

Lüke and power residue sequence diffusers^{a)}

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Conventional Schroeder diffusers have been successfully used for many years. However, their frequency range is limited by the flat plate effect that occurs when all the wells radiate in phase. This occurs at harmonics of p times the design frequency f_0 , where p is the small prime that is used to generate the structure. A typical diffuser, using $p=7$ and $f_0=500$ Hz, has an upper frequency limit of only 3.5 kHz. Achieving a first flat plate frequency above 20 kHz requires a prime equal to at least 41 and results in diffusers that are too big to be practical in most applications. This paper suggests an alternative approach using number theoretic sequences that, although short in length, are based on large integers. Two new sequences, Type-II Lüke and power residue, have this desired characteristic. They are investigated using both simple models and the more exact boundary element method. The results show the flat plate effect is moved to much higher frequencies as expected. For Lüke sequences at certain frequencies, redirection rather than dispersion is achieved. Modulation techniques can be used to mitigate these problems. Power residue sequences perform the best, providing good diffusion and a flat plate frequency outside the audible range.

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I. INTRODUCTION

In the 1970s, Schroeder introduced the concept of using maximum length sequences in diffuser design to improve sound diffusion in concert halls and reverberation chambers.¹ Since then, a variety of diffusers has been developed.² A well known and widely applied class of diffusers is one that consists of wells with the depths being determined by an integer-based pseudorandom sequence. The most common examples are quadratic residue diffusers (QRDs) or primitive root diffusers (PRDs).^{2,3} A cross-section through such a device is shown in Fig. 1(a).

When sound is incident on this surface, the wave has to travel different distances before it is reflected back out of each well. Thus the reflected waves display different phases as they exit the wells. By choosing well depths which result in appropriate radiated phases, scattering via interference between these reflected waves is achieved.

The depth of the n th well d_n in the diffuser is set using a pseudorandom sequence:²

$$d_n = \frac{s_n \lambda_0}{2P}, \quad (1)$$

where s_n is the n th term of the pseudorandom sequence, λ_0 is the design wavelength, and P is the integer the sequence has generated using, e.g., the prime p for QRDs.

Unfortunately, at certain specific frequencies, the wells of these diffusers radiate in phase and the whole structure will reflect sound as though it is a flat surface. This flat plate effect arises because there is a simple integer relationship

between the different well depths. To illustrate this, consider a $N=7$ QRD which is based on the sequence $[0,1,4,2,2,4,1]$. Because of this construction, there will be a frequency for which the wells corresponding to 1 in the sequence will equal half a wavelength. At this frequency, all the reflected waves radiated from the diffuser wells will be in phase, due to their integer relationship. The lowest frequency that this flat plate effect is noticed is usually Pf_0 , where f_0 is the design frequency.² The flat plate effect is also seen at harmonics of this lowest frequency. Unfortunately, for both the QRD and PRD, the prime number is directly linked with the period of the sequence and therefore the number of wells in the diffuser N . For the QRD $P=N$ while for the PRD $P=N+1$. One solution is to move the first flat plate frequency to a high enough frequency so that it is not of concern. For QRDs and PRDs, however, in order to move the flat plate effect to higher frequencies, one is forced to use much longer sequences, which results in wider diffusers. The experience of the last 30 years shows that short period diffusers are used much more often than wide diffusers, not least because such surfaces are cheaper to make and install.

Angus suggested the use of non-integer based sequences to remove this problem,⁴ but the physical realization of the diffusers suggested is problematic. Angus also suggested ways of mitigating the problem using orthogonal modulation schemes.⁵ Another solution to this problem is to use numerical optimization where a computer searches for the best well depths.⁶ Because the computer no longer uses depths that are integer based, then the flat plate effect is not a problem. However, the optimization process is a rather brute-force design method, and consequently, a more elegant solution based on number theory was sought.

It is suggested that by utilizing integer-based sequences that have small periods, but are generated using larger inte-

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gers, the flat plate effect can be avoided. Two possible sequences are investigated: Type-II Lüke and power residue. The paper begins by outlining the Fourier prediction model used as it enables the underlying principles to be more easily understood. Then the general principles behind the sequences are given and their performance considered using this model. Finally, a boundary element model (BEM) is used to gain more accurate predictions.

II. THEORY

Consider a structure with a distribution of reflection coefficients R_n across its surface. The structure is considered to be extruded in one direction, so that significant diffraction only occurs in one plane. This simplifies the prediction and interpretation of the results. For normal incidence sound, the far field reflected pressure, p , can be found using²

$$p(\theta) = \sum_{n=0}^{N-1} R_n e^{-inkd_n \sin(\theta)}, \quad (2)$$

where θ is the angle of reflection, n is the well number, R_n is the reflection coefficient of the n th well, k the wavenumber and N the number of wells. Note that this is a discrete Fourier transform using $kd \sin(\theta)$, and for this reason the prediction model is often referred to as a Fourier model.

III. SEQUENCES

The goal of a diffuser design is usually a uniform scattered pressure distribution, and therefore a structure that has reflection coefficients whose Fourier transform has a uniform magnitude should diffuse well. The Wiener-Khinchin theorem states that the square of the magnitude of the sequence's Fourier transform is equal to the Fourier transform of its autocovariance (or autocorrelation) function. As a result a sequence of reflection coefficients, whose autocorrelation function is a Kronecker delta function, will display good diffusion properties. Consequently, pseudo-random number sequences should be a good choice, because they display this ideal property, and many have been devised for use in other areas of engineering and science.

The reflection coefficients of the wells are given by

$$R_n(f) = \exp \left[i2\pi \frac{s_n f}{P f_0} \right], \quad (3)$$

where f is the frequency and f_0 is the design frequency of the diffuser.

For some frequencies, all the phases of the reflection coefficients will become multiples of 2π , and so all the reflection coefficients will be 1. This occurs when the fraction $f/(P f_0)$ is an integer. These flat plate frequencies, as shown in Fig. 1(b), are given by

$$f_m = m P f_0, \quad \text{where } m = 1, 2, 3, \dots \quad (4)$$

While in theory every structure displays an infinite number of flat plate frequencies, in reality it is rare for there to be more than one of these frequencies in the bandwidth of concern. Therefore, it is common to refer to the first of them as "the" flat plate frequency.

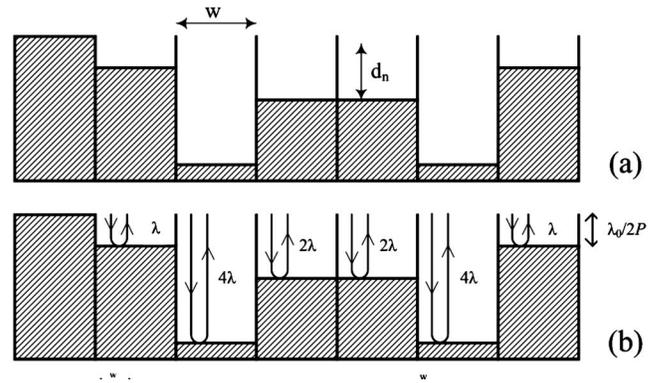


FIG. 1. (a) One period of a quadratic residue diffuser (QRD) of well width w and depth of the n th well, d_n . (b) The flat plate effect occurs when a multiple of $\lambda/2$ exactly fits in all the wells.

Of interest in this paper are power residue and Type-II Lüke sequences, which are both manipulations of primitive root sequences.

A. Type-II Lüke sequences

Type-II Lüke sequences are generated in families which have $p-1$ members, each of which could be used to make a diffuser. For any given prime p the n th terms of the sequences are given by^{7,8}

$$s_n^{(r)} = (\alpha^n(p-1) + rnp) \bmod p(p-1), \quad (5)$$

where α is the primitive root of the prime p , r denotes the family member and mod indicates the least non-negative remainder. The sequences are generated via the integer $P = p(p-1)$ and have a period length of $N = p-1$. A necessary condition is $0 \leq n, r \leq p-2$.

The reflection coefficients of Type-II Lüke sequences have the following, two valued, autocorrelation magnitudes:

$$|G_{r,r}(\tau)| = \begin{cases} p-1, & \tau = 0 \\ 1, & -\frac{p-1}{2} \leq \tau \leq \frac{p-1}{2} \quad \tau \neq 0, \end{cases} \quad (6)$$

where G is the autocorrelation and τ is the autocorrelation delay variable. This autocorrelation function is the same as that of a PRD of the same period. Note that it is the autocorrelation magnitude that is constant for $|\tau| > 0$, which is unusual for sequences used in diffuser design. Table I shows the properties for an example sequence based $p=7$.

TABLE I. Characteristics of the autocorrelation function for some cases of Type-II Lüke and power residue sequences.

Sequence	p	M	N	$ G $ ($\tau=0$)	$ G $ ($\tau \neq 0$)
Quadratic residue	7	n/a	7	7	0
Primitive root	7	n/a	6	6	1
Type-II luke	7	n/a	6	6	1
Power residue	19	2	9	9	2.236
Power residue	37	4	9	9	2.646
Power residue	73	8	9	9	2.828

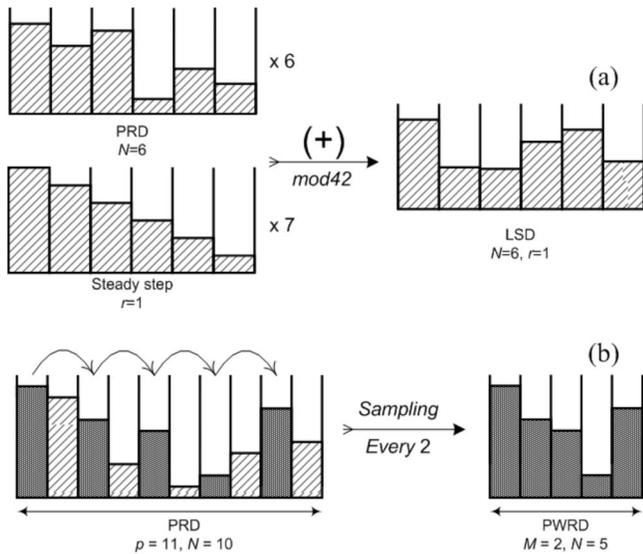


FIG. 2. Schematic showing the generation of (a) Lüke ($p=7, r=1$) and (b) power residue ($p=11, M=2, r=0$) sequences.

Essentially the Type-II Lüke sequences are formed by superposing the primitive root sequence q of prime p :

$$q_n = a^n \text{ mod } p \quad (7)$$

and a steady step sequence t of the same period:

$$t_n = rn \text{ mod } p - 1 \quad (8)$$

with r giving the step size, as shown in Fig. 2.

This is possible because a linear ramp can be added to any number sequence, provided the period is correct, without changing the autocorrelation properties.⁸

For the above reason, every primitive root sequence can be considered to be the first sequence ($r=0$) of each Type-II Lüke sequence family. From any primitive root sequence a set of $p-2$ new Type-II Lüke sequences can be generated, each one with a different step size.

To give an example for $p=7, \alpha=3$ and $r=0$ the primitive root sequence is $q=[1, 3, 2, 6, 4, 5]$. The phases of the reflection coefficients of this sequence, at the design frequency f_0 , are $2\pi/7, 2\pi 3/7, 2\pi 2/7$, etc. The flat plate effect will occur when all phases are multiples of 2π ; this can be accomplished by multiplying all the arguments by a factor of 7, which happens when the incident wave is of frequency $f=7f_0$ as previously discussed.

On the other hand for $p=7, \alpha=3$ and $r=1$ the Type-II Lüke sequence is given by $s=[6, 25, 26, 15, 10, 23]$. The phases of the reflection coefficients in this case, at the design frequency f_0 , are $2\pi 6/42, 2\pi 25/42$, etc. For all the phases in this case to become multiples of 2π they need to be multiplied by a factor 42 which happens when the incident wave is of frequency $f=42f_0$. Therefore, by using Type-II Lüke sequences it is possible to increase the frequency at which all the wells radiate in phase by a factor of 6.

B. Power residue sequence

For a prime number p that can be expressed in the form:

$$p = MN + 1, \quad (9)$$

where M and N are integers, M power residue sequences of period N can be generated using the equation⁸

$$s_n^{(r)} = (\alpha^r \beta^n) \text{ mod } p, \quad (10)$$

where $0 \leq r < M, 0 \leq n < N, \alpha$ is a primitive root of p and β is α raised to the power of M ($\beta = \alpha^M$).

In the case that a set of N integers $D=[d_1, d_2, \dots, d_N]$ are modulo an integer p they are said to form an integer difference set if every integer $h \neq 0$ can be expressed in exactly λ ways in the form:

$$d_i - d_j \equiv h \text{ mod } p \quad (11)$$

The properties of the difference set are usually represented using the nomenclature (p, N, λ) .

If, and only if, the power residue sequence forms a cyclic difference set (p, N, λ) , then the reflection coefficients that it generates displays two level autocorrelation magnitudes.⁸

$$|G_{r,r}(\tau)| = \begin{cases} N, & \tau = 0 \\ \sqrt{N - \frac{N-1}{M} + \frac{1}{M}}, & \frac{N-1}{2} \leq \tau \leq \frac{N-1}{2} \\ (\tau \neq 0), \end{cases} \quad (12)$$

where Table I gives a few examples, and demonstrates that power residue sequences display worse autocorrelation properties than QRD, PRD and Type-II Lüke sequences, as the out-of-phase magnitude is always greater than 1 and becomes greater as M increases.

Essentially, power residue sequences are under-sampled primitive root sequences, with a sample taken every M th coefficient, with a different starting point, as shown in Fig. 2(b).

The starting point is set by r . For instance, for $p=11$, the primitive root is 2, and the primitive root sequence is $q=[1, 2, 4, 8, 5, 10, 9, 7, 3, 6]$. For $M=2$, and $r=0$ every other coefficient is taken to form the power residue sequence starting from the first $s^{(0)}=[1, 4, 5, 9, 3]$ while for $r=1$ the starting point is the second $s^{(1)}=[2, 8, 10, 7, 6]$. Note that the one sequence is the inverse of the other. If the coefficients of $s^{(1)}$ are cyclically shifted back 2 positions, it becomes $[10, 7, 6, 2, 8]$. So in this case, the two power residue sequences are connected via the equation

$$s^{(0)} = p - s^{(1)}. \quad (13)$$

This connection between the two sequences results in pairs of diffusers in a family that performs similarly, because pairs have reflection coefficients with opposite phases.

To postpone the flat plate effect to a higher frequency, sequences with larger p are needed. There are three cases that form cyclic difference sets and need to be considered.⁸

$M = 2$ and N odd

$M = 4$ and $N = j^2$ where j is odd

$M = 8$ and $p = 8j^2 + 1 = 62m^2 + 9$

where j and m are odd (14)

Since the goal is to push the flat plate effect to higher frequencies, the most promising case is the last as it combines higher primes p with the shortest sequences possible. The first case that falls under this category is

$M = 8, n = 3, m = 1 \Rightarrow p = 73.$

This generates a short sequence typical of the length used in practical Schroeder diffusers (period $N=9$) but with a prime number generator of 73. One such a sequence is $s^{(1)} = [5, 10, 20, 40, 7, 14, 28, 56, 39]$, which is the second of the family ($r=1$). The higher prime number gives a first flat plate frequency of 73 times the design frequency.

IV. SCHROEDER DIFFUSERS

In order to evaluate the performances of the two new sequences, diffusers were simulated and predictions of their scattered pressure distribution were made. From these their diffusion coefficient² was calculated and comparisons with other integer based diffusers such as the standard QRD and PRD were made.

As the main reason for choosing these new sequences was the fact that they are both based on large prime number generators and display two level autocorrelation properties, according to a Fourier model, this model will initially be used to estimate their relative performance.

A. Type-II Lüke diffusers

Type-II Lüke sequences are formed by the addition of a step sequence to a primitive root sequence. Consequently, diffusers that are generated with steady step sequences of opposing inclinations can be paired as they perform similarly. This leaves one sequence that cannot be paired, the middle one which is generated for $r=M/2$.

The case of Type-II Lüke sequence diffusers (LSD) generated by the integer $P=42$ is considered. These are diffusers of period $N=6$ and well width approximately 4.2 cm. Their design frequency is $f_0=500$ Hz. Eight periods of the diffuser are used. This gives a structure with an overall width of 2 m.

The diffusion coefficient will be used initially to estimate the overall performance of the diffusers. This coefficient has values from 0 to 1, where 0 represents perfect specular reflection (no diffusion) and 1 represents uniform scattered distribution into all angles of reflection⁹ (complete diffusion).

Figure 3 displays the diffusion coefficients of some diffusers of this family of LSDs along with the equivalent PRD of the same characteristics. Since the design frequency used is 500 Hz the PRD is expected to display a flat plate effect at 7500 Hz=3.5 kHz while the LSDs are expected to display their first flat plate effect at 42×500 Hz=21 kHz.

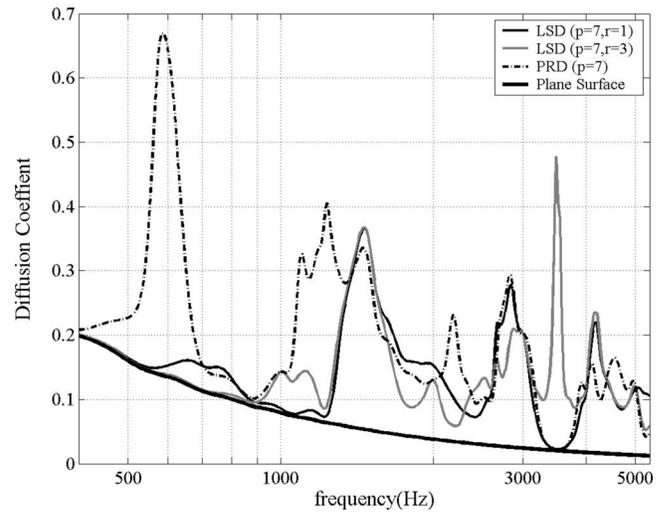


FIG. 3. Autocorrelation diffusion coefficient, as predicted using the Fourier model, of different types of periodic Lüke and primitive root diffusers of the same total width.

Unfortunately the LSD with $r=1$ displays a dip in the diffusion coefficient similar to the PRD's flat plate effect at 3.5 kHz. This is because the $r=1$ LSD causes redirection rather than diffusion at this frequency. The reflection coefficients at 3.5 kHz have phases of $0, \pi/3, 2\pi/3, \pi, 4\pi/3, 5\pi/3$ which have equal phase shift increment of $\pi/3$ from one well to the next. This constant phase increment of the reflection coefficients is why the main reflected lobe is redirected into another direction; it is identical to the phase shifts used to beam steer loudspeaker arrays. This behavior is inherent in Lüke sequence diffusers because they are formed by adding a PRD to a linear stepped ramp. At the frequency in question, all the reflection coefficients of the base PRD are equal to 1 with a phase shift of 0, leaving only the linear stepped ramp. Essentially the PRD disappears and the diffuser acts like a tilted flat plate.

This can be seen in Fig. 4 where the scattered intensity distribution from one period of this LSD with $r=1$ is compared to that from a plane surface of the same size and shows that the diffuser is redirecting instead of scattering the incident wave.

All the LSDs of the family display this behavior with the exception of the middle one (in this case $r=3$) (Fig. 3) which appears to be dispersing the incident wave uniformly. How-

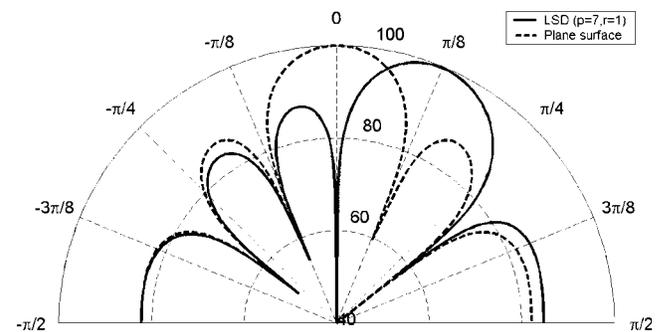


FIG. 4. Scattered intensity distribution (dB) at the tilted flat plate frequency of a Lüke diffuser.

ever, a closer inspection reveals that the reflection coefficients at this frequency are simply +1 and -1 one after the other (representing a steady phase shift of π). Based on the Fourier model, cancellation in the specular reflection direction occurs. However, mutual interactions between adjacent wells will tend to “smooth out” the surface pressure distribution and reduce the cancellation in real surfaces.

All sequences of the same family perform almost identically when considered in a 1/3rd octave band. They vary in their performance at specific frequencies and in the overall variation of their diffusion coefficient with frequency. The diffusers that have $r=2$ and $r=4$ display more variation of diffusion with frequency and have many dips in the diffusion coefficient. For this reason they are considered to perform worse than $r=1$, $r=3$, and $r=5$. However, it remains to be verified whether the $r=3$ LSD will perform as well as predicted by the Fourier model.

Another aspect worth taking into consideration is the maximum depth of the diffusers because of the space it removes from the room. For this family of LSDs $r=1$ displays the smallest maximum depth of 21.3 cm which is considerably smaller than that of the equivalent PRD which is 29.5 cm, for the given design frequency. However, although LSD ($r=1$) appears to be the most promising of its family it does not seem to perform any better than the equivalent PRD.

The dips that are evident in the diffusion coefficients of all three structures around 1.2 and 2.4 kHz are due to the periodicity caused by the repetition of the base diffuser 8 time. Because of this, the structures can be considered as 8 point sources spaced 25 cm apart which will generate additional minima due to the grating lobes generated by that periodicity.

B. Power residue diffusers

Power residue diffusers (PWRDs) of the same family can be separated into pairs that diffuse similarly as well. Each diffuser is paired with its inverse, which is found in the same family $M/2$ sequences away ($|r_1 - r_2| = M/2$).

The cases of PWRD of period $N=9$ are taken into consideration. They can be generated by the following cases:

$$M = 2, N = 9 \Rightarrow p = 19$$

$$M = 4, N = 9 \Rightarrow p = 37$$

$$M = 5, N = 9 \Rightarrow p = 73$$

Their well width was set to approximately 4.4 cm so that a structure of 5 periods was 2 m long and their design frequency was also set to 500 Hz. This allows direct comparison of their diffusion performance with the Lüke sequence diffusers. Given their design frequency and their prime number generator their flat plate frequencies are expected to be 8.5, 18.5 and 37.5 kHz, respectively.

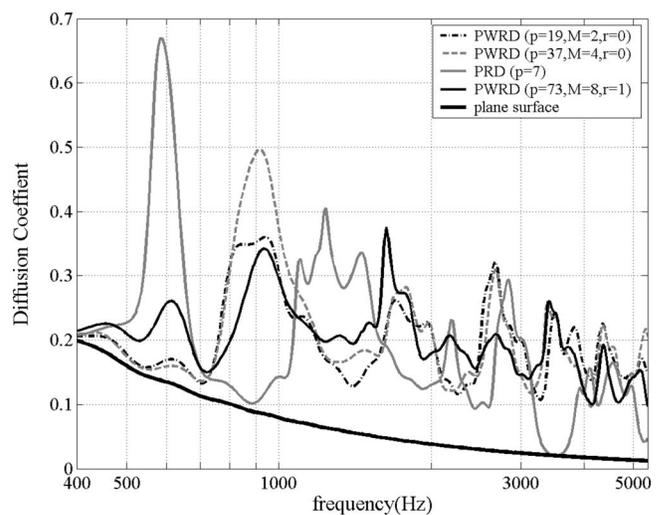


FIG. 5. Autocorrelation diffusion coefficient, as predicted using the Fourier model, of different types of periodic power residue and primitive root diffusers of the same total width.

Figure 5 shows the performance of one case of each PWRD family compared to 8 periods of a PRD ($p=7$). Their flat plate frequency is, as expected, far beyond the frequency range of the PRD, and that of the graph. However, they seem to perform worse than the PRD at some frequencies. However, they display a more uniform performance over the whole frequency range. Again the diffusers all display poor performance at just over 700 Hz due to the repetition of a single diffuser.

These three diffusers were chosen over others of their family because they performed better. Both PWRD ($p=19$) performed identically on their own, as one is the inverse of the other. The other two PWRDs were chosen because they appeared to perform better, or as well as, the rest of the diffusers in their family. Overall, the most promising are the diffusers of larger integer number ($p=37$ and $p=73$) as they display more uniform diffusion over the bandwidth. Furthermore, the fact that there are more than two diffusers in these families allows more design options. The shallowest diffuser is the PWRD based on the prime $p=73$ which displays a maximum depth of 26.4 cm. The other cases have maximum depths around 30 cm.

C. Boundary element method (BEM)

Having predicted the performance of the diffusers using the Fourier method, and established their overall behavior, a more detailed and exact BEM¹⁰ simulation was used for prediction as it has been shown to give accurate results for Schroeder diffusers.¹¹ The design frequency and dimensions of the diffusers are the same as given previously and the results are displayed in Figs. 6 and 7. The diffusion coefficients display a broadly similar pattern to those of the Fourier model but they seem to have more variation with frequency.

Figure 6 displays the performance of LSDs in comparison with the PRD of the same size. It is evident from this

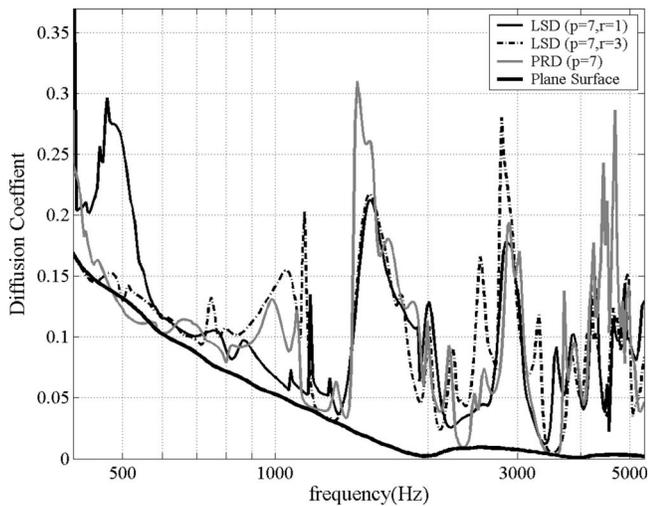


FIG. 6. Autocorrelation diffusion coefficient, as predicted using the boundary element model, of different types of periodic Lüke and primitive root diffusers of the same total width.

graph that the middle diffuser of the family ($r=3$) performs almost identically to the PRD at the flat plate frequency of the latter. This contradicts the earlier prediction of good diffusion from the Fourier model (Fig. 3). On the other hand, the initial prediction of the behavior of the other LSD ($r=1$) is quite accurate. It is also evident that none of the LSDs perform any better than the PRD.

Figure 7 shows the behavior of PWRD of period $N=9$ with the same characteristics that were used for the Fourier model in comparison with the PRD of period $N=6$ in $1/3$ rd octave bands. Once again, the results display a similar pattern to that of the Fourier model. Unlike the LSDs the PWRDs have no problematic frequencies within this bandwidth in particular $p=73$ has; better diffusion than all the other ones.

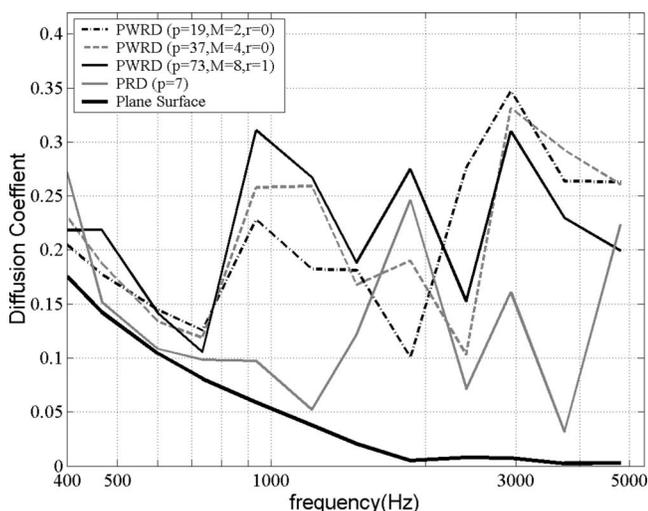


FIG. 7. One-third octave band autocorrelation diffusion coefficient, as predicted using the boundary element model, of different types of periodic power residue and primitive root diffusers of the same total width.

V. MODULATION AND PERIODICITY

To cover large surface areas more than one period of a diffuser structure is commonly used. The repetition of the sequences introduces the problem of periodicity. Periodicity causes harmonics to be created in the autocorrelation function this; creates sharper grating lobes and as a result a less uniform scattering distribution.^{5,12}

One method of dealing with the effects of periodicity is to modulate the base sequence with another.^{5,12} In order to modulate two sequences a binary pseudorandom sequence is required. The binary sequence defines the order in which the sequences will be placed with 1 corresponding to the first sequence and 0 to the second.

There are three major ways of choosing the second sequence to be used in the modulation. These are as follows:

- (1) **Using the inverse of the base sequence:** An inverse sequence is created by subtracting the original sequence from the integer that it was generated from. So, for example, the sequence that will generate the inverse diffuser of the LSD with $P=42$ and $r=1$ [6,25,26,15,10,23] is calculated by subtracting this sequence's coefficients from its integer number generator, 42 in this case, to give [36,17,16,27,32,19] as the inverse.
- (2) **Using the base sequence in reverse order:** Another technique for modulation is to use the same sequence but in reverse order. In practice this is easily achieved by rotating the diffuser in its plane such that its left becomes its right.¹³ The modulation is essentially a diffuser and its mirror image. For example, for LSD with $P=42$ and $r=1$ [6,25,26,15,10,23] the mirror diffuser is simply [23,10,15,26,25,6]. This method has the added advantage of the overall structure having the same depth as the base diffuser; in addition it only requires one base diffuser. However, it only works if there is a degree of asymmetry in the base diffuser.
- (3) **Using a different sequence to that of the base diffuser:** In principle any alternate sequence may be used, but it is usual to use one that is performing better than the base sequence at the frequencies where it is performing badly. For example, for the base sequence above, a suitable sequence would be $P=42$ and $r=5$ [6,11,40,15,38,37] because it complements the base sequence in performance.

A. Type-II Lüke diffusers

As shown above, at some frequencies the LSDs simply redirect the sound because they act like beam steerers. In general, diffusers should be dispersing sound and not simply redirecting it. An effective solution is to modulate the diffuser with another that, at the problematic frequencies, redirects sounds into another angle. Such a diffuser could be the inverse or the mirror image of the first diffuser or an LSD from the same family constructed from a step sequence of opposing inclination. Figure 8 displays the scattered distribution from such composite structures at this frequency. The main lobe of the periodic diffuser has been substituted by

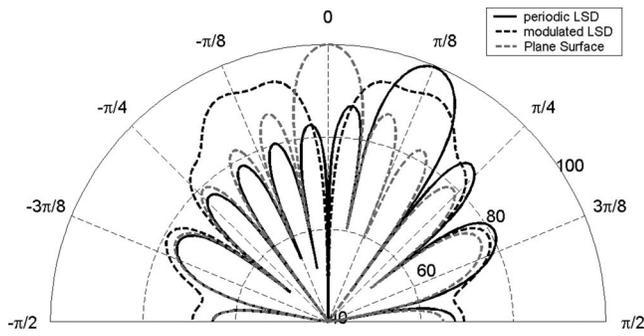


FIG. 8. Scattered intensity distribution (dB) at the flat plate frequency for two periods of LSD ($p=7, r=1$) periodic and modulated with its inverse in comparison with a plane surface of the same width.

two wider lobes of less energy. Thus the incident wave is scattered more uniformly in comparison to the periodic diffruser.

The binary sequence $[1,0,0,1,1,0,1,0]$ was used to modulate the base diffuser, LSD ($r=1, P=42$), in the three different ways discussed earlier. Modulation in all three ways improves the overall performance of the diffuser, as shown in Fig. 9, because the diffusion coefficient is higher for all frequencies compared to the periodic case.

It is important to note that for the periodic case and the modulation with the mirror diffuser the maximum depth is 21.3 cm while for the inverse it is 29.5 cm and for the LSD of opposing inclination ($r=5$) it is 32.7 cm. From these three modulations the one with the inverse diffuser and the other with the LSD ($r=5$) seem to disperse best. However, if the maximum depth is taken into account, the fact that the modulated with the mirror diffuser will take up less space from the volume of the room could make it more desirable for some applications. If the lost volume is of no concern, it is important to note that the modulation with the other LSD manages to treat the notable dip of the diffusion coefficient that is evident in the other cases around 2.4 kHz.

The phenomenon of periodicity could be used to treat the problem of beam steering occurring with the LSD. Con-

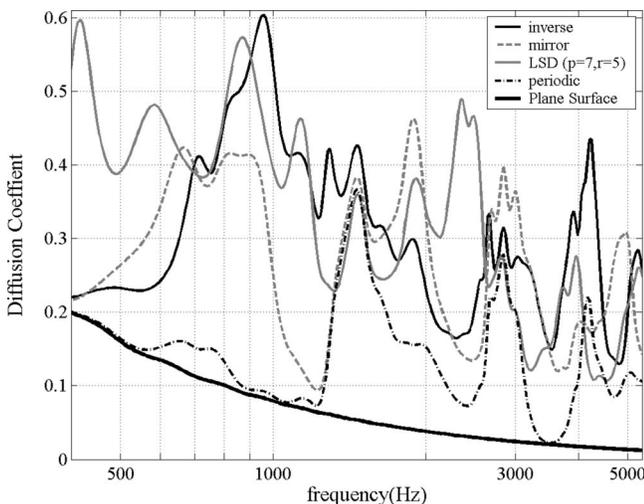


FIG. 9. Autocorrelation diffusion coefficient, as predicted using the Fourier model, of different modulations of Lüke diffusers of the same total width.

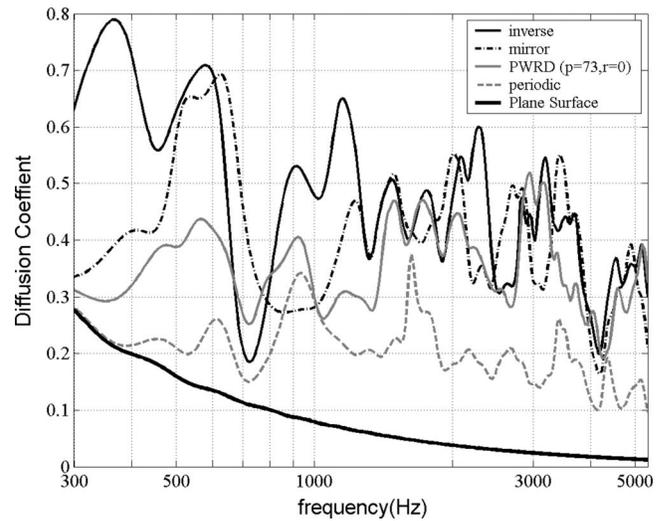


FIG. 10. Autocorrelation diffusion coefficient, as predicted using the Fourier model, of different modulations of power residue diffusers of the same total width.

sider a structure that is composed of the periodic repetition of LSDs. If the maximum reflection lobe could be set on the angle that a minimum of the periodicity pattern occurs, it could be cancelled out. Unfortunately, in order for that to be accomplished a large number of periods must be considered while the wells must be thinned down to unrealistic values. Even in the cases that uniform diffusion is achieved at the flat plate frequencies, the performance of the diffuser is worse than poor for the frequencies up to that.

B. Power residue diffusers

PWRDs do not present any notable problems until their flat plate frequency. So the only problem that needs dealing with is periodicity. The widely used modulation with the diffusers inverse can be applied in this case. The modulations with the mirror diffuser and another diffuser of the same family can be used as well. The binary sequence $[1,0,1,1,0]$ has been used for the modulation.

Figure 10 shows the diffusion coefficient of a PWRD of period $N=9$, prime number generator $p=73$ and $r=1$. All modulations diffuse much better than the periodic case, in addition modulation with the inverse diffuser performs more uniformly compared to the others.

VI. DISCUSSION

This paper has presented two new types of diffusers, estimated their performance, and showed improved performance compared to a standard PRD. But which is the best sequence, Type-II Lüke or power residue?

The PWRDs seem to be the best. As Fig. 11 shows, both PWRDs in their modulated version have better diffusion coefficients than the other diffusers. They have no problematic frequencies, where they are unable to scatter the incident wave, and they have a more uniform diffusion coefficient over this bandwidth. The PWRD generated when $p=73$ could be preferable as it displays steadier diffusion with frequency.

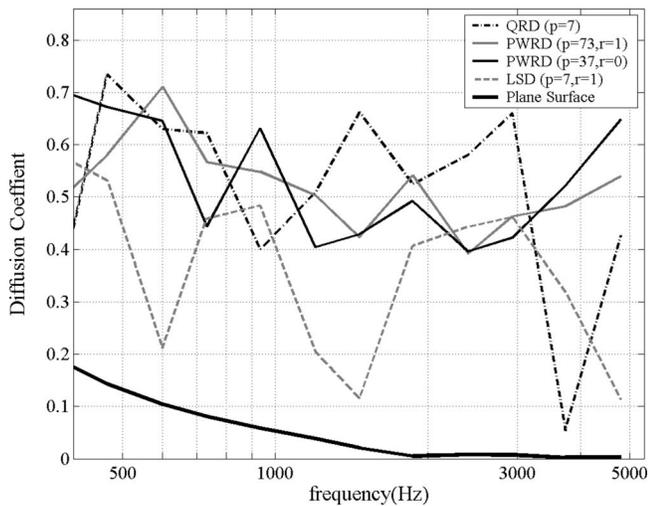


FIG. 11. One-third octave band autocorrelation diffusion coefficient, as predicted using the BEM, of different types of modulated diffusers of the same total width.

The LSD, shown in the above figure, is more ambiguous. It is evident that they do not diffuse as well as the QRD and the PRD except at their flat plate frequencies (3.5 kHz). The periodic version of the LSDs has a tilted flat plate at that frequency, but when modulated manages to avoid the problem. Modulated the LSD is preferred to a PRD or QRD because it is well behaved until the flat plate frequency at 21 kHz.

VII. CONCLUSIONS

This paper has proposed the use of Type-II Lüke and power residue sequences for the design of number theoretic diffusers with a small numbers of wells per period. The logical and mathematical background has been presented and their performance has been predicted with both the Fourier and the boundary element methods. Classic Schroeder diffusers based on quadratic residue and primitive root sequences suffer from flat plate frequencies where no scattering is achieved. Type-II Lüke and power residue sequences use

larger numbers to generate the sequence, and consequently their flat plate frequencies are at much higher frequencies, often outside the audible range for practical diffusers.

The results show that Type-II Lüke sequences act like beam steerers at some frequencies, and consequently at these frequencies diffusion is poor. Modulation techniques have been presented to mitigate this problem. However, the performance of Power Residue sequences has been found to meet the initial demands for more-uniform diffusion and very high flat plate frequency and with the addition of modulation their performance is greatly improved.

Finally both types of diffuser are shown to display better diffusion characteristics when modulated than standard modulated quadratic and power residue diffusers.

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