----] [-----] [---------------┬---------- R --------|
           |                                                        |         F1.2 |
           |                                                        └---------- S --------|
           |12  F1.0                                                     Q2.2 |
p8         |------] [----------------------------------------------------------(   )--------|
           |13  F1.2                                                     Q2.3 |
p10        |------] [----------------------------------------------------------(   )--------|
           |14  F0.0                                                     Q2.0 |
Assumption |------] [---------------------------------------------------┬------(   )--------|
           |                                                          |      Q2.1 |
           |                                                          └------(   )--------|
           |                                                                          |
```

Figure 7.28. The LLD for the TPLC, shown in Fig. 7.27.

## SECTION A6

## THE FORBIDDEN STATE PROBLEM

## THE C-TPM RULE METHOD

### 7.2.1.6. The C-TPM Rule Method

#### 7.2.1.6.1. Synthesis of Supervisory Controller

Note that in this method a set of "*Token Passing Marking rules*" (TPM rules) are used to obtain the controlled model from the uncontrolled model. Note that in this case, the TPM rules are assumed to be given. In fact, the forbidden state specifications are converted into related TPM rules. In these rules some markings of the uncontrolled model are identified for restricting the firing of some of the controllable transitions. However, in this case when the controlled model of the system is constructed it is necessary to verify its correctness by using reachability graph (RG) analysis. In this method, it is not clear whether the closed-loop model obtained is maximally permissive. However, it may be proved by comparing the uncontrolled behaviour with the controlled behaviour, but this process requires further RG analysis for the uncontrolled model which may be computationally prohibitive for complex systems. Recall that the synthesis of supervisory controller in the C-TPM rule method is divided into following steps:

Step 1- Design the uncontrolled model of the system using APNs

Step 2- Convert the forbidden state specifications, given, into related TPM rules

Step 3- Construct the controlled model of the system by combining the uncontrolled model and the TPM rules

Step 4- Generate the reachability graph (RG) of the controlled model

Step 5- Check whether the controlled model behaves according to the specifications: If it does not behave according to the specifications, go to the step 2 and make necessary corrections

Step 6- Implement the supervisor (the controlled model) on a PLC as LLDs

### 7.2.1.6.1.1. Step 2 - Convert the forbidden state specifications into related TPM rules

Consider the manufacturing system introduced in section 7.2.1. Note that the uncontrolled APN model of the manufacturing system, representing the uncontrolled system behaviour, is shown in Fig. 7.3. This represents the first design step of the C-TPM rule method. The forbidden state specifications are as follows:

1. Operate the sort solenoid only when there is space in the assembly chute and there is a ring at the sort area.
2. Operate the assembly solenoid only when there is space at the assembly area and there is a ring in the assembly chute.

These specifications can be re-stated in the ' if ... then ... ' rule format as follows:

1. *if* *<there is a ring at the sort area>* AND *<the assembly chute is empty>*
   *then* *<operate the sort solenoid>*

2. *if* *<there is a ring in the assembly chute>* AND *<the assembly area is empty>*
   *then* *<operate the assembly solenoid>*

Now, in order to obtain the TPM rules for these two specifications, the *if* part of the specifications can be represented by related markings and the *then* part of the specifications can be represented by related controllable transitions of the uncontrolled model of the manufacturing system. Firstly, consider the first specification. In the *if* part of the first TPM rule, *<there is a ring at the sort area>* can be represented by $<M(p_2) = 1>$ because when there is a token in place $p_2$ this means that there is a ring at the sort area. Similarly, $<M(p_3) = 1>$ can represent *<the assembly chute is empty>* because when there is a token in place $p_3$ this means that the assembly chute is empty. Now consider the *then* part of the first TPM rule. As can be seen from the uncontrolled model of the manufacturing system when there is a token in place $p_8$, the action Q2.2 is active, i.e., the sort solenoid is switched *on*. To control this process the controllable transition $t_6$ with the firing condition (event) $\chi_6$ is used. That is, when $t_6$ fires, the sort solenoid is switched *on*. Therefore, the *then* part of the specification 1, i.e., *<operate the sort solenoid>*, is represented with <transition $t_6$ is to be enabled> in the *then* part of the TPM rule 1. As a result, the TPM rule 1 for the specification 1 is as shown below. Now consider the second specification. In the *if* part of the second TPM rule *<there is a ring in the assembly chute>* can be represented by <$M(p_4) = 1$> because when there is a token in place $p_4$ this means that there is a ring in the assembly chute. Similarly, <$M(p_5) = 1$> can represent *<the assembly area is empty>* because when there is a token in place $p_5$ this means that the assembly area is empty. Now consider the *then* part of the second TPM rule. Using the similar approach, the *then* part of the specification 2, i.e., *<operate the assembly solenoid>*, can be represented by <transition $t_7$ is to be enabled> in the *then* part of the TPM rule 2. As a result, the TPM rule 2 for the specification 2 is shown below.

1. *if* <$M(p_2) = 1$> AND <$M(p_3) = 1$>
*then* <transition $t_6$ is to be enabled>

2. *if* <$M(p_4) = 1$> AND <$M(p_5) = 1$>
*then* <transition $t_7$ is to be enabled>

#### 7.2.1.6.1.2. Step 3 - Construct the controlled model of the system by combining the uncontrolled model and the TPM rules

The controlled model (i.e., the supervisor), shown in Fig. 7.29, for the *C-TPM rule method* is obtained by using the uncontrolled APN model, shown in Fig. 7.3 and the TPM rules obtained. In order to implement the TPM rule 1, the enabling arcs $En(p_2, t_6)$ and $En(p_3, t_6)$ are connected from places $p_2$ and $p_3$ to the controllable transition $t_6$. Similarly, to implement the TPM rule 2, the enabling arcs $En(p_4, t_7)$ and $En(p_5, t_7)$ are connected from places $p_4$ and $p_5$ to the controllable transition $t_7$. This yields the supervisor (the controlled model) for the manufacturing system in the C-TPM rule method.

$\chi_1 = \overline{I0.0}$

$\chi_2 = I0.0\&\overline{I0.1}$

$\chi_3 = \overline{I0.0}$

$\chi_4 = I0.2$

$\chi_5 = \overline{I0.2}$

$\chi_6 = 1$

$\chi_7 = 1$

T1: 0.7 sec.

T2: 1.5 sec.

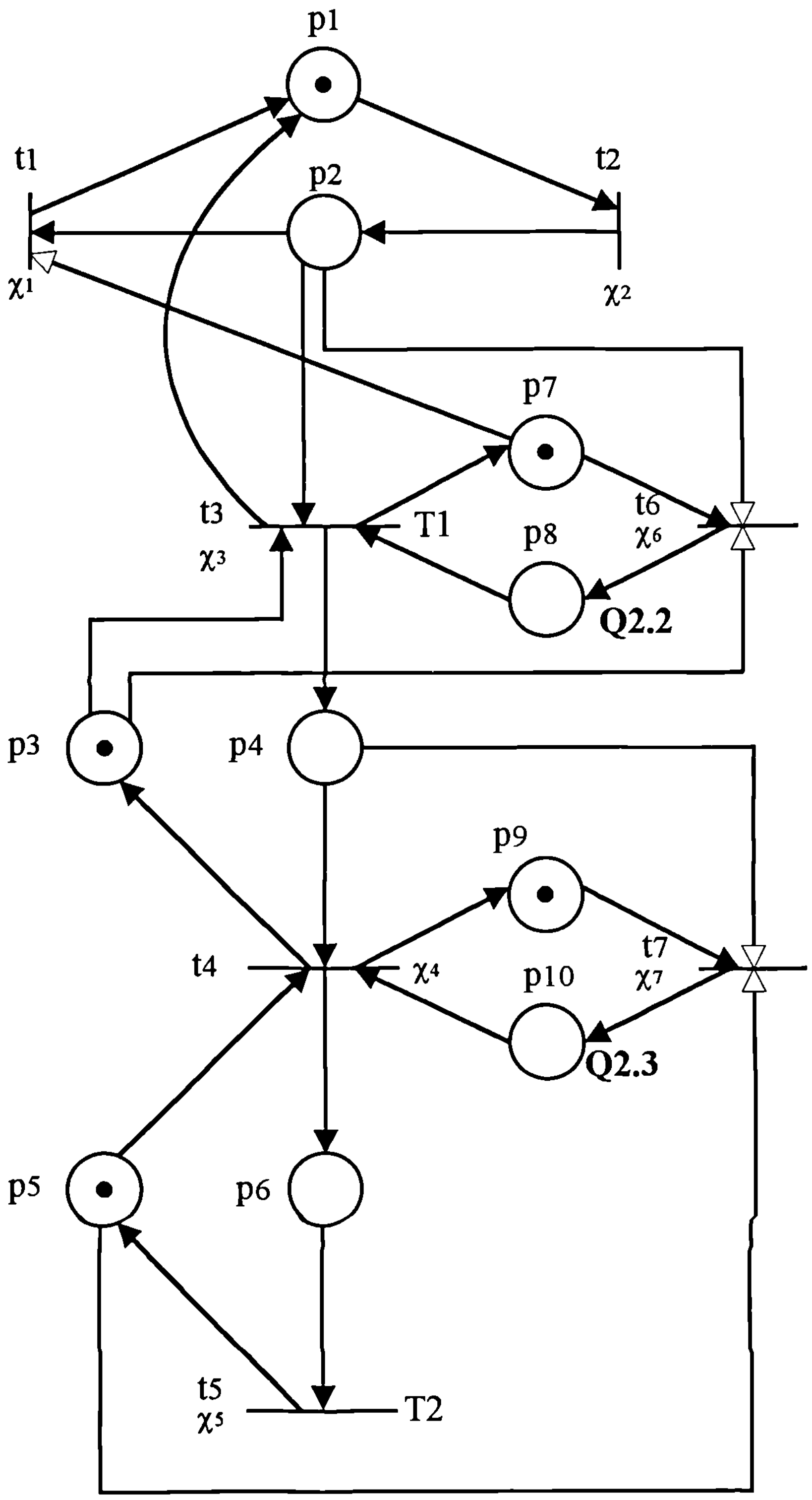


Figure 7.29. The supervisor (controlled model) for the manufacturing system in the C-TPM rule method.

### 7.2.1.6.1.3. Step 4 - Generate the reachability graph (RG) of the controlled model

The reachability graph (RG) of the supervisor (controlled model) is shown in Fig. 7.30, where there are twenty-three arcs, representing the firing of transitions in the controlled model, and there are twelve nodes $M = \{ M_0, M_1, M_2, ..., M_{11} \}$, representing the all possible markings reachable from the initial marking $M_0$. Table 7.10 provides detailed information about the RG nodes. Note that for simplicity reasons only the events, which are associated with the transitions, are shown in the RG. Therefore the events (firing conditions) $\chi = \{ \chi_1, \chi_2, ....., \chi_7 \}$ in the RG represent the firing of corresponding transitions $T = \{ t_1, t_2, ....., t_7 \}$ respectively. It is also important to note that although it is not explicitly written in the RG, time delays 0.7 sec. and 1.5 sec. are associated with the firing of transitions $t_3$ and $t_5$ respectively.

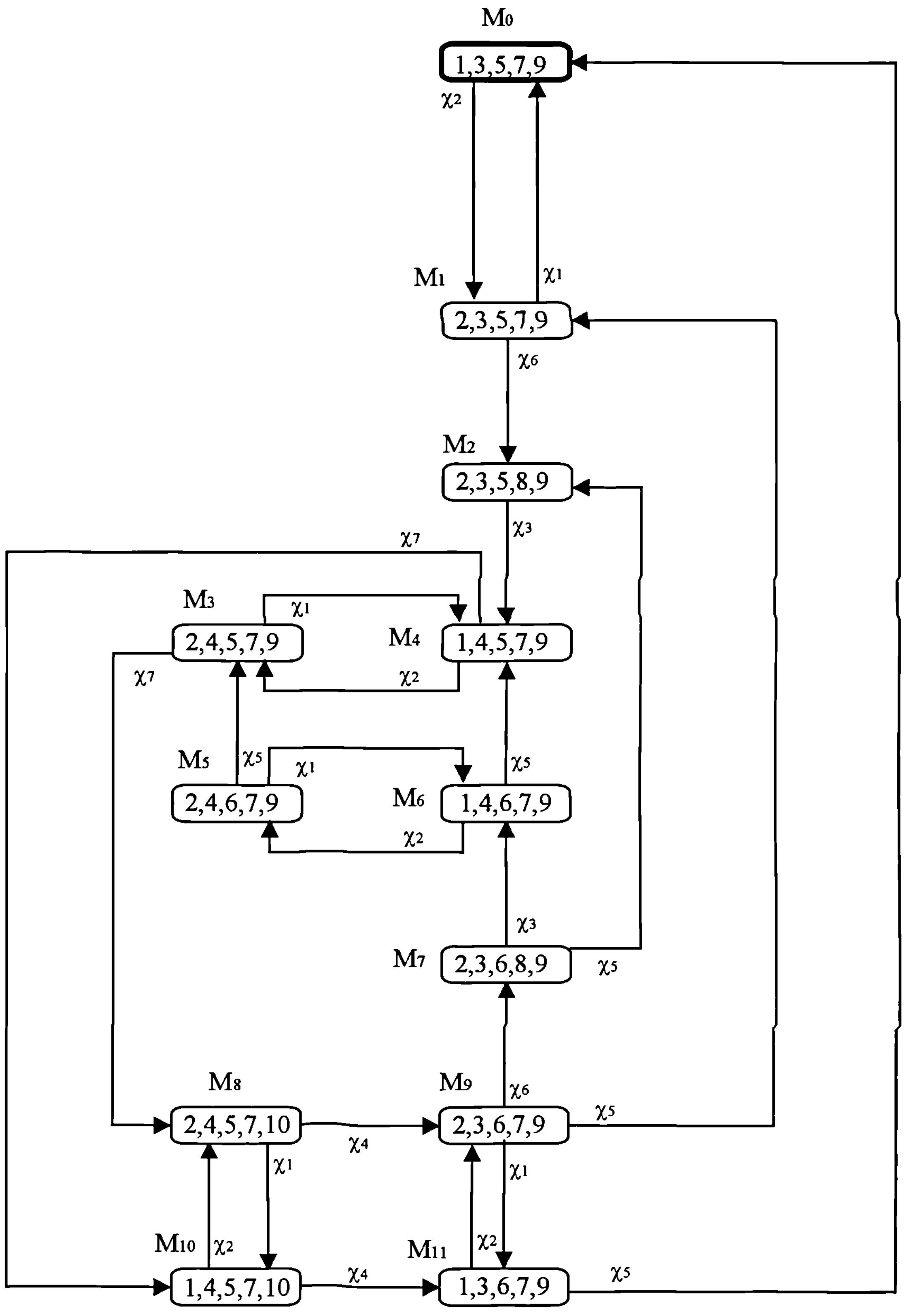


Figure 7.30. The reachability graph (RG) of the controlled APN model, shown in Fig. 7.29.

| Marking | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ | $p_7$ | $p_8$ | $p_9$ | $p_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $M_0 = (1,3,5,7,9)$ | 1 | | 1 | | 1 | | 1 | | 1 | |
| $M_1 = (2,3,5,7,9)$ | | 1 | 1 | | 1 | | 1 | | 1 | |
| $M_2 = (2,3,5,8,9)$ | | 1 | 1 | | 1 | | | 1 | 1 | |
| $M_3 = (2,4,5,7,9)$ | | 1 | | 1 | 1 | | 1 | | 1 | |
| $M_4 = (1,4,5,7,9)$ | 1 | | | 1 | 1 | | 1 | | 1 | |
| $M_5 = (2,4,6,7,9)$ | | 1 | | 1 | | 1 | 1 | | 1 | |
| $M_6 = (1,4,6,7,9)$ | 1 | | | 1 | | 1 | 1 | | 1 | |
| $M_7 = (2,3,6,8,9)$ | | 1 | 1 | | | 1 | | 1 | 1 | |
| $M_8 = (2,4,5,7,10)$ | | 1 | | 1 | 1 | | 1 | | | 1 |
| $M_9 = (2,3,6,7,9)$ | | 1 | 1 | | | 1 | 1 | | 1 | |
| $M_{10} = (1,4,5,7,10)$ | 1 | | | 1 | 1 | | 1 | | | 1 |
| $M_{11} = (1,3,6,7,9)$ | 1 | | 1 | | | 1 | 1 | | 1 | |

Table 7.10. The markings of the reachability graph (RG).

### 7.2.1.6.1.4. Step 5 - Check whether the controlled model behaves according to the specifications: If it does not behave according to the specifications, go to the step 2 and make necessary corrections

As can be seen from the RG, shown in Fig. 7.30, the controlled model (supervisor) behaves according to the specifications given. This is because there are no markings representing a forbidden state in the RG. For this particular example, since the RG shown in Fig. 7.30 is identical with the FRRG obtained in the previous methods, it can be concluded that the supervisor obtained is not only the correct but it is also maximally permissive, i.e., it does not unnecessarily constrain the behaviour of the system.

### 7.2.1.6.1.5. Step 6 - Implement the supervisor (the controlled model) on a PLC as LLDs

In order to convert the supervisor (the controlled model) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code.

As a result to convert the supervisor, given in Fig. 7.29, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2 are assigned to the places P = { $p_1$, $p_2$, ....., $p_{10}$ } of the APN model respectively. The on delay timers T1 with 0.7 sec. time delay and T2 with 1.5 sec. time delay are assigned only to the timed-transitions $t_3$ and $t_5$. After the TPLC is obtained as shown in Fig. 7.31, it is then converted into the LLD code, as shown in Fig. 7.32, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The rungs 1, 2, 3, ....., 9 represent the transitions T = { $t_1$, $t_2$, ...., $t_7$ } of the supervisor. Then, action places $p_8$ and $p_{10}$ are represented by rungs 10 and 11 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 12. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

$\chi_1 = \overline{I0.0}$

$\chi_2 = I0.0 \& \overline{I0.1}$

$\chi_3 = \overline{I0.0}$

$\chi_4 = I0.2$

$\chi_5 = \overline{I0.2}$

$\chi_6 = 1$

$\chi_7 = 1$

T1: 0.7 sec.

T2: 1.5 sec.

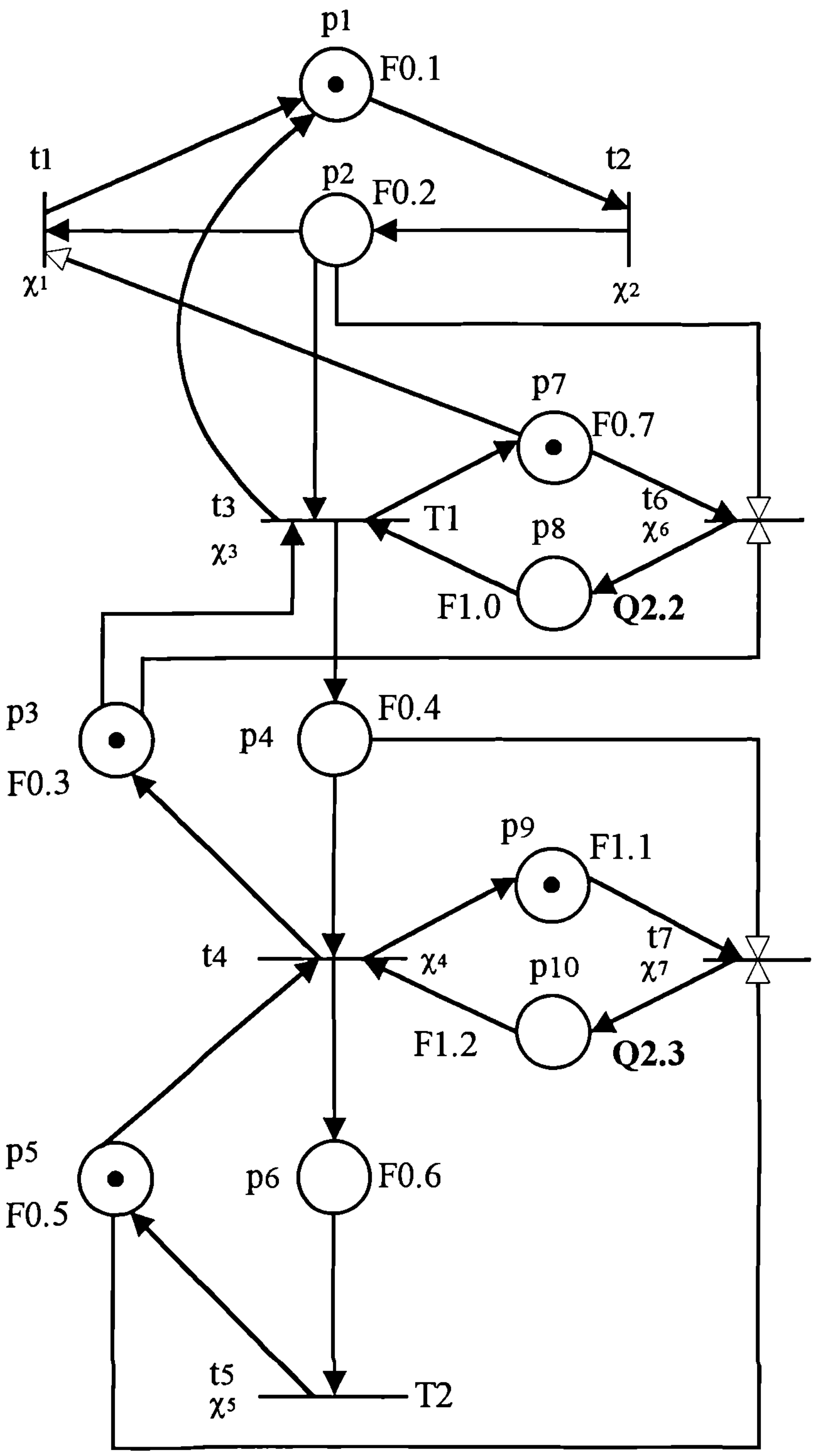


Figure 7.31. The TPLC for the supervisor shown in Fig. 7.29.

```
               | 0  F0.0                                              F0.1   |
initialisation | ------]/[------------------------------------------+---------- S --------|
               |                                                    |    F0.3    |
               |                                                    |---------- S --------|
               |                                                    |    F0.5    |
               |                                                    |---------- S --------|
               |                                                    |    F0.7    |
               |                                                    |---------- S --------|
               |                                                    |    F1.1    |
               |                                                    |---------- S --------|
               |                                                    |    F0.0    |
               |                                                    +---------- S --------|
               | 1  F0.2  F0.7   I0.0                                    F0.2   |
t1             | ------] [-----] [-----]/[--------------------------+---------- R -------|
               |                                                    |    F0.1    |
               |                                                    +---------- S --------|
               | 2  F0.1  I0.0   I0.1                                    F0.1   |
t2             | ------] [-----] [-----]/[--------------------------+---------- R --------|
               |                                                    |    F0.2    |
               |                                                    +---------- S --------|
               |
               | 3  F0.2  F0.3  F1.0  I0.0                        T1: 0.7 sec.  |
t3             | ------] [-----] [-----] [-----]/[------------------------------ SR -------|
               | 4  F0.2  F0.3  F1.0  I0.0     T1                        F0.2   |
t3             | ------] [-----] [-----] [-----]/[-----] [------------+---------- R --------|
               |                                                    |    F0.3    |
               |                                                    |---------- R --------|
               |                                                    |    F1.0    |
               |                                                    |---------- R -------|
               |                                                    |    F0.1    |
               |                                                    |---------- S --------|
               |                                                    |    F0.4    |
               |                                                    |---------- S --------|
               |                                                    |    F0.7    |
               |                                                    +---------- S --------|
               | 5  F0.4  F0.5  F1.2  I0.2                               F0.4   |
t4             | ------] [-----] [-----] [-----] [--------------------+---------- R --------|
               |                                                    |    F0.5    |
               |                                                    |---------- R -------|
               |                                                    |    F1.2    |
               |                                                    |---------- R -------|
               |                                                    |    F0.3    |
               |                                                    |---------- S --------|
               |                                                    |    F0.6    |
               |                                                    |---------- S --------|
               |                                                    |    F1.1    |
               |                                                    +---------- S --------|
               | 6  F0.6  I0.2                                    T2: 1.5 sec.  |
t5             | ------] [-----]/[-------------------------------------------- SR --------|
               | 7   F0.6 I0.2   T2                                      F0.6   |
t5             | ------] [-----]/[-----] [--------------------------+---------- R --------|
               |                                                    |    F0.5    |
               |                                                    +---------- S --------|
              | 8  F0.2 F0.3  F0.7                                  F0.7      |
t6            | ------] [----] [-----] [-----------------------------+---------- R ---------|
              |                                                     |         F1.0      |
              |                                                     +---------- S ---------|
              |                                                                        |
              | 9  F0.4 F0.5  F1.1                                  F1.1      |
t7            | ------] [----] [-----] [-----------------------------+---------- R ---------|
              |                                                     |         F1.2      |
              |                                                     +---------- S ---------|
              | 10  F1.0                                            Q2.2      |
p8            | ------] [-----------------------------------------------------(    )--------|
              | 11  F1.2                                            Q2.3      |
p10           | ------] [-----------------------------------------------------(    )--------|
              | 12  F0.0                                            Q2.0      |
Assumption    | ------] [---------------------------------------------+-------(    )--------|
              |                                                     |       Q2.1      |
              |                                                     +-------(    )--------|
              |                                                                        |
```

Figure 7.32. The LLD for the TPLC, shown in Fig. 7.31.

## SECTION B

## THE DESIRED STRING PROBLEM

## 7.3. THE DESIRED STRING PROBLEM

In this section, the manufacturing system is extended by introducing metallic rings and a metallic sensor in the system. This is required to show how the desired string problem may arise in the case of low level control problems and to show how these problems can be solved by the methods introduced in the Chapter 5. This section (Section B) is organised as follows: firstly the extended manufacturing system is introduced together with a) the reversible deterministic desired string problem, b) the reversible nondeterministic desired string problem. Secondly, since there is a forbidden state problem to solve before solving these two problems, in the section B1 the forbidden state problem is solved. Then, in the section B2, the reversible deterministic desired string problem is solved and finally in the section B3, the reversible nondeterministic desired string problem is solved.

### 7.3.1. Problem Description

The Manufacturing System, shown in Fig. 7.33, represents a multi-component sorting and assembly processes that can be controlled by virtually any PLC. The upper conveyor and the lower conveyor are driven by the upper conveyor motor (Actuator 1) and the lower conveyor motor (Actuator 2) respectively. A random selection of metallic pegs, metallic rings and plastic rings are placed on the upper conveyor. The metallic and plastic components need to be identified. This is achieved by an inductive sensor (Sensor 4). The rings and pegs also need to be identified and separated. This is done by two sensors, a proximity sensor (Sensor 1) and an infra-red reflective sensor (Sensor 2). By using these three sensors a distinction can be made between the metallic pegs, the plastic rings and the metallic rings. By means of the sort solenoid (Actuator 3), rings can be ejected down the assembly chute, which can have up to five rings. Metallic pegs, meanwhile, continue on the upper conveyor and are deflected down the feeder chute. The feeder chute automatically feeds pegs onto the lower conveyor. An infra-red emitter/detector (Sensor 3) is used to determine whether or not the assembly area is empty. If it is, the

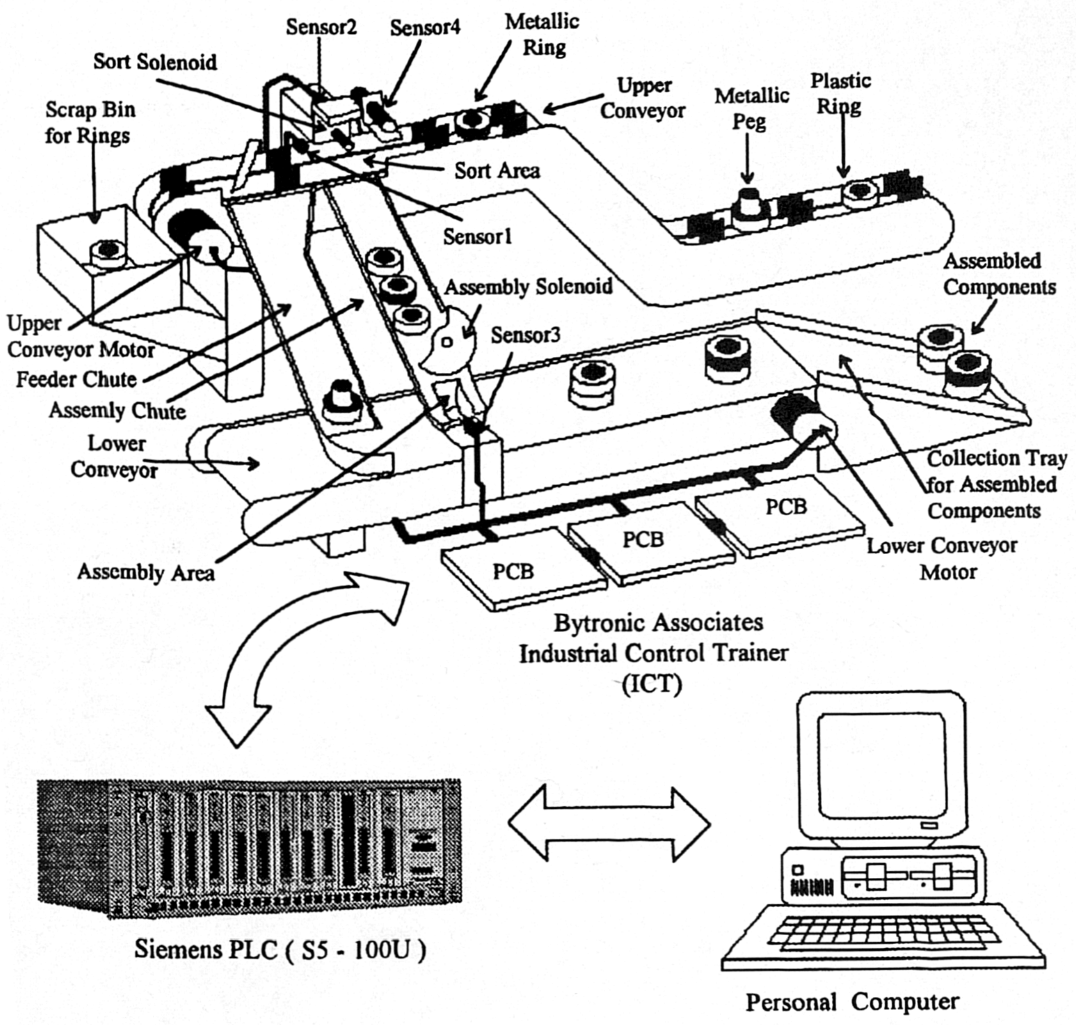


Figure 7.33. Multi-component discrete manufacturing system.

assembly solenoid (Actuator 4) is used to dispense a ring from the assembly chute into the assembly area. The assembly area is positioned just above the lower conveyor and, when a metallic peg passes, the peg engages with the hole in the ring and the two components are assembled. The lower conveyor is used to carry completed components into the collection tray. A Siemens PLC (S5-100U) is used to control the process, and a PC-based package called 'Quadriga' is used to program the PLC. PLC inputs and outputs are given in Table 7.11 and in Table 7.12 respectively.

| PLC Inputs | Sensor No. | Definition |
|---|---|---|
| I0.0 | Sensor 1 | Detects a ring or a peg at the sort area |
| I0.1 | Sensor 2 | Detects a peg at the sort area |
| I0.2 | Sensor 3 | Detects a ring in the assembly area |
| I0.3 | Sensor 4 | Detects metallic components (rings or pegs) on the upper conveyor |

Table 7.11. PLC inputs.

| PLC Outputs | Actuator No. | Definition |
|---|---|---|
| Q2.0 | Actuator 1 | Upper conveyor motor |
| Q2.1 | Actuator 2 | Lower conveyor motor |
| Q2.2 | Actuator 3 | Sort solenoid |
| Q2.3 | Actuator 4 | Assembly solenoid |

Table 7.12. PLC outputs.

For simplification purposes it is assumed that the assembly chute can have up to three rings (plastic or metallic) at a time. It is also assumed that when the system is switched on, both the upper conveyor motor and the lower conveyor motor are switched on automatically.

As the *reversible deterministic desired string problem*, the manufacturing system is required to produce assemblies in a sequence as follows: The first assembly must have a metallic ring and the next one must have a plastic ring and then this process should carry on in this reversible deterministic sequence in a repeating fashion.

As the *reversible <u>nondeterministic</u> desired string problem*, the manufacturing system is required to produce assemblies as follows: Firstly, the system must allow metallic rings to be used for making assemblies as many as it takes until a plastic ring comes along at the sort area. Next, the system must use exactly the same number of plastic rings as the metallic rings that have been used previously. Then this process should carry on starting with the next set of metallic rings and so on. Note that in the beginning if there are no metallic rings appearing at the sort area, then none of the plastic rings appearing at the sort area is allowed to be used. In simple terms if the system uses, say, three metallic rings, that appear at the sort area one after another to make assemblies and if a plastic ring appears at the sort area, then the system must be obliged to use three plastic rings in total before using another set of metallic rings and plastic rings consequently. If the system uses, say, 25 metallic rings that appear at the sort area one after another and if a plastic ring appears at the sort area, then the system must be obliged to use 25 plastic rings in total before using another set of metallic rings and plastic rings consequently. This process should carry on in this reversible nondeterministic fashion.

Before solving these two desired string problems, it is necessary to solve the forbidden state problem related to the manufacturing system, in order to obtain an untreated APN model to be used as a basis for solving the desired string problems. This is because the system has a forbidden state problem as well as the desired string problems. According to the methodologies introduced in the Chapter 5, before solving the desired string problems, the forbidden state problem, if there is any, must be solved. In this case, the forbidden state specifications are as follows:

1. Operate the sort solenoid only when there is space in the assembly chute, which is allowed to have up to three rings at a time, and there is a ring (metallic or plastic) at the sort area.

2. Operate the assembly solenoid only when there is space at the assembly area and there is a ring (metallic or plastic) in the assembly chute.

## SECTION B1

## THE C-TPM RULE METHOD FOR SOLVING THE FORBIDDEN STATE PROBLEM

### 7.3.2. The C-TPM Rule Method for Solving the Forbidden State Problem

In order to solve the forbidden state problem for the manufacturing system, the C-TPM rule method is used, because it provides the simplest possible supervisor for the forbidden state problems among the six methods introduced in the Chapter 3 and 4. Note that the supervisor obtained in this section becomes the untreated model for the desired string problems.

#### 7.3.2.1. Synthesis of Supervisory Controller

Recall that the synthesis of supervisory controller in the C-TPM rule method is divided into following steps:

Step 1- Design the uncontrolled model of the system using APNs

Step 2- Convert the forbidden state specifications, given, into related TPM rules

Step 3- Construct the controlled model of the system by combining the uncontrolled model and the TPM rules

Step 4- Generate the reachability graph (RG) of the controlled model

Step 5- Check whether the controlled model behaves according to the specifications: If it does not behave according to the specifications, go to the step 2 and make necessary corrections

Step 6- Implement the supervisor (the controlled model) on a PLC as LLDs

##### 7.3.2.1.1. Step 1 - *Design the uncontrolled model of system using APNs*

By using the place invariant method, i.e., by introducing monitor places to define the token capacity of the physical places, and the concurrent composition, i.e., by merging the transitions with the same events, the uncontrolled model is obtained as an APN as shown in Fig. 7.34, where there are twenty-two places, $P = \{ p_1, p_2, ..., p_{22} \}$ and nineteen transitions $T = \{ t_1, t_2, ...., t_{19} \}$, with which the firing conditions, $\chi = \{ \chi_1, \chi_2, ....., \chi_{19} \}$ are associated respectively. Note that when the firing condition of a transition is 1, it is not shown in the uncontrolled APN model. Places $p_3$, $p_6$, $p_8$, $p_{11}$, $p_{14}$ and $p_{17}$ are monitor places, as defined in the place invariant method, and they represent the capacity of the physical places. Transitions $t_8$, $t_9$, $t_{14}$ and $t_{15}$ are obtained by using the concurrent composition. In the uncontrolled APN model transitions $t_8$ and $t_9$ are timed-transitions with 0.7 sec. time delay and similarly transitions $t_{16}$ and $t_{17}$ are timed-transitions with 1.5 sec. time delay. Note that actions Q2.2 and Q2.3 are assigned to places $p_{19}$ and $p_{21}$ respectively. They represent the sort solenoid and the assembly solenoid operations respectively. The initial marking of the uncontrolled model is $M_0 = ( 0, 0, 1, 0, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 1 )^T$ or simply $M_0 = ( 3, 6, 8, 11, 14, 17, 20, 22 )$. This means that initially, there are no rings (plastic or metallic) in the manufacturing system and both the sort solenoid and the assembly solenoid are *off*. Note that transitions $t_{18}$ and $t_{19}$ are controllable transitions. In fact the objective in this case is to come up with a supervisor to decide when to fire the controllable transitions $t_{18}$ and $t_{19}$ such that the forbidden state specifications are met. Note that the uncontrolled APN model, shown in Fig. 7.34 is safe, i.e., 1-bounded, live, reversible, and conservative.

Places $p_{19}$ and $p_{20}$ represent the *on* and *off* states of the sort solenoid respectively. Likewise, places $p_{21}$ and $p_{22}$ represent the *on* and *off* states of the assembly solenoid. A token in places $p_3$, $p_6$, $p_8$, $p_{11}$, $p_{14}$ and $p_{17}$ represent the available spaces at the sort area, in front of the metallic sensor, in the 3rd place of the assembly chute, in the 2nd place of the assembly chute, in the 1st place of the assembly chute and in the assembly area respectively. A token in places $p_1$ depicts the presence of a metallic peg at the sort area, while a token in places $p_2$ shows the presence of a metallic ring at the sort area and a token in places $p_4$ shows the presence of a plastic ring at the sort area. A token in places $p_7$, $p_{10}$ and $p_{13}$ represents a metallic ring in the 3rd, 2nd and 1st place of the assembly chute respectively. Similarly, places $p_9$, $p_{12}$ and $p_{15}$ represents a plastic ring in 3rd, 2nd and 1st place in the assembly chute respectively. A token in place $p_{16}$ represents a metallic ring in the assembly area, while a token in place $p_{18}$ represents a plastic ring in the assembly area. Initially, both solenoids are *off* and there are no rings in the manufacturing system. Note that the meaning of the places used in the uncontrolled APN model is provided in Table 7.13.

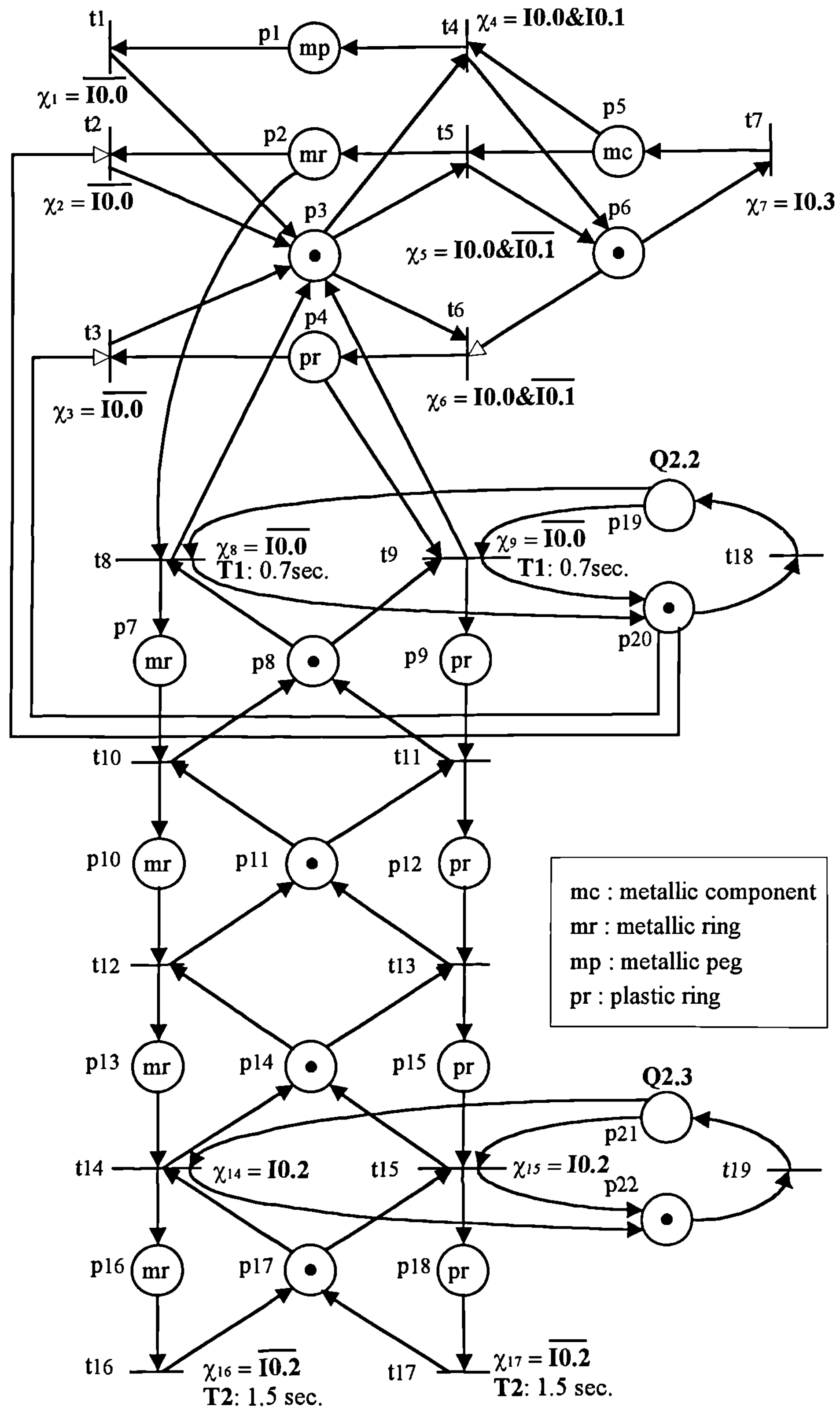

t1
p1
mp
t4
$\chi_4 = \text{I0.0\&I0.1}$
$\chi_1 = \overline{\text{I0.0}}$
t2
p2
mr
t5
p5
mc
t7
$\chi_2 = \overline{\text{I0.0}}$
p3
$\chi_5 = \text{I0.0\&}\overline{\text{I0.1}}$
p6
$\chi_7 = \text{I0.3}$
p4
t6
t3
pr
$\chi_3 = \overline{\text{I0.0}}$
$\chi_6 = \text{I0.0\&}\overline{\text{I0.1}}$
Q2.2
p19
t8
$\chi_8 = \overline{\text{I0.0}}$
T1: 0.7sec.
t9
$\chi_9 = \overline{\text{I0.0}}$
T1: 0.7sec.
t18
p7
mr
p8
p9
pr
p20
t10
t11
p10
mr
p11
p12
pr
mc : metallic component
mr : metallic ring
mp : metallic peg
pr : plastic ring
t12
t13
p13
mr
p14
p15
pr
Q2.3
p21
t14
$\chi_{14} = \text{I0.2}$
t15
$\chi_{15} = \text{I0.2}$
p22
t19
p16
mr
p17
p18
pr
t16
$\chi_{16} = \overline{\text{I0.2}}$
T2: 1.5 sec.
t17
$\chi_{17} = \overline{\text{I0.2}}$
T2: 1.5 sec.

Figure 7.34. The uncontrolled model of the manufacturing system as an APN.

| Places | Interpretation |
|---|---|
| $p_1$ | A metallic peg (mp) at the sort area |
| $p_2$ | A metallic ring (mr) at the sort area |
| $p_3$ | Component ( mp, mr or pr) capacity of the sort area |
| $p_4$ | A plastic ring (pr) at the sort area |
| $p_5$ | A metallic component (either a peg or a ring) in front of the metallic sensor |
| $p_6$ | Component (mp or mr) capacity in front of the metallic sensor |
| $p_7$ | A metallic ring (mr) in the 3rd place of the assembly chute |
| $p_8$ | Ring (mr or pr) capacity in the 3rd place of the assembly chute |
| $p_9$ | A plastic ring (pr) in the 3rd place of the assembly chute |
| $p_{10}$ | A metallic ring (mr) in the 2nd place of the assembly chute |
| $p_{11}$ | Ring (mr or pr) capacity in the 2nd place of the assembly chute |
| $p_{12}$ | A plastic ring (pr) in the 2nd place of the assembly chute |
| $p_{13}$ | A metallic ring (mr) in the 1st place of the assembly chute |
| $p_{14}$ | Ring (mr or pr) capacity in the 1st place of the assembly chute |
| $p_{15}$ | A plastic ring (pr) in the 1st place of the assembly chute |
| $p_{16}$ | A metallic ring (mr) in the assembly area |
| $p_{17}$ | Ring (mr or pr) capacity in the assembly area |
| $p_{18}$ | A plastic ring (pr) in the assembly area |
| $p_{19}$ | *On* state of the sort solenoid |
| $p_{20}$ | *Off* state of the sort solenoid |
| $p_{21}$ | *On* state of the assembly solenoid |
| $p_{22}$ | *Off* state of the assembly solenoid |

Table 7.13. The meaning of the places in the uncontrolled APN model.

When the presence of a metallic component (ring or peg) is detected, i.e., $\chi_7$ = I0.3, in front of the metallic sensor and there is not any metallic component, i.e., $M(p_6) = 1$, transition $t_7$ fires by removing a token from place $p_6$ and by depositing a token into place $p_5$. This means that there is a metallic component (metallic peg or metallic ring) in front of the metallic sensor, i.e., $M(p_5) = 1$. When there is a metallic component in front of the metallic sensor and there is space at the sort area, i.e., $M(p_3) = 1$, if the presence of a metallic peg is detected, i. e., $\chi_4$ = I0.0 & I0.1, then transition $t_4$ fires by removing a token each from places $p_3$ and $p_5$ and by depositing a token each into places $p_1$ and $p_6$. This means that the metallic component, i.e., $M(p_5) = 1$, is identified as a metallic peg and it is now at the sort area, i.e., $M(p_1) = 1$. Since no action is taken in the presence of a metallic peg at the sort area, the metallic peg (mp) clears the sort area through transition

$t_1$ with $\chi_1$ = I0.0. When $t_1$ fires, it removes a token from place $p_1$ and deposits a token into place $p_3$.

When there is a metallic component in front of the metallic sensor, i.e., $M(p_5) = 1$, and there is space at the sort area, i.e., $M(p_3) = 1$, if the presence of a metallic ring is detected, i.e., $\chi_5 = \mathrm{I0.0}\ \&\ \overline{\mathrm{I0.1}}$, then transition $t_5$ fires by removing a token each from places $p_3$ and $p_5$ and by depositing a token each into places $p_2$ and $p_6$. This means that the metallic component, i.e., M ($p_5$) = 1, is identified as a metallic ring and it is now at the sort area, i.e., $M(p_2) = 1$. When there is a metallic ring at the sort area, i.e., $M(p_2) = 1$, either it clears the sort area through transition $t_2$ or it is ejected into the assembly chute through transition $t_8$. When there is a metallic ring (mr) at the sort area, i.e., $M(p_2) = 1$ and the sort solenoid is *off*, i.e., $M(p_{20}) = 1$, if the absence of a ring is detected, i.e., $\chi_2 = \overline{\mathrm{I0.0}}$, then transition $t_2$ fires by *removing a token from place* $p_2$ *and by depositing a token* into place $p_3$. This means that the metallic ring cleared the sort area. When there is a metallic ring at the sort area, i.e., $M(p_2) = 1$, the sort solenoid is *on*, i.e., $M(p_{19}) = 1$ and there is space in the assembly chute, i.e., $M(p_8) = 1$, if the absence of a ring is detected, i.e., $\chi_8 = \overline{\mathrm{I0.0}}$, then the timed transition $t_8$ is being fired for 0.7 sec., after which a token each is removed from places $p_2$, $p_8$ and $p_{19}$ and a token each is deposited into places $p_3$, $p_7$ and $p_{20}$. This means that the metallic ring is ejected into the assembly chute by clearing the sort area and by occupying the 3rd place of the assembly chute. Note that after transition $t_8$ fires the sort solenoid is switched *off*, i.e., $M(p_{20}) = 1$.

When the sort area is empty, i.e., $M(p_3) = 1$ and there is not any metallic components, i.e., $M(p_6) = 1$, if the presence of a ring is detected, $\chi_8 = \mathrm{I0.0}\ \&\ \overline{\mathrm{I0.1}}$, then transition $t_6$ fires by removing a token from place $p_3$ and by depositing a token into place $p_4$. This means that there is a plastic ring (pr) at the sort area, i.e., $M(p_4) = 1$. When there is a plastic ring at the sort area, i.e., $M(p_4) = 1$, either it clears the sort area through transition $t_3$ or it is ejected into the assembly chute through transition $t_9$. When there is a plastic ring at the sort area, i.e., $M(p_4) = 1$ and the sort solenoid is *off*, i.e., $M(p_{20}) = 1$, if the absence of a ring is detected, i.e., $\chi_3 = \overline{I0.0}$, then transition $t_3$ fires by removing a token from place $p_4$ and by depositing a token into place $p_3$. This means that the plastic ring cleared the sort area. When there is a plastic ring at the sort area, i.e., $M(p_4) = 1$, the sort solenoid is on, i.e., $M(p_{19}) = 1$, and there is space in the assembly chute, i.e., $M(p_8) = 1$, if the absence of a ring is detected, i.e., $\chi_9 = \overline{I0.0}$, then the timed transition $t_9$ is being fired for 0.7 sec., after which a token each is removed from places $p_4$, $p_8$ and $p_{19}$ and a token each is deposited into places $p_3$, $p_9$ and $p_{20}$. This means that the plastic ring is ejected from the sort area into the assembly chute, by clearing the sort area and by occupying the 3rd place of the assembly chute. Note that after transition $t_9$ fires the sort solenoid is switched *off*, i.e., $M(p_{20}) = 1$.

When there is a metallic ring in the 3rd place of the assembly chute, i.e., $M(p_7) = 1$, and there is not any rings in the 2nd place of the assembly chute, i.e., $M(p_{11}) = 1$, transition $t_{10}$ fires by removing a token each from places $p_7$ and $p_{11}$ and by depositing a token each into places $p_8$ and $p_{10}$. This means that the metallic ring slides one place below from the 3rd place to the 2nd place in the assembly chute. When there is a metallic ring in the 2nd place of the assembly chute, i.e., $M(p_{10}) = 1$, and there is not any rings in the 1st place of the assembly chute, i.e., $M(p_{14}) = 1$, transition $t_{12}$ fires by removing a token each from places $p_{10}$ and $p_{14}$ and by depositing a token each into places $p_{11}$ and $p_{13}$. This means that the metallic ring slides one place below from the 2nd place to the 1st place in the assembly chute. The same applies to plastic rings in a similar way as follows: When there is a plastic ring in the 3rd place of the assembly chute, i.e., $M(p_9) = 1$, and there is not any rings in the 2nd place of the assembly chute, i.e., $M(p_{11}) = 1$, transition $t_{11}$ fires by removing a token each from places $p_9$ and $p_{11}$ and by depositing a token each into places $p_8$ and $p_{12}$. This means that the plastic ring slides one place below from the 3rd place to the 2nd place in the assembly chute. When there is a plastic ring in the 2nd place of the assembly chute, i.e., $M(p_{12}) = 1$, and there is not any rings in the 1st place of the assembly chute, i.e., $M(p_{14}) = 1$, transition $t_{13}$ fires by removing a token each from places $p_{12}$ and $p_{14}$ and by depositing a token each into places $p_{11}$ and $p_{15}$. This means that the plastic ring slides one place below from the 2nd place to the 1st place in the assembly chute.

When there is a metallic ring in the 1st place of the assembly chute, i.e., $M(p_{13}) = 1$, there is space at the assembly area, i.e., $M(p_{17}) = 1$ and the assembly solenoid is *on*, i.e., $M(p_{21}) = 1$, if the presence of a ring is detected at the assembly area, i.e., $\chi_{14} = I0.2$, then transition $t_{14}$ fires by removing a token each from places $p_{13}$, $p_{17}$ and $p_{21}$ and by depositing a token each into places $p_{14}$, $p_{16}$ and $p_{22}$. This means that the metallic ring is put into the assembly area from the 1st place of the assembly chute. At the same time the assembly solenoid is switched *off*. The same applies to the plastic ring in the 1st place of the assembly chute, in a similar way as follows: When there is a plastic ring in the 1st place of the assembly chute, i.e., $M(p_{15}) = 1$, there is space at the assembly area, i.e., $M(p_{17}) = 1$ and the assembly solenoid is *on*, i.e., $M(p_{21}) = 1$, if the presence of a ring is detected at the assembly area, i.e., $\chi_{15} = I0.2$, then transition $t_{15}$ fires by removing a token each from places $p_{15}$, $p_{17}$ and $p_{21}$ and by depositing a token each into places $p_{14}$, $p_{18}$ and $p_{22}$. This means that the plastic ring is put into the assembly area from the 1st place of the assembly chute. At the same time the assembly solenoid is switched *off*.

If there is a metallic ring at the assembly area, i.e., $M(p_{16}) = 1$, and a peg engages with the hole in the ring, $\chi_{16} = I0.2$, then it takes 1.5 sec. for the metallic ring and the peg to be assembled and to clear the assembly area. Similarly if there is a plastic ring at the assembly area, i.e., $M(p_{18}) = 1$, and a peg engages with the hole in the ring, $\chi_{17} = I0.2$, then it takes 1.5 sec. for the plastic ring and the peg to be assembled and to clear the assembly area. After an assembly is being made, the assembly area becomes empty, i.e., $M(p_{17}) = 1$.

### 7.3.2.1.2. Step 2 - Convert the forbidden state specifications into related TPM rules

The forbidden state specifications are as follows:

1. Operate the sort solenoid only when there is space in the assembly chute and there is a ring (metallic or plastic) at the sort area.
2. Operate the assembly solenoid only when there is space at the assembly area and there is a ring (metallic or plastic) in the assembly chute.

These specifications can be re-stated in the ' if ... then ... ' rule format as follows:

```
1.    if     <there is a metallic ring OR a plastic ring at the sort area>
             AND <there is space in the assembly chute>
    then     <operate the sort solenoid>

2.    if     <there is a metallic ring OR a plastic ring in the assembly chute>
             AND <the assembly area is empty>
    then     <operate the assembly solenoid>
```

Note that these specifications can be re-written by separating the metallic ring and the plastic ring parts one from another as follows:

```
1.i).  if    <there is a metallic ring at the sort area>
             AND <there is space in the assembly chute>
    then     <operate the sort solenoid>
      OR
  ii). if    <there is a plastic ring at the sort area>
             AND <there is space in the assembly chute>
    then     <operate the sort solenoid>

2.i).  if    <there is a metallic ring in the assembly chute>
             AND <the assembly area is empty>
    then     <operate the assembly solenoid>
      OR
  ii). if    <there is a plastic ring in the assembly chute>
             AND <the assembly area is empty>
    then     <operate the assembly solenoid>
```

These forbidden state specifications can be represented in terms of markings of the uncontrolled APN model. By simply replacing statements of the above rules with the markings, the following TPM rules can be obtained:

1.i) *if* <$M(p_2) = 1$> AND <$M(p_8) = 1$>
*then* <transition $t_{18}$ is to be enabled>
*OR*
ii) *if* <$M(p_4) = 1$> AND <$M(p_8) = 1$>
*then* <transition $t_{18}$ is to be enabled>

2.i) *if* <$M(p_{13}) = 1$> AND <$M(p_{17}) = 1$>
*then* <transition $t_{19}$ is to be enabled>
*OR*
ii) *if* <$M(p_{15}) = 1$> AND <$M(p_{17}) = 1$>
*then* <transition $t_{19}$ is to be enabled>

### 7.3.2.1.3. Step 3 - Construct the controlled model of the system by combining the uncontrolled model and the TPM rules

The controlled model (i.e., the supervisor), shown in Fig. 7.35, is obtained by using the uncontrolled APN model, shown in Fig. 7.34 and the TPM rules obtained. Since the rule 1 is split into two parts and contains an *or* operation, transition $t_{18}$ of the uncontrolled APN model is duplicated to accommodate the *or* operation within the Petri net formalism and replaced with transitions $t_{18}$ and $t_{19}$ within the controlled APN model. Similarly, since the rule 2 is split into two parts and contains an *or* operation, transition $t_{19}$ of the uncontrolled APN model is duplicated to accommodate the *or* operation within the Petri net formalism and replaced with transitions $t_{20}$ and $t_{21}$ within the controlled APN model. Therefore the APM rules are modified as follows:

1.i) *if* <$M(p_2) = 1$> AND <$M(p_8) = 1$>
*then* <transition $t_{18}$ is to be enabled>
ii) *if* <$M(p_4) = 1$> AND <$M(p_8) = 1$>
*then* <transition $t_{19}$ is to be enabled>

2.i) *if* <$M(p_{13}) = 1$> AND <$M(p_{17}) = 1$>
*then* <transition $t_{21}$ is to be enabled>
ii) *if* <$M(p_{15}) = 1$> AND <$M(p_{17}) = 1$>
*then* <transition $t_{20}$ is to be enabled>

In order to implement the TPM rule 1.i, the enabling arcs $En(p_2, t_{18})$ and $En(p_8, t_{18})$ are connected from places $p_2$ and $p_8$ to the controllable transition $t_{18}$. To implement the TPM rule 1.ii, the enabling arcs $En(p_4, t_{19})$ and $En(p_8, t_{19})$ are connected from places $p_4$ and $p_8$ to the controllable transition $t_{19}$. Similarly, to implement the TPM rule 2.i, the enabling arcs $En(p_{13}, t_{21})$ and $En(p_{17}, t_{21})$ are connected from places $p_{13}$ and $p_{17}$ to the controllable transition $t_{21}$. To implement the TPM rule 2.ii, the enabling arcs $En(p_{15}, t_{20})$ and $En(p_{17}, t_{20})$ are connected from places $p_{15}$ and $p_{17}$ to the controllable transition $t_{20}$. This yields the supervisor (the controlled model) for the manufacturing system.

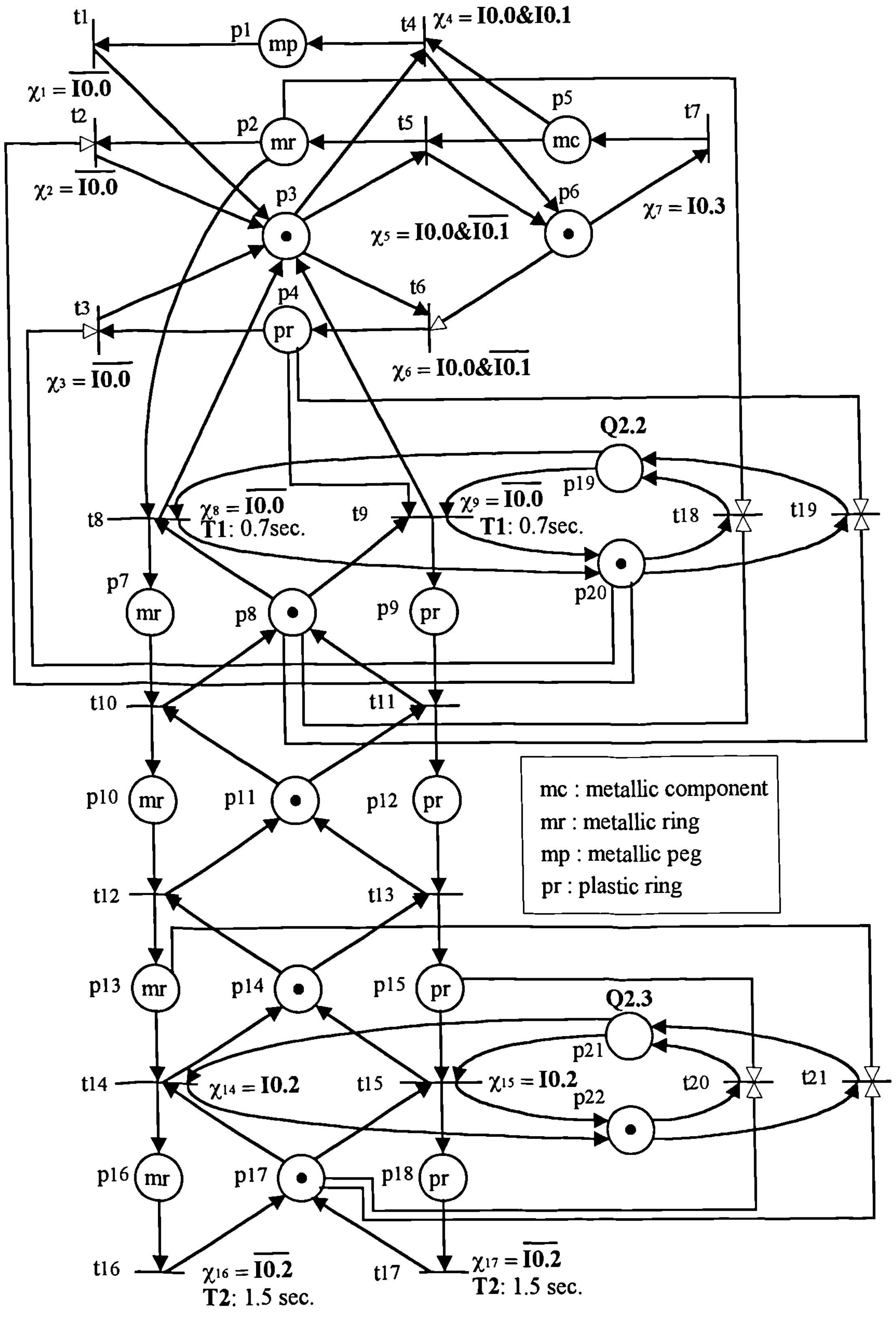


Figure 7.35. The supervisor (controlled model) for the manufacturing system.

#### 7.3.2.1.4. Step 4 - Generate the reachability graph (RG) of the controlled model

The reachability graph (RG) of the supervisor (controlled model) is given in the Appendix A, there are 924 nodes M = { $M_0$, $M_1$, $M_2$, ..., $M_{923}$ }, representing the all possible markings reachable from the initial marking $M_0 = (3, 6, 8, 11, 14, 17, 20, 22)$. Note that in order to obtain this RG, a software package, called ARP2-4.exe (Maziero, 1995), is used. Therefore the RG and the reachable markings are given as they are produced as outputs from the mentioned program. It is also important to note that although it is not explicitly written in the RG, 0.7 sec. time delay is associated with timed transitions $t_8$ and $t_9$ and similarly, 1.5 sec. time delay is associated with timed transitions $t_{16}$ and $t_{17}$.

#### 7.3.2.1.5. Step 5 - Check whether the controlled model behaves according to the specifications: If it does not behave according to the specifications, go to the step 2 and make necessary corrections

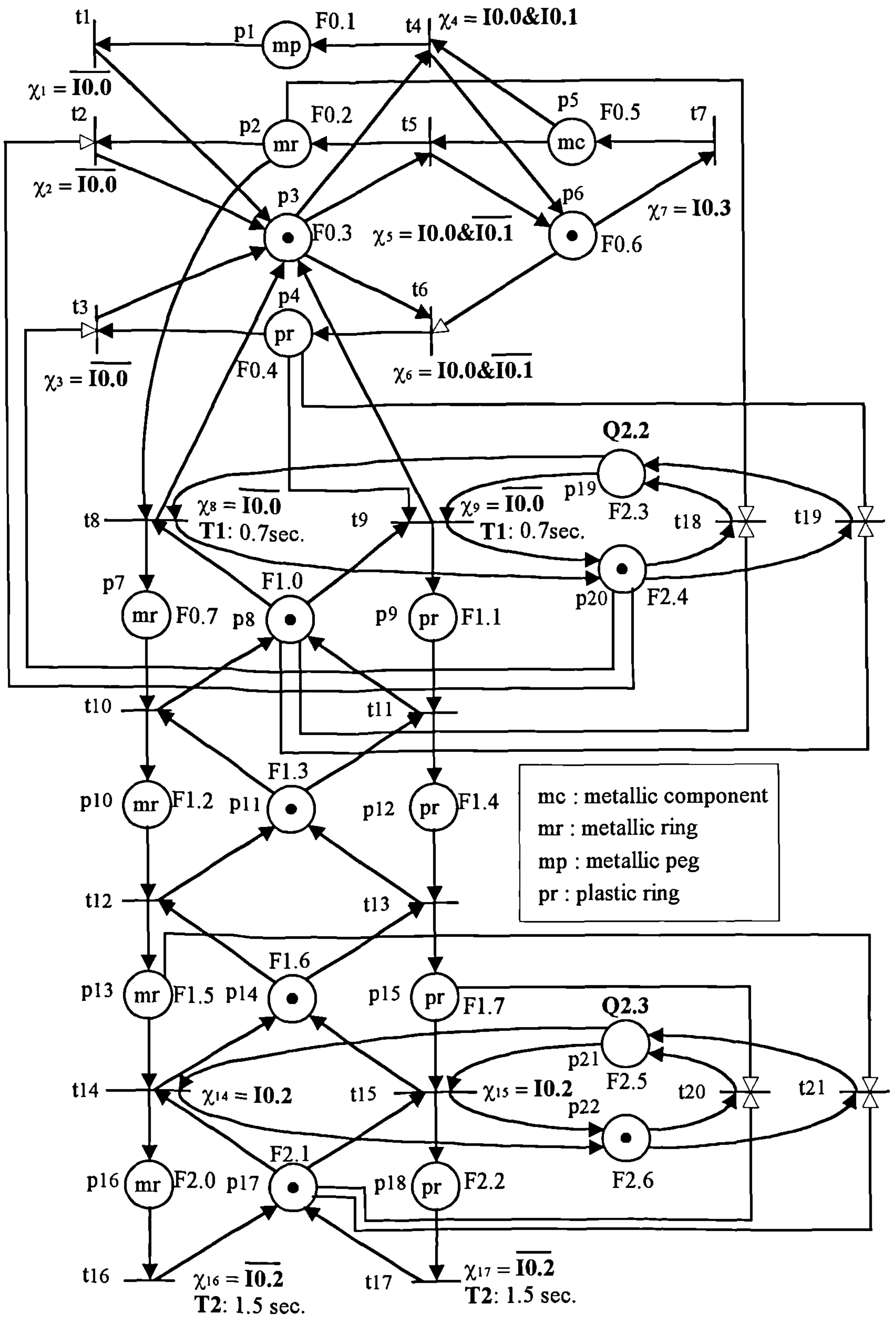
It can be seen from the RG, given in the Appendix A, that the controlled model (supervisor) behaves according to the specifications given. This is because there are no markings representing a forbidden state in the RG. In other words, all the markings that appear in the RG represent admissible markings, that do not contradict the forbidden state specifications given.

#### 7.3.2.1.6. Step 6 - Implement the supervisor (the controlled model) on a PLC as LLDs

In order to convert the supervisor (the controlled model) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code.

As a result to convert the supervisor, given in Fig. 7.35, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2, F1.3, F1.4, F1.5, F1.6, F1.7, F2.0, F2.1, F2.2, F2.3, F2.4, F2.5 and F2.6 are assigned to the places $P = \{ p_1, p_2, ....., p_{22} \}$ of the controlled APN model respectively. The on delay timer T1 with 0.7 sec. time delay is assigned to the timed transitions $t_8$ and $t_9$ and the on delay timer T2 with 1.5 sec. time delay is assigned to the timed transitions $t_{16}$ and $t_{17}$. After the TPLC is obtained as shown in Fig. 7.36, it is then converted into the LLD code, as shown in Fig. 7.37, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.4.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The rungs from 1 to 21 represent the transitions $T = \{ t_1, t_2, ...., t_{21} \}$ of the supervisor. The timing requirement for the timed transitions $t_8$ and $t_9$ is represented by the rung 22 and similarly, the timing requirement for the timed transitions $t_{16}$ and $t_{17}$ is represented by the rung 23. Then, action places $p_{19}$ and $p_{21}$ are represented by rungs 24 and 25 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 26. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.



| | |
|---|---|
| mc | : metallic component |
| mr | : metallic ring |
| mp | : metallic peg |
| pr | : plastic ring |

Figure 7.36. The TPLC for the supervisor shown in Fig. 7.35.

```
                | 0 F0.0                                          F0.3      |
initialisation  |------]/[------------------------------------+---------- S --------|
                |                                             |          F0.6     |
                |                                             |---------- S --------|
                |                                             |          F1.0     |
                |                                             |---------- S --------|
                |                                             |          F1.3     |
                |                                             |---------- S --------|
                |                                             |          F1.6     |
                |                                             |---------- S --------|
                |                                             |          F2.1     |
                |                                             |---------- S --------|
                |                                             |          F2.4     |
                |                                             |---------- S --------|
                |                                             |          F2.6     |
                |                                             |---------- S --------|
                |                                             |          F0.0     |
                |                                             L---------- S --------|
                | 1  F0.1 I0.0                                           F0.1     |
      t1        |------] [-----]/[----------------------------+---------- R --------|
                |                                             |          F0.3     |
                |                                             L---------- S --------|
                | 2  F0.2 F2.4  I0.0                                     F0.2     |
      t2        |------] [-----] [-----]/[---------------------+---------- R --------|
                |                                             |          F0.3     |
                |                                             L---------- S --------|
                | 3  F0.4 F2.4  I0.0                                     F0.4     |
      t3        |------] [-----] [-----]/[---------------------+---------- R --------|
                |                                             |          F0.3     |
                |                                             L---------- S --------|
                | 4  F0.3  F0.5  I0.0  I0.1                              F0.3     |
      t4        |------] [-----] [-----] [-----] [-------------+---------- R --------|
                |                                             |          F0.5     |
                |                                             |---------- R --------|
                |                                             |          F0.1     |
                |                                             |---------- S --------|
                |                                             |          F0.6     |
                |                                             L---------- S --------|
                | 5  F0.3  F0.5  I0.0  I0.1                              F0.3     |
      t5        |------] [-----] [-----] [-----]/[-------------+---------- R --------|
                |                                             |          F0.5     |
                |                                             |---------- R --------|
                |                                             |          F0.2     |
                |                                             |---------- S --------|
                |                                             |          F0.6     |
                |                                             L---------- S --------|
                | 6  F0.3  F0.6  I0.0  I0.1                              F0.3     |
      t6        |------] [-----] [-----] [-----]/[-------------+---------- R --------|
                |                                             |          F0.4     |
                |                                             L---------- S --------|
                |                                                                 |
     | 7  F0.6  I0.3                                          F0.6     |
t7   |------] [-----] [----------------------------------+----------- R ---------|
     |                                                   |        F0.5     |
     |                                                   +----------- S ---------|
     | 8  F0.2  F1.0  F2.3  I0.0   T1                          F0.2     |
t8   |------] [-----] [-----] [-----]/[-----] [----------+----------- R ---------|
     |                                                   |        F1.0     |
     |                                                   |----------- R ---------|
     |                                                   |        F2.3     |
     |                                                   |----------- R ---------|
     |                                                   |        F0.3     |
     |                                                   |----------- S ---------|
     |                                                   |        F0.7     |
     |                                                   |----------- S ---------|
     |                                                   |        F2.4     |
     |                                                   +----------- S ---------|
     | 9  F0.4  F1.0  F2.3  I0.0   T1                          F0.4     |
t9   |------] [-----] [-----] [-----]/[-----] [----------+----------- R ---------|
     |                                                   |        F1.0     |
     |                                                   |----------- R ---------|
     |                                                   |        F2.3     |
     |                                                   |----------- R ---------|
     |                                                   |        F0.3     |
     |                                                   |----------- S ---------|
     |                                                   |        F1.1     |
     |                                                   |----------- S ---------|
     |                                                   |        F2.4     |
     |                                                   +----------- S ---------|
     | 10 F0.7  F1.3                                          F0.7     |
t10  |------] [-----] [----------------------------------+----------- R ---------|
     |                                                   |        F1.3     |
     |                                                   |----------- R ---------|
     |                                                   |        F1.0     |
     |                                                   |----------- S ---------|
     |                                                   |        F1.2     |
     |                                                   +----------- S ---------|
     | 11 F1.1  F1.3                                          F1.1     |
t11  |------] [-----] [----------------------------------+----------- R ---------|
     |                                                   |        F1.3     |
     |                                                   |----------- R ---------|
     |                                                   |        F1.0     |
     |                                                   |----------- S ---------|
     |                                                   |        F1.4     |
     |                                                   +----------- S ---------|
     | 12 F1.2  F1.6                                          F1.2     |
t12  |------] [-----] [----------------------------------+----------- R ---------|
     |                                                   |        F1.6     |
     |                                                   |----------- R ---------|
     |                                                   |        F1.3     |
     |                                                   |----------- S ---------|
     |                                                   |        F1.5     |
     |                                                   +----------- S ---------|
     |                                                                  |
     | 13  F1.4  F1.6                                         F1.4   |
t13  |------] [-----] [-------------------------------------+------- R -------|
     |                                                     |       F1.6   |
     |                                                     |------- R -------|
     |                                                     |       F1.3   |
     |                                                     |------- S -------|
     |                                                     |       F1.7   |
     |                                                     +------- S -------|
     | 14  F1.5  F2.1  F2.5   I0.2                         F1.5   |
t14  |------] [-----] [-----] [-----] [---------------------+------- R -------|
     |                                                     |       F2.1   |
     |                                                     |------- R -------|
     |                                                     |       F2.5   |
     |                                                     |------- R -------|
     |                                                     |       F1.6   |
     |                                                     |------- S -------|
     |                                                     |       F2.0   |
     |                                                     |------- S -------|
     |                                                     |       F2.6   |
     |                                                     +------- S -------|
     | 15  F1.7  F2.1  F2.5   I0.2                         F1.7   |
t15  |------] [-----] [-----] [-----] [---------------------+------- R -------|
     |                                                     |       F2.1   |
     |                                                     |------- R -------|
     |                                                     |       F2.5   |
     |                                                     |------- R -------|
     |                                                     |       F1.6   |
     |                                                     |------- S -------|
     |                                                     |       F2.2   |
     |                                                     |------- S -------|
     |                                                     |       F2.6   |
     |                                                     +------- S -------|
     | 16  F2.0  I0.2   T2                                 F2.0   |
t16  |------] [-----]/[-----] [-----------------------------+------- R -------|
     |                                                     |       F2.1   |
     |                                                     +------- S -------|
     | 17  F2.2  I0.2   T2                                 F2.2   |
t17  |------] [-----]/[-----] [-----------------------------+------- R -------|
     |                                                     |       F2.1   |
     |                                                     +------- S -------|
     | 18  F0.2  F1.0  F2.4                                F2.4   |
t18  |------] [-----] [-----] [-----------------------------+------- R -------|
     |                                                     |       F2.3   |
     |                                                     +------- S -------|
     | 19  F0.4  F1.0  F2.4                                F2.4   |
t19  |------] [-----] [-----] [-----------------------------+------- R -------|
     |                                                     |       F2.3   |
     |                                                     +------- S -------|
     | 20  F1.7  F2.1  F2.6                                F2.6   |
t20  |------] [-----] [-----] [-----------------------------+------- R -------|
     |                                                     |       F2.5   |
     |                                                     +------- S -------|
     |                                                                   |
```

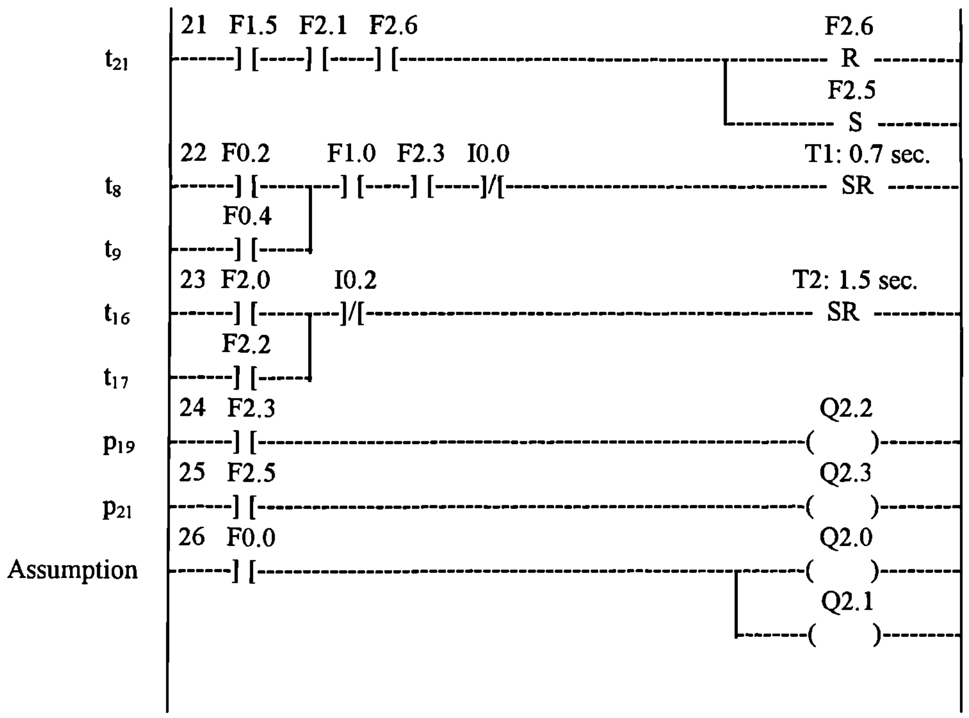


Figure 7.37. The LLD for the TPLC, shown in Fig. 7.36.

## SECTION B2

## THE REVERSIBLE DETERMINISTIC DESIRED STRING PROBLEM

### 7.3.3. The Reversible Deterministic Desired String Problem

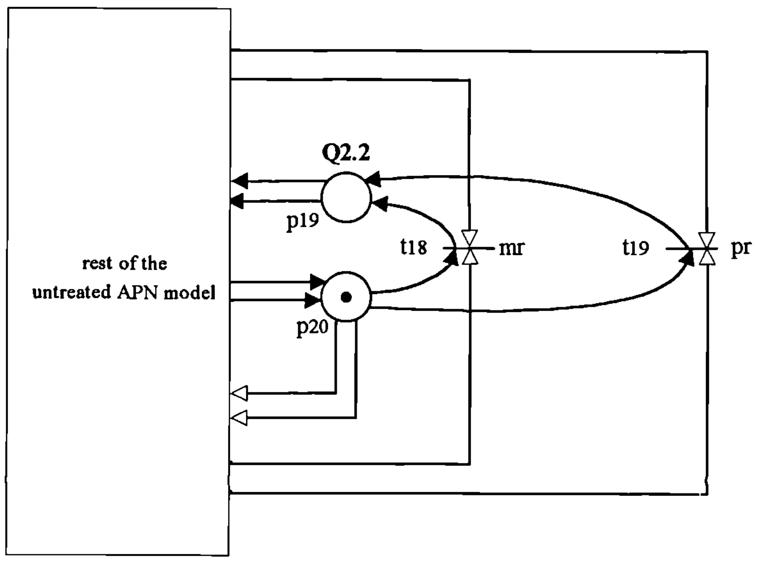
Recall that, as stated in the section 7.3.1, the reversible deterministic desired string specification for the manufacturing system is as follows: The first assembly must have a metallic ring and the next one must have a plastic ring and then this process should carry on in this reversible deterministic sequence in a repeating fashion.

Note that Fig. 7.35 represents the supervisor that allows rings (metallic or plastic) to be put into the assembly chute for the purpose of making assemblies in a mixed order, i.e., the order of the metallic rings and the plastic rings is not specified. Therefore, this supervisor becomes an *untreated model* for solving the desired string problem stated above. The objective in this section is to provide a deterministic specification APN to represent the desired string specification as a net structure and then to combine this specification APN with the untreated model by using the *concurrent composition*. This process yields the *supervised model* that satisfies the desired string specification. The supervised model is then converted into LLD code for implementation on a PLC.

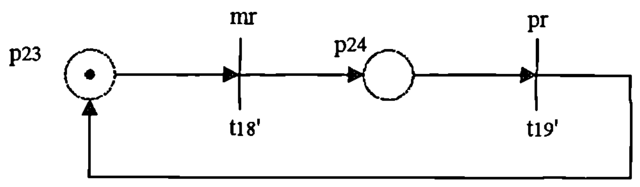
#### 7.3.3.1 Designing the supervised model

The reversible deterministic desired string specification, as given above, means that the system must accept the metallic rings (mr) and the plastic rings (pr) into the assembly chute in a repeating order one after another as long as it goes: mr - pr - mr - pr - mr .......
In Fig. 7.35, transition $t_{18}$ is responsible for ejecting a metallic ring into the assembly chute and similarly, transition $t_{19}$ is responsible for ejecting a plastic ring into the assembly chute. Therefore, these two transitions are required to be ordered in a way that they comply with the desired string specification given. Since ordering the firing of these two transitions is enough to solve the desired string problem, the rest of the untreated model is not required to be considered. This is shown in Fig. 7.38.(a), where transition $t_{18}$ is related to the metallic rings (mr) and transition $t_{19}$ is related to the plastic rings (pr).

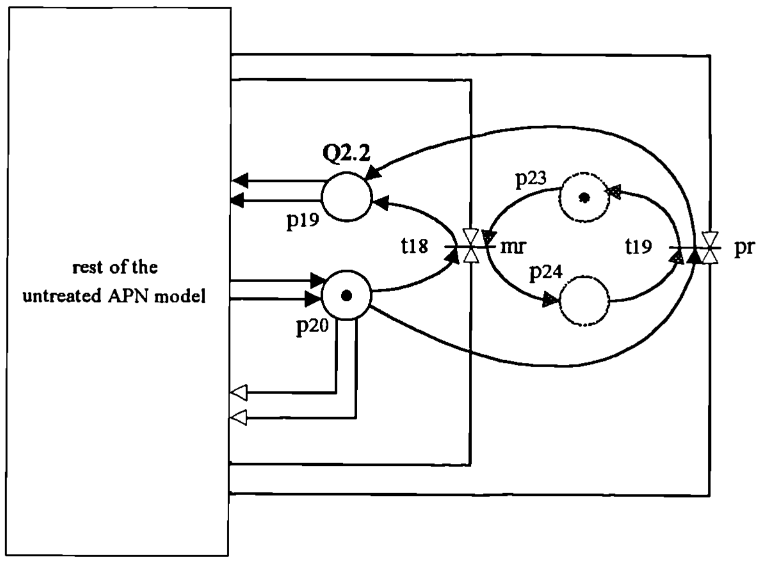
The reversible deterministic desired string specification given can be represented by a specification APN as shown in Fig. 7.38.(b), where there are two places P = { $p_{23}$, $p_{24}$ } and two transitions T = { $t_{18}$', $t_{19}$' }. In the specification APN, transition $t_{18}$' is related to a metallic ring (mr) and transition $t_{19}$' is related to a plastic ring (pr). It can be seen from the specification APN that transitions $t_{18}$' and $t_{19}$' fire one after another, representing a metallic ring (mr) and a plastic ring (pr) being accepted in the assembly chute in a repeating fashion as required by the specification. Now, the next thing to do is to combine this specification APN with the untreated model in order to obtain the supervised model. Therefore, the supervised model, as shown in Fig. 7.38.(c), is obtained by combining the untreated APN model, shown in Fig. 7.38.(a), with the specification APN, shown in Fig. 7.38.(b). To do this the concurrent composition concept is used by merging transition $t_{18}$ of the untreated model with the transition $t_{18}$' of the specification APN, as transition $t_{18}$ in the supervised model and similarly, by merging transition $t_{19}$ of the untreated model with the transition $t_{19}$' of the specification APN, as transition $t_{19}$ in the supervised model. The complete supervised model containing the all APN places and transitions is shown in Fig. 7.39.



(a)

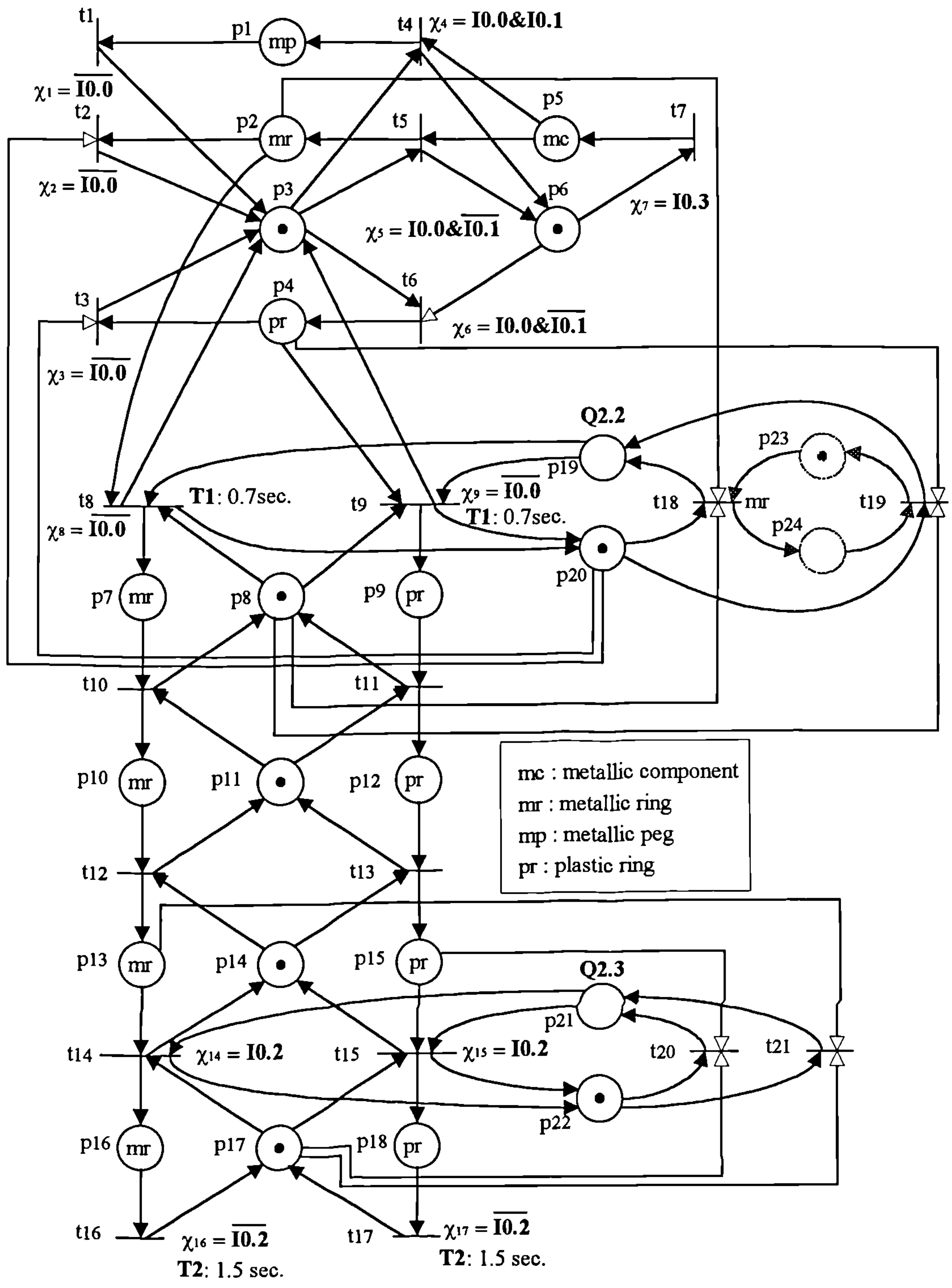


(b)



(c)

Figure 7.38. (a) The untreated model of the manufacturing system.
(b) The reversible deterministic specification APN. (c) The supervised model of the system.



| |
|---|
| mc : metallic component |
| mr : metallic ring |
| mp : metallic peg |
| pr : plastic ring |

Figure 7.39. The supervised model (supervisor) of the system.

### 7.3.2.2. Implementing the supervised model (supervisor) on a PLC as LLDs

In order to convert the supervised model into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code.

As a result to convert the supervisor, given in Fig. 7.39, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2, F1.3, F1.4, F1.5, F1.6, F1.7, F2.0, F2.1, F2.2, F2.3, F2.4, F2.5 and F2.6 are assigned to the places $P = \{ p_1, p_2, \ldots\ldots, p_{22} \}$ respectively. The on delay timer T1 with 0.7 sec. time delay is assigned to the timed transitions $t_8$ and $t_9$ and the on delay timer T2 with 1.5 sec. time delay is assigned to the timed transitions $t_{16}$ and $t_{17}$. After the TPLC is obtained as shown in Fig. 7.40, it is then converted into the LLD code, as shown in Fig. 7.41, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The rungs from 1 to 21 represent the transitions $T = \{ t_1, t_2, \ldots., t_{21} \}$ of the supervisor. The timing requirement for the timed transitions $t_8$ and $t_9$ is represented by the rung 22 and similarly, the timing requirement for the timed transitions $t_{16}$ and $t_{17}$ is represented by the rung 23 Then, action places $p_{19}$ and $p_{21}$ are represented by rungs 24 and 25 respectively. Finally, the assumption that said “when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation”, is realised by the final rung 26. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

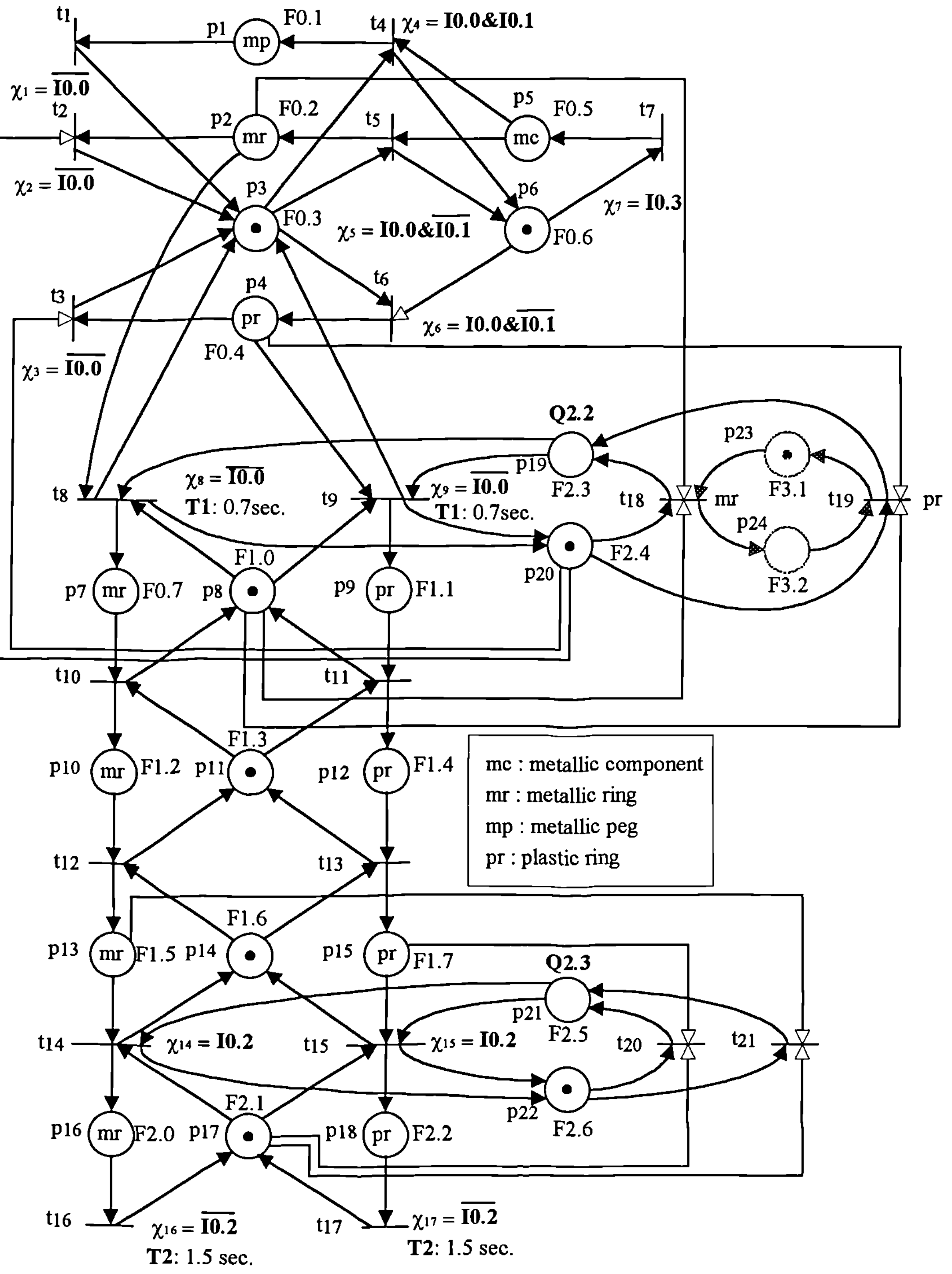
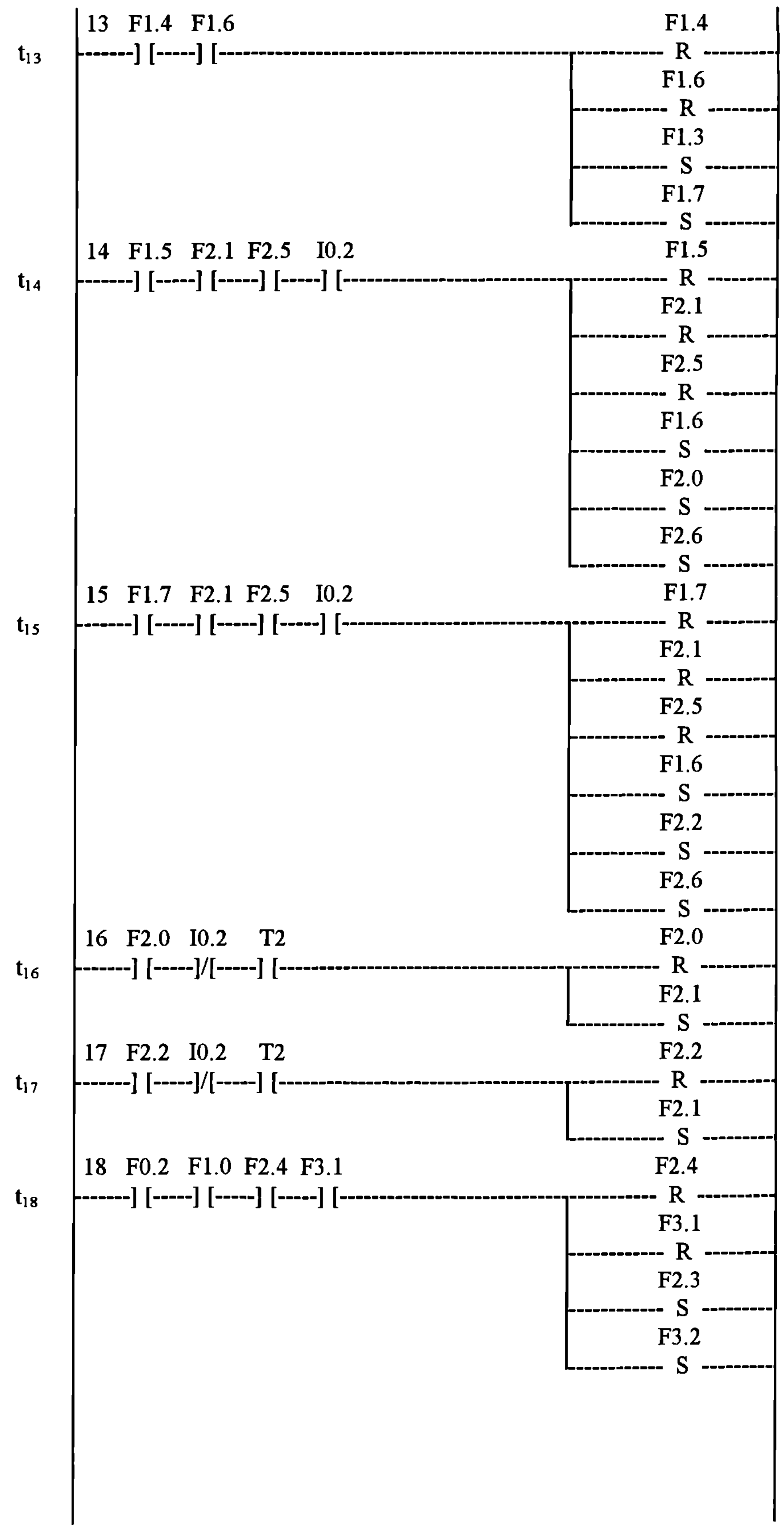


Figure 7.40. The TPLC for the supervised model, shown in Fig. 7.39.

```
               | 0  F0.0                                                  F0.3      |
initialisation |-------]/[--------------------------------------------+------ S ------|
               |                                                      |       F0.6    |
               |                                                      |------ S ------|
               |                                                      |       F1.0    |
               |                                                      |------ S ------|
               |                                                      |       F1.3    |
               |                                                      |------ S ------|
               |                                                      |       F1.6    |
               |                                                      |------ S ------|
               |                                                      |       F2.1    |
               |                                                      |------ S ------|
               |                                                      |       F2.4    |
               |                                                      |------ S ------|
               |                                                      |       F2.6    |
               |                                                      |------ S ------|
               |                                                      |       F3.1    |
               |                                                      |------ S ------|
               |                                                      |       F0.0    |
               |                                                      +------ S ------|
               | 1   F0.1  I0.0                                               F0.1    |
 t1            |------] [-----]/[-------------------------------------+------ R ------|
               |                                                      |       F0.3    |
               |                                                      +------ S ------|
               | 2   F0.2 F2.4  I0.0                                          F0.2    |
 t2            |------] [-----] [-----]/[-----------------------------+------ R ------|
               |                                                      |       F0.3    |
               |                                                      +------ S ------|
               | 3   F0.4 F2.4  I0.0                                          F0.4    |
 t3            |------] [-----] [-----]/[-----------------------------+------ R ------|
               |                                                      |       F0.3    |
               |                                                      +------ S ------|
               | 4  F0.3  F0.5  I0.0  I0.1                                    F0.3    |
 t4            |------] [-----] [-----] [-----] [---------------------+------ R ------|
               |                                                      |       F0.5    |
               |                                                      |------ R ------|
               |                                                      |       F0.1    |
               |                                                      |------ S ------|
               |                                                      |       F0.6    |
               |                                                      +------ S ------|
               | 5  F0.3  F0.5  I0.0  I0.1                                    F0.3    |
 t5            |------] [-----] [-----] [-----]/[---------------------+------ R ------|
               |                                                      |       F0.5    |
               |                                                      |------ R ------|
               |                                                      |       F0.2    |
               |                                                      |------ S ------|
               |                                                      |       F0.6    |
               |                                                      +------ S ------|
               | 6  F0.3  F0.6  I0.0  I0.1                                    F0.3    |
 t6            |------] [-----] [-----] [-----]/[---------------------+------ R ------|
               |                                                      |       F0.4    |
               |                                                      +------ S ------|
               |                                                                      |
     | 7  F0.6  I0.3                                     F0.6        |
t7   |------] [-----] [-------------------------------+--------- R --------|
     |                                                |          F0.5        |
     |                                                +--------- S --------|
     | 8  F0.2  F1.0  F2.3  I0.0   T1                        F0.2        |
t8   |------] [-----] [-----] [-----]/[-----] [-------+--------- R --------|
     |                                                |          F1.0        |
     |                                                +--------- R --------|
     |                                                |          F2.3        |
     |                                                +--------- R --------|
     |                                                |          F0.3        |
     |                                                +--------- S --------|
     |                                                |          F0.7        |
     |                                                +--------- S --------|
     |                                                |          F2.4        |
     |                                                +--------- S --------|
     | 9  F0.4  F1.0  F2.3  I0.0   T1                        F0.4        |
t9   |------] [-----] [-----] [-----]/[-----] [-------+--------- R --------|
     |                                                |          F1.0        |
     |                                                +--------- R --------|
     |                                                |          F2.3        |
     |                                                +--------- R --------|
     |                                                |          F0.3        |
     |                                                +--------- S --------|
     |                                                |          F1.1        |
     |                                                +--------- S --------|
     |                                                |          F2.4        |
     |                                                +--------- S --------|
     | 10  F0.7  F1.3                                            F0.7        |
t10  |------] [-----] [-------------------------------+--------- R --------|
     |                                                |          F1.3        |
     |                                                +--------- R --------|
     |                                                |          F1.0        |
     |                                                +--------- S --------|
     |                                                |          F1.2        |
     |                                                +--------- S --------|
     | 11  F1.1  F1.3                                            F1.1        |
t11  |------] [-----] [-------------------------------+--------- R --------|
     |                                                |          F1.3        |
     |                                                +--------- R --------|
     |                                                |          F1.0        |
     |                                                +--------- S --------|
     |                                                |          F1.4        |
     |                                                +--------- S --------|
     | 12  F1.2  F1.6                                            F1.2        |
t12  |------] [-----] [-------------------------------+--------- R --------|
     |                                                |          F1.6        |
     |                                                +--------- R --------|
     |                                                |          F1.3        |
     |                                                +--------- S --------|
     |                                                |          F1.5        |
     |                                                +--------- S --------|
     |                                                                       |
```



```
     13  F1.4  F1.6                                      F1.4
t13  ------] [-----] [-------------------------------------- R ------
                                                  |      F1.6
                                                  |----- R ------
                                                  |      F1.3
                                                  |----- S ------
                                                  |      F1.7
                                                  |----- S ------
     14  F1.5  F2.1  F2.5  I0.2                          F1.5
t14  ------] [-----] [-----] [-----] [------------------ R ------
                                                  |      F2.1
                                                  |----- R ------
                                                  |      F2.5
                                                  |----- R ------
                                                  |      F1.6
                                                  |----- S ------
                                                  |      F2.0
                                                  |----- S ------
                                                  |      F2.6
                                                  |----- S ------
     15  F1.7  F2.1  F2.5  I0.2                          F1.7
t15  ------] [-----] [-----] [-----] [------------------ R ------
                                                  |      F2.1
                                                  |----- R ------
                                                  |      F2.5
                                                  |----- R ------
                                                  |      F1.6
                                                  |----- S ------
                                                  |      F2.2
                                                  |----- S ------
                                                  |      F2.6
                                                  |----- S ------
     16  F2.0  I0.2   T2                                 F2.0
t16  ------] [-----]/[-----] [-------------------------- R ------
                                                  |      F2.1
                                                  |----- S ------
     17  F2.2  I0.2   T2                                 F2.2
t17  ------] [-----]/[-----] [-------------------------- R ------
                                                  |      F2.1
                                                  |----- S ------
     18  F0.2  F1.0  F2.4  F3.1                          F2.4
t18  ------] [-----] [-----] [-----] [------------------ R ------
                                                  |      F3.1
                                                  |----- R ------
                                                  |      F2.3
                                                  |----- S ------
                                                  |      F3.2
                                                  |----- S ------
```

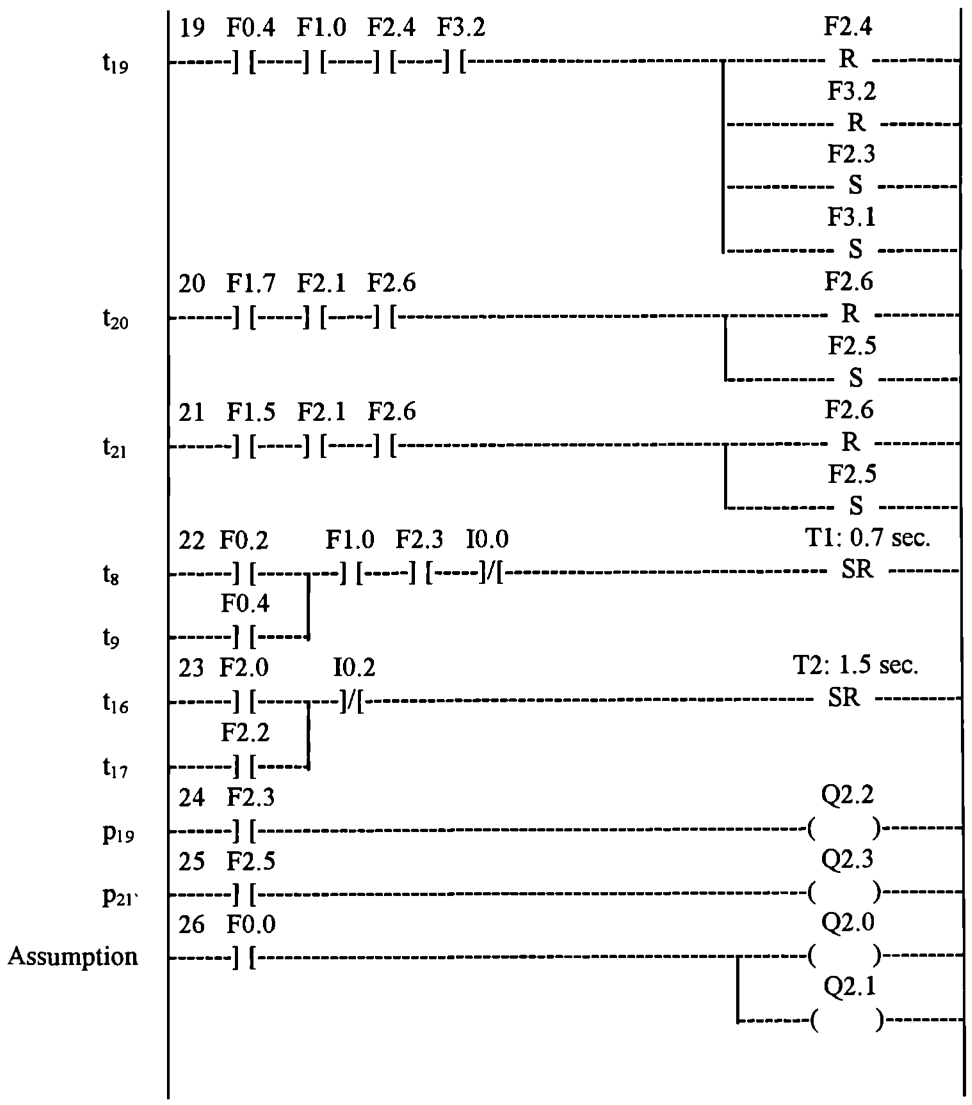


Figure 7.41. The LLD for the TPLC, shown in Fig. 7.40.

## SECTION B3

## THE REVERSIBLE NONDETERMINISTIC DESIRED STRING PROBLEM

### 7.3.4. The Reversible Nondeterministic Desired String Problem

Recall that, as stated in the section 7.3.1, the reversible nondeterministic desired string problem for the manufacturing system is as follows: Firstly, the system must allow metallic rings to be used for making assemblies as many as it takes until a plastic ring comes along at the sort area. Next, the system must use exactly the same number of plastic rings as the metallic rings that have been used previously. Then, this process should carry on starting with the next set of metallic rings and so on. Note that in the beginning if there are no metallic rings appearing at the sort area, then none of the plastic rings appearing at the sort area is allowed to be used. In simple terms if the system uses, say, three metallic rings, that appear at the sort area one after another, to make assemblies and if a plastic ring appears at the sort area, then the system must be obliged to use three plastic rings in total before using another set of metallic rings and plastic rings consequently. If the system uses, say, 25 metallic rings that appear at the sort area one after another and if a plastic ring appears at the sort area, then the system must be obliged to use 25 plastic rings in total before using another set of metallic rings and plastic rings consequently. This process should carry on in this reversible nondeterministic fashion.

Note that Fig. 7.35 represents the supervisor that allows rings (metallic or plastic) to be put into the assembly chute for the purpose of making assemblies in a mixed order, i.e., the order of the metallic rings and the plastic rings is not specified. Therefore, this supervisor becomes an *untreated model* for solving the nondeterministic desired string specification, stated above. The objective in this section is to provide a nondeterministic specification APN to represent the nondeterministic desired string specification as a net structure and then to combine this specification APN with the untreated model by using the *concurrent composition*. This process yields the *supervised model* that satisfies the desired string specification. The supervised model is then converted into LLD code for implementation on a PLC.

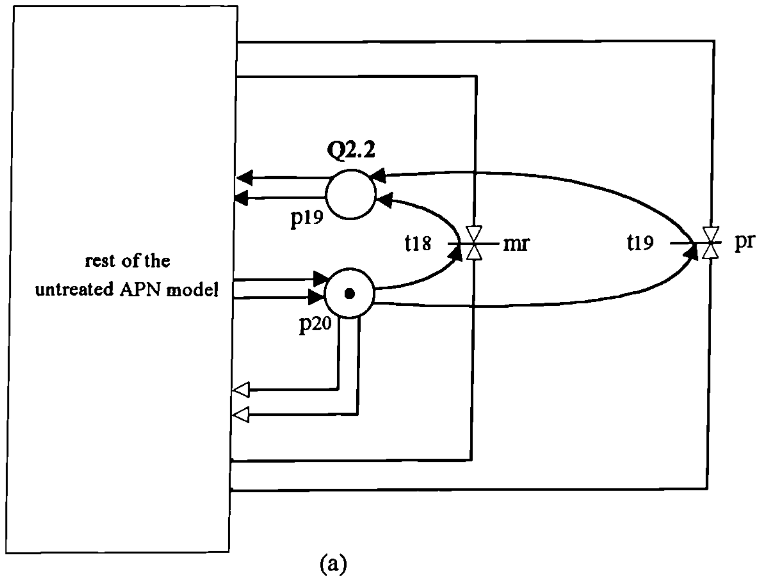
### 7.3.4.1 Designing the supervised model

In Fig. 7.35, transition $t_{18}$ is responsible for ejecting a metallic ring into the assembly chute and similarly, transition $t_{19}$ is responsible for ejecting a plastic ring into the assembly chute. Therefore, these two transitions are required to be ordered in a way that they comply with the desired string specification given. Since ordering the firing of these two transitions is enough to solve the reversible nondeterministic desired string problem, the rest of the untreated model is not required to be considered. This is shown in Fig. 7.42.(a), where transition $t_{18}$ is related to the metallic rings (mr) and transition $t_{19}$ is related to the plastic rings (pr).

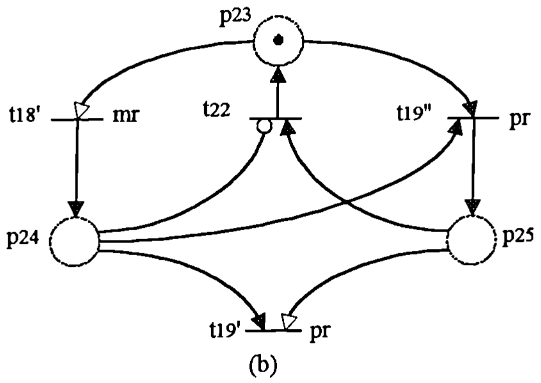
The reversible nondeterministic desired string specification given can be represented by a nondeterministic specification APN as shown in Fig. 7.42.(b), where there are three places $P = \{ p_{23}, p_{24}, p_{25} \}$ and four transitions $T = \{ t_{18}', t_{19}', t_{19}'', t_{22} \}$. Note that transition $t_{18}'$ is related to metallic rings (mr), while transitions $t_{19}'$ and $t_{19}''$ are related to plastic rings (pr). Initially, place $p_{23}$ has a token and only transition $t_{18}'$ is enabled, i.e., only metallic rings (mr) are allowed to be put into the assembly chute. Once the transition $t_{18}'$ fires, transitions $t_{18}'$ and $t_{19}''$ are enabled. In other words, metallic rings or plastic rings can be used to make assemblies. Every firing of transition $t_{18}'$ puts a token into place $p_{24}$ and re-enables transitions $t_{18}'$ and $t_{19}''$. Upon the first firing of transition $t_{19}''$ (pr), transition $t_{18}'$ is disabled for this turn of the string, i.e., no more metallic rings are allowed to be used and transition $t_{19}'$ (pr) can fire repeatedly until the tokens in place $p_{24}$ are consumed. When there is not any tokens left in place $p_{24}$ and there is a token in place $p_{25}$, this represents the end of a nondeterministic string. In this case, transition $t_{22}$ is enabled and it fires by removing a token from place $p_{25}$ and by depositing a token in place $p_{23}$. This means that the nondeterministic specification APN goes back to its initial marking and another nondeterministic string can be generated in a similar way.

In order to obtain the supervised model, it is necessary to combine this specification APN with the untreated model. Therefore, the supervised model, as shown in Fig.

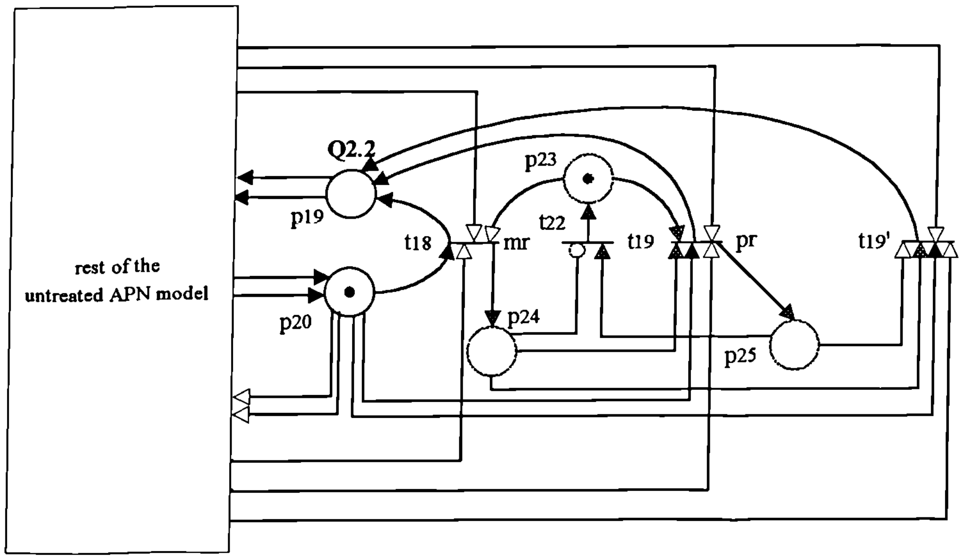
7.42.(c), is obtained by combining the untreated APN model, shown in Fig. 7.42.(a), with the specification APN, shown in Fig. 7.42.(b). To do this the concurrent composition concept is used as follows: Firstly, transition $t_{19}$ of the untreated APN model is duplicated as $t_{19}$ and $t_{19}$'. Secondly, transition $t_{18}$ of the untreated model is merged with transition $t_{18}$' of the specification APN, as transition $t_{18}$ in the supervised model, transition $t_{19}$ of the untreated model is merged with the transition $t_{19}$'' of the specification APN, as transition $t_{19}$ in the supervised model and transition $t_{19}$' of the untreated model is merged with the transition $t_{19}$' of the specification APN, as transition $t_{19}$' in the supervised model. The complete supervised model containing the all APN places and transitions is shown in Fig. 7.43.



(a)



(b)



(c)

Figure 7.42. (a) The untreated model of the manufacturing system. (b) The reversible nondeterministic specification APN, representing the given desired string specification. (c) The supervised model of the system.

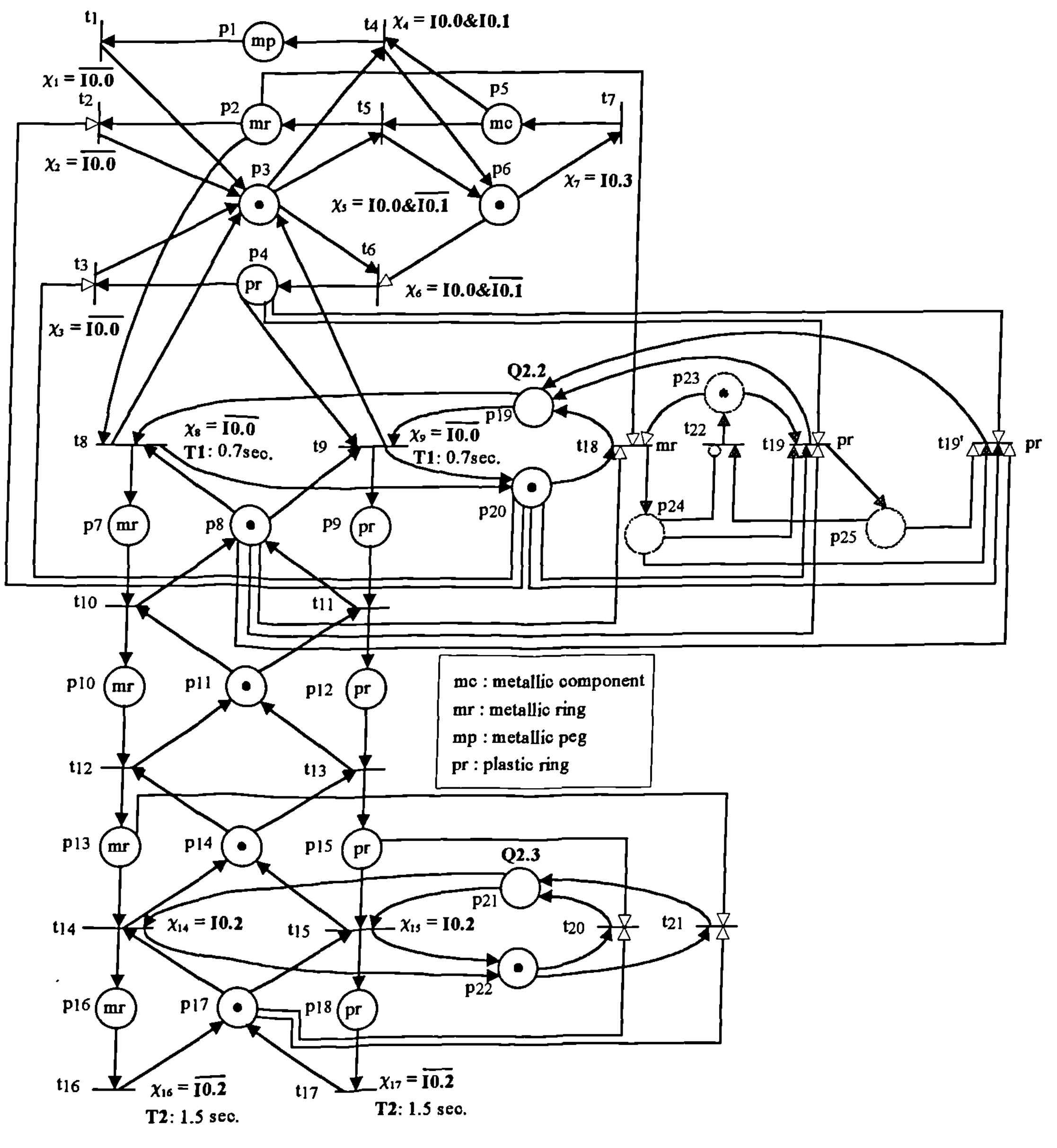


Figure 7.43. The supervised model (supervisor) of the system.

### 7.3.4.2. Implementing the controlled model (supervisor) on a PLC as LLDs

In order to convert the controlled model(supervisor) into ladder logic diagram (LLD) for implementation on a programmable logic controller (PLC), the token passing logic methodology, as described in chapter 6, is used. This means that firstly the supervisor is converted into a token passing logic controller (TPLC) by assigning flags to places, whose token capacity is one, by assigning counters to places, whose token capacity is bigger than or equal to one and by assigning on delay timers to the timed-transitions to realise the timing requirements. Secondly, the TPLC obtained is converted into LLDs for implementation on a PLC. To do this a direct mapping is used from TPLC to LLD code.

As a result to convert the supervisor, given in Fig. 7.43, into a TPLC, flags F0.1, F0.2, F0.3, F0.4, F0.5, F0.6, F0.7, F1.0, F1.1, F1.2, F1.3, F1.4, F1.5, F1.6, F1.7, F2.0, F2.1, F2.2, F2.3, F2.4, F2.5, F2.6, F3.1 and F3.2 are assigned to the places P = { $p_1$, $p_2$, ....., $p_{22}$, $p_{23}$, $p_{25}$} respectively. Counter C1 is also assigned to place $p_{24}$. The on delay timer T1 with 0.7 sec. time delay is assigned to the timed transitions $t_8$ and $t_9$ and the on delay timer T2 with 1.5 sec. time delay is assigned to the timed transitions $t_{16}$ and $t_{17}$. After the TPLC is obtained as shown in Fig. 7.44, it is then converted into the LLD code, as shown in Fig. 7.45, by using a direct mapping from TPLC to LLD. This code is written for a Siemens S5-100U PLC. The LLD symbols for Siemens S5-100U are defined in Table 7.5.

The LLD code is structured in such a way that the rung 0 initialises the system by means of the initialisation flag F0. The rungs from 1 to 23 represent the transitions T = { $t_1$, $t_2$, ...., $t_{18}$ , $t_{19}$, $t_{19}$', $t_{20}$, $t_{21}$, $t_{22}$}. The timing requirement for the timed transitions $t_8$ and $t_9$ is represented by the rung 24 and similarly, the timing requirement for the timed transitions $t_{16}$ and $t_{17}$ is represented by the rung 25. Then, action places $p_{19}$ and $p_{21}$ are represented by rungs 26 and 27 respectively. Finally, the assumption that said "when the system is switched on the upper conveyor motor (action Q2.0) and the lower conveyor motor (action Q2.1) must be in operation", is realised by the final rung 28. By adopting this concept further clarity can be added to the system documentation and it is very easy to understand and modify the LLD code if necessary.

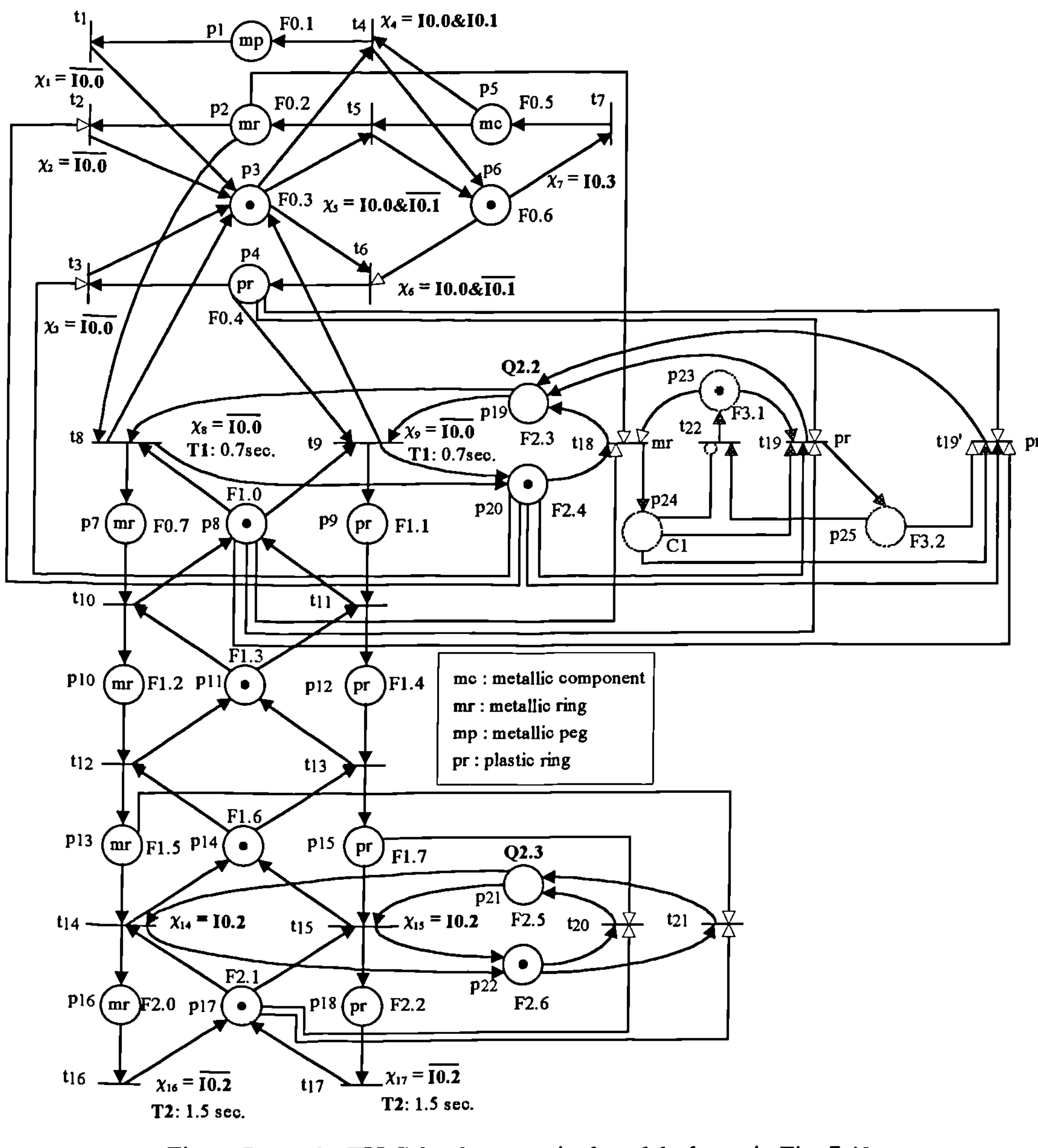


Figure 7.44. The TPLC for the supervised model, shown in Fig. 7.43.

```
               | 0 F0.0                                     F0.3      |
initialisation |-------]/[---------------------------+---------- S ---------|
               |                                     |         F0.6      |
               |                                     |---------- S ---------|
               |                                     |         F1.0      |
               |                                     |---------- S ---------|
               |                                     |         F1.3      |
               |                                     |---------- S ---------|
               |                                     |         F1.6      |
               |                                     |---------- S ---------|
               |                                     |         F2.1      |
               |                                     |---------- S ---------|
               |                                     |         F2.4      |
               |                                     |---------- S ---------|
               |                                     |         F2.6      |
               |                                     |---------- S ---------|
               |                                     |         F3.1      |
               |                                     |---------- S ---------|
               |                                     |         F0.0      |
               |                                     +---------- S ---------|
               | 1   F0.1  I0.0                                F0.1      |
t1             |-------] [-----]/[-------------------+---------- R ---------|
               |                                     |         F0.3      |
               |                                     +---------- S ---------|
               | 2   F0.2 F2.4  I0.0                           F0.2      |
t2             |-------] [-----] [-----]/[-----------+---------- R ---------|
               |                                     |         F0.3      |
               |                                     +---------- S ---------|
               | 3   F0.4 F2.4  I0.0                           F0.4      |
t3             |-------] [-----] [-----]/[-----------+---------- R ---------|
               |                                     |         F0.3      |
               |                                     +---------- S ---------|
               | 4  F0.3  F0.5  I0.0  I0.1                     F0.3      |
t4             |-------] [-----] [-----] [-----] [---+---------- R ---------|
               |                                     |         F0.5      |
               |                                     |---------- R ---------|
               |                                     |         F0.1      |
               |                                     |---------- S ---------|
               |                                     |         F0.6      |
               |                                     +---------- S ---------|
               | 5  F0.3  F0.5  I0.0  I0.1                     F0.3      |
t5             |-------] [-----] [-----] [-----]/[---+---------- R ---------|
               |                                     |         F0.5      |
               |                                     |---------- R ---------|
               |                                     |         F0.2      |
               |                                     |---------- S ---------|
               |                                     |         F0.6      |
               |                                     +---------- S ---------|
               | 6  F0.3  F0.6  I0.0  I0.1                     F0.3      |
t6             |-------] [-----] [-----] [-----]/[---+---------- R ---------|
               |                                     |         F0.4      |
               |                                     +---------- S ---------|
    |7   F0.6  I0.3                                          F0.6     |
t7  |------] [-----] [----------------------------------+---------- R --------|
    |                                                   |         F0.5     |
    |                                                   +---------- S --------|
    |8  F0.2  F1.0  F2.3  I0.0   T1                              F0.2     |
t8  |------] [-----] [-----] [-----]/[-----] [----------+---------- R --------|
    |                                                   |         F1.0     |
    |                                                   +---------- R --------|
    |                                                   |         F2.3     |
    |                                                   +---------- R --------|
    |                                                   |         F0.3     |
    |                                                   +---------- S --------|
    |                                                   |         F0.7     |
    |                                                   +---------- S --------|
    |                                                   |         F2.4     |
    |                                                   +---------- S --------|
    |9   F0.4  F1.0  F2.3  I0.0   T1                              F0.4     |
t9  |------] [-----] [-----] [-----]/[-----] [----------+---------- R --------|
    |                                                   |         F1.0     |
    |                                                   +---------- R --------|
    |                                                   |         F2.3     |
    |                                                   +---------- R --------|
    |                                                   |         F0.3     |
    |                                                   +---------- S --------|
    |                                                   |         F1.1     |
    |                                                   +---------- S --------|
    |                                                   |         F2.4     |
    |                                                   +---------- S --------|
    |10  F0.7  F1.3                                          F0.7     |
t10 |------] [-----] [----------------------------------+---------- R --------|
    |                                                   |         F1.3     |
    |                                                   +---------- R --------|
    |                                                   |         F1.0     |
    |                                                   +---------- S --------|
    |                                                   |         F1.2     |
    |                                                   +---------- S --------|
    |11  F1.1  F1.3                                          F1.1     |
t11 |------] [-----] [----------------------------------+---------- R --------|
    |                                                   |         F1.3     |
    |                                                   +---------- R --------|
    |                                                   |         F1.0     |
    |                                                   +---------- S --------|
    |                                                   |         F1.4     |
    |                                                   +---------- S --------|
    |12  F1.2  F1.6                                          F1.2     |
t12 |------] [-----] [----------------------------------+---------- R --------|
    |                                                   |         F1.6     |
    |                                                   +---------- R --------|
    |                                                   |         F1.3     |
    |                                                   +---------- S --------|
    |                                                   |         F1.5     |
    |                                                   +---------- S --------|
    |                                                                         |
      | 13  F1.4  F1.6                                      F1.4     |
t13   |------] [-----] [--------------------------------+---- R -----|
      |                                                 |   F1.6     |
      |                                                 +---- R -----|
      |                                                 |   F1.3     |
      |                                                 +---- S -----|
      |                                                 |   F1.7     |
      |                                                 +---- S -----|
      |                                                              |
      | 14  F1.5  F2.1  F2.5  I0.2                          F1.5     |
t14   |------] [-----] [-----] [-----] [----------------+---- R -----|
      |                                                 |   F2.1     |
      |                                                 +---- R -----|
      |                                                 |   F2.5     |
      |                                                 +---- R -----|
      |                                                 |   F1.6     |
      |                                                 +---- S -----|
      |                                                 |   F2.0     |
      |                                                 +---- S -----|
      |                                                 |   F2.6     |
      |                                                 +---- S -----|
      |                                                              |
      | 15  F1.7  F2.1  F2.5  I0.2                          F1.7     |
t15   |------] [-----] [-----] [-----] [----------------+---- R -----|
      |                                                 |   F2.1     |
      |                                                 +---- R -----|
      |                                                 |   F2.5     |
      |                                                 +---- R -----|
      |                                                 |   F1.6     |
      |                                                 +---- S -----|
      |                                                 |   F2.2     |
      |                                                 +---- S -----|
      |                                                 |   F2.6     |
      |                                                 +---- S -----|
      |                                                              |
      | 16  F2.0  I0.2   T2                                 F2.0     |
t16   |------] [-----]/[-----] [------------------------+---- R -----|
      |                                                 |   F2.1     |
      |                                                 +---- S -----|
      |                                                              |
      | 17  F2.2  I0.2   T2                                 F2.2     |
t17   |------] [-----]/[-----] [------------------------+---- R -----|
      |                                                 |   F2.1     |
      |                                                 +---- S -----|
      |                                                              |
      | 18  F0.2  F1.0  F2.4  F3.1                          F2.4     |
t18   |------] [-----] [-----] [-----] [----------------+---- R -----|
      |                                                 |   F2.3     |
      |                                                 +---- S -----|
      |                                                 |   C1       |
      |                                                 +---- CU ----|
      |                                                              |
           | 19  F0.4  F1.0  F2.4  F3.1  C1                          F2.4      |
t19        |------] [-----] [-----] [-----] [-----] [-------------------------- R ---------|
           |                                                 |          F3.1      |
           |                                                 |----------- R ---------|
           |                                                 |           C1       |
           |                                                 |---------- CD ---------|
           |                                                 |          F2.3      |
           |                                                 |----------- S ---------|
           |                                                 |          F3.2      |
           |                                                 |----------- S ---------|
           | 20  F0.4  F1.0  F2.4  F3.2  C1                          F2.4      |
t19'       |------] [-----] [-----] [-----] [-----] [-------------------------- R ---------|
           |                                                 |          F2.3      |
           |                                                 |----------- S ---------|
           |                                                 |           C1       |
           |                                                 |---------- CD ---------|
           | 21  F1.7  F2.1  F2.6                                     F2.6      |
t20        |------] [-----] [-----] [------------------------------------ R ---------|
           |                                                 |          F2.5      |
           |                                                 |----------- S ---------|
           | 22  F1.5  F2.1  F2.6                                     F2.6      |
t21        |------] [-----] [-----] [------------------------------------ R ---------|
           |                                                 |          F2.5      |
           |                                                 |----------- S ---------|
           | 23  F3.2  C1                                             F3.2      |
t22        |------] [-----]/[--------------------------------------------- R ---------|
           |                                                 |          F3.1      |
           |                                                 |----------- S ---------|
           | 24 F0.2      F1.0  F2.3  I0.0                       T1: 0.7 sec.  |
t8         |------] [-------+---] [-----] [-----]/[-------------------------- SR --------|
           |    F0.4        |                                                  |
t9         |------] [-------+                                                  |
           | 25 F2.0      I0.2                                   T2: 1.5 sec.  |
t16        |------] [-------+---]/[-------------------------------------------- SR ---------|
           |    F2.2        |                                                  |
t17        |------] [-------+                                                  |
           | 26  F2.3                                                 Q2.2      |
p19        |------] [-------------------------------------------------(    )--------|
           | 27  F2.5                                                 Q2.3      |
p21        |------] [-------------------------------------------------(    )--------|
           | 28  F0.0                                                 Q2.0      |
Assumption |------] [----------------------------------------------+-----(    )--------|
           |                                                 |        Q2.1      |
           |                                                 |-------(    )--------|
           |                                                                   |
```

Figure 7.45. The LLD for the TPLC, shown in Fig. 7.44.

## 7.4. DISCUSSION

In this chapter a discrete manufacturing system has been considered to illustrate the applicability, strengths and drawbacks of the design techniques proposed in this thesis. It is important to point out that this chapter has shown how low level manufacturing control problems can be solved with the methods proposed in addition to high level control problem considered in the previous chapters. Both the forbidden state and the desired string problems have been considered.

In the case of the *forbidden state problem*, the results obtained for the six different methods, namely, the inhibitor arc method, the enabling arc method, the intermediate place method, the APN-SM method, U-TPM rule method and C-TPM rule method, can be compared in terms of the number of places, and transitions that have been used in the supervisor for each method as well as the ladder logic diagram (LLD) code generated from these supervisors. It is evident from Table 7.13 that the efficiency of any implementation is a function of both the places and transitions. A large number of places and transitions leads to a large number of LLD rungs. Furthermore, it is evident that the first three methods, i.e., the inhibitor arc method, the enabling arc method and the intermediate place method, have large numbers of places and transitions, and thus give rise to large LLD solution. The APN-SM method has a smaller number of places but similar number of transitions and thus give rise to a medium size LLD solution. Finally, both the U-TPM and C-TPM rule methods have small numbers of places and transitions, thus giving highly efficient LLD solution.

| | Inhibitor arc method | Enabling arc method | intermediate place method | APN-SM method | U-TPM rule method | C-TPM rule method |
|---|---|---|---|---|---|---|
| number of places used | 22 | 22 | 24 | 12 | 10 | 10 |
| number of transitions used | 30 | 32 | 30 | 23 | 9 | 7 |
| number of LLD rungs produced | 36 | 38 | 36 | 29 | 15 | 13 |

Table 7.14. The number of places, transitions that has been used and LLD rungs that has been produced in the supervisors.

Once again it is obvious from the Table 7.14 that first four methods, namely inhibitor arc method, enabling arc method, intermediate place method and APN-SM method, suffer from the state explosion problem. The effect of the state explosion problem on these four methods is two-fold. The first one is that the computation of the supervisor becomes very difficult as the system becomes bigger. The second effect is that the bigger the system, the bigger the number of places and transitions required to be used in the supervisor. However, in these methods the supervisors obtained are *maximally permissive*, i.e., they do not unnecessarily constrain the system behaviour and *nonblocking*, i.e., they do not contradict the specifications given. In addition, the supervisors obtained are correct by construction. In the case of the U-TPM rule method the state explosion problem has an effect only on the computation of the supervisor. That is, the computation of the supervisor becomes very difficult as the system becomes bigger. However, the number of places and transitions used in the supervisor does not increase exponentially in the size of the model. In this case, the supervisor is still maximally permissive, nonblocking and correct by construction. Finally, the C-TPM rule method does not suffer from state explosion problem in the design phase. However, the correctness of the supervisor obtained must be verified by using reachability graph (RG) analysis. In return this could still pose a problem, because for very big systems the RG of the system could still be very big. In this case, the supervisor is not necessarily maximally permissive. In order to check this property, it is necessary to carry out further RG analysis on the uncontrolled model and check the results with the RG obtained for the controlled model.

In the case of the *desired string problem*, it has been shown how this type of problems may arise in low level manufacturing systems and how they can be solved efficiently by using the methods introduced in this thesis.

In this case, both the reversible deterministic desired string problem and the reversible nondeterministic desired string problem are considered. Since the system required the forbidden state problems to be solved before solving the desired string problems, the forbidden state problem has been solved by using the C-TPM rule method. This has proved once again that although the C-TPM rule method provides very low number of markings because of using controlled model for analysis as oppose to uncontrolled model, it can be very difficult to verify the design. This is because the bigger the system, the bigger the number of reachable states.

After solving the forbidden state problems both the reversible deterministic desired string problem and the reversible nondeterministic desired string problem have been solved easily. This is because it is not difficult to represent the desired strings (deterministic or nondeterministic) as a specification APN. After the desired string is represented by a specification APN, the supervised model can be obtained by using concurrent composition.

Note that all the LLDs provided in this chapter have all been tested on a real manufacturing system and have been proven to be correct in controlling the system.

# CHAPTER 8

# CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

# CHAPTER 8

# CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

## 8.1. CONCLUSIONS

A new approach to the real time supervisory control of discrete event systems (DES) is proposed. The original contributions consist of two major parts:

i) the extension of existing Petri net based control design techniques, to allow the formal design of compiled supervisors for both the forbidden state specifications and the desired string specifications.

ii) the development of a conversion technique from the Petri net based supervisors into ladder logic diagrams (LLDs) for the implementation of the corresponding supervisors on programmable logic controllers (PLCs).

Formal design of Petri-net-based compiled supervisors for DES control problems and their efficient implementations are a challenging problem. Little work has been done on both the formal design and their formal LLD implementations on PLCs. Because of this fact the research carried out has aimed at producing techniques to address these two important issues and to fill the gap between the theory and practice.

To solve the design problem in the case of the forbidden state specifications a family of solutions have been proposed, namely, the inhibitor arc method, the enabling arc method, the intermediate place method, the APN-SM method, U-TPM rule method and C-TPM rule method. Because of having to consider the whole state space, i.e., all possible markings, of a system in terms of the reachability graph of the model, the first four methods, i.e., inhibitor arc method, enabling arc method, intermediate place method and APN-SM method, suffer from the state explosion problem. The effect of the state explosion problem on these four methods is two-fold. The first one is that the computation of the supervisor becomes very difficult as the system becomes bigger. The second effect is that the bigger the system, the bigger the number of places and transitions required to be used in the supervisor. However, in these methods the supervisors obtained are *maximally permissive*, i.e., they do not unnecessarily constrain the system behaviour and *nonblocking*, i.e., they do not contradict the specifications given. In addition, the supervisors obtained are correct by construction.

In the case of the U-TPM rule method the state explosion problem has an effect only on the computation of the supervisor. That is, the computation of the supervisor becomes very difficult as the system becomes bigger. However, the number of places and transitions used in the supervisor does not increase exponentially in the size of the model. In this case, the supervisor is still maximally permissive, nonblocking and correct by construction. Finally, the C-TPM rule method does not suffer from the state explosion problem in the design phase. However, the correctness of the supervisor obtained must be verified by using reachability graph (RG) analysis. In return this could still pose a problem, because for very big systems the RG of the system could still be very big. In this case, the supervisor may not be maximally permissive. In order to check this property, it is necessary to carry out further RG analysis on the uncontrolled model and check the results with the RG obtained for the controlled model.

To solve the design problem in the case of the desired string specifications a technique has been proposed. In this technique, the concurrent composition as proposed by Giua (Giua and DiCesare, 1991), is used to integrate the model of a system with the specification model, which represents the desired string. This method has been proposed as an alternative to the use of formal language concepts. The technique proposed can be

used when the desired string requires a deterministic specification APN as well as a nondeterministic specification APN.
For implementation, the supervisors obtained are converted into LLDs to be implemented on any PLC. This is done through the use of a general methodology, which has been proposed for converting Petri nets into LLDs. This process unifies theory and application and provides guidelines to control synthesis and control enforcement.

The treatment of examples has illustrated the details of both the synthesis techniques and the implementation issues, and has provided the strengths and drawbacks of the proposed design techniques in relation to one another.

Note that the results obtained can be applied to a wide range of DESs, for which in their models the following classes of Petri nets may be required: untimed, timed, safe, i.e., 1-bounded, and k-bounded Petri nets.

It has been shown that the results obtained in this thesis can be applied to high level manufacturing control, where the role of the supervisor is to coordinate factory-wide control of machines, and to low-level manufacturing control, where the role of the supervisor is to arrange low-level interaction between the control devices, such as motors, actuators, etc. Although in this thesis only discrete manufacturing systems are considered as examples, the results obtained can be applied to other DESs as well.

Note that these results are based on the assumption that the DESs considered are *controllable*, i.e., there is a sufficient number of discrete event actuators, motors, etc. available, and *observable*, i.e., there is a sufficient number of discrete event sensors available within the system.

In conclusion it is postulated that the design of Petri-net-based compiled supervisors for the control of DESs can be achieved in a formal way by using a family of techniques, each of which has its own strengths and drawbacks. New and novel solutions have been proposed. The design and implementation tools proposed represent a significant contribution.

## 8.2. RECOMMENDATIONS FOR FURTHER WORK

The objective of this thesis was to investigate the formal design techniques for the control of DESs using supervisory control theory and Petri net concepts, and consequently to implement the resulting designs as LLDs on PLCs. It is recommended that further work be carried out in the following areas:

1. More efficient modelling techniques may be used to construct Petri net models with less places. The current models used make use of the place invariant techniques in the modelling phase to represent the physical constraints. The place invariant techniques require the use of so called monitor places for each constraint. Note that the bigger the number of places in a model, the bigger the state space of the model will be. Naturally, as the state space of a system becomes bigger the computational effort to find the controller will become bigger. As an alternative, inhibitor arcs may be used to represent the physical constraints in stead of monitor places.

2. In this research it was assumed that the system considered is controllable in the sense that there are enough motors, actuators, etc. and observable in the sense that there are enough sensors in the system. However, it is also possible to study systems, which are partially controllable and/or partially observable. Therefore another important area of further work may be in this direction.

3. In this research it was also assumed that the motors, actuators, etc. and the sensors of the systems are in a perfect working order, i.e., not faulty in their operations. However, in real systems, it is natural that they fail from time to time. Therefore, the

extension of the present framework may include the fault tolerant and reconfigurable control system design.

4. Finally, a software package may be produced to facilitate the modelling, analysis, control synthesis and automatic code generation processes.

## APPENDIX A

**Reachable markings (states) of the supervisor (controlled model) shown in Fig. 7. 35, in the Chapter 7:**

The following table represents the reachable markings of the supervisor, shown in Fig. 7.35, in the chapter 7. Note that

$$M_0 \quad : \{p_{11}, \quad p_{14}, \quad p_{17}, \quad p_{20}, \quad p_{22}, \quad p_3, \quad p_6, \quad p_8\}$$

means initially (i.e., $M_0$), there is a token each in places $p_{11}$, $p_{14}$, $p_{17}$, $p_{20}$, $p_{22}$, $p_3$, $p_6$, and $p_8$.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_0$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_6$, | $p_8$} |
| $M_1$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_4$} |
| $M_2$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{19}$, | $p_4$} |
| $M_3$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{22}$, | $p_8$, | $p_{19}$, | $p_4$, | $p_5$} |
| $M_4$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$ | $p_{20}$, | $p_{22}$, | $p_3$, | $p_5$, | $p_9$} |
| $M_5$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{12}$, | $p_5$} |
| $M_6$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{15}$, | $p_5$} |
| $M_7$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_3$, | $p_8$, | $p_{15}$, | $p_{21}$, | $p_5$} |
| $M_8$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{18}$, | $p_5$} |
| $M_9$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_5$} |
| $M_{10}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_1$} |
| $M_{11}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_8$, | $p_1$, | $p_5$} |
| $M_{12}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_2$} |
| $M_{13}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{19}$, | $p_2$} |
| $M_{14}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{22}$, | $p_8$, | $p_{19}$, | $p_2$, | $p_5$} |
| $M_{15}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_5$, | $p_7$} |
| $M_{16}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{10}$, | $p_5$} |
| $M_{17}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{13}$, | $p_5$} |
| $M_{18}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_3$, | $p_8$, | $p_{13}$, | $p_{21}$, | $p_5$} |
| $M_{19}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{16}$, | $p_5$} |
| $M_{20}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_1$, | $p_{16}$} |
| $M_{21}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_6$, | $p_8$, | $p_{16}$} |
| $M_{22}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{16}$, | $p_4$} |
| $M_{23}$ | : {$p_{11}$, | $p_{14}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{16}$, | $p_{19}$, | $p_4$} |
| $M_{24}$ | : {$p_{11}$, | $p_{14}$, | $p_{22}$, | $p_8$, | $p_{16}$, | $p_{19}$, | $p_4$, | $p_5$} |
| $M_{25}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_{16}$, | $p_5$, | $p_9$} |
| $M_{26}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{12}$, | $p_{16}$, | $p_5$} |
| $M_{27}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_8$, | $p_{15}$, | $p_{16}$, | $p_5$} |
| $M_{28}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_1$, | $p_{15}$, | $p_{16}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{29}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{16}$} |
| $M_{30}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{15}$} |
| $M_{31}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{21}$} |
| $M_{32}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{18}$} |
| $M_{33}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{18}$, | $p_{4}$} |
| $M_{34}$ | : {$p_{11}$, | $p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{18}$, | $p_{19}$, | $p_{4}$} |
| $M_{35}$ | : {$p_{11}$, | $p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{36}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{37}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{5}$} |
| $M_{38}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{5}$} |
| $M_{39}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{15}$, | $p_{18}$} |
| $M_{40}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{18}$} |
| $M_{41}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{4}$} |
| $M_{42}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{4}$} |
| $M_{43}$ | : {$p_{11}$, | $p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{19}$, | $p_{4}$} |
| $M_{44}$ | : {$p_{11}$, | $p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{19}$, | $p_{21}$, | $p_{4}$} |
| $M_{45}$ | : {$p_{11}$, | $p_{17}$, | $p_{8}$, | $p_{15}$, | $p_{19}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{46}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{47}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{5}$} |
| $M_{48}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{21}$} |
| $M_{49}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{21}$} |
| $M_{50}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{18}$} |
| $M_{51}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$} |
| $M_{52}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{4}$} |
| $M_{53}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{19}$, | $p_{4}$} |
| $M_{54}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{55}$ | : {$p_{11}$, | $p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{56}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{15}$, | $p_{5}$, | $p_{9}$} |
| $M_{57}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{5}$} |
| $M_{58}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{15}$} |
| $M_{59}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$} |
| $M_{60}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{4}$} |
| $M_{61}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{19}$, | $p_{4}$} |
| $M_{62}$ | : {$p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{19}$, | $p_{21}$, | $p_{4}$} |
| $M_{63}$ | : {$p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{19}$, | $p_{4}$} |
| $M_{64}$ | : {$p_{11}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{4}$} |
| $M_{65}$ | : {$p_{11}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{66}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{67}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{5}$} |
| $M_{68}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{18}$} |
| $M_{69}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{18}$} |
| $M_{70}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{4}$} |
| $M_{71}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{4}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{72}$ | : {$p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{73}$ | : {$p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{74}$ | : {$p_{17}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{19}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{75}$ | : {$p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{76}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{77}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{5}$, | $p_{9}$} |
| $M_{78}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{9}$} |
| $M_{79}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{9}$} |
| $M_{80}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{15}$, | $p_{9}$} |
| $M_{81}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{15}$, | $p_{21}$, | $p_{9}$} |
| $M_{82}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{18}$, | $p_{9}$} |
| $M_{83}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{9}$} |
| $M_{84}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{4}$, | $p_{9}$} |
| $M_{85}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{86}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{4}$, | $p_{5}$} |
| $M_{87}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{4}$, | $p_{5}$} |
| $M_{88}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{89}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |
| $M_{90}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{4}$, | $p_{5}$} |
| $M_{91}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |
| $M_{92}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{4}$} |
| $M_{93}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |
| $M_{94}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |
| $M_{95}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{96}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{9}$} |
| $M_{97}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{4}$} |
| $M_{98}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{99}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{100}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{4}$, | $p_{9}$} |
| $M_{101}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{102}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{4}$, | $p_{5}$} |
| $M_{103}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{4}$, | $p_{9}$} |
| $M_{104}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{105}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{15}$, | $p_{9}$} |
| $M_{106}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{15}$, | $p_{21}$, | $p_{9}$} |
| $M_{107}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{18}$, | $p_{9}$} |
| $M_{108}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{18}$} |
| $M_{109}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$} |
| $M_{110}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{15}$} |
| $M_{111}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{15}$, | $p_{21}$} |
| $M_{112}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{18}$} |
| $M_{113}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{18}$, | $p_{5}$} |
| $M_{114}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{1}$, | $p_{15}$, | $p_{21}$, | $p_{5}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{115}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{15}$, | $p_{5}$} |
| $M_{116}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{5}$} |
| $M_{117}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{18}$, | $p_{5}$} |
| $M_{118}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{15}$, | $p_{18}$, | $p_{5}$} |
| $M_{119}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{9}$} |
| $M_{120}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{5}$, | $p_{9}$} |
| $M_{121}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{122}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{123}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{5}$} |
| $M_{124}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{15}$, | $p_{5}$, | $p_{9}$} |
| $M_{125}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{5}$} |
| $M_{126}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{5}$, | $p_{9}$} |
| $M_{127}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{2}$, | $p_{9}$} |
| $M_{128}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{2}$, | $p_{9}$} |
| $M_{129}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{2}$} |
| $M_{130}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{19}$, | $p_{2}$} |
| $M_{131}$ | : {$p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{19}$, | $p_{2}$, | $p_{21}$} |
| $M_{132}$ | : {$p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{19}$, | $p_{2}$} |
| $M_{133}$ | : {$p_{11}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{2}$} |
| $M_{134}$ | : {$p_{11}$, | $p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{19}$, | $p_{2}$} |
| $M_{135}$ | : {$p_{11}$, | $p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{19}$, | $p_{2}$, | $p_{21}$} |
| $M_{136}$ | : {$p_{11}$, | $p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{18}$, | $p_{19}$, | $p_{2}$} |
| $M_{137}$ | : {$p_{11}$, | $p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{18}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{138}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{139}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{18}$, | $p_{5}$} |
| $M_{140}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{13}$, | $p_{18}$, | $p_{5}$} |
| $M_{141}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{13}$, | $p_{18}$} |
| $M_{142}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{18}$} |
| $M_{143}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{13}$} |
| $M_{144}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{21}$} |
| $M_{145}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{21}$, | $p_{4}$} |
| $M_{146}$ | : {$p_{11}$, | $p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{19}$, | $p_{21}$, | $p_{4}$} |
| $M_{147}$ | : {$p_{11}$, | $p_{17}$, | $p_{8}$, | $p_{13}$, | $p_{19}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{148}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{149}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{5}$} |
| $M_{150}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{21}$} |
| $M_{151}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{21}$} |
| $M_{152}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{16}$} |
| $M_{153}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{4}$} |
| $M_{154}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{4}$} |
| $M_{155}$ | : {$p_{11}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{4}$} |
| $M_{156}$ | : {$p_{11}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{157}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{158}$ | $: \{p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{16},$ | $p_{5}\}$ |
| $M_{159}$ | $: \{p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{1},$ | $p_{12},$ | $p_{15},$ | $p_{16}\}$ |
| $M_{160}$ | $: \{p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{16}\}$ |
| $M_{161}$ | $: \{p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{16},$ | $p_{4}\}$ |
| $M_{162}$ | $: \{p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{16},$ | $p_{19},$ | $p_{4}\}$ |
| $M_{163}$ | $: \{p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{16},$ | $p_{19},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{164}$ | $: \{p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{12},$ | $p_{15},$ | $p_{16},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{165}$ | $: \{p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{12},$ | $p_{15},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{166}$ | $: \{p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{12},$ | $p_{15},$ | $p_{21},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{167}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{1},$ | $p_{12},$ | $p_{15}$ | $p_{21},$ | $p_{9}\}$ |
| $M_{168}$ | $: \{p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{6},$ | $p_{12},$ | $p_{15}$ | $p_{21},$ | $p_{9}\}$ |
| $M_{169}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{12}$ | $p_{18},$ | $p_{9}\}$ |
| $M_{170}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{15}$ | $p_{18},$ | $p_{9}\}$ |
| $M_{171}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{15},$ | $p_{18}$ | $p_{4},$ | $p_{9}\}$ |
| $M_{172}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{15},$ | $p_{18},$ | $p_{4},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{173}$ | $: \{p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{18},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{174}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{12},$ | $p_{18},$ | $p_{4},$ | $p_{9}\}$ |
| $M_{175}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{12},$ | $p_{18},$ | $p_{4},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{176}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{12},$ | $p_{15},$ | $p_{21},$ | $p_{4},$ | $p_{9}\}$ |
| $M_{177}$ | $: \{p_{17},$ | $p_{20},$ | $p_{12},$ | $p_{15},$ | $p_{21},$ | $p_{4},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{178}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{1},$ | $p_{12},$ | $p_{18},$ | $p_{9}\}$ |
| $M_{179}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{1},$ | $p_{15},$ | $p_{18},$ | $p_{9}\}$ |
| $M_{180}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{15},$ | $p_{18},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{181}$ | $: \{p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{12},$ | $p_{15},$ | $p_{18},$ | $p_{5}\}$ |
| $M_{182}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{12},$ | $p_{18},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{183}$ | $: \{p_{17},$ | $p_{20},$ | $p_{1},$ | $p_{12},$ | $p_{15},$ | $p_{21},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{184}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{12},$ | $p_{15},$ | $p_{2},$ | $p_{21},$ | $p_{9}\}$ |
| $M_{185}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{12},$ | $p_{18},$ | $p_{2},$ | $p_{9}\}$ |
| $M_{186}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{15},$ | $p_{18},$ | $p_{2},$ | $p_{9}\}$ |
| $M_{187}$ | $: \{p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{18},$ | $p_{2}\}$ |
| $M_{188}$ | $: \{p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{18},$ | $p_{19},$ | $p_{2}\}$ |
| $M_{189}$ | $: \{p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{18},$ | $p_{19},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{190}$ | $: \{p_{17},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{19},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{191}$ | $: \{p_{17},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{19},$ | $p_{2},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{192}$ | $: \{p_{14},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{18},$ | $p_{19},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{193}$ | $: \{p_{11},$ | $p_{22},$ | $p_{8},$ | $p_{15},$ | $p_{18},$ | $p_{19},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{194}$ | $: \{p_{11},$ | $p_{17},$ | $p_{22},$ | $p_{8},$ | $p_{15},$ | $p_{19},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{195}$ | $: \{p_{11},$ | $p_{17},$ | $p_{8},$ | $p_{15},$ | $p_{19},$ | $p_{2},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{196}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{15},$ | $p_{21},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{197}$ | $: \{p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{198}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{8},$ | $p_{1},$ | $p_{10},$ | $p_{15},$ | $p_{21}\}$ |
| $M_{199}$ | $: \{p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{21}\}$ |
| $M_{200}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{18}\}$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{201}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$} |
| $M_{202}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{4}$} |
| $M_{203}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{4}$} |
| $M_{204}$ | : {$p_{11}$, | $p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{19}$, | $p_{4}$} |
| $M_{205}$ | : {$p_{11}$, | $p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{206}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{13}$, | $p_{5}$, | $p_{9}$} |
| $M_{207}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{5}$} |
| $M_{208}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$} |
| $M_{209}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$} |
| $M_{210}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{4}$} |
| $M_{211}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{19}$, | $p_{4}$} |
| $M_{212}$ | : {$p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{19}$, | $p_{21}$, | $p_{4}$} |
| $M_{213}$ | : {$p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{19}$, | $p_{4}$} |
| $M_{214}$ | : {$p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{215}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{216}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{16}$, | $p_{9}$} |
| $M_{217}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{16}$, | $p_{9}$} |
| $M_{218}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{15}$, | $p_{16}$, | $p_{9}$} |
| $M_{219}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{9}$} |
| $M_{220}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{221}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{222}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{16}$, | $p_{4}$, | $p_{9}$} |
| $M_{223}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{224}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{15}$, | $p_{16}$, | $p_{9}$} |
| $M_{225}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{226}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{5}$} |
| $M_{227}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{228}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{16}$, | $p_{2}$, | $p_{9}$} |
| $M_{229}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{9}$} |
| $M_{230}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{2}$} |
| $M_{231}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |
| $M_{232}$ | : {$p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{233}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{234}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{15}$, | $p_{5}$, | $p_{7}$} |
| $M_{235}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{236}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{237}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{238}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{5}$} |
| $M_{239}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{5}$} |
| $M_{240}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{15}$} |
| $M_{241}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$} |
| $M_{242}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{4}$} |
| $M_{243}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{19}$, | $p_{4}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{244}$ | $: \{p_{17},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{19},$ | $p_{21},$ | $p_{4}\}$ |
| $M_{245}$ | $: \{p_{14},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{18},$ | $p_{19},$ | $p_{4}\}$ |
| $M_{246}$ | $: \{p_{11},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{13},$ | $p_{18},$ | $p_{19},$ | $p_{4}\}$ |
| $M_{247}$ | $: \{p_{11},$ | $p_{22},$ | $p_{8},$ | $p_{13},$ | $p_{18},$ | $p_{19},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{248}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{13},$ | $p_{18},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{249}$ | $: \{p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{18},$ | $p_{5}\}$ |
| $M_{250}$ | $: \{p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{1},$ | $p_{12},$ | $p_{13},$ | $p_{18}\}$ |
| $M_{251}$ | $: \{p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{18}\}$ |
| $M_{252}$ | $: \{p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{18},$ | $p_{4}\}$ |
| $M_{253}$ | $: \{p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{18},$ | $p_{19},$ | $p_{4}\}$ |
| $M_{254}$ | $: \{p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{18},$ | $p_{19},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{255}$ | $: \{p_{17},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{19},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{256}$ | $: \{p_{17},$ | $p_{8},$ | $p_{12},$ | $p_{13},$ | $p_{19},$ | $p_{21},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{257}$ | $: \{p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{12},$ | $p_{13},$ | $p_{21},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{258}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{1},$ | $p_{12},$ | $p_{13},$ | $p_{21},$ | $p_{9}\}$ |
| $M_{259}$ | $: \{p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{6},$ | $p_{12},$ | $p_{13},$ | $p_{21},$ | $p_{9}\}$ |
| $M_{260}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{12},$ | $p_{13},$ | $p_{21},$ | $p_{4},$ | $p_{9}\}$ |
| $M_{261}$ | $: \{p_{17},$ | $p_{20},$ | $p_{12},$ | $p_{13},$ | $p_{21},$ | $p_{4},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{262}$ | $: \{p_{17},$ | $p_{20},$ | $p_{1},$ | $p_{12},$ | $p_{13},$ | $p_{21},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{263}$ | $: \{p_{17},$ | $p_{20},$ | $p_{6},$ | $p_{12},$ | $p_{13},$ | $p_{2},$ | $p_{21},$ | $p_{9}\}$ |
| $M_{264}$ | $: \{p_{17},$ | $p_{20},$ | $p_{12},$ | $p_{13},$ | $p_{2},$ | $p_{21},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{265}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{12},$ | $p_{16},$ | $p_{2},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{266}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{15},$ | $p_{16},$ | $p_{2},$ | $p_{5},$ | $p_{9}\}$ |
| $M_{267}$ | $: \{p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{16},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{268}$ | $: \{p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{269}$ | $: \{p_{17},$ | $p_{20},$ | $p_{8},$ | $p_{12},$ | $p_{15},$ | $p_{2},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{270}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{18},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{271}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{15},$ | $p_{18},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{272}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{15},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{273}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{8},$ | $p_{15},$ | $p_{2},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{274}$ | $: \{p_{11},$ | $p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{18},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{275}$ | $: \{p_{11},$ | $p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{276}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{277}$ | $: \{p_{14},$ | $p_{17},$ | $p_{22},$ | $p_{8},$ | $p_{12},$ | $p_{19},$ | $p_{2},$ | $p_{5}\}$ |
| $M_{278}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{12},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{279}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{15},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{280}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{1},$ | $p_{15},$ | $p_{7}\}$ |
| $M_{281}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{15},$ | $p_{7}\}$ |
| $M_{282}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{3},$ | $p_{6},$ | $p_{15},$ | $p_{21},$ | $p_{7}\}$ |
| $M_{283}$ | $: \{p_{11},$ | $p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{18},$ | $p_{7}\}$ |
| $M_{284}$ | $: \{p_{11},$ | $p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{7}\}$ |
| $M_{285}$ | $: \{p_{11},$ | $p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{4},$ | $p_{7}\}$ |
| $M_{286}$ | $: \{p_{11},$ | $p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{287}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{4}$, | $p_{5}$} |
| $M_{288}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{4}$, | $p_{5}$} |
| $M_{289}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{290}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{291}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{292}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{5}$, | $p_{9}$} |
| $M_{293}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{9}$} |
| $M_{294}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{9}$} |
| $M_{295}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{9}$} |
| $M_{296}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{21}$, | $p_{9}$} |
| $M_{297}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{16}$, | $p_{9}$} |
| $M_{298}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{16}$, | $p_{4}$, | $p_{9}$} |
| $M_{299}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{300}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{301}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{302}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{9}$} |
| $M_{303}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{4}$} |
| $M_{304}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{305}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{306}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{4}$, | $p_{9}$} |
| $M_{307}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{308}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{4}$, | $p_{5}$} |
| $M_{309}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{4}$, | $p_{9}$} |
| $M_{310}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{311}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{9}$} |
| $M_{312}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{21}$, | $p_{9}$} |
| $M_{313}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{16}$, | $p_{9}$} |
| $M_{314}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{16}$} |
| $M_{315}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{16}$, | $p_{5}$} |
| $M_{316}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{15}$, | $p_{16}$, | $p_{5}$} |
| $M_{317}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{318}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{319}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{5}$} |
| $M_{320}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{13}$, | $p_{5}$, | $p_{9}$} |
| $M_{321}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{5}$} |
| $M_{322}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{5}$, | $p_{9}$} |
| $M_{323}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{2}$, | $p_{9}$} |
| $M_{324}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{2}$, | $p_{9}$} |
| $M_{325}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{2}$} |
| $M_{326}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{19}$, | $p_{2}$} |
| $M_{327}$ | : {$p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{19}$, | $p_{2}$, | $p_{21}$} |
| $M_{328}$ | : {$p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |
| $M_{329}$ | : {$p_{11}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{330}$ | : {$p_{11}$, | $p_{22}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{331}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{332}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{5}$} |
| $M_{333}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{16}$} |
| $M_{334}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$} |
| $M_{335}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{4}$} |
| $M_{336}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{4}$} |
| $M_{337}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{338}$ | : {$p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{339}$ | : {$p_{17}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{19}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{340}$ | : {$p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{341}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{342}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{18}$, | $p_{9}$} |
| $M_{343}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{18}$, | $p_{9}$} |
| $M_{344}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{18}$, | $p_{9}$} |
| $M_{345}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |
| $M_{346}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{347}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |
| $M_{348}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |
| $M_{349}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{350}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{18}$, | $p_{9}$} |
| $M_{351}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{352}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{5}$} |
| $M_{353}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{354}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{355}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{356}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{2}$} |
| $M_{357}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_{2}$} |
| $M_{358}$ | : {$p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{359}$ | : {$p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{360}$ | : {$p_{17}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{19}$, | $p_{2}$, | $p_{21}$, | $p_{5}$} |
| $M_{361}$ | : {$p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{362}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{363}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{16}$, | $p_{7}$} |
| $M_{364}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{16}$, | $p_{7}$} |
| $M_{365}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{15}$, | $p_{16}$, | $p_{7}$} |
| $M_{366}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{367}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{4}$, | $p_{7}$} |
| $M_{368}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{7}$} |
| $M_{369}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{4}$} |
| $M_{370}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{18}$, | $p_{4}$} |
| $M_{371}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{18}$, | $p_{4}$} |
| $M_{372}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{373}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{10},$ | $p_{18},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{374}$ | $: \{p_{17},$ | $p_{20},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{21},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{375}$ | $: \{p_{11},$ | $p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{18},$ | $p_{4},$ | $p_{7}\}$ |
| $M_{376}$ | $: \{p_{11},$ | $p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{18},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{377}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{15},$ | $p_{21},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{378}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{15},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{379}$ | $: \{p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{380}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{15},$ | $p_{16},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{381}$ | $: \{p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{16},$ | $p_{4},$ | $p_{5}\}$ |
| $M_{382}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{3},$ | $p_{6},$ | $p_{12},$ | $p_{7}\}$ |
| $M_{383}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{12},$ | $p_{4},$ | $p_{7}\}$ |
| $M_{384}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{12},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{385}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{12},$ | $p_{16},$ | $p_{4},$ | $p_{7}\}$ |
| $M_{386}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{12},$ | $p_{16},$ | $p_{4},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{387}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{1},$ | $p_{15},$ | $p_{16},$ | $p_{7}\}$ |
| $M_{388}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{15},$ | $p_{16},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{389}$ | $: \{p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{10},$ | $p_{15},$ | $p_{16},$ | $p_{5}\}$ |
| $M_{390}$ | $: \{p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{10},$ | $p_{15},$ | $p_{5}\}$ |
| $M_{391}$ | $: \{p_{17},$ | $p_{20},$ | $p_{8},$ | $p_{1},$ | $p_{10},$ | $p_{15},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{392}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{10},$ | $p_{18},$ | $p_{5}\}$ |
| $M_{393}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{13},$ | $p_{18},$ | $p_{5}\}$ |
| $M_{394}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{13},$ | $p_{5}\}$ |
| $M_{395}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{8},$ | $p_{1},$ | $p_{13},$ | $p_{21},$ | $p_{5}\}$ |
| $M_{396}$ | $: \{p_{11},$ | $p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{16},$ | $p_{5}\}$ |
| $M_{397}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{8},$ | $p_{1},$ | $p_{10},$ | $p_{5}\}$ |
| $M_{398}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{15},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{399}$ | $: \{p_{11},$ | $p_{17},$ | $p_{20},$ | $p_{1},$ | $p_{15},$ | $p_{21},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{400}$ | $: \{p_{11},$ | $p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{18},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{401}$ | $: \{p_{11},$ | $p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{402}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{1},$ | $p_{12},$ | $p_{7}\}$ |
| $M_{403}$ | $: \{p_{14},$ | $p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{12},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{404}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{1},$ | $p_{12},$ | $p_{16},$ | $p_{5},$ | $p_{7}\}$ |
| $M_{405}$ | $: \{p_{14},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{12},$ | $p_{16},$ | $p_{2},$ | $p_{7}\}$ |
| $M_{406}$ | $: \{p_{11},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{15},$ | $p_{16},$ | $p_{2},$ | $p_{7}\}$ |
| $M_{407}$ | $: \{p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{16},$ | $p_{2}\}$ |
| $M_{408}$ | $: \{p_{17},$ | $p_{20},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{2}\}$ |
| $M_{409}$ | $: \{p_{17},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{19},$ | $p_{2}\}$ |
| $M_{410}$ | $: \{p_{17},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{15},$ | $p_{19},$ | $p_{2},$ | $p_{21}\}$ |
| $M_{411}$ | $: \{p_{14},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{10},$ | $p_{18},$ | $p_{19},$ | $p_{2}\}$ |
| $M_{412}$ | $: \{p_{11},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{13},$ | $p_{18},$ | $p_{19},$ | $p_{2}\}$ |
| $M_{413}$ | $: \{p_{11},$ | $p_{17},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{13},$ | $p_{19},$ | $p_{2}\}$ |
| $M_{414}$ | $: \{p_{11},$ | $p_{17},$ | $p_{6},$ | $p_{8},$ | $p_{13},$ | $p_{19},$ | $p_{2},$ | $p_{21}\}$ |
| $M_{415}$ | $: \{p_{11},$ | $p_{14},$ | $p_{22},$ | $p_{6},$ | $p_{8},$ | $p_{16},$ | $p_{19},$ | $p_{2}\}$ |

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|---|---|---|---|---|---|---|---|---|
| $M_{416}$ | : $\{p_{11}$, | $p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}\}$ |
| $M_{417}$ | : $\{p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{16}$, | $p_{5}$, | $p_{7}\}$ |
| $M_{418}$ | : $\{p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{16}$, | $p_{5}\}$ |
| $M_{419}$ | : $\{p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{5}\}$ |
| $M_{420}$ | : $\{p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{13}$, | $p_{16}\}$ |
| $M_{421}$ | : $\{p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{16}\}$ |
| $M_{422}$ | : $\{p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{4}\}$ |
| $M_{423}$ | : $\{p_{11}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{4}\}$ |
| $M_{424}$ | : $\{p_{11}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{4}$, | $p_{5}\}$ |
| $M_{425}$ | : $\{p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{426}$ | : $\{p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{5}\}$ |
| $M_{427}$ | : $\{p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{16}\}$ |
| $M_{428}$ | : $\{p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}\}$ |
| $M_{429}$ | : $\{p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{4}\}$ |
| $M_{430}$ | : $\{p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{4}\}$ |
| $M_{431}$ | : $\{p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{4}$, | $p_{5}\}$ |
| $M_{432}$ | : $\{p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{433}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{434}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{9}\}$ |
| $M_{435}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{9}\}$ |
| $M_{436}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{4}$, | $p_{9}\}$ |
| $M_{437}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{4}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{438}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{439}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{2}$, | $p_{9}\}$ |
| $M_{440}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{2}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{441}$ | : $\{p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{9}\}$ |
| $M_{442}$ | : $\{p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{9}\}$ |
| $M_{443}$ | : $\{p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{9}\}$ |
| $M_{444}$ | : $\{p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{445}$ | : $\{p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{446}$ | : $\{p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{9}\}$ |
| $M_{447}$ | : $\{p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{9}\}$ |
| $M_{448}$ | : $\{p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}\}$ |
| $M_{449}$ | : $\{p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{5}\}$ |
| $M_{450}$ | : $\{p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{2}\}$ |
| $M_{451}$ | : $\{p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{2}\}$ |
| $M_{452}$ | : $\{p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}\}$ |
| $M_{453}$ | : $\{p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{7}\}$ |
| $M_{454}$ | : $\{p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{5}$, | $p_{7}\}$ |
| $M_{455}$ | : $\{p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{7}\}$ |
| $M_{456}$ | : $\{p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{7}\}$ |
| $M_{457}$ | : $\{p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{7}\}$ |
| $M_{458}$ | : $\{p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{7}\}$ |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{459}$ | : {$p_{17}$, | $p_{20}$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_4$, | $p_5$, | $p_7$} |
| $M_{460}$ | : {$p_{17}$, | $p_{20}$, | $p_1$, | $p_{12}$, | $p_{13}$, | $p_{21}$, | $p_5$, | $p_7$} |
| $M_{461}$ | : {$p_{17}$, | $p_{20}$, | $p_6$, | $p_{12}$, | $p_{13}$, | $p_2$, | $p_{21}$, | $p_7$} |
| $M_{462}$ | : {$p_{17}$, | $p_{20}$, | $p_{12}$, | $p_{13}$, | $p_2$, | $p_{21}$, | $p_5$, | $p_7$} |
| $M_{463}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{16}$, | $p_2$, | $p_5$, | $p_7$} |
| $M_{464}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{16}$, | $p_2$, | $p_5$, | $p_7$} |
| $M_{465}$ | : {$p_{20}$, | $p_{22}$, | $p_8$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_2$, | $p_5$} |
| $M_{466}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_8$, | $p_{10}$, | $p_{15}$, | $p_2$, | $p_5$} |
| $M_{467}$ | : {$p_{17}$, | $p_{22}$, | $p_8$, | $p_{10}$, | $p_{15}$, | $p_{19}$, | $p_2$, | $p_5$} |
| $M_{468}$ | : {$p_{17}$, | $p_8$, | $p_{10}$, | $p_{15}$, | $p_{19}$, | $p_2$, | $p_{21}$, | $p_5$} |
| $M_{469}$ | : {$p_{14}$, | $p_{22}$, | $p_8$, | $p_{10}$, | $p_{18}$, | $p_{19}$, | $p_2$, | $p_5$} |
| $M_{470}$ | : {$p_{11}$, | $p_{22}$, | $p_8$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_2$, | $p_5$} |
| $M_{471}$ | : {$p_{11}$, | $p_{17}$, | $p_{22}$, | $p_8$, | $p_{13}$, | $p_{19}$, | $p_2$, | $p_5$} |
| $M_{472}$ | : {$p_{11}$, | $p_{17}$, | $p_8$, | $p_{13}$, | $p_{19}$, | $p_2$, | $p_{21}$, | $p_5$} |
| $M_{473}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_3$, | $p_{13}$, | $p_{21}$, | $p_5$, | $p_7$} |
| $M_{474}$ | : {$p_{17}$, | $p_{20}$, | $p_3$, | $p_8$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_5$} |
| $M_{475}$ | : {$p_{17}$, | $p_{20}$, | $p_6$, | $p_8$, | $p_1$, | $p_{10}$, | $p_{13}$, | $p_{21}$} |
| $M_{476}$ | : {$p_{17}$, | $p_{20}$, | $p_3$, | $p_6$, | $p_8$, | $p_{10}$, | $p_{13}$, | $p_{21}$} |
| $M_{477}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_6$, | $p_8$, | $p_{10}$, | $p_{16}$} |
| $M_{478}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{10}$, | $p_{16}$, | $p_4$} |
| $M_{479}$ | : {$p_{14}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{10}$, | $p_{16}$, | $p_{19}$, | $p_4$} |
| $M_{480}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_6$, | $p_8$, | $p_{10}$, | $p_{19}$, | $p_4$} |
| $M_{481}$ | : {$p_{14}$, | $p_{22}$, | $p_8$, | $p_{10}$, | $p_{16}$, | $p_{19}$, | $p_4$, | $p_5$} |
| $M_{482}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_{10}$, | $p_{16}$, | $p_5$, | $p_9$} |
| $M_{483}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_1$, | $p_{10}$, | $p_{16}$, | $p_9$} |
| $M_{484}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_6$, | $p_{10}$, | $p_{16}$, | $p_9$} |
| $M_{485}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_3$, | $p_6$, | $p_{13}$, | $p_{16}$, | $p_9$} |
| $M_{486}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_{13}$, | $p_{16}$, | $p_4$, | $p_9$} |
| $M_{487}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{16}$, | $p_4$, | $p_5$, | $p_9$} |
| $M_{488}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_{10}$, | $p_{16}$, | $p_4$, | $p_9$} |
| $M_{489}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{16}$, | $p_4$, | $p_5$, | $p_9$} |
| $M_{490}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_1$, | $p_{13}$, | $p_{16}$, | $p_9$} |
| $M_{491}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_1$, | $p_{13}$, | $p_{16}$, | $p_5$, | $p_9$} |
| $M_{492}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_1$, | $p_{10}$, | $p_{16}$, | $p_5$, | $p_9$} |
| $M_{493}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_{10}$, | $p_{16}$, | $p_2$, | $p_9$} |
| $M_{494}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_6$, | $p_{13}$, | $p_{16}$, | $p_2$, | $p_9$} |
| $M_{495}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{16}$, | $p_2$, | $p_5$, | $p_9$} |
| $M_{496}$ | : {$p_{20}$, | $p_{22}$, | $p_8$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_2$, | $p_5$} |
| $M_{497}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_8$, | $p_{12}$, | $p_{13}$, | $p_2$, | $p_5$} |
| $M_{498}$ | : {$p_{17}$, | $p_{20}$, | $p_8$, | $p_{12}$, | $p_{13}$, | $p_2$, | $p_{21}$, | $p_5$} |
| $M_{499}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_8$, | $p_{12}$, | $p_{16}$, | $p_2$, | $p_5$} |
| $M_{500}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_8$, | $p_{15}$, | $p_{16}$, | $p_2$, | $p_5$} |
| $M_{501}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_2$, | $p_5$, | $p_9$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{502}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{503}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{504}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{505}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{506}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{507}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{508}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{509}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{4}$} |
| $M_{510}$ | : {$p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{21}$, | $p_{4}$} |
| $M_{511}$ | : {$p_{17}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{512}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{513}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{9}$} |
| $M_{514}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{9}$} |
| $M_{515}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{9}$} |
| $M_{516}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{517}$ | : {$p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{518}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{9}$} |
| $M_{519}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{520}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$} |
| $M_{521}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{16}$} |
| $M_{522}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$} |
| $M_{523}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{13}$} |
| $M_{524}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{13}$, | $p_{21}$} |
| $M_{525}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{16}$, | $p_{5}$} |
| $M_{526}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{13}$, | $p_{16}$, | $p_{5}$} |
| $M_{527}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{5}$} |
| $M_{528}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{21}$} |
| $M_{529}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{16}$, | $p_{2}$} |
| $M_{530}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{2}$} |
| $M_{531}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{2}$} |
| $M_{532}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{2}$, | $p_{21}$} |
| $M_{533}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{16}$, | $p_{2}$} |
| $M_{534}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{16}$, | $p_{2}$, | $p_{5}$} |
| $M_{535}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{5}$} |
| $M_{536}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{2}$, | $p_{5}$} |
| $M_{537}$ | : {$p_{11}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |
| $M_{538}$ | : {$p_{11}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{539}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{540}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{5}$} |
| $M_{541}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{5}$} |
| $M_{542}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$} |
| $M_{543}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$} |
| $M_{544}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{4}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{545}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{4}$} |
| $M_{546}$ | : {$p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{547}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{5}$, | $p_{9}$} |
| $M_{548}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{9}$} |
| $M_{549}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{9}$} |
| $M_{550}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{4}$, | $p_{9}$} |
| $M_{551}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{552}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{5}$, | $p_{9}$} |
| $M_{553}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{9}$} |
| $M_{554}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{555}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{4}$, | $p_{5}$} |
| $M_{556}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{5}$} |
| $M_{557}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{2}$} |
| $M_{558}$ | : {$p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{2}$} |
| $M_{559}$ | : {$p_{17}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{2}$, | $p_{21}$} |
| $M_{560}$ | : {$p_{14}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |
| $M_{561}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{19}$, | $p_{2}$} |
| $M_{562}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{563}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{5}$, | $p_{7}$} |
| $M_{564}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{13}$, | $p_{5}$, | $p_{7}$} |
| $M_{565}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{7}$} |
| $M_{566}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{7}$} |
| $M_{567}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{21}$, | $p_{7}$} |
| $M_{568}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{16}$, | $p_{7}$} |
| $M_{569}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{570}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{571}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{7}$} |
| $M_{572}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{573}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{4}$, | $p_{7}$} |
| $M_{574}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{575}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{21}$, | $p_{7}$} |
| $M_{576}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{16}$, | $p_{7}$} |
| $M_{577}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{7}$} |
| $M_{578}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{579}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{580}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{13}$, | $p_{5}$, | $p_{7}$} |
| $M_{581}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{2}$, | $p_{7}$} |
| $M_{582}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{7}$} |
| $M_{583}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{584}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{2}$, | $p_{7}$} |
| $M_{585}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{2}$} |
| $M_{586}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{2}$, | $p_{5}$} |
| $M_{587}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{588}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{589}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{16}$, | $p_{2}$, | $p_{5}$} |
| $M_{590}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$} |
| $M_{591}$ | : {$p_{14}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{592}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{593}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{16}$, | $p_{7}$} |
| $M_{594}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{16}$, | $p_{7}$} |
| $M_{595}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{16}$, | $p_{7}$} |
| $M_{596}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$} |
| $M_{597}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{4}$} |
| $M_{598}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{4}$} |
| $M_{599}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{600}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{601}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{9}$} |
| $M_{602}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{9}$} |
| $M_{603}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{9}$} |
| $M_{604}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{605}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{606}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{9}$} |
| $M_{607}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{608}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$} |
| $M_{609}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{610}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{611}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{7}$} |
| $M_{612}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{4}$, | $p_{7}$} |
| $M_{613}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{614}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{615}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{616}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{16}$, | $p_{7}$} |
| $M_{617}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{16}$} |
| $M_{618}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{5}$} |
| $M_{619}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{620}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{7}$} |
| $M_{621}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{5}$, | $p_{7}$} |
| $M_{622}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{623}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{624}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{625}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{2}$} |
| $M_{626}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |
| $M_{627}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{628}$ | : {$p_{17}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{629}$ | : {$p_{17}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{19}$, | $p_{2}$, | $p_{21}$, | $p_{5}$} |
| $M_{630}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{631}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{7}$} |
| $M_{632}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{7}$} |
| $M_{633}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{7}$} |
| $M_{634}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{635}$ | : {$p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{636}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{7}$} |
| $M_{637}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{638}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{639}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{640}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$} |
| $M_{641}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{5}$} |
| $M_{642}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{5}$} |
| $M_{643}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{644}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{645}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{646}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{5}$, | $p_{7}$} |
| $M_{647}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{7}$} |
| $M_{648}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{7}$} |
| $M_{649}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{4}$, | $p_{7}$} |
| $M_{650}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{651}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{5}$, | $p_{7}$} |
| $M_{652}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{7}$} |
| $M_{653}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{654}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{655}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{7}$} |
| $M_{656}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{7}$} |
| $M_{657}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{658}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{659}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{660}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{661}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{662}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{2}$, | $p_{7}$} |
| $M_{663}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{664}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{5}$} |
| $M_{665}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{18}$} |
| $M_{666}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$} |
| $M_{667}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{4}$} |
| $M_{668}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_{4}$} |
| $M_{669}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{670}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{671}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{9}$} |
| $M_{672}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{9}$} |
| $M_{673}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{674}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{675}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{676}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{677}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{678}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |
| $M_{679}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{5}$} |
| $M_{680}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{2}$} |
| $M_{681}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_{2}$} |
| $M_{682}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{683}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{684}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{7}$} |
| $M_{685}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{7}$} |
| $M_{686}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{687}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{688}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{689}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{690}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{691}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$} |
| $M_{692}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{13}$, | $p_{18}$, | $p_{7}$} |
| $M_{693}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{13}$, | $p_{18}$, | $p_{7}$} |
| $M_{694}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{695}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{696}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{697}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{698}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{699}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{700}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{18}$, | $p_{7}$} |
| $M_{701}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{18}$, | $p_{7}$} |
| $M_{702}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{703}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{704}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{705}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{706}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{707}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{708}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{7}$} |
| $M_{709}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{7}$} |
| $M_{710}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{7}$} |
| $M_{711}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{712}$ | : {$p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{713}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{7}$} |
| $M_{714}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{715}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{5}$, | $p_{7}$} |
| $M_{716}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{7}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{717}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{7}$} |
| $M_{718}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{4}$, | $p_{7}$} |
| $M_{719}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{720}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{5}$, | $p_{7}$} |
| $M_{721}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{7}$} |
| $M_{722}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{723}$ | : {$p_{17}$, | $p_{20}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$} |
| $M_{724}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{18}$, | $p_{2}$, | $p_{5}$} |
| $M_{725}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$} |
| $M_{726}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{727}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{728}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{7}$} |
| $M_{729}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{7}$} |
| $M_{730}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{731}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{,15}$ | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{732}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{733}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{734}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{735}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{736}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{737}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{738}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{739}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{7}$} |
| $M_{740}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{7}$} |
| $M_{741}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{4}$, | $p_{7}$} |
| $M_{742}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{743}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{5}$, | $p_{7}$} |
| $M_{744}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{2}$, | $p_{7}$} |
| $M_{745}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{746}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{7}$} |
| $M_{747}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{7}$} |
| $M_{748}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{749}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{750}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{751}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{752}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{753}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{21}$} |
| $M_{754}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{18}$, | $p_{2}$} |
| $M_{755}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{13}$, | $p_{18}$, | $p_{2}$} |
| $M_{756}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{19}$, | $p_{2}$} |
| $M_{757}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{2}$, | $p_{7}$} |
| $M_{758}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{7}$} |
| $M_{759}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{760}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{2}$, | $p_{7}$} |
| $M_{761}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{762}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{7}$} |
| $M_{763}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{7}$} |
| $M_{764}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{765}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{766}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{767}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{768}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{769}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$} |
| $M_{770}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{771}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{772}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{773}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{9}$} |
| $M_{774}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{9}$} |
| $M_{775}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{9}$} |
| $M_{776}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{777}$ | : {$p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{778}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{9}$} |
| $M_{779}$ | : {$p_{17}$, | $p_{20}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{780}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{5}$, | $p_{9}$} |
| $M_{781}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{9}$} |
| $M_{782}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{9}$} |
| $M_{783}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{4}$, | $p_{9}$} |
| $M_{784}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{785}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{5}$, | $p_{9}$} |
| $M_{786}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{9}$} |
| $M_{787}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{788}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{789}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{9}$} |
| $M_{790}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{9}$} |
| $M_{791}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{9}$} |
| $M_{792}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{793}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{794}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{9}$} |
| $M_{795}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{796}$ | : {$p_{14}$, | $p_{17}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{19}$, | $p_{2}$} |
| $M_{797}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{13}$, | $p_{2}$, | $p_{21}$} |
| $M_{798}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{16}$, | $p_{2}$} |
| $M_{799}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{16}$, | $p_{2}$} |
| $M_{800}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{2}$} |
| $M_{801}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{2}$, | $p_{21}$} |
| $M_{802}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{18}$, | $p_{2}$} |

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|---|---|---|---|---|---|---|---|---|
| $M_{803}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{2}$} |
| $M_{804}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{13}$, | $p_{2}$, | $p_{21}$, | $p_{9}$} |
| $M_{805}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{16}$, | $p_{2}$, | $p_{9}$} |
| $M_{806}$ | : {$p_{11}$, | $p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{2}$, | $p_{9}$} |
| $M_{807}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{15}$, | $p_{21}$, | $p_{7}$} |
| $M_{808}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{18}$, | $p_{7}$} |
| $M_{809}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{18}$} |
| $M_{810}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{811}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{812}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{813}$ | : {$p_{14}$, | $p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{814}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{815}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{9}$} |
| $M_{816}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{9}$} |
| $M_{817}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |
| $M_{818}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{819}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{820}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{821}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{13}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{822}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{18}$} |
| $M_{823}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$} |
| $M_{824}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{4}$} |
| $M_{825}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{4}$} |
| $M_{826}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{4}$, | $p_{5}$} |
| $M_{827}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{828}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{9}$} |
| $M_{829}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{9}$} |
| $M_{830}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |
| $M_{831}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{832}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{833}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{834}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{835}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$} |
| $M_{836}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{5}$} |
| $M_{837}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{2}$} |
| $M_{838}$ | : {$p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{2}$} |
| $M_{839}$ | : {$p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{19}$, | $p_{2}$, | $p_{5}$} |
| $M_{840}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{841}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{7}$} |
| $M_{842}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{7}$} |
| $M_{843}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{844}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{845}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |

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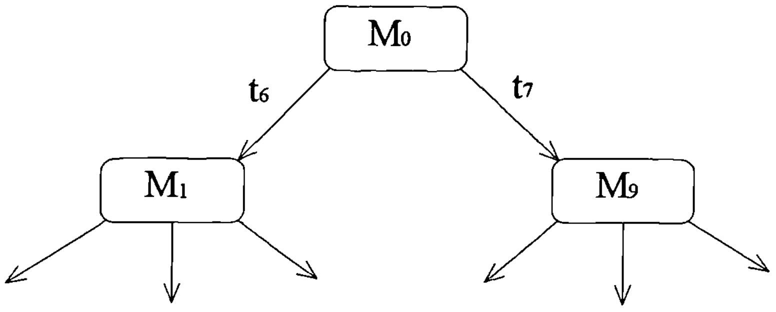
|---|---|---|---|---|---|---|---|---|
| $M_{846}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{847}$ | : {$p_{20}$, | $p_{22}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{848}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{10}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$} |
| $M_{849}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{15}$, | $p_{18}$, | $p_{7}$} |
| $M_{850}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{15}$, | $p_{18}$, | $p_{7}$} |
| $M_{851}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{852}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{853}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{854}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{855}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{856}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{18}$, | $p_{7}$} |
| $M_{857}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{18}$, | $p_{7}$} |
| $M_{858}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{859}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{860}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{861}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{862}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{863}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{7}$} |
| $M_{864}$ | : {$p_{17}$, | $p_{20}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{7}$} |
| $M_{865}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{7}$} |
| $M_{866}$ | : {$p_{17}$, | $p_{20}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{867}$ | : {$p_{17}$, | $p_{20}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{868}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{7}$} |
| $M_{869}$ | : {$p_{17}$, | $p_{20}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{7}$} |
| $M_{870}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{7}$} |
| $M_{871}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{7}$} |
| $M_{872}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{4}$, | $p_{7}$} |
| $M_{873}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{874}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{5}$, | $p_{7}$} |
| $M_{875}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{7}$} |
| $M_{876}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{877}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{7}$} |
| $M_{878}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{7}$} |
| $M_{879}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{7}$} |
| $M_{880}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |
| $M_{881}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{7}$} |
| $M_{882}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{7}$} |
| $M_{883}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{884}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{885}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{7}$} |
| $M_{886}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{7}$} |
| $M_{887}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{7}$} |
| $M_{888}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{7}$} |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $M_{889}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{7}$} |
| $M_{890}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{7}$} |
| $M_{891}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{7}$} |
| $M_{892}$ | : {$p_{20}$, | $p_{22}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$} |
| $M_{893}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{894}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{895}$ | : {$p_{17}$, | $p_{20}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{5}$, | $p_{9}$} |
| $M_{896}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{9}$} |
| $M_{897}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{9}$} |
| $M_{898}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{4}$, | $p_{9}$} |
| $M_{899}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{900}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{5}$, | $p_{9}$} |
| $M_{901}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{9}$} |
| $M_{902}$ | : {$p_{17}$, | $p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{903}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{9}$} |
| $M_{904}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{9}$} |
| $M_{905}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{9}$} |
| $M_{906}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{907}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{5}$, | $p_{9}$} |
| $M_{908}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{9}$} |
| $M_{909}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{16}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{910}$ | : {$p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{15}$, | $p_{2}$, | $p_{21}$} |
| $M_{911}$ | : {$p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{12}$, | $p_{18}$, | $p_{2}$} |
| $M_{912}$ | : {$p_{11}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{18}$, | $p_{2}$} |
| $M_{913}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{15}$, | $p_{2}$, | $p_{21}$, | $p_{9}$} |
| $M_{914}$ | : {$p_{11}$, | $p_{14}$, | $p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{915}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{916}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{9}$} |
| $M_{917}$ | : {$p_{20}$, | $p_{22}$, | $p_{3}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{9}$} |
| $M_{918}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{9}$} |
| $M_{919}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{4}$, | $p_{5}$, | $p_{9}$} |
| $M_{920}$ | : {$p_{20}$, | $p_{22}$, | $p_{1}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{5}$, | $p_{9}$} |
| $M_{921}$ | : {$p_{20}$, | $p_{22}$, | $p_{6}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{9}$} |
| $M_{922}$ | : {$p_{20}$, | $p_{22}$, | $p_{12}$, | $p_{15}$, | $p_{18}$, | $p_{2}$, | $p_{5}$, | $p_{9}$} |
| $M_{923}$ | : {$p_{11}$, | $p_{17}$, | $p_{20}$, | $p_{6}$, | $p_{8}$, | $p_{15}$, | $p_{21}$, | $p_{4}$} |

**The reachability graph (RG) of the supervisor (controlled model) shown in Fig. 7.35, in the Chapter 7:**

The following table represents the reachability graph of the supervisor, shown in Fig. 7.35, in the chapter 7. Note that $\mathbf{M_0}$ : ($t_6$ : $M_1$) ($t_7$ : $M_9$) means that from the initial marking $M_0$, either transition $t_6$ or transition $t_7$ may fire. If transition $t_6$ fires, then the new marking will be $M_1$. Alternatively, if transition $t_7$ fires, then the new marking will be $M_9$. This can also be seen from the following figure.



| | | | |
|---|---|---|---|
| $\mathbf{M_0}$ | : ($t_6$: $M_1$) | ($t_7$: $M_9$) | |
| $\mathbf{M_1}$ | : ($t_{19}$: $M_2$) | ($t_3$: $M_0$) | ($t_7$: $M_{90}$) |
| $\mathbf{M_2}$ | : ($t_7$: $M_3$) | ($t_9$: $M_{83}$) | |
| $\mathbf{M_3}$ | : ($t_9$: $M_4$) | | |
| $\mathbf{M_4}$ | : ($t_{11}$: $M_5$) | ($t_4$: $M_{119}$) | ($t_5$: $M_{806}$) |
| $\mathbf{M_5}$ | : ($t_{13}$: $M_6$) | ($t_4$: $M_{109}$) | ($t_5$: $M_{803}$) |
| $\mathbf{M_6}$ | : ($t_{20}$: $M_7$) | ($t_4$: $M_{110}$) | ($t_5$: $M_{800}$) |
| $\mathbf{M_7}$ | : ($t_{15}$: $M_8$) | ($t_4$: $M_{111}$) | ($t_5$: $M_{801}$) |
| $\mathbf{M_8}$ | : ($t_{17}$: $M_9$) | ($t_4$: $M_{112}$) | ($t_5$: $M_{802}$) |
| $\mathbf{M_9}$ | : ($t_4$: $M_{10}$) | ($t_5$: $M_{12}$) | |
| $\mathbf{M_{10}}$ | : ($t_1$: $M_0$) | ($t_7$: $M_{11}$) | |
| $\mathbf{M_{11}}$ | : ($t_1$: $M_9$) | | |
| $\mathbf{M_{12}}$ | : ($t_{18}$: $M_{13}$) | ($t_2$: $M_0$) | ($t_7$: $M_{275}$) |
| $\mathbf{M_{13}}$ | : ($t_7$: $M_{14}$) | ($t_8$: $M_{284}$) | |
| $\mathbf{M_{14}}$ | : ($t_8$: $M_{15}$) | | |
| $\mathbf{M_{15}}$ | : ($t_{10}$: $M_{16}$) | ($t_4$: $M_{577}$) | ($t_5$: $M_{584}$) |
| $\mathbf{M_{16}}$ | : ($t_{12}$: $M_{17}$) | ($t_4$: $M_{522}$) | ($t_5$: $M_{585}$) |
| $\mathbf{M_{17}}$ | : ($t_{21}$: $M_{18}$) | ($t_4$: $M_{523}$) | ($t_5$: $M_{531}$) |
| $\mathbf{M_{18}}$ | : ($t_{14}$: $M_{19}$) | ($t_4$: $M_{524}$) | ($t_5$: $M_{532}$) |
| $\mathbf{M_{19}}$ | : ($t_{16}$: $M_9$) | ($t_4$: $M_{20}$) | ($t_5$: $M_{533}$) |
| $\mathbf{M_{20}}$ | : ($t_1$: $M_{21}$) | ($t_{16}$: $M_{10}$) | ($t_7$: $M_{396}$) |
| $\mathbf{M_{21}}$ | : ($t_{16}$: $M_0$) | ($t_6$: $M_{22}$) | ($t_7$: $M_{19}$) |

| | | | | |
|---|---|---|---|---|
| $M_{22}$ | : ($t_{16}$: $M_1$) | ($t_{19}$: $M_{23}$) | ($t_3$: $M_{21}$) | ($t_7$: $M_{290}$) |
| $M_{23}$ | : ($t_{16}$: $M_2$) | ($t_7$: $M_{24}$) | ($t_9$: $M_{297}$) | |
| $M_{24}$ | : ($t_{16}$: $M_3$) | ($t_9$: $M_{25}$) | | |
| $M_{25}$ | : ($t_{11}$: $M_{26}$) | ($t_{16}$: $M_4$) | ($t_4$: $M_{313}$) | ($t_5$: $M_{805}$) |
| $M_{26}$ | : ($t_{13}$: $M_{27}$) | ($t_{16}$: $M_5$) | ($t_4$: $M_{314}$) | ($t_5$: $M_{798}$) |
| $M_{27}$ | : ($t_{16}$: $M_6$) | ($t_4$: $M_{28}$) | ($t_5$: $M_{799}$) | |
| $M_{28}$ | : ($t_1$: $M_{29}$) | ($t_{16}$: $M_{110}$) | ($t_7$: $M_{316}$) | |
| $M_{29}$ | : ($t_{16}$: $M_{30}$) | ($t_6$: $M_{154}$) | ($t_7$: $M_{27}$) | |
| $M_{30}$ | : ($t_{20}$: $M_{31}$) | ($t_6$: $M_{42}$) | ($t_7$: $M_6$) | |
| $M_{31}$ | : ($t_{15}$: $M_{32}$) | ($t_6$: $M_{923}$) | ($t_7$: $M_7$) | |
| $M_{32}$ | : ($t_{17}$: $M_0$) | ($t_6$: $M_{33}$) | ($t_7$: $M_8$) | |
| $M_{33}$ | : ($t_{17}$: $M_1$) | ($t_{19}$: $M_{34}$) | ($t_3$: $M_{32}$) | ($t_7$: $M_{89}$) |
| $M_{34}$ | : ($t_{17}$: $M_2$) | ($t_7$: $M_{35}$) | ($t_9$: $M_{82}$) | |
| $M_{35}$ | : ($t_{17}$: $M_3$) | ($t_9$: $M_{36}$) | | |
| $M_{36}$ | : ($t_{11}$: $M_{37}$) | ($t_{17}$: $M_4$) | ($t_4$: $M_{107}$) | ($t_5$: $M_{914}$) |
| $M_{37}$ | : ($t_{13}$: $M_{38}$) | ($t_{17}$: $M_5$) | ($t_4$: $M_{108}$) | ($t_5$: $M_{911}$) |
| $M_{38}$ | : ($t_{17}$: $M_6$) | ($t_4$: $M_{39}$) | ($t_5$: $M_{912}$) | |
| $M_{39}$ | : ($t_1$: $M_{40}$) | ($t_{17}$: $M_{110}$) | ($t_7$: $M_{118}$) | |
| $M_{40}$ | : ($t_{17}$: $M_{30}$) | ($t_6$: $M_{41}$) | ($t_7$: $M_{38}$) | |
| $M_{41}$ | : ($t_{17}$: $M_{42}$) | ($t_{19}$: $M_{64}$) | ($t_3$: $M_{40}$) | ($t_7$: $M_{94}$) |
| $M_{42}$ | : ($t_{19}$: $M_{43}$) | ($t_{20}$: $M_{923}$) | ($t_3$: $M_{30}$) | ($t_7$: $M_{87}$) |
| $M_{43}$ | : ($t_{20}$: $M_{44}$) | ($t_7$: $M_{55}$) | ($t_9$: $M_{80}$) | |
| $M_{44}$ | : ($t_{15}$: $M_{34}$) | ($t_7$: $M_{45}$) | ($t_9$: $M_{81}$) | |
| $M_{45}$ | : ($t_{15}$: $M_{35}$) | ($t_9$: $M_{46}$) | | |
| $M_{46}$ | : ($t_{11}$: $M_{47}$) | ($t_{15}$: $M_{36}$) | ($t_4$: $M_{106}$) | ($t_5$: $M_{913}$) |
| $M_{47}$ | : ($t_{15}$: $M_{37}$) | ($t_4$: $M_{48}$) | ($t_5$: $M_{910}$) | |
| $M_{48}$ | : ($t_1$: $M_{49}$) | ($t_{15}$: $M_{108}$) | ($t_7$: $M_{123}$) | |
| $M_{49}$ | : ($t_{15}$: $M_{50}$) | ($t_6$: $M_{97}$) | ($t_7$: $M_{47}$) | |
| $M_{50}$ | : ($t_{13}$: $M_{40}$) | ($t_{17}$: $M_{51}$) | ($t_6$: $M_{92}$) | ($t_7$: $M_{37}$) |
| $M_{51}$ | : ($t_{13}$: $M_{30}$) | ($t_6$: $M_{52}$) | ($t_7$: $M_5$) | |
| $M_{52}$ | : ($t_{13}$: $M_{42}$) | ($t_{19}$: $M_{53}$) | ($t_3$: $M_{51}$) | ($t_7$: $M_{86}$) |
| $M_{53}$ | : ($t_{13}$: $M_{43}$) | ($t_7$: $M_{54}$) | ($t_9$: $M_{79}$) | |
| $M_{54}$ | : ($t_{13}$: $M_{55}$) | ($t_9$: $M_{77}$) | | |
| $M_{55}$ | : ($t_{20}$: $M_{45}$) | ($t_9$: $M_{56}$) | | |
| $M_{56}$ | : ($t_{11}$: $M_{57}$) | ($t_{20}$: $M_{46}$) | ($t_4$: $M_{105}$) | ($t_5$: $M_{128}$) |
| $M_{57}$ | : ($t_{20}$: $M_{47}$) | ($t_4$: $M_{58}$) | ($t_5$: $M_{129}$) | |
| $M_{58}$ | : ($t_1$: $M_{59}$) | ($t_{20}$: $M_{48}$) | ($t_7$: $M_{125}$) | |
| $M_{59}$ | : ($t_{20}$: $M_{49}$) | ($t_6$: $M_{60}$) | ($t_7$: $M_{57}$) | |
| $M_{60}$ | : ($t_{19}$: $M_{61}$) | ($t_{20}$: $M_{97}$) | ($t_3$: $M_{59}$) | ($t_7$: $M_{102}$) |
| $M_{61}$ | : ($t_{20}$: $M_{62}$) | ($t_7$: $M_{73}$) | ($t_9$: $M_{897}$) | |
| $M_{62}$ | : ($t_{15}$: $M_{63}$) | ($t_7$: $M_{74}$) | ($t_9$: $M_{168}$) | |
| $M_{63}$ | : ($t_{13}$: $M_{64}$) | ($t_{17}$: $M_{53}$) | ($t_7$: $M_{75}$) | ($t_9$: $M_{169}$) |
| $M_{64}$ | : ($t_{17}$: $M_{43}$) | ($t_7$: $M_{65}$) | ($t_9$: $M_{170}$) | |

| | | | | | |
|---|---|---|---|---|---|
| $\mathbf{M_{65}}$ | : ($t_{17}$: $M_{55}$) | ($t_9$: $M_{66}$) | | | |
| $\mathbf{M_{66}}$ | : ($t_{11}$: $M_{67}$) | ($t_{17}$: $M_{56}$) | ($t_4$: $M_{179}$) | ($t_5$: $M_{186}$) | |
| $\mathbf{M_{67}}$ | : ($t_{17}$: $M_{57}$) | ($t_4$: $M_{68}$) | ($t_5$: $M_{187}$) | | |
| $\mathbf{M_{68}}$ | : ($t_1$: $M_{69}$) | ($t_{17}$: $M_{58}$) | ($t_7$: $M_{181}$) | | |
| $\mathbf{M_{69}}$ | : ($t_{17}$: $M_{59}$) | ($t_6$: $M_{70}$) | ($t_7$: $M_{67}$) | | |
| $\mathbf{M_{70}}$ | : ($t_{17}$: $M_{60}$) | ($t_{19}$: $M_{71}$) | ($t_3$: $M_{69}$) | ($t_7$: $M_{173}$) | |
| $\mathbf{M_{71}}$ | : ($t_{17}$: $M_{61}$) | ($t_7$: $M_{72}$) | ($t_9$: $M_{917}$) | | |
| $\mathbf{M_{72}}$ | : ($t_{17}$: $M_{73}$) | ($t_9$: $M_{915}$) | | | |
| $\mathbf{M_{73}}$ | : ($t_{20}$: $M_{74}$) | ($t_9$: $M_{165}$) | | | |
| $\mathbf{M_{74}}$ | : ($t_{15}$: $M_{75}$) | ($t_9$: $M_{166}$) | | | |
| $\mathbf{M_{75}}$ | : ($t_{13}$: $M_{65}$) | ($t_{17}$: $M_{54}$) | ($t_9$: $M_{76}$) | | |
| $\mathbf{M_{76}}$ | : ($t_{13}$: $M_{66}$) | ($t_{17}$: $M_{77}$) | ($t_4$: $M_{178}$) | ($t_5$: $M_{185}$) | |
| $\mathbf{M_{77}}$ | : ($t_{13}$: $M_{56}$) | ($t_4$: $M_{78}$) | ($t_5$: $M_{127}$) | | |
| $\mathbf{M_{78}}$ | : ($t_1$: $M_{79}$) | ($t_{13}$: $M_{105}$) | ($t_7$: $M_{126}$) | | |
| $\mathbf{M_{79}}$ | : ($t_{13}$: $M_{80}$) | ($t_6$: $M_{103}$) | ($t_7$: $M_{77}$) | | |
| $\mathbf{M_{80}}$ | : ($t_{11}$: $M_{59}$) | ($t_{20}$: $M_{81}$) | ($t_6$: $M_{100}$) | ($t_7$: $M_{56}$) | |
| $\mathbf{M_{81}}$ | : ($t_{11}$: $M_{49}$) | ($t_{15}$: $M_{82}$) | ($t_6$: $M_{96}$) | ($t_7$: $M_{46}$) | |
| $\mathbf{M_{82}}$ | : ($t_{11}$: $M_{50}$) | ($t_{17}$: $M_{83}$) | ($t_6$: $M_{91}$) | ($t_7$: $M_{36}$) | |
| $\mathbf{M_{83}}$ | : ($t_{11}$: $M_{51}$) | ($t_6$: $M_{84}$) | ($t_7$: $M_4$) | | |
| $\mathbf{M_{84}}$ | : ($t_{11}$: $M_{52}$) | ($t_3$: $M_{83}$) | ($t_7$: $M_{85}$) | | |
| $\mathbf{M_{85}}$ | : ($t_{11}$: $M_{86}$) | ($t_3$: $M_4$) | | | |
| $\mathbf{M_{86}}$ | : ($t_{13}$: $M_{87}$) | ($t_{19}$: $M_{54}$) | ($t_3$: $M_5$) | | |
| $\mathbf{M_{87}}$ | : ($t_{19}$: $M_{55}$) | ($t_{20}$: $M_{88}$) | ($t_3$: $M_6$) | | |
| $\mathbf{M_{88}}$ | : ($t_{15}$: $M_{89}$) | ($t_{19}$: $M_{45}$) | ($t_3$: $M_7$) | | |
| $\mathbf{M_{89}}$ | : ($t_{17}$: $M_{90}$) | ($t_{19}$: $M_{35}$) | ($t_3$: $M_8$) | | |
| $\mathbf{M_{90}}$ | : ($t_{19}$: $M_3$) | ($t_3$: $M_9$) | | | |
| $\mathbf{M_{91}}$ | : ($t_{11}$: $M_{92}$) | ($t_{17}$:.$M_{84}$) | ($t_3$: $M_{82}$) | ($t_7$: $M_{95}$) | |
| $\mathbf{M_{92}}$ | : ($t_{13}$: $M_{41}$) | ($t_{17}$: $M_{52}$) | ($t_{19}$: $M_{63}$) | ($t_3$: $M_{50}$) | ($t_7$: $M_{93}$) |
| $\mathbf{M_{93}}$ | : ($t_{13}$: $M_{94}$) | ($t_{17}$: $M_{86}$) | ($t_{19}$: $M_{75}$) | ($t_3$: $M_{37}$) | |
| $\mathbf{M_{94}}$ | : ($t_{17}$: $M_{87}$) | ($t_{19}$: $M_{65}$) | ($t_3$: $M_{38}$) | | |
| $\mathbf{M_{95}}$ | : ($t_{11}$: $M_{93}$) | ($t_{17}$: $M_{85}$) | ($t_3$: $M_{36}$) | | |
| $\mathbf{M_{96}}$ | : ($t_{11}$: $M_{97}$) | ($t_{15}$: $M_{91}$) | ($t_3$: $M_{81}$) | ($t_7$: $M_{99}$) | |
| $\mathbf{M_{97}}$ | : ($t_{15}$: $M_{92}$) | ($t_{19}$: $M_{62}$) | ($t_3$: $M_{49}$) | ($t_7$: $M_{98}$) | |
| $\mathbf{M_{98}}$ | : ($t_{15}$: $M_{93}$) | ($t_{19}$: $M_{74}$) | ($t_3$: $M_{47}$) | | |
| $\mathbf{M_{99}}$ | : ($t_{11}$: $M_{98}$) | ($t_{15}$: $M_{95}$) | ($t_3$: $M_{46}$) | | |
| $\mathbf{M_{100}}$ | : ($t_{11}$: $M_{60}$) | ($t_{20}$: $M_{96}$) | ($t_3$: $M_{80}$) | ($t_7$: $M_{101}$) | |
| $\mathbf{M_{101}}$ | : ($t_{11}$: $M_{102}$) | ($t_{20}$: $M_{99}$) | ($t_3$: $M_{56}$) | | |
| $\mathbf{M_{102}}$ | : ($t_{19}$: $M_{73}$) | ($t_{20}$: $M_{98}$) | ($t_3$: $M_{57}$) | | |
| $\mathbf{M_{103}}$ | : ($t_{13}$: $M_{100}$) | ($t_3$: $M_{79}$) | ($t_7$: $M_{104}$) | | |
| $\mathbf{M_{104}}$ | : ($t_{13}$: $M_{101}$) | ($t_3$: $M_{77}$) | | | |
| $\mathbf{M_{105}}$ | : ($t_1$: $M_{80}$) | ($t_{11}$: $M_{58}$) | ($t_{20}$: $M_{106}$) | ($t_7$: $M_{124}$) | |
| $\mathbf{M_{106}}$ | : ($t_1$: $M_{81}$) | ($t_{11}$: $M_{48}$) | ($t_{15}$: $M_{107}$) | ($t_7$: $M_{122}$) | |
| $\mathbf{M_{107}}$ | : ($t_1$: $M_{82}$) | ($t_{11}$: $M_{108}$) | ($t_{17}$: $M_{119}$) | ($t_7$: $M_{121}$) | |

| | | | | |
|---|---|---|---|---|
| $M_{108}$ | : $(t_1: M_{50})$ | $(t_{13}: M_{39})$ | $(t_{17}: M_{109})$ | $(t_7: M_{117})$ |
| $M_{109}$ | : $(t_1: M_{51})$ | $(t_{13}: M_{110})$ | $(t_7: M_{116})$ | |
| $M_{110}$ | : $(t_1: M_{30})$ | $(t_{20}: M_{111})$ | $(t_7: M_{115})$ | |
| $M_{111}$ | : $(t_1: M_{31})$ | $(t_{15}: M_{112})$ | $(t_7: M_{114})$ | |
| $M_{112}$ | : $(t_1: M_{32})$ | $(t_{17}: M_{10})$ | $(t_7: M_{113})$ | |
| $M_{113}$ | : $(t_1: M_8)$ | $(t_{17}: M_{11})$ | | |
| $M_{114}$ | : $(t_1: M_7)$ | $(t_{15}: M_{113})$ | | |
| $M_{115}$ | : $(t_1: M_6)$ | $(t_{20}: M_{114})$ | | |
| $M_{116}$ | : $(t_1: M_5)$ | $(t_{13}: M_{115})$ | | |
| $M_{117}$ | : $(t_1: M_{37})$ | $(t_{13}: M_{118})$ | $(t_{17}: M_{116})$ | |
| $M_{118}$ | : $(t_1: M_{38})$ | $(t_{17}: M_{115})$ | | |
| $M_{119}$ | : $(t_1: M_{83})$ | $(t_{11}: M_{109})$ | $(t_7: M_{120})$ | |
| $M_{120}$ | : $(t_1: M_4)$ | $(t_{11}: M_{116})$ | | |
| $M_{121}$ | : $(t_1: M_{36})$ | $(t_{11}: M_{117})$ | $(t_{17}: M_{120})$ | |
| $M_{122}$ | : $(t_1: M_{46})$ | $(t_{11}: M_{123})$ | $(t_{15}: M_{121})$ | |
| $M_{123}$ | : $(t_1: M_{47})$ | $(t_{15}: M_{117})$ | | |
| $M_{124}$ | : $(t_1: M_{56})$ | $(t_{11}: M_{125})$ | $(t_{20}: M_{122})$ | |
| $M_{125}$ | : $(t_1: M_{57})$ | $(t_{20}: M_{123})$ | | |
| $M_{126}$ | : $(t_1: M_{77})$ | $(t_{13}: M_{124})$ | | |
| $M_{127}$ | : $(t_{13}: M_{128})$ | $(t_2: M_{79})$ | $(t_7: M_{813})$ | |
| $M_{128}$ | : $(t_{11}: M_{129})$ | $(t_2: M_{80})$ | $(t_{20}: M_{913})$ | $(t_7: M_{810})$ |
| $M_{129}$ | : $(t_{18}: M_{130})$ | $(t_2: M_{59})$ | $(t_{20}: M_{910})$ | $(t_7: M_{268})$ |
| $M_{130}$ | : $(t_{20}: M_{131})$ | $(t_7: M_{190})$ | $(t_8: M_{871})$ | |
| $M_{131}$ | : $(t_{15}: M_{132})$ | $(t_7: M_{191})$ | $(t_8: M_{864})$ | |
| $M_{132}$ | : $(t_{13}: M_{133})$ | $(t_{17}: M_{796})$ | $(t_7: M_{192})$ | $(t_8: M_{857})$ |
| $M_{133}$ | : $(t_{17}: M_{134})$ | $(t_7: M_{193})$ | $(t_8: M_{850})$ | |
| $M_{134}$ | : $(t_{20}: M_{135})$ | $(t_7: M_{194})$ | $(t_8: M_{281})$ | |
| $M_{135}$ | : $(t_{15}: M_{136})$ | $(t_7: M_{195})$ | $(t_8: M_{282})$ | |
| $M_{136}$ | : $(t_{17}: M_{13})$ | $(t_7: M_{137})$ | $(t_8: M_{283})$ | |
| $M_{137}$ | : $(t_{17}: M_{14})$ | $(t_8: M_{138})$ | | |
| $M_{138}$ | : $(t_{10}: M_{139})$ | $(t_{17}: M_{15})$ | $(t_4: M_{808})$ | $(t_5: M_{759})$ |
| $M_{139}$ | : $(t_{12}: M_{140})$ | $(t_{17}: M_{16})$ | $(t_4: M_{809})$ | $(t_5: M_{754})$ |
| $M_{140}$ | : $(t_{17}: M_{17})$ | $(t_4: M_{141})$ | $(t_5: M_{755})$ | |
| $M_{141}$ | : $(t_1: M_{142})$ | $(t_{17}: M_{523})$ | $(t_7: M_{393})$ | |
| $M_{142}$ | : $(t_{17}: M_{143})$ | $(t_6: M_{371})$ | $(t_7: M_{140})$ | |
| $M_{143}$ | : $(t_{21}: M_{144})$ | $(t_6: M_{203})$ | $(t_7: M_{17})$ | |
| $M_{144}$ | : $(t_{14}: M_{21})$ | $(t_6: M_{145})$ | $(t_7: M_{18})$ | |
| $M_{145}$ | : $(t_{14}: M_{22})$ | $(t_{19}: M_{146})$ | $(t_3: M_{144})$ | $(t_7: M_{289})$ |
| $M_{146}$ | : $(t_{14}: M_{23})$ | $(t_7: M_{147})$ | $(t_9: M_{296})$ | |
| $M_{147}$ | : $(t_{14}: M_{24})$ | $(t_9: M_{148})$ | | |
| $M_{148}$ | : $(t_{11}: M_{149})$ | $(t_{14}: M_{25})$ | $(t_4: M_{312})$ | $(t_5: M_{804})$ |
| $M_{149}$ | : $(t_{14}: M_{26})$ | $(t_4: M_{150})$ | $(t_5: M_{797})$ | |
| $M_{150}$ | : $(t_1: M_{151})$ | $(t_{14}: M_{314})$ | $(t_7: M_{319})$ | |

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|---|---|---|---|---|---|
| $\mathbf{M_{151}}$ | : $(t_{14}: M_{152})$ | $(t_{6}: M_{303})$ | $(t_{7}: M_{149})$ | | |
| $\mathbf{M_{152}}$ | : $(t_{13}: M_{29})$ | $(t_{16}: M_{51})$ | $(t_{6}: M_{153})$ | $(t_{7}: M_{26})$ | |
| $\mathbf{M_{153}}$ | : $(t_{13}: M_{154})$ | $(t_{16}: M_{52})$ | $(t_{19}: M_{213})$ | $(t_{3}: M_{152})$ | $(t_{7}: M_{300})$ |
| $\mathbf{M_{154}}$ | : $(t_{16}: M_{42})$ | $(t_{19}: M_{155})$ | $(t_{3}: M_{29})$ | $(t_{7}: M_{301})$ | |
| $\mathbf{M_{155}}$ | : $(t_{16}: M_{43})$ | $(t_{7}: M_{156})$ | $(t_{9}: M_{218})$ | | |
| $\mathbf{M_{156}}$ | : $(t_{16}: M_{55})$ | $(t_{9}: M_{157})$ | | | |
| $\mathbf{M_{157}}$ | : $(t_{11}: M_{158})$ | $(t_{16}: M_{56})$ | $(t_{4}: M_{224})$ | $(t_{5}: M_{229})$ | |
| $\mathbf{M_{158}}$ | : $(t_{16}: M_{57})$ | $(t_{4}: M_{159})$ | $(t_{5}: M_{230})$ | | |
| $\mathbf{M_{159}}$ | : $(t_{1}: M_{160})$ | $(t_{16}: M_{58})$ | $(t_{7}: M_{226})$ | | |
| $\mathbf{M_{160}}$ | : $(t_{16}: M_{59})$ | $(t_{6}: M_{161})$ | $(t_{7}: M_{158})$ | | |
| $\mathbf{M_{161}}$ | : $(t_{16}: M_{60})$ | $(t_{19}: M_{162})$ | $(t_{3}: M_{160})$ | $(t_{7}: M_{221})$ | |
| $\mathbf{M_{162}}$ | : $(t_{16}: M_{61})$ | $(t_{7}: M_{163})$ | $(t_{9}: M_{904})$ | | |
| $\mathbf{M_{163}}$ | : $(t_{16}: M_{73})$ | $(t_{9}: M_{164})$ | | | |
| $\mathbf{M_{164}}$ | : $(t_{16}: M_{165})$ | $(t_{4}: M_{903})$ | $(t_{5}: M_{908})$ | | |
| $\mathbf{M_{165}}$ | : $(t_{20}: M_{166})$ | $(t_{4}: M_{896})$ | $(t_{5}: M_{901})$ | | |
| $\mathbf{M_{166}}$ | : $(t_{15}: M_{76})$ | $(t_{4}: M_{167})$ | $(t_{5}: M_{184})$ | | |
| $\mathbf{M_{167}}$ | : $(t_{1}: M_{168})$ | $(t_{15}: M_{178})$ | $(t_{7}: M_{183})$ | | |
| $\mathbf{M_{168}}$ | : $(t_{15}: M_{169})$ | $(t_{6}: M_{176})$ | $(t_{7}: M_{166})$ | | |
| $\mathbf{M_{169}}$ | : $(t_{13}: M_{170})$ | $(t_{17}: M_{79})$ | $(t_{6}: M_{174})$ | $(t_{7}: M_{76})$ | |
| $\mathbf{M_{170}}$ | : $(t_{11}: M_{69})$ | $(t_{17}: M_{80})$ | $(t_{6}: M_{171})$ | $(t_{7}: M_{66})$ | |
| $\mathbf{M_{171}}$ | : $(t_{11}: M_{70})$ | $(t_{17}: M_{100})$ | $(t_{3}: M_{170})$ | $(t_{7}: M_{172})$ | |
| $\mathbf{M_{172}}$ | : $(t_{11}: M_{173})$ | $(t_{17}: M_{101})$ | $(t_{3}: M_{66})$ | | |
| $\mathbf{M_{173}}$ | : $(t_{17}: M_{102})$ | $(t_{19}: M_{72})$ | $(t_{3}: M_{67})$ | | |
| $\mathbf{M_{174}}$ | : $(t_{13}: M_{171})$ | $(t_{17}: M_{103})$ | $(t_{3}: M_{169})$ | $(t_{7}: M_{175})$ | |
| $\mathbf{M_{175}}$ | : $(t_{13}: M_{172})$ | $(t_{17}: M_{104})$ | $(t_{3}: M_{76})$ | | |
| $\mathbf{M_{176}}$ | : $(t_{15}: M_{174})$ | $(t_{3}: M_{168})$ | $(t_{7}: M_{177})$ | | |
| $\mathbf{M_{177}}$ | : $(t_{15}: M_{175})$ | $(t_{3}: M_{166})$ | | | |
| $\mathbf{M_{178}}$ | : $(t_{1}: M_{169})$ | $(t_{13}: M_{179})$ | $(t_{17}: M_{78})$ | $(t_{7}: M_{182})$ | |
| $\mathbf{M_{179}}$ | : $(t_{1}: M_{170})$ | $(t_{11}: M_{68})$ | $(t_{17}: M_{105})$ | $(t_{7}: M_{180})$ | |
| $\mathbf{M_{180}}$ | : $(t_{1}: M_{66})$ | $(t_{11}: M_{181})$ | $(t_{17}: M_{124})$ | | |
| $\mathbf{M_{181}}$ | : $(t_{1}: M_{67})$ | $(t_{17}: M_{125})$ | | | |
| $\mathbf{M_{182}}$ | : $(t_{1}: M_{76})$ | $(t_{13}: M_{180})$ | $(t_{17}: M_{126})$ | | |
| $\mathbf{M_{183}}$ | : $(t_{1}: M_{166})$ | $(t_{15}: M_{182})$ | | | |
| $\mathbf{M_{184}}$ | : $(t_{15}: M_{185})$ | $(t_{2}: M_{168})$ | $(t_{7}: M_{895})$ | | |
| $\mathbf{M_{185}}$ | : $(t_{13}: M_{186})$ | $(t_{17}: M_{127})$ | $(t_{2}: M_{169})$ | $(t_{7}: M_{894})$ | |
| $\mathbf{M_{186}}$ | : $(t_{11}: M_{187})$ | $(t_{17}: M_{128})$ | $(t_{2}: M_{170})$ | $(t_{7}: M_{893})$ | |
| $\mathbf{M_{187}}$ | : $(t_{17}: M_{129})$ | $(t_{18}: M_{188})$ | $(t_{2}: M_{69})$ | $(t_{7}: M_{892})$ | |
| $\mathbf{M_{188}}$ | : $(t_{17}: M_{130})$ | $(t_{7}: M_{189})$ | $(t_{8}: M_{886})$ | | |
| $\mathbf{M_{189}}$ | : $(t_{17}: M_{190})$ | $(t_{8}: M_{884})$ | | | |
| $\mathbf{M_{190}}$ | : $(t_{20}: M_{191})$ | $(t_{8}: M_{234})$ | | | |
| $\mathbf{M_{191}}$ | : $(t_{15}: M_{192})$ | $(t_{8}: M_{235})$ | | | |
| $\mathbf{M_{192}}$ | : $(t_{13}: M_{193})$ | $(t_{17}: M_{277})$ | $(t_{8}: M_{236})$ | | |
| $\mathbf{M_{193}}$ | : $(t_{17}: M_{194})$ | $(t_{8}: M_{237})$ | | | |

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|---|---|---|---|---|
| $M_{194}$ | : ($t_{20}$: $M_{195}$) | ($t_8$: $M_{279}$) | | |
| $M_{195}$ | : ($t_{15}$: $M_{137}$) | ($t_8$: $M_{196}$) | | |
| $M_{196}$ | : ($t_{10}$: $M_{197}$) | ($t_{15}$: $M_{138}$) | ($t_4$: $M_{807}$) | ($t_5$: $M_{758}$) |
| $M_{197}$ | : ($t_{15}$: $M_{139}$) | ($t_4$: $M_{198}$) | ($t_5$: $M_{753}$) | |
| $M_{198}$ | : ($t_1$: $M_{199}$) | ($t_{15}$: $M_{809}$) | ($t_7$: $M_{391}$) | |
| $M_{199}$ | : ($t_{15}$: $M_{200}$) | ($t_6$: $M_{369}$) | ($t_7$: $M_{197}$) | |
| $M_{200}$ | : ($t_{12}$: $M_{142}$) | ($t_{17}$: $M_{201}$) | ($t_6$: $M_{370}$) | ($t_7$: $M_{139}$) |
| $M_{201}$ | : ($t_{12}$: $M_{143}$) | ($t_6$: $M_{202}$) | ($t_7$: $M_{16}$) | |
| $M_{202}$ | : ($t_{12}$: $M_{203}$) | ($t_{19}$: $M_{480}$) | ($t_3$: $M_{201}$) | ($t_7$: $M_{287}$) |
| $M_{203}$ | : ($t_{19}$: $M_{204}$) | ($t_{21}$: $M_{145}$) | ($t_3$: $M_{143}$) | ($t_7$: $M_{288}$) |
| $M_{204}$ | : ($t_{21}$: $M_{146}$) | ($t_7$: $M_{205}$) | ($t_9$: $M_{295}$) | |
| $M_{205}$ | : ($t_{21}$: $M_{147}$) | ($t_9$: $M_{206}$) | | |
| $M_{206}$ | : ($t_{11}$: $M_{207}$) | ($t_{21}$: $M_{148}$) | ($t_4$: $M_{311}$) | ($t_5$: $M_{324}$) |
| $M_{207}$ | : ($t_{21}$: $M_{149}$) | ($t_4$: $M_{208}$) | ($t_5$: $M_{325}$) | |
| $M_{208}$ | : ($t_1$: $M_{209}$) | ($t_{21}$: $M_{150}$) | ($t_7$: $M_{321}$) | |
| $M_{209}$ | : ($t_{21}$: $M_{151}$) | ($t_6$: $M_{210}$) | ($t_7$: $M_{207}$) | |
| $M_{210}$ | : ($t_{19}$: $M_{211}$) | ($t_{21}$: $M_{303}$) | ($t_3$: $M_{209}$) | ($t_7$: $M_{308}$) |
| $M_{211}$ | : ($t_{21}$: $M_{212}$) | ($t_7$: $M_{255}$) | ($t_9$: $M_{435}$) | |
| $M_{212}$ | : ($t_{14}$: $M_{213}$) | ($t_7$: $M_{256}$) | ($t_9$: $M_{259}$) | |
| $M_{213}$ | : ($t_{13}$: $M_{155}$) | ($t_{16}$: $M_{53}$) | ($t_7$: $M_{214}$) | ($t_9$: $M_{217}$) |
| $M_{214}$ | : ($t_{13}$: $M_{156}$) | ($t_{16}$: $M_{54}$) | ($t_9$: $M_{215}$) | |
| $M_{215}$ | : ($t_{13}$: $M_{157}$) | ($t_{16}$: $M_{77}$) | ($t_4$: $M_{216}$) | ($t_5$: $M_{228}$) |
| $M_{216}$ | : ($t_1$: $M_{217}$) | ($t_{13}$: $M_{224}$) | ($t_{16}$: $M_{78}$) | ($t_7$: $M_{227}$) |
| $M_{217}$ | : ($t_{13}$: $M_{218}$) | ($t_{16}$: $M_{79}$) | ($t_6$: $M_{222}$) | ($t_7$: $M_{215}$) |
| $M_{218}$ | : ($t_{11}$: $M_{160}$) | ($t_{16}$: $M_{80}$) | ($t_6$: $M_{219}$) | ($t_7$: $M_{157}$) |
| $M_{219}$ | : ($t_{11}$: $M_{161}$) | ($t_{16}$: $M_{100}$) | ($t_3$: $M_{218}$) | ($t_7$: $M_{220}$) |
| $M_{220}$ | : ($t_{11}$: $M_{221}$) | ($t_{16}$: $M_{101}$) | ($t_3$: $M_{157}$) | |
| $M_{221}$ | : ($t_{16}$: $M_{102}$) | ($t_{19}$: $M_{163}$) | ($t_3$: $M_{158}$) | |
| $M_{222}$ | : ($t_{13}$: $M_{219}$) | ($t_{16}$: $M_{103}$) | ($t_3$: $M_{217}$) | ($t_7$: $M_{223}$) |
| $M_{223}$ | : ($t_{13}$: $M_{220}$) | ($t_{16}$: $M_{104}$) | ($t_3$: $M_{215}$) | |
| $M_{224}$ | : ($t_1$: $M_{218}$) | ($t_{11}$: $M_{159}$) | ($t_{16}$: $M_{105}$) | ($t_7$: $M_{225}$) |
| $M_{225}$ | : ($t_1$: $M_{157}$) | ($t_{11}$: $M_{226}$) | ($t_{16}$: $M_{124}$) | |
| $M_{226}$ | : ($t_1$: $M_{158}$) | ($t_{16}$:.$M_{125}$) | | |
| $M_{227}$ | : ($t_1$: $M_{215}$) | ($t_{13}$: $M_{225}$) | ($t_{16}$: $M_{126}$) | |
| $M_{228}$ | : ($t_{13}$: $M_{229}$) | ($t_{16}$: $M_{127}$) | ($t_2$: $M_{217}$) | ($t_7$: $M_{265}$) |
| $M_{229}$ | : ($t_{11}$: $M_{230}$) | ($t_{16}$: $M_{128}$) | ($t_2$: $M_{218}$) | ($t_7$: $M_{266}$) |
| $M_{230}$ | : ($t_{16}$: $M_{129}$) | ($t_{18}$: $M_{231}$) | ($t_2$: $M_{160}$) | ($t_7$: $M_{267}$) |
| $M_{231}$ | : ($t_{16}$: $M_{130}$) | ($t_7$: $M_{232}$) | ($t_8$: $M_{878}$) | |
| $M_{232}$ | : ($t_{16}$: $M_{190}$) | ($t_8$: $M_{233}$) | | |
| $M_{233}$ | : ($t_{16}$: $M_{234}$) | ($t_4$: $M_{877}$) | ($t_5$: $M_{882}$) | |
| $M_{234}$ | : ($t_{20}$: $M_{235}$) | ($t_4$: $M_{870}$) | ($t_5$: $M_{875}$) | |
| $M_{235}$ | : ($t_{15}$: $M_{236}$) | ($t_4$: $M_{863}$) | ($t_5$: $M_{868}$) | |
| $M_{236}$ | : ($t_{13}$: $M_{237}$) | ($t_{17}$: $M_{278}$) | ($t_4$: $M_{856}$) | ($t_5$: $M_{861}$) |

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|---|---|---|---|---|
| $M_{237}$ | : ($t_{10}$: $M_{238}$) | ($t_{17}$: $M_{279}$) | ($t_{4}$: $M_{849}$) | ($t_{5}$: $M_{854}$) |
| $M_{238}$ | : ($t_{17}$: $M_{239}$) | ($t_{4}$: $M_{822}$) | ($t_{5}$: $M_{837}$) | |
| $M_{239}$ | : ($t_{20}$: $M_{197}$) | ($t_{4}$: $M_{240}$) | ($t_{5}$: $M_{408}$) | |
| $M_{240}$ | : ($t_{1}$: $M_{241}$) | ($t_{20}$: $M_{198}$) | ($t_{7}$: $M_{390}$) | |
| $M_{241}$ | : ($t_{20}$: $M_{199}$) | ($t_{6}$: $M_{242}$) | ($t_{7}$: $M_{239}$) | |
| $M_{242}$ | : ($t_{19}$: $M_{243}$) | ($t_{20}$: $M_{369}$) | ($t_{3}$: $M_{241}$) | ($t_{7}$: $M_{379}$) |
| $M_{243}$ | : ($t_{20}$: $M_{244}$) | ($t_{7}$: $M_{338}$) | ($t_{9}$: $M_{782}$) | |
| $M_{244}$ | : ($t_{15}$: $M_{245}$) | ($t_{7}$: $M_{339}$) | ($t_{9}$: $M_{774}$) | |
| $M_{245}$ | : ($t_{12}$: $M_{246}$) | ($t_{17}$: $M_{480}$) | ($t_{7}$: $M_{340}$) | ($t_{9}$: $M_{343}$) |
| $M_{246}$ | : ($t_{17}$: $M_{204}$) | ($t_{7}$: $M_{247}$) | ($t_{9}$: $M_{344}$) | |
| $M_{247}$ | : ($t_{17}$: $M_{205}$) | ($t_{9}$: $M_{248}$) | | |
| $M_{248}$ | : ($t_{11}$: $M_{249}$) | ($t_{17}$: $M_{206}$) | ($t_{4}$: $M_{350}$) | ($t_{5}$: $M_{355}$) |
| $M_{249}$ | : ($t_{17}$: $M_{207}$) | ($t_{4}$: $M_{250}$) | ($t_{5}$: $M_{356}$) | |
| $M_{250}$ | : ($t_{1}$: $M_{251}$) | ($t_{17}$: $M_{208}$) | ($t_{7}$: $M_{352}$) | |
| $M_{251}$ | : ($t_{17}$: $M_{209}$) | ($t_{6}$: $M_{252}$) | ($t_{7}$: $M_{249}$) | |
| $M_{252}$ | : ($t_{17}$: $M_{210}$) | ($t_{19}$: $M_{253}$) | ($t_{3}$: $M_{251}$) | ($t_{7}$: $M_{347}$) |
| $M_{253}$ | : ($t_{17}$: $M_{211}$) | ($t_{7}$: $M_{254}$) | ($t_{9}$: $M_{816}$) | |
| $M_{254}$ | : ($t_{17}$: $M_{255}$) | ($t_{9}$: $M_{814}$) | | |
| $M_{255}$ | : ($t_{21}$: $M_{256}$) | ($t_{9}$: $M_{433}$) | | |
| $M_{256}$ | : ($t_{14}$: $M_{214}$) | ($t_{9}$: $M_{257}$) | | |
| $M_{257}$ | : ($t_{14}$: $M_{215}$) | ($t_{4}$: $M_{258}$) | ($t_{5}$: $M_{263}$) | |
| $M_{258}$ | : ($t_{1}$: $M_{259}$) | ($t_{14}$: $M_{216}$) | ($t_{7}$: $M_{262}$) | |
| $M_{259}$ | : ($t_{14}$: $M_{217}$) | ($t_{6}$: $M_{260}$) | ($t_{7}$: $M_{257}$) | |
| $M_{260}$ | : ($t_{14}$: $M_{222}$) | ($t_{3}$: $M_{259}$) | ($t_{7}$: $M_{261}$) | |
| $M_{261}$ | : ($t_{14}$: $M_{223}$) | ($t_{3}$: $M_{257}$) | | |
| $M_{262}$ | : ($t_{1}$: $M_{257}$) | ($t_{14}$: $M_{227}$) | | |
| $M_{263}$ | : ($t_{14}$: $M_{228}$) | ($t_{2}$: $M_{259}$) | ($t_{7}$: $M_{264}$) | |
| $M_{264}$ | : ($t_{14}$: $M_{265}$) | ($t_{2}$: $M_{257}$) | | |
| $M_{265}$ | : ($t_{13}$: $M_{266}$) | ($t_{16}$: $M_{813}$) | ($t_{2}$: $M_{215}$) | |
| $M_{266}$ | : ($t_{11}$: $M_{267}$) | ($t_{16}$: $M_{810}$) | ($t_{2}$: $M_{157}$) | |
| $M_{267}$ | : ($t_{16}$: $M_{268}$) | ($t_{18}$: $M_{232}$) | ($t_{2}$: $M_{158}$) | |
| $M_{268}$ | : ($t_{18}$: $M_{190}$) | ($t_{2}$: $M_{57}$) | ($t_{20}$: $M_{269}$) | |
| $M_{269}$ | : ($t_{15}$: $M_{270}$) | ($t_{18}$: $M_{191}$) | ($t_{2}$: $M_{47}$) | |
| $M_{270}$ | : ($t_{13}$: $M_{271}$) | ($t_{17}$: $M_{276}$) | ($t_{18}$: $M_{192}$) | ($t_{2}$: $M_{37}$) |
| $M_{271}$ | : ($t_{17}$: $M_{272}$) | ($t_{18}$: $M_{193}$) | ($t_{2}$: $M_{38}$) | |
| $M_{272}$ | : ($t_{18}$: $M_{194}$) | ($t_{2}$: $M_{6}$) | ($t_{20}$: $M_{273}$) | |
| $M_{273}$ | : ($t_{15}$: $M_{274}$) | ($t_{18}$: $M_{195}$) | ($t_{2}$: $M_{7}$) | |
| $M_{274}$ | : ($t_{17}$: $M_{275}$) | ($t_{18}$: $M_{137}$) | ($t_{2}$: $M_{8}$) | |
| $M_{275}$ | : ($t_{18}$: $M_{14}$) | ($t_{2}$: $M_{9}$) | | |
| $M_{276}$ | : ($t_{13}$: $M_{272}$) | ($t_{18}$: $M_{277}$) | ($t_{2}$: $M_{5}$) | |
| $M_{277}$ | : ($t_{13}$: $M_{194}$) | ($t_{8}$: $M_{278}$) | | |
| $M_{278}$ | : ($t_{13}$: $M_{279}$) | ($t_{4}$: $M_{402}$) | ($t_{5}$: $M_{760}$) | |
| $M_{279}$ | : ($t_{10}$: $M_{239}$) | ($t_{20}$: $M_{196}$) | ($t_{4}$: $M_{280}$) | ($t_{5}$: $M_{757}$) |

| | | | | |
|---|---|---|---|---|
| $M_{280}$ | : ($t_1$: $M_{281}$) | ($t_{10}$: $M_{240}$) | ($t_{20}$: $M_{807}$) | ($t_7$: $M_{398}$) |
| $M_{281}$ | : ($t_{10}$: $M_{241}$) | ($t_{20}$: $M_{282}$) | ($t_6$: $M_{367}$) | ($t_7$: $M_{279}$) |
| $M_{282}$ | : ($t_{10}$: $M_{199}$) | ($t_{15}$: $M_{283}$) | ($t_6$: $M_{368}$) | ($t_7$: $M_{196}$) |
| $M_{283}$ | : ($t_{10}$: $M_{200}$) | ($t_{17}$: $M_{284}$) | ($t_6$: $M_{375}$) | ($t_7$: $M_{138}$) |
| $M_{284}$ | : ($t_{10}$: $M_{201}$) | ($t_6$: $M_{285}$) | ($t_7$: $M_{15}$) | |
| $M_{285}$ | : ($t_{10}$: $M_{202}$) | ($t_3$: $M_{284}$) | ($t_7$: $M_{286}$) | |
| $M_{286}$ | : ($t_{10}$: $M_{287}$) | ($t_3$: $M_{15}$) | | |
| $M_{287}$ | : ($t_{12}$: $M_{288}$) | ($t_{19}$: $M_{291}$) | ($t_3$: $M_{16}$) | |
| $M_{288}$ | : ($t_{19}$: $M_{205}$) | ($t_{21}$: $M_{289}$) | ($t_3$: $M_{17}$) | |
| $M_{289}$ | : ($t_{14}$: $M_{290}$) | ($t_{19}$: $M_{147}$) | ($t_3$: $M_{18}$) | |
| $M_{290}$ | : ($t_{16}$: $M_{90}$) | ($t_{19}$: $M_{24}$) | ($t_3$: $M_{19}$) | |
| $M_{291}$ | : ($t_{12}$: $M_{205}$) | ($t_9$: $M_{292}$) | | |
| $M_{292}$ | : ($t_{12}$: $M_{206}$) | ($t_4$: $M_{293}$) | ($t_5$: $M_{323}$) | |
| $M_{293}$ | : ($t_1$: $M_{294}$) | ($t_{12}$:.$M_{311}$) | ($t_7$: $M_{322}$) | |
| $M_{294}$ | : ($t_{12}$: $M_{295}$) | ($t_6$: $M_{309}$) | ($t_7$: $M_{292}$) | |
| $M_{295}$ | : ($t_{11}$: $M_{209}$) | ($t_{21}$: $M_{296}$) | ($t_6$: $M_{306}$) | ($t_7$: $M_{206}$) |
| $M_{296}$ | : ($t_{11}$: $M_{151}$) | ($t_{14}$: $M_{297}$) | ($t_6$: $M_{302}$) | ($t_7$: $M_{148}$) |
| $M_{297}$ | : ($t_{11}$: $M_{152}$) | ($t_{16}$: $M_{83}$) | ($t_6$: $M_{298}$) | ($t_7$: $M_{25}$) |
| $M_{298}$ | : ($t_{11}$: $M_{153}$) | ($t_{16}$: $M_{84}$) | ($t_3$: $M_{297}$) | ($t_7$: $M_{299}$) |
| $M_{299}$ | : ($t_{11}$: $M_{300}$) | ($t_{16}$: $M_{85}$) | ($t_3$: $M_{25}$) | |
| $M_{300}$ | : ($t_{13}$: $M_{301}$) | ($t_{16}$: $M_{86}$) | ($t_{19}$: $M_{214}$) | ($t_3$: $M_{26}$) |
| $M_{301}$ | : ($t_{16}$: $M_{87}$) | ($t_{19}$: $M_{156}$) | ($t_3$: $M_{27}$) | |
| $M_{302}$ | : ($t_{11}$: $M_{303}$) | ($t_{14}$: $M_{298}$) | ($t_3$: $M_{296}$) | ($t_7$: $M_{305}$) |
| $M_{303}$ | : ($t_{14}$: $M_{153}$) | ($t_{19}$: $M_{212}$) | ($t_3$: $M_{151}$) | ($t_7$: $M_{304}$) |
| $M_{304}$ | : ($t_{14}$: $M_{300}$) | ($t_{19}$: $M_{256}$) | ($t_3$: $M_{149}$) | |
| $M_{305}$ | : ($t_{11}$: $M_{304}$) | ($t_{14}$: $M_{299}$) | ($t_3$: $M_{148}$) | |
| $M_{306}$ | : ($t_{11}$: $M_{210}$) | ($t_{21}$: $M_{302}$) | ($t_3$: $M_{295}$) | ($t_7$: $M_{307}$) |
| $M_{307}$ | : ($t_{11}$: $M_{308}$) | ($t_{21}$: $M_{305}$) | ($t_3$: $M_{206}$) | |
| $M_{308}$ | : ($t_{19}$: $M_{255}$) | ($t_{21}$: $M_{304}$) | ($t_3$: $M_{207}$) | |
| $M_{309}$ | : ($t_{12}$: $M_{306}$) | ($t_3$: $M_{294}$) | ($t_7$: $M_{310}$) | |
| $M_{310}$ | : ($t_{12}$: $M_{307}$) | ($t_3$: $M_{292}$) | | |
| $M_{311}$ | : ($t_1$: $M_{295}$) | ($t_{11}$: $M_{208}$) | ($t_{21}$: $M_{312}$) | ($t_7$: $M_{320}$) |
| $M_{312}$ | : ($t_1$: $M_{296}$) | ($t_{11}$: $M_{150}$) | ($t_{14}$: $M_{313}$) | ($t_7$: $M_{318}$) |
| $M_{313}$ | : ($t_1$: $M_{297}$) | ($t_{11}$: $M_{314}$) | ($t_{16}$: $M_{119}$) | ($t_7$: $M_{317}$) |
| $M_{314}$ | : ($t_1$: $M_{152}$) | ($t_{13}$: $M_{28}$) | ($t_{16}$: $M_{109}$) | ($t_7$: $M_{315}$) |
| $M_{315}$ | : ($t_1$: $M_{26}$) | ($t_{13}$: $M_{316}$) | ($t_{16}$: $M_{116}$) | |
| $M_{316}$ | : ($t_1$: $M_{27}$) | ($t_{16}$:.$M_{115}$) | | |
| $M_{317}$ | : ($t_1$: $M_{25}$) | ($t_{11}$: $M_{315}$) | ($t_{16}$: $M_{120}$) | |
| $M_{318}$ | : ($t_1$: $M_{148}$) | ($t_{11}$: $M_{319}$) | ($t_{14}$: $M_{317}$) | |
| $M_{319}$ | : ($t_1$: $M_{149}$) | ($t_{14}$: $M_{315}$) | | |
| $M_{320}$ | : ($t_1$: $M_{206}$) | ($t_{11}$: $M_{321}$) | ($t_{21}$: $M_{318}$) | |
| $M_{321}$ | : ($t_1$: $M_{207}$) | ($t_{21}$: $M_{319}$) | | |
| $M_{322}$ | : ($t_1$: $M_{292}$) | ($t_{12}$: $M_{320}$) | | |

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|---|---|---|---|---|
| $M_{323}$ | : $(t_{12}: M_{324})$ | $(t_{2}: M_{294})$ | $(t_{7}: M_{506})$ | |
| $M_{324}$ | : $(t_{11}: M_{325})$ | $(t_{2}: M_{295})$ | $(t_{21}: M_{804})$ | $(t_{7}: M_{501})$ |
| $M_{325}$ | : $(t_{18}: M_{326})$ | $(t_{2}: M_{209})$ | $(t_{21}: M_{797})$ | $(t_{7}: M_{497})$ |
| $M_{326}$ | : $(t_{21}: M_{327})$ | $(t_{7}: M_{359})$ | $(t_{8}: M_{740})$ | |
| $M_{327}$ | : $(t_{14}: M_{328})$ | $(t_{7}: M_{360})$ | $(t_{8}: M_{457})$ | |
| $M_{328}$ | : $(t_{13}: M_{329})$ | $(t_{16}: M_{796})$ | $(t_{7}: M_{361})$ | $(t_{8}: M_{364})$ |
| $M_{329}$ | : $(t_{16}: M_{134})$ | $(t_{7}: M_{330})$ | $(t_{8}: M_{365})$ | |
| $M_{330}$ | : $(t_{16}: M_{194})$ | $(t_{8}: M_{331})$ | | |
| $M_{331}$ | : $(t_{10}: M_{332})$ | $(t_{16}: M_{279})$ | $(t_{4}: M_{387})$ | $(t_{5}: M_{406})$ |
| $M_{332}$ | : $(t_{16}: M_{239})$ | $(t_{4}: M_{333})$ | $(t_{5}: M_{407})$ | |
| $M_{333}$ | : $(t_{1}: M_{334})$ | $(t_{16}: M_{240})$ | $(t_{7}: M_{389})$ | |
| $M_{334}$ | : $(t_{16}: M_{241})$ | $(t_{6}: M_{335})$ | $(t_{7}: M_{332})$ | |
| $M_{335}$ | : $(t_{16}: M_{242})$ | $(t_{19}: M_{336})$ | $(t_{3}: M_{334})$ | $(t_{7}: M_{381})$ |
| $M_{336}$ | : $(t_{16}: M_{243})$ | $(t_{7}: M_{337})$ | $(t_{9}: M_{790})$ | |
| $M_{337}$ | : $(t_{16}: M_{338})$ | $(t_{9}: M_{788})$ | | |
| $M_{338}$ | : $(t_{20}: M_{339})$ | $(t_{9}: M_{780})$ | | |
| $M_{339}$ | : $(t_{15}: M_{340})$ | $(t_{9}: M_{772})$ | | |
| $M_{340}$ | : $(t_{12}: M_{247})$ | $(t_{17}: M_{291})$ | $(t_{9}: M_{341})$ | |
| $M_{341}$ | : $(t_{12}: M_{248})$ | $(t_{17}: M_{292})$ | $(t_{4}: M_{342})$ | $(t_{5}: M_{354})$ |
| $M_{342}$ | : $(t_{1}: M_{343})$ | $(t_{12}: M_{350})$ | $(t_{17}: M_{293})$ | $(t_{7}: M_{353})$ |
| $M_{343}$ | : $(t_{12}: M_{344})$ | $(t_{17}: M_{294})$ | $(t_{6}: M_{348})$ | $(t_{7}: M_{341})$ |
| $M_{344}$ | : $(t_{11}: M_{251})$ | $(t_{17}: M_{295})$ | $(t_{6}: M_{345})$ | $(t_{7}: M_{248})$ |
| $M_{345}$ | : $(t_{11}: M_{252})$ | $(t_{17}: M_{306})$ | $(t_{3}: M_{344})$ | $(t_{7}: M_{346})$ |
| $M_{346}$ | : $(t_{11}: M_{347})$ | $(t_{17}: M_{307})$ | $(t_{3}: M_{248})$ | |
| $M_{347}$ | : $(t_{17}: M_{308})$ | $(t_{19}: M_{254})$ | $(t_{3}: M_{249})$ | |
| $M_{348}$ | : $(t_{12}: M_{345})$ | $(t_{17}: M_{309})$ | $(t_{3}: M_{343})$ | $(t_{7}: M_{349})$ |
| $M_{349}$ | : $(t_{12}: M_{346})$ | $(t_{17}: M_{310})$ | $(t_{3}: M_{341})$ | |
| $M_{350}$ | : $(t_{1}: M_{344})$ | $(t_{11}: M_{250})$ | $(t_{17}: M_{311})$ | $(t_{7}: M_{351})$ |
| $M_{351}$ | : $(t_{1}: M_{248})$ | $(t_{11}: M_{352})$ | $(t_{17}: M_{320})$ | |
| $M_{352}$ | : $(t_{1}: M_{249})$ | $(t_{17}: M_{321})$ | | |
| $M_{353}$ | : $(t_{1}: M_{341})$ | $(t_{12}: M_{351})$ | $(t_{17}: M_{322})$ | |
| $M_{354}$ | : $(t_{12}: M_{355})$ | $(t_{17}: M_{323})$ | $(t_{2}: M_{343})$ | $(t_{7}: M_{771})$ |
| $M_{355}$ | : $(t_{11}: M_{356})$ | $(t_{17}: M_{324})$ | $(t_{2}: M_{344})$ | $(t_{7}: M_{770})$ |
| $M_{356}$ | : $(t_{17}: M_{325})$ | $(t_{18}: M_{357})$ | $(t_{2}: M_{251})$ | $(t_{7}: M_{769})$ |
| $M_{357}$ | : $(t_{17}: M_{326})$ | $(t_{7}: M_{358})$ | $(t_{8}: M_{763})$ | |
| $M_{358}$ | : $(t_{17}: M_{359})$ | $(t_{8}: M_{761})$ | | |
| $M_{359}$ | : $(t_{21}: M_{360})$ | $(t_{8}: M_{454})$ | | |
| $M_{360}$ | : $(t_{14}: M_{361})$ | $(t_{8}: M_{455})$ | | |
| $M_{361}$ | : $(t_{13}: M_{330})$ | $(t_{16}: M_{277})$ | $(t_{8}: M_{362})$ | |
| $M_{362}$ | : $(t_{13}: M_{331})$ | $(t_{16}: M_{278})$ | $(t_{4}: M_{363})$ | $(t_{5}: M_{405})$ |
| $M_{363}$ | : $(t_{1}: M_{364})$ | $(t_{13}: M_{387})$ | $(t_{16}: M_{402})$ | $(t_{7}: M_{404})$ |
| $M_{364}$ | : $(t_{13}: M_{365})$ | $(t_{16}: M_{382})$ | $(t_{6}: M_{385})$ | $(t_{7}: M_{362})$ |
| $M_{365}$ | : $(t_{10}: M_{334})$ | $(t_{16}: M_{281})$ | $(t_{6}: M_{366})$ | $(t_{7}: M_{331})$ |

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|---|---|---|---|---|---|
| $M_{366}$ | : $(t_{10}: M_{335})$ | $(t_{16}: M_{367})$ | $(t_3: M_{365})$ | $(t_7: M_{380})$ | |
| $M_{367}$ | : $(t_{10}: M_{242})$ | $(t_{20}: M_{368})$ | $(t_3: M_{281})$ | $(t_7: M_{378})$ | |
| $M_{368}$ | : $(t_{10}: M_{369})$ | $(t_{15}: M_{375})$ | $(t_3: M_{282})$ | $(t_7: M_{377})$ | |
| $M_{369}$ | : $(t_{15}: M_{370})$ | $(t_{19}: M_{244})$ | $(t_3: M_{199})$ | $(t_7: M_{374})$ | |
| $M_{370}$ | : $(t_{12}: M_{371})$ | $(t_{17}: M_{202})$ | $(t_{19}: M_{245})$ | $(t_3: M_{200})$ | $(t_7: M_{373})$ |
| $M_{371}$ | : $(t_{17}: M_{203})$ | $(t_{19}: M_{246})$ | $(t_3: M_{142})$ | $(t_7: M_{372})$ | |
| $M_{372}$ | : $(t_{17}: M_{288})$ | $(t_{19}: M_{247})$ | $(t_3: M_{140})$ | | |
| $M_{373}$ | : $(t_{12}: M_{372})$ | $(t_{17}: M_{287})$ | $(t_{19}: M_{340})$ | $(t_3: M_{139})$ | |
| $M_{374}$ | : $(t_{15}: M_{373})$ | $(t_{19}: M_{339})$ | $(t_3: M_{197})$ | | |
| $M_{375}$ | : $(t_{10}: M_{370})$ | $(t_{17}: M_{285})$ | $(t_3: M_{283})$ | $(t_7: M_{376})$ | |
| $M_{376}$ | : $(t_{10}: M_{373})$ | $(t_{17}: M_{286})$ | $(t_3: M_{138})$ | | |
| $M_{377}$ | : $(t_{10}: M_{374})$ | $(t_{15}: M_{376})$ | $(t_3: M_{196})$ | | |
| $M_{378}$ | : $(t_{10}: M_{379})$ | $(t_{20}: M_{377})$ | $(t_3: M_{279})$ | | |
| $M_{379}$ | : $(t_{19}: M_{338})$ | $(t_{20}: M_{374})$ | $(t_3: M_{239})$ | | |
| $M_{380}$ | : $(t_{10}: M_{381})$ | $(t_{16}: M_{378})$ | $(t_3: M_{331})$ | | |
| $M_{381}$ | : $(t_{16}: M_{379})$ | $(t_{19}: M_{337})$ | $(t_3: M_{332})$ | | |
| $M_{382}$ | : $(t_{13}: M_{281})$ | $(t_6: M_{383})$ | $(t_7: M_{278})$ | | |
| $M_{383}$ | : $(t_{13}: M_{367})$ | $(t_3:.M_{382})$ | $(t_7: M_{384})$ | | |
| $M_{384}$ | : $(t_{13}: M_{378})$ | $(t_3: M_{278})$ | | | |
| $M_{385}$ | : $(t_{13}: M_{366})$ | $(t_{16}: M_{383})$ | $(t_3: M_{364})$ | $(t_7: M_{386})$ | |
| $M_{386}$ | : $(t_{13}: M_{380})$ | $(t_{16}: M_{384})$ | $(t_3: M_{362})$ | | |
| $M_{387}$ | : $(t_1: M_{365})$ | $(t_{10}: M_{333})$ | $(t_{16}: M_{280})$ | $(t_7: M_{388})$ | |
| $M_{388}$ | : $(t_1: M_{331})$ | $(t_{10}: M_{389})$ | $(t_{16}: M_{398})$ | | |
| $M_{389}$ | : $(t_1: M_{332})$ | $(t_{16}: M_{390})$ | | | |
| $M_{390}$ | : $(t_1: M_{239})$ | $(t_{20}: M_{391})$ | | | |
| $M_{391}$ | : $(t_1: M_{197})$ | $(t_{15}: M_{392})$ | | | |
| $M_{392}$ | : $(t_1: M_{139})$ | $(t_{12}: M_{393})$ | $(t_{17}: M_{397})$ | | |
| $M_{393}$ | : $(t_1: M_{140})$ | $(t_{17}: M_{394})$ | | | |
| $M_{394}$ | : $(t_1: M_{17})$ | $(t_{21}: M_{395})$ | | | |
| $M_{395}$ | : $(t_1: M_{18})$ | $(t_{14}: M_{396})$ | | | |
| $M_{396}$ | : $(t_1: M_{19})$ | $(t_{16}: M_{11})$ | | | |
| $M_{397}$ | : $(t_1: M_{16})$ | $(t_{12}: M_{394})$ | | | |
| $M_{398}$ | : $(t_1: M_{279})$ | $(t_{10}: M_{390})$ | $(t_{20}: M_{399})$ | | |
| $M_{399}$ | : $(t_1: M_{196})$ | $(t_{10}: M_{391})$ | $(t_{15}: M_{400})$ | | |
| $M_{400}$ | : $(t_1: M_{138})$ | $(t_{10}: M_{392})$ | $(t_{17}: M_{401})$ | | |
| $M_{401}$ | : $(t_1: M_{15})$ | $(t_{10}: M_{397})$ | | | |
| $M_{402}$ | : $(t_1: M_{382})$ | $(t_{13}: M_{280})$ | $(t_7: M_{403})$ | | |
| $M_{403}$ | : $(t_1: M_{278})$ | $(t_{13}: M_{398})$ | | | |
| $M_{404}$ | : $(t_1: M_{362})$ | $(t_{13}: M_{388})$ | $(t_{16}: M_{403})$ | | |
| $M_{405}$ | : $(t_{13}: M_{406})$ | $(t_{16}: M_{760})$ | $(t_2: M_{364})$ | $(t_7: M_{463})$ | |
| $M_{406}$ | : $(t_{10}: M_{407})$ | $(t_{16}: M_{757})$ | $(t_2: M_{365})$ | $(t_7: M_{464})$ | |
| $M_{407}$ | : $(t_{16}: M_{408})$ | $(t_{18}: M_{756})$ | $(t_2: M_{334})$ | $(t_7: M_{465})$ | |
| $M_{408}$ | : $(t_{18}: M_{409})$ | $(t_2: M_{241})$ | $(t_{20}: M_{753})$ | $(t_7: M_{466})$ | |

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|---|---|---|---|---|
| $M_{409}$ | : $(t_{20}: M_{410})$ | $(t_7: M_{467})$ | $(t_8: M_{717})$ | |
| $M_{410}$ | : $(t_{15}: M_{411})$ | $(t_7: M_{468})$ | $(t_8: M_{709})$ | |
| $M_{411}$ | : $(t_{12}: M_{412})$ | $(t_{17}: M_{561})$ | $(t_7: M_{469})$ | $(t_8: M_{701})$ |
| $M_{412}$ | : $(t_{17}: M_{413})$ | $(t_7: M_{470})$ | $(t_8: M_{693})$ | |
| $M_{413}$ | : $(t_{21}: M_{414})$ | $(t_7: M_{471})$ | $(t_8: M_{566})$ | |
| $M_{414}$ | : $(t_{14}: M_{415})$ | $(t_7: M_{472})$ | $(t_8: M_{567})$ | |
| $M_{415}$ | : $(t_{16}: M_{13})$ | $(t_7: M_{416})$ | $(t_8: M_{568})$ | |
| $M_{416}$ | : $(t_{16}: M_{14})$ | $(t_8: M_{417})$ | | |
| $M_{417}$ | : $(t_{10}: M_{418})$ | $(t_{16}: M_{15})$ | $(t_4: M_{576})$ | $(t_5: M_{583})$ |
| $M_{418}$ | : $(t_{12}: M_{419})$ | $(t_{16}: M_{16})$ | $(t_4: M_{521})$ | $(t_5: M_{529})$ |
| $M_{419}$ | : $(t_{16}: M_{17})$ | $(t_4: M_{420})$ | $(t_5: M_{530})$ | |
| $M_{420}$ | : $(t_1: M_{421})$ | $(t_{16}: M_{523})$ | $(t_7: M_{526})$ | |
| $M_{421}$ | : $(t_{16}: M_{143})$ | $(t_6: M_{422})$ | $(t_7: M_{419})$ | |
| $M_{422}$ | : $(t_{16}: M_{203})$ | $(t_{19}: M_{423})$ | $(t_3: M_{421})$ | $(t_7: M_{508})$ |
| $M_{423}$ | : $(t_{16}: M_{204})$ | $(t_7: M_{424})$ | $(t_9: M_{485})$ | |
| $M_{424}$ | : $(t_{16}: M_{205})$ | $(t_9: M_{425})$ | | |
| $M_{425}$ | : $(t_{11}: M_{426})$ | $(t_{16}: M_{206})$ | $(t_4: M_{490})$ | $(t_5: M_{494})$ |
| $M_{426}$ | : $(t_{16}: M_{207})$ | $(t_4: M_{427})$ | $(t_5: M_{450})$ | |
| $M_{427}$ | : $(t_1: M_{428})$ | $(t_{16}: M_{208})$ | $(t_7: M_{449})$ | |
| $M_{428}$ | : $(t_{16}: M_{209})$ | $(t_6:.M_{429})$ | $(t_7: M_{426})$ | |
| $M_{429}$ | : $(t_{16}: M_{210})$ | $(t_{19}: M_{430})$ | $(t_3: M_{428})$ | $(t_7: M_{448})$ |
| $M_{430}$ | : $(t_{16}: M_{211})$ | $(t_7: M_{431})$ | $(t_9: M_{442})$ | |
| $M_{431}$ | : $(t_{16}: M_{255})$ | $(t_9: M_{432})$ | | |
| $M_{432}$ | : $(t_{16}: M_{433})$ | $(t_4: M_{441})$ | $(t_5: M_{446})$ | |
| $M_{433}$ | : $(t_{21}: M_{257})$ | $(t_4: M_{434})$ | $(t_5: M_{439})$ | |
| $M_{434}$ | : $(t_1: M_{435})$ | $(t_{21}: M_{258})$ | $(t_7: M_{438})$ | |
| $M_{435}$ | : $(t_{21}: M_{259})$ | $(t_6: M_{436})$ | $(t_7: M_{433})$ | |
| $M_{436}$ | : $(t_{21}: M_{260})$ | $(t_3: M_{435})$ | $(t_7: M_{437})$ | |
| $M_{437}$ | : $(t_{21}: M_{261})$ | $(t_3: M_{433})$ | | |
| $M_{438}$ | : $(t_1: M_{433})$ | $(t_{21}: M_{262})$ | | |
| $M_{439}$ | : $(t_2: M_{435})$ | $(t_{21}: M_{263})$ | $(t_7: M_{440})$ | |
| $M_{440}$ | : $(t_2: M_{433})$ | $(t_{21}: M_{264})$ | | |
| $M_{441}$ | : $(t_1: M_{442})$ | $(t_{16}: M_{434})$ | $(t_7: M_{445})$ | |
| $M_{442}$ | : $(t_{16}: M_{435})$ | $(t_6: M_{443})$ | $(t_7: M_{432})$ | |
| $M_{443}$ | : $(t_{16}: M_{436})$ | $(t_3: M_{442})$ | $(t_7: M_{444})$ | |
| $M_{444}$ | : $(t_{16}: M_{437})$ | $(t_3: M_{432})$ | | |
| $M_{445}$ | : $(t_1: M_{432})$ | $(t_{16}: M_{438})$ | | |
| $M_{446}$ | : $(t_{16}: M_{439})$ | $(t_2: M_{442})$ | $(t_7: M_{447})$ | |
| $M_{447}$ | : $(t_{16}: M_{440})$ | $(t_2: M_{432})$ | | |
| $M_{448}$ | : $(t_{16}: M_{308})$ | $(t_{19}: M_{431})$ | $(t_3: M_{426})$ | |
| $M_{449}$ | : $(t_1: M_{426})$ | $(t_{16}: M_{321})$ | | |
| $M_{450}$ | : $(t_{16}: M_{325})$ | $(t_{18}: M_{451})$ | $(t_2: M_{428})$ | $(t_7: M_{496})$ |
| $M_{451}$ | : $(t_{16}: M_{326})$ | $(t_7: M_{452})$ | $(t_8: M_{747})$ | |

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|---|---|---|---|---|---|
| $M_{452}$ | : ($t_{16}$: $M_{359}$) | ($t_{8}$: $M_{453}$) | | | |
| $M_{453}$ | : ($t_{16}$: $M_{454}$) | ($t_{4}$: $M_{746}$) | ($t_{5}$: $M_{751}$) | | |
| $M_{454}$ | : ($t_{21}$: $M_{455}$) | ($t_{4}$: $M_{739}$) | ($t_{5}$: $M_{744}$) | | |
| $M_{455}$ | : ($t_{14}$: $M_{362}$) | ($t_{4}$: $M_{456}$) | ($t_{5}$: $M_{461}$) | | |
| $M_{456}$ | : ($t_{1}$: $M_{457}$) | ($t_{14}$: $M_{363}$) | ($t_{7}$: $M_{460}$) | | |
| $M_{457}$ | : ($t_{14}$: $M_{364}$) | ($t_{6}$: $M_{458}$) | ($t_{7}$: $M_{455}$) | | |
| $M_{458}$ | : ($t_{14}$: $M_{385}$) | ($t_{3}$: $M_{457}$) | ($t_{7}$: $M_{459}$) | | |
| $M_{459}$ | : ($t_{14}$: $M_{386}$) | ($t_{3}$: $M_{455}$) | | | |
| $M_{460}$ | : ($t_{1}$: $M_{455}$) | ($t_{14}$: $M_{404}$) | | | |
| $M_{461}$ | : ($t_{14}$: $M_{405}$) | ($t_{2}$: $M_{457}$) | ($t_{7}$: $M_{462}$) | | |
| $M_{462}$ | : ($t_{14}$: $M_{463}$) | ($t_{2}$: $M_{455}$) | | | |
| $M_{463}$ | : ($t_{13}$: $M_{464}$) | ($t_{16}$: $M_{738}$) | ($t_{2}$: $M_{362}$) | | |
| $M_{464}$ | : ($t_{10}$: $M_{465}$) | ($t_{16}$: $M_{735}$) | ($t_{2}$: $M_{331}$) | | |
| $M_{465}$ | : ($t_{16}$: $M_{466}$) | ($t_{18}$: $M_{726}$) | ($t_{2}$: $M_{332}$) | | |
| $M_{466}$ | : ($t_{18}$: $M_{467}$) | ($t_{2}$: $M_{239}$) | ($t_{20}$: $M_{723}$) | | |
| $M_{467}$ | : ($t_{20}$: $M_{468}$) | ($t_{8}$: $M_{715}$) | | | |
| $M_{468}$ | : ($t_{15}$: $M_{469}$) | ($t_{8}$: $M_{707}$) | | | |
| $M_{469}$ | : ($t_{12}$: $M_{470}$) | ($t_{17}$: $M_{562}$) | ($t_{8}$: $M_{699}$) | | |
| $M_{470}$ | : ($t_{17}$: $M_{471}$) | ($t_{8}$: $M_{663}$) | | | |
| $M_{471}$ | : ($t_{21}$: $M_{472}$) | ($t_{8}$: $M_{564}$) | | | |
| $M_{472}$ | : ($t_{14}$: $M_{416}$) | ($t_{8}$: $M_{473}$) | | | |
| $M_{473}$ | : ($t_{10}$: $M_{474}$) | ($t_{14}$: $M_{417}$) | ($t_{4}$: $M_{575}$) | ($t_{5}$: $M_{582}$) | |
| $M_{474}$ | : ($t_{14}$: $M_{418}$) | ($t_{4}$: $M_{475}$) | ($t_{5}$: $M_{528}$) | | |
| $M_{475}$ | : ($t_{1}$: $M_{476}$) | ($t_{14}$: $M_{521}$) | ($t_{7}$: $M_{527}$) | | |
| $M_{476}$ | : ($t_{14}$: $M_{477}$) | ($t_{6}$: $M_{509}$) | ($t_{7}$: $M_{474}$) | | |
| $M_{477}$ | : ($t_{12}$: $M_{421}$) | ($t_{16}$: $M_{201}$) | ($t_{6}$: $M_{478}$) | ($t_{7}$: $M_{418}$) | |
| $M_{478}$ | : ($t_{12}$: $M_{422}$) | ($t_{16}$: $M_{202}$) | ($t_{19}$: $M_{479}$) | ($t_{3}$: $M_{477}$) | ($t_{7}$: $M_{507}$) |
| $M_{479}$ | : ($t_{12}$: $M_{423}$) | ($t_{16}$: $M_{480}$) | ($t_{7}$: $M_{481}$) | ($t_{9}$: $M_{484}$) | |
| $M_{480}$ | : ($t_{12}$: $M_{204}$) | ($t_{7}$: $M_{291}$) | ($t_{9}$: $M_{294}$) | | |
| $M_{481}$ | : ($t_{12}$: $M_{424}$) | ($t_{16}$: $M_{291}$) | ($t_{9}$: $M_{482}$) | | |
| $M_{482}$ | : ($t_{12}$: $M_{425}$) | ($t_{16}$: $M_{292}$) | ($t_{4}$: $M_{483}$) | ($t_{5}$: $M_{493}$) | |
| $M_{483}$ | : ($t_{1}$: $M_{484}$) | ($t_{12}$: $M_{490}$) | ($t_{16}$: $M_{293}$) | ($t_{7}$: $M_{492}$) | |
| $M_{484}$ | : ($t_{12}$: $M_{485}$) | ($t_{16}$: $M_{294}$) | ($t_{6}$: $M_{488}$) | ($t_{7}$: $M_{482}$) | |
| $M_{485}$ | : ($t_{11}$: $M_{428}$) | ($t_{16}$: $M_{295}$) | ($t_{6}$: $M_{486}$) | ($t_{7}$: $M_{425}$) | |
| $M_{486}$ | : ($t_{11}$: $M_{429}$) | ($t_{16}$: $M_{306}$) | ($t_{3}$: $M_{485}$) | ($t_{7}$: $M_{487}$) | |
| $M_{487}$ | : ($t_{11}$: $M_{448}$) | ($t_{16}$: $M_{307}$) | ($t_{3}$: $M_{425}$) | | |
| $M_{488}$ | : ($t_{12}$: $M_{486}$) | ($t_{16}$: $M_{309}$) | ($t_{3}$: $M_{484}$) | ($t_{7}$: $M_{489}$) | |
| $M_{489}$ | : ($t_{12}$: $M_{487}$) | ($t_{16}$: $M_{310}$) | ($t_{3}$: $M_{482}$) | | |
| $M_{490}$ | : ($t_{1}$: $M_{485}$) | ($t_{11}$: $M_{427}$) | ($t_{16}$: $M_{311}$) | ($t_{7}$: $M_{491}$) | |
| $M_{491}$ | : ($t_{1}$: $M_{425}$) | ($t_{11}$: $M_{449}$) | ($t_{16}$: $M_{320}$) | | |
| $M_{492}$ | : ($t_{1}$: $M_{482}$) | ($t_{12}$: $M_{491}$) | ($t_{16}$: $M_{322}$) | | |
| $M_{493}$ | : ($t_{12}$: $M_{494}$) | ($t_{16}$: $M_{323}$) | ($t_{2}$: $M_{484}$) | ($t_{7}$: $M_{505}$) | |
| $M_{494}$ | : ($t_{11}$: $M_{450}$) | ($t_{16}$: $M_{324}$) | ($t_{2}$: $M_{485}$) | ($t_{7}$: $M_{495}$) | |

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|---|---|---|---|---|---|
| $M_{495}$ | : $(t_{11}: M_{496})$ | $(t_{19}: M_{501})$ | $(t_{2}: M_{425})$ | | |
| $M_{496}$ | : $(t_{16}: M_{497})$ | $(t_{18}: M_{452})$ | $(t_{2}: M_{426})$ | | |
| $M_{497}$ | : $(t_{18}: M_{359})$ | $(t_{2}: M_{207})$ | $(t_{21}: M_{498})$ | | |
| $M_{498}$ | : $(t_{14}: M_{499})$ | $(t_{18}: M_{360})$ | $(t_{2}: M_{149})$ | | |
| $M_{499}$ | : $(t_{13}: M_{500})$ | $(t_{16}: M_{276})$ | $(t_{18}: M_{361})$ | $(t_{2}: M_{26})$ | |
| $M_{500}$ | : $(t_{16}: M_{272})$ | $(t_{18}: M_{330})$ | $(t_{2}: M_{27})$ | | |
| $M_{501}$ | : $(t_{11}: M_{497})$ | $(t_{2}: M_{206})$ | $(t_{21}: M_{502})$ | | |
| $M_{502}$ | : $(t_{11}: M_{498})$ | $(t_{14}: M_{503})$ | $(t_{2}: M_{148})$ | | |
| $M_{503}$ | : $(t_{11}: M_{499})$ | $(t_{16}: M_{504})$ | $(t_{2}: M_{25})$ | | |
| $M_{504}$ | : $(t_{11}: M_{276})$ | $(t_{2}: M_{4})$ | | | |
| $M_{505}$ | : $(t_{12}: M_{495})$ | $(t_{16}: M_{506})$ | $(t_{2}: M_{482})$ | | |
| $M_{506}$ | : $(t_{12}: M_{501})$ | $(t_{2}: M_{292})$ | | | |
| $M_{507}$ | : $(t_{12}: M_{508})$ | $(t_{16}: M_{287})$ | $(t_{19}: M_{481})$ | $(t_{3}: M_{418})$ | |
| $M_{508})$ | : $(t_{16}: M_{288}$ | $(t_{19}: M_{424})$ | $(t_{3}: M_{419})$ | | |
| $M_{509}$ | : $(t_{14}: M_{478})$ | $(t_{19}: M_{510})$ | $(t_{3}: M_{476})$ | $(t_{7}: M_{520})$ | |
| $M_{510}$ | : $(t_{14}: M_{479})$ | $(t_{7}: M_{511})$ | $(t_{9}: M_{514})$ | | |
| $M_{511}$ | : $(t_{14}: M_{481})$ | $(t_{9}: M_{512})$ | | | |
| $M_{512}$ | : $(t_{14}: M_{482})$ | $(t_{4}: M_{513})$ | $(t_{5}: M_{518})$ | | |
| $M_{513}$ | : $(t_{1}: M_{514})$ | $(t_{14}: M_{483})$ | $(t_{7}: M_{517})$ | | |
| $M_{514}$ | : $(t_{14}: M_{484})$ | $(t_{6}: M_{515})$ | $(t_{7}: M_{512})$ | | |
| $M_{515}$ | : $(t_{14}: M_{488})$ | $(t_{3}: M_{514})$ | $(t_{7}: M_{516})$ | | |
| $M_{516}$ | : $(t_{14}: M_{489})$ | $(t_{3}: M_{512})$ | | | |
| $M_{517}$ | : $(t_{1}: M_{512})$ | $(t_{14}: M_{492})$ | | | |
| $M_{518}$ | : $(t_{14}: M_{493})$ | $(t_{2}:.M_{514})$ | $(t_{7}: M_{519})$ | | |
| $M_{519}$ | : $(t_{14}: M_{505})$ | $(t_{2}: M_{512})$ | | | |
| $M_{520}$ | : $(t_{14}: M_{507})$ | $(t_{19}: M_{511})$ | $(t_{3}: M_{474})$ | | |
| $M_{521}$ | : $(t_{1}: M_{477})$ | $(t_{12}: M_{420})$ | $(t_{16}: M_{522})$ | $(t_{7}: M_{525})$ | |
| $M_{522}$ | : $(t_{1}: M_{201})$ | $(t_{12}: M_{523})$ | $(t_{7}: M_{397})$ | | |
| $M_{523}$ | : $(t_{1}: M_{143})$ | $(t_{21}: M_{524})$ | $(t_{7}: M_{394})$ | | |
| $M_{524}$ | : $(t_{1}: M_{144})$ | $(t_{14}: M_{20})$ | $(t_{7}: M_{395})$ | | |
| $M_{525}$ | : $(t_{1}: M_{418})$ | $(t_{12}: M_{526})$ | $(t_{16}: M_{397})$ | | |
| $M_{526}$ | : $(t_{1}: M_{419})$ | $(t_{16}: M_{394})$ | | | |
| $M_{527}$ | : $(t_{1}: M_{474})$ | $(t_{14}: M_{525})$ | | | |
| $M_{528}$ | : $(t_{14}: M_{529})$ | $(t_{18}: M_{559})$ | $(t_{2}: M_{476})$ | $(t_{7}: M_{642})$ | |
| $M_{529}$ | : $(t_{12}: M_{530})$ | $(t_{16}: M_{585})$ | $(t_{18}: M_{560})$ | $(t_{2}: M_{477})$ | $(t_{7}: M_{589})$ |
| $M_{530}$ | : $(t_{16}: M_{531})$ | $(t_{18}: M_{537})$ | $(t_{2}: M_{421})$ | $(t_{7}: M_{590})$ | |
| $M_{531}$ | : $(t_{18}: M_{413})$ | $(t_{2}: M_{143})$ | $(t_{21}: M_{532})$ | $(t_{7}: M_{536})$ | |
| $M_{532}$ | : $(t_{14}: M_{533})$ | $(t_{18}: M_{414})$ | $(t_{2}: M_{144})$ | $(t_{7}: M_{535})$ | |
| $M_{533}$ | : $(t_{16}: M_{12})$ | $(t_{18}: M_{415})$ | $(t_{2}: M_{21})$ | $(t_{7}: M_{534})$ | |
| $M_{534}$ | : $(t_{16}: M_{275})$ | $(t_{18}: M_{416})$ | $(t_{2}: M_{19})$ | | |
| $M_{535}$ | : $(t_{14}: M_{534})$ | $(t_{18}: M_{472})$ | $(t_{2}: M_{18})$ | | |
| $M_{536}$ | : $(t_{18}: M_{471})$ | $(t_{2}: M_{17})$ | $(t_{21}: M_{535})$ | | |
| $M_{537}$ | : $(t_{16}: M_{413})$ | $(t_{7}: M_{538})$ | $(t_{8}: M_{595})$ | | |

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|---|---|---|---|---|
| $M_{538}$ | : $(t_{16}: M_{471})$ | $(t_{8}: M_{539})$ | | |
| $M_{539}$ | : $(t_{10}: M_{540})$ | $(t_{16}: M_{564})$ | $(t_{4}: M_{616})$ | $(t_{5}: M_{624})$ |
| $M_{540}$ | : $(t_{16}: M_{541})$ | $(t_{4}: M_{617})$ | $(t_{5}: M_{625})$ | |
| $M_{541}$ | : $(t_{21}: M_{474})$ | $(t_{4}: M_{542})$ | $(t_{5}: M_{557})$ | |
| $M_{542}$ | : $(t_{1}: M_{543})$ | $(t_{21}: M_{475})$ | $(t_{7}: M_{556})$ | |
| $M_{543}$ | : $(t_{21}: M_{476})$ | $(t_{6}: M_{544})$ | $(t_{7}: M_{541})$ | |
| $M_{544}$ | : $(t_{19}: M_{545})$ | $(t_{21}: M_{509})$ | $(t_{3}: M_{543})$ | $(t_{7}: M_{555})$ |
| $M_{545}$ | : $(t_{21}: M_{510})$ | $(t_{7}: M_{546})$ | $(t_{9}: M_{549})$ | |
| $M_{546}$ | : $(t_{21}: M_{511})$ | $(t_{9}: M_{547})$ | | |
| $M_{547}$ | : $(t_{21}: M_{512})$ | $(t_{4}: M_{548})$ | $(t_{5}: M_{553})$ | |
| $M_{548}$ | : $(t_{1}: M_{549})$ | $(t_{21}: M_{513})$ | $(t_{7}: M_{552})$ | |
| $M_{549}$ | : $(t_{21}: M_{514})$ | $(t_{6}: M_{550})$ | $(t_{7}: M_{547})$ | |
| $M_{550}$ | : $(t_{21}: M_{515})$ | $(t_{3}: M_{549})$ | $(t_{7}: M_{551})$ | |
| $M_{551}$ | : $(t_{21}: M_{516})$ | $(t_{3}: M_{547})$ | | |
| $M_{552}$ | : $(t_{1}: M_{547})$ | $(t_{21}: M_{517})$ | | |
| $M_{553}$ | : $(t_{2}: M_{549})$ | $(t_{21}: M_{518})$ | $(t_{7}: M_{554})$ | |
| $M_{554}$ | : $(t_{2}: M_{547})$ | $(t_{21}: M_{519})$ | | |
| $M_{555}$ | : $(t_{19}: M_{546})$ | $(t_{21}: M_{520})$ | $(t_{3}: M_{541})$ | |
| $M_{556}$ | : $(t_{1}: M_{541})$ | $(t_{21}: M_{527})$ | | |
| $M_{557}$ | : $(t_{18}: M_{558})$ | $(t_{2}: M_{543})$ | $(t_{21}: M_{528})$ | $(t_{7}: M_{641})$ |
| $M_{558}$ | : $(t_{21}: M_{559})$ | $(t_{7}: M_{628})$ | $(t_{8}: M_{648})$ | |
| $M_{559}$ | : $(t_{14}: M_{560})$ | $(t_{7}: M_{629})$ | $(t_{8}: M_{632})$ | |
| $M_{560}$ | : $(t_{12}: M_{537})$ | $(t_{16}: M_{561})$ | $(t_{7}: M_{591})$ | $(t_{8}: M_{594})$ |
| $M_{561}$ | : $(t_{12}: M_{413})$ | $(t_{7}: M_{562})$ | $(t_{8}: M_{611})$ | |
| $M_{562}$ | : $(t_{12}: M_{471})$ | $(t_{8}: M_{563})$ | | |
| $M_{563}$ | : $(t_{12}: M_{564})$ | $(t_{4}:.M_{620})$ | $(t_{5}: M_{662})$ | |
| $M_{564}$ | : $(t_{10}: M_{541})$ | $(t_{21}: M_{473})$ | $(t_{4}: M_{565})$ | $(t_{5}: M_{581})$ |
| $M_{565}$ | : $(t_{1}: M_{566})$ | $(t_{10}: M_{542})$ | $(t_{21}: M_{575})$ | $(t_{7}: M_{580})$ |
| $M_{566}$ | : $(t_{10}: M_{543})$ | $(t_{21}: M_{567})$ | $(t_{6}: M_{573})$ | $(t_{7}: M_{564})$ |
| $M_{567}$ | : $(t_{10}: M_{476})$ | $(t_{14}: M_{568})$ | $(t_{6}: M_{571})$ | $(t_{7}: M_{473})$ |
| $M_{568}$ | : $(t_{10}: M_{477})$ | $(t_{16}: M_{284})$ | $(t_{6}: M_{569})$ | $(t_{7}: M_{417})$ |
| $M_{569}$ | : $(t_{10}: M_{478})$ | $(t_{16}: M_{285})$ | $(t_{3}: M_{568})$ | $(t_{7}: M_{570})$ |
| $M_{570}$ | : $(t_{10}: M_{507})$ | $(t_{16}: M_{286})$ | $(t_{3}: M_{417})$ | |
| $M_{571}$ | : $(t_{10}: M_{509})$ | $(t_{14}: M_{569})$ | $(t_{3}: M_{567})$ | $(t_{7}: M_{572}$ |
| $M_{572}$ | : $(t_{10}: M_{520})$ | $(t_{14}: M_{570})$ | $(t_{3}: M_{473})$ | |
| $M_{573}$ | : $(t_{10}: M_{544})$ | $(t_{21}: M_{571})$ | $(t_{3}: M_{566})$ | $(t_{7}: M_{574})$ |
| $M_{574}$ | : $(t_{10}: M_{555})$ | $(t_{21}: M_{572})$ | $(t_{3}: M_{564})$ | |
| $M_{575}$ | : $(t_{1}: M_{567})$ | $(t_{10}: M_{475})$ | $(t_{14}: M_{576})$ | $(t_{7}: M_{579})$ |
| $M_{576}$ | : $(t_{1}: M_{568})$ | $(t_{10}: M_{521})$ | $(t_{16}: M_{577})$ | $(t_{7}: M_{578})$ |
| $M_{577}$ | : $(t_{1}: M_{284})$ | $(t_{10}: M_{522})$ | $(t_{7}: M_{401})$ | |
| $M_{578}$ | : $(t_{1}: M_{417})$ | $(t_{10}: M_{525})$ | $(t_{16}: M_{401})$ | |
| $M_{579}$ | : $(t_{1}: M_{473})$ | $(t_{10}: M_{527})$ | $(t_{14}: M_{578})$ | |
| $M_{580}$ | : $(t_{1}: M_{564})$ | $(t_{10}: M_{556})$ | $(t_{21}: M_{579})$ | |

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|---|---|---|---|---|
| $M_{581}$ | : ($t_{10}$: $M_{557}$) | ($t_2$: $M_{566}$) | ($t_{21}$: $M_{582}$) | ($t_7$: $M_{643}$) |
| $M_{582}$ | : ($t_{10}$: $M_{528}$) | ($t_{14}$: $M_{583}$) | ($t_2$: $M_{567}$) | ($t_7$: $M_{644}$) |
| $M_{583}$ | : ($t_{10}$: $M_{529}$) | ($t_{16}$: $M_{584}$) | ($t_2$: $M_{568}$) | ($t_7$: $M_{588}$) |
| $M_{584}$ | : ($t_{10}$: $M_{585}$) | ($t_2$: $M_{284}$) | ($t_7$: $M_{587}$) | |
| $M_{585}$ | : ($t_{12}$: $M_{531}$) | ($t_{18}$: $M_{561}$) | ($t_2$: $M_{201}$) | ($t_7$: $M_{586}$) |
| $M_{586}$ | : ($t_{12}$: $M_{536}$) | ($t_{18}$: $M_{562}$) | ($t_2$: $M_{16}$) | |
| $M_{587}$ | : ($t_{10}$: $M_{586}$) | ($t_2$: $M_{15}$) | | |
| $M_{588}$ | : ($t_{10}$: $M_{589}$) | ($t_{16}$: $M_{587}$) | ($t_2$: $M_{417}$) | |
| $M_{589}$ | : ($t_{12}$: $M_{590}$) | ($t_{16}$: $M_{586}$) | ($t_{18}$: $M_{591}$) | ($t_2$: $M_{418}$) |
| $M_{590}$ | : ($t_{16}$: $M_{536}$) | ($t_{18}$: $M_{538}$) | ($t_2$: $M_{419}$) | |
| $M_{591}$ | : ($t_{12}$: $M_{538}$) | ($t_{16}$: $M_{562}$) | ($t_8$: $M_{592}$) | |
| $M_{592}$ | : ($t_{12}$: $M_{539}$) | ($t_{16}$: $M_{563}$) | ($t_4$: $M_{593}$) | ($t_5$: $M_{623}$) |
| $M_{593}$ | : ($t_1$: $M_{594}$) | ($t_{12}$: $M_{616}$) | ($t_{16}$: $M_{620}$) | ($t_7$: $M_{622}$) |
| $M_{594}$ | : ($t_{12}$: $M_{595}$) | ($t_{16}$: $M_{611}$) | ($t_6$: $M_{614}$) | ($t_7$: $M_{592}$) |
| $M_{595}$ | : ($t_{10}$: $M_{596}$) | ($t_{16}$: $M_{566}$) | ($t_6$: $M_{609}$) | ($t_7$: $M_{539}$) |
| $M_{596}$ | : ($t_{16}$: $M_{543}$) | ($t_6$: $M_{597}$) | ($t_7$: $M_{540}$) | |
| $M_{597}$ | : ($t_{16}$: $M_{544}$) | ($t_{19}$: $M_{598}$) | ($t_3$: $M_{596}$) | ($t_7$: $M_{608}$) |
| $M_{598}$ | : ($t_{16}$: $M_{545}$) | ($t_7$: $M_{599}$) | ($t_9$: $M_{602}$) | |
| $M_{599}$ | : ($t_{16}$: $M_{546}$) | ($t_9$: $M_{600}$) | | |
| $M_{600}$ | : ($t_{16}$: $M_{547}$) | ($t_4$: $M_{601}$) | ($t_5$: $M_{606}$) | |
| $M_{601}$ | : ($t_1$: $M_{602}$) | ($t_{16}$: $M_{548}$) | ($t_7$: $M_{605}$) | |
| $M_{602}$ | : ($t_{16}$: $M_{549}$) | ($t_6$: $M_{603}$) | ($t_7$: $M_{600}$) | |
| $M_{603}$ | : ($t_{16}$: $M_{550}$) | ($t_3$: $M_{602}$) | ($t_7$: $M_{604}$) | |
| $M_{604}$ | : ($t_{16}$: $M_{551}$) | ($t_3$: $M_{600}$) | | |
| $M_{605}$ | : ($t_1$: $M_{600}$) | ($t_{16}$: $M_{552}$) | | |
| $M_{606}$ | : ($t_{16}$: $M_{553}$) | ($t_2$: $M_{602}$) | ($t_7$: $M_{607}$) | |
| $M_{607}$ | : ($t_{16}$: $M_{554}$) | ($t_2$: $M_{600}$) | | |
| $M_{608}$ | : ($t_{16}$: $M_{555}$) | ($t_{19}$: $M_{599}$) | ($t_3$: $M_{540}$) | |
| $M_{609}$ | : ($t_{10}$: $M_{597}$) | ($t_{16}$: $M_{573}$) | ($t_3$: $M_{595}$) | ($t_7$: $M_{610}$) |
| $M_{610}$ | : ($t_{10}$: $M_{608}$) | ($t_{16}$: $M_{574}$) | ($t_3$: $M_{539}$) | |
| $M_{611}$ | : ($t_{12}$: $M_{566}$) | ($t_6$: $M_{612}$) | ($t_7$: $M_{563}$) | |
| $M_{612}$ | : ($t_{12}$: $M_{573}$) | ($t_3$: $M_{611}$) | ($t_7$: $M_{613}$) | |
| $M_{613}$ | : ($t_{12}$: $M_{574}$) | ($t_3$: $M_{563}$) | | |
| $M_{614}$ | : ($t_{12}$: $M_{609}$) | ($t_{16}$: $M_{612}$) | ($t_3$: $M_{594}$) | ($t_7$: $M_{615}$) |
| $M_{615}$ | : ($t_{12}$: $M_{610}$) | ($t_{16}$: $M_{613}$) | ($t_3$: $M_{592}$) | |
| $M_{616}$ | : ($t_1$: $M_{595}$) | ($t_{10}$: $M_{617}$) | ($t_{16}$: $M_{565}$) | ($t_7$: $M_{619}$) |
| $M_{617}$ | : ($t_1$: $M_{596}$) | ($t_{16}$: $M_{542}$) | ($t_7$: $M_{618}$) | |
| $M_{618}$ | : ($t_1$: $M_{540}$) | ($t_{16}$: $M_{556}$) | | |
| $M_{619}$ | : ($t_1$: $M_{539}$) | ($t_{10}$: $M_{618}$) | ($t_{16}$: $M_{580}$) | |
| $M_{620}$ | : ($t_1$: $M_{611}$) | ($t_{12}$: $M_{565}$) | ($t_7$: $M_{621}$) | |
| $M_{621}$ | : ($t_1$: $M_{563}$) | ($t_{12}$: $M_{580}$) | | |
| $M_{622}$ | : ($t_1$: $M_{592}$) | ($t_{12}$: $M_{619}$) | ($t_{16}$: $M_{621}$) | |
| $M_{623}$ | : ($t_{12}$: $M_{624}$) | ($t_{16}$: $M_{662}$) | ($t_2$: $M_{594}$) | ($t_7$: $M_{638}$) |

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|---|---|---|---|---|
| $M_{624}$ | : $(t_{10}: M_{625})$ | $(t_{16}: M_{581})$ | $(t_2: M_{595})$ | $(t_7: M_{639})$ |
| $M_{625}$ | : $(t_{16}: M_{557})$ | $(t_{18}: M_{626})$ | $(t_2: M_{596})$ | $(t_7: M_{640})$ |
| $M_{626}$ | : $(t_{16}: M_{558})$ | $(t_7: M_{627})$ | $(t_8: M_{656})$ | |
| $M_{627}$ | : $(t_{16}: M_{628})$ | $(t_8: M_{654})$ | | |
| $M_{628}$ | : $(t_{21}: M_{629})$ | $(t_8: M_{646})$ | | |
| $M_{629}$ | : $(t_{14}: M_{591})$ | $(t_8: M_{630})$ | | |
| $M_{630}$ | : $(t_{14}: M_{592})$ | $(t_4:.M_{631})$ | $(t_5: M_{636})$ | |
| $M_{631}$ | : $(t_1: M_{632})$ | $(t_{14}: M_{593})$ | $(t_7: M_{635})$ | |
| $M_{632}$ | : $(t_{14}: M_{594})$ | $(t_6: M_{633})$ | $(t_7: M_{630})$ | |
| $M_{633}$ | : $(t_{14}: M_{614})$ | $(t_3: M_{632})$ | $(t_7: M_{634})$ | |
| $M_{634}$ | : $(t_{14}: M_{615})$ | $(t_3: M_{630})$ | | |
| $M_{635}$ | : $(t_1: M_{630})$ | $(t_{14}: M_{622})$ | | |
| $M_{636}$ | : $(t_{14}: M_{623})$ | $(t_2: M_{632})$ | $(t_7: M_{637})$ | |
| $M_{637}$ | : $(t_{14}: M_{638})$ | $(t_2: M_{630})$ | | |
| $M_{638}$ | : $(t_{12}: M_{639})$ | $(t_{16}: M_{645})$ | $(t_2: M_{592})$ | |
| $M_{639}$ | : $(t_{10}: M_{640})$ | $(t_{16}: M_{643})$ | $(t_2: M_{539})$ | |
| $M_{640}$ | : $(t_{16}: M_{641})$ | $(t_{18}: M_{627})$ | $(t_2: M_{540})$ | |
| $M_{641}$ | : $(t_{18}: M_{628})$ | $(t_2: M_{541})$ | $(t_{21}: M_{642})$ | |
| $M_{642}$ | : $(t_{14}: M_{589})$ | $(t_{18}: M_{629})$ | $(t_2: M_{474})$ | |
| $M_{643}$ | : $(t_{10}: M_{641})$ | $(t_2: M_{564})$ | $(t_{21}: M_{644})$ | |
| $M_{644}$ | : $(t_{10}: M_{642})$ | $(t_{14}: M_{588})$ | $(t_2: M_{473})$ | |
| $M_{645}$ | : $(t_{12}: M_{643})$ | $(t_2: M_{563})$ | | |
| $M_{646}$ | : $(t_{21}: M_{630})$ | $(t_4: M_{647})$ | $(t_5: M_{652})$ | |
| $M_{647}$ | : $(t_1: M_{648})$ | $(t_{21}: M_{631})$ | $(t_7: M_{651})$ | |
| $M_{648}$ | : $(t_{21}: M_{632})$ | $(t_6: M_{649})$ | $(t_7: M_{646})$ | |
| $M_{649}$ | : $(t_{21}: M_{633})$ | $(t_3: M_{648})$ | $(t_7: M_{650})$ | |
| $M_{650}$ | : $(t_{21}: M_{634})$ | $(t_3: M_{646})$ | | |
| $M_{651}$ | : $(t_1: M_{646})$ | $(t_{21}: M_{635})$ | | |
| $M_{652}$ | : $(t_2: M_{648})$ | $(t_{21}: M_{636})$ | $(t_7: M_{653})$ | |
| $M_{653}$ | : $(t_2: M_{646})$ | $(t_{21}:.M_{637})$ | | |
| $M_{654}$ | : $(t_{16}: M_{646})$ | $(t_4: M_{655})$ | $(t_5: M_{660})$ | |
| $M_{655}$ | : $(t_1: M_{656})$ | $(t_{16}: M_{647})$ | $(t_7: M_{659})$ | |
| $M_{656}$ | : $(t_{16}: M_{648})$ | $(t_6: M_{657})$ | $(t_7: M_{654})$ | |
| $M_{657}$ | : $(t_{16}: M_{649})$ | $(t_3: M_{656})$ | $(t_7: M_{658})$ | |
| $M_{658}$ | : $(t_{16}: M_{650})$ | $(t_3: M_{654})$ | | |
| $M_{659}$ | : $(t_1: M_{654})$ | $(t_{16}: M_{651})$ | | |
| $M_{660}$ | : $(t_{16}: M_{652})$ | $(t_2: M_{656})$ | $(t_7: M_{661})$ | |
| $M_{661}$ | : $(t_{16}: M_{653})$ | $(t_2: M_{654})$ | | |
| $M_{662}$ | : $(t_{12}: M_{581})$ | $(t_2: M_{611})$ | $(t_7: M_{645})$ | |
| $M_{663}$ | : $(t_{10}: M_{664})$ | $(t_{17}: M_{564})$ | $(t_4: M_{692})$ | $(t_5: M_{697})$ |
| $M_{664}$ | : $(t_{17}: M_{541})$ | $(t_4: M_{665})$ | $(t_5: M_{680})$ | |
| $M_{665}$ | : $(t_1: M_{666})$ | $(t_{17}: M_{542})$ | $(t_7: M_{679})$ | |
| $M_{666}$ | : $(t_{17}: M_{543})$ | $(t_6: M_{667})$ | $(t_7: M_{664})$ | |

| | | | | |
|---|---|---|---|---|
| $\mathbf{M}_{667}$ | : $(t_{17}: M_{544})$ | $(t_{19}: M_{668})$ | $(t_{3}: M_{666})$ | $(t_{7}: M_{678})$ |
| $\mathbf{M}_{668}$ | : $(t_{17}: M_{545})$ | $(t_{7}: M_{669})$ | $(t_{9}: M_{672})$ | |
| $\mathbf{M}_{669}$ | : $(t_{17}: M_{546})$ | $(t_{9}: M_{670})$ | | |
| $\mathbf{M}_{670}$ | : $(t_{17}: M_{547})$ | $(t_{4}: M_{671})$ | $(t_{5}: M_{676})$ | |
| $\mathbf{M}_{671}$ | : $(t_{1}: M_{672})$ | $(t_{17}: M_{548})$ | $(t_{7}: M_{675})$ | |
| $\mathbf{M}_{672}$ | : $(t_{17}: M_{549})$ | $(t_{6}: M_{673})$ | $(t_{7}: M_{670})$ | |
| $\mathbf{M}_{673}$ | : $(t_{17}: M_{550})$ | $(t_{3}: M_{672})$ | $(t_{7}: M_{674})$ | |
| $\mathbf{M}_{674}$ | : $(t_{17}: M_{551})$ | $(t_{3}: M_{670})$ | | |
| $\mathbf{M}_{675}$ | : $(t_{1}: M_{670})$ | $(t_{17}: M_{552})$ | | |
| $\mathbf{M}_{676}$ | : $(t_{17}: M_{553})$ | $(t_{2}: M_{672})$ | $(t_{7}: M_{677})$ | |
| $\mathbf{M}_{677}$ | : $(t_{17}: M_{554})$ | $(t_{2}: M_{670})$ | | |
| $\mathbf{M}_{678}$ | : $(t_{17}: M_{555})$ | $(t_{19}: M_{669})$ | $(t_{3}: M_{664})$ | |
| $\mathbf{M}_{679}$ | : $(t_{1}: M_{664})$ | $(t_{17}: M_{556})$ | | |
| $\mathbf{M}_{680}$ | : $(t_{17}: M_{557})$ | $(t_{18}: M_{681})$ | $(t_{2}: M_{666})$ | $(t_{7}: M_{691})$ |
| $\mathbf{M}_{681}$ | : $(t_{17}: M_{558})$ | $(t_{7}: M_{682})$ | $(t_{8}: M_{685})$ | |
| $\mathbf{M}_{682}$ | : $(t_{17}: M_{628})$ | $(t_{8}: M_{683})$ | | |
| $\mathbf{M}_{683}$ | : $(t_{17}: M_{646})$ | $(t_{4}: M_{684})$ | $(t_{5}: M_{689})$ | |
| $\mathbf{M}_{684}$ | : $(t_{1}: M_{685})$ | $(t_{17}: M_{647})$ | $(t_{7}: M_{688})$ | |
| $\mathbf{M}_{685}$ | : $(t_{17}: M_{648})$ | $(t_{6}: M_{686})$ | $(t_{7}: M_{683})$ | |
| $\mathbf{M}_{686}$ | : $(t_{17}: M_{649})$ | $(t_{3}: M_{685})$ | $(t_{7}: M_{687})$ | |
| $\mathbf{M}_{687}$ | : $(t_{17}: M_{650})$ | $(t_{3}: M_{683})$ | | |
| $\mathbf{M}_{688}$ | : $(t_{1}: M_{683})$ | $(t_{17}: M_{651})$ | | |
| $\mathbf{M}_{689}$ | : $(t_{17}: M_{652})$ | $(t_{2}: M_{685})$ | $(t_{7}: M_{690})$ | |
| $\mathbf{M}_{690}$ | : $(t_{17}: M_{653})$ | $(t_{2}: M_{683})$ | | |
| $\mathbf{M}_{691}$ | : $(t_{17}: M_{641})$ | $(t_{18}: M_{682})$ | $(t_{2}: M_{664})$ | |
| $\mathbf{M}_{692}$ | : $(t_{1}: M_{693})$ | $(t_{10}: M_{665})$ | $(t_{17}: M_{565})$ | $(t_{7}: M_{696})$ |
| $\mathbf{M}_{693}$ | : $(t_{10}: M_{666})$ | $(t_{17}: M_{566})$ | $(t_{6}: M_{694})$ | $(t_{7}: M_{663})$ |
| $\mathbf{M}_{694}$ | : $(t_{10}: M_{667})$ | $(t_{17}: M_{573})$ | $(t_{3}: M_{693})$ | $(t_{7}: M_{695})$ |
| $\mathbf{M}_{695}$ | : $(t_{10}: M_{678})$ | $(t_{17}: M_{574})$ | $(t_{3}: M_{663})$ | |
| $\mathbf{M}_{696}$ | : $(t_{1}: M_{663})$ | $(t_{10}: M_{679})$ | $(t_{17}: M_{580})$ | |
| $\mathbf{M}_{697}$ | : $(t_{10}: M_{680})$ | $(t_{17}: M_{581})$ | $(t_{2}: M_{693})$ | $(t_{7}: M_{698})$ |
| $\mathbf{M}_{698}$ | : $(t_{10}: M_{691})$ | $(t_{17}: M_{643})$ | $(t_{2}: M_{663})$ | |
| $\mathbf{M}_{699}$ | : $(t_{12}: M_{663})$ | $(t_{17}: M_{563})$ | $(t_{4}: M_{700})$ | $(t_{5}: M_{705})$ |
| $\mathbf{M}_{700}$ | : $(t_{1}: M_{701})$ | $(t_{12}: M_{692})$ | $(t_{17}: M_{620})$ | $(t_{7}: M_{704})$ |
| $\mathbf{M}_{701}$ | : $(t_{12}: M_{693})$ | $(t_{17}: M_{611})$ | $(t_{6}: M_{702})$ | $(t_{7}: M_{699})$ |
| $\mathbf{M}_{702}$ | : $(t_{12}: M_{694})$ | $(t_{17}: M_{612})$ | $(t_{3}: M_{701})$ | $(t_{7}: M_{703})$ |
| $\mathbf{M}_{703}$ | : $(t_{12}: M_{695})$ | $(t_{17}: M_{613})$ | $(t_{3}: M_{699})$ | |
| $\mathbf{M}_{704}$ | : $(t_{1}: M_{699})$ | $(t_{12}: M_{696})$ | $(t_{17}: M_{621})$ | |
| $\mathbf{M}_{705}$ | : $(t_{12}: M_{697})$ | $(t_{17}: M_{662})$ | $(t_{2}: M_{701})$ | $(t_{7}: M_{706})$ |
| $\mathbf{M}_{706}$ | : $(t_{12}: M_{698})$ | $(t_{17}: M_{645})$ | $(t_{2}: M_{699})$ | |
| $\mathbf{M}_{707}$ | : $(t_{15}: M_{699})$ | $(t_{4}: M_{708})$ | $(t_{5}: M_{713})$ | |
| $\mathbf{M}_{708}$ | : $(t_{1}: M_{709})$ | $(t_{15}: M_{700})$ | $(t_{7}: M_{712})$ | |
| $\mathbf{M}_{709}$ | : $(t_{15}: M_{701})$ | $(t_{6}: M_{710})$ | $(t_{7}: M_{707})$ | |

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|---|---|---|---|---|
| $M_{710}$ | : ($t_{15}$: $M_{702}$) | ($t_3$: $M_{709}$) | ($t_7$: $M_{711}$) | |
| $M_{711}$ | : ($t_{15}$: $M_{703}$) | ($t_3$: $M_{707}$) | | |
| $M_{712}$ | : ($t_1$: $M_{707}$) | ($t_{15}$: $M_{704}$) | | |
| $M_{713}$ | : ($t_{15}$: $M_{705}$) | ($t_2$: $M_{709}$) | ($t_7$: $M_{714}$) | |
| $M_{714}$ | : ($t_{15}$: $M_{706}$) | ($t_2$: $M_{707}$) | | |
| $M_{715}$ | : ($t_{20}$: $M_{707}$) | ($t_4$: $M_{716}$) | ($t_5$: $M_{721}$) | |
| $M_{716}$ | : ($t_1$: $M_{717}$) | ($t_{20}$: $M_{708}$) | ($t_7$: $M_{720}$) | |
| $M_{717}$ | : ($t_{20}$: $M_{709}$) | ($t_6$: $M_{718}$) | ($t_7$: $M_{715}$) | |
| $M_{718}$ | : ($t_{20}$: $M_{710}$) | ($t_3$: $M_{717}$) | ($t_7$: $M_{719}$) | |
| $M_{719}$ | : ($t_{20}$: $M_{711}$) | ($t_3$: $M_{715}$) | | |
| $M_{720}$ | : ($t_1$: $M_{715}$) | ($t_{20}$: $M_{712}$) | | |
| $M_{721}$ | : ($t_2$: $M_{717}$) | ($t_{20}$: $M_{713}$) | ($t_7$: $M_{722}$) | |
| $M_{722}$ | : ($t_2$: $M_{715}$) | ($t_{20}$: $M_{714}$) | | |
| $M_{723}$ | : ($t_{15}$: $M_{724}$) | ($t_{18}$: $M_{468}$) | ($t_2$: $M_{197}$) | |
| $M_{724}$ | : ($t_{12}$: $M_{725}$) | ($t_{17}$: $M_{586}$) | ($t_{18}$: $M_{469}$) | ($t_2$: $M_{139}$) |
| $M_{725}$ | : ($t_{17}$: $M_{536}$) | ($t_{18}$: $M_{470}$) | ($t_2$: $M_{140}$) | |
| $M_{726}$ | : ($t_{16}$: $M_{467}$) | ($t_8$: $M_{727}$) | | |
| $M_{727}$ | : ($t_{16}$: $M_{715}$) | ($t_4$: $M_{728}$) | ($t_5$: $M_{733}$) | |
| $M_{728}$ | : ($t_1$: $M_{729}$) | ($t_{16}$: $M_{716}$) | ($t_7$: $M_{732}$) | |
| $M_{729}$ | : ($t_{16}$: $M_{717}$) | ($t_6$: $M_{730}$) | ($t_7$: $M_{727}$) | |
| $M_{730}$ | : ($t_{16}$: $M_{718}$) | ($t_3$: $M_{729}$) | ($t_7$: $M_{731}$) | |
| $M_{731}$ | : ($t_{16}$: $M_{719}$) | ($t_3$: $M_{727}$) | | |
| $M_{732}$ | : ($t_1$: $M_{727}$) | ($t_{16}$: $M_{720}$) | | |
| $M_{733}$ | : ($t_{16}$: $M_{721}$) | ($t_2$: $M_{729}$) | ($t_7$: $M_{734}$) | |
| $M_{734}$ | : ($t_{16}$: $M_{722}$) | ($t_2$: $M_{727}$) | | |
| $M_{735}$ | : ($t_{10}$: $M_{466}$) | ($t_2$: $M_{279}$) | ($t_{20}$: $M_{736}$) | |
| $M_{736}$ | : ($t_{10}$: $M_{723}$) | ($t_{15}$: $M_{737}$) | ($t_2$: $M_{196}$) | |
| $M_{737}$ | : ($t_{10}$: $M_{724}$) | ($t_{17}$: $M_{587}$) | ($t_2$: $M_{138}$) | |
| $M_{738}$ | : ($t_{13}$: $M_{735}$) | ($t_2$: $M_{278}$) | | |
| $M_{739}$ | : ($t_1$: $M_{740}$) | ($t_{21}$: $M_{456}$) | ($t_7$: $M_{743}$) | |
| $M_{740}$ | : ($t_{21}$: $M_{457}$) | ($t_6$: $M_{741}$) | ($t_7$: $M_{454}$) | |
| $M_{741}$ | : ($t_{21}$: $M_{458}$) | ($t_3$: $M_{740}$) | ($t_7$: $M_{742}$) | |
| $M_{742}$ | : ($t_{21}$: $M_{459}$) | ($t_3$: $M_{454}$) | | |
| $M_{743}$ | : ($t_1$: $M_{454}$) | ($t_{21}$: $M_{460}$) | | |
| $M_{744}$ | : ($t_2$: $M_{740}$) | ($t_{21}$: $M_{461}$) | ($t_7$: $M_{745}$) | |
| $M_{745}$ | : ($t_2$: $M_{454}$) | ($t_{21}$: $M_{462}$) | | |
| $M_{746}$ | : ($t_1$: $M_{747}$) | ($t_{16}$: $M_{739}$) | ($t_7$: $M_{750}$) | |
| $M_{747}$ | : ($t_{16}$: $M_{740}$) | ($t_6$: $M_{748}$) | ($t_7$: $M_{453}$) | |
| $M_{748}$ | : ($t_{16}$: $M_{741}$) | ($t_3$: $M_{747}$) | ($t_7$: $M_{749}$) | |
| $M_{749}$ | : ($t_{16}$: $M_{742}$) | ($t_3$: $M_{453}$) | | |
| $M_{750}$ | : ($t_1$: $M_{453}$) | ($t_{16}$: $M_{743}$) | | |
| $M_{751}$ | : ($t_{16}$: $M_{744}$) | ($t_2$: $M_{747}$) | ($t_7$: $M_{752}$) | |
| $M_{752}$ | : ($t_{16}$: $M_{745}$) | ($t_2$: $M_{453}$) | | |

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|---|---|---|---|---|---|
| $M_{753}$ | : ($t_{15}$: $M_{754}$) | ($t_{18}$: $M_{410}$) | ($t_2$: $M_{199}$) | ($t_7$: $M_{723}$) | |
| $M_{754}$ | : ($t_{12}$: $M_{755}$) | ($t_{17}$: $M_{585}$) | ($t_{18}$: $M_{411}$) | ($t_2$: $M_{200}$) | ($t_7$: $M_{724}$) |
| $M_{755}$ | : ($t_{17}$: $M_{531}$) | ($t_{18}$: $M_{412}$) | ($t_2$: $M_{142}$) | ($t_7$: $M_{725}$) | |
| $M_{756}$ | : ($t_{16}$: $M_{409}$) | ($t_7$: $M_{726}$) | ($t_8$: $M_{729}$) | | |
| $M_{757}$ | : ($t_{10}$: $M_{408}$) | ($t_2$: $M_{281}$) | ($t_{20}$: $M_{758}$) | ($t_7$: $M_{735}$) | |
| $M_{758}$ | : ($t_{10}$: $M_{753}$) | ($t_{15}$: $M_{759}$) | ($t_2$: $M_{282}$) | ($t_7$: $M_{736}$) | |
| $M_{759}$ | : ($t_{10}$: $M_{754}$) | ($t_{17}$: $M_{584}$) | ($t_2$: $M_{283}$) | ($t_7$: $M_{737}$) | |
| $M_{760}$ | : ($t_{13}$: $M_{757}$) | ($t_2$: $M_{382}$) | ($t_7$: $M_{738}$) | | |
| $M_{761}$ | : ($t_{17}$: $M_{454}$) | ($t_4$: $M_{762}$) | ($t_5$: $M_{767}$) | | |
| $M_{762}$ | : ($t_1$: $M_{763}$) | ($t_{17}$: $M_{739}$) | ($t_7$: $M_{766}$) | | |
| $M_{763}$ | : ($t_{17}$: $M_{740}$) | ($t_6$: $M_{764}$) | ($t_7$: $M_{761}$) | | |
| $M_{764}$ | ($t_{17}$: $M_{741}$) | ($t_3$: $M_{763}$) | ($t_7$: $M_{765}$) | | |
| $M_{765}$ | ($t_{17}$: $M_{742}$) | ($t_3$: $M_{761}$) | | | |
| $M_{766}$ | : ($t_1$: $M_{761}$) | ($t_{17}$: $M_{743}$) | | | |
| $M_{767}$ | : ($t_{17}$: $M_{744}$) | ($t_2$: $M_{763}$) | ($t_7$: $M_{768}$) | | |
| $M_{768}$ | : ($t_{!7}$: $M_{745}$) | ($t_2$: $M_{761}$) | | | |
| $M_{769}$ | : ($t_{17}$: $M_{497}$) | ($t_{18}$: $M_{358}$) | ($t_2$: $M_{249}$) | | |
| $M_{770}$ | : ($t_{11}$: $M_{769}$) | ($t_{17}$: $M_{501}$) | ($t_2$: $M_{248}$) | | |
| $M_{771}$ | : ($t_{12}$: $M_{770}$) | ($t_{17}$: $M_{506}$) | ($t_2$: $M_{341}$) | | |
| $M_{772}$ | : ($t_{15}$: $M_{341}$) | ($t_4$: $M_{773}$) | ($t_5$: $M_{778}$) | | |
| $M_{773}$ | : ($t_1$: $M_{774}$) | ($t_{15}$: $M_{342}$) | ($t_7$: $M_{777}$) | | |
| $M_{774}$ | : ($t_{15}$: $M_{343}$) | ($t_6$: $M_{775}$) | ($t_7$: $M_{772}$) | | |
| $M_{775}$ | : ($t_{15}$: $M_{348}$) | ($t_3$: $M_{774}$) | ($t_7$: $M_{776}$) | | |
| $M_{776}$ | : ($t_{15}$: $M_{349}$) | ($t_3$: $M_{772}$) | | | |
| $M_{777}$ | : ($t_1$: $M_{772}$) | ($t_{15}$: $M_{353}$) | | | |
| $M_{778}$ | : ($t_{15}$: $M_{354}$) | ($t_2$: $M_{774}$) | ($t_7$: $M_{779}$) | | |
| $M_{779}$ | : ($t_{15}$: $M_{771}$) | ($t_2$: $M_{772}$) | | | |
| $M_{780}$ | : ($t_{20}$: $M_{772}$) | ($t_4$: $M_{781}$) | ($t_5$: $M_{786}$) | | |
| $M_{781}$ | : ($t_1$: $M_{782}$) | ($t_{20}$: $M_{773}$) | ($t_7$: $M_{785}$) | | |
| $M_{782}$ | : ($t_{20}$: $M_{774}$) | ($t_6$: $M_{783}$) | ($t_7$: $M_{780}$) | | |
| $M_{783}$ | : ($t_{20}$: $M_{775}$) | ($t_3$: $M_{782}$) | ($t_7$: $M_{784}$) | | |
| $M_{784}$ | : ($t_{20}$: $M_{776}$) | ($t_3$: $M_{780}$) | | | |
| $M_{785}$ | : ($t_1$: $M_{780}$) | ($t_{20}$: $M_{777}$) | | | |
| $M_{786}$ | : ($t_2$: $M_{782}$) | ($t_{20}$: $M_{778}$) | ($t_7$: $M_{787}$) | | |
| $M_{787}$ | : ($t_2$: $M_{780}$) | ($t_{20}$: $M_{779}$) | | | |
| $M_{788}$ | : ($t_{16}$: $M_{780}$) | ($t_4$: $M_{789}$) | ($t_5$: $M_{794}$) | | |
| $M_{789}$ | : ($t_1$: $M_{790}$) | ($t_{16}$: $M_{781}$) | ($t_7$: $M_{793}$) | | |
| $M_{790}$ | : ($t_{16}$: $M_{782}$) | ($t_6$: $M_{791}$) | ($t_7$: $M_{788}$) | | |
| $M_{791}$ | : ($t_{16}$: $M_{783}$) | ($t_3$: $M_{790}$) | ($t_7$: $M_{792}$) | | |
| $M_{792}$ | : ($t_{16}$: $M_{784}$) | ($t_3$: $M_{788}$) | | | |
| $M_{793}$ | : ($t_1$: $M_{788}$) | ($t_{16}$: $M_{785}$) | | | |
| $M_{794}$ | : ($t_{16}$: $M_{786}$) | ($t_2$: $M_{790}$) | ($t_7$: $M_{795}$) | | |
| $M_{795}$ | : ($t_{16}$: $M_{787}$) | ($t_2$: $M_{788}$) | | | |

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|---|---|---|---|---|---|
| $M_{796}$ | : ($t_{13}$: $M_{134}$) | ($t_7$: $M_{277}$) | ($t_8$: $M_{382}$) | | |
| $M_{797}$ | : ($t_{14}$: $M_{798}$) | ($t_{18}$: $M_{327}$) | ($t_2$: $M_{151}$) | ($t_7$: $M_{498}$) | |
| $M_{798}$ | : ($t_{13}$: $M_{799}$) | ($t_{16}$: $M_{803}$) | ($t_{18}$: $M_{328}$) | ($t_2$: $M_{152}$) | ($t_7$: $M_{499}$) |
| $M_{799}$ | : ($t_{16}$: $M_{800}$) | ($t_{18}$: $M_{329}$) | ($t_2$: $M_{29}$) | ($t_7$: $M_{500}$) | |
| $M_{800}$ | : ($t_{18}$: $M_{134}$) | ($t_2$: $M_{30}$) | ($t_{20}$: $M_{801}$) | ($t_7$: $M_{272}$) | |
| $M_{801}$ | : ($t_{15}$: $M_{802}$) | ($t_{18}$: $M_{135}$) | ($t_2$: $M_{31}$) | ($t_7$: $M_{273}$) | |
| $M_{802}$ | : ($t_{17}$: $M_{12}$) | ($t_{18}$: $M_{136}$) | ($t_2$: $M_{32}$) | ($t_7$: $M_{274}$) | |
| $M_{803}$ | : ($t_{13}$: $M_{800}$) | ($t_{18}$: $M_{796}$) | ($t_2$: $M_{51}$) | ($t_7$: $M_{276}$) | |
| $M_{804}$ | : ($t_{11}$: $M_{797}$) | ($t_{14}$: $M_{805}$) | ($t_2$: $M_{296}$) | ($t_7$: $M_{502}$) | |
| $M_{805}$ | : ($t_{11}$: $M_{798}$) | ($t_{16}$: $M_{806}$) | ($t_2$: $M_{297}$) | ($t_7$: $M_{503}$) | |
| $M._{806}$ | : ($t_{11}$: $M_{803}$) | ($t_2$: $M_{83}$) | ($t_7$: $M_{504}$) | | |
| $M_{807}$ | : ($t_1$: $M_{282}$) | ($t_{10}$: $M_{198}$) | ($t_{15}$: $M_{808}$) | ($t_7$: $M_{399}$) | |
| $M_{808}$ | : ($t_1$: $M_{283}$) | ($t_{10}$: $M_{809}$) | ($t_{17}$: $M_{577}$) | ($t_7$: $M_{400}$) | |
| $M_{809}$ | : ($t_1$: $M_{200}$) | ($t_{12}$: $M_{141}$) | ($t_{17}$: $M_{522}$) | ($t_7$: $M_{392}$) | |
| $M_{810}$ | : ($t_{11}$: $M_{268}$) | ($t_2$: $M_{56}$) | ($t_{20}$: $M_{811}$) | | |
| $M_{811}$ | : ($t_{11}$: $M_{269}$) | ($t_{15}$: $M_{812}$) | ($t_2$: $M_{46}$) | | |
| $M_{812}$ | : ($t_{11}$: $M_{270}$) | ($t_{17}$: $M_{504}$) | ($t_2$: $M_{36}$) | | |
| $M_{813}$ | : ($t_{13}$: $M_{810}$) | ($t_2$: $M_{77}$) | | | |
| $M_{814}$ | : ($t_{17}$: $M_{433}$) | ($t_4$: $M_{815}$) | ($t_5$: $M_{820}$) | | |
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