THE UNIVERSITY OF SALFORD

A Model of the Construction Project Selection and Bidding Decision

A Thesis submitted for
the degree of Doctor of Philosophy

by

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"If the organism carries a *small scale model* of external reality and of its own possible actions within its head, it is able to carry out various alternatives, conclude which is the best of them, react to future situations before they arise, utilize the knowledge of past events in dealing with the present and in the future, and in every way react in a fuller, safer and more competent manner to the emergencies which face it".

CRAIK, K.J.W. (1943) *The Nature of Explanation*
Cambridge University Press

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A MODEL OF THE CONSTRUCTION PROJECT SELECTION AND BIDDING DECISION

ABSTRACT

The thesis considers one of the central problems of corporate planning for a construction company, the project selection and bidding decision, and a model is developed for the entire decision environment.

The nature of decision systems is examined and considered to consist of the identification, evaluation and selection from a range of options. Corporate decisions are discussed leading to the conclusion that a suitable model is needed.

A basic model is proposed in which three outcome criteria consisting of people, money and property are required to be assessed, the values of the outcome criteria being determined by four project characteristics. Some approaches to the solution of multiple criteria problems are examined.

The implications of time are next considered and the use of Gottinger's sequential machines examined as a means of modelling the complexities involved. Non-deterministic aspects of the problem are introduced which, together with dynamical considerations, suggest a model of intermediate complexity to be appropriate.

The final chapters of the thesis concentrate on some ways in which the computational burden associated with the model can be reduced. The role of decision strategies is examined as a means of identifying the most suitable options. The suitability of probabilistic approaches to modelling non-deterministic aspects is investigated and an empirical analysis of three sets of bidding data is made to examine some possible simplifying assumptions.

CHAPTER 1

Introduction

1 INTRODUCTION

One of the most important decisions facing a company in the western world is the price to charge for its products or services. For the construction company this decision usually forms a part of the process of bidding for future projects. The selection of projects and deciding on suitable bid levels has attracted a considerable amount of research, the majority of which is aimed at maximising profits through optimal bidding. A recent study by Lansley (1983), however, has found very little evidence of the application of optimal bidding techniques by construction companies. One criticism is that such techniques often rely on "... oversimplified assumptions of the existence of data that are never actually available" (Wagner,1971,p1273). What is needed, according to Wagner, is a substantive and robust analysis of the practical issues to identify the truly pivotal factors involved to enable assessment of the practical application of formal solution techniques.

The objective of the first part of the research described in this thesis was to conduct just such an analysis. Chapters 2 to 6 examine the issues surrounding the project selection and bidding decision. All pivotal factors are identified and a preliminary assessment of some potential appropriate solution techniques is made.

The second part of the research, described in Chapters 7 to 10, concentrates on the application of one particular solution technique, statistical modelling, and a multivariate approach is examined through the simultaneous analysis of three case studies.

The absence of any comparable study of the global aspects covered by the first part of the research discouraged the proposition of any *a priori* hypothesis. It was considered therefore that any findings should be regarded as essentially *post hoc*. The subsequent statistical analyses contain several hypotheses some of which, for brevity, have not been formalised. Full reference, however, is made to previous work for comparative purposes.

CHAPTER 2

Analysis of decision making in construction companies

2 ANALYSIS OF DECISION MAKING IN CONSTRUCTION COMPANIES

2.1 Types of Decisions

Ansoff (1965,p18) has identified three categories of decisions, strategic, administrative and operating. Strategic decisions "... are primarily concerned with external rather than internal problems of the firm" involving the consideration of such issues as the nature of the firm's objectives and goals, and diversification strategies. Administrative decisions are concerned with "... structuring the firm's resources in a way which creates a maximum performance potential". One part of the administrative problem is concerned with organisation: structuring of authority and responsibility relationships, work flows, information flows, distribution channels, and location of facilities. The other part is concerned with the acquisitionand development of resources: development of raw material sources, personnel training and development, financing and acquisition of facilities and equipment. Operating decisions usually absorb the bulk of the firm's energy and attention, the object being to "... maximise profitability of current operations" involving such major decisions as resource allocation, scheduling of operations, supervision of performance and applying control actions.

These categories, however, are essentially derived from the functional divisions of the organisation, typical of the "management" approach. Management has been defined as "... those senior managers whose decisions influence policy and affect the organisation's relationships with its external environment", (Bullock & Stallybrass,1977,p366). This corresponds to Ansoff's "strategic decisions". It then follows that "administrative" and "operating" decisions are carried out by middle management "... whose decisions affect the internal functions of production, accounting and finance, marketing, personnel research and development", and operational management at the foreman or supervisory levels (Bullock & Stallybrass,1977,p366).

Recent studies of leading companies, by Peters & Waterman (1982) for instance, suggest that the more successful organisations have little regard for such a strict managerial hierachy. Many of the companies investigated were found to have a very loose decision making structure, apart from the firm centralisation of "core" values.

Another classification of decisions is made by Fellows et al (1983,p11)in the distinction between "strategic" and "tactical" decisions. These are defined as "what shall we do?" and "how shall we do it?" decisions respectively. The former includes attempts to answer such questions as, "what are we trying to achieve?"; "what are our objectives?"; "what opportunities are open to us?"; "what are our strengths and weaknesses?"; "what are our current strategies?"; "what strategic choices do we have?"; and "what should we do?" Tactical decisions are the operational decisions involved in estimating, buying and accounting. Strategic decisions are additionally defined as those which are very important to the company, although, as Ansoff (1965,p18) observes "... depending on its position, the firm may find operating decisions to be more important than strategic ones".

The term strategic in this sense seems to imply the devising of a kind of prosthetic "formula", the application of which, despite possible short term problems, will provide some long-term benefits. Hence, in the military sense - "losing the battle but winning the war".

Some criticisms of the Fellows et al classification are apparent. The reference to strategic decisions as being very important and to tactical decisions as being operational indicates a functional managerial influence similar to Ansoff's classifications. Also, the military analogy has been found by Peters & Waterman to be inappropriate in their study of successful companies.

Despite the wealth of literature on strategy, there appears to be very little documentation on the actual decisions available to a company.

One major aspect is in the relationship with other organisations. These factors identified by Fellows et al (1983) are (a) acquisition or merger, (b) joint venture and (c) licences or agencies. The acquisition or merger involves the permanent unification with another organisation, the joint venture is a project-specific temporary unification and the licence/agency is a project/process-specific temporary indirect unification.

Another major aspect is the internal organisation of the company, the arrangement of the physical, human and financial resources, often termed the organisational structure.

One vital recurring decision for the construction organisation, and that to which this thesis is devoted, is the selection of suitable projects. The majority of construction companies rely almost exclusively on project work obtained by competitive tender. The type and location of projects obtained is by far the most important factor in determining the direction of the organisation, (Lansley,1979).

2.2 Making the 'Right' Decisions

One view of the decision making process is that it is essentially a question of selection from a set of options. Whilst it is theoretically possible to select from a standard (and probably infinite) set of options in all decision situations, such an approach is hardly practicable. It is necessary, therefore, to reduce the set of options to a more convenient size. It is also necessary, as an aid to selection, to make some evaluation of each option. This three stage process is called the decision choice process (after Johnson & Scholes,1984) involving the identification and evaluation of options and the selection of decisions.

2.2.1 Identification of Options

The identification of interesting options is often a function of the evaluation and selection process. In practice, possible options which are difficult to evaluate may be omitted whilst options more easily evaluated may be consistently included. Another factor influencing the inclusion of a potential option is the quality and quantity of information needed and available, and the associated time and costs. Time is also a factor in constraining the number of options that can be identified, depending on the speed of identification. A further factor is the ability and the preconceptions of the optioner himself which will be related to some extent to his education, experience and motivation.

Fortunately, the number of types of realistic options are relatively limited for the construction organisation. The major difficulty lies in the evaluation stage. However, some recent findings by Lansley et al (1980) suggest that flexibility is an increasingly important attribute for the success or survival of the construction organisation. One aspect of flexibility would seem to be the willingness to bring an increasing variety of options into consideration.

A popular approach to the option identification problem is to apply a feasibility technique which allows a cost effective procedure to reduce the option set. This involves quickly sifting out the least likely options before employing a more sophisticated evaluation. An often recommended procedure is to bring the companies' objectives and policies to bear on the problem. A policy to concentrate entirely on house building, for instance, would certainly be a very cheap option identification aid, but its effectiveness in identifying all the best options will depend on the policy formulation procedure.

2.2.2 Evaluation of Options

Evaluation of each option implies that some knowledge is available of the future outcome of the decision option. As in the option identification problem, the extent of this knowledge will depend on the quality and quantity of information available, and the associated cost and time.

Accuracy of evaluation will also depend on the evaluator. A further aspect of option evaluation is that the outcome of a decision is not necessarily independent of the decision maker, who may well participate in the implementation process.

In order to best help the selection procedure, each option will need to be evaluated in a similar manner, which implies the presence of some criteria.

2.2.3 Selection

Having identified and evaluated the various options available, the selection process ought to be relatively straightforward. Difficulties occur in accomodating conflicting criteria, particularly those evaluated non-quantitatively. The problem can also be exacerbated by the need to make several decisions, either simultaneously or sequentially. This latter aspect is a decision choice process in itself, involving the identification and evaluation of *sets* of decisions.

Once again, information, cost, time and the ability of the selector are important aspects.

2.3 Corporate Decision Systems

2.3.1 Generally

Construction companies have, for some time now, been urgently recommended to exercise some forethought before taking decisions and subsequent action. Argenti (1974) and Ansoff (1965) in the general business field, for instance, and Grinyer (1972), Diepeveen & Benes (1978), Lansley (1981), Fellows et al (1983) and many others in the construction literature have all advocated the application of long term planning as the basis for short term effective action as "... without planning, a course of action becomes (if not completely aimless) a succession of random changes in direction", (Brech,1975,ch12). Holderman, in his "Characteristics of an Unsuccessful Contractor" has even suggested that "... most companies that fail do so because they did not do adequate planning", (Holderman,1984,p18).

Corporate planning has been defined as simply "... basing decisions on purpose, facts and considered estimates", (Koontz & O'Donnell,1972);
or, organisationally, as the "... systematic process of determining a firm's goals and objectives for at least three years ahead and developing strategies ... to achieve these objectives", (Rajab,1981,p1);
or, managerially, as "... a continuous process of making entrepreneurial decisions systematically and with the best possible knowledge of their futurity, organizing systematically the effort needed to carry out these

decisions", (Drucker,1955). Rajab (1981,p177) has also noted the difference between formal and informal planning.

The notion of objectives is often stated to be inextricably bound up in the corporate planning systems. Cheetham (1980), for instance, has been a strong advocate of management by objectives (MBO) for construction companies.

Murray (1978) identifies six basic evolutionary corporate planning models; Allison's (1971) "Rational Actor" model, in which the decision maker is a kind of "super-person" who always behaves in a perfectly rational manner; the "Organisation Process" model, which emphasises the impact of processes and procedures of organisations on the strategic planning process in the tradition of organisational theory; the "Bureaucratic Politics" model, in which decision-making is assumed to be a political process wherein agreement is reached through bargaining games; Steinbrunner's "Cybernetic" model, in which the central focus of the decision process is the business of eliminating the variety inherent in any significant decision problem; Steinbrunner's "Cognitive" model, containing cognitive models and belief structures modified by inputs from the real world; and Mintzberg's (1975) "Contingency" model, in which alternative explanations are provided for phenomena under different conditions.

Many benefits have been claimed for corporate planning systems. Rajab (1981), for instance, in investigating the nature and extent of corporate planning in construction companies, found that corporate planning "... makes managers think about the future and the effects of decisions on the future; encourages an understanding of the company aims leading to a better understanding of operations; focusses managers' attention on developing the business; quicker decision-making; better co-operation between departments; increased competency of managers by making them face up to key decisions", (Rajab,1981,p167).

2.3.2 Use in Construction Companies

There appears to be little use of formal decision systems in any industry. The results of operations research practice, for instance, "... are not well regarded or used by decision-makers", (Bonder,1979,p209). Particular problems have been found in introducing operations research into such activities as marketing and devising competitive strategies (such as product pricing and bidding), (Wagner,1971,p1269).

Wong (1978), Stark (1976) and Lansley (1983), amongst others, have found that contractors do not favour the use of bidding models. Barnard's research (1981) found that, in the construction industry, "... in common with most industries, there is little use made of corporate planning".

Humphrey's study (1977) of 18 Merseyside construction companies, found little evidence of the operation of formal policies, although annual turnover forecasting was widely practised together with cash flow forecasts at monthly and quarterly intervals, mainly to ensure the availability of capital to finance projects.

Cusack's investigation (1981) of decision making in construction companies, six in some depth, found "... the picture emerging from these investigations is one of intuitive decision-making situations based mainly on experience..." (Cusack,1981,p14). The study did find, however, that plenty of information was available but not in the right form:

Rajab (1981) did locate five companies using corporate planning systems, but was unable to determine whether the use of these systems benefitted the companies. Some differences were observed between the systems operated by the companies themselves, and the systems recommended in the literature. For instance, "... a substantial number of companies did not carry out very systematic internal appraisals", (Rajab,1981,p163). The relevance of this was not clear, however, as the researchers were unable to determine that the companies would be "... more profitable if they plan exactly as suggested in the books because there is no proof of this", adding that there was "... no real reason to believe that it should be true", (Rajab,1981,p177).

Another study of 23 construction companies between 1970 and 1976 found a "... considerable variability between companies' performance and policies", (South,1979,p292).

Studies of organisations outside the construction industry suggest that "... the most important contribution of corporate planning systems are actually in the 'process' rather than in the 'decision' realm" in that "... they create a network of information that would not otherwise be available", (Bahrami,1981,p4).

Apart from isolated cases, such as Cheetham (1980), MBO has been found to be of benefit to "... a large regional construction company", although other recent studies suggest that most construction companies are using a form of contingency rather than long-range planning, (Edwards & Harris,1979;Lansley et al, 1980).

2.3.3 Reasons for Lack of Use

Rajab has identified five major problems associated with corporate planning systems in construction organisations: co-ordination of aims and objectives of various units in the organisation; communication problems; forecasting results and accuracy; restrictions due to capital policies; and political or economic uncertainty overseas (Rajab,1981,p166).

Fryer (1977) has suggested that lack of managerial skills could be responsible. In a survey of 29 managers in construction companies, he found that, although decision-making was the second highest rated skill (after "social" skills), such decisions were normally concerned with short term, day-to-day issues rather than strategic aspects of management.

Many of the problems may well be due to special characteristics of the industry itself. Economists, for instance, have frequently failed to understand the industry due to "... its extremely complex technological and institutional constraints; imperfection of knowledge about future markets; lack of an adequate theory of human capital; concentration on the demand side because of historical excess capacity; lack of importance of time in neo-classical production theory; the local nature of the industry; and the small effect on the economy prior to 1950", (Burton,1972,p1). The effects of this can be far reaching for, in Burton's view "... many current national problems can be traced ... to the fact that economists and ... operations research specialists have not provided the level of understanding of the construction industry necessary for the solution to these problems", (Burton,1972,p2).

Many of the difficulties in objective decision-making appear to stem from the complexity of the construction process. The "immense number of variables" involved, (Park,1966), and the "uncertain environment" (Cusack,1981,p14) result in " the absence of ... necessary data for managerial decisions", (Burton,1972,p86). Difficulties in accurately assessing long term demand and the non-continuous volume of work from clients, particularly the government, makes long term forecasting and planning "so much guesswork", (Goodlad,1974,p73).

Another aspect is the limited amount of time the decision-maker has available to make each decision. Prosper (1984,p24) has noted the difficulties in finding the time to apply 'correct' management techniques.

The combination of lack of relevant information and lack of time seems to be a big factor in restricting the use of formal decision systems.

Problems have also been encountered in the relevancy of the techniques available, a particular problem being the involvement of specialists. Some criticisms of operations research, for instance, are of "... the relevance of current mathematical developments" and that "... techniques and methods are being developed by individuals who have more of a disciplinary allegiance to mathematics and economics", (Bonder,1979,p210), resulting in there being "... too much optimisation, the results of which are usually irrelevant to decision-making", (Jensen,1976).

2.4 Aspects of a Decision System

2.4.1. Scope

The characteristics of an effective decision system are essentially those attributed to effective management but, as Ball observes, these are not easy to define in any unique sense

> "... clearly part of the process of defining effectiveness is by results ... [but] success also has a time dimension.

> Short term success can mean long term disaster. The tasks and decisions of management have themselves different time horizons which have in some way to be brought together to show some index of effectiveness. But even when we believe that this can be done, it is not enough to stop there since, in the social system, both of the organisation and of the wider social system of which the organisation is part, it is not a matter of indifference as to how results are achieved. To some degree, this is because managerial behaviour will be governed by acceptable social values and modes of behaviour and these values may change over time," (Ball, 1977, p4).

In identifying, evaluating and selecting decision options, therefore, it is necessary to consider the interaction between decisions and the environment (social system) over time. One view is that the organisation simply responds to direct environmental "stimulus", providing a service to satisfy the demands of the environment, the "outside-in" approach. A more recent recommendation in the construction literature is to adopt a more aggressive policy of attempting to influence the environment by promotional activities for instance, the "inside-out" approach (Ewing,1968,ch6). Rajab's study (1981) of construction companies indicated that both approaches are necessary.

The construction company's environment is often conveniently divided into two separate groups, the internal and the external environment. Different companies need to consider different environments. In Dressel's view (1965), the essential differences between companies are in their "capacity, size and structure". Commonality, however, does exist in such basic resources as people, property and finance.

The shift in emphasis in environmental perception in recent years has been marked, perhaps even on the scale of a Kuhn paradigm (Cotsgrove,1980). Table 2.1 indicates some of the changes noted by Cotsgrove. Ansoff's retitling (1979) of the firm as an "environmental serving organisation" further evidences the alternative approach. It follows, therefore, that a decision system will ultimately "... need to recognise cultural, political and social inputs in an open system, renegotiated environment", (Murray,1980,p200). These considerations lead to the increasing necessity to analyse both the internal and external environments to identify power groups and individual values

Table 2.1 Competing environmental perceptions

	Dominant social paradigm	Alternative paradigm
Core values	Material (economic growth)	Non-material (self-actualisation)
	Natural environment valued as resource	Natural environment intrinsically valued
	Domination over nature	Harmony with nature
Economy	Market forces	Public interest
	Risk and reward	Safety
	Rewards for achievement	Incomes related to need
	Differentials	Egalitarian
	Individual self-help	Collective/social provision
Polity	Authoritative structures: (experts influential)	Participative structures: (citizen/worker involvement)
	Hierarchical	Non-hierarchical
	Law and order	Liberation
Society	Centralised	Decentralised
	Large-scale	Small-scale
	Associational	Communal
	Ordered	Flexible
Nature	Ample reserves	Earth's resources limited
	Natural hostile/neutral	Nature benign
	Environment controllable	Nature delicately balanced
Knowledge	Confidence in science and technology	Limits to science
	Rationality of means	Rationality of ends
	Separation of fact/value, thought/feeling	Integration of fact/value, thought/feeling

Source: Cotgrove (1980, p129, table 2)

(Johnson & Scholes,1984) and pursue social objectives (Andrew,1973,p18). It is important, as Toffler suggests, to recognise in organisations "... an array of goals other than economic ones and growing increasingly sensitive to changes in the non-economic environment", (Toffler,1971,p409).

Bahrami's studies (1981) of 14 corporate planning systems found these consistent features in all the systems: they facilitated the adaption of the company's strategic posture to the emerging opportunities and threats to its environment; an integrative function by facilitating communication and flows of information; and a control function to implement strategic priorities by evaluating proposals and monitoring performance. The construction industry, it has been observed, has not been noted for its speed of reaction to environmental events, such as changes in demand. "A sudden and substantial increase or decrease in demand in a major sector or geographical area has not normally been matched as quickly by an appropriate increase or decrease in capacity", (Campbell et al,1974,p21). However, as Sidwell (1984,p22) comments, moving into new and unfamiliar markets places greater strain on the efficiency and skills of the company. What appears to be needed is some preparedness for a future state. Uncertainty of the exact nature of future environmental states is a big problem in this respect, but, ironically, as Ansoff points out, the greater the uncertainty the greater the need to be prepared.

Lansley (1981) suggests that construction companies who followed the 'traditional doctrines' in the 1970's either went out of business or diminished in size. The only firms who survived were those who were flexible and responsive to the needs of the changing market. Diepeveen et al (1985,p113) suggest that contractors "... should evaluate future technological developments which may affect the business structure" implying that "scenario writing" may be an effective approach. It is suggested that management should work out "... two or more possible future alternatives which are intrinsically consistent", (Benes & Diepeveen,1985,p29). This recommendation closely resembles contingency planning, previously found to be successfully employed by some construction companies, but *ahead* of, instead of *after*, environmental changes. One approach to this is through the concept of "weak signals", (Ansoff,1984,ch5.4) where the effects of possible changes in the environment are examined. Another, interdependent, approach is by simulation studies.

One final aspect of the scope of a decision system is the criticism by Murray (1980) of Ansoff's approach to strategic management in that a "rational-actor" model is assumed, that is the decision maker is seen as a dedicated remote "super-person" dedicated to some optimising or maximising strategy. The increasing amount of decentralisation of decision making currently being reported, together with the sometimes rather irrational and decentralised method of the 'excellent' companies (Peters & Waterman, 1982) does indeed suggest that Ansoff's assumption may be misplaced. In terms of a decision system this implies the existence of several option selection procedures. What really seems to be needed is a system "... that can inform the executive as to the likely effects of decision strategies that he has himself formulated" and therefore "... permit a manager to evaluate decisions that satisfy his personalised rationality", (Wagner,1971,p1269).

2.4.2 Practical Needs

Analysis of attempts to introduce decision systems into construction organisations reveals that certain practical aspects need to be considered.

The major problem is in the cost of implementing and monitoring the system. This will depend on the depth to which the decision maker is prepared to go. Limiting the set of options, limiting the number of evaluation criteria, approximating option evaluations, simplifying selection procedures illustrate possible approaches. Neale (1985) recommends the adoption of simple systems with a minimum data demand. Cusack found no shortage of data, but what was missing was a "...quick and accurate method of analysis that enables alternative solutions to be compared", (Cusack,1981,p29). Levinson has suggested using a combination of formal and informal methods by allowing "... the operations research department to solve those fragments of a total problem that are amenable to quantative formulation. The sub-optimised solutions can then be considered [by the decision maker] together with intangibles, the unquantifiable elements of the problem. The executive decision will, in some cases, be based partly on the operations research solutions, partly on other data produced by the company, and partly on the judgement and intuition of the management", (Levinson,1953). What is proposed is an economic trade-off between more elaborate models that require greater data processing and more approximate models that need less data to apply", (Wagner,1971,p1268). The issue is, of course. centred on the tensions between risk and cost, the reconciliation of which is a decision problem in itself.

The number of decision options is a measure of the versatility of the system as decision makers "... need alternatives that can provide them with more flexibility over time", (Bonder,1979,p211). Retaining the flexibility of decision options has been dealt with to some extent by Rosenhead et al (1972), Merkhofer (1977) and Pye (1978) by focusing on the size of the alternative action space available to the decision maker, the flexibility being reduced to zero when a specific alternative is chosen. As Merkhofer (1977) notes "... all flexibility is lost when an irrevocable commitment is made to a specific alternative". Clearly, some compromise between versatility and cost is necessary and "... although the versatility ideas are still imprecise, and methods are not available to assist in their implementation, we can and should persue the spirit of the concept in our planning and analysis support to decision making", (Bonder,1979,p222).

Risk is also a problem associated with the option evaluation process. Estimating the outcome of decisions is bound to be a rough and ready business, especially when the outcomes are often only fully realised at some quite distant time in the future. Unfortunately, the construction industry is particularly vulnerable in this respect. The methods of obtaining work and the length of contracts, for instance, together with the fragmented nature of the industry, the customised product and the unstable nature of the environment in which the construction process takes place make risk assessment particularly unattractive. In fact "...one of the main reasons for the high failure rate [of construction companies] is the under-estimation of risks", (Langford & Wong,1979,p21). It is possible that risk assessment can be improved by formal feedback systems but, in many cases, the decision maker has to rely on more subjective information.

2.4.3 System Design

In designing a decision system "... a sensitive system of indicators geared to measuring the achievement of social and cultural goals, and integrated with economic indicators ... is an absolute precondition",(Toffler,1971,p413). Informational support, it is suggested, would come from a Strategic Data Base (SDB) representing the major conclusions regarding the environment and the organisation's clientele (King & Cleland,1978,p95). A Management Information System (MIS) is a form of SDB, being "... specifically designed to formally present information required to support managerial decision making", (Booth,1981,p5).

The properties of a MIS include: provision of information from both internal and external sources necessary to support a range of specific management activities and decisions; provision of information in a manner and at a time relevant to managerial decision making; and flexibility to adapt to and accomodate organisational and environmental change", (Booth,1981,p6). A MIS in support of the strategic planning process would, it is suggested, provide information on the general environment, economic, technical and political (including legislation);factors of productions; and competition, future demand for products/services, policies of competitors etc.

One approach to MIS design is through analysis of the current decision making process. There are, however, some limitations in this approach as "... it results in an essentially static, rational view of decision making; users descriptions are biased towards expectations; it tends to rationalise decision making, over-simplify goals and under-play uncertainty; modelling of uncertain/complex phenomena involves simplification; and it is difficult to foresee information needs to support future decisions", (Booth,1981,p46). Booth suggests that a contingency framework focussing on the 'if - then' relationships of the problem situation would provide a more appropriate starting point.

Information systems are normally associated with some type of environmental scanning activity. Aguilar has identified four types, undirected viewing, involving considerable orientation by the scanner in selection of particular sources; conditional viewing, where the scanner is sensitive to particular types of data; informal search, where information wanted is actively sought; and formal search, a programmed or quasi-programmed search to a pre-established plan, procedure or methodology, (Aguilar,1967). Etzioni (1967) advocates a method of mixed scanning involving broad surveys of the problem area and detailed investigation of areas adjudged to merit such attention.

An implicit prerequisite in any information system is to provide adequate forecasts of future events. This is a particular difficulty in the construction industry where operations are often short-run and on a project basis because of the need to continuously re-allocate with shifts in market demand. Gill has even opined that "... it is not possible to forecast plans from one project through a succession of projects" (Gill,1968). There are, however, techniques available to enable some predictions to be made. Raiffa (1968), for instance, has shown how probability theory can be employed in general decision situations involving risk. Benjamin (1969), Langford & Wong (1979), Wolf & Kalley (1983) andothers have attempted to introduce aspects of the decision-maker's preferences into a probabilistic approach by means of utility theory. Still others have conducted simulation studies (eg.Bennett & Fine, 1980; Morrison & Stevens, 1980).

Before designing a MIS an understanding of the major underlying characteristcs of the system is needed. Booth refers to this as the "conceptual design stage" and "... of fundamental importance in MIS design" and which requires that "... a clear understanding of the decision environment and process is developed" (Booth,1981,p232). In such complex and dynamic conditions as those prevailing in the construction industry, one approach is to model the complexities involved.

2.5 Decision Models

Several models have been proposed for the construction industry decision maker but, as Stark observes in relation to project bidding, "It is common for research papers to develop a thesis, usually in the form of a mathematical model, without adequate mention or consideration of underlying assumptions and characteristics of the bidding environment. In many instances, assumptions are demonstrably untenable in the market places I have experienced", (Stark,1976,p22). A similar observation has also been recorded by Lange who, in reviewing pricing stategy, concludes that "... despite the enormous literature concerned with pricing, economists have devoted relatively little space to the consideration of pricing in the construction industry. The literature that does deal with construction pricing concentrates on the formulation of optimal bidding stategies for contracts, while largely neglecting the fundamental issue of presenting a detailed analysis of the interaction of the chief factors, both quantifiable and unquantifiable, that influence the contractor's bidding decision" (Lange,1973,p91).

"What is needed is a model that reflects the truly pivotal factors in the environment being modelled, especially with regard to the types and amounts of available data and the ability to process this information rapidly enough to be useful to the decision maker" (Wagner,1971,p1273). The construction literature reveals no existence of any such substantive approach to decision model building.

The foregoing analysis has revealed, however, that the decision process can be considered to be in three stages, identification of options, evaluation of options and selection of the best option. Each of these stages contains its own problems and involves some knowledge of the future. The contingency approach suggests that the identification of options should be widened to consider not only the options presently available, but also options that may become available. The evaluation of options is essentially a report on the likely changes in the future environment as a result of the choice of each option. The selection process will involve consideration of several, probably conflicting, criteria representing interesting aspects of the environment.

One operational characteristic of the decision model is concerned with the sequencing of the three stages - is it necessary to identify all options prior to evaluation and is it necessary to evaluate all options prior to selection? Booth suggests that the evaluation of options is normally done *as they are identified* as "... search activity is often conducted within the constraints of time and cost" (Booth,1981,p133). This approach logically leads to an iterative model of decision making where each option is in turn identified, evaluated and compared with the previously best selection. This comparison will determine whether the previously 'best' selection should be replaced by the current option or not. Such a procedure has the great practical advantage of allowing the decision maker to search amongst a feasible set of options of his own choosing for as long as he wishes. The basic model then, illustrated in Fig 2.1, is envisaged as an iterative process occuring within, and interacting with, the environment.

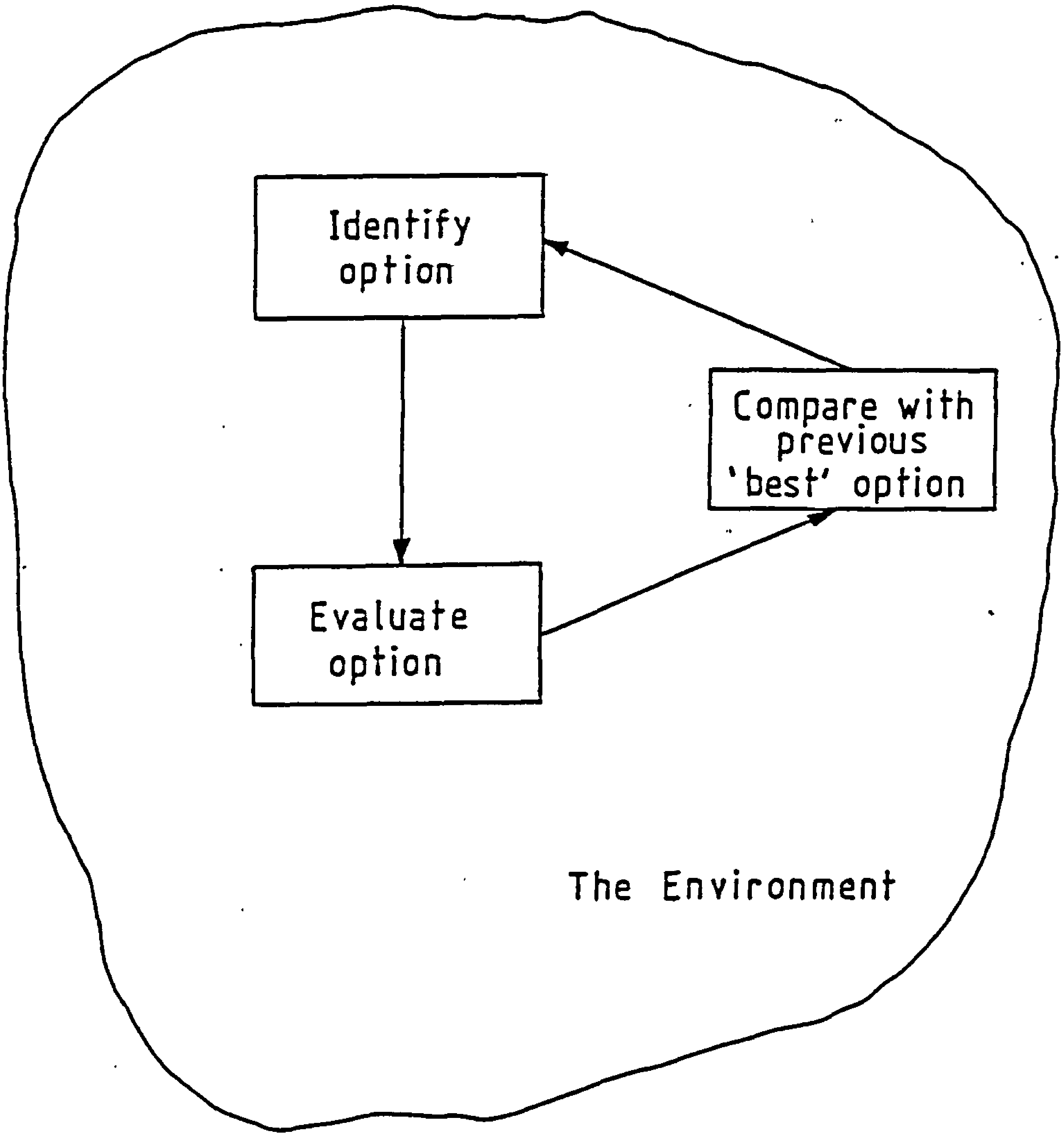
Lindblom's (1959) "The Science of Muddling Through" involves a similar incremental procedure. Some of the problems associated with this approach include the possibility that an important variable is missed; policies may be overlooked; it may reinforce indifference to new technologies; and that it relies on satisfactory present policies, continuity in the nature of the problem and continuity in the means for dealing with the problem (Dror,1964). It is necessary, therefore, to identify all of the variables involved and the variety of policies available in an open system contingent on environmental change.

Criticisms levelled at the "muddling through" approach, such as the use of subjective evaluations, stopping at the first 'good looking' selection and late responses to problems unresolved by earlier decisions (Grinyer,1972, p9) are idiosyncratic of the decision maker rather than the incrementation procedure. The advantages claimed of the incremental approach are, however, relevant to the proposed model in that it is intended to be relatively quick and efficient, more flexible, does not demand explicit objectives and makes use of the decision maker's experience (Grinyer,1972,p9).

2.6 Conclusions

There are many decisions faced by a construction company in the ordinary course of its business, ranging from the major strategic decisions taken by top management to operational decisions taken at lower levels. Recent research suggests that the more successful companies place more emphasis on the decision than on the level of management concerned with the decision. Formal management oriented decision aids, such as corporate planning systems, have received little attention from construction companies. Major difficulties appear to stem from organisational issues and, perhaps, more importantly, knowledge acquisition. These two interrelated aspects involve

Fig 2.1 Basic Decision Model



difficulties in co-ordination of aims and objectives and communication in the former and time constrained informational needs in the latter.

Particularly difficult problems in the construction industry are due to its complex and uncertain nature. Another major informational difficulty encountered by a construction company is the necessity to forecast events over the life span of a project and beyond.

The basic model proposed in Fig 2.1 represents a possible basis of a practical decision system. In order to develop the model further, it has been found necessary to concentrate on one particular type of decision, referred to as "... one of the central problems of corporate planning" (Bischoff,1976,p1), the project selection decision. In so doing, an approach often found in the operations research literature has been adopted, in which the uncertain and dynamical aspects of the model are treated separately. The following chapter, therefore, deals with the complexities of a static/deterministic project selection model.

CHAPTER 3

Deterministic project selection models

3. DETERMINISTIC PROJECT SELECTION MODELS

3.1 Introduction

This chapter aims to specify the construction company's project selection decision problem within the framework proposed in Chapter 2. In order to do this it has been found expedient to restrict considerations to 'wisdom' aspects of the problem, that is 'perfect knowledge' is assumed to exist. The effect of this is that only the *kinds* of knowledge that are directly relevant to the problem are examined here (indirect aspects and knowledge acquisition are dealt with in Chapter 5). A further expedient has been to largely ignore time-dependent aspects such as cash flow and the impact of current decisions on future decisions (these are dealt with in Chapter 4). The chapter concludes with the consideration of some possible approaches to developing a solution technique.

3.2 The Decision Environment

A decision model has been proposed in Chapter 2 in which decisions and actions are viewed as a process within, and interacting with, an 'environment'. This view necessarily implies a contextual definition of the environment as anything which affects or which is affected by the decisions or actions. The decision environment would, therefore, include such bodies as clients, competitors and even governments, in addition to personnel, property and finance thought to exist generally within the organisation.

'Resources' are defined here as that part of the decision environment which are not at the decision-maker's disposal in accomodating the decision. Resources would, therefore, usually include personnel, property and finance but not clients or competitors, (Ansoff,1965,p17).

A further point is in the distinction between decisions, actions and outcomes. For the purpose of this thesis, a decision is regarded as a process involving the three components of option identification, evaluation and selection. Actions are presumed to take place once a decision has been made. Implementation is taken to be synonymous with action. Outcomes are a set of environmental states or changes in states associated with a particular decision-action sequence. With the perfect knowledge assumption, the interest is in the (predictable) relationship between the decision and the outcome, and 'action' is simply subsumed within 'outcome'.

The environment is continually changing. The effect of a decision, then, is to produce an outcome which is different to that of other decisions, or no decision. The project selection decision problem is essentially targetted at identifying the decision which will result in the most favourable outcome.

The decision maker is interested in two facets of the environment, those aspects of the environment which generate work opportunities (projects), and those aspects of the environment which are affected by the decision (outcomes).

3.3 The Outcome Environment

The outcome environment consists essentially of people (aspirational environment) and property (non-aspirational environment). The aspirational environment includes workmen, managers, administrators, executives and directors within the organisation (internal aspirational) and shareholders, clients, sub-contractors, and competitors outside the organisation (external aspirational). The aspirational environment can be further subdivided into individuals and groups.

The non-aspirational environment is often classified into monetary aspects (eg. liquid assets and cash) and non-monetary aspects (eg. buildings, land, plant and equipment).

A further convenient distinction between outcome environments concerns those aspects directly affected by the decision (resources) and those indirectly affected by the decision. The latter environments include competitors and the project generating environment.

3.3.1 People

The extent to which people are affected by the decision process depends on their aspirations, expectations, attitudes and personal philosophies. These attributes are termed by Johnson & Scholes (1984,p116) as "values". Rockeach (1973) makes a finer definition wherein "attitudes" are considered to reflect a level of affect towards a specific object or situation whilst "values" are thought to transcend objects and situations and be connected with the satisfaction of higher order personal needs, thus occupying a more central position in an individual's personality, make-up and cognitive system. This distinction has been found useful by Hackett & Guion (1985), for example, where absenteeism was found to be a result of a decision process involving the individual's personal values rather than attitudes to job satisfaction.

Recent studies indicate that the performance of tasks within an organisation fulfils some essential psycho-sociological needs of the individual. Kahn (1971), for instance, found that about three quarters of employed men and the majority of employed women would carry on working even if they did not need a wage. The major reasons for this were considered to be due to the presence of friends at work and the fact that the occupation helped to reduce boredom.

Many interrelated factors have been associated with individual and group needs: activity, meaning, reward and social status (Kahn,1981), for instance. In the construction organisation context non-monetary objectives such as "leisure" or "partaking in civic duties" (Fellows et al,1983,p40), "maintaining a way of life" (Hillebrandt,1974), "personal security" (Fellows et al,1983,p18) and "serving the general community" (Barnard,1981) are valued. Attitudes to such objectives, however, would seem to be tempered by the current state of need fulfilment of the individual.

One study of operative motivation (MacKenzie & Harris,1984) has used Maslow's hierachical need state structure as a framework for comparing operatives' and managers' views on operative motivation. Maslow's theory implies the existence of five states of need: psychological; safety; belonging; esteem; and self actualisation. An individual is said to progress through each state, from psychological to self actualisation, as the needs associated with each state are satisfied. MacKenzie & Harris's results together, with the ranking of operatives' views of the importance of incentives from an earlier study by Wilson (1979), is shown in Table 3.1. These results, although indicative of the type of factors affecting operatives, strongly suggest the inapplicability of Maslow's system in providing collectively mutually exclusive need states for the operatives. It is possible, however, that *individual* operatives may provide a better fit.

Despite extensive research into human behaviour, a brief summary of which is included in Fryer (1985), there is little concensus on any basic explanatory theory. The fundamental problem may be in the inconsistency of the human decision process.

Group behaviour presents no less of a problem. In many respects, the needs and aspirations of groups are identical to those of individuals. The interactions of individuals within groups are of particular concern, however, power and social context being important factors. Recent studies by Tjosvold (1985) suggest that social contexts involving co-operative, individual or competitive related activities were more important than vested power.

Inter-individual and inter-group relationships are usually referred to in terms of such manifestations as "politics" or "power". Here the tendency is to rely entirely on overtly expressed values of power groups such as unions (Johnson & Scholes,1984).

"Corporate harmony" has been implied to be a characteristic of a successfully progressing company (Fellows et al,1983,p48). Recent studies by Peter & Waterman (1982), however, have found instances of some very successful companies thriving on internal competition.

One power group that has attracted particular interest in the construction literature is that of senior management. Managers have an additional role in the organisation, which is to be formally responsible for resources. Insofar as human resources are concerned, this responsibility requires a concern for welfare (Lansley et al,1980,p43), satisfying employees (Moore,1984,p20) and their aspirations (Barnard,1981) and encouraging and supporting individual growth and development (Fryer,1985). The basis of this responsibility

Table 3.1 Comparison of operative incentive rankings

Theoretical Ranking (After Maslow)	Operative Ranking (A.J.Wilson)	Management Ranking
Physiological Needs		
Earnings	3	1
Short travel to and from work	7	-
Safety Needs		
Physical/Safety/Working Conditions	1	7
Welfare Conditions	2	6
Job Security	18	4
Belonging Needs		
Friendliness of site	4=	10
Work with people as a team	12	9
Work on a well organised site	4=	2
Good relations with management	14	3
Fringe benefits	15	8
Needs for esteem		
Recognition from management/workmates	10	-
Working for a successful company	18	-
Working for a modern company	15=	-
Need for self-actualisation		
Challenge in the job	17	-
Job freedom	9	1
Plenty of time for personal/family life	6	5
Prospects for promotion	21	12=
Opportunities for training	20	12=
Ability to make use of, and develop, skills	8	-

Source: Mackenzie & Harris (1984,Table 3) is, according to Fryer (1985), in the provision of secure employment (though not highly rated in MacKenzie & Harris,Table 3.1), a friendly and co-operative atmosphere and fair compensation for the efforts of employees. Much of this managerial task is covered by such functional terms as personnel management, health and safety, and labour relations.

Conflicts that exist within and between resources are particularly notable where managerial responsibilities are concerned. The conflict between personal and company interests has been discussed by Cyert & Marsh (1963), although Hillebrandt (1974,p90) considers such conflicts to be minimal in construction companies where there is "... a substantial overlap of ownership and control". The major area of conflict appears to be in the management and control of monetary and non-monetary resources.

3.3.2 Control of Resources

The construction industry, according to Sidwell (1984), "... relies heavily on the flexibility and initiative of its people" and as a result "... firms which [rely] on standardised systems and procedures [are] particularly restrained in this repect" (Lansley et al,1980,p43). Controls based on "performance standards and direct supervision" have been found to be less constraining. What appears to be needed is a means by which people can obtain "... clear and consistent views of their own roles, the roles of colleagues and the firm's objectives" and "... co-ordinate their activities", (Lansley et al,1980.p43). There are times, though, when rather more than communicative and co-ordinating support are needed. One such occasion is in the preparation and management of change, particularly when resistance to change is anticipated. In this case, the system can be used to *manipulate* resources. Similar manipulation activities occurs in *balancing* resources. A company may, for instance, increase monetary resources at the expense of human resource development, and *vice versa*. The provision of such manipulatory facilities are obtained through control systems, usually embedded in the *organisational structure* of the company.

"A wide variey of organisational structures exists in the construction environment" (Lansley et al,1979,pt3,p74). Lansley et al have identified four basic structure types: ideal beaurocratic, with high control and integration; mechanistic, with high control and low integration; organic, with low control and high integration; and anarchic, with low control and low integration. Their study of 26 national and regional construction companies involved in general contracting, housing and services found "... national firms tending to display relatively high levels of control but with no one structure being favoured by any of the different types of firm although ... there is a suggestion that smaller firms had the most organic structures" (Lansley et al, 1979,pt3,p67). Performance, however, was found to be "strongly related" to higher levels of integration, whilst control was of "little importance".

The apparent lack of influence of control may be due to the existence of an "adhocracy" form (Mintzberg,1979), typically found in construction organisations because of the temporary and diverse nature of project activities (Ireland,1985,p60). As a result, construction organisations have been urged to concentrate on developing structures and systems which enable effective "... location of technical and specialist support and systems on site", "... integration between staff and their activities" and "... communication of information" (Lansley et al,1980,p43).

The relationship between corporate decisions and organisation structure has been extensively studied by Chandler (1961) and others, resulting in "... the now accepted thesis" (Smith,1985,p176) that the organisation of the enterprise develops to match its decisions. Studies by Newcombe (1976), however, of "a number of construction companies of various sizes and types" found that delays in developing an appropriate organisation structure can be fatal. Ansoff (1965,p179) has proposed the adoption of an administrative strategy to manage the organisational evolution of the firm. Such an administrative strategy could, according to Ansoff, be "... elaborated further into specific organisational relationships and provisions for growth of organisational resources" (Ansoff,1965,p179). Several researchers, however, have noted a distinct lack of application of administrative strategy in construction companies, evidenced by "... the lack of suitable teaching and training material which could be used to develop the abilities of the managers" (Lansley et al, 1979,pt1,p65), for instance. The reason for this may be that the very factors responsible for the existence of adhocracies mitigate against controlled organisational development.

Peters & Waterman's study (1982) of "excellent" companies suggests a simple organisational form of the adhocratic type to be most appropriate. This study found senior managers to be relatively few in number and demands; the focus of the attention to be on people, particularly the customer and the workers, and the product; a bias for action, to cause and react quickly to changing circumstances.

3.3.3 Property

Property, termed "physical resources" by Johnson & Scholes (1984), consists of such physical assets as land, buildings, machines and materials. The extent to which such property is directly affected by the project selection decision is often minimal, except perhaps in the case of very large or unusual projects, as many effects are of a temporary nature. Some of the more permanent effects can be the need to increase the size of the head office to accomodate an expanding permanent staff, which may involve the acquisition of further land and buildings. Plant and materials are normally acquired for the duration of the project, although the residue of some large items of plant, such as a tower crane or batching plant, will have an impact. The acquisition of plant or manufacturing facilities for larger projects can have longer term implications in generating possible decision options involving permanent and separate business operations.

3.3.4 Money

Monetary resources are usually classified into long term/medium term finance. Long term finance is used "... to purchase buildings, plant and equipment and to carry stocks of materials" (Harris & McCaffer, 1983,p312). Short term finance is used to overcome immediate cash flow problems, such as the purchase of materials, plant hire and payment of sub-contractors (Harris & McCaffer,1983,p312). The project selection decision will, therefore, predominantly affect short term finance and generally only indirectly affect long term finance. Typical sources of long and short term finance are given in Figs. 3.1 and 3.2.

Fig 3.1 Sources of Long-Term Finance

Source: Harris & McCaffer (1983,p311)

Fig 3.2 Sources of Short-Term Finance

Source: Harris & McCaffer (1963,p313)

The acquisition of finance generates benefits (assets) and costs (liabilities). The liabilities incurred in the acquisition of finance consists of internal liabilities, for example debts owed to ordinary and preference shareholders, and external liabilities, such as sums owed to debenture holders, the Inland Revenue (for taxes), banks (for loans and overdrafts), and trade creditors (Adam,1965,p226). Liabilities can also be short term (current liabilities), such as thise payable to trade creditors, or long term (deferred liabilities), usually more than one year (Adam,1985,p226).

Assets can similarly be divided into current and fixed depending on the time period involved. A further relevant distinction is between liquid and illiquid assets. Working capital comprises the liquid or near-liquid assets needed to lubricate the daily transactions of business. It is represented by the difference between current assets and current liabilities, and is locked up in a continuous cycle, shown in Fig 3.3 (Harris & McCaffer,1983,p315).

3.3.5 Interrelationships in the Outcome Environment

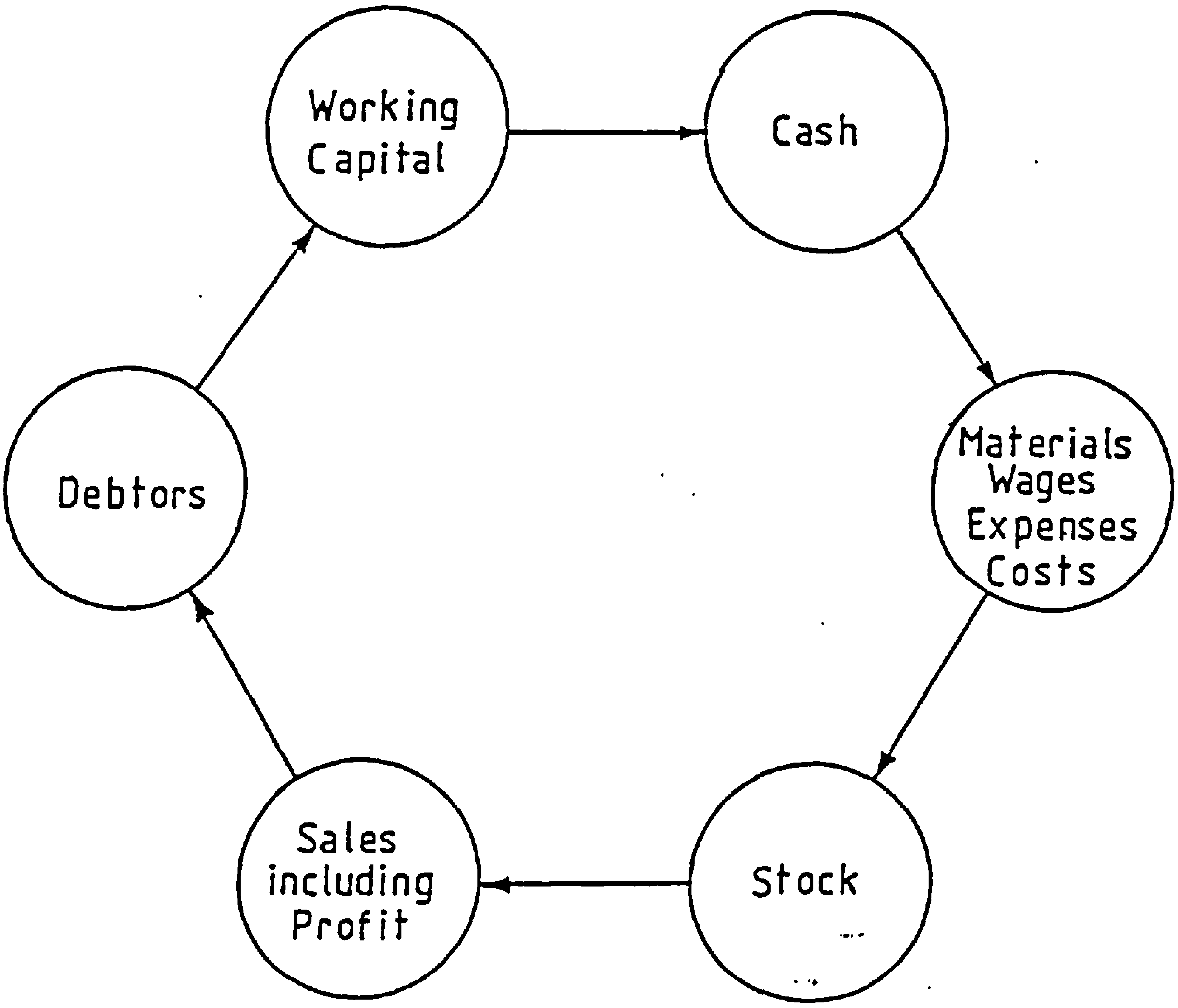
Many aspects of the outcome environment are interrelated and often conflicting. A common feature is the clash of interests between power groups, such as senior management and unions, where changes in the environment which are beneficial to one group are detrimental to the other. Similarly improvements in levels of financial resources of one groupof people usually implies a reduction in financial resources in another. The successful progress of the organisation depends exclusively on the balance of benefits received by these sections of the outcome environment.

3.3.6 Measures of Benefits

The degree of benefit derived depends upon the development state of the environment at the time. The effect of earning £1, for instance, depends on the number of pounds already earned.

Measures of satisfaction can be obtained by means of questionnaires to provide ranked priorities. Lansley et al (1979,Appendix

Fig 3.3 The Working Capital Cycle



Source: Harris & McCaffer (1983,p316)

F,p23) have obtained group measures of job satisfaction, company satisfaction and company commitment (Fig. 3.4).

The development states of an individual or a group of people are not yet sufficiently understood to allow any universal classification.

Some of the benefits normally associated with internal and external individuals and groups are tabulated using Maslow's needs/drives hierachy in Fig 3.5. Although Maslow's system has many defects, the figure serves to illustrate the general proposition that the size of benefit is dependent on the state reached.

Measures of monetary benefits are well accepted. 'Profit' and 'turnover' are of major interest, but several other descriptive statistics are used. Calvert (1981) has identified the ratios commonly used:

1 Current assets to current liabilities (working capital ratio)
2 Liquid assets to current liabilities
3 Outstanding debts to sales
4 Illiquid assets to sales moving annual total
5 Cash to current liabilities
6 Current profit to invested capital
7 Current profit to sales
8 Direct labour to turnover
9 Average credit period
10 Overhead percentage

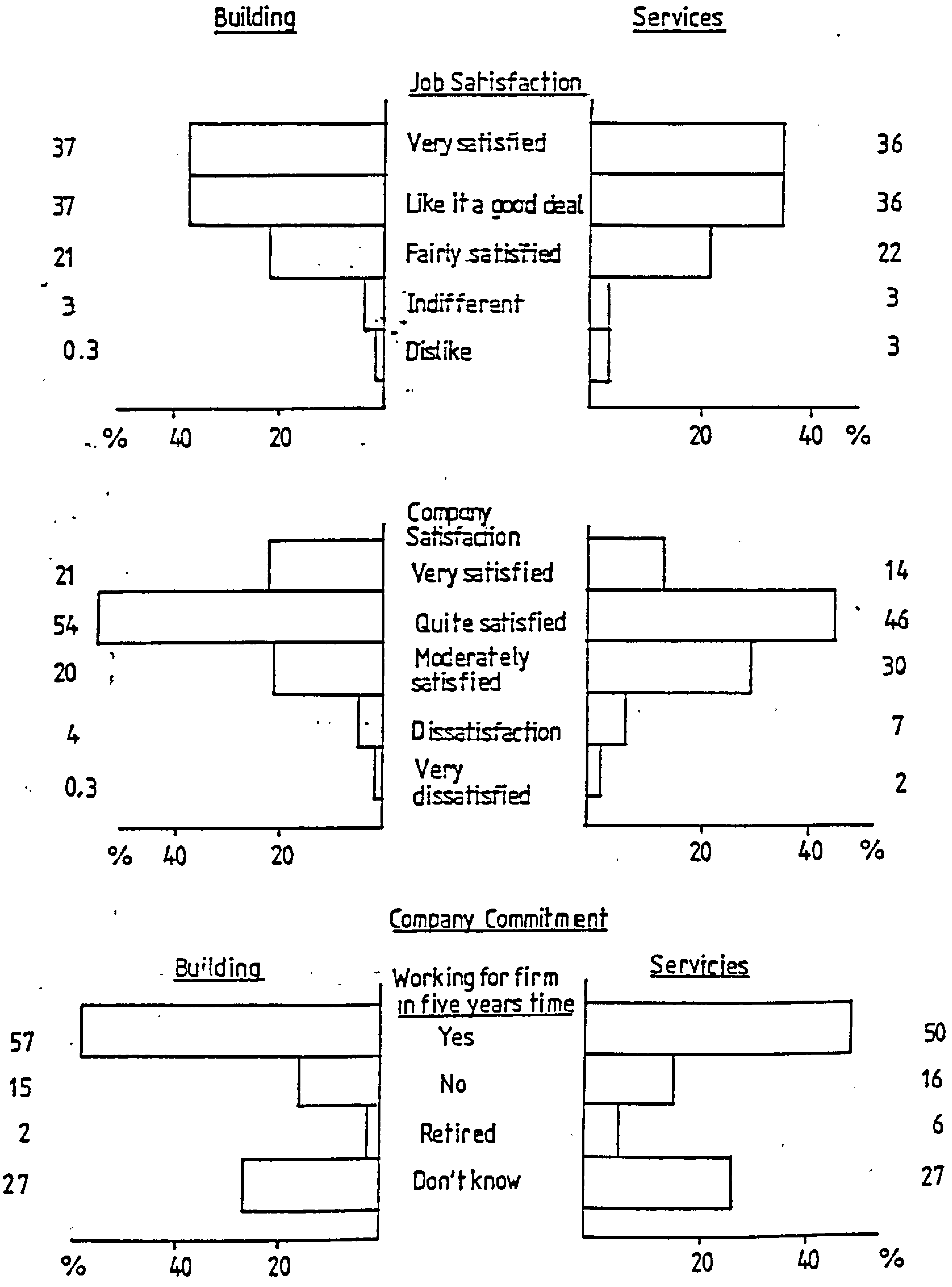
Taxation and cash flow are two further considerations. Real property is normally evaluated in monetary terms. Depreciation is an important factor in such evaluations and also affects taxation liabilities.

3.4 Project Characteristics

3.4.1 Relationship between Project Characteristics and the Outcome Environment

The types and kinds of projects undertaken by a construction organisation have a considerable impact on its outcome environment. Deliberate moves into a new area of work have been found to be an

Fig. 3.4 Measures of satisfaction with job and company



Source: Lansley et al (1979,Appendix F,p23)

Figure 3.5: Development status

Maslow's Needs/Drives Hierachy owner Development Status	Recipient	Individual		Group	
		Internal	External	Internal	External
		Workmen Forman Site Agent Executives Manager Director Owner	Shareholder Customer User Others	Gang Site/ regional, national Organisation Other power groups - unions	Attached power groups - Shareholders Unattached power groups - sub-contractors, suppliers, unions ecological, client government, bodies
Survival	pay/cover overheads meet debts obtain minimum work	stay alive food and drink clothing, shelter	stay alive food and drink	stay in business keep licence stay together minimum workload	stay in business stay together minimum workload
Safety	steady work steady income resolution of basic grievances	job security	secure income	assets steady workload minimum growth minimum profits and turnover maintain market share minimum power	steady workload minimum power
Love (belonging)	stability increasing income stimulation participation in decisions (influence)	job satisfaction personal development		job satisfaction growth of assets, profit, turnover and market share	power growth
Esteem	respect status prestige philanthropy community service advisory (non power)			growth	
Self-actualisation	philanthropy assured future				

important means of developing mobile management, for instance (Lansleyet al,1979,pt1,p59). Project value also has an important bearing on financial resources.

The characteristics of projects are necessarily a function of the construction industry itself, which "... encompasses a whole range of diverse activities such as civil engineering works, building works, public and private sector work, large capital utilities, development work and refurbishment" involving contracts obtained under "... competition, negotiation or a variety of cost reimbursement forms" (Sidwell,1984,p22).

In examining the nature of industries generally, Smith has suggested key factors as being "... the type of products produced and the market served, the technology of production and the nature of the materials required" (Smith,1985,p3). The diverse nature of the construction industry, however, makes these factors rather difficult to define.

Hillebrandt defines the construction product as basically "... the service of moving earth and material, of assembling and managing the whole process". However, as this service and management varies according to the technical processes involved, the industry is viewed as consisting of many sub-industries "... coming under the umbrella of the industry concept" (Hillebrandt,1974,p24). A more appropriate analysis is proposed by Hillebrandt to be the *market* "... in which a group of firms, whose products are more or less substitutes for each other, operates" (Hillebrandt,1974,p27). Lange (1978) has termed these determinants, with the addition of industry branch 'sub-markets'. Size, complexity and industry branch are often referred to as "type" (Lansley et al,1980) or "size and type of work" (Harrison,1982), Lansley et al (1980) proposing "client" as a further sub-market.

These sub-markets, ie type of work, client, location and competitors, define the "nature of the work" (Mannerings,1970) coming from the construction company's "immediate environment" (Foster,1974). A recent study by the Building Economics Research Unit (Cusack,1981) has found that the sub-markets collectively account for over 97% of reasons underlying the decision to tender for projects.

3.4.2 Type of Work

The type of work available in the construction industry is reflected in the activities of large construction companies "... ranging from general building and civil engineering to materials manufacturing, property development, trade sepcification, and even open cast mining", together with "... peripheral services such as materials supply, plant hire and ... project management" (Fellows et al,1983,p1). A building company's services can include "... a building on its site; a building for assembly on a site provided by the client; an assembly service, for a building designed on commission to the client; or one of a series of contributory services brought together and co-ordinated on behalf of the client to erect a building to a design commissioned by him" (Jepson & Nicholson,1972,p5).

Types of buildings are usually denoted by function: residential, commercial, industrial, educational and recreational being typical groupings. The building's function will largely be associated with benefits to the consumer.

The physical and monetary size of a project affects the company's resources and particularly management and finance. Productivity has been associated with project size (Clark & Lorenzoni,1985). Large contracts can develop managerial skills, for instance, provided the personnel have reached a suitable stage of development (Harrison,1982). Monetary resources similarly need to be sufficiently high to withstand cash flow pressures.

Lansley et al (1979,pt3,p5) have found that the technology of the project, expressed in terms of size, complexity and method of construction required, significantly affects organisational and managerial aspects of the company when an unfamiliar technology is involved. In such situations, the organisational structure tended to become more flexible. Different organisation structures occurred on large contracts involving many complex tasks (eg hospitals and hotels) than on smaller and less complex contracts. Distinctions between organisation structures for civil engineering and building projects were not found to be significant, however, although many building companies considered the acquisition of technical expertise and understanding of the commercial aspects of civil engineering to be an insurmountable difficulty.

'Technology', although of potentially great value in expressing relationships between project characteristics and the outcome environment, has been found difficult to completely define. Theoretical developments are still needed in this respect (Lansley et al,1979,pt1,p64).

3.4.3 Types of Client

Jepson & Nicholson (1972,p4) have identified four types of client: "... a speculator, investing in building for profit; a public body, investing in building on behalf of or for the benefit of the community; an occupier with a family or commerical activity or an industrial process to house; and a person or body seeking a monument". Public construction demand is about 30 to 40 per cent of overall demand (Diepeveen,1985,p111).

Two main client influences on outcomes have been found to be "ability to pay" and "relations" (South,1979). The latter also includes the client's advisors, the architect, engineers and quantity surveyor. Relations have been found to affect the contractor's efficiency and cause delays in settling variations and the final account, (Cusack,1981).

The factors related to the client organisation and the construction project are procurement methods and contractual arrangements. Ireland (1985) has identified six procurement methods in common use: a single lump sum contract on a fully documented project; provisional or partial quantities contracts; cost reimbursement (cost-plus); package deal (design and construct or turnkey); construction management; and project management. There appear to be no satisfactory criteria which uniquely separate each procurement method. However, four important aspects have been suggested to be "... the arrangements for determining the cost of the project and identifying the contractor to be used; the roles and relationships of the specialists used, including the possibility of having the contractor available to contribute to the design; the process structure adopted, including such aspects as the overlap of design and construction, the use of multiple prime contracts and the staging of these; and details included in the conditions of contract such as provision for extensions of time for industrial disputation of inclement weather etc" (Ireland,1985,p77).

The value (cost to the client) of the project is considered to be related to the cost determination and contractor selection method used. Negotiated contracts are generally agreed to increase value, whilst open tenders generally decrease value (Smith,1979). However, as Adrian (1973,p370) observes, the price offered by the contractor needs to be comparable with potential competitors, even in the absence of direct competition.

The contractor's contribution to the design necessarily affects his resources. The overlap of design and construction generally reflects the desire for speedy completion. The speed at which the project is needed has important repercussions on estimating resources (Mannerings,1970; South,1979) as well as production resources.

The conditions of contract mainly affect risk (see Chapter 4). Specific instructions regarding the type (eg sub-contractors, materials) and use (access, storage, permissible working times) of resources have direct implications.

Some specific clients may have a special interest in the company through a previous relationship, for instance. In these cases, the client may have a particular influence in selecting the second lowest tender, for example, and thereby modifying the market value principle.

3.4.4 Geographical Location

Dressel (c.1980,p14) distinguishes between "home" markets (consisting of town area, region, county, province and country) and "abroad" (consisting of neighbouring country, developing countries and overseas).

Lansley et al's (1979) studies of several construction companies in the South East, South West and East Midlands of Britain revealed that most firms worked over a small area, mainly within a maximum radius of forty miles from their base. Even national contracting firms were found to have interests centred upon their local or regionally based units, which generally adhered to similar boundaries (Lansley et al,1979,pt2,p21).

The distance of the project from the company's local base affects operatives, who appreciate short travel and welcome the extra free time it produces (see Table 3.1, ranking 7 and 6 respectively). Local craft can, of course, be employed, but this often adversely affects productivity (Clark & Lorenzoni,1985) and hence monetary resources.

Transportation costs are another important factor, together with the costs incurred by non-productive travelling time and subsistence allowances. The organisational structure can also be affected by the need to make special communication arrangements between the site and local, regional or head offices. Remote sites and overseas projects can have special influences due to weather conditions and cultural differences.

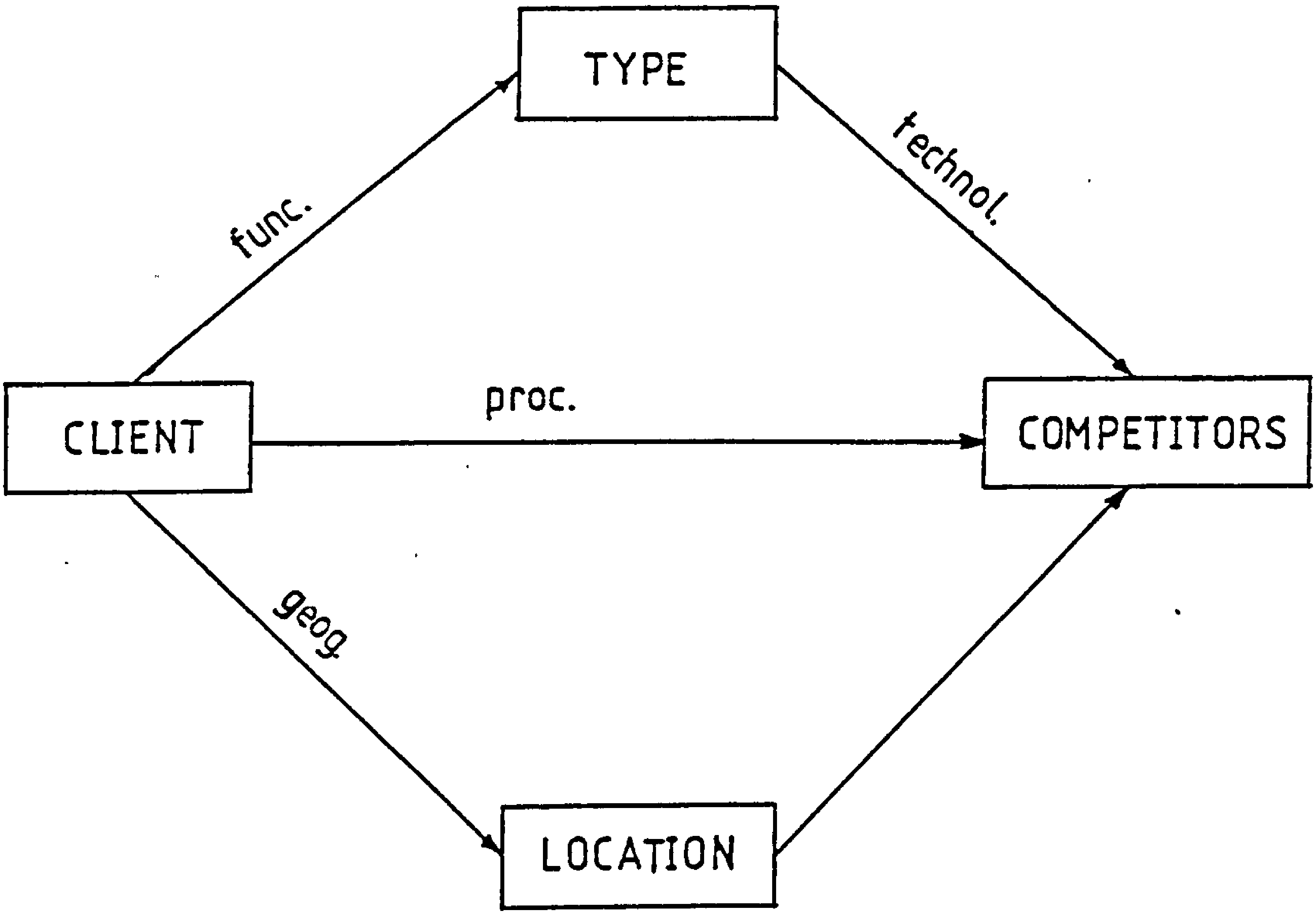
3.4.5 Competitors

From the market viewpoint it is the company's competitors that determine the value of the project and, as the deterministic model presupposes that competitors' bids are known, then the value must also be known. The factors influencing bid levels of competitors will be the same as the company's factors. Ease of entry to the industry or market, for instance, simply reflects the position where the option to enter the industry or market is associated with beneficial and preferential outcomes.

3.4.6 Summary

Four project characteristics, type, client, location and competitors, have been considered in relation to influences on the outcome environment. These characteristics are themselves interrelated as, for instance, certain types of client always want certain types of buildings, or always build in the same locality. A simple causal model is shown in Fig 3.6. A more complex model would accomodate possible relationships between type and location (eg. nuclear power stations

Fig.3.6 Relationships between project characteristics



should be on remote sites), competitors and clients (eg. the existence of a well established package dealer may influence the procurement type decision), competition and type (eg. a company's known ability to produce certain pre-fabricated components may influence the design) etc. Other environmental factors such as governmental and social legislation also affect the project characteristics. These are dealt with in the next Chapter as 'indirect' influences.

Fig 3.7 summarises the relationships between the project characteristics and the outcome environment briefly introduced in this Chapter.

Fig 3.8 shows the essentials of the project selection model, consisting of the available projects, decision and outcome, their influences on each other together with indirect environmental influences. The dotted influence line between outcomes and projects is the dynamical link required for the dynamic models examines later. The requirements of the contingent approach to decision making demand that *potential* options which may not currently be actual project opportunities are considered, and these have been accomodated in the decision 'box' for this reason.

3.5 Selection Criteria

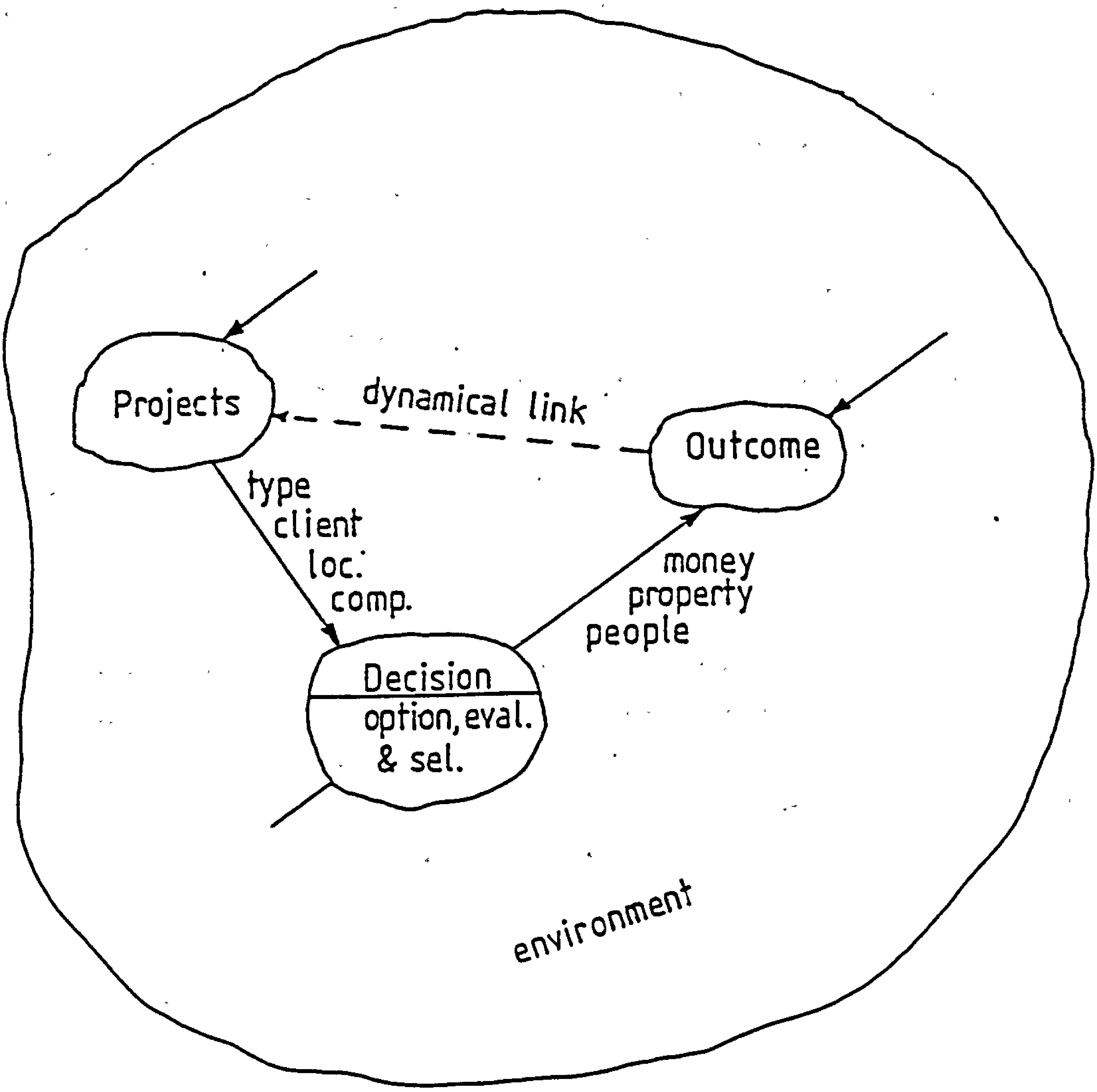
The project selection decision is a result of considerations of the beneficial effects of the decision on the outcome environment. The decision-maker, however, places differing levels of emphasis upon different aspects of the outcome environment. The internal outcome environment, for instance, is usually of more concern than the external outcome environment. These aspects of the environment are, effectively, project selection criteria and the degree of emphasis placed upon each criterion is indicative of its relevance to the problem. Relevant criteria are closely associated with the decision-maker's objectives and goals.

The primary objective of the company has been said to be "... in the continued existence and further development of the company" (Dressel,c.1980,p2). Special objectives involve market, supply, production, financial, personnel and organisational aims

Figure 3.7: Relationships between project characteristics and the outcome environment

Project Characteristics	Outcome Environment		
	People	Money	Real Property
Type (function)	Consumer		Materials (technology)
(size)	Management	(size) Finance	Plant (technology)
(technology)	Organisation & Management	(technology) Profit	Head Office (size)
	Sub-Contractors		
	Suppliers (technology)		
Client	Estimators (procedure)	(ability to pay) Finance	Materials (contract)
	Designers (procedure)	Profit (Procedure)	
	Sub-Contractors (contract)	Finance (procedure)	
Location	Operations (travel, free time) Organisation & Mangement (communications) Local Labour, Sub-Contractors, Suppliers	Finance, Profit (cost of transport) (weather) (accomodation and subsistence) (production) (communications)	Transportation Vehicles Communication devices-telephone
Competitors		Value (finance, profit)	

Fig.3.8 Project selection model



(Dressel,c.1980). These are dealt with here as monetary, non-monetary and market related objectives.

3.5.1 Monetary Objectives

The desired changes in levels of monetary resources are usually expressed in terms of profits or profitability. The conventional economist interpretation of company objectives is in the maximisation of profits, although Simon (1960) has found profit 'satisficing' to be a more appropriate description of the general business aspiration.

Profit maximisation has been considered by many writers in the construction industry to be a rather naive view of the project objective (see Fellows et al,1983,p40; Hillebrandt,1974,p89; Woodward,1975,p170, for instance). It has been suggested that the company's primary objective is to make "adequate" profits (Hillebrandt,1974,p92), "normal" profits (Hillebrandt,1974,p93), "modest" profits (Fellows et al,1983,p40), "target" profits (Niss,1965; Hillebrandt,1974,p89) or minimize losses. Profits have been measured in absolute terms, or as the level of return on investments (Barnard,1981; Mannerings,1970). A further distinction is between before and after tax profits.

An alternative approach concentrates on the growth of earnings (Barnard,1981), commonly referred to as 'turnover', where similar objectives such as "target" turnover (Hillebrandt,1974,p91; Niss,1965) or "limited expansion" (Hillebrandt,1974,p89) have been identified, often involving annual turnover forecasts (Humphreys,1977).

3.5.2 Other Objectives

A frequently reported objective concerns the utilisation of resources. This includes the efficient use of resources (Fryer,1985) such as labour and materials (Barnard,1981; Niss,1965) and filling plant capacity (Benson,1979). Maintaining the size of the workforce (Cusack,1981) or keeping key workers (Niss,1965) have also been found to be important objectives.

Objectives involving people in the environment have already been discussed to some extent in this chapter in identifying human developmental characteristics. Other objectives include serving the client well (Fryer,1985; Barnard,1981) also the general community (Barnard,1981) by maintaining and improving quality and service (Niss,1965) and avoiding such activities which endanger the environment and public health (Fryer,1985). Retaining the confidence of suppliers and sub-contractors has also been found to be important (Moore,1984,p19).

3.5.3 Market Related Objectives

Objectives are sometimes more conveniently expressed indirectly in terms of the market, instead of as specific desired states in the outcome environment. These aspirations, termed product-market scope (Ansoff,1965,p98); include such objectives as increases in market share (Fellows et al,1983,p118; Barnard,1981), staying in existing markets (Adrian,1973,p371) such as construction type (Cooke,1981) or location (Foster,1974), entering new markets (Woodward,1975,p170; Foster,1974) and growth in a number of markets (Barnard,1981; Fellows et al,1983,p27). Product-market scope is, however, a 'means' rather than an 'end' and, as such, constitutes more of a 'strategy' than an 'objective' (Ansoff,1965,p100). The decision model adopted in this thesis relies purely on the consideration of outcomes arising from the decision. A strategy is, therefore, regarded as a means of preempting the workings of such a model by some globalising mechanism such as, in this case, attributing market related factors to outcomes. Such a globalising process, it is maintained, would be only enabled *through* the model. A useful purpose can be served, however, in comparing known successful strategies with those revealed by the model as part of the validation process. These are described in Chapter 6.

3.5.4 Multiple Objectives

Studies by Bengtsson (1985) have failed to reveal the existence of any single unambiguous company goal. "Multiple objectives are required if the relations of an organisation to the environment are to be understood" (Bengtsson,1985,p33). Conflicting objectives were found, particularly between monetary objectives (eg. "profit within own area of responsibility", "company profit") and human needs and aspirations (eg. " stimulating tasks and internal training programme", "job security", "work environment", "safety" etc.). The emphasis placed on each objective, the 'goal profile', was found to differ between individuals. An analysis of 86 individuals in construction companies suggested differences in goal profiles to be associated with different managerial hierachical levels in the organisation and different quantities of expertise. There is, as Bengtsson observes, no simple method for evaluating goal profiles.

3.6 Selection Strategies

Selection strategies can be classed as either 'rational' or 'irrational'. In both cases the objective must be to make the best possible choice from a set of alternatives. The identification of rational strategies is referred to as the "optimisation problem" (Bullock & Stallybrass,1977,p444), which is usually expressed in mathematical terms, involving the minimisation of some function (the objective function or criterion function). In problems where several criteria exist, this function is variously referred to as the "vector criterion", "multivariate criterion" or a "multidimensional criterion" (Mood,1983,p55) When several criteria are combined into a single criteria the resulting criterion is termed the 'scalar criterion'.

Methods of handling optimisation problems containing several criteria are called "multiple criteria" or "multi-attribute" methods.

3.6.1 Multiple Criteria Decision Evaluation

There is a practical need, in the face of multiple decision criteria, to develop a strategy for "... presenting all those potential courses of action which might reasonably be regarded as attractive without obscuring what is at best likely to be a complex decision by the presence of many less desirable possibilities" (Kmietowicz & Pearson,1981,p106).

In recent years, an increasing number of papers have appeared concerned with both the theory and practice of multiple criteria decision making. French and Dutch authors, in particular, have made significant contributions to the field. Whilst the majority of early formal work appears to have emanated from psychologists and management scientists in the USA, many potential areas of application exist. Public policy decisions, for example, frequently involve the consideration of a wide range of consequences which affect many different groups of people in different places and in different ways.

It is apparent, however, that no single multiple criteria technique exists which is unambiguously superior to all others, the most appropriate method or combination of methods varying from problem to problem, (Kmietowicz & Pearson,1981,p106).

One possible approach is to construct a multi-attribute function covering the whole range of monetary and non-monetary outcomes which could potentially arise from the decision to be taken. The best known methods for reducing multiple consequences to a single dimension are in financial appraisal and cost-benefit analysis. In the former case, market prices are used to evaluate the different consequences of a course of action. In the latter, account is taken of consequences which may not have a market and for which shadow prices have to be substituted.

The multi-attribute function, termed the "global preference function" by Ibbs and Crandall (1982,p191), can be formulated in several ways. Keeney & Raiffa (1976) have considered four of these formulations: additive; multilinear; multiplicative; and general.

MacCrimmon (1973) has identified four major categories of solution technique: weighting methods; sequential elimination methods; mathematical programming methods; and spatial proximity methods. At one point or another, a weighting method is central to the evaluation procedure of most practical multiple criteria decision making models, the main difference between methods being the techniques used for establishing the weights (Kmietowicz & Pearson,1981,p107).

3.6.2 Evaluation of the Preference Function

Examination of the construction industry literature reveals two approaches to the formation of the global preference function evaluation process in the project selection problem. Although both of these approaches are presented in a non-deterministic context, they, nevertheless, serve to illustrate the potential use of the multi-attribute technique.

Study 1 (Fellows & Langford,1980 and Fellows et al,1983,ch3)

Fellows & Langford give an example of a weighted additive multi-attribute utility function approach to the construction project selection problem. They consider five courses of action (outcomes): returning the tender documents; submitting a cover price; providing detailed estimates and a tender conversion; preparing a tender based upon approximate estimates; or reworking the tender. Each of these possible outcomes is evaluated on five criteria: speed of obtaining solution; accuracy of solution; cost of solution; client/consultant consideration (risk, attitudes etc); and benefits, success potential to firm (profitability potential, employment of resources, continuity of work etc). Adjustment for the relative importance of each criterion is made by a utility weighting device before values are summed into an aggregated "outcome evaluation". Under the reasonable assumption that the decision-maker is interested in maximising the utility of the various consequences resulting from his decision, the 'best' decision will be that associated with the highest aggregated outcome evaluation. "Sensitivity tests" are recommended by the authors in assessing the effect of changes in the utility weightings and "the criteria themselves".

Study 2 (Ibbs & Crandall,1982)

Ibbs & Crandall have considered the use of weighted additive and multiplicative multi-attribute functions. In their example there are three (unspecified) decision options, the outcomes of which are evaluated as three criteria: profit return; contract size; and "regulatory aspects".

The multiplicative formulation relies on the existence of Utility Independence (UI) and Multi Utility Independence (MUI). Tests for the existence of these states involves the construction of a standard lottery to check the decision maker's indifference to relative changes in permutations of criteria values. In the additive formulation MUI only is a necessary condition.

Weighting factors in the multiplicative formulation are required for all attributes together with an attribute independent scaling factor. As Ibbs & Crandall show, the value of this attribute independent scaling factor can be computed from the attribute values and weightings and, in particular, that when the attribute independent scaling factor obtains a zero value the multiplicative formulation reduces to the additive form. Indeed it would seem that the attribute independent scaling factor can be used as a decision variable in determining the method of aggregation to adopt.

The resulting global preference function value is computed for each decision and, as in the previous Study, the 'best' decision is considered to be that associated with the highest valued global preference function.

In addition to the recommendations for sensitivity tests on the weighting factors and the criteria used, several further observations are made. Precise definition of the decision variables is considered to be important in order to avoid confusion of the variables measured. 'Profit return', for instance, may have a variety of valid submeasures, such as absolute monetary profit or return on investment, which reflect different value levels. The independence checks, involving all permutations of the criterion variables can become "unwieldy" in the presence of more than "five or six" criteria. If none of the independence conditions can be found among the decision criteria, other steps may be necessary. Special formulations are available, but difficult, or some method of criteria parsimony applied by isolating offending variables or combining variables by an orthogonal technique such as factor analysis.

Three final operational comments are made. Firstly, some decision-makers have difficulty in expressing a preference function for the criteria. Secondly, there are situations in which decision alternatives are assigned scalar values outside the original limits of the analysis. It could be, for instance, that the decision maker considers a criterion variable to be more important than another criterion variable which has already been assigned a maximum value on the weighting index. Finally, a related feature, and one which appears to be a recurring problem in the utility approach, is the instability of utility values in the weighting scheme. These values, it seems, must be continually monitored as the firm's preferences change.

3.7 Conclusions

The project selection problem, in its deterministic and static form, centres on the consideration of three fundamental and interrelated aspects of the outcome environment, people, money and physical property. Four interrelated project characteristics, type, client, location and competitors have a significant influence on these environmental aspects.

It is considered that, in order to pursue the successful progress of the company, the 'best' decision should be the one which will most benefit all these often competing aspects of the outcome environment. This will normally entail the evaluation of criteria including profits, profitability, earnings and turnover for monetary outcomes, the type, value and usage of physical resources, the usefulness and aspirations of individuals and groups within the organisation, the level of the client and community satisfaction of the company's quality and service and the confidence of the suppliers and sub-contractors. The value of these criteria, it is argued, will be relative to previous levels, referred to as the developmental state.

The final stage of the decision model outlined in Chapter 2 involves the comparison of the evaluated criteria with those of an alternative decision. Where only one criterion exists, profit for instance, the comparison is trivial. The presence of multiple criteria is, however, clearly an essential feature of the problem. Before any attempt can be made at a solution, it is apparent that some method of weighting must be applied. This is regarded as an aspect of the selection phase of the problem in order to preserve the generality of the evaluation phase.

CHAPTER 4

Time dependent aspects

The proposal to combine the weighted criteria into a single scale is, perhaps, the most controversial issue. The major criticism with this approach is that, in reducing the multiple dimensional consequences of an act to a single dimensional evaluation, information is inevitably lost. "If there is no doubt about the rates at which decision makers are prepared to trade off different consequences against each other at all different levels of attainment of those consequences, then reduction to a single dimension should cause no great problems", (Kmietowicz & Pearson,1981,p106). Ibbs & Crandall's study indicates that this is not likely to be the case, some difficulty being encountered in acquiring consistent trade-off values.

Kmietowicz & Pearson (1981,p106) suggest two possible approaches to this problem. One is the comparison and ranking of selection strategies on the basis of more than one decision. The other is in the reduction of the dimensionality of the problem by fixing acceptable weights to the different consequences of strategies and the exploration of the implications of some tolerance in the precise values of those weights.

Both of the studies reported have adopted the latter approach in attempting to attach weights to the various criterion variables and in recommending some sensitivity analyses. The need to perform extensive independence checks and the difficulty in assigning weights suggests that the approach may not be without its problems, even in the deterministic model.

It is noted, however, that, whatever solution technique is employed, some formalisation of the decision process does have value in helping to clarify both what is being aimed at and the relative importance of conflicting goals. Furthermore, communication with other decision makers and affected parties is facilitated if some framework for presenting and comparing the consequences of different courses of action exist

4. TIME-DEPENDENT ASPECTS

4.1 Introduction

The previous chapter was concerned with the relationship between the project selection decision and the immediate environment of the decision-maker, assuming perfect knowledge and, generally, without regard to time-dependent aspects of the problem. The purpose of this chapter is to explore such time-dependent aspects whilst still maintaining focus on the immediate environment and assuming perfect knowledge.

Time-dependent aspects of the decision making process have two major implications in terms of the project selection model. One is the causal relationship between the outome environment and projects, shown in Fig. 3.8 as the dynamical link. The other implication is that time introduces a new dimension to the problem.

4.2 The Relationship between the Outcome Environment and Projects

The previous chapter examined the problem of selecting a project or set of simultaneous projects which would best benefit the outcome environment without regard to any further selections that may be required in the future. However, it is clear that future decisions will be significantly affected by current decisions. The problem can, therefore, be restated as that of selecting the set of *sequential* projects which will best benefit the outcome environment. Problems involving sequential decisions are said to be dynamical problems (Bullock & Stallybrass,1977,p184). The dynamical version of the project decision problem demands knowledge of the effect of the outcome environment associated with each project on the quantity and characteristics of future projects.

The generation of project opportunities is normally regarded to be a result of some marketing activity. Until relatively recently, marketing has not generally been considered appropriate for construction companies, as they belong to a service industry largely waiting to be asked for their services (Sidwell,1984). Lansley et al (1979,pt3,p78), however, found some rather more aggressive contractors who actively sought opportunities through market research and by cultivating contacts with prospective clients. It was also found that contractors who normally adopted a passive attitude became more aggressive when work was short (Lansley et al,1979,pt3,p78).

The marketing aspects of the project selection decision are only a small part of the total possible marketing effort, and very little literature is available on the subject.

An obvious point of interest is the client, who is part of the outcome environment *and* project characteristics. Enhanced benefits for a particular client may well generate further project opportunities. Jepson & Nicholson (1972) term this general strategy "image building" or the development of "goodwill". Such enhancement necessarily implies reduced benefits in other aspects of the outcome environment. It may require, for instance, "... the unquestioning assumption of liability for the errors and failings of employees and associates and that profit may thus be lower" (Jepson & Nicholson,1972,p78). The opposite approach, termed 'milking' the project, is aimed at enhancing non-client aspects of the outcome environment and with the possible consequence of reduced future project opportunities or modified project characteristics.

This means of influencing potential clients extends to other aspects of the outcome environment. Knowledge of benefits received by one client may influence other potential clients. Benefits received by a section of the community may also influence potential clients in a similar way. Even benefits obtained internally, such as the wellbeing of the workforce, may engender a special attitude in potential clients towards the benefits they may receive should they wish to employ the company.

Unless clients possess perfect knowledge, then communication of events that take place in the outcome environment to potential clients is an important issue. Such communication usually implies some advertising or promotional activity by the company. The project selection decision can provide some assistance in this through the selection of certain prestigious projects, for instance, or projects associated with well publicised designers or causes. In these cases, enhanced promotional benefits may be preferred to short term monetary gain.

4.3 The Time Dimension

The effect of time considerations is to introduce an additional dimension to the problem, as changes in the decision environment occur at different points in time.

Major implications occur in the outcome environment. The extent of benefits received by people is, as discussed in the last Chapter, dependent on the stage of development reached at the time. These benefits are received continuously, in the case of human resources, resulting in continually changing developmental and aspirational states. Also, as people join and leave the organisation, fluctuations in quantity as well as quality occur.

Performance levels are affected by time, instanced by such phenomena as the learning curve. The time of the year can affect output by up to 50% (Cusack,1981,p54). Overtime working and bonus schemes also provide other time related aspects. Construction sites suffer particular difficulties in the co-ordination of sequential activities as operatives frequently change work places (Bennett & Fine,1980).

The acquisition of a new project involves some degree of disruption to personnel, depending on the project characteristics, although craft based organisations, such as construction companies, have been said to respond quickly and effectively to major changes in demand, (Burton,1972,p72). Lansley et al (1979), however, has found evidence of resistance and reluctance to change due to incompatible individual development strategies (rigidity of views and attitudes); the frequency of changes, especially those not providing benefits to the individuals; lack of involvement in the decision, causing the change or its implementation; and poor communication of decisions and their anticipated effects on the individuals concerned.

The extent and frequency of change has an effect on the organisational structure of the company, a tendency to a more flexible structure occuring with increasing change (Lansley et al,1980) depending on the present size of the company (Lansley et al,1979). Movements in monetary resources are primarily linked with the project duration, resulting in the consideration of cash flow implications. Cash flow has been found to be affected mostly by such project characteristics as the size of the project, the type of project (eg. speculative housing), the client (eg. private work) and bias in the progress valuations (caused by "front end loading") (South,1979). The client's conditions of contract further impact on cash flow by restricting income (retention) and reimbursement of changes in the value of money (fluctuations). The cost of creating liquid assets (interest rates) and the timing of payments to suppliers, sub-contractors, shareholders etc are further important aspects.

Time effects on physical property include the provision of temporary buildings and the acquisition, maintenance and disposal of materials and plant. These aspects are normally treated in monetary terms, such as depreciation or sinking fund provisions.

Changes also occur in the project characteristics. Design modifications take place during the course of construction. These affect the management and organisation of the work, productivity, cash flow and even the contract period. Changes in client or consultant personnel can affect working relationships, sometimes quite dramatically. Such changes (eg change in Quantity Surveying personnel) can also affect monetary resources.

4.4 Implications for Evaluation and Selection

Consideration of time related aspects of project selection introduces the notion that outcomes take place over time. These outcomes can be regarded as being discrete events or as continuously developing, as represented by discrete or continuous time systems. An outcome, in these terms, is effectively the state of the outcome environment *at some point in time* after the decision has been made. Points of time that are of interest occur in the short, medium and long term, (Adrian,1973,p370). For construction companies short term is associated with the duration of a particular project, medium term has been said to be about three years (Bahrami,1981), although in recent changing times twelve to eighteen months has been considered more appropriate (Benes & Diepeveen,1985), and long term some distant future of a minimum of ten years ahead. Such distinctions are quite arbitrary however. Cash flow analysis, for instance, attempts to predict monetary outcomes at quite frequent, usually monthly, intervals and is particularly useful in identifying times when financial problems will occur. Ideally, a system would indicate the state of the outcome environment *at any moment in time.*

The influence of the outcome environment on project opportunities raises special strategic issues. Sacrificing profits to enhance opportunities, for instance, is one such strategy (Fellows et al,1983,p40,p188). Strategies aimed at stabilising profits, return on investments and turnover are also evident (Barnard,1981). Development strategies also exist to enable exploitation of opportunities (Fellows et al,1983), particularly organisational development (Lansley et al,1979). Marketing strategies of this nature are termed forward integration strategies (Moss,1981).

These considerations suggest that the project selection decision will require evaluation of the state of the *project generating environment* in addition to the outcome environment. Strategic decisions also require evaluations of the *nature of changes* in the relevant environment. This would require knowledge of the rate of change or the existence of trends for instance. It is contended, however, that, as noted in the previous chapter, such strategy oriented aspects are necessarily concerned with behaviours manifested by, rather than incorporated into, the model, except as a simplifying expedient.

In the 'systems' context, changes in the environment can be regarded as an iterative process and any decisions affecting these changes are adjustment processes. The competitive economy, which is a particular organisational form in which all members as regarded as acting in competition with each other, has received some attention in this respect. The competitive economy has been described as consisting of "... agents, involved in a competitive process, who act in response to their changing 'environments' and to actions by other agents resulting in 'messages' (prices)" (Gottinger,1983,p178). "... An adjustment process in this organisation is a kind of scheme or process which this organisation reveals at each iteration and which would satisfy certain properties to the best of all members of this organisation" (Gottinger,1983,p178). In this context, an adjustment process can be viewed as "... a sequence of aggregated actions (behaviour patterns) taken by each agent" (Gottinger,1983,p178).

For a different class of environment, Hurwicz (1959) has studied adjustment processes in terms of differential equations in which agents respond to messages from other agents including themselves (memorising). The behavioural pattern of such an economic system can be studied in terms of a particular social welfare function satisfying an optimality criterion given an environment of a particular kind. On the basis of the adjustment process new states will be generated up to a point where the final state is compatible with the welfare criterion. Some important results in this area have been obtained notably by Hurwicz (1959) and Gottinger (1983,p179), although always depending on some simplifying assumption such as the existence of a 'classical' environment or a Pareto-like stabilising tendency. Major criticisms of these results turn on informational efficiency, giving rise to controversies about the choice of economic systems, and the goal-compatible behaviour patterns of economic agents in which a competitive system is satisfied, given the classical environment by assuming profit and utility maximisation.

A further difficulty with this approach is that the agents in the project selection environment do not simply respond to environmental stimuli, but attempt to influence stimulatory parts of the environment (evidenced by the 'marketing' effort), neither do the agents act in a purely competitive manner.

The complexities of the various interactive elements in the decision environment over time have been modelled by Gottinger (1983) by a device termed a 'sequential machine'. The sequential machine is a finite-state dynamic system possessing five general characteristics:

> "(1) *A Set of Inputs*, eg. those changing parameters of the environment which will affect the system behaviour in a predictable way.
>
> (2) *A Set of Outputs*, ie. those parameters which act upon the environment leaving observable changes in the relationship between the system and the environment.
>
> (3) *A Set of States*, ie. those internal parameters which determine the relationship between inputs and outputs and which may reveal all necessary information embodied in the part.
>
> (4) *A State Transition Function* which determines the dynamics of how the state will change when the system is fed by various inputs.

> (5) *The Output Function* which determines what output the system will yield with a given input when in a given state. "
>
> (Gottinger, 1983, p17)

A sequential machine is then defined as a function $f:\Sigma A \rightarrow B$ where A is the basic input set, B is the basic output set and $f(a_1,\dots,a_n)=b_n$ is the next input at time j $(1 \leq j \leq n)$. A is a nonempty set of ΣA, ie. $\Sigma A=\{(a_1,\dots,a_n):n \geq 1 \text{ and } a_j \in A)$. Looking inside the machine, a circuit C is defined as a quintuple (A,B,Z,λ,δ), where Z is the (nonempty) set of internal states, $\lambda:A \times Z \rightarrow Z$ is the next-state function, and $\delta:A \times Z \rightarrow B$ is the next output function

$$C_z(a_1)=\delta(a_1,z);$$
$$C_z(a_1,\dots,a_n)=C_{\lambda(a_1,z)}(a_2,\dots,a_n) \text{ for } n \geq 2$$

The basic idea of a sequential circuit C is then

$$\begin{tikzcd} C: z=z_0 \arrow[r, "a_1"] & z_1 \arrow[r, "a_2"] \arrow[d] & z_2 \arrow[r, "a_3"] \arrow[d] & \dots \arrow[r, "a_n"] & z_n \arrow[d] \\ & b_1 & b_2 \dots\dots\dots & & b_n \end{tikzcd}$$

Gottinger's perspective is to consider sequential machines as basic analogues for modelling complex 'humanistic' systems (organisations), and to treat adjustment processes in terms of transformations on the set of states of a machine. One consequence of the sequential machine concept is that any biological, ecological, or economic system evolving in time can be viewed as a transformational semigroup (tsg) in which time is an irreversible resource.

The first task in building such a machine is to decompose the system into component parts or sub-systems. This is done by first identifying the external state vector $x_t=[x_{1,t},x_{2,t},\dots,x_{n,t}]$ representing exogeneous factors driving the system from 'outside'. In terms of the project selection process, these exogeneous factors include such indirect influences as government policy and social attidudes. Exogeneous factors are not incorporated into the decomposition.

The next step is to choose a kind of partition of the overall system into parts that comprise the main activities of the system. Gottinger suggests that this can be achieved by a decomposition into three types of machine, a message machine, a decision machine and a payoff machine. In terms of project selection this implies project opportunities and characteristics (message machine), project selection/rejection (decision machine) and the outcome environment (payoff machine). The payoff machine, for instance, would resemble the configuration shown in Fig 4.1. Each part enclosed in dotted lines is itself an automaton called a component. The interaction of all components with feedback constitutes the realisation of the entire system. The transformations relating to each component are each described by a set of structural equations taking into account imput or feedback stimuli from other components. Each stimulus for a component is composed of an external stimulus, together with all state-output configurations of all previous components plus the feedback responses to subsequent components. The overall design complexity is determined by the structural complexity of the components and the computational complexity of the interaction between components, the length of computational strings to arrive at solutions. The control complexity is a kind of complexity that satisfies some bounds in the performance boxes in order to keep the system in harmony and stability. A refinement of the payoff machine shown in Fig. 4.1 would further decompose the outcome environment into individual people, separate types of monetary assets and individual items of property.

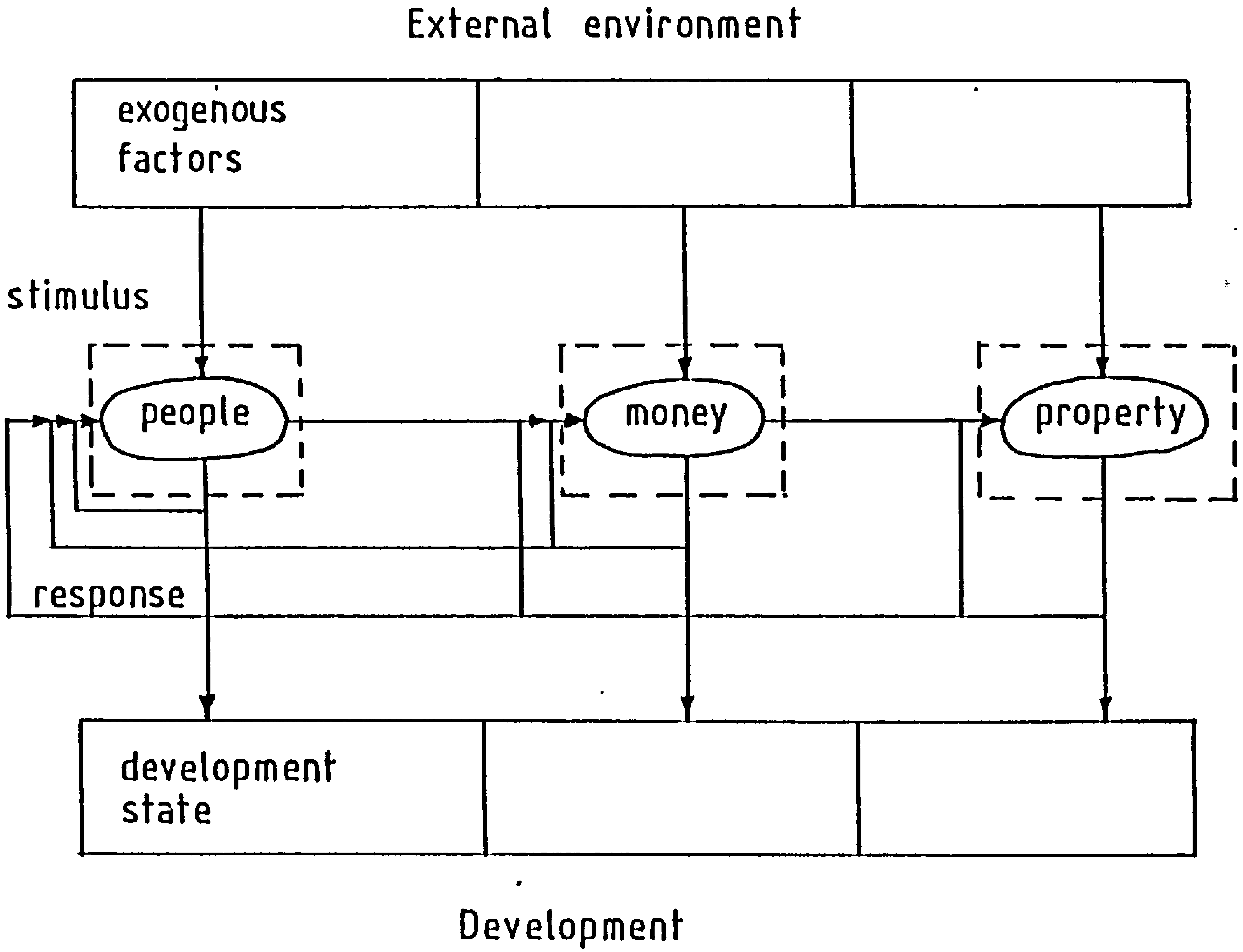
Fig. 4.1 illustrates a cascade decomposition. Other types of decomposition are available such as serial and parallel decomposition. Machines can also be decomposed in similar ways or combination of ways. Fig. 4.2. indicates a possible machine configuration for project selection. General environmental factors (external stimuli) affect all the machines involved. The projects and their characteristics affect the decision machine. The outcome environment (payoff machine) loops back to influence (future) environmental influences.

The project selection process is then, in terms of this model, controlled by the decision machine, the key developmental results being formed in the payoff machine. The solution to the problem must be in operating the decision machine in such a manner as to obtain the 'best' set of developmental states.

Three immediate problems arise in the model procedure examined above:

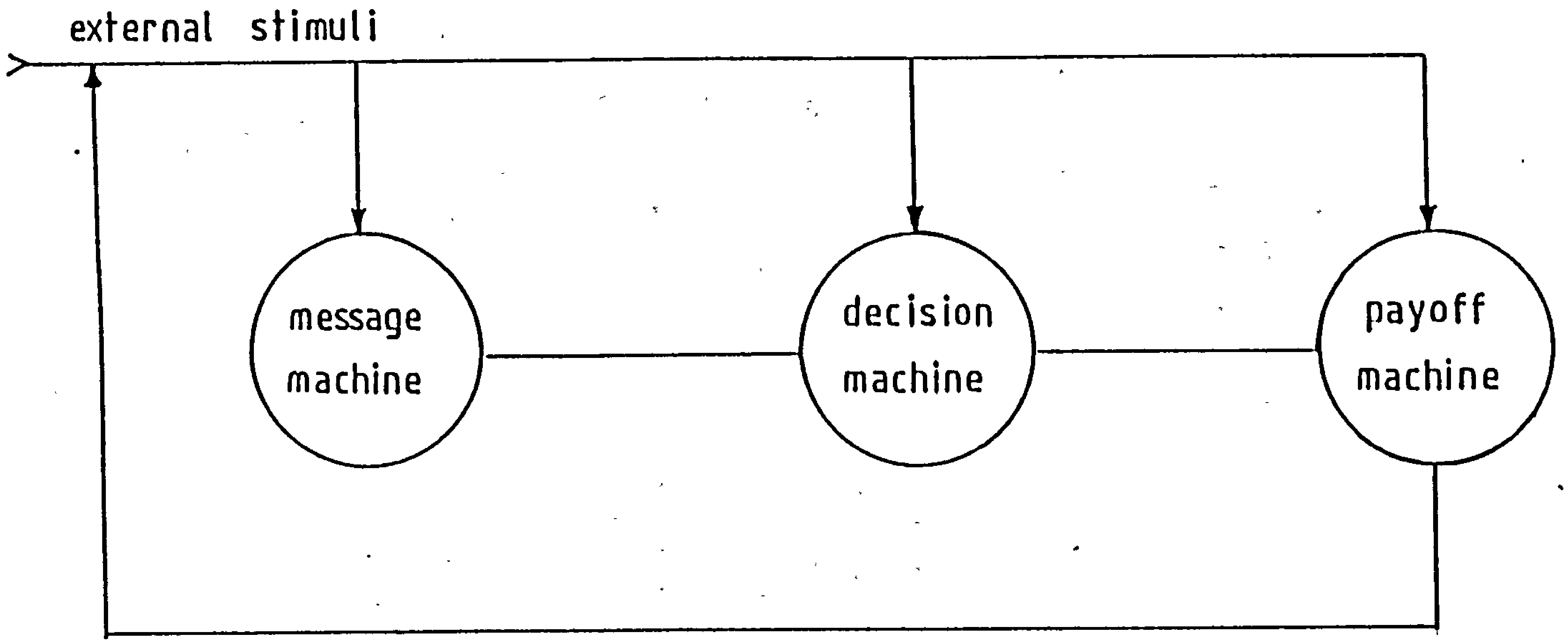
1. As the number of components (particulary in the payoff machine) and sufficiently strong connections among components increase, the

Fig 4.1 The payoff machine



after Gottinger (1983)

Fig 4.2 Project selection machine configuration



behaviour of the system becomes increasingly obscured by complex interactions which resemble very much non-linearities in the total system's behaviour in correspondence with size.

2. The structure and size of components themselves present a potential source of complexity depending on whether and to which extent a component system is sensitive to disturbances, errors, threshold phenomena etc.

3. As the number of components and interdependencies in the system enhances, increasingly longer sequences of calculations are required to deduce the behaviour of the system which results in computational complexity.

The solution to these three problems would, as Gottinger observes, "... enable us to determine the complexity of the system on-line, as it is running from some initial time to some target time in the future. But knowing the complexity would permit us to design control strategies which are effective in guiding the system toward relative stability or harmony" (Gottinger,1983,p117). In order to understand the complexity of the systems it is important, that "... we should be able to understand the strongly connected, coupled nature of its subsystems. For this purpose we need a measure of complexity that reflects the structural performance of each of the connected subsystems in terms of state space configurations, plus the number of computational links that are established among the various subsystems and that reflect the richness of state representations in the global trajectory space of the entire system" (Gottinger,1983,p119).

A recent approach to dealing with the computational complexities involved has been to reconfigure the computer hardware in such a way as to arrange the processing elements to match the structure of the problem. This has resulted in an entirely new kind of computer, the "connection machine" (Hills,1985) which, in terms of the project selection problem, implies a separate processor for each aspect of the problem environment, eg. each individual member of the organisation.

It is clear from the literature, however, that insufficient knowledge is yet available to determine the complexity of the system to the degree recommended by Gottinger. Even the assumption that the objective is to

"...guide the system toward relative stability and harmony" is as yet untested.

4.5 Conclusions

This chapter has outlined the dynamical and dimensional effects of time on the project selection problem. An approach of conceptualising the problem along the lines of Gottinger's sequential machine has been proposed as a means of handling the complexities involved. It is anticipated that this, together with some type of multi-attribute analysis discussed in the previous chapter, may form the basis of a useful and realistic approach to developing appropriate solution techniques. Before attempting such a task, however, it is necessary to consider a further (and most critical) aspect of the problem - the effects of imperfect knowledge.

CHAPTER 5

Non-deterministic project selection models

5 NON-DETERMINISTIC PROJECT SELECTION MODELS

5.1 Introduction

The aim of this chapter is to consider the implications of relaxing the 'perfect knowledge' assumption. The effect of this is to reduce our decision-maker to a mere mortal faced, as will be seen, with a task of rather unearthly proportions.

Imperfect knowledge introduces uncertainties, in the most general sense, into the problem. One such uncertainty, the value of the project, is of particular relevance in the competitive situation in that the decision-maker can no longer be certain of obtaining the project he has selected. Problems involving this characteristic are normally termed auction, bidding or tendering problems upon which a body of literature already exists. The following extract from Woodward (1975) is, perhaps, a suitable introduction to the subject.

> "To many people, the whole subject of bidding and tendering appears to defy analysis and is cloaked in a certain amount of mystery. One reason for this is that there are so many variables that are not well understood when they interact, that it becomes very difficult to make specific predictions. Some of the more important of these variables are related to the cost of the completion of the work, for example the price of materials, labour rates, labour productivity, plant usage, ground conditions, weather, variations instructed as to the detailed work to be completed and additional costs associated with delays caused by shortage of materials or lack of information. A second reason for the mystery surrounding bidding is that it is a very sensitive area to many contractors and they are unwilling to discuss it. This is largely due to the desire to keep methods of estimating work and the prices used confidential to the company, but a more subtle and seldom quoted reason for secrecy is that many contractors do not themselves fully appreciate how their prices are arrived at, and may indeed have very little or no systematic approach to bidding whatever."
>
> (Woodward, 1975, p168).

5.2 Imperfect Knowledge

The model of the project selection decision process thus far developed relies on the decision maker 'knowing' the exact effect of potential decisions on the outcome environment, placing some value on each of these effects and selecting the 'best' decision by comparing these values. The reality of the situation, however, is that 'exact' effects can seldom, if ever, be predicted. The difference between predicted and actual outcomes, termed 'errors' in this thesis, are, therefore, a direct result of the imperfect knowledge of the decision maker. The nature and magnitude of these errors have a significant impact on the problem.

If the 'real world' is defined as the 'prototype' (after Aris,1978) and the individual's perception of the real world is defined as the 'model' (after Kelly,1955) then the fundamental cause of all errors is in the discrepancy between the prototype and the model. This does not , of course, imply that a model generating no predictive errors must necessarily be a perfect compositional mapping of the prototype (although such a model may often be considered to be a 'perfect' model). The formula for predicting the expansion of a piece of metal, for instance, is not, although virtually error free, in any way composed of the actual physical process involved. Although, in essence, simple or 'top-down' predictive models ('simple' in this context is equated with 'elegant') are highly desirable computationally, the instability associated with non-physical prototypes (people) invariably inhibits their development. The logical action, therefore, is to adopt the 'bottom-up' approach of concentrating on the compositional aspects of the prototype, at least until 'top-down' modelling is sufficiently well-advanced.

Compositional discrepancies between the prototype and the individual's model are due to the prototype information received or not received by the individual together with the individual's ability or inability to model the prototype once the information is received. Interdependencies have been found to exist between the prototype, the individual and information. Skitmore (1985), for instance, in a brief review of the psychology of expertise has noted the profound differences in the way experts and novices handle contextual information.

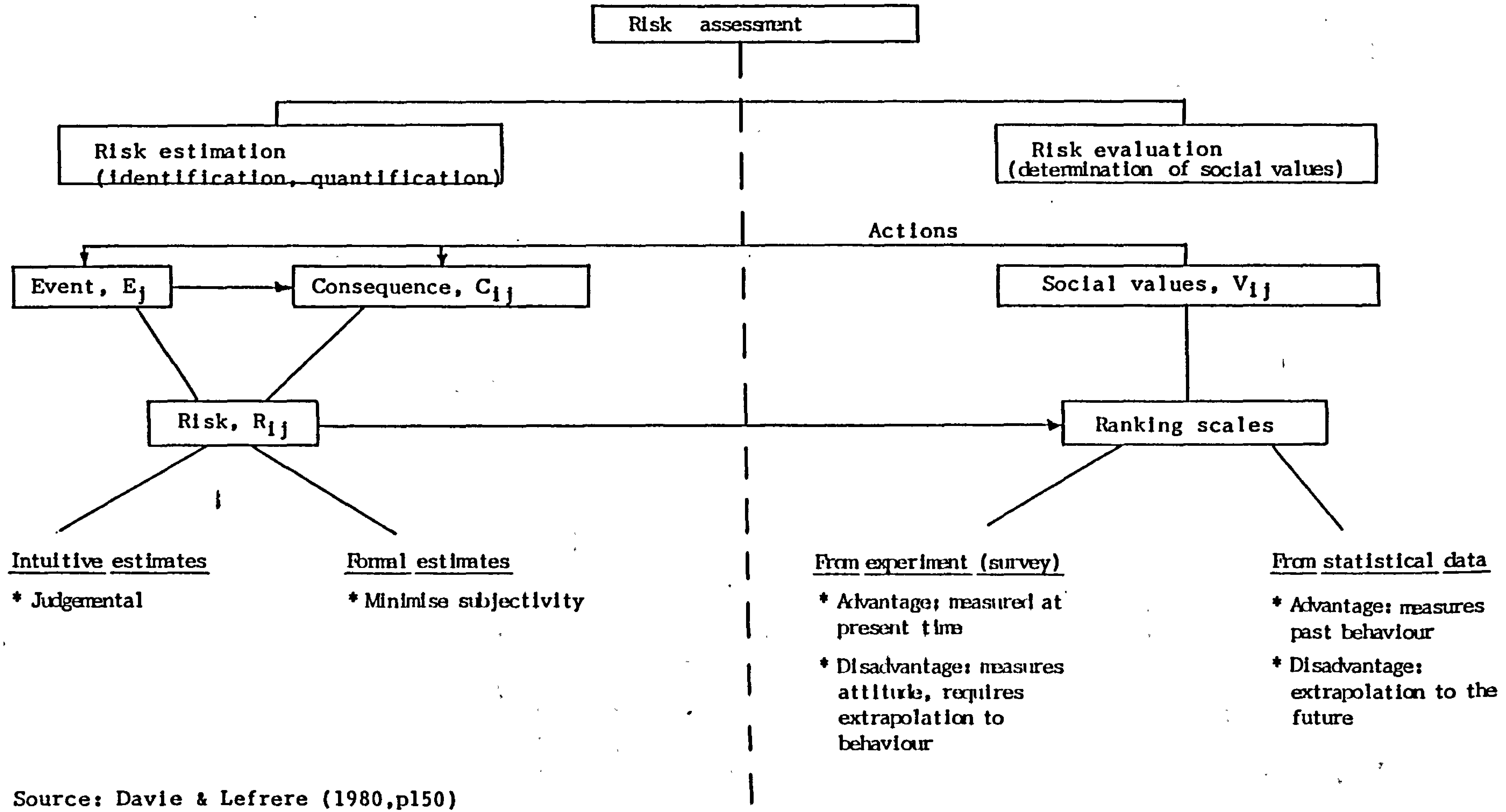
In the context of the project selection decision, discrepancies between the prototype and the decision-maker's model are frequently termed uncertainties (in the general sense). One type of uncertainty has been identified as "inherent uncertainty" (Bennett & Barnes,1979), due to "chance variations" (Gates,1971), "chance events" (Woodward,1975,p12) or "lack of predictability" (Ireland,1985,p62) in the prototype and resulting in "an inherent inability to forecast positively the efficiency, and therefore the production rate for any given crew for any given operation" (Gates,1971,p277) for example. Inherent uncertainty is, therefore, intended to represent the cause of those errors which cannot be avoided in any way, in other words, the limit to which the model can approximate the prototype.

Some knowledge of the nature of the prototype/model misfit, however, can be useful. A measure of this misfit is called 'risk'. Risk is essentially a measure of the strength of belief that an event will or will not occur, and is usually expressed as the chance or probability of occurance. The definition of risk is usually extended to all events to which the chance of their occurance can be quantified in some way. The definition of uncertainty is then confined to inherent uncertainties for which no measure of their chance occurance is available.

A general structure of risk assesment is given by Otway and Paranka (1980) reproduced in Fig 5.1. This structure separates the tasks of risk estimation, based on intuitive or formal estimates, from risk evaluation, based on experimental or statistical data, the object in this case being to gauge the environmental consequence of risk acceptance. Risk evaluation is a complex process of determining the meaning, or value, of the estimated risks to those affected, referred to by Häfele (1974) as the "embedding" of risks into the "sociosphere". Evaluation has been considered by Otway & Pahner (1980) to be a process of ranking risks so that their total effects, both objective and subjective, may be compared for acceptability.

Attitudes to risk vary between individuals from cautious to adventurous. Studies by Dickenson (1979) suggect that risk experienced people tend to be more cautious in their attitude to risk. Building contractors, although no evidence is yet available, are also thought to be generally risk-averse and this, too, may be a result of risk exposure. Whilst not wishing to make risky decisions, however, it would appear that people in the construction industry are able to adequately cope with risky situations. Indeed, the Tavistock Institute's study (1966) found evidence of people actually thriving in risky and uncertain situations.

Fig 5.1 The general structure of risk assessment



Source: Davie & Lefrere (1980,p150)

5.2.1 Coping with Risk and Uncertainty

There are two recommended approaches to dealing with risks and uncertainties. One approach is to devise methods of exploiting the situation, by adapting a more flexible posture, for instance. The alternative is to reduce the effects of risks and uncertainties, especially those effects associated with environmental disbenefits, by either reallocation or by improving the prototype/decision-maker's model fit. Reallocation can involve sub-contracting sections of the work, insuring risks via external agencies or by partly reallocating to the client by means of contingency allowances. Improving the model fit involves either changing the model or changing the prototype in some way.

The quality of the model is, as mentioned earlier, dependent on the interrelated aspects of context, the modeller and the information received by the modeller. The context of the project selection problem is the entire decision environment which includes not only the immediate outcome environment but the whole of the project generating environment. The ability of the modeller and the information he receives depends upon this contextual state. Lansley et al (1980), for instance, have commented on the importance of past experience in reducing uncertainties and the need for training to handle unfamiliar problems by familiarisation with possible future environments. Turbulent environments create special organisational problems in this respect, resulting in "difficulties in obtaining a sufficiently detailed understanding of the changing business environment" due to staff being "unable to keep abreast of change or pursue activities critical to the firm's survival, particularly marketing and industrial relations", (Lansley et al,1980,p43). The 'customised' nature of the construction process is a particular source of uncertainty, resulting in "lack of routine" and "unfamiliarity" (Ireland,1985,p62).

Ansoff (1984), in stressing the need for familiarisation with current events and preparation for future events, suggests the development of surveillance systems for detecting "weak signals" of future movements in the environment. Fellows et al include such action in their recommendations for "external appraisal" to "forecast the pattern of demand and competition" by identifying competitive, political, economic, social and technological trends [in the project generating environment]"

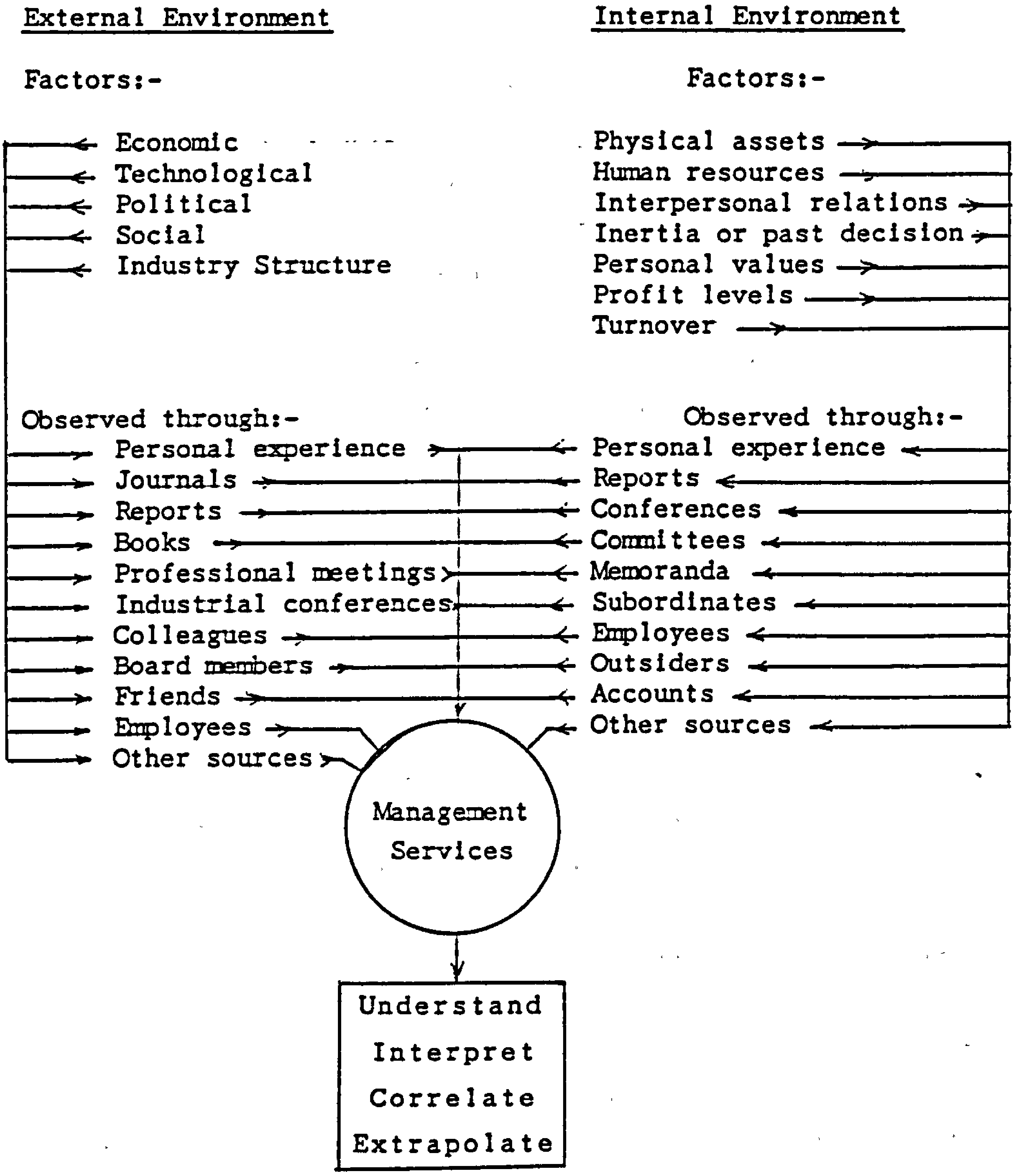
(Fellows et al,1983,p44). Fellows et al's "internal appraisal" covers internal aspects of the outcome environment. Foster (1974) indicates the major sources of information of these environmental states (Fig 5.2) as inputs to his management services system. As Erikson & Boyer (1976) observe, however, such information is not always readily available, mainly due to the competitive nature of the environment. A further problem, as noted in Chapter 2, is that the information does not always exist in the most appropriate form. Gilchrist (1984), for instance, discusses the problems of aggregated information which may be simply averages of some highly variable observations. The information may also contain errors which implies the same considerations as errors occuring in the decision problems generally. Further difficulties occur in interpreting and communicating information to the decision maker and assimilation of the information by the decision maker. These activities, as Bunn (1975) observes, require a careful synthesis of information, predictions and opinions.

The cost of information is a subject which has attracted many writers and, in the context of this thesis, forms a part of the overall cost of the decision making process. It is possible, of course, that information costs may be distributed amongst the other information users where identical information is needed but, as previously noted, such compatibility seldom exists in current systems.

5.2.2 Behaviour of the Prototype

Empirical research in the physical sciences is directed towards the discovery of "laws of nature", underlying environmental behaviour. Belief in the existence and immutability of those laws of nature is axiomatic in fundamental scientific research. The origins of laws of nature may have motivational implications for the scientist in the worthiness of the enterprise and optimism of its outcome. "Raffiniert ist der Herrgott, aber boshaft ist er nicht" (Subtle is the Lord, but malicious He is not - Albert Einstein). The origins of such laws are, however, of little relevance to most current scientific research, which is essentially concerned with the more pragmatic task of predicting behaviour. Laws of nature in this context often implies the scientist to be a passive observer of events 'caused' by the laws, in the way that an astronomer may observe the apparent movements of celestial

Fig 5.2 Basic sources of information



Adapted from Foster (1974)

bodies, or that we have some control over 'conditions' (in experimental work for instance). In these cases, the view is essentially fatalistic in that events are predetermined generally or predetermined in a given set of conditions.

The belief that there are laws of nature waiting to be discovered is, however, more a product of faith than of rationality (Pais,1982). There are also occasions where the observer can have a good deal of influence over the occurance of future events. Typical of this is in the occurance of observer bias in psychological and sociological work. Another example is in the interaction between predictions and outcomes. Whilst a piece of metal expands under heat in a way that is independent of any predictions about its ultimate state, the same cannot be said in situations where the experimentee is aware of the experimentor's predictions.

Interaction between predictions and outcomes exist in the project selection problem. In many cases, the decision maker's prediction, say that a certain profit will be achieved, will be treated as a target. Similarly, predictions of low rates of personnel development can generate low expectation of personal progress. Complete interaction between prediction and outcome is termed a 'self-fulfilling prophesy' in which the prediction can be said to be the cause of the outcome. Some further discussion on this aspect is to be found in Skitmore (1981a).

The extent to which the outcome environment can be manipulated into its predicted state depends on the decision making processes of individuals in the outcome environment. Major inhibitors within this manipulation process include errors, unforseen and unpredictable events, lack of information and human factors (Cusack,1981) which are precisely the factors associated with the lack of fit between the prototype and the decision maker's model.

5.2.3 Conclusions

Lack of perfect knowledge opens up whole new areas of the project selection problem. As the decision maker can no longer be regarded to have direct access to the 'real world' he must attempt to create an internalised version from his perception of that world. The closeness to which the internalised model aligns with the real world, it is argued, determines the quality of the model and hence the quality of decisions. The degree of alignment is considered to be dependent on the contextually interdependent aspects of the modeller and the information received by the modeller.

Little is known of the abilities demanded of the modeller except that experience, training and, perhaps, some inate characteristics are beneficial. Informational requirements are, on the other hand, rather better known. Information directly relevant to the problem is, however, never complete. Some kinds of information are either too costly to obtain or simply unobtainable. These practicalities dictate the need for relatively inexpensive information concerning the entire decision environment. Further issues centre on the accuracy and usefulness of information and its relationship with the decision maker.

5.3 The Decision Environment

Bahrami (1981) considers the general 'contextual' environment of an organisation to consist of: economic factors, including inflation, interest and exchange rates; political factors, including the political ideology of the government and political developments on the international scale; social factors, including changes in life styles and values; technological factors, including the impact of new technology on specific industries; and legal and legislative factors, including the impact of government legislation on such matters as 'lead in petrol' on oil and motor vehicle companies (Bahrami,1981,p80). Smith (1985) actually reduces these five factors into four by combining legislative with political factors. These factors have been variously termed the organisation's "background environment" (Foster,1974) or the environment "outside the building process" (Tavistock Institute,1966).

Political and economic factors are, as Fellows et al (1982,p45) note, interdependent. A construction organisation at the local level is affected by the need to obtain planning approvals and also by the state of local public sector building programmes. Besides being an important client to the construction industry, the national government acts indirectly through legislation on safety, tax, noise, employment etc. Construction demand is closely related to the health of the economy at local, national and international levels. Economic trends in regional development, the treatment of urban decay, the regulation of the economy, the manipulation of interest rates, and national economic growth and decline are important factors (Fellows et al,1982). Foster (1974), however, considers that the effect of government policy and its economic consequences on the industry's output "would not be sufficient to disrupt a long range planning exercise", a rather surprising statement considering events taking place at this time.

Social factors, which include demographic movements, changes in education, working hours, housing, leisure, retirement, sports promotion and holiday patterns, are said to have an "enormous" impact on the decision environment (Fellows et al,1982,p46). Pressure groups, such as those concerned with conservation, have also become a particularly important social phenomenon.

The combination of economic, political and social factors has been identified by the Tavistock Institute (1966) with sources of uncertainty attributable to goverment departments, planning authorities, public bodies, client organisations and the general public. The same combination has been studied recently by Murray (1980) in respect of social values concluding that the type and stability of values are dependent upon the interrelationships between social structure, policy and culture.

Technological factors mainly affect the construction organisation's site or business systems. There are also technological implications in the demand for certain types of construction due to increased need for computer component manufacturing facilities for instance. Foster (1974) suggests that there are important informational implications caused by changes in technology, knowledge being needed in particular of the impact of new materials, new methods and processes, and working in new environments. Fryer (1985), however, maintains that "the construction industry does not have to cope with rapid technical change, but the market for buildings is changeable and unpredictable" (Fryer,1985,p14).

The implications of these major environmental factors on the market, or in this thesis' terminology, the project generating environment, are examined in more detail in the next section.

5.3.1 The Project Generating Environment

Successive governments of the post-war era have gradually changed in their policies from direct intervention towards demand management (Budd,1978). Direct intervention still exists in the construction industry in the form of public client organisations such as the Property Services Agency and regulative action through building, fire and safety regulations and planning legislation. Indirect measures occur in the form of grants and fiscal policies which affect the demand for construction, the building land market, capital money markets and industrial and competitive practise.

"Changes in government policy as a result of either internal or external matters or events can and do affect the demand for construction services" (Moore.1984,p21). The size of the government budget and changes in public expenditure policy involving cuts, reflation attempts and moratoria on cash limits, are particularly significant for the industry (Lansley et al,1979,pt1,p9). Foreign policies influence events on the international scene, in construction for military projects for instance. Entry to the European Economic Community and associated changes in taxation, legislation for equal opportunities and protection of employment are further important influences (Lansley et al,1979,pt1,p9).

Many important economic features are related to the health of the economy (Fellows et al,1982,p3). Balance of payments problems, sterling crises, high interest and mortgage rates, high inflation, rising unemployment and low economic growth being particular examples (Lansley et al,1979,pt1,p9). Economic factors are, in Clark & Lorenzoni's view (1985), of prime importance as the decision to build is "... the result of a favourable analysis of the marketing situation that shows future increased product demands or the need for new and different products or the research department may develop new products with high sales potential, or new government or social requirements may dictate the need for new facilities ... the ultimate reason, in virtually every case, is economic (Clarke & Lorenzoni,1985,p2).

Two particular instances of economic factors are given by Moore (1984). One is in the effect of the oil crisis, resulting in a concentration of construction activity in offshore projects, in terminal sites such as

Aberdeen, energy saving schemes, energy creating projects such as nuclear power stations, and projects involving new types of energy generation. The other example concerns the growth of container transport and the consequent need for projects involving port facilities, storage and roads.

Social factors influencing the demand for construction arise from the basic needs and aspirations of individuals and groups of people. In general terms these influences are connected with leisure, education, shelter, mobility and environmental concern (Moore,1984).

Constructional implications associated with leisure include such facilities as sports and recreational centres, swimming pools, squash courts etc.. The increasing popularity of dining out involves the construction of clubs, extensions to hotels and public houses, and car parks. Holiday activities create the demand for extensions and alterations to hotels, additional aiport buildings and runways, and docks and landing facilities for overseas travel.

Educational and domicilary building activity is affected by changes in population levels and the state of the existing building stock. A declining birthrate implies less demand for schools and teacher training colleges, for instance, whilst deteriorating housing involves the renovation or replacement of dwellings in the form of new housing estates together with associated amenities.

Increased mobility and the greater use of the motor car has created a need for the provision of more suburban facilities - supermarkets, department stores, and surface and multistorey car parks.

Environmental pressure groups have had a significant impact on the nature of construction projects, such as motorways, roads, airports, nuclear power stations, and waste disposal plants.

Science and technology, and also the products of research and development, have provided a further impact on the nature of demand for construction projects. Technologically based production facilities, such as those involved in the manufacture of computer hardware and other electronic equipment or those utilising such products (such as computer centres or robotic assembly lines) are typical examples.

5.3.2 Predicting Project Opportunities

Predicting project opportunities clearly involves obtaining knowledge of the frequency and characteristics of future projects. Such knowledge is, invariably, acquired through market intelligence. Market intelligence can be obtained directly from clients and architects, although this approach has its problems, for, as Jepson & Nicholson observe, "... Architects' offices react unfavourably to the visits of *salesmen*, and the levels at which interviews are usually negotiated with local authorities may be of more use to suppliers than to contractors" (Jepson & Nicholson,1972,p51,author's emphasis).

An alternative approach is to seek market intelligence indirectly. One such indirect method invoving records of approved planning applications has been found to be significantly associated with the volume of subsequent work (SEBI,c1965). Majid (1967) has studied "with some success" (Jepson & Nicholson,1972,p50) this use of planning applications in predicting housing trends in the North West of England. Difficulties exist, however, in that "Local Authority procedure is not uniform and once data are required for a locality extending across Local Authority boundaries, then standardisation of the format of applications and centralised processing of a periodic summarised return from authorities becomes necessary" (Jepson & Nicholson,1972,p50). A further problem is that not all relevant information on project characteristics is available. There are, however, some commercial organisations who exist to provide this type of information to contractors.

The more long term changes in the project generating environment can be predicted by assessing trends in the demand for construction. Such assessments depend on knowledge of future changes in the economic, political, sociological and technological factors affecting demand. Political trends may be observed from statements made by influential politicians (see, for instance, Freeson,1977), Acts of Parliament, white and green papers etc. Economic trends may be revealed by economic indicators including population statistics, indicators of regional prosperity, industrial structure, investment etc. (see, for instance, Lansley et al,1979,pt2,p22). Demographic analysis of census statistics have been used to predict changes in housing demand, for example (Parry-Lewis,1968).

Many of the informational sources used for evaluating aspects of the market are detailed by Harris & McCaffer (1983,p182-188), including such periodicals as 'The Economist', 'National Institute Economic Review'; government publications such as the 'Monthly Bulletin of Construction Statistics', 'Housing Statistics', 'Sample Census Statistics', 'Regional Economic Reports' 'Local Planning Reports' and 'Monthly Digest of Statistics'; and other publications such as 'BMP Weekly Publication' and 'Construction Trends'.

The extent to which the organisation attempts to predict the action of the project generating environment is largely dependent on its marketing policies. Rajab's survey (1981) suggests that the most attention is paid to externally compiled statistics (100%), external market researchers (33%), trend projections (14%), consumer surveys (11%) and economic modelling (11%), commercial intelligence being preferred to government reports (Rajab,1981,p164). Most forecasts and information on the external environment were found to be obtained on a regional basis by lower management (Rajab,1981,p163).

The general level of accuracy of predictions does not seem to be very good, particularly in current economic conditions. "The hard facts of a collapsing market, growing unemployment, high interest rates, and the dismantling of impressive building concerns have taught us that our forecasts of the early seventies were far from accurate ... changes in social, political and economic factors have been so rapid in recent years that forecasting on the basis of extrapolation of existing trends is now far from reliable" (Benes & Diepeveen,1985,p27). Causes of demand fluctuations have often been attributed to government measures to alter the level of activity in construction (eg. public work moratoria) or the whole economy through construction (by public expenditure cuts) (Campbell et al,1974). More recently, however, there has been some evidence to suggest that the government has tried to stabilise demands on the industry by planning its expenditure and operating a more effective system of monitoring and controlling local authority expenditure (Cannon,1978,p13). Whether the government has been successful in this is not clear for, although it is suggested that there are "very few examples of turbulent environments in construction" (Harding,1985,p220) and the construction environment only moderately uncertain (Brown,1974), Diepeveen (1985) is still adamant that public sector demand is very unstable despite efforts to create stability. One possibility is that the environmental changes occuring are of a more fundamental nature outside governmental control, for as Lansley et al (1979) observe "... the 1970's have witnessed a series of probably fundamental changes both in the determinants and in the structure of demand more sporadic and unpredictable than have hitherto been experienced since the war" (Lansley et al,1979,introduction).

5.3.3 Predictions in the Outcome Environment

The bulk of the literature dealing with predictions in the outcome environment is concerned with monetary aspects and particularly predictions of expenditure by the contractor, usually termed the cost estimate. Moyles (1978), for instance, has considered in some detail the factors affecting the accuracy of this cost estimate under the groupings labour, materials, plant, sub-contractors, overheads and profit. Fellows et al (1982) use similar groupings as a means of identifying fixed, variable and semi-variable costs (in the economic sense) together with direct (project related) and indirect costs.

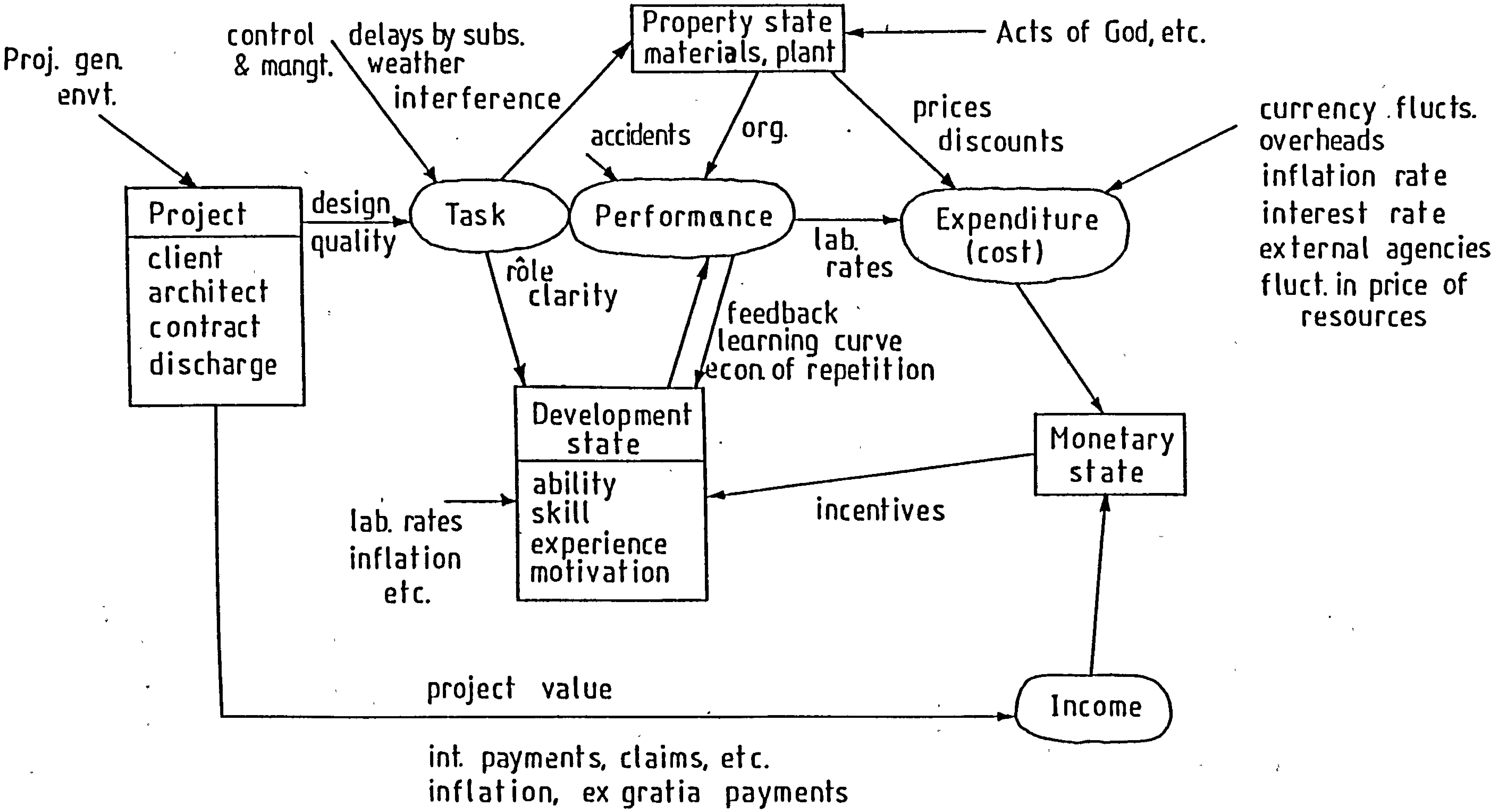
Fig 5.3 indicates the basic relationship of the factors in the outcome environment proposed in this thesis, which is essentially concerned with project characteristics and their influence on people, money and property. The central issue in this model in determining costs is that of productivity or performance which, together with the nature of the tasks generated by the project, largely determine expenditure.

5.3.4 The Influence of Project Characteristics

The project characteristics are seen as influencing the tasks to be done and the income to be received by the participants in the construction process.

The information available for predicting the extent and nature of the tasks is contained formally in tender documents in the competitive situation. These, in the UK, may comprise the drawings and specification and/or bills of quantities. The accuracy of task predictions has been found to be affected by errors and omissions in the drawings (Ormerod,1984) or by misinterpretation of contract

Fig 5.3 Relationship of factors in the outcome environment



requirements (Gates,1971) over, for instance, the suitability of "equal substitutes" (Gates,1971) or "quality of work" (Moyles,1973,p51; Bennett & Ormerod,1984). Mistakes occur through computational errors (Langford & Wong,1979) and omission and commission (Gates,1971). Quantity errors are of particular concern (Langford & Wong,1979) resulting in undermeasurement or omission of items (Park,1966). The generality of quantity descriptions also provides a source of error, the extent of which has been said to be dependent on the level of measurement (Bennett & Barnes,1979). It is possible, however, that misestimates in quantities can sometimes be anticipated (Stark,1976). Many criticisms have been made of the lack of association between quantity items and the nature of the construction task. Task oriented quantities have been proposed to improve predictability (Flanagan,1980 and Thompson,1981, for instance).

Further project associated information regarding tasks can be obtained by site visits and enquiries to the client or his advisors. The kind of relationship with the client and his delegates has a bearing on this and future events (Cauwelaert & Heynig,1978).

Difficulties occur due to "unknown work" (Bennett & Ormerod,1984), which in the post-contract period can be a result of design changes, which are mostly attributable to client requests (Diekmann & Nelson,1985), involving "major scope changes" (Clark & Lorenzoni,1985). Delays caused by the client and architect are a further factor (Langford & Wong,1979) which can, in some cases, develop into delays of a more permanent nature, such as financial failure of the client (Langford & Wong,1979), resulting in the early discharge of the contract.

Many of the project factors which influence tasks also have implications in predicting income. The main determinant of income is the price offered by the contractor at the tender stage, this price being determined by the project selection decision, one consideration of which is the predicted monetary state caused by the amount of expenditure incurred. The heavy emphasis usually placed on monetary aspects, together with the difficulties involved in predicting the market value of the 'customised' construction poject, often results in a situation where predicted price and cost are closely related. The construction contractual arrangements are, in many respects, aimed at monitoring this relationship by providing some expenditure related income adjustments. These are mainly project based in covering such events as errors and ommissions in drawings, specification and quantities, and client interference in the form of design chages. Environmental changes can also be accomodated in reimbursements for inflation and other unforseen events. Income fluctuations are, however, seldom exactly the same as fluctuations in expenditure as some form of surrogate measure is often employed. Design changes, for instances, are usually valued by quantity related measures rather than by an exact record of expense incurred. Time related aspects are also important as, in construction contracts, income is received some time after the expenditure has taken place. These cash flow aspects can be exacerbated by delays in client payments, particularly where outstnding claims are involved.

5.3.5 Tasks and Performance

The ways in which tasks are defined and executed are attributable to a combination of project characteristics and other events of differing degrees of predictability. The weather, especially in the early stages of the construction process, is clearly an important factor, as is the extent and quality of management and control (Gates,1971; Duff,1976). Delays in obtaining management approvals is a source of variability (Bennett & Ormerod,1984). Adequate advance planning by means of network analysis, for instance, can improve predictability but there is no evidence of its widespread use in the construction process (Cusack,1981). The complex and dynamic arrangement of tasks on a construction site is, as demonstrated by Bennett & Fine's (1980) simulation studies, an important cause of variability.

Performance is influenced by the task and its associated working conditions (Manson,1985), the state of materials and plant resources and the development and aspirational state of the operations. The degree of difficulty presented by the task is difficult to predict due to the variability in tasks and condition between projects. As Moyles observes "... a complete workforce is seldom transferred from job to job so whilst a competent site agent may obtain favourable results in one situation, there is no guarantee that this programme will be repeated in another" (Moyles,1973,p46). Predicting learning curves or the economies of repetition for similar reasons is also problematic (Foster,1974). Performance is linked to incentives (Moyles,1973,p49) although "... the cost of the work often remains the same" (Moyles,1973,p49), the saving in time and the reduction of people utilised, however, can affect expenditure by reducing on-costs and overheads. Labour outputs can also vary from area to area as some parts of the country have "... traditionally poor output and strong union militancy" (Moyles,1973,p49), although these are "... fairly well recognised by local and national contractors with experience of working in those areas" (Moyles,1973,p50). Labour unrest (Gates,1971), disputes (Barnes,1974) and strikes (Clark & Lorenzoni,1985) are particularly sensitive factors in performance predictions.

Fryer (1985) intimates several personal factors - personality, experience, motivation, ability and skill, and stress to be important determinants of human performance. Other factors include role clarity and feedback in addition to task demands. Feedback, in general, has not been thought to be adequate in construction (Flanagan,1980), although measurement techniques of work study and activity sampling do exist.

Efficient working is also dependent on the requisite materials being delivered at the right time and in the desired place (Moyles,1973,p50). The use of power tools and other items of plant has a major impact on productivity (Niss,1965; Cusack,1981). The occurance of illness and accidents are further factors (Clark & Lorenzoni,1985).

Many other factors are involved in determining performance levels, a complete review of which is beyond the scope of this thesis. It is clear, however, from this brief review, that performance rates are "highly variable" (Bennett & Ormerod,1984) and their prediction is probably the most uncertain aspect of the entire project selection problem.

5.3.6 Outcome States

There is very little reference in the constuction literature to the prediction of human development. Such literature as does exist is concerned primarily with human development as a means to further improve the company's monetary state, via improved efficiency and productivitiy for instance. Human development in such cases may be measured in terms of responsibility, position in the organisational hierachy, remuneration, courses attended and qualifications acquired. A more indirect measure of human development (but more directly relevant to the monetary state), particularly at the operative level, is productivity, which is, to some extent, a reflection of the individual's abilities and skills, experience and motivation. Predicting human development *per se*, however, may involve the assessment of other factors more directly associated with well-being. The only such measures that appear to be available at the moment seem to be evaluated intuitively, in that "... the workforce is reasonably content", or by the frequency of disputes, which, as has been noted earlier, may be regarded by some individuals and groups to be aspirational events in themselves. The only other alternative that seems to available lies in the use of questionnaires, although this approach has currently been restricted to research activities.

Predictions of property states are, in the project context, normally confined to changes in the materials and plant levels. Various control techniques are available to influence the level of stocks and which are applied with varying degrees of success. Plant and particularly materials supply has been shown to be virtually perfectly elastic (Burton,1972), although "... delays do occur resulting in variabilities caused by late deliveries" (Bennett & Ormerod,1984; South,1979), mechanical breakdown and malfunction of equipment are a further source of variability (Bennett & Ormerod,1984).

The price of labour and materials and plant combine to impact on monetary states. Material substitutes can affect the accuracy of predictions, for "... a careful search of the market alternatives may yield better goods at competitive prices" (Harris & McCaffer,1983,p181). The acceptability of such substitutes to the client, however, is not necessarily assured, though, and is in itself a further cause of prediction error (Gates,1971). The fluctuating prices of materials is a cause for concern (Gates,1971; Case,1972), although trends can be predicted to some extent (Cauwelaert & Heynig,1978). Fluctuations in discount levels are a further difficulty.

The effect of labour costs on monetary levels is dependent on productivity and rates of pay. Rates of pay can fluctuate for many reasons, (Caulewaert & Heynig,1978), including changes in government legislation, union agreements, responsibility and incentive schemes. Labour related expenses also exist in the form of overtime payments, supervision costs, national insurance contribution etc.

Further fluctuating expenses stem from changing interest rates (Barnes,1974), inflation (Bennett & Ormerod,1984) and currency rates (Clark & Lorenzoni,1985). Prediction errors can also occur in the failure to allow a sufficient amount for overheads (Park,1966).

5.3.7 The Effect of Sub-Contracting

The incidence of sub-contracting has greatly increased in the post-war era "... probably due to the complexity of modern buildings, labour shortage, structuring of large firms and taxation policy" (Moyles,1973,p59). There are exceptions, however, as some companies are "... actively pursuing closer links with sub-contractors through acquisition" (Lansley et al,1979,pt1,p59). These changes are seen, by Lansley et al, as an indication of the companies' preference "... to increase control over their operational environment through 'internalising' the activities upon which they are dependent or by attempting to reduce uncertainty by externalising parts of their business" (Lansley et al,1979,pt1,p59).

The extent to which control is lessened is largely dependent on the relationship between the company and the sub-contractor. The continued use of a sub-contractor should be beneficial in this respect, although this is not always possible due to contract requirements (nominated sub-contracts, for instance) or financial considerations. The degree of uncertainty is similarly dependent on familiarity with the sub-contractor. An important source of uncertainty occurs when the contractor has to estimate sub-contract prices instead of obtaining quotations.

Delays by sub-contractors have been found to be a particular cause of prediction error (Langford & Wong,1979).

5.4 The Prediction Process

The very need to predict future environmental states generates processes which themselves affect the decision environment. The degree of accuracy of predictions, dependent on the contextual relationship between the predictor and his information, largely determines the level of effect.

Predicting movements in the project generating environment is usually regarded as a marketing function. Little is known of the personnel and monetary consequences of this relatively recent activity in construction companies.

The functional process of predicting changes in the outcome environment, especially the monetary aspects, is better documented. Predicting future project related costs is normally performed by the estimator or estimating department. The size of the estimating department is often related to the company's turnover (Humphreys,1977), the amount of resources needed being determined by project characteristics such as the type and size of project, its location and the time available for tendering, together with the estimator's expertise and information. The time available for tendering has been found to be particularly important (Cusack,1981). The frequency of estimates appears to be season related, Humphreys' (1977) study noting a peak period between February and March during which time a total of 14 to 16 estimates were being compiled simultaneously.

The direct effect of the estimating process on monetary states has been investigated by several researchers. Broemser's (1968) analysis of one construction company found the cost of estimating to represent 9.1% of total assets and 1.8% of total receipts, this figure being equivalent to 0.18% of the value of each project estimated, as only 10% of the estimated projects were actually obtained. Park (1966) places this figure higher at between 0.5% and 2.0% of project value, offering a "... rule of thumb used by some contractors on large projects" to be "... total estimating cost = 0.005 x estimated direct materials costs + 0.015 x estimated direct labour costs" (Park,1966), equivalent to 1% on a $1m contract. Rubey & Milner (1966) suggest that, for a "good bid", estimating costs would be "perhaps 1% of the total bid". Harrison (1981) is of the view that estimating costs vary quite widely between

0.1% and 10% of project value, depending on the degree of repetition involved and the experience of the estimator.

Larew (1976) has used a multiple regression analysis technique to identify possible causal associations in the time spent by one contractor in estimating 22 project costs in the early 1960's. The results of this analysis suggest that high activity may be caused by the presence of many speciality items: excessively detailed specifications, and reference to exotic standards; high quality finish; the insistence that contractors satisfy owners' every desire; and where the contractor seems to be held responsible for the errors and omissions of the designers. Medium levels of activity are associated with some specialities, but not to an excess; reasonable and understandable contract documents; the requirements for a good and workmanlike finish; the contractor being responsible only for work shown on plans and in the specifications; where the designer accepts responsibility for the contract documents; and where the contract assures a fair, prompt and impartial mediation of disputes. Low activity is associated with very few specialities; abbreviated specification; open structures with low quality of finish; simplicity in every respect; and relatively straight-forward production work is needed. Estimating time was also shown to be less for "in town" projects and more for "out of town" projects, due to the necessity for the estimator to visit the site. Niss (1976) has also obtained evidence that estimating costs are a logarithmic function of project value.

All of the research indicated above, however, was conducted prior to the introduction of computer aids. Since that time, several inexpensive estimating systems have become available on the construction market and have attracted some considerable interest. One such system is known to be installed in over 1,000 locations at present (Hunt,1986). The effect of this on project estimating costs is likely to be quite dramatic.

5.4.1 Accuracy of Predictions

As has been previously discussed, the accuracy of predictions relating to the project generating environment is not thought to be generally very good. There are no specific figures available in the literature concerning accuracy levels achieved and some research in this direction, particularly in relation to project characteristics, the abilities of the predictor and information used would be advantageous.

Accuracy in predicting the outcome environment is, however, known to be rather poor. Estimating construction costs is probably the most researched area and the process has been found in practise very "approximate and crude" (Benjamin,1979) relying on "haphazard" methods often "grossly in error" (Neil,1978). Ashworth & Skitmore (1983) have examined estimating accuracy in some detail, considering the various measures available and noting the extensive use of subjective judgement involved. Their findings suggest that the extent of complexities and uncertainties in the process results in accuracy being determined more by the ability of the predictor than the project information avaialble. In view of the facility to 'control' work to some extent in the post prediction period, it is reasonable to assume that the cost estimate can often be considered to be self-fulfilling to some extent, in which case only a reasonable figure is needed. There is, however, a bias in the process due to the need to avoid excessively low estimates of expenditure. Such low estimates are avoided by including "contingency allowances" (Harrison,1981) or "risk premiums" (Portsmouth Polytechnic,1974; Barnes,1974) to cover possible errors, particularly those caused by uncertainty or risk. This can result in uncertain cost being "padded two or more fold" (Case,1972). Clearly there are limits to this procedure as increasing estimates of expenditure in this way will result in a totally false impression of the predicted monetary state. A further difficulty is that the predicted price of the project, if based on expenditure predictions formulated in this way, may well be in excess of the price the client wishes to pay. One way of avoiding this is to "... exclude any allowance for such unpredictable events as strikes, bad weather, major scope changes, acts of God, currency fluctuations etc from the estimate" and any other item with less than a 50/50 chance of occurance, and making full allowance for other events (Clark & Lorenzoni,1985,p117).

An alternative approach is to consider the probability of occurance of an event and the cost associated with its occurance, the product of which is the "expectation value" to be incorporated into that estimate (Gates,1971,p277). The suitability of this approach depends on the risk attitude of the decision-maker who may or may not be happy with this averaging technique. Case (1972) and Stacey (1979) have both proposed a method of deriving a probability distibution of cost estimates from an indication of the most likely lower and upper bounds of the predicted cost of individual items. Such a technique is anticipated to accomodate the preferences of decision-makers to varying risk attitudes.

5.5 Selection Strategies and the Non- Deterministic Model

Kmietowicz & Pearman (1981,p105) have considered three models for decision-making with multiple criteria. These models cover situations of uncertainty, risk and "incomplete knowledge" (where criteria can be ranked only).

Although they acknowledge that it may ultimately be possible to develop a strategy for handling simultaneously the twin problems of uncertainty and multiple criteria, "current practice is nowhere near this point".

Four approaches encountered in the construction literature are concerned exclusively with the second model where a cardinal scale is used to assess the relative importance of different criteria. The first two of these approaches, by Hillebrandt (1974,ch13) and Benjamin (1969) attempts to evaluate preferences directly from the decision-maker, based on Shackle's (1952) degree of potential surprise in the former and utility theory in the latter. The two other approaches, by Fellows & Langford (1980) and Ibbs & Crandall (1982) use a multi-attribute utility function.

5.5.1 Study 1 (Hillebrandt,1974,ch13)

Hillebrandt's approach is to first construct an index of potential surprise of a specified profit/loss outcome for a specified bid value. The next step is to identify the profit and loss outcome on the potential surprise index which will generate the greatest stimulus to the decision-maker. These outcomes, called standardised focus gain and standardised focus loss are then evaluated on a gambler indifference map to ascertain the ultimate focus gain. Ultimate focus gains are found for all possible bid values and the bid value associated with the maximum ultimate focus gain is adjudged to be the best decision.

A further development proposed by Hillebrandt is to allow for competitive aspects of the problem by plotting the ultimate focus gains and the degree of potential surprise of obtaining the contract associated with each bid value on a bidding indifference map.

Although seemingly complicated, Hillebrandt's approach is simply a formalised intuitive procedure. In accomodating a whole range of subjective judgements, the approach represents an admirable attempt to avoid many of the criticisms normally levelled at bidding decision aids. The benefits of this form of analysis are suggested by Hillebrandt as being "... helpful for understanding the reasons for decisions on tender prices; in locating the reasons for difference of opinion between persons sharing the entrepreneurial function within contracting firms; in assessing how the bidding theory based on the probability approach can help in tendering decisions; and hence, altogether, in making the process of tender decisions more logical and efficient" (Hillebrandt,1974,p179).

5.5.2 Study 2 (Benjamin,1969)

Benjamin identifies three aspects of the selection strategy problem (a) a probability distribution to express the relationship between the cost estimate and the actual monetary cost of performing the work, (b) a non-linear utility function which scales the decision-maker's preference for different amounts of money and (c) a means of assessing the probability of obtaining the project with different bid amounts.

In deriving the probability distribution of the cost estimate, it is proposed that such factors as sub-contractors' bids, materials availability and costs, labour availability and productivity, methods of performing the work, season in which the work is done, location of job, type of building or type of construction and supervisory capacity are taken into account.

The utility function "... transforms the monetary value of the outcome to a different scale which satisfies the decision- maker's ordering of preferences for different amounts of profit or loss with a given bid". Further work by Willenbrock (1972) has developed the work into a sophisticated technique.

In estimating the probability of obtaining the project the characteristics of the project and the "bidding environment" are considered.

5.5.3 Study 3 (Fellows & Langford,1980)

The value on the five criteria speed, accuracy, cost, client and benefits are assessed by the decision-maker. The aggregated outcome evaluations, together with the probability of each outcome are then analysed by means of a decision tree. Sensitivity analysis is recommended to assist in identifying the best decision route.

5.5.4 Study 4 (Ibbs & Crandall,1982)

The values of the three criteria profit return, project size and "regulatory aspects" are assumed to be derived by "conventional procedures".

Estimating scalar values for the preference function is suggested to be an often "imprecise task". For a decision problem such as the search for new business markets, the American authors recommend sources such as 'Dun & Bradstreet' statistics as providing some relative indication of possible expected profit margins. Various government agencies and owner's representatives may supply future bidding volume and project size information. In the final analysis, though, the authors maintain that it is the "... decision-maker who, with the assistance of the decision-analyst, must make these estimates".

In their example, the mean estimated scalar value and the estimated standard deviation about that mean is assumed. A sensitivity analysis of the decision alternative is then conducted by a Monte Carlo simulation procedure for one hundred iterations.

5.5.5 Conclusions

Of the four selected published accounts of non-deterministic approaches to the construction project selection problem with several criteria, the first, by Hillebrandt, relies entirely on the decision maker's judgement.

Benjamin's approach estimates the monetary effect of a decision based on some knowledge of the probability distributions of estimated/actual costs and 'our' bid/competitors' bids. These probability distributions are suggested to be obtained as far as possible by objective analysis. The estimated value of the preference function is derived from some non-linear function of the estimated monetary value of the project.

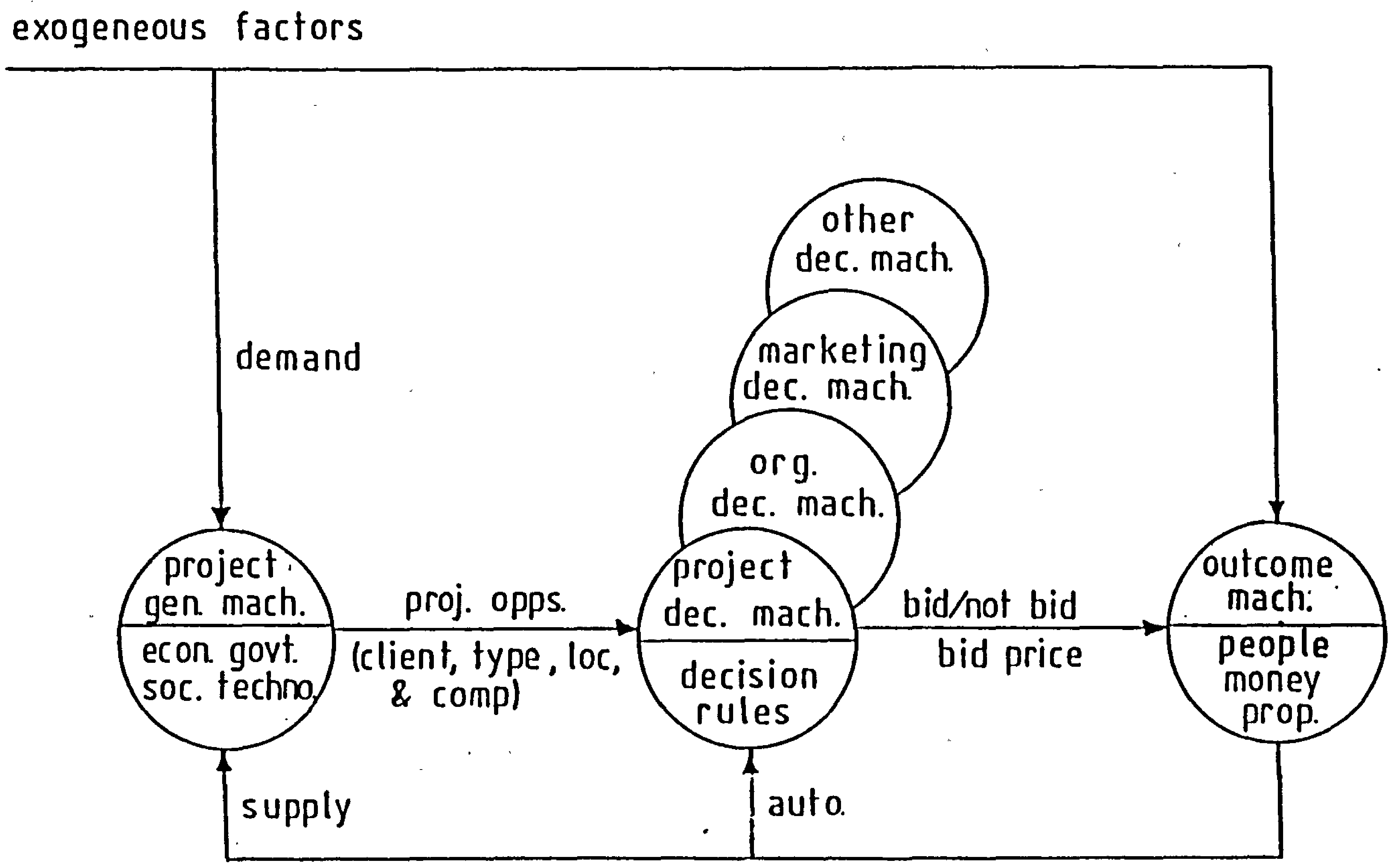
Fellows & Langford and Ibbs & Crandall both estimate values of individual criteria by relying mainly on the decision maker's judgement. Sensitivity tests are recommended to test the reliability of outcome evaluations.

The additional difficulties associated with the multiple criteria project selection problem in the non-deterministic model are in the reliable assessment of criteria values. Whilst an assessment by empirical means would seem desirable, the frequent recourse to the decision-maker's judgement in these assessments prompts the view expressed by Hillebrandt (1974,p184) that "... most of the factors are so varied and qualitative that it seems better to use judgement for quite wide groups of variables together".

5.6 Dynamical Aspects

The effects of imperfect knowledge are essentially two-fold. One factor is that uncertainties in the immediate decision environment introduce the necessity to consider more indirect influences, particularly in the project generating environment. These indirect influences also result in considerations of other aspects including organisation and control together with marketing decisions. In terms of the sequential machine approach discussed in Chapter 4, this would suggest a revised model along the lines of Fig 5.4 Here the project decision machine envisaged as being one of several such decision machines including an organisation and marketing decision machine all

Fig 5.4 The project decision system environment



causally related to the project generating machine and the outcome machine. The project generating machine consists of a set of four internal 'states', economic, governmental, societal and technological, which are determined by exogeneous variables such as government ministerial policy decisions, changing international economic conditions, shifts in societal attitudes and general technological developments, generally influencing the demand for construction work. Other determinants of the project generating machine states emanate endogenously from the outcome machine in what may loosly be termed as marketing inputs. The outputs of the project generating machine determine the frequency and characteristics of project opportunities for input into the project decision machine. The project decision machine output in the non-deterministic situation consists not only of the dichotomous project selection variable, but also of other features which may determine project acquisition, such as the price and contract duration offered. The result of the project decision, together with such exogenous factors as societal aspirations, interest and inflation rates, forms an input into the outcome machine affecting its three states of people, money and property. The connection between the outcome machine and the project generating machine forms the dynamical link in the system and the control link (shown dotted) suggests a means by which the decision machine can be automated to respond in a rational manner to changes in outcome states.

The second factor turns on the predictive difficulties associated with the system components. The five general characteristics of each machine, inputs, outputs, internal states, state transition and output functions, have some degree of unpredictability, usually extremely high. The way in which environmental changes affect project opportunities is largely unpredictable and little appears to be known of the effect of project characteristics on the state of personal wellbeing. Perhaps the most knowledge that is available concerns the effect of project characteristics on monetary outcomes, although, as discussed in this Chapter, there is still a considerable degree of uncertainty involved. These predictive difficulties are further exacerbated by the need to forecast future events, preferably over a period of several years.

The situation is, however, not entirely hopeless. One of the great contributions of J. von Neumann (1969) is to have proved the fact that a predictable system can be built from unpredicatable parts. An outstanding example of this is Bennett & Fine's (1980) construction project simulator (CPS) which, by repeated simulation of a stochastic construction process model containing many highly varying elements, generates fairly stable probability distributions of project cost and duration. The CPS typifies what Gottinger (1983) has termed dynamic systems of "intermediate complexity". Gottinger outlines four points on which he would like to see computer models developed in order to cope successfully with systems of "intermediate complexity". Firstly, the model should be aimed at achieving improvements rather than optimality. Secondly, sensitivity analysis should be preferred to formal statistical hypotheses testing. Thirdly, the computer model should consist of an interaction between human beings and machines, And, finally, the system should be integrated as far as possible with other similar such systems.

These 'points' coincide well with the work described in the last section in prescribing a judgement-related system with a facility to observe the effect of errors by means of sensitivity analyses. The improvement-related approach has already been proposed in Chapter 2, Fig. 2.1, where options are evaluated consecutively for improvements.

Relationships with other systems has already been considered, and these other systems have been introduced in Fig. 5.4 as other decision machines, the organisational and marketing machines being seen as particularly relevant. Computer simulation, an essential ingredient in systems of intermediate complexity, also aligns with the need to anticipate changes in the environ ment in contingency planning or other 'scenario' type approaches to flexible management.

CHAPTER 6

The project decision system

6 THE PROJECT DECISION SYSTEM

6.1 Introduction

This chapter completes the specification of the project decision system by defining the various options available to the decision-maker. Some means are discussed whereby the system can be utilised in the search for decisions which will best enhance the outcome environment. Further references are made to the construction literature to determine possible appropriate search methodologies by means of decision rules and strategies. The chapter concludes with a final proposal for a project decision system with an indication of some devices necessary for its practical operation.

6.2 Decision Options

The decision options considered in this section are restricted to the set of outputs associated with the project decision machine shown in Fig 5.4. Options that are associated directly with organisational and control decisions, marketing decisions etc are outside the scope of this thesis. These other decisions, however, interact with the project decision and references will be made where the interaction occurs.

Ansoff (1965) recommends five possible alternative courses of action in project screening: reject the project; provisionally accept the project; add it to the reserve list of approved projects; remove a project from the reserve list and replace it with the same project; and remove an active project, discontinue it and provisionally accept the present project (Ansoff,1965,p183). As contractual issues prevent the practical implementation of any of these alternatives once the construction organisation is formally employed by the client, the relevance of these options will be restricted to the pre-contract period and, in particular, the tendering (bidding) period.

The limited amount of time available for bidding limits the use of reserve lists to a minimum, the usual options being restricted to acceptance or rejection of the project. Harris & McCaffer (1983) suggest that this decision to bid must be made at three possible points in the bidding process: during the pre-selection stage, if a pre-selection process is being used; after receiving all the contract documents; and after the cost estimate has been prepared.

The decision that the project is not required generates two options, not to bid or to bid with a 'cover' price. The decision that the project is required generates several options , including alternative price levels and what Simmonds (1968) terms "non-price features" such as quality, contract duraction and design facilities (Clark & Lorenzoni,1985). The break-down of price also creates options for front loading (South,1979; Diekmann et al,1982) to enhance cash flow. Further options are theoretically available in the combinations of price, non-price features and loadings.

6.3 Selection of 'Best' Options

The basic decision model proposed in Fig. 2.1 involves the sequential identification, evaluation and comparison of decision options as a means of locating the 'best' option. In terms of the project decision system, this would entail the evaluation of various bid levels, non-price features and loading arrangements, and all combinations of these, for a sequence of project opportunities over a suitable period (about 1½ years). The uncertainties in the system environment imply that the frequency and nature of project opportunities, changes in the outcome environment and exogenous variables together with their interrelationships are not known very well.

One approach to finding the best set of options in these circumstances would be to evaluate every conceivable combination of options and events. There are, however, clearly extremely severe computational difficulties in this approach. An alternative procedure is to confine the process in some way to a reduced set of alternatives. This approach effectively introduces a further decision into the system ie. which option or set of options to evaluate next. This decision is termed here the option identification decision (OID) and a procedure for determining the option identification decision is termed an option identification decision rule (OIDR).

The literature reveals many (non-random) OIDR's, often of a qualitative nature, that are applied by construction project decision makers. In almost every case the options considered are whether the project is wanted and, if so, the level at which to bid. The next section examines the nature of the OIDR's commonly employed.

6.4 Option Identification Decision Rules

The magnitude of a decision, it has been said, "... will normally be judged in terms of resource commitment its implementation will require and the risk factors associated with this commitment in relation to the expected outcome" (Cusack,1981,p24). The substantial effect of project decisions on resources together with the associated high levels of risk and uncertainty known to exist, indicate that the project decision is of paramount importance. The guidelines for making such decisions are determined by the organisation's business policy. Business policy statements are of the type "When faced with a situation of the type X, always choose course of action A, rather than B or C ..." (Kempner,1971,p62), and as such would seem to be eminently suitable potential OIDR's. The rather subjective manner in which policies are formulated "... as a result of moral, political, aesthetic or personal considerations rather than as a result of logical and scientific analysis" (Kempner,1971,p63) and their influence in the project evaluation and selection process in determining the choice amongst multiple criteria strengthens this view.

Policies are normally devised to coincide with the Managing Director's objectives (Lansley et al,1979,Appendix E,p11-13), the board of directors and senior management (Cusack,1981,p39) or in consultation within a group of companies (South,1979).

Business strategies appear to perform a similar role to policies. Strategies have been defined as "broad policies" (Bahrami,1981,vol2,p5) describing "patterns of decisions" (Andrews,1980) resulting from or enabling the "continuous process of making entrepreneurial decisions" (Drucker,1959). The distinction between strategies and policies, on this basis, is not very clear and, indeed, some "considerable confusion" is known to exist (Kempner,1971,p63). It has been said that "... policy decisions refer more often to the character and nature that the company wishes to adopt, while strategy refers to the means to be employed in bringing about these desired characteristics" (Kempner,1971,p63). This suggests that, in terms of the project decision system, the view on what constitutes the most desirable states in the outcome machine is determined by policy whilst the internal mechanics of the decision machine which generate these states is determined by strategy. In other words, the OIDR's are determined by strategy and the 'best' set of OIDR's are determined by an 'optimal' strategy.

The high level of uncertainty surrounding the project decision problem has an important effect on strategies and policies (Frazer,1981), mainly due to prediction difficulties (Benes & Diepeveen,1985). Uncertainties can occur exogenously by changes in the levels of demand, for instance, and endogenously by the results of the decision mechanism. The combination of these situations is considered here in terms of high/low risk exogenous factors and strategies.

6.5 Low Risk Exogenous Factors and Strategies

Harding (1985) has identified two types of low risk exogenous sets of factors, the placid, randomised environment and the placid clustered environment. Low risk strategies in the placid randomised environment are essentially "stick with the knitting" (Peters & Waterman,1982) in terms of project characteristics with attention mainly focused on the outcome environment especially in terms of production (Lansley et al,1981) and backward integration (Moss,1981). Typical objectives in these situations are to "... achieve target profits and monetary return" (Niss,1965) and limited or selected growth (Fellows et al,1983; Barnard,1981; Porter,1980,pxvii) with some preference for growth (Fellows et al,1983). The emphasis is on the correspondence between projects and resources, which Moss terms the focusing effect (Moss,1981) resulting in responses to project opportunities limited and influenced by the size of the company (Jarman,1978; Lansley et al,1979,pt3,p2). This simple response mechanism, termed "operational" (Lansley et al,1979,pt3,p2) or "tactical" (Harding,1975,p216) can influence the type of work chosen (South,1979), although smaller builders appear to be less affected (Jarman,1978).

The focusing effect has been observed in most construction organisation strategies. South's examination of 23 construction companies, between 1970 and 1976, found "little effort to change markets" except from public to private housebuilding (South,1979). Most companies had limits on work types, geographical range from head office and value ranges of work. In comparing differing sized construction companies, Jarman concluded that large companies "... do not tackle business ventures with little knowledge or expertise" (Jarman,1978,p171). Medium sized companies "... usually concentrated in one area, both spatially and by product" (Jarman,1978,p165). Small companies specialised in maintenance and repairs which the large companies generally avoided unless as a rolling programme. House builders, on the other hand, could do "... any type of job within reason, local authority housing ... small industrial units ... being specified alternatives" (Jarman,1978,p163).

Outcome oriented strategies are essentially concerned with the need for work reflected in the level of utilisation of resources. Low risk strategies recognise limitations in capability and capacity, concentrating on the availability of resources by means of capability profiles (Fellows et al,1983,p18) for instance. Project work load has been found to be the single most important factor in determining the bidding decision (Mannerings,1970) affecting the organisation's ability to tender and resulting in the decision to return a tender, reduce margins or be more selective in projects (South,1979).

The availability of personnel has been found to be a major factor affecting the bidding decision (Niss,1965), one reason being due to the policy of keeping the work force together (Niss,1965). Plant and skilled operative constraints are said to exist (Cusack,1981), craft shortages occuring because of redeployment difficulties caused by barriers to entry between crafts (Burton,1972). Utilisation of equipment is one objective of construction companies (Niss,1965) and materials availability has been found to affect the bidding decision as it can influence the contract duration and thus the level of project overheads (South,1979). Low risk monetary oriented strategies include minimising capital usage (Rajab,1981) or utilising available capital sufficiently to avoid financial over-extension (Park,1971,p24.4), an essential constraint being that the project must generate sufficient profit to cover overheads (Humphreys,1977).

Harding's second type of low risk exogenous factors, the placid clustered environment, implies the existence of some predictable movements in the project generating environment which can be exploited by the organisation. Such identifiable trends can lead to low risk demand related pricing strategies (South,1979) based on forecasts of future project opportunities and levels of competition (South,1979). Medium sized organisations for instance, being more susceptible to down-turns in demand (Jarman,1978), may respond by increasing specialisation (Jarman,1978) or competitive advantage. A more coherent market oriented strategy can be adopted to enhance company image and reputation and, to some extent, influence demand by adopting a passive attitude on claims (Harris & McCaffer,1983,p181), co-operating with clients in improving the design and contract (Harris & McCaffer,1983,p182) and generally increasing client satisfaction (Rajab,1981; Peters & Waterman,1982).

Low risk responses to change in demand imply that output trends will be a delayed and smoothed version of demand trends. Evidence of this lagged and damped effect (South,1979) suggest that low risk exogenous factors and strategies typify the construction industry.

6.6 High Risk Exogenous Factors and Strategies

High risk exogenous factors are associated with Harding's (1985) disturbed reactive environment and Ansoff's (1979) turbulent fields. Turbulent fields imply a gross increase in relative uncertainty where effects are amplified and become unpredictable; where there is little relationship between a decision, its resulting effects and the next decision to be made; and where the future appears to be disjointed and discontinuous (Harding,1985,p220). There are, fortunately, few examples of turbulent environments in the practice of construction (Harding,1985,p220), the construction environment being found to be "moderately uncertain" (Brown,1974) and not significantly disruptive "... other than through the inflationary mechanism" (Foster,1974). Turbulence, however, can be induced if the organisational structure is inappropriate to the demands placed on it, or if the quality of management declines (Harding,1985,p220). Low risk strategies of the kind outlined in the previous section can have similar effects.

Swift change in demand can cause "overheating" in the industry. With low risk strategies "... the various parts of the [construction] industry cannot increase or decrease output as quickly as orders. But when orders increase too rapidly, the industry becomes overheated and contractors spread work forward and *vice versa* when demand is inadequate" (Campbell et al,1974,p17). The results of such overheating or inadequate demand can be observed by such indicators as the ratio of vacancies to unemployed, level of brick stocks, the rate of increase of earnings and bid prices in construction compared with other industries, architect's new commissions, and bankruptcies in the industry (Campbell et al,1974).

Perseverance with low risk strategies in high risk environments can have undesirable consequences in the form of defensive take-overs (Jarman,1978) or liquidation (Lansley et al,1980; Jarman,1978). There are difficulties, however, despite this knowledge, in changing strategies to accomodate fluctuations in demand because of commitment to the focusing effect (South,1979) and general resistance to change (Lansley et al,1979). The project decision model implies that commitment to the focusing effect is inappropriate except as one means of identifying plausible options. The organisation and management of change is considered to be a function of the organisation machine (Fig. 5.4).

The high risk environment has a twofold effect on the organisation. The fluctuating and unpredictable changes in demand represent increased and varied opportunities and threats. Successful low risk strategies are associated with avoiding threats and concentrating on survival (see, for instance, Fellows et al,1983,p188; Hillebrandt,1974,p89,p92; Woodward,1975,p170; Harrison,1982), whilst high risk strategies attempt to exploit opportunities as a means of further growth (see, for instance, Lansley et al,1979,pt3,p86; Niss,1965; Fellows et al,1983,p188; Rajab,1981). The former strategies can lead to controlled regression (Benes & Diepeveen,1985), decline (Fellows et al,1983,p32), the desire for stability (Niss,1965; Rajab,1981) or a constant volume of work (Niss,1965). Growth strategies lead to the desire to progress and expand (Niss,1965,p92) by increasing the level of operations (Woodward,1975,p170), size and turnover (Fellows et al,1983,p188).

In high risk environments, both low and high risk strategies require a certain 'fleetness of foot' regarding market orientation. Low risk strategies are essentially passive in that attempts to influence project generation are kept to a minimum except in times of need. The accent is on market specialism until circumstances dictate otherwise. Stability strategies (Lansley,1979,pt3,p87), emphasis on quality of product (Rajab,1981; Niss,1965), strategies to keep out competitors (Woodward,1970) and maintain or increase market share (Foster,1974; Cusack,1981; Fellows et al,1983,p188) are typical. Entry into new markets, when necessary, can present difficulties, however, depending on the costs of starting up and the chances of obtaining more work in that market (Foster,1974). One approach to this difficulty is the "foot in the door" strategy (Foster,1974) involving a gradual transition between markets. Information on new markets is clearly a vital factor.

The main emphasis with low risk strategies in high risk environments, however, is in the outcome environment. The chances of survival can be improved by increased productivity (Harris & McCaffer,1983,p157; Cusack,1981; Clark & Lorenzoni,1985,p55,pp64-72) by better and more flexible use of resources through subcontracting (Foster,1974) or redeployment of workmen (Cusack,1981), for instance, to increase efficiency and minimise costs (Niss,1965). Increased flexibility has been found in at least two cases, plant and staff work load, to minimise the impact of resource levels on the bidding decision (South,1979).

High risk strategies, under the definition implied here, are associated with aggressive market oriented behaviour. It has been suggested that priority to market related objectives is essential for the modern company (Lansley et al,1980). Such forward integration strategies (Moss,1981) include expansion and diversification (Dressel,1965,p14; Fellows et al,1983,p26) developing merger potential (Fellows et al,1983,p49) and growth through acquisition (Lansley et al,1979,pt3,p78). Grinyer (1972) has postulated the incremental nature of strategic development in diagrammatic form, from existing expansion and diversification to conglomerate diversification (Fig. 6.1). Strategic developments on this scale demand close integration of bidding, organisational and marketing activities in order to be successful. Newcombe (1976), for instance, found incompatibilities between organisational and marketing strategies can have a terminal effect on

Fig 6.1 Alternative strategies

	Existing services type of construction, or product	New but related services, type of construction or product	New and largely unrelated products
Existing clients in same geographic area	Existing strategy	Expansion	Expansion
Existing clients in new geographic area	Expansion	Expansion	Diversification
New clients in same geographic area	Expansion	Diversification	Conglomerate diversification
New clients in new geographic area	Expansion	Diversification	Conglomerate diversification

"Construction firms are normally well advised to explore strategic alternatives in the sequence indicated by the arrows".

Source: Grinyer (1972,p9,Fig1)

the organisation. Organisational flexibility is needed to prevent incompatibility occuring (Lansley et al,1979,pt1,p15).

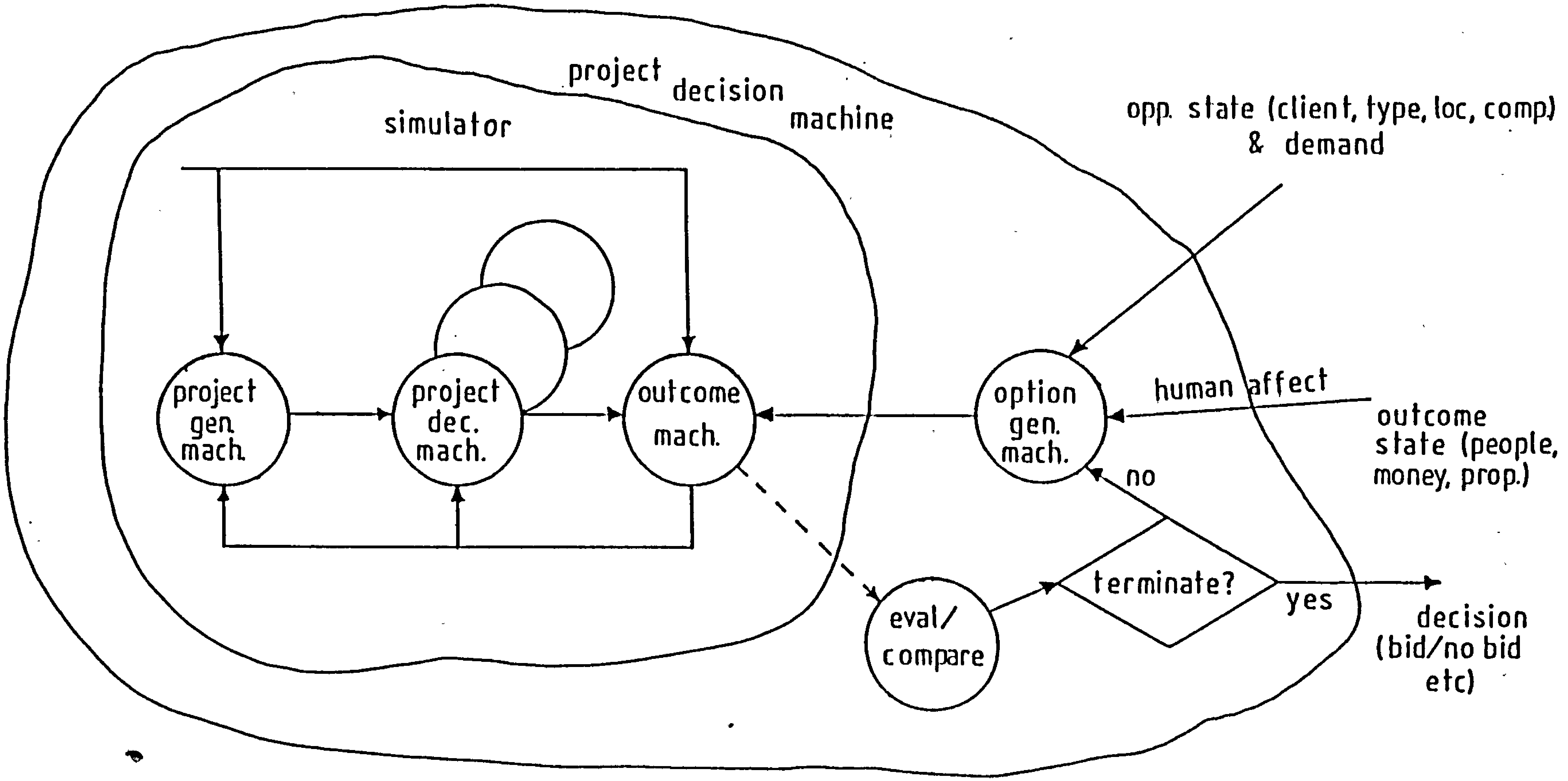
High risk strategies continually seek new fields of endeavour (Moore,1984,p20) and attempt to create their own demand (Sidwell,1984). Such objectives necessarily imply the existence of good environmental information and active marketing together with appropriate management and co-operation of those involved in the decision environment to overcome long run limitations of size, organisation and markets (Moss,1981).

6.7 Conclusions

The previous examination of the total project decision problem reveals the need to assess multiple conflicting criteria under dynamic and uncertain conditions. The appropriate decision is selected by evaluation and comparisons of options over time. The options to be evaluated include the decision of whether or not the project is wanted and, if it is wanted, the level of price to offer. Options occur due to the presence of project opportunities generated by the general demand for construction together with the marketing efforts associated with previous decisions. The result of the project decision is to influence changes in the outcome environment. The uncertain nature of all aspects of the decision problem, and particularly those important factors which lie in the future, indicates that values and strengths of relationships will need to be estimated. The combined complexities and uncertainties of the problem suggest that some simulation model may be most appropriate, allowing the implication of inaccurate estimates to be examined. Fig. 5.4 proposes a schematic system of the project decision incorporating major aspects of the problem in terms of project generating, project decision and outcome machines. A schemata for the project decision machine is shown in Fig 6.2 incorporating a means of determining future events by simulation.

The project decision machine contains an option generator and evaluator/comparator. The option generator is activated by incoming project opportunities and construction demands together with details of the outcome states. Human input is available through the 'people'

Fig 6.2 The project decision machine



outcome state. The option generator contains option identification rules (strategies) whilst the evaluator/comparator contains rules for comparing multiple criteria (policies). The simulator is a small scale model of the project decision model, designed to simulate future environments and decisions. The project decision machine contained in the simulator has exactly the same configuration as the project decision machine illustrated in Fig. 5.4. The project decision machine operates, therefore, in a recursive manner, each decision machine containing a nested version of itself. The satisfaction of some termination criterion results in the output of the currently 'best' decision. Decision reliability is estimated by some form of sensitivity analysis.

A typical medium term decision horizon for a medium sized construction organisation would seem to be about 1½ years, during which time the organisation will typically receive about 75 project opportunities. An exhaustive search of all possible options will certainly present some serious problems for a sub speed-of-lightcomputer, even when the options are restricted to two, to bid or not to bid for each opportunity. The number of iterations in this case are 2^{75} which, even assuming an extremely fast 1 nanosecond per iteration, will still take almost 1.2 million years of computer time! A method of limited search is clearly necessary.

Searching problems occur in expert systems, problem solvers and robot controllers, where the search is for operations that transform the current state into a state that meets some desired criteria.

Sowa (1984) has identified seven "certain features" of searching problems of this nature:

> "*Depth.* Every search is characterized by a depth n, which is the number of steps from start to finish. If a problem has no solution, n may be infinite. If a problem has multiple solutions, n may vary for different search paths.
>
> *Branching Factor.* At each step in the search there may be many possible ways to proceed. The branching factor k is the number of options at each step.
>
> *Direction.* A search may be *data-directed* when it goes forward from the original data, it may be goal-directed when it starts at the goal and goes backward, or it may be *bi-directional* when it does some searching from both ends.

> *Scheduling*. A *breadth-first* search proceeds along all options in parallel, and a depth-first search takes one option at each step, until it reaches either a goal or a dead end then backs up to take a previously untried option. A *best-first* search keeps an *agenda* of options to try, and uses an *evaluation* function to choose the most promising one at each step.
>
> *Pruning*. An exhaustive search tests all possibilities, and *pruning* eliminates the unlikely ones. In *forward pruning* some of the options are rejected before any searching is done. In *backward pruning* information gained while searching one branch is used to select or reject alternatives on other branches. The *alpha-beta algorithm* is a common form of backward pruning for searching game trees.
>
> *Termination*. To determine when a goal is reached there must be some criterion for testing whether the current state is the end. For some searches the criterion is the binary choice ... for other searches there is a measure of goodness for each state, and the criterion is either to find the best state or find one that exceeds a certain threshold.
>
> *Heuristics*. A systematic guessing strategy can guide or speed up a search. Heuristics may select the best option to try, reducing the branching factor by pruning options or test the current state against the termination criteria".
>
> (Sowa, 1984, p198)

Strategies for controlling complexity include the introduction of constraints to eliminate redundant or dead-end files; shallow searches; special cases; generate and test plausible solutions; large knowledge bases; and special hardware (Sowa,1984). Various systems combine these strategies. The chess computer, Belle, which has reached master level in competition with human players, uses the alpha-beta algorithm to prune options, an arbitrary cut-off to limit the search, special cases for forcing sequences, heuristics for ordering the search and evaluating positions, a knowledge base of opening moves derived from grand master practice and parallel hardware for generating legal moves and computing the value of a position (Condon & Thompson,1982).

Many of these devices are also applicable to the project selection problem:

1. The problem is likely to have multiple solutions, especially after sensitivity testing, so n may vary depending on the search path.

2. The branching factor k is potentially very large, even infinite, if all real numbers are considered for the set of potential bids. Sensitivity testing, however, is unlikely to indicate that prices within the range ±½% or greater will be sufficient.

3. The project decision model represented in Fig. 5.4 and Fig. 6.2 implies the use of a data directed search, no specific goal other than an 'improvement' in outcome state being defined. it is possible, however, that some abitrary goal such as target profit or turnover may be found desirable in which case a goal directed or bid directed search could be used.

4. A breadth first search could be used to secure short-term solutions say the best bid to make for the current opportunity irrespective of future opportunities. Sensitivity tests may well reveal that increasing uncertainties of future events reduce their impact on current decisions. The various strategies outlined in this Chapter indicate possible heuristic evaluation functions for the best first search. The indications are that a consistent "stick with the knitting" market strategy combined with a resource utilisation strategy would suffice unless faced with undue changes in demand in which case a broader search may be more appropriate.

5. Forward pruning could be utilised to remove obvious opportunities that are well outside the organisation's area of expertise. Backward pruning is a possibility where the selection of several contiguous projects is patently linked to create work load or cash flow overload. The alpha-beta algorithm has a potential application in bidding by enabling some pruning to be made based on the likely actions of competitors leading to a rapid decision not to bid against a leading competitor for instance.

6. The incremental approach proposed by the model implies that lines of option search are improving the results. A termination procedure might be adopted, for instance, based on a non-improvement heuristic resulting perhaps in an entirely different search strategy.

7. One constraint is immediately apparent - there will be a limit below which certain resources cannot fall. Monetary resources, for example, will have a minimum level. A minimum resource constraint may

also be necessary on a continuous basis. A certain sequence of projects could easily generate this state. Sensitivity tests may well reveal the possibility of constraint violation for most combinations of projects.

CHAPTER 7

Statistical models

7 STATISTICAL MODELS

7.1 Introduction

The previous Chapter introduced some ideas to ease the computational burden of the project decision problem mainly centering on possible search heuristics for 'best' decisions. This chapter examines another possibility, the use of statistical models, as a means of reducing some aspects of the problem at least to manageable proportions.

7.2 Uncertainty, Risk and Probability

It is clear from the preceding Chapters that uncertainties and risks are likely to be major aspects of the project decision problem. One approach to dealing with these aspects, sensitivity analysis, has already been discussed. Sensitivity analyses, however, exist in many forms although all such techniques are essentially concerned with investigating the effects of uncertainty by inducing peturbationsin the data. It is important, therefore, as Bacarreza (1973,p129) observes "... to develop techniques that will permit representation of the uncertainties ... as accurately as possible". A popular technique for representing uncertainty is by a statistical model in which some variable aspects of the problem are modelled as random events. Heisenberg's "uncertainty principle" is an obvious example. The advantage of this approach is that the assumption of purely random events implies a special kind of stability, indeed, as Pierce (1980) asserts, "... nothing could be imagined to be more systematic". A further advantage is that the statistical approach, enables a potentially substantial amount of theoretical knowledge to be applied to the problem.

There are, unfortunately, few events (if any) which occur in the prototype which will be truly random in the statistical sense,, as there are equally few events that are truly deterministic. The best that can be expected, therefore, is a statistical model which will 'reasonably' map some aspect of the prototype. The 'reasonableness' of the mapping is to be judged by the consequent performance of the decision system, and not necessarily the degree with which the model 'fits' the prototype. This will, of course, depend on the 'robustness' of the system in the same sense that statistical tests for means (robust) are less reliant on the assumption of the 'Normal' distribution than tests for variance (non-robust).

The first task, however, is to identify those aspects of the system likely to be amenable to statistical modelling and the type of model which may be most appropriate. The remainder of this Chapter, therefore, contains an examination of the literature relating to the statistical nature of variables contained in the project decision model outlined in the previous Chapters.

7.3 Construction Demand and Project Characteristics

Most modellers consider total demand to be infinite (Atkins,1975) and that project opportunities occur in an infinite (Oren & Rothkopf,1975) or unending sequence (Agnew,1972). Benson (1970) assumes the number of opportunitities occuring in a given time interval is known or estimated from previous experience without reference to any frequency distribution. Hossein's (1977) empirical analysis of 106 projects, however, found the frequency distribution of the size of project opportunities to be Exponential as adjudged by the chi-square test at the 99% confidence level.

Ortega-Reichert (1968) suggested the work content of the project to be a random variable. This approach has been developed by Morrison & Stevens (1980) in a simulation study involving the random generation of such constructional features as gross floor area, number of storeys, column centres, building plan, the occurence and size of basements and types of roofs. Project opportunities were, in this case, generated randomly from a distribution based on the total industry turnover target, average project size and duration.

Models representing the number of competitors bidding for a given project start with Friedman (1956) who suggested a Poisson distribution to be appropriate, an assumption subsequently adopted, but not tested, by Hossein (1977), for instance. Friedman also suggested several methods of estimating the 'average' number of bidders. "In many cases there is information available to a company about the intentions of its competitors. This information combined with the experience of the executives of the company may give a good estimate of the number of bidders" (Friedman,1956,p109), an approach reiterated by Rubey & Milner (1966) with particular reference to the type and size of the project involved.

A further approach considered by Friedman, based on an assumed relationship between project size and the number of bidders, was to regress the number of bidders against the company's cost estimates on previous bids. Such a regression has been applied by Wade & Harris's (1976) analysis of 136 bid tabulations from three small to medium sized general construction contractors located and working in Ann Arbor, Michegan, indicating that a logarithmic relationship between the number of bidders and project value.

A similar analysis has been conducted by Gates (1967), however, but with very poor predictive results. Sugrue's (1977) analysis confirmed this by finding no significant relationships between project size, number of bidders, number of suppliers and sub-contractors involved. Park (1966) has suggested that a non-linear relationship may exist but no evidence has been found to adequately support this notion. Skitmore's (1981) analysis of bidding data indicated a relationship between the number of bidders and market conditions, although no model was developed. Such lack of predictive models has led researchers to conduct simulation studies based on a randomised number of bidders (Rickwood,1972).

The identities of competitors vary, according to Benjamin (1969), with the type and size of project, the client and the location. Some evidence was found in support of this in "... the records of the contractor who provided data for this study". However, Morin & Clough's (1969) "... inspection of real world data ... showed that different competitors were met on different classes of work", the extent of such differences though, were not revealed. Wade & Harris (1976) have suggested that the identities of the various individual and combinations of competitors can be treated probabilistically whilst Shaffer & Micheau (1971,p116) briefly mention a predictive technique termed the multidistribution model (MD) which "... represents the local structure of the construction industry, a structure which allows the contractor to predict with a high level of confidence who his competitors will be on a specific project". The details of this model are apparently given in Casey & Shaffer (1964), a publication to which this writer has been unable to gain access.

Difficulties in predicting the identity of certain competitors, particularly those of whom the company has little or no knowledge, has led to the separate treatment of "strangers" and "key" competitors. These and other competitive aspects, however, are examined later in this chapter.

7.4 The Outcome Environment

Whilst no evidence of probabilistic approaches to modelling human development and aspirations was located, some considerable literature is available concerning the physical and particularly monetary aspects of the outcome environment. Monetary aspects are dependent on two major factors, the probability of acquiring the project and the probability of the occurance of certain monetary states conditional upon the project being acquired or lost. This latter factor is examined first, on the assumption that the project will be acquired, in terms of expenditure (cost) and income probabilities. The probability of project acquisition will be considered in the final section of this Chapter.

7.4.1 Expenditure

7.4.1.1 Cost and Estimated Cost Variables

Although some models assume future project expenditure to be known with certainty (Agnew,1972; Gates,1960; Park,1966; Broemser,1968; Edelman,1965; Morin & Clough,1969, for instance) it is clear that this is far from the case in the prototype. Several attempts have been made to formulate the problem in a quantitative manner which allows treatment of the variation between expenditure and estimates. One approach adopts the concept of 'true' cost (Whittaker,1970), sometimes expressed as God's cost (McCaffer,1976a) or the Devil's cost (Fine,1974). This is essentially that of Friedman's (1956) approach who takes the view that "... the true cost can only be known after the job has been completed" and assumes that the distribution of the ratio 's' of the true cost to the estimated cost can be determined from the contractor's records, this ratio being "... clearly a random variable" (Benjamin,1972). As the estimate is often assumed to be "correct on average" (Capen et al,1971), the population mean of this ratio is, therefore, unity with a dispersion, according to determinists, of zero (Casey & Shaffer,1964). Weverberg (1978) also refers to the random variable 'S' as the ratio of real and estimated costs, where real costs are implied to equal true costs.

A different perspective is provided by McCaffer (1976b) whose model is derived on the basis that "... different estimators will obviously assess the effects of factors on costs differently and hence a number of estimators are liable to produce a range of [estimated] costs". This suggests a probability distribution of estimates around some mean. This mean has been termed 'the likely cost' (Cauwelaert & Heynig,1979) and several simulation studies (Fine & Hackmar,1970; Rickwood,1972; Morrison & Stevens,1980, for instance) have been conducted on this basis. The advantage of the likely cost approach is that each project cost estimate can be considered to be a random value drawn from a distribution of possible cost estimates unique to each project, whilst Friedman's approach implies one distribution to apply irrespective of non-random differences that may occur between projects. This, according to Benjamin (1969) is an important factor for, in his view, "... there is no single distribution of the ratio of true cost to estimated cost that applies to all jobs without regard to the characteristics of the job". Curtis & Maines (1973) have similar reservations regarding the Friedman approach.

An alternative line has been adopted by Park (1966) who takes the view that the actual project costs are distributed about the estimated costs, this distribution being regarded by Vegara (1977) as symmetrical around estimated costs with actual costs being equal, on average, to estimated costs. This approach suggests that the estimated cost is somehow the target figure, a possibility discussed earlier.

The difference between the 'true' versus 'likely' cost and the 'actual' cost models is essentially that "... some authors consider estimated costs as a stochastic variable and the true cost as non-stochastic, [whilst], others take the true cost as being stochastic and estimated cost to be non-stochastic" (Naert & Weverberg,1978,p362). In statistical terms, this difference, according to Naert & Weverberg, "... basically boils down to taking a classical versus a Bayesian point of view".

An alternative way of dealing with this difference has been to treat both the costs *and* the estimates as random variables (Fuerst,1977; Rothkopf,1980). There is some justification for this approach for, as Fine & Hackemar (1970) demonstrate, variability in estimates of production and costs exists both before and after the event as "... estimates are guesses at future costs and accounts are guesses at past costs". In their view, the two variables may not be strongly causally dependent, certainly as far as feedback is concerned, for "... in theory the estimator's guess should be based on accounting data and should be obtained from these by a process of data manipulation and calculation. In practice data of this kind are of little concern to anyone involved in the process" (Fine & Hackemar,1970p1).

7.4.1.2 The Suitability of a Statistical Model

Whittaker noted in 1970 "... that the estimated cost is a stochastic variable has been recognised by almost all previous investigations" (Whittaker,1970). There has been no discernable change in circumstances since that time, Fuerst (1976) suggesting the cost estimate to be a random variable; Vegara (1977) using probabilistic estimates to treat the cost of the project as a random variable; Carr (1982) assuming estimates of total cost to be random variables; and Rothkopf (1980) considering both cost and estimates to be random variables. Very little evidence exists, however, to support the assumptions and some criticisms have been recorded. Curtis & Maines (1973), for instance, consider that cost estimates are not random but depend on the company position. Ortega-Reichert (1968) implies the cost to be conditioned by the work content of the project. Stark & Mayer (1971,p474) suggest that "... cost dependencies can exist in which economies are anticipated from executing certain combinations of contracts eg. the efficient utilisation of supervisory personnel for nearby construction sites. Firms which contract for large construction projects and hence consider fewer bid opportunities might expect their costs to be independent. However, as the number of opportunities to bid increase, cost interactions between projects are more pronounced".

Several aspects of costs and cost estimates have been proposed that may be amenable to statistical modelling. These have been studied, to some extent, by Green (1978) and Vegara & Boyer (1974). Bacarreza (1973) considers the main random variables to consist of: cost percentage of, for instance, labour, materials, plant and overheads; cost 'variances'; cost curves; bidding curves; and contract duration. At a more detailed level, random variables have been said to include the following:

1. Labour Costs

Site activity has been modelled as a series of stochastic independent events by Fine (1970) and subsequently by Bennett & Fine (1980), Bennett & Ormerod (1984) and Wilson (1982) amongst others. Armstrong (1972) has also modelled trends in wages, outputs and performance standards and the ease or difficulty of performing the work in a similar manner. Benjamin (1969) suggests that "... labour costs associated with the costs of performing the work are random variables whose behaviour may be described by a probability distribution" and Gates (1971) has suggested some typical coefficients of variation (Table 7.1).

2. Material Costs

Trends in material costs have been modelled stochastically by Armstrong (1972), Benjamin (1969) suggesting that the theory of stochastic processes (time series analysis in particular) may be used to predict the probability distribution of the cost of materials at the time they are to be purchased.

3. Sub-Contractors' Costs

Benjamin (1969) suggests that the receipt of a low bid from a subcontractor could be treated as a Poisson arrival and the relative amount by which it is a low bid could be described by some other probability distribution.

Table 7.1 Coefficients of variation V% of estimated productivity for various operations in building construction industry

Operation	V %
Unloading and stacking	
Packaged material	10
Brick, block	10
Loose lumber, bars	20
Site improvements	
Clearing	10
Grubbing	20
Remove pavements	15
Pipe culverts	15
Rip-rap	15
Fine grading	10
Paving	15
Sidewalks	10
Power lines	20
Fence	10
Manual excavation	
Loosen soil	15
Shovel and cast	15
Backfill	10
Compact	20
Wheelbarrow	15
Mechanical excavation	
Small equipment	20
Medium equipment	20
Large equipment	15
Truck haul	10
Drill blastholes	15
Sheet piles; drive	
Brace and pull	15
Foundation piles	20
Precast concrete	
Manufacture	10
Erect	15
Insulation	10
Plastering	
Lath to walls	7.5
Lath to ceilings	10
Stucco netting	10
Plaster	10
Stucco and gunite	10
Painting	
Walls, floors, ceilings	7.5
Doors, windows, trim	10
Structural steel	10
Wallpapering	20
Floor, ceilings and wall tile	
Ceramic, quarry, structural	10
Asbestos, asphalt, acoustic	7.5
Glazing	10
Concrete formwork	
Fabricate	10
Erect	10
Strip, clean, oil	10
Repair	20
Concreting	
Placing	15
Finishing	15
Curing	10
Reinforcing steel	
Fabricate bars	10
Set bars	7.5
Wire mesh	10
Structural steel	
Fabricate	15
Erect	10
Deck and siding	15
Masonry	
Trim stone	20
Set stone	10
Lay brick, block & tile	7.5
Point	20
Rough carpentry	
Cutting, ripping, drilling	
Manual	15
Power	10
Framing	
Install	7.5
Remove	15
Flooring, sheathing	10
Plaster board	7.5
Finish carpentry	
Exterior siding	10
Exterior trim	20
Interior panelling	10
Interior trim	15
Cabinet work	15
Stairs	20
Hang doors	15
Install windows	15
Roofing	
Shingle	15
Built-up tar & gravel	10
Flashing	10
Electrical work	
Conduit, cable & wire	10
Install fixtures	15
Buried cable	10
Plumbing	
Exterior piping	15
Interior piping & tubing	10
Install fixtures	10
Cut and thread	10
Heating and air conditioning	
Pipe and duct runs	10
Fittings	10
Insulation	7.5
Install fixtures	15

Source: Gates (1971, p294, Table 2)

4. Quantity Related Costs

Gates (1971) has proposed that errors caused by mistakes are distributed triangularly and symmetrically. Mistakes, according to Gates, include "... gross mistakes, foolish mistakes and unpardonable mistakes due to carelessness or ignorance". "Quantity take-off results in mistakes from plan reading, measurement and related arithmetic as well as ambiguous or incomplete plans. Carelessness results in missing some quantities and even items of work" (Gates,1971,p278). Fine & Hackemar (1970) and Grinyer & Whittaker (1973) also advocate the use of probabilistic models for mistakes of this nature.

5. Effects of Weather and Seasons

Benjamin (1969) suggests that "... since the occurrance of different weather conditions in different seasons are random variables, the costs associated with changes of weather are also random", a view endorsed by Hillebrandt (1974) and adopted in Armstrong's (1972) simulation studies.

6. Costs of Estimating

Leech & Jenkins (1978) have performed a stochastic simulation of the tendering system using a distribution of activity times, based on evidence provided by Leech & Earthrowl (1972).

7. Additional Costs

Gates (1971) considers that additional costs required by the Engineer can be dealt with by probabilistic modelling. The acceptability of substitutes is also recommended for treatment in a similar manner (Gates,1971).

8. Other Costs

Benjamin (1979) suggests that other costs, such as insurance costs, bonding costs and fringe benefits, which are functions of the above costs are also random variables.

Statistical approaches to modelling cost and estimate variability have not gone unchallenged, however. Hillebrandt (1974), in distiguishing between risk (where probabilities can be determined) and uncertainty (where probabilities cannot be determined) suggests that most of the foregoing aspects are uncertain rather than risky and, therefore, considers it "... questionable whether probability is the right tool" (Hillebrandt,1974,p183).

The criticism, however, is not that probabilistic modelling is inappropriate *per se* but that estimation of the parameters involved is likely to be the difficulty.

7.4.1.3 Estimation of Parameters

The complete specification of any probability distribution involves three parameters which have been referred to as shape, spread and location (Spiegelhalter,1983). Although the distribution of costs and estimated cost is, as Naert & Weverberg (1978) point out, not observable, several indications are available.

7.4.1.4 Shape

A wide range of shape parameters have been assumed by modellers of project costs. Vickrey's (1961) early work on the application of game theory to auctions assumes costs to be Uniformly distributed, an assumption also adopted by later researchers in this field (Griesmer et al,1967, for instance). Fine (1974) and Hackemar (1970) also adopt the uniform assumption in their simulation studies of construction projects, together with McCaffer (1976b), Cauwelaert & Heynig (1979) and Harris & McCaffer (1983), who all assume "... the range of estimates produced to be the likely cost ±A%" (Harris & McCaffer,1983,p226) or in the range B to 1/B, where B = (100 + A)/100 (Fine,1974). Beckmann (1974), Naert & Weverberg (1978), Rickwood (1972) and Mitchell (1977) assume a Normal (Gaussian) distribution of cost estimates to be a reasonable assumption particularly "... in those situations where a cost estimate is the sum of a large number of cost components" (Mitchell,1977,p192), as implied by the Central Limit Theorem (Case,1972).

Capen et al (1971) and Zinn et al (1975), on the other hand, prefer the Lognormal distribution on the assumption that the cost estimate is the product of variables, Smith & Case (1975) opting for the Loglogistic model for similar reasons. Rothkopf (1969, 1980), Oren & Rothenkopf (1975) and Zinn et al (1975) assume the two parameter Weibull distribution to be "... particularly appropriate because it is a limiting distribution in the theory of extreme value statistics" (Rothkopf,1969,p363), expressing surprise that the Weibull model is not adopted more frequently.

An alternative approach has been to model components of cost as individual probability distributions. An early example of this is Case (1972) who assumed each cost component to be represented by a Beta distribution. A similar procedure has been advocated by Stacey (1979) and others. Spooner (1974) has suggested using the triangular distribution to model cost components, an approach adopted by Wilson (1982), mainly for its simplicity. Recent simulation studies by Bennett & Fine (1980) and Bennett & Ormerod (1984) use a variety of probability distributions to represent the variability of cost components. The resulting total cost and estimate probability distributions in these cases is clearly determined by the degree of dependency between the component cost variables. No studies of these dependencies have, however, been reported.

Construction companies, it appears, have little knowledge of the frequency distribution of costs and estimates as generally "... no use is made of statistics and probability to systematically evaluate the uncertainty and risk inherent in construction" (Neil,1978), perhaps because methods of estimating costs "... do not attempt to quantify the variability of actual costs" (Larew,1976). Benjamin (1969) has suggested the use of three methods to determine the distribution of the total cost of performing the work "... by convolution of Normally distributed random variables whose elemental distributions are determined by multiple regression; covolution of Beta distributed random variables whose elemental distributions are determined subjectively by the construction cost estimates; and by examination of historical data without regard to the elements or activities of which the job is completed". The first method requires the assumption of stochastic independence which, in Benjamin's view is "not unreasonable". Ashworth's (1977) attempts to apply the method, however, encountered severe difficulties in devising suitable explanatory variables. The second method also relies, as mentioned above, on the independence assumption, but no practical applications have been located. The third method, direct assessment, also presents some difficulties for, as Whittaker (1970) has found, details of actual project cost are "... often not available to the contractor since bulk buying and general stores complicate the task of determining the cost for the project". Further difficulties in estimate/actual cost comparisons are created by post-estimate design changes, for instance.

Despite the difficulties associated with direct assessment, several attempts have been made to determine the probability distribution of costs and estimates in this way by analysis of the ratios of estimated to actual costs for a sample of completed projects. The earliest example of this is by Friedman (1956) who found the Gamma distribution "frequently furnishes a fit" for projects of an unspecified nature. Gates' (1967) analysis of 110 projects completed by a large highway contractor for the Connecticut State Highway Department between 1963 and 1965 found the actual/estimated construction cost ratios to be approximately Normally distributed. Morin & Clough's (1969) analysis of a "limited sample" of a contractor's cost and estimating data found the distribution of the ratio of actual to estimated cost to be symmetrical. Whittaker's (1970) analysis of 153 construction projects completed by a contractor between 1968 and 1969 showed that "... the use of the Uniform distribution for cost estimates is consistent with, and provides an adequate description of, the real system". Leech & Earthrowl (1972), from a limited amount of information, and Smith & Case (1975) have found a multiplicative model with Lognormal distributed estimates to furnish a reasonable approximation of an actual auction for oil tracts.

An alternative approach, based on the assumption that the difference between different bids for a project is largely determined by the random nature of the costs and estimates, is to estimate the shape of the distribution of bids. McCaffer (1976b), however, suggests that this may be misleading and Skitmore's (1981) analysis of 269 building contracts, indicating a systematic parameter change more closely related to income (price) than costs, would seem to offer some confirmation.

7.4.1.5 Spread

It has been said that contractors should estimate with an error of considerably less than 10% of their total final cost (Rubey & Milner,1966) and, generally, of the order of ±5%, given a set of quantities and sub-contractors' quotations (Park,1966). Experience in process engineering contracts suggest that ±5% is a reasonable figure (Liddle,1979). An opinion survey taken amongst construction contractors at a seminar in Loughborough also confirmed the view that ±5% is generally appropriate, notwithstanding the lack of supporting data (Moyles,1973). Simulation studies have indicated higher figures to more closely represent those acually obtained by a leading contractor. ±8% to ±11% (Fine & Hackemar,1970), usually rounded to ±10% (Fine,1974) has been quoted, together with figures between ±5% and ±15% (Hackemar,1970). Morrison & Steven's (1980) simulation used ±20% labour rates, ±10% materials and ±30% output, the results indicating a mean accuracy of 5% to 7.5%.

Barnes & Law (1974) judge the average spread for process plant contractors to have an average coefficient of variation (cv) of 7%, the performance of particular companies "... varying widely from this average, certainly from 4% to 15%". Beeston (1974) reported a cv 4% found by one civil engineering contractor after "... careful analysis of the extent of agreement among his estimators" when estimating the same project. Gates' (1967) analysis indicated an approximate cv 7.5% whilst other researchers suggest a cv 5.5% for engineering services "from experience" (Case,1972) or a cv 2% from an analysis of a "... limited sample of contractors' cost and estimate data" (Morin & Clough,1969).

Barnes (1971), in attempting to overcome some of the difficulties mentioned earlier, has used the ratio of the actual total cost to the estimated total cost multiplied by the ratio of the tender sum to the final account to measure spread. His analysis of data collected for 160 completed British construction contracts indicates a cv 5.8%.

7.4.1.6 Location

In common with most statistical models, the mean or expected error is normally taken to be the same as the 'true' value. Where the ratios of actual to estimated costs are used, this implies a value of unity, an assumption made by Rothkopf (1980) for instance. Naert & Weverberg's (1978) "... discussions with executives frequently engaged in closed sealed bidding [for construction projects] suggest that the expected value of the ratios is often close to unity". Willenbrock's (1972) analysis of data supplied by a road contractor for 20 completed projects, however, showed a 3% increase in costs over the estimate after deducting change orders and claims.

Theoretical considerations of the competitive aspects of the situation implies that some bias must be present in order to avoid the effects of the "winner's curse". Winner's curse is said to apply in situations where acquisition of the project is not independent of the cost estimate. Thus, for instance, an underestimate of costs is associated with, and partially responsible for, the acquisition of a project resulting in the expected value of estimated costs conditional upon acquiring the project being somewhat lower than the unconditional expected value of the estimated costs. Friedman (1956) has proposed a means of unbiasing estimating costs but, as Simmonds (1968b) observes, the distribution of actual/estimated cost ratios *is* the conditional distribution and, therefore, already debiased to some extent. The situation is, however, complicated by factors associated with competitive aspects, for, as Weverberg (1981) and others have shown, the degree of bias is likely to be closely related (negatively correlated) to the probability of acquiring the project.

Winner's curse may also be an artifact of the model employed. Dependencies among errors of estimation between bidders and differing amounts of information (is bidders with different distribution parameters) can remove the effects of the winner's curse completely (Winkler & Brooks,1978). Dependency between estimated and actual costs can also have a similar effect if the estimated cost, as discussed previously, has some predetermining effect on the actual cost of the project.

7.4.1.7 Relationships between the Probability Distribution of Expenditure and Other Factors

It has been suggested that the probability distribution of cost will change with the project characteristics. As "... the variance of the probability distribution of cost is an indication of the riskiness of the job" (Benjamin,1969), then it follows that increases in spread are a function of increases in risk. Risk has been said to depend on the type of project (Case,1972), lower risk levels being associated with a company's specialty work and less complex projects (Benjamin,1969) and higher risk levels with large and complex projects (Neil,1978). Both large and small projects, and projects demanding either low or high work intensity, have been associated with high risk (Broemser,1968), although Benjamin considers low intensity projects to produce lower risks. High levels of subcontracting are usually associated with low risk (Broemser,1968), providing quotations have been obtained in advance.

One approach to reducing spread is by increasing the estimating effort. Smart (1976) has discussed the possible effect of increasing tendering effort in reducing the variability between costs and estimates.

7.4.1.8 Summary of Overall Expenditure Distribution Parameters

Project expenditure has been modelled extensively by probability distributions. Table 7.2 summarises, in alphabetical order, the overall shape, spread and location parameters adopted in the cases reviewed in this section.

7.4.2 Income and Cash Flow

Income is normally assumed to be some function of the value of the bid, the majority of modellers assuming a one to one relationship. It is clear, however, that a one to one assumption is far from realistic in the prototype, many factors influencing changes between the bid value and the income ultimately received. Most of the factors involved are dealt with on a contractual basis, remuneration often being provided for unpredicted events such as inflation, additional work caused by

Table 7.2 Distribution parameters for costs/estimates

Modeller	Shape	Spread	Location
Barnes (1971)[h]		cv 5.8%	
Barnes & Lau (1974)[a]		cv 4-15%	
Beckmann(1974)[a]	Normal		
Beeston (1974)[g]		cv 4%	
Capen et al (1971)[a]	Lognormal		
Case (1972)[e]		cv 5.5%	
Cauwelaert & Heynig (1979)[a]	Uniform	± A%	
Fine (1974)[b]	Uniform	± 10%	
Fine & Hackemar (1970)[b]	Uniform	± 8-10%	
Friedman (1956)[c]	Gamma		
Gates (1967)[d]	Normal	cv 7.5%	
Greismer et al (1967)[a]	Uniform		
Hackemar (1970)[b]		± 5-15%	
Harris & M^cCaffer (1983)[a]	Uniform	± A%	
Leech & Earthrowl (1972)[c]	Lognormal		
Liddle (1979)[a]		± 5%	
Mitchell (1979)[a]	Normal		
Morin & Clough (1969)[c]	Symmetrical	cv 2%	1.0 (median)
Morrison & Stevens (1980)[a]		± 5-7½%(mean)	
Moyles (1973)[f]		± 5%	
Naert & Weverberg (1978)[i]			close to 1
Oren & Rothkopf (1975)[a]	Weibull		
Park (1966)[a]		±5%	
Rickwood (1972)[b]	Normal		
Rothkopf (1969)[a]	Weibull		1.0 (exp.val)
Rothkopf (1980)[a]	Weibull		
Rubey & Milner (1966)[a]		less than 10%	
Smith & Case (1975)[c]	Lognormal		
Smith & Case (1975)[c]	Loglogistic		
Vickrey (1961)[a]	Uniform		
Whittaker (1970)[e]	Uniform		
Willenbrock (1972)[j]			+ 3%

[a] assumed for theoretical purposes
[b] assumed for simulation purposes
[c] source of data unknown
[d] analysis of 110 USA road projects
[e] analysis of 153 UK construction projects
[f] opinion survey of UK contractors
[g] analysis of extent of agreement between UK construction estimators
[h] analysis of 160 British construction projects
[i] discussion with Dutch construction companies
[j] analysis of 20 USA road projects

design changes, uncovering suspected (incorrectly) faulty work, changes caused by fire and flood, andquantity errors and delays outside the company's control. Further income may accrue outside the contractual position in the form of ex gratia payments for perhaps exceptionally inclement weather, interest received on invested capital, receipts from the leasing of advertising space on hoarding etc. Of these variations in income only one factor, the incidence of design changes, has been modelled statistically.

Profit, however, is another matter. Estimates of profit are, invariably, taken as the difference between the bid and the cost estimate, usually in percentage terms, and the probability distribution of profit as the difference between the bid (a constant) and the cost (a variable). It is clear, though, that the probability distribution of profit is the difference between the two variables, income and expenditure. As the mechanism of the bidding process often is recognised as being such that the bid is a multiple of a cost estimate (Curtis & Maines,1973), it would seem appropriate that income, expenditure and profit are regarded as being a function of the estimated cost, conditional on the multiple (mark-up) applied. The degree of mark-up will also have a bearing on the likelihood of acquiring the project, an issue which is dealt with later in this Chapter.

Models of income and expenditure over time, cash flow models, have been developed. The DHSS's curve formulae, for instance, provide a deterministic approximation of expenditure flows based on projected work value modelled continuously over the project duration, for differing sizes of projects. Similar models have been used by Atkins (1975) to represent the "cash flow pattern" for differing project sizes and types. The only probabilistic model that has been identified in the work reviewed in this Chapter is that adopted by Kangari & Boyer (1981) in the form of a Beta distribution.

7.4.3 Conclusion

Project opportunities and the outcome environment, particularly expenditure, for an individual company have received some attention from statistical modellers encountered in the literature and the indications are that the assumptions implied by the statistical approaches, ie the existence of random variables with estimable parameters, may well be appropriate in some instances.

A particularly significant factor in determining the state of the project decision system is whether the project will be acquired or not. This factor would seem to be dependent to some degree on the estimated cost of completing the project and the mark-up applied. Other companies also have a considerable influence in the ultimate appropriation of the project. The next section examines some approaches to modelling the behaviour of these competitors before finally considering probabilistic aspects of project acquisition

7.5 Modelling Competitors' Bids

Researchers since Friedman (1956) have assumed that "... by keeping a record of the competitors' past bids it is possible to evaluate its bidding habits ... by tracking competition we can develop its bidding behaviour [and] that history usually can be used as a basis for predicting competitive bid levels just as statistical sampling is used to predict election results" (McCall,1977). The prediction of election results, however, by these means, has sometimes achieved spectacular failures. It is not surprising, therefore, to find some criticism of this approach insofar as construction project bidding is concerned. One major criticism is that the events taking place are not truly random in the classical statistical sense as "... the basis of classical statistical theory is that there is an experiment that can be repeated many times in order to gather data from which the parameters of the probability distribution of some random variable of interest can be estimated. A sequence of bidding situations is not really a sequence of performances of the same experiment [and] each job is unique" (Benjamin,1972). Whilst there is no denying the truth of the statement, the same can equally be said for all observed phenomena, as discussed earlier in this Chapter. Neoclassical theory simply utilises statistical techniques on the assumption that the underlying mechanisms in the data 'reasonably' resemble the statistical premises, the degree of 'reasonablness' being determined pragmatically rather than by 'goodness of fit'.

Empirical criticisms are more serious. It has been claimed, for instance, that the assumption of randomness is invalid "... as we know that many subjective factors influence bidding behaviour" (Curtis & Maines,1974,p181). Spooner (1971) has also suggested that "... a random selection process is not a rational representation of behaviour" in these circumstances, a view endorsed by Simon (1957,1979) who also believes that the uncertainty of bidding can and will not be solved through the adoption of probabilistic techniques. The evidence upon which these views are based, however, appears to be rather more circumstantial than factual, and there is a substantial body of opinion holding the opposite view. The general opinion relies on the existence of stable distributions of cost generating stable distributions of bids (Beckman,1974) based on "... the long recognised fact that there is a stability in mass data, although the mass is comprised of erratic individual cases" (Gates,1960,p22). "This fundamental precept ...", according to Gates, "... underlies the basis of actuarial science and is the foundation of the insurance industry" for instance.

A second, and related, criticism concerns what has been termed "... one of the most serious limitations of the statistical approach" which is "... the basic assumption that competitors will follow the same general bidding patterns in the future that they have in the past" (Park,1962). An individual competitor may, for instance, "... change his strategy, thus rendering past data about him misleading" (Beeston,1983,p114). This criticism is essentially aimed at all inductive approaches, for induction necessarily extrapolates from past events to future events. Induction, however, does not attempt merely to extend into the future repetitions that have occured in the past, but rather by creating the conception of a mechanism [model] that explains the past from which the future may be deduced (Adler,1963,p236). McCaffer's (1976a) analysis of individual bidders suggests that some contractors behave in a matter not entirely consistent with the random model. His series of time based tests, however, were generally inconclusive and further research may well indicate possible predictor variables in this respect. Despite their reservations, both Park and Beeston concede that in the absence of other information, probably the best guide to the future is the past and that statistical modelling may provide benefits to the organisation.

Some models do, in fact, incorporate other information such as project characteristics and measures to deal with the degrading effects of time, and some of these will be examined later in this section. Two other aspects, which are related to some extent, dependent and non-serious bids, together with problems caused by data acquisition are also addressed later in this section.

7.5.1 The Behaviour of Competitors

The project decision model, proposed in this thesis, is intended to apply to any construction organisation, including competitors. It is clear, however, that the information and computational burden problems implied by attempting to incorporate complex models of competitors' behaviour will be largely insurmountable. This section, therefore, examines some relatively simple models of competitors' behaviour, both collectively and individually.

7.5.2 Collective Behaviour

Probably the most detailed and sophisticated analysis of individual and collective bidding behaviour of construction companies has been made by McCaffer (1976a), who found, from his Belgian data, "... substantial evidence that existing bidding processes are little more than random". The reasons for this may be attributable to the mechanism of the bidding process by which "... each competitor calculates his cost estimate, which is a random sample taken from his cost estimate distribution, and multiplies it by his mark-up" (Curtis & Maines,1973). The implication, therefore, is that each competitor's cost estimate is taken from the same distribution and that the mark-up may also be treated as being taken from a distribution of mark-ups in a similar manner.

The notion of some commonality between competitors' cost estimates does seem to hold some attraction for, as Park observes, "... taking a single job, most of the competing contractors can also be expected to encounter roughly the same costs of performing the work; they are all subject to the same costs of operation, have access to the same labour supply, use the same types of equipment, obtain supplies and materials from the same sources, and have somewhat comparable, if not equal, supervisory capabilities" (Park,1972,p24.1). As a result, it is claimed that "... in general building, where there is not a large element of highly specialised work and where there is a number of contractors of similar efficiences, especially in areas where staff and labour move from company to company, a simplification assumes that the 'likely cost' of a contract to each company is similar" (Harris & McCaffer, 1983,p226). Many modellers have consequently adopted the assumption that each bidder has identical estimated costs (Larew,1976), true costs (Rothkopf,1980; Rickwood,1972) or that estimated costs are similar (Broemser,1968) or vary around some common mean (Oren & Rothkopf,1975; Morrison & Stevens,1980).

The application of statistical models to the cost/estimate variable has already been examined in the previous section. The variability of mark-ups has been modelled in a stochastic simulation by Rickwood (1972) by the Normal distribution. Grinyer & Whittaker (1973) found mark-ups to vary very little within firms (6.8% ± 0.35%) and their discussions with other firms have "... confirmed the impression that mark-ups do not vary greatly between firms". A similar analysis of Shaffer & Micheau's data (1971) however indicates an average mark-up of 5.40% (1.844 standard deviation), quite different figures. Similarities between bidders were observed in Whittaker's (1970) study of UK construction companies who "... used almost identical methods of determining costs and then all used almost the same percentage mark-up to arrive at their bid prices" inviting the conclusion that "... different firms attempt to place the same value on a specified contract. The differences that occur between estimated costs are primarily attributable to uncertainty" and that "... the statistical techniques which average the behaviour of competitors and aggregate the results of past competitions are the most appropriate methods with which to study the situation".

Many models assume competitors' actions to be "purely random" (Morin & Clough,1969) and, therefore, amenable to treatment as random variables and description by appropriate probability density distributions. The distribution of competitors' bids is sometimes expressed in terms of the distribution of the bid/cost estimate ratios, where the cost estimate value is that known by one of the bidders, or bid/average bid ratios (Whittaker,1970; McCaffer,1976a; Carr,1982). It then follows that

"... each time the decision maker bids on a contract against n competitors, a sample of the size n is drawn from this distribution of competitor bid to cost ratios" (Sugrue,1980,p500).

The assumption that all bidders take their bids from the same distribution enables estimation of the distribution parameters to be made by direct observation of the bids entered for each project.

7.5.2.1 Shape

Vickrey's (1961) early work assumes that all bids are drawn from the same Uniform distribution. Stochastic simulation studies by Fine & Hackemar (1970) have used the assumption that bids are taken from a Uniform distribution claiming that bids generated in this way compared very favourably with the distribution of bids found in the Costain construction company records. Cauwelaert & Heynig have also assumed a Uniform distribution for mathematical convenience, although they do claim that the assumption is "... perhaps not far from the truth" (Caulewaert & Heynig,1979,p15). Whittaker's (1970) analysis of bids for 153 construction projects by four companies between 1968 and 1969 purported to show the Uniform distribution to be a reasonable model. Whittaker's method of analysis, dividing each bid on each contract by the mean bid for that contract and pooling the resulting ratios, has been severely criticised as being "invalid" (McCaffer,1976a) mainly because of the distorting effect of the standardising procedure used and the information loss caused by pooling.

Several models have been proposed based on the assumption that bids are taken from a Normal distribution (Alexander,1970; Emond,1971; Mitchell,1977 and Carr,1983, for instance). Morrison & Stevens (1980) also adopt this assumption in their stochastic simulations. Benjamin & Meador (1979) point out that it is the bid/cost estimate ratios that are often taken to be Normally distributed. McCaffer's (1976a) study of bids for 384 road and 190 building projects in Belgium found the Normal distribution to be the most appropriate model, especially for the building projects. Various trend analyses were performed by McCaffer on these data but with little success, inviting the conclusion that the assumption of randomness was perhaps reasonable. Cauwelaert & Heynig (1979,p18), in reviewing McCaffer's work, suggested that the conclusions regarding the Normal distribution and randomness were "...

consistent with the work of other researchers* neglecting, unfortunately, to provide any further information.

Park (1966) has used a statistical model of bid/cost estimate ratios that is positively skewed, a model considered to be appropriate by Beeston (1974) for bids for construction projects required by the Property Services Agency. The degree of skewness however, according to Beeston, was only slight and "for practical purposes" a Normal distribution would suffice. Another report on McCaffer's roads data proposed an identical conclusion (McCaffer & Pettit,1976).

The assumption of a Lognormal distribution to model bids is, according to Weverberg (1982), "... not doing too bad ... at least as a first approximation". Klein (1976), for instance, has assumed the Lognormal distribution to be appropriate for bids and Capen et al (1971) have adopted the distribution in modelling estimate value/bid ratios for oil tracts. There would seem, in fact, to be a degree of consensus regarding the Lognormal assumption for oil and mineral tract bids (Arps,1965; Brown,1966 and Crawford,1970). The consistency in the standard deviation implied by the Lognormal assumption has also been observed by Hansmann & Rivett (1959) and Pelto (1970) in their analyses of oil tracts and mineral rights sales.

Friedman (1956) suggests a Gamma distribution to be generally appropriate, an assumption adopted by Dogherty & Nozaki (1975) for oil tract bids. Analysis of pooled bid/cost estimate ratios for 545 civil engineering and 63 mechanical engineering projects has indicated a Gamma distribution to be the best fit (followed by the Lognormal and Normal distributions) (Hossein,1977).

Finally, Oren & Rothkopf (1975) have proposed a two parameter Weibull distribution to be a suitable model of bids in auctions generally. A summary of shape parameters is given in Table 7.3.

7.5.2.2 Spread and Location

Several researchers have estimated the average spread of bids for individual contracts. These estimates are given without discussion in Table 7.3.

Table 7.3 Distribution parameters for bids

Modeller	Shape	Spread	Location
AICBOR (1967)[o]		cv 6.8%	
Alexander (1970)[d]	Normal		
Arps (1965)[d]	Lognormal		
Barnes (1971)[m]		cv 6.5%	
Beeston (1974)[i]	Pos. skewed	cv 5.2-6%	
Brown (1966)[d]	Lognormal		
Capen et al (1971)[d]	Lognormal		
Cauwelaert & Heynig (1979)[a]	Uniform		
Cauwelaert & Heynig (1979)[g]	Normal		
Crawford (1970)[d]	Lognormal		
Dougherty & Nozaki (1975)[d]	Gamma		
Emond (1971)[d]	Normal		
Fine & Hackemar (1970)[b]	Uniform	cv 5%	
Friedman (1956)[a]	Gamma		
Grinyer & Whittaker (1973)[c]	Uniform	cv 6.04%	
Hossein (1977)[k]	Gamma		
Klein (1976)[d]	Lognormal		
M^{c}Caffer (1976a)[a]	Normal	cv 6.5%	
M^{c}Caffer (1976a)[a]	Normal	cv 7.5%	
M^{c}Caffer (1976a)[a]	Normal	cv 8.4%	
M^{c}Caffer & Pettit (1976)[j]	Pos. skewed	cv 8.4%	
Mitchell (1977)[a]	Normal		
Morrison & Stevens (1980)[e]	Normal	19.1% av.range	
Oren & Rothkopf (1975)[a]	Weibull		
Park (1966)[h]	Pos.skewed		
Pelto (1970)[d]	Lognormal		
Shaffer & Micheau (1971)[p]		cv 7.65%	
Skitmore (1981a)[l]		cv 6.4%	
Weverberg (1982)[a]	Lognormal		
Whittaker (1970)[c]	Uniform		1.068

[a] Assumed for theoretical purposes
[b] Analysis of an 'adequate' sample of UK construction projects
[c] Analysis of 153 UK government construction projects
[d] USA oil and mineral tracts - source of data unknown
[e] Assumed for simulation studies
[f] Analysis of 183 Belgian building projects
[g] "consistent with work of other researchers"
[h] USA construction projects - source of data unknown
[i] Large sample of PSA projects
[j] Analysis of 384 Belgian roads contracts
[k] Analysis of 545 US civil engineering and 63 mechanical engineering projects
[l] Analysis of 269 UK building projects
[m] Analysis of 159 UK construction projects
[n] Analysis of 16 Belgian bridges projects
[o] Analysis of 213 UK motorway projects
[p] Analysis of 50 USA construction projects

The location parameter will, of course, depend upon the size of the project. In the absence of any further information, the location parameter may be estimated for each project from the bids for that project. Location parameters for bid/cost estimate ratios represent, under the assumptions of the collective model, a measure of the relationship between a company's cost estimate and bid, in other words, the mark-up.

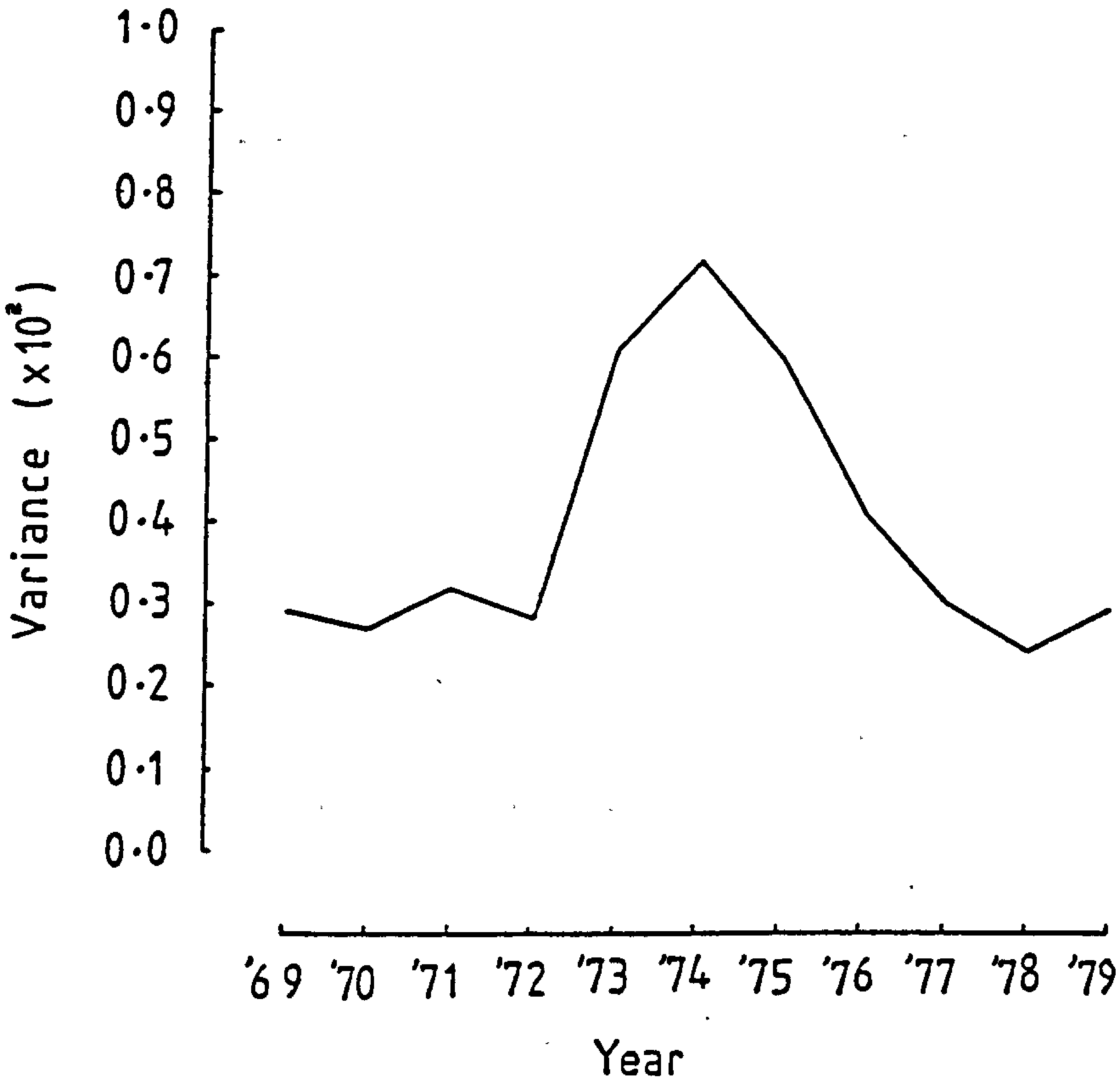
7.5.2.3 Relationships between the Probability Distributions of Bids and Other Factors

Johnston's (1978) analysis of bids for road projects found a significant positive skewness during the years 1970 to 1972,and a slightly negative skewness during the years 1973 to 1975, a change that Johnston attributed to the changing volume of project opportunities. Skitmore's (1981a) analysis of bidding projects data, however, found an opposite trend to exist. Further analysis by Skitmore of parts of bids suggests some relationship of an indeterminate nature may exist between skewness and market conditions.

The spread of bids has been analysed against project value by McCaffer (1976a) and Skitmore (1981a) and a possible but unconfirmed negative correlation obtained. A similar negative correlation has been observed by Morrison(1984) and Flanagan & Norman(1985). Beeston (1983) has suggested that changes in bid spread may be associated with changes inconditions over a few months, the rate of change being an important factor. Skitmore's (1981) analysis over time shows a dramatic increase in spread in the year 1974 (Fig. 7.1), which coincides with some rather extreme movements in the market at that time. Further analysis by Skitmore of parts of bids, implies some relationship of an indeterminate nature to exist between spread and market conditions.

It would seem perhaps that, in view of the indications revealed above, that some further studies of the influence of market conditions may be beneficial.

Fig.7.1 Variance over time



7.5.2.4 Distribution of Low Bids

An alternative approach is to model the winning bids as a probability density function. In this case, the usual procedure is to model the winning bid/estimated project value ratios (Hansmann & Rivett,1975) or the winning bid/cost estimate ratios (Ackoff & Sasieni,1968; Sugrue,1977 & 1980). These ratios are often assumed to follow a Normal distribution (Ackoff & Sasieni,1968; Sugrue,1980), an assumption tested empirically by Beeston (1983) and Sugrue (1977), the latter's chi-square test failing to reject the Normal assumption for 68 road low bid/cost estimate ratios. A slightly different version by Sasieni et al (1959) considers the ratios (B - K)/K, where B is the winning bid and K the cost estimate, to also follow a Normal distribution.

Weverberg (1977) has considered, in some detail, two possible procedures, maximum likelihood and an iterative minimum mean square error procedure, for estimating two parameters of the joint distribution of estimated costs and lowest opposing bids. It was concluded that the minimum mean square error method compared favourably with maximum likelihood estimation, although the method of maximum likelihood was particularly appropriate in estimating parameters of multivariate Lognormal distributions, as would be expected. An unfortunate aspect of this study was that both methods resulted in considerable estimation errors, especially for medium sized samples (Weverberg,1977,p197), although it was considered that with "fairly good" *a priori* knowledge of the parameters of the marginal distribution of estimated costs "... estimation of the remaining parameters of the joint distribution would be much easier and more efficient".

Three sets of data have been published which allow some analysis. Broemser's (1968) data from one contractor bidding for 76 USA construction projects indicates that the frequency distribution of low bid/cost estimate ratios has a sample mean of 0.993 with a standard deviation of 5.49%. Similar data published by Shaffer & Micheau (1971) from one contractor bidding for 50 USA building projects have a sample mean of 0.991 (8.19% standard deviation). Benjamin & Meador's (1979) data covers 131 USA construction projects over a three year period, the distribution of low bid/cost estimates (which the authors assume to be Normal) has a sample mean of 0.996 (6.8% standard deviation). The similarity of the means of these independently obtained ratios is striking and strongly suggests that the expected value of the cost estimate may be quite close to the project value as defined by the lowest bid.

7.5.3 Individual Competitors

Broemser (1968) has observed that, although bidders' cost estimates may well be very similar due to common factors and that "... these common factors would probably account for a large proportion of the volume of the job, the things that would be different among contractors would be the management skills in planning and using labour, materials, equipment and subcontractors". In addition, the bids depend upon the competitors' mark-up which will reflect "... the bidding policies which are chosen to achieve their own objectives" (Mercer & Russell,1969). It follows, therefore, that "... every competitor will exhibit different bidding characteristics; some bid consistently high, some bid consistently low, some spread their bids uniformly over a wide range and some may bid within a fairly well-defined and narrow limits (Park,1972,p24-27). Differences in level of bid (ie consistently high or consistently low) have been termed 'proximity' differences (Skitmore,1981b), said to reflect the relative efficiency (McCaffer,1976a) or 'competitive advantage' of competitors. Competitive advantage, according to Fuerst (1977) includes differences between the "... methods used, the efficiency and availability of equipment, ownership of supply sources, proximity to home office or sites of current contracts, and managerial skill in performing the work. Both policy dictated mark-up decisions and competitive advantage have been modelled as random variables" (Mercer & Russell,1969; Fuerst,1977).

Several researchers have modelled individual bids, starting with Friedman (1956) and including Taylor (1963) and Morin & Clough (1969). The approaches are similar to that of modelling competitors collectively in that competitors' bid/cost estimate ratios are obtained and probability density functions fitted to the ensuing frequency distributions (Friedman,1956; Taylor,1963; Benjamin,1972, for instance). Beeston (1982) has suggested using D ratios, in a similar manner to Sasieni et al, where D = {(lowest bid - estimated cost)/estimated cost} expressed as a percentage. Morin & Clough (1969), on the other hand, have used the relative frequencies of competitors' bid/cost estimate to own bid/cost estimate ratios.

Whilst all of the distribution parameters postulated for collective models necessarily apply to individual bidders, some modellers have proposed probability distributions specifically for the individual case. Griesmer et al (1967) assume bidders draw from a Uniform distribution unique to each, and Winkler & Brooks (1980) have proposed models in which differing amounts of information (ie different variances) exist between bidders. Capen et al (1971), Curtis & Maines (1973) and Fuerst (1977) have attempted to derive parameter estimates for each bidder by simulation techniques. Weverberg (1982) has used a multivariate technique to estimate parameters of coalition bidding for oil leases, assuming the winning bid to be a constant. Skitmore (1982) has proposed a multivariate approach to a part of the parameter estimation problem, involving the solution of two sets of simultaneous equations to determine the variances and the relative means of the log transformation of bid values. Multivariate methods, however, often rely on the assumption of independence between bidders.

7.5.3.1 The Independence Assumption

Most modellers since Friedman (1956) assume that errors in an individual bidder's cost estimates are independent of errors in previous cost estimates (within bidders) and also independent of errors in other bidders' cost estimates (between bidders). This assumption has also been generously applied to bids within and between bidders and also the true cost or actual cost/cost estimate and bid/cost estimate ratios. True or actual cost/estimated cost interactions have already been discussed. The way in which competitors behave in bidding may be influenced by several factors. These factors are considered to be those associated with the project decision environment. The effects of collusion are treated separately.

7.5.3.2 The Project Decision Environment

Several models have been proposed which incorporate features of the project generating environment in order to utilise any error trends within bidders. Park (1980), for instance, has suggested that models of competitors' bids should incorporate the effects of changes in market conditions and Whittaker (1970), in using a discounted cash flow technique for the time element, has adjusted the assumption that past bidding behaviour of competitors is a good indicator of future behaviour to take account of the state of the market. Carr & Sandahl (1978) have used multiple regression analysis (MRA) to predict the lowest bid of any competitor by incorporating a variable representing the "economic environment". Neufville et al (1977) have found "economic conditions" to be an important factor affecting bidding behaviour.

Project characteristics have also been recommended as predictor variables (Christenson,1965; Broemser,1968; Benjamin,1972; Neufville et al,1977; Sugrue,1977; Carr & Sandahl,1978; Morin & Clough,1969). Relationships with the class of construction have been postulated (Shaffer & Micheau,1971 and Cooke,1981), Morin & Clough (1969) finding the ratios of bids by one contractor to cost estimates by another contractor to have a mean of 1.133, 1.232 and 1.333 for three classes of work. The influence of project size on bidding behaviour has been analysed by McCaffer (1976a), who found no correlations, Harvey (1979), whose MRA attempt to predict low bid/engineers' estimate ratios from variables including job size and Lange (1973) who found a sharp drop in his SECLOW quantities (the percentage difference between the lowest and the second lowest bidders) associated with the size of 451 Massachusetts projects. An analysis by Neufville et al (1977) also found the size of project to be important. Pelto (1971) has fitted a complicated function to bidding data (for oil tracts) involving project location, a variable also used in Harvey's MRA. Surprisingly no studies have been documented using the client as a predictor variable.

Several writers have considered the effect of competitors. The number of bidders has been associated with the distribution of bids by McCaffer (1976a) (inconclusively) and Pelto (1971) in his model. Benjamin (1970) and Harvey (1979) have also used the number of bidders as a predictor variable in their MRA's, although Broemser (1968), in a similar study, found the number of bidders to be of no statistical significance in his regression model for predicting the distribution of low bids. Carr & Sandahl (1978) include the "make-up of competitors" in their MRA to predict low bids. A further discussion on the effect of the number and identify of bidders is provided later in this section.

Several researchers have considered the implications of each bidder adopting similar (non-random) strategies (Rothkopf,1969; Oren & Rothkopf,1975; Banerjee & Ghosh,1969, for instance). Whilst some results of theoretical importance have been obtained, they only apply under certain restrictive assumptions. Some of these assumptions are of significance in the construction bidding situation for, as has been observed, when considering a competitor's reaction to a bidder's new strategy "... the degree of reaction will probably depend on the number of institutional factors not represented by the model [including] the speed and certainty with which competitors can discern a policy change and the extent to which the competitors in one auction are likely to be the same as the competitors in the succeeding auctions" (Oren & Rothkopf,1975,p1088).

Some evidence also exists which indicates that likely outcomes have a bearing on competitors' behaviour. In Sheldon's investigations in the process plant industry "... nine [managers] stated that each contract was unique, hence information relating to past contracts would only be of use if each firm offered homogeneous equipment and technologies, and if contracts were undertaken at similar sites in similar conditions. The conclusion of most firms was that evidence on past bidding patterns was too difficult to quantify" (Sheldon,1982,p12). It is interesting to note, however, that whatever information was available was used by these organisations. Sugrue (1977) has also examined the effect of union labour on the distribution of low bid/cost estimate ratios for his 68 road projects. No differences were found between the unionised and non-unionised projects in this respect.

The degrading effects of time have been accomodated in Morin & Clough's (1969) model by weighting the more recent data. A similar weighting was also applied to the bids of those competitors who most frequently competed for the same projects.

The association of bids with aspects of the decision environment has clearly been of interest to researchers in the field. The use of multivariate analyses would seem to be particularly apropriate in examining potential correlations between bids and likely characteristics of the project generating and outcome environments. Little consensus is apparent as yet on the impact of any of the predictor variables employed except perhaps project size, which has received frequent attention. Further work on this aspect of the problem would appear to be desirable. In Benjamin's words "... one of the most important directions for future research in the competitive bidding area is in the development of satisfactory multivariable statistical models to predict the behaviour of the competition in the bidding situation" (Benjamin,1972,p328).

7.5.3.3 Collusion

All of the models consulted rely on the assumption that no collaboration takes place between the bidders. However, as Mitchell (1977) has pointed out "... in any real-life bidding situation, there are many complicating factors, not least the possibility of collusion".

Sheldon (1982) has examined the aspect of collusion in some detail. In view of the uncertainty of competitive bidding and the degree of interdependence between firms engendered by such uncertainty, Sheldon holds that bidding may be conducted *a priori* through collusive agreements. He considers that such agreements would be "... an attractive means of maintaining a steady flow of work and achieving higher joint, risk-adjusted, discounted profits". Little evidence of collusive agreements seems to be available however, which is perhaps to be expected. Sheldon's view of the process plant industry is that the variety of process areas in contracting and also periodic excess capacity would be a destabilising factor in any such agreements. Barriers to entry of the industry are also discussed but it is concluded that "... the ability of firms to actually raise bid prices in excess of an average cost is a function of the buyer's sensitivity to price and non-price factors in a bid, rather than a function of the barriers to entry, and hence the ability of firms to actually limit prices is curtailed by the buyer's power". Insofar as the construction industry is concerned, collusive bidding seems even less likely than the process plant industry as barriers to entry are far less severe and the proliferation of projects is extensive, especially small projects. Collusion, if practised at all in the construction industry, must surely be restricted to a very limited number of specialised projects.

A more realistic proposition is that correlations exist between bidders due to some commonality between companies. "Common training, experience and information" (Winkler & Brooks,1978) particularly in a "localised construction context" (Stark & Mayer,1969) support the view that "positive correlations seem to be more appropriate than negative correlations" (Winkler & Brooks,1980). The vary mechanism of the data generating process, where cost estimates are drawn independently, providing a basis for assessing competitors' bids, implies a dependency of some kind (Weverberg,1981).

Flanagan & Norman's analysis of bids entered by three construction companies for 39 county council projects, found a discernable trend "... between specific contractors when bidding in competition, and that this trend can be expected to vary with different types of work, with work of different value ranges, or as a result of varying workloads of the contractors" (Flanagan & Norman,1982,p29). An empirical analysis of 68 USA road contracts, on the other hand, found no evidence of any correlation between bidders (Sugrue,1977).

7.5.3.4 Non-Serious and Unrealistic Bids

Whittaker (1970), as a result of his interviews with several construction companies, reported that "... the management concerned stated that all bids were 'serious and competitive' ... these were contracts that the company would have liked to win". McCaffer (1976a), however, who has some considerable experience in this field, has warned that some allowance may be needed for unrealistic bids in modelling competitors' bids.

One type of non-serious bid is known as the 'cover price', where the bidder enters a bid the value of which is advised by a competitor. The Institute of Quantity Surveyors (IQS) Sussex branch (1979), in an opinion survey involving "... a few individuals earning their living in preparing bills of quantities, estimates, managing contracts and business", found that cover prices are taken notwithstanding attempts to prevent the practice, adding that "... the responses showed a marked unanimity". The report concluded, however, that the cover prices "did not distort market prices". Daniels (1978), in describing the work of the Builders' Conference, revealed that bidders admitted to the use of 'cover prices' because of the cost of bidding, the high risk of losing, not wishing to offend and the short period allowed for building preparation. Moyles (1973) has suggested that, because of these constraints "... contractors will usually give detailed attention only to desirable contracts", the remainder being "... prepared in a more approximate manner with a risk allowance to cover for unforseen circumstances and for the less accurate method of estimating". Indeed, discussions at a conference entitled "Estimating, the Way Ahead" (1979), organised by the Building Trades Journal, openly revealed the practice of taking such cover prices, discussing alternative methods of acquiring such prices

The methods adopted by researchers in discounting these non-bona-fide bids have been inconsistent. Southwell's attempts (1971) to model bid sets simply excludes non-serious bids without further comment. Franks (1970) in comparing the variability of students' estimates with bids obtained for several "live" projects, arbitrarily excludes the upper of 20% of bids as being probably non serious. Morrison & Stevens (1980) have considered excluding the highest two bids in each set, whilst Whittaker's analysis (1970) of 153 contracts excluded all bids exceeding the average bid by a factor of 6 and any obviously abnormal sets (for instance where one bid was more than 21% higher than the next highest bid) were eliminated. Whittaker also imposed an additional restriction by including only the bids which satisfied the condition

$$(\text{highest bid} - \text{lowest bid})/\text{mean bid} \qquad < 24\%$$

Pimm (1974), on the other hand, along with the majority of bidding strategists, does not advocate rejecting bids that look "wrong", although he suggests excluding bids his own firm know to be wrong because of arithmetical or judgemental errors.

McCaffer (1976a) claims to have discovered the presence of outliers during the performance of the Anderson-Darling test, due to the formation of unexpectedly long tails in the analysis. Since his data appeared to have been drawn from a general Normal distribution, a test developed by Grubbs (1950) is recommended. The test, however, has been criticised as inappropriate in this case, as the sample sizes are too small and that the presence of outliers is more likely to be indicative of a wrongly assumed shape parameter than an 'unrealistic' bid (Skitmore,1981a).

Johnston (1978), far from eliminating suspect bids, considers them to be of great importance and, in calculating skewness, has suggested a possible correlation with the industry's work load. Analysis by Skitmore (1981) of a different set of data has rejected Johnston's findings.

A further reason for retaining the so-called 'unrealistic' bids is that some companies have been found to have quite distinct bidding behaviour and what appears to be an unrealistic bid may be a genuine bid in some cases. In any event, non-serious bidders are not likely to have any effect on low bid models (Weverberg,1981; Beeston,1983). It is concluded, therefore, that, in the absence of any reliable predictor of known non-serious bids, it would be advisable to retain all bids in the model.

7.5.4 Data Limitations

A major criticism of models of competitors' bidding behaviour is based on the difficulty in obtaining the necessary data. Friedman's (1956) model is particularly susceptible to this criticism, demanding, as it does, the collection of bid/cost estimate ratios against each competitor in order to construct a frequency distribution of sufficient dimensions to enable a probability density function to be fitted. Such a quantity of data does not seem to be generally available in the construction industry (Grinyer & Whittaker,1973), a difficulty considered by some to bring into question the entire applicability of bidding models in the industry (Cooke,1981,p61). The situation deteriorates further when considering combinations of specified competitors due to the reduced amount of data available (Beeston,1983) to assess the joint probability distribution of each possible sub-set of competitors, 2^n for n competitors (Christenson,1965). Added to this are the typical characteristics of the construction bidding situation in that "... past histories of bidding behaviour are relatively short and only a small number of potential competitors participate in a particular contract" (Weverberg,1981). It is not surprising to find that "... the experience of the contractor studies seems to indicate that it is of little value to try and estimate the distribution of the bid-cost ratios of known competitors ... [as] there are relatively few competitors who are bid against often enough to provide sufficient information to estimate these distributions with any confidence" (Benjamin,1972,p328). A further difficulty that has been encountered is that not all (if any) competitors may be known for a project.

Data difficulties and unknown competitors have been anticipated by Friedman's (1956) collective competitors model, termed 'the average bidder', where all competitors are assumed to behave in a similar manner, that is, their bids are considered to be drawn from identical distributions. The majority of empirical studies rely on the collective competitors model, the individual competitor model being restricted to competitors encountered most frequently ('key' competitors), mainly due to the extreme difficulties involved in obtaining stable parameter estimates for individual competitors (see Capen et al,1971; Curtis & Maines,1973 and Fuerst,1977, for instance).

Apart from resorting to modelling the distribution of low bids, with the accompanying loss of information, only two approaches appear to be feasible. The first is to use the collective competitor model on the assumption that competing bidders do behave in a similar manner, and the second is to adopt a multi-variable approach along the lines of Weverberg (1982) and Skitmore (1982). There are some grounds for accepting the first approach to be reasonable for "... although for some companies quite distinct bidding behaviour in terms of mean and spread are found, pooling of companies into 'average competitors' does not seem to be a major cause of bias: for many companies behaviour is sufficiently similar" (Weverberg,1982,p26). In the context of the bidding problem it would seem that differences in spread are of more concern than differences in mean (Weverberg,1982,p62). Insofar as the second approach is concerned, data can be collected on all contracts irrespective of whether the collector enters a bid or not, thereby reducing the informational problems of the uni-variate approach normally employed. Success is not guaranteed, however, for, as Weverberg (1982) observes "... naive approaches based on pairwise independence and assuming univariate analyses are inevitably quite unreliable. Even using multivariate methods differences in bidding behaviour are not easy to detect".

7.5.5 Project Acquisition

The allocation of projects by potential clients depends on the client's allocation criteria which may include such factors as price, speed, quality, reliability, flexibility and control. These criteria determine the global procurement methods chosen (such as traditional, design and build, or management contract), the construction companies to be involved, and the value of the project. It is generally assumed that the price of the project is the client's main interest and, in the majority of cases, this is reflected in the competitive traditional approach where the lowest bidder is awarded the project. Such an approach is not the universal practice however. There are instances of projects being awarded to construction companies offering shorter durations or, directly or indirectly, greater reliability, better quality and greater financial security. On some occasions the second or even third lowest bidder has been known the acquire the project. One reason for this is the view that the lowest bidder may have entered a 'suicidally' low bid due to, perhaps, some gross deficiency in his cost estimate. Cauwelaert & Heynig's (1978) 'Belgian' solution proposes a method of identifying such low bids in order to avoid allocating projects to these bidders.

Simmonds (1968a) has proposed a method of modelling the various features offered by the company in its attempt to acquire a new project in terms of mark-up or mark-up equivalent. By this method, non-price features relative to competitors,are evaluated subjectively for their likely effect on project acquisition.

Very little evidence appears to be available on the impact of non-price features and the allocation of project to bidders other than those entering the lowest bid. Benjamin's (1969) analysis of 125 construction projects found only one case of a project being awarded to anyone other than the lowest bidder.

7.5.6 Factors Affecting the Likelihood of Entering the Lowest Bid

The competitive pressures in the construction industry, it has been said, are probably more intense than any other industry (Park,1972,p24.1). In the presence of such competition it is not altogether surprising to find that "... judging from the attitudes of some companies, competitive bidding does not result in competition based upon costs or profit margins, but actually produces a lottery in which the inherent uncertainty of the process decides the winner" (Whittaker,1970). Indeed, McCaffer (1976a) has found "... substantial evidence that existing bidding processes are little more than random". Pim's (1974) analysis of the number of projects awarded to four construction companies indicates that the average number of projects acquired is generally the reciprocal of the average number of bidders competing, the proportion that would only be won by 'chance' (Table 7.4).

Table 7.4 Frequency of low bids

Company	No. of Projects	No. of bids	Bids per project	% win by chance	% actually won
A	41	249	6.1	16.4	17.0
B	36	183	7.0	14.2	15.4
C	19	88	4.6	21.6	21.1
D	35	202	5.8	21.6	17.1

Source: Pim (1974,p541)

This would suggest an extremely simple model in which the probability P of entering the lowest bid is the reciprocal of n, the total number of bidders. The value of n, however, may not be known with certainty but may, as has been discussed earlier, itself be modelled by a probability density function say f(n).

Research by Broemser (1969), however, indicates that n is not significantly correlated with P, although many consider n to be a very important factor (Park,1962, for instance).

Empirical attempts to link other factors with P have also met with limited success. Gates (1967) and McCaffer (1976a), for instance, have examined (inconclusively) the influence of project size, and Broemser's (1968) MRA using several predictor variables was unable to explain most of the variance, concluding that "... we expect most of the remaining variance, ie, the standard error of 5.18%, is due to the difference between cost estimates".

A great deal of attention has centered at the theoretical level on the actions of individual competitors, information about which has been regarded as "critical" (Griesmer et al,1963). Most models implicitly require the analyst to develop probability distributions for any competitor's bid (Neufville et al,1977), which means that the model builder is faced with "... the problem of explicating probability laws for opposing bids" (Weverberg,1981). Carr (1982) has shown that differences in assumptions of the spreads of opposing bids can have significant effects on results, although asymmetrical information (different spreads) produce a "very messy" theory (Klein,1976). Edelmann (1965) and Flanagan & Norman (1982) have, nevertheless, derived a matrix of award probabilities for each bidder.

In recognition of the informational difficulties, Park (1966) has suggested considering individual competitors when six or less are present and Morin & Clough (1969) have used the 'key' competitor analysis for those competitors encountered on in least 40% of bidding situations.

The most popular factor that has been associated with P is the difference between the bid and cost estimate, commonly termed the 'mark-up', as this represents "... the underlying assumption of bidding theory in that for each marginal change of mark-up there is a corresponding change in the probability [P] of success " (Cooke,1981,p61).

7.5.7 The Probability (P) of Entering the Lowest Bid as a Function of Mark-Up

It has been assumed that a company may estimate the prior probability of P for a "particular bid", this probability being determined from the company's expectations of its competitors' bids. It follows, therefore, that P "... will vary continuously with the amount bid which may be varied almost continuously" (Benjamin,1970). Edelmann's (1965) model relies on the intuitive assessment of the value of P as a function of the bid. An alternative approach has been to fit a curve to the percentage cumulative observed rate of bidding success for a given cost plus a percentage mark-up (Benson,1970). Friedman (1956) has suggested that a value for P for a particular project can be estimated by combining the probabilities of underbidding each individual competitor. Friedman's model has been described many times, probably the most cogent description being that of Fuerst (1976):

> "Assume for a letting under consideration that the bid of each competitor, $C_i, i=1,\ldots,n$, is independently drawn from $g_i(b_i|c)$ - a probability density function of the bid of C_i, conditional upon the actual cost,c. Also, any competitor winning the contract is assumed to have the same actual cost. Therefore
>
> $$P(C_1 \text{ wins}) = \int_{b_1=0}^{\infty} g_1(b_1) \int_{b_2=b_1}^{\infty} g_2(b_2 \ldots \int_{b_n=b_1}^{\infty} g_n(b_n)db_n \ldots db_2,db_1$$
>
> in which, for notational ease, $g_i(b_i)$ has been written for $g_i(b_i|c)$"
>
> (Fuerst,1976,p174).

If bidder C_1's bid is replaced by cost estimate c_1, plus a mark-up m then the above becomes

$$P(C_1 \text{ wins}|m) = \int_{c_1+m=0}^{\infty} f_1(c_1)+m. \int_{b_2=c_1+m}^{\infty} g_2(b_2) \ldots \int_{b_n=c_1+m}^{\infty} g_n(b_n)db_n \ldots db_2,dc_1$$

Friedman then advocates obtaining $f_1(c_1)$ and $g_i(b_i)$ empirically, based on observations from past lettings, from the frequency distributions of actual cost/cost estimate and bid/cost estimate ratios respectively. However, as Fuerst (and others) observe "... both a cost estimate and a competitor's bid should be considered random variables, and the density function of the ratio of two random variables is almost always complexly related to the density functions of the individual random variables". A further problem that occurs with the use of ratios is that, if the ratio of bids and cost estimates is to be considered as truly independent of the bids and cost estimates, then the ratio must always be constant, as proved by Rothkopf (1980).

Two alternative approaches appear to be available in circumventing these problems. One approach adopted by Grinyer & Whittaker (1973) assumes that the ratio of bids to average bids, for any project, form a Uniform density function and then estimating, via a combination of managerial judgement and past data, the value of the mean of the density function for each contract to be bid. Grinyer & Whittaker's assumption of Uniformly distributed bids has, however, been rejected by McCaffer (1976a) on methodological grounds, as previously mentioned. McCaffer (1976a) has also questioned the accuracy of prediction of the mean bid by this approach.

The other approach has been to utilise the distribution of low bids, irrespective of the identity of the bidders, by compiling a frequency distribution of low bid/cost estimate ratios. This approach, whilst overcoming the problems to some extent, suffers from informational loss, as already discussed. It has, however, been claimed to be increasingly beneficial as individual competitors' distributions differ (Weverberg,1981,p19). This must clearly depend on the predictability of the identity of the individual competitors. Estimates of P based on the low bid distribution are likely to be rather poor in the absence of a consistently keen, but absent, competitor, for instance.

A third and, as yet untried, approach avoids the use of ratios entirely in estimating function parameters by a multi-variate technique (Skitmore, 1982). The advantage of this approach is that it avoids the usual problems associated with ratios by dealing with the log values of the bid and cost estimate variables, thus enabling these variables to be handled separately. Some aspects of this approach are examined in the next Chapters.

A further difficulty that arises with the above formulae is that bids, as has been discussed, are not expected to be independent, for several reasons. No models have been proposed, however, to deal with this problem in situations where more than two bidders are involved.

Two theoretical conclusions are of interest with the Friedman model. Firstly, where symmetric information exists (identical functions) for all competitors, a value of P can be estimated by order statistics (Curtis & Maines,1973; McCaffer,1976a; Klein,1976 and Mitchell,1977, for instance); and secondly, it has been shown that, under certain restrictive conditions, the expected value of the winning bid "... is surely equal to the true value" of the project (Wilson,1979 and Milgrom,1979, for instance).

Summary and Conclusions

This chapter has examined in some detail the possibility of simplifying some aspects of the project selection decision by means of statistical models. The frequency distribution of the size of the project opportunities has been found, in one case, to be Exponential and the work content of the projects modelled as a series of random variables. A Poisson model has been proposed to model the distribution of the number of competitors involved and several regression models have been devised to predict the number of competitors, none of which appears to have been been particularly successful. A predictive technique termed the multidistribution model (MD) has been used to predict the identitiy of competitors and further models have been developed which purport to give the probability of certain competitors being present in a project bidding situation.

Actual and estimated costs have been extensively modelled stochastically, the degree of independence between these two variables being a major debate. Individual aspects of costs and estimated costs have been considered amenable to statistical modelling, including labour, materials, sub-contractors, quantity related costs, weather and seasons, costs of estimating and additional costs. Many proposals have been reviewed defining the nature of cost distributions and these are summarised in Table 7.2.

Relatively little attention has been paid to modelling project income in a statistical manner, except that a Beta distribution has been applied to cash flows.

The bidding behaviour of competitors, both collectively and individually, has been the subject of many statistical models, the distributional characteristics of which are summarised in Table 7.3. Some possible effects of market conditions have been noted. The distribution of low bids in relation to cost estimates has also been treated in a similar manner, a Normal probability density function often being considered appropriate. An interesting result arising from the analysis of three sets of published data indicates the expected value of low bid/cost estimate ratio to be approximately unity.

The behaviour of individual competitors has been modelled separately in some cases. The assumption that the behaviour of bidders is independent of events, including the actions of competitors, 'economic conditions', type and possibly size of project and location has been questioned as being an oversimplification although little evidence appears to be available to determine the significance of this. The use of MRA has been recommended in identifying correlations of these events with individual bids.

The possibility of collusion has also been discussed and is considered to be rarely practised in the construction industry. It is thought, however, that a subverted form of collusion may exist because of commonalities between companies. Non-serious and unrealistic bids have also been considered for possible separate treatment with the conclusion that, in the absence of any identification procedures, such bids may be more fruitfully retained in any general analysis.

Data limitations appear to be severe except for collective competitors, low bids and certain key competitors. A multi-variate technique, however, has been proposed which may alleviate the problem.

Project allocation has been considered to depend on many possible client controlled factors and a method has been reviewed which aggregates these factors into a mark-up adjustment.

The effect of the mark-up on the probability (P) of entering the lowest bid has been found to be contained in a model first proposed by Friedman (1956). The model essentially requires some knowledge of the probability distribution of bids for each competitor and the probability distribution of actual/estimated costs of the decision-maker's organisation. The problems associated with bid/estimated cost and actual/estimated cost ratios indicate that an alternative approach may be required. The use of collective and low bid model has been considered and the multi-variate approach again identified as a possible suitable alternative.

Statistical models would appear to be reasonable approximations of many aspects of the project decision environment, if only because of the volume of studies reported, only a sample of which, have been reviewed in this Chapter. Perhaps the most important aspect of the entire project decision system is whether a project is acquired or not, as this event will have a considerable impact on the state of the outcome environment, both initially and particularly over a period of time. The liklihood of project acquisition would seem, as has been seen, to be a strong candidate for modelling in a statistical manner in terms of the probability of entering the lowest bid for the project. An estimate of this probability, if it can be obtained with sufficient accuracy, could then be applied to the model states previously outlined by inputing a probabilistic element to those states of the model that have hitherto been regarded as conditional upon acquisition.

The remaining Chapters describe an empirical study investigating the suitability of simple statistical modelling, including some of those reviewed in this Chapter, of aspects of the project selection and bidding problem.

CHAPTER 8

Analysis of bidding data

8 ANALYSIS OF BIDDING DATA

8.1 Introduction

This chapter contains a summary of the first part of an empirical analysis of bidding data for construction projects. The data consisted of three sets (referred to as Cases 1,2,3) of bids entered for projects in two geographical locations of the UK. Details are provided in Appendix B.

Three aspects are examined to identify the suitability of modelling the data in a simple statistical manner, the frequency of project values, the frequency of the number of bids entered for each project and the distribution of bid values for each project. The second part of the analysis, which examined aspects involving the individual bidders, is continued in Chapter 9.

8.2 Project Values

8.2.1 Distribution of project values

Figures 8.1 to 8.3 show the frequency of project values (lowest bids) as histograms for each of the 3 cases studied. Cases 2 & 3 provide data for all projects let in two geographical regions whilst Case 1 covers only projects in which bids were entered by one firm. The histograms for Cases 2 & 3 suggests that an exponential model may well approximate the data and a chi-square test may also indicate a good fit (cf. Hossein, 1977). However, closer inspection of the left-hand tails of Cases 2 & 3 (Figures 8.4 & 8.5) suggest that lower valued contracts do not behave as expected by the exponential distribution as the frequencies flatten out and start to fall with very low value projects. Several parametric distributions thought to have a similar shape to the shape indicated by the histograms were fitted to the data and tested for goodness of fit.

8.2.2 The Exponential model

The Exponential distribution was fitted to the standardised values $y_i = x_i/\bar{x}$ and the observed frequency of y_i compared with the expected

Fig 8.1 Case 1: Frequency distribution of project size (lowest bid)

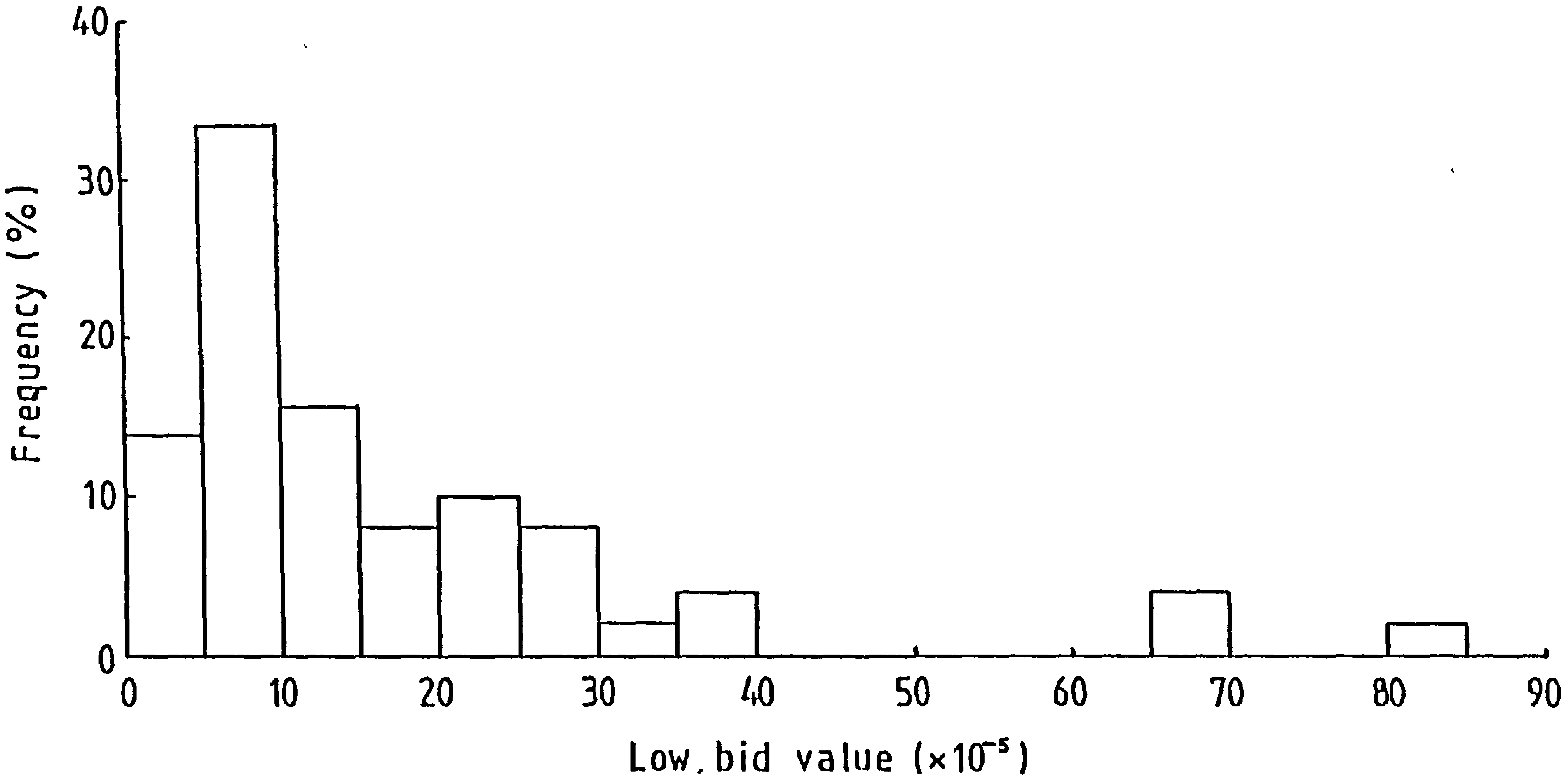


Fig.8.2 Case 2: Frequency distribution of project size (low bids)

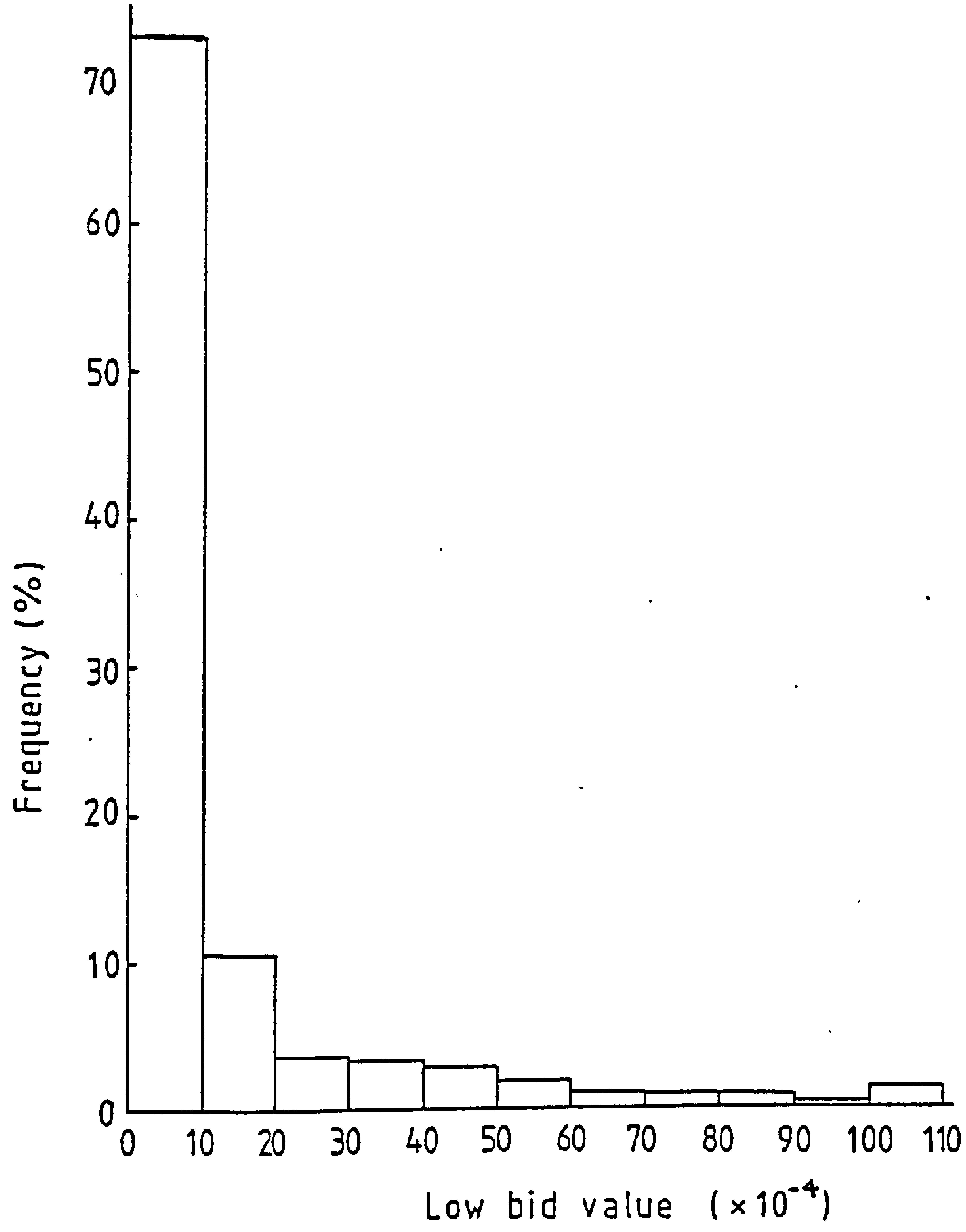


Fig 8.3 Case 3: Frequency distribution of project size (low bids)

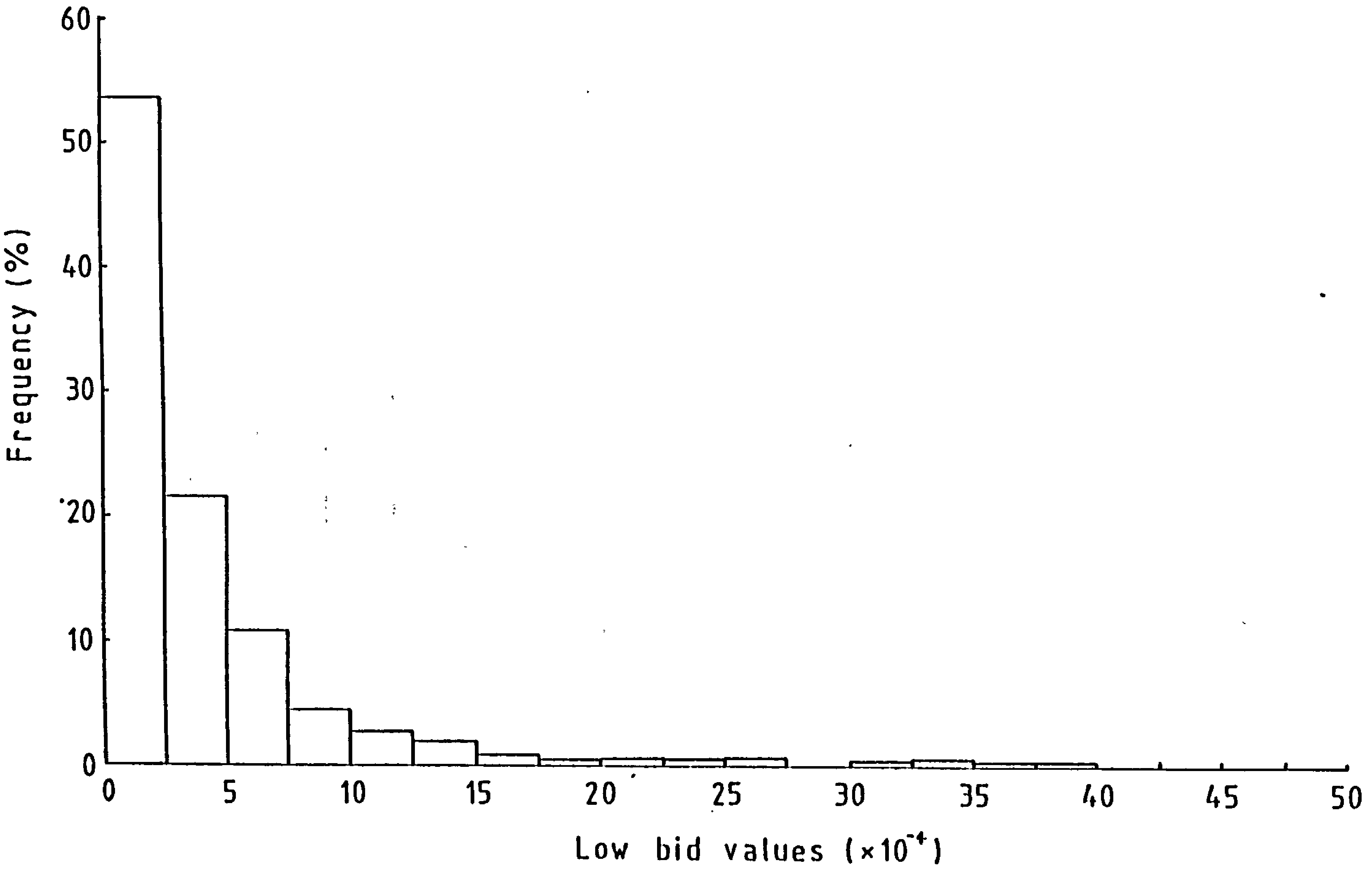


Fig 8.4 Case 2: LH tail of frequency distribution of project size (low bids)

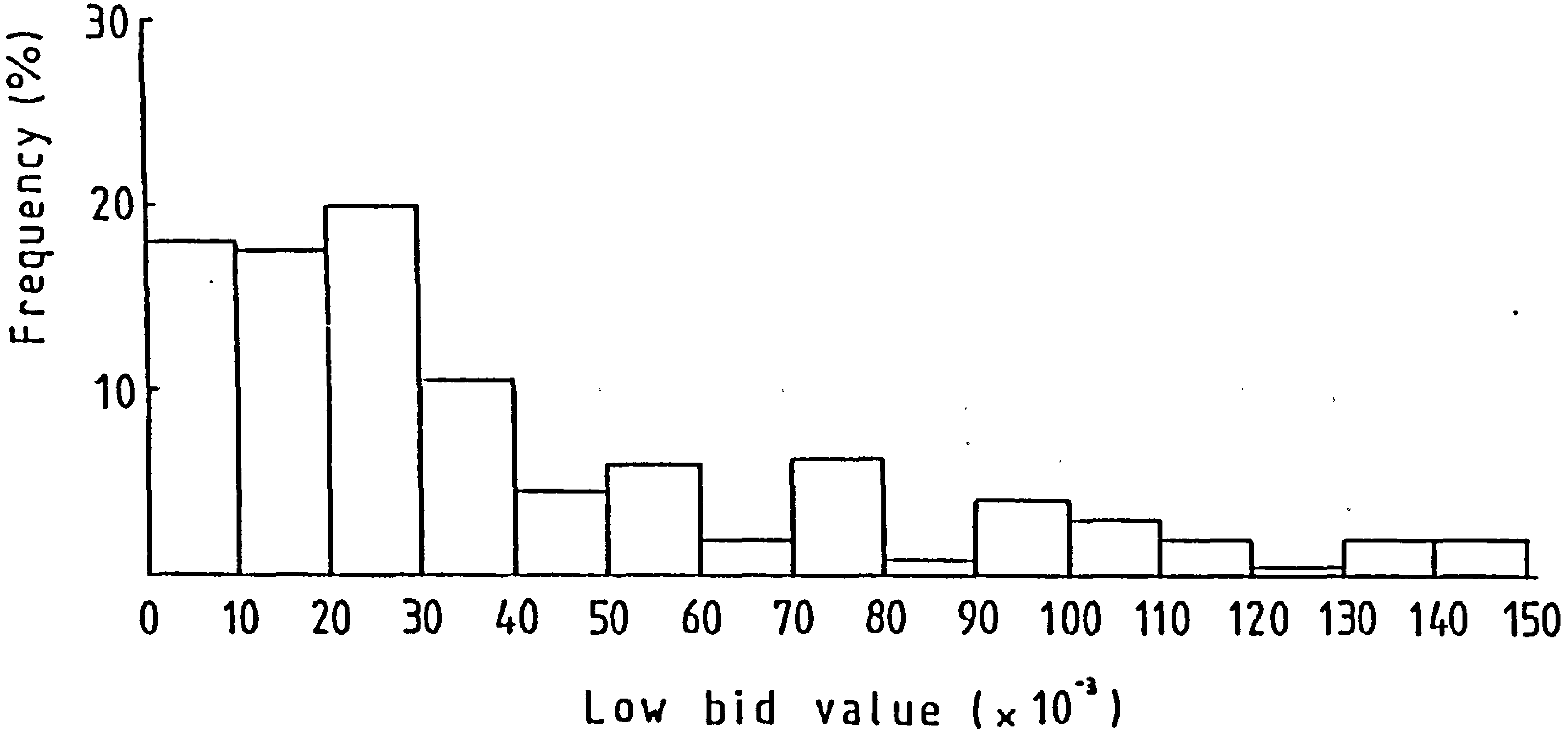
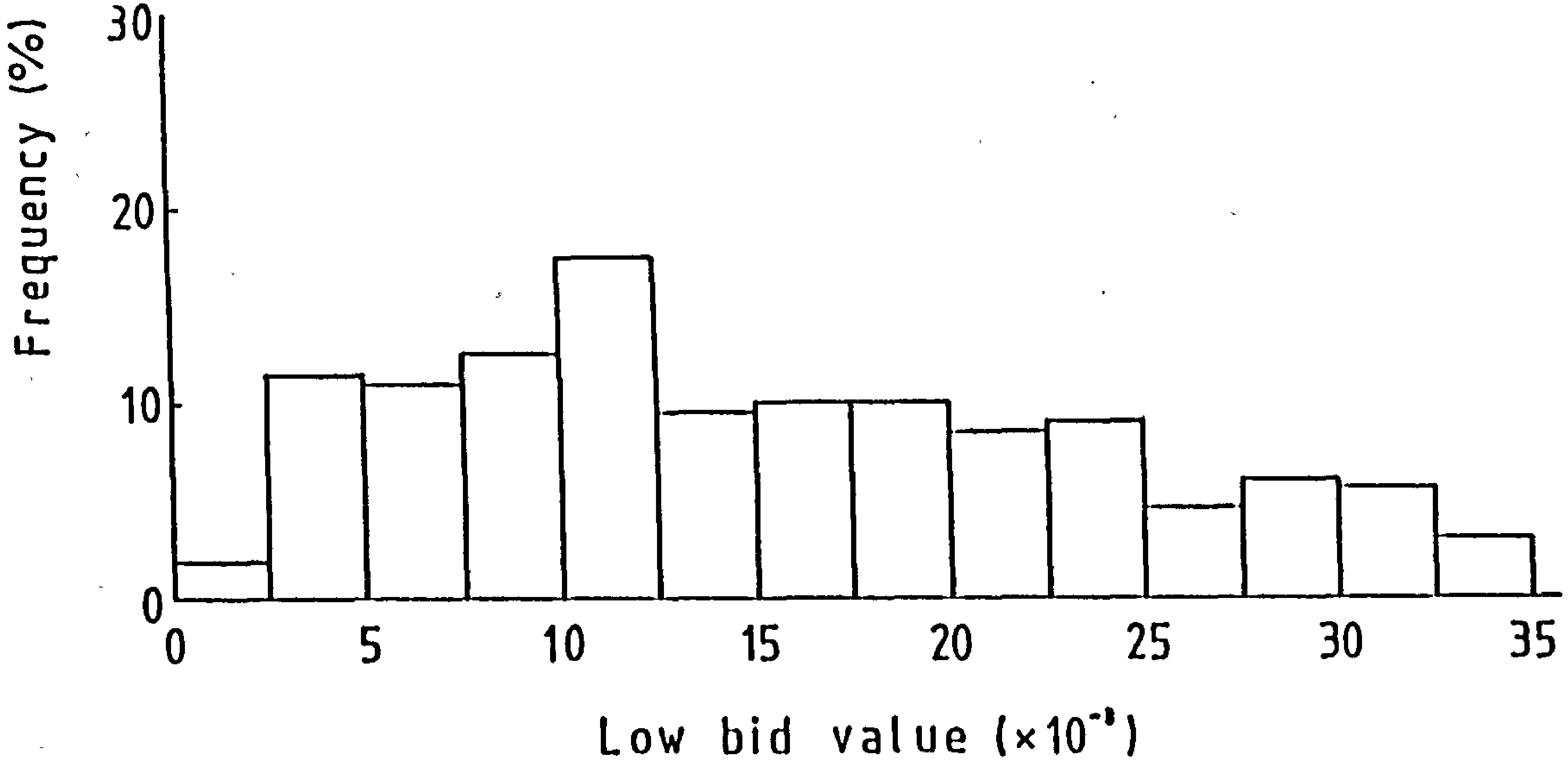


Fig 8.5 Case 3: LH tail of frequency distribution of project size (low bid) up to 250000



frequency $\Pr(y_i) = 1-e^{-y_i}$. For Case 1, both the chi-square test ($X^2_{(6)} = 8.9$) and the Kolmogorov-Smirnov test ($K\text{-}S_{(51)} = 0.16$) do not reject the null hypothesis at the 5% level. Case 2 ($X^2_{(16)} = 164.6$), $K\text{-}S_{(218)} = 0.28$) and Case 3 ($X^2_{(24)} = 92.5$; $K\text{-}S_{(373)} = 0.12$), however, do reject the null hypothesis at the 5% level. This result is not surprising for it is known that, for the Exponential distribution $\mu_y=\sigma_y$ which implies for these standardized values that s_y approximates to unity, which clearly does not apply in Cases 2 and 3.

8.2.3 The Gamma model

The Gamma distribution was fitted to the values x_i such that $f(x) = \lambda(\lambda x)^{k-1}\, e^{-\lambda x}/\Gamma(k)$, where $k = \bar{x}^2/s_x^2$ and $\lambda = \bar{x}/s_x^2$. The observed frequency of x_i was compared with the expected frequency of $f(x)$. In all cases except the K-S test for Case 1 the null hypothesis was rejected at the 5% level. This result is also not surprising as, for values of k less than unity, the shape of the Gamma distribution is a reverse J shape similar to that of the Exponential distribution. Such a shape, as was previously noted, is not likely to be very representative of the distribution of the observed values.

8.2.4 The Beta model

The Beta distribution of the first kind was fitted to the standardised values $y_i = (x_i - \min x)/(\max x - \min x)$

$$f(y) = \frac{\Gamma(t)}{\Gamma(r)\Gamma(t-r)}\; y^{r-1}\,(1-y)^{t-r-1},$$ values for r and t being

estimated from $\bar{y} = \dfrac{r}{t}$ and $s_y^2 = r(t-r)/t^2(t+1)$. The observed frequency of y_i was compared with the expected frequency of $f(y)$. In all cases both the chi-square and K-S test reject the null hypothesis at the 5% level. A comparison of the observed coefficient of skewness Y_{1y} and that expected of the Beta distribution ie.

$$Y_1 = \frac{1}{\sigma^3}\,\frac{r}{t}\left\{\frac{(r+2)(r-1)}{(t+2)(t+1)} - \frac{3r(r+1)}{t(t+1)} + \frac{2r^2}{t^2}\right\}$$

indicates the differences (Table 8.1).

Table 8.1: Project values and the Beta distribution - comparison of expected and observed coefficients of skewness

Statistic	Case		
	1	2	3
Expected γ_{1y}	1.25	3.01	2.73
Observed γ_{1y}	2.31	4.34	5.51

8.2.5 The Normal model (log transformation)

The Normal distribution was fitted to the standardized log values, $z_i = (y_i - \bar{y})/s_y$ where $y_i = \ell n(x_i)$ and the observed frequency of z_i compared with the expected frequency of N(0,1). All tests except the K-S test for Case 2 failed to reject the null hypothesis at the 5% level.

8.2.6 The Normal model (log-log transformation)

The normal distribution was fitted to the standardized log-log values, $z_i = (y_i - \bar{y})/s_y$, where $y_i = \ell n\,[\ell n(x_i)]$, and the observed frequency of z_i compared with the expected frequency of N(0,1). All tests failed to reject the null hypothesis at the 5% level. The chi-square tests of the standardized values for Case 2 is given in Table 8.2.

Table 8.2: Case 2 - Log-log transformation contract size (low bid) distribution

Chi-Square Test of Fit
Normal Distribution

From	To	Obs	Z	PHI	EXP CUM	EXP	X^2
-INF	-1.750	11	-1.750000	0.040059	8.732896	8.732896	0.588552
-1.750	-1.500	5	-1.500000	0.066807	14.563970	5.831074	0.118449
-1.500	-1.000	16	-1.000000	0.158655	34.586845	20.022875	0.808252
-1.000	-0.750	8	-0.750000	0.226627	49.404763	14.817917	3.137013
-0.750	-0.500	26	-0.500000	0.308538	67.261183	17.856421	3.713952
-0.500	-0.250	28	-0.250000	0.401294	87.482021	20.220838	2.992723
0.250	0.000	23	0.000000	0.500000	109.000000	21.517979	0.102072
0.000	0.250	16	0.250000	0.598706	130.517979	21.517979	1.415007
0.250	0.500	16	0.500000	0.691462	150.738817	20.220838	0.881045
0.500	0.750	17	0.750000	0.773373	168.595237	17.856421	0.041075
0.750	1.000	14	1.000000	0.841345	183.413155	14.817917	0.045147
1.000	1.250	14	1.250000	0.894350	194.968349	11.555195	0.517263
1.250	1.500	7	1.500000	0.933193	203.436030	8.467681	0.254389
1.500	+INF	17	+INF	1.000000	218.000000	14.563970	0.407461
		218				218.000000	15.022399

Chi-square of 15.022399 (13 df), has probability of 0.305937

8.2.7 Conclusions

Table 8.3 summarises the results of the various models applied to the project values. As can be seen from the table, only the Log-lognormal model appears to fit all cases without rejection by the tests applied.

Table 8.3: Modelling project values

Statistic	Case		
	1	2	3
Exponential ($y_i=x_i/\bar{x}$)			
$\bar{y}$	1.00	1.00	1.00
s_y	1.008	2.031	1.596
χ^2	$8.88^{*}_{(6)}$	$164.6_{(16)}$	$92.5_{(24)}$
K-S	$0.16^{*}_{(51)}$	$0.28_{(218)}$	$0.12_{(373)}$
Gamma			
$\bar{x}$	1639369	138398	457650
s_x	1652622	281142	730440
$k=\bar{x}^2/s_x^2$	0.984	0.242	0.393
$\lambda=\bar{x}/s_x^2$	6.002 E-07	1.751 E-06	8.578 E-07
χ^2	$23.23_{(7)}$	$37.78_{(10)}$	$89.45_{(10)}$
K-S	$0.16^{*}_{(51)}$	$0.269_{(218)}$	$0.265_{(373)}$
Beta $\{y_i=(x_i-\min x/(\max x-\min x)\}$			
min x	248733	1172	8941
max x	7831865	2257024	8553300
$\bar{y}$	0.183	0.061	0.053
s_y	0.218	0.125	0.085
r	0.395	0.163	0.305
t	2.153	2.678	5.808
χ^2	$26.16_{(6)}$	$28.34_{(9)}$	$84.995_{(10)}$
K-S	$0.232_{(51)}$	$0.388_{(218)}$	$0.309_{(373)}$
Lognormal			
$\bar{y}$	13.945	10.731	12.421
s_y	0.834	1.463	1.066
χ^2	$9.29^{*}_{(6)}$	$32.76_{(14)}$	$12.68^{*}_{(17)}$
K-S	$0.109^{*}_{(51)}$	$0.073^{*}_{(218)}$	$0.038^{*}_{(373)}$
Log-lognormal $\{y_i=\ell n(\ell n(x_i))\}$			
$\bar{y}$	2.633	2.364	2.516
s_y	0.059	0.137	0.086
χ^2	$6.68^{*}_{(5)}$	$15.02^{*}_{(13)}$	$7.37^{*}_{(16)}$
K-S	$0.107^{*}_{(51)}$	$0.055^{*}_{(218)}$	$0.021^{*}_{(373)}$

* Null hypothesis not rejected at 5% level

8.3 The Number of Bidders

8.3.1 The distribution of the number of bidders per project

Friedman (1956) has suggested that the number of bidders k might have a Poisson distribution. That is if λ is the estimated number of bidders then:

$$g(k) = \lambda^{k-1} e^{-\lambda}/k$$

Estimates of $\lambda = \bar{x}$, where x_i is the number of bids recorded for each project, were obtained from the data and the Poisson function g(x) fitted (Figures 8.6 & 8.7). In all cases the null hypothesis was rejected by the chi-square test at the 5% level.

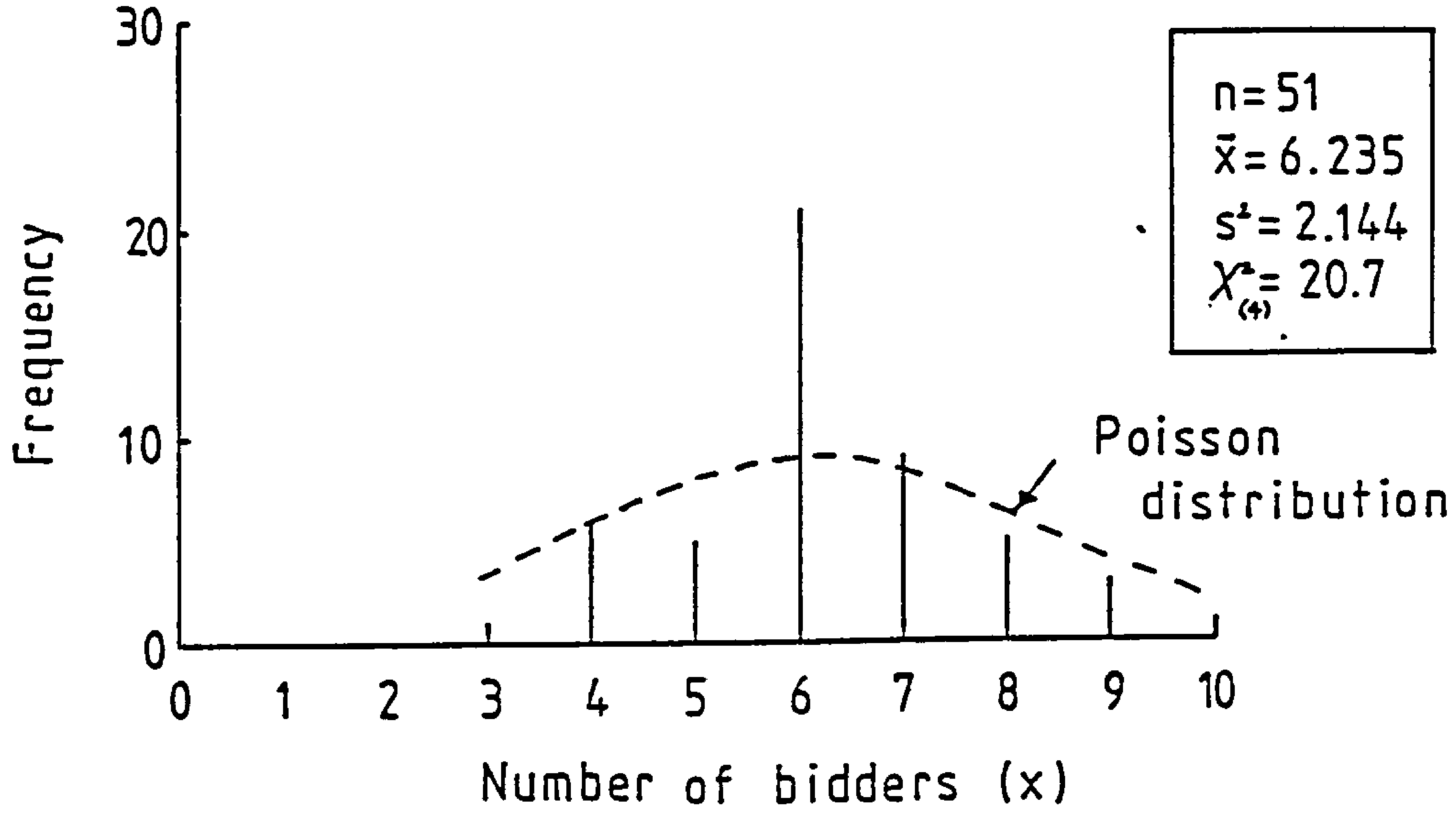
Friedman (1956) has further suggested that better estimates of λ may be obtained by predictions obtained from regressing the project value on the number of bidders. A regression was performed on the data of the project values (low bids) on the number of bids entered for each project (Table 8.4).

Table 8.4: Results of regression of project value on the number of bids per project

Case	α	β	Prod.mom corr.coeff	$t(\beta=0)$
1	5.825	2.501 E-07	0.282	$2.06^{*}_{(49)}$
2	5.245	3.037 E-07	0.378	$6.00^{*}_{(216)}$
3	4.918	4.729 E-07	0.178	$3.48^{*}_{(371)}$

* rejects null hypothesis at 5% level.

Fig.8.6 Case 1: Frequency of bids per project



Case 2: Frequency of bids per project

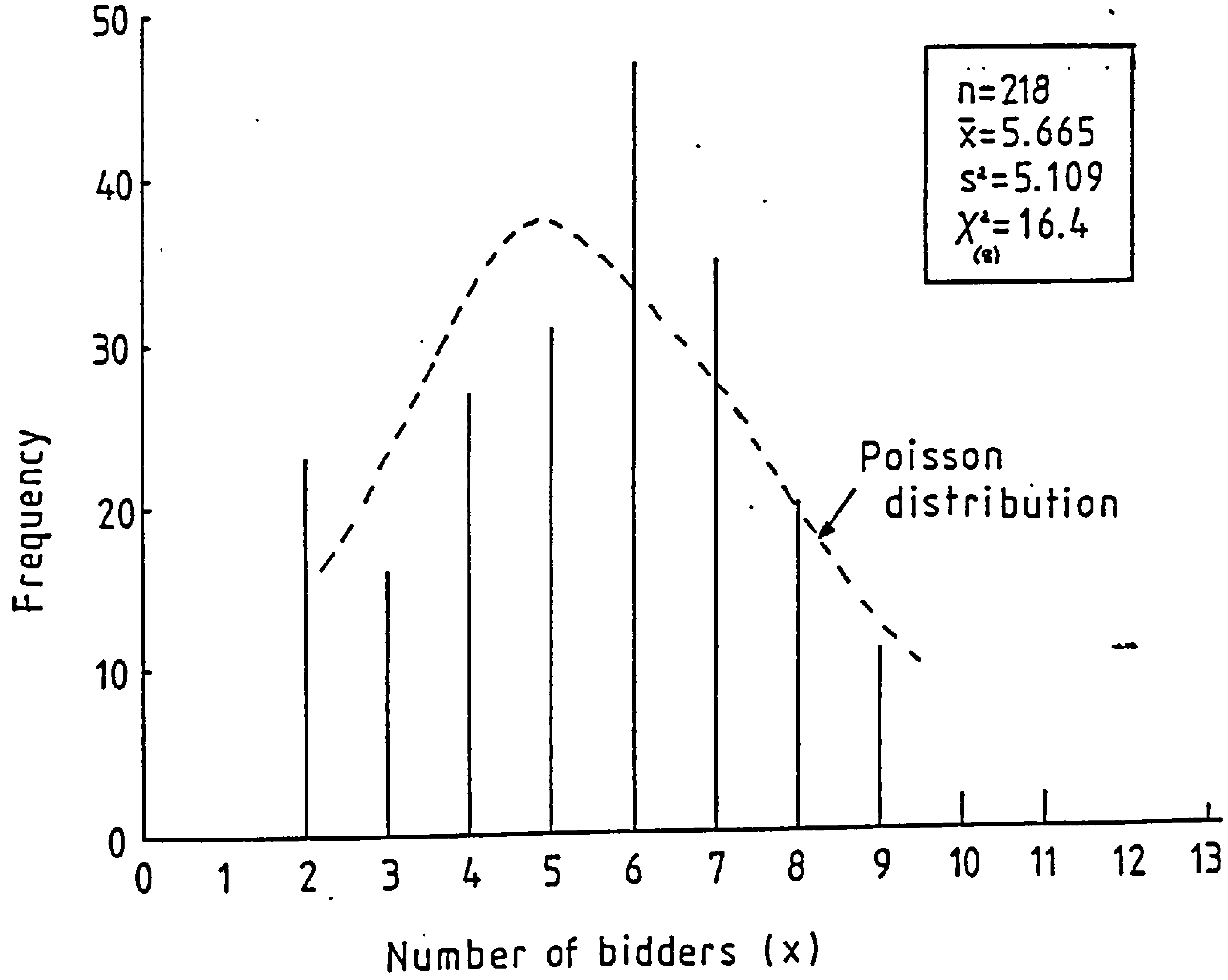
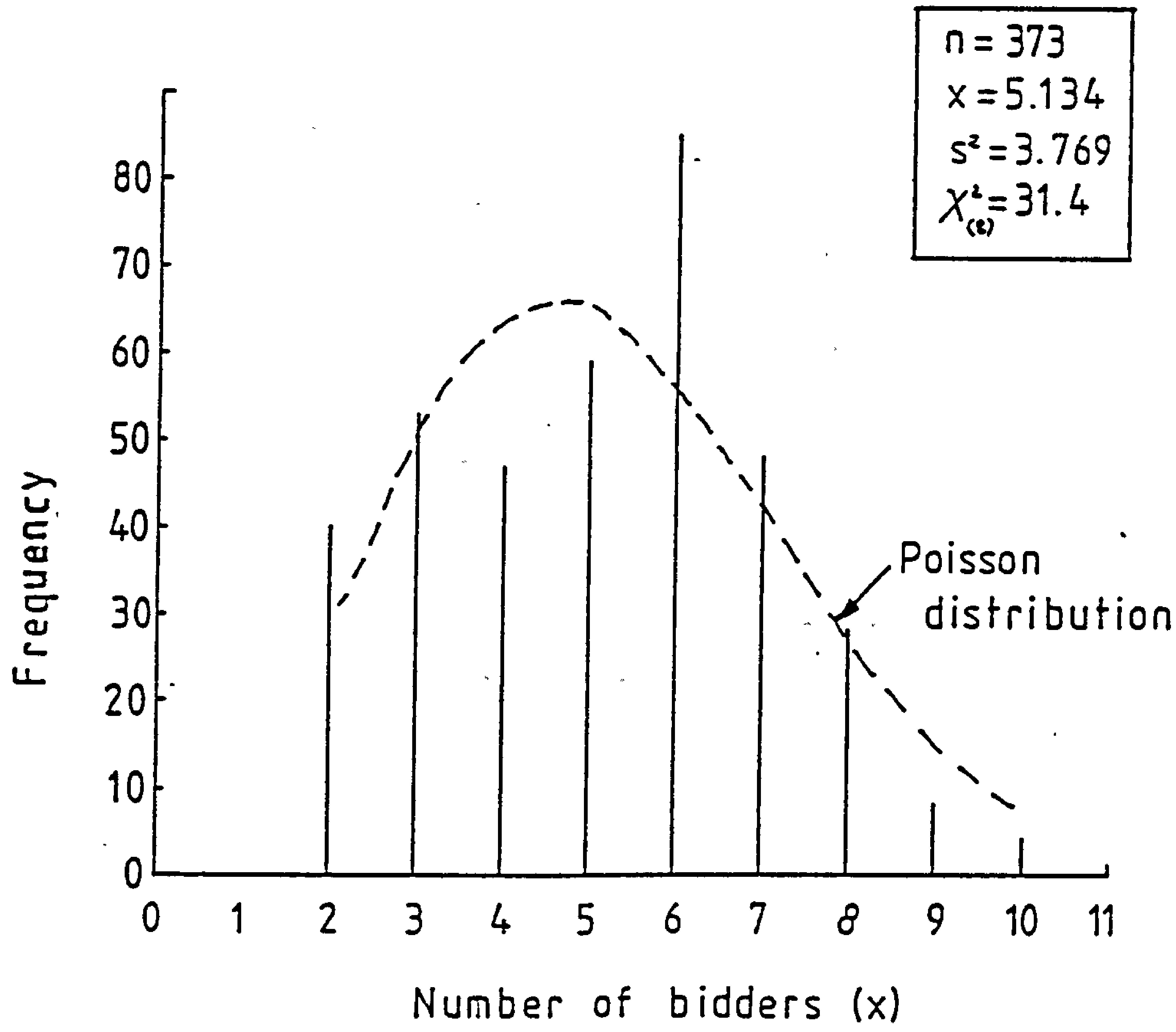


Fig.8.7 Case 3: Frequency of bids per project



Although the product moment correlation coefficient was quite low, the t-test rejection of $\beta = 0$ hypothesis was taken to be an indication that some relationship between project value and the number of bids per contract may exist. However, such small values of β were found difficult to handle computationally and, in view of the advantages found with the log transformation of project values in the previous section, it was decided to use the log project values to predict the number of bids per project. The average log project values were calculated for projects containing x=1,2,.. bids. As Figure 8.8 shows, there appear to be marked differences in absolute values between the three cases although a similar trend is apparent in each case. For ease of comparison the log project values were plotted against the number of bidders per project and a polynomial least squares regression line fitted (Figures 8.9-8.11). Although some evidence was found in the Case 3 data of a curvilinear relationship, it was decided for the sake of simplicity to adopt a linear model. The results of the linear regression of log project value on the number of bidders per project are given in Table 8.5.

A second regression was performed, forcing the regression through zero for comparative purposes (Table 8.6).

Now having obtained a least squares linear prediction of the number of bids for a project, Friedman's assumption that the actual number of bids is a Poisson distribution can be tested as follows:-
Let the observed number of bids for project i be x_i
and the expected number of bids for project i be λ_i
then the probability that $0 < X_i < x_i$ is

$$Pr(x_i) = \Phi \chi^2 \{2(x_i+1)\} \, 2(\lambda_i)$$

and for all projects (i = 1,2,....,c), $Pr(x_i)$ will be uniformly distributed between zero and unity. The calculations for Case 1 are shown in Tables 8.7 and 8.8. Table 8.6 summarizes the results in all three cases.

Fig 8.8 Number of bidders by average project value

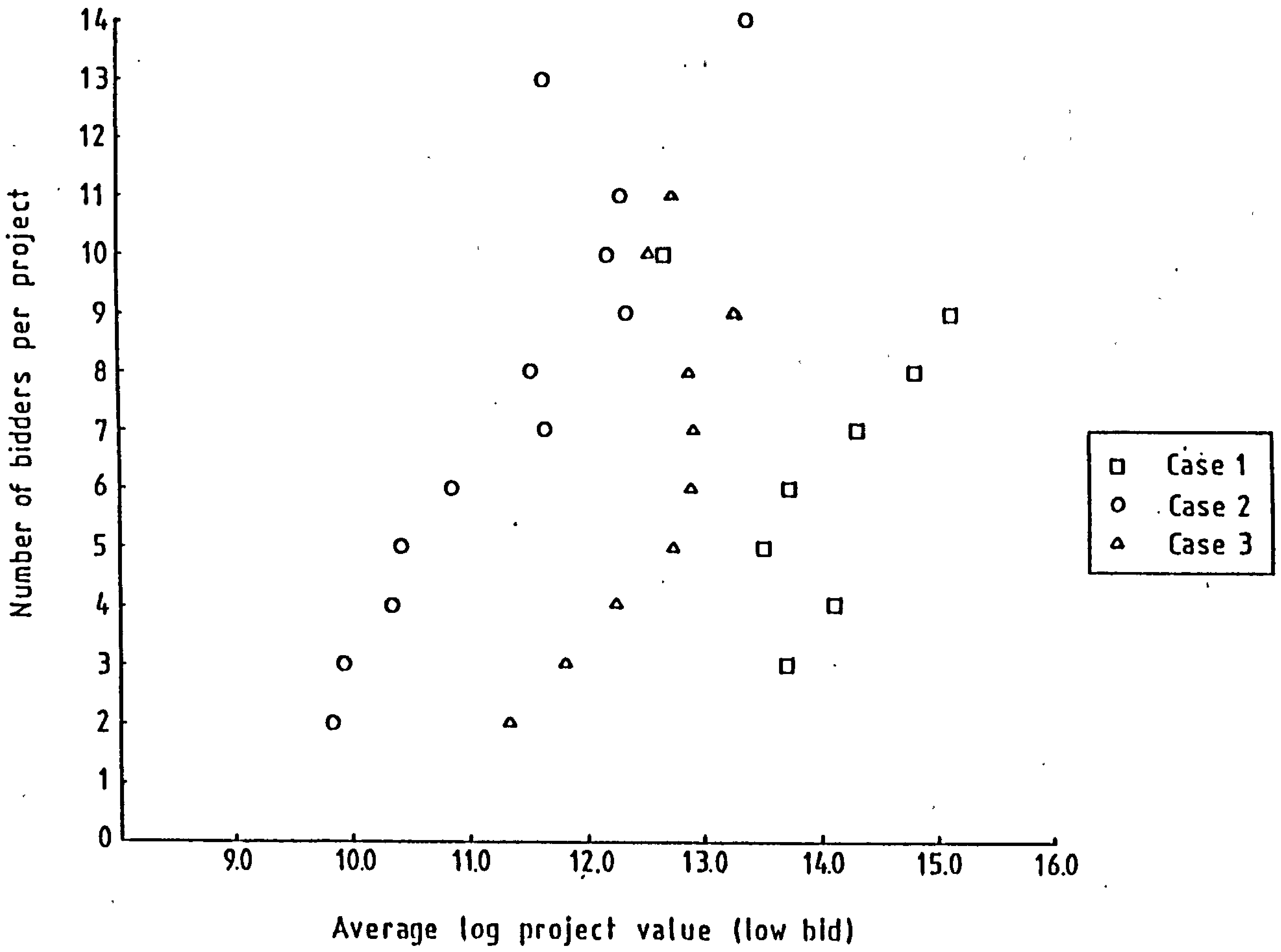


Fig 8.9 CASE1: NUMBER OF BIDDERS V. LOG CONTRACT VALUE

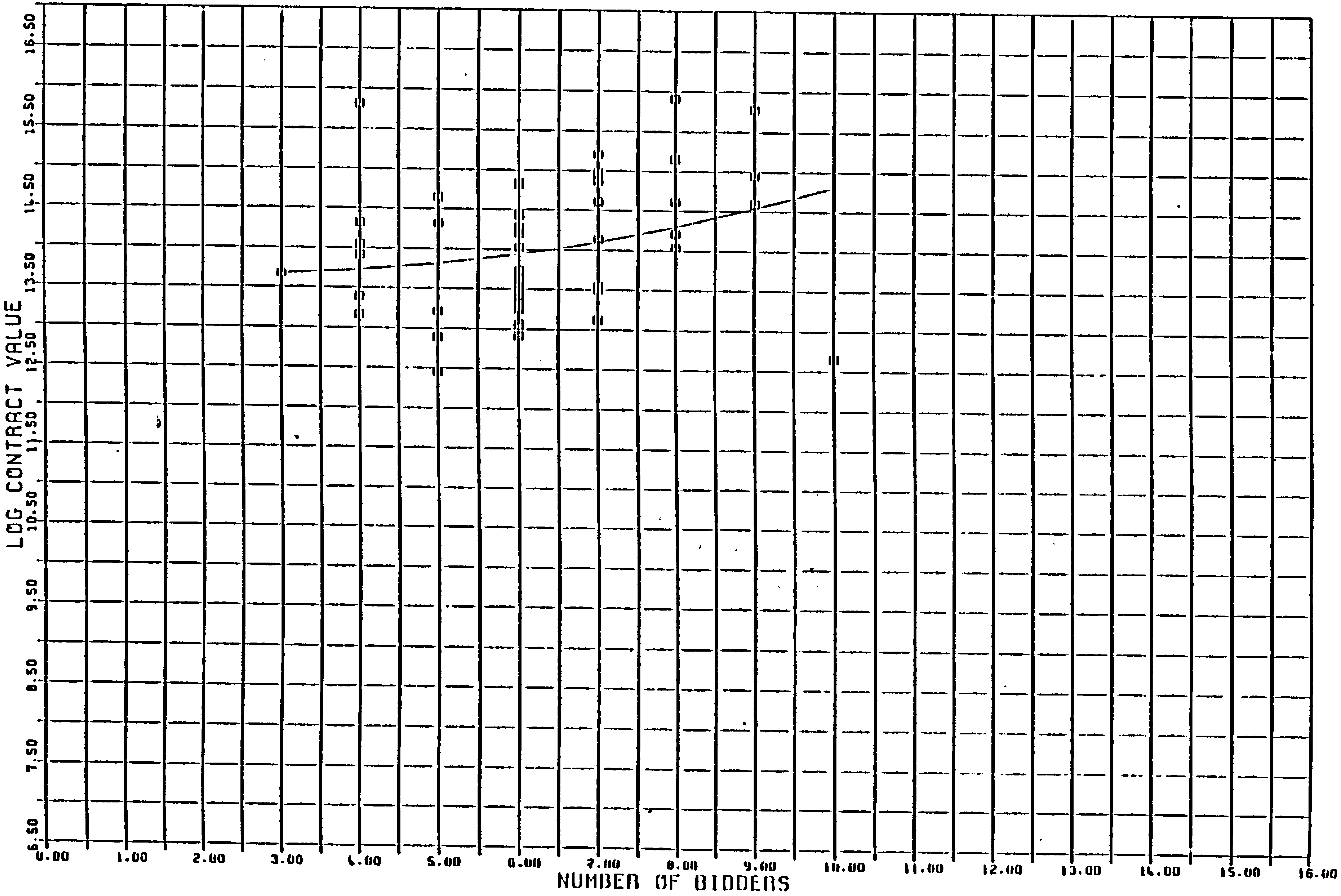


Fig 8.10 CASE2: NUMBER OF BIDDERS V. LOG CONTRACT VALUE

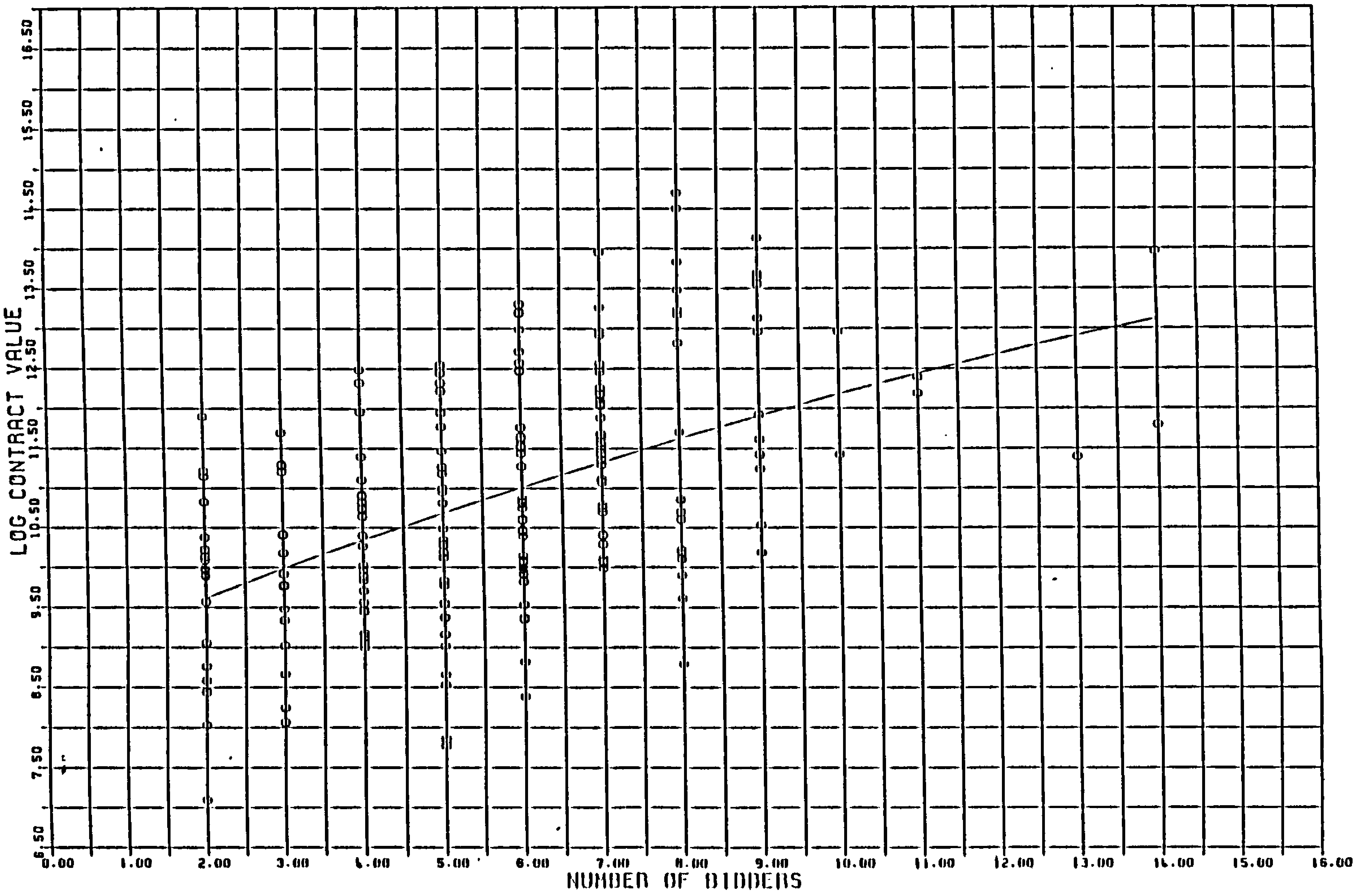


Fig 8.11

CASE3: NUMBER OF BIDDERS V. LOG CONTRACT VALUE

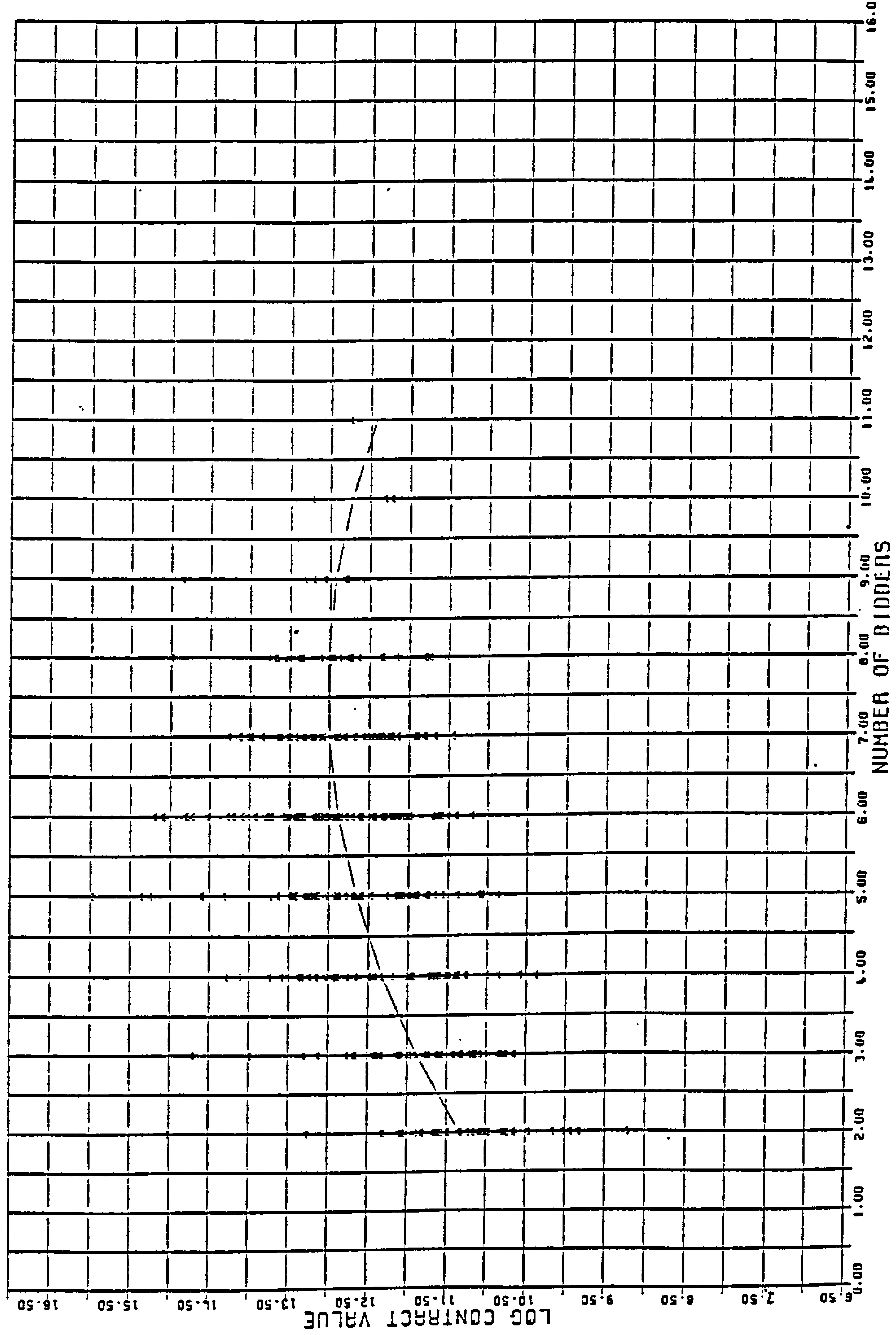


Table 8.5: Distribution of observed number of bidders around regression prediction

$n_j = \alpha + \beta y_{ij}$	Case		
	1	2	3
Regression Results			
$y_{ij} = \ell n\ (x_{ij})$			
Prod. mom. corr. coeff.	0.2714	0.489	0.460
α (SE)	-0.4129 (3.3737)	-2.4455 (0.9931)	-5.2745 (1.0483)
β (SE)	0.4767 (0.2415)	0.75585 (0.0917)	0.8380 (0.0841)
t-test (β = 0)	$1.97^{*}_{(49)}$	$8.24_{(216)}$	$9.97_{(371)}$
Residuals			
Test for Poisson distn.			
X^2	$45.1_{(4)}$	$29.83_{(9)}$	$63.53_{(9)}$
K-S	$0.231_{(51)}$	$0.134_{(219)}$	$0.136_{(372)}$
Test for Normal distn.			
X^2	$10.11^{*}_{(5)}$	$8.49^{*}_{(9)}$	$10.06^{*}_{(9)}$
K-S	$0.117^{*}_{(51)}$	$0.060^{*}_{(217)}$	$0.041^{*}_{(372)}$

* Null hypothesis not rejected

Table 8.6: Distribution of observed number of bidders around regression prediction (forced through zero)

$n_j = \beta y_{ij}$	Case 1	2	3
Regression results			
$y_{ij} = \ell n\ (x_{ij})$			
Prod. mom. corr. coeff.	0.2714	0.489	0.460
α (SE)	0 (0)	0 (0)	0 (0)
β (SE)	0.4472 (0.0143)	0.5321 (0.0125)	0.4164 (0.0074)
t-test ($\beta = 0$)	$31.66_{(50)}$	$42.56_{(219)}$	$56.20_{(372)}$
Residuals			
Test for Poisson distn.			
X^2	$41.77_{(4)}$	$48.7_{(9)}$	$57.10_{(9)}$
k-s	$0.226_{(51)}$	$0.113_{(217)}$	$0.107_{(278)}$
Test for Normal distn.			
X^2	$10.11^{*}_{(5)}$	$14.78^{*}_{(9)}$	$17.67_{(9)}$
k-s	$0.113^{*}_{(51)}$	$0.072^{*}_{(217)}$	$0.071_{(372)}$

* Null hypothesis not rejected

Table 8.7: Regression predictions of number of bids per project (Case 1) and Poisson probability Pr(^{n}j) of residuals

Project (j)	No Bids (n_j)	Value $\ell n\ (x_{ij})$	Pred n_j	Pr (n_j)
1	6	14.186	6.350	0.5503
2	4	13.126	5.845	0.3064
3	7	14.052	6.286	0.7037
4	6	13.366	5.959	0.6129
5	6	12.872	5.724	0.6506
6	9	14.565	6.531	0.8747
7	7	14.936	6.707	0.6422
8	4	15.747	7.094	0.1646
9	6	13.678	6.108	0.5890
10	4	13.877	6.203	0.2588
11	6	14.381	6.443	0.5355
12	6	13.935	6.230	0.5694
13	4	13.346	5.950	0.2918
14	6	13.279	5.918	0.6195
15	6	14.173	6.344	0.5513
16	3	13.644	6.092	0.1432
17	10	12.010	5.790	0.6401
18	6	13.010	5.790	0.6401
19	9	14.866	6.674	0.8620
20	8	15.874	7.155	0.7087
21	7	15.132	6.801	0.6283
22	6	13.259	5.908	0.6210
23	5	14.292	6.401	0.3836
24	8	13.995	6.636	0.5053
25	6	14.785	6.636	0.5053
26	7	13.060	5.813	0.7693
27	4	14.239	6.375	0.2380
28	7	14.460	6.529	0.6686
29	6	13.122	5.843	0.6315
30	6	14.786	6.636	0.5052
31	6	13.304	5.929	0.6176
32	6	13.196	5.878	0.6259
33	6	13.584	6.063	0.5962
34	7	14.550	6.524	0.6693
35	6	13.619	6.080	0.5935
36	6	13.520	6.032	0.6011
37	9	15.707	7.075	0.8228
38	5	13.140	5.851	0.4698
39	7	13.197	5.878	0.7887
40	8	14.552	6.524	0.7887
41	6	14.223	6.368	0.5474
42	8	15.103	6.787	0.7564
43	6	13.352	5.953	0.6139
44	5	14.628	6.561	0.3603
45	4	14.000	6.261	0.2516
46	7	14.835	6.660	0.6493
47	8	14.151	6.333	0.8110
48	7	13.456	6.002	0.7437
49	5	12.424	5.510	0.5272
50	5	12.791	5.685	0.4975
51	6	13.176	5.869	0.6274
Total:	318		318.000	

Table 8.8: Test for Uniform distribution of Pr(^{n}j) [Case 1]

Chi Square Test of Fit

Uniform distribution

From	To	Obs. Freq. (0)	Z	PHI	EXPCUM	Exp. Freq. (E)	$X^2 = (0-E)^2/E$
0.000	0.292	6	0.300	0.300	15.300	15.300	5.653
0.292	0.498	5	0.500	0.500	25.500	10.200	2.651
0.498	0.596	11	0.600	0.600	30.600	5.100	6.825
0.596	0.669	16	0.700	0.700	35.700	5.100	23.296
0.789	1.000	13	1.000	1.000	51.000	15.300	0.346
		51				51.000	38.771

Chi-Square of 38.771242 (4 df), has prob. of 0.000000

Kolmogorov-Smirnov Statistic = 0.231157

Critical Value at 5% level is approximately 0.190000.

Rejected

8.3.2 Conclusions

Table 8.5 indicates that the regression does go some way towards predicting the number of bids from the project value (lowest bid), the β coefficient being significantly different to zero for Cases 2 and 3. The Poisson model for the distribution of the observed number of bidders around the predicted value was not found to be appropriate by the tests used. The Normal model, however, was found to be satisfactory.

8.4 The Distribution of Bids for Each Project

8.4.1 Shape

8.4.1.1 First impressions

A first impression of the shape of the distribution of bids was gained by calculating the weighted average of the third and fourth moments of the bids for each project as these moments are independent of the first and second moments.

The weighted averages of the coefficients of skewness and kurtosis were obtained in the manner of McCaffer (1976a) as follows:

$$\bar{Y}_1 = \frac{1}{N} \sum_{j=1}^{c} Y_{1j} k_j$$

$$\bar{Y}_2 = \frac{1}{N} \sum_{j=1}^{c} Y_{2j} k_j$$

where Y_1 and Y_2 are the coefficients of skewness and kurtosis respectively (see Appendix A), k is the number of bids entered for projects j=1,2, ..., c and

Table 8.9: Average coefficients of skewness and kurtosis

No. of Bids (k)	No. of projects containing bids (m_k)	Total (km_k)	Av. skewness Coeffient $(\overline{Y}_{1k})$	Weighted Total $(km_k \overline{Y}_{1k})$	Av. kurtosis Coefficient $(\overline{Y}_{2k})$	Weighted Total $(km_k \overline{Y}_{2k})$
Case 1:						
3	1	3	-1.054	-3.162		
4	6	24	0.138	3.324	0.040	0.958
5	5	25	0.678	16.951	-1.129	-28.231
6	21	126	0.191	24.027	-0.075	- 9.480
7	9	63	0.796*	50.164	0.648	40.805
8	5	40	0.837*	33.484	0.128	5.123
9	3	27	0.775	20.923	1.130	30.517
10	1	10	0.449	4.486	-0.756	- 7.557
Total:	51	318	$\overline{Y}_1$ = 0.472*	150.197	$\overline{Y}_2$ = 0.102	32.135

* null hypothesis rejected at 5% level

No. of Bids (k)	No. of projects containing bids (m_k)	Total (km_k)	Av. skewness Coefficient ($\bar{Y}_{1k}$)	Weighted Total ($km_k \bar{Y}_{1k}$)	Av. kurtosis Coefficient ($\bar{Y}_{2k}$)	Weighted Total ($km_k \bar{Y}_{2k}$)
Case 2:						
2	23	46				
3	16	48	-0.118	-5.658		
4	27	108	-0.097	-10.495	0.012	1.329
5	31	155	0.103	16.022	0.454	70.393
6	47	282	0.325*	91.784	0.424	119.534
7	35	245	0.119	29.107	-0.085	-20.825
8	20	160	0.171	27.425	-0.242	-38.674
9	12	108	0.303	32.738	0.813	87.834
10	2	20	0.307	6.149	0.009	0.186
11	2	22	0.004	0.088	-1.166	-25.650
13	1	13	0.371	4.825	-0.835	-10.856
14	2	28	0.564	15.791	2.383*	66.729
Total:	218	1235	$\bar{Y}_1 = 0.175$*	207.778	$\bar{Y}_2 = 0.219$	250.002

* null hypothesis rejected at 5% level

No. of Bids (k)	No. of projects containing bids (m_k)	Total (km_k)	Av. skewness Coefficient ($\bar{Y}_{1k}$)	Weighted Total ($km_k \bar{Y}_{1k}$)	Av. kurtosis Coefficient ($\bar{Y}_{2k}$)	Weighted Total ($km_k \bar{Y}_{2k}$)
Case 3:						
2	40	80				
3	53	159	-0.066	-10.479		
4	47	188	0.585*	110.024	0.786*	147.785
5	59	295	0.303*	89.298	0.841*	248.021
6	85	510	0.188	95.793	0.043	21.879
7	48	336	0.256	85.882	0.078	26.188
8	28	224	0.325*	72.883	-0.178	-38.690
9	8	72	0.351	25.307	0.826	59.456
10	4	40	0.284	11.373	-0.303	-12.115
11	1	11	0.948	10.423	0.489	5.381
Total:	373	1915	$\bar{Y}_1 = 0.267$*	490.503	$\bar{Y}_2 = 0.273$*	457.905

* null hypothesis rejected at 5% level

$$N = \sum_{j=1}^{c} k_j$$

The full calculation is shown in Table 8.8.

An approximate test for departure from the normal distribution was made from the standard error of the coefficient of skewness ($SE\gamma_1 = \sqrt{(6/df)}$) and kurtosis ($SE\gamma_2 = \sqrt{(24/df)}$) where $df = km_k$. Under the null hypothesis that the bids are normally distributed it was judged that $SE\gamma_1$ and $SE\gamma_2$ would have to be less than 1.96. Some values greater than 1.96 were recorded as indicated in Table 8.9, and in particular the significantly positive skewness coefficients in all cases. Reference to previously published work suggest that most data of this kind may be skewed in a similar manner (Table 8.10).

Table 8.10: Summary of weighted shape statistic for project bids

Analysis	Av. Skewness Coeff.	df	Av. Kurtosis Coeff.	df
McCaffer's (1976a) bridges	-0.012	108	0.642	108
Skitmore (1981)	0.165*	1631	0.204	1590
Case 2	0.175*	1189	0.219	1141
McCaffer's (1976a) roads	0.210*	1721	0.200	1721
Case 3	0.267*	1835	0.273*	1676
Case 1	0.472*	318	0.102	315
McCaffer's (1976a) buildings	0.518*	1114	0.082	1114

* null hypothesis rejected at 5% level

8.4.1.2 The relationship between the coefficient of skewness and the number of bidders

It was considered possible that the number of bidders for the project may be associated in some way with the coefficient of skewness. A test was made for correlation by a linear regression. As Table 8.11 indicates, some correspondence does appear to exist between increasing numbers of bidders and increasing skewness, although the test used did not reject the null hypothesis. It was also considered that the observed correlation, such as it was, may well be a result of the confounding effects of project value.

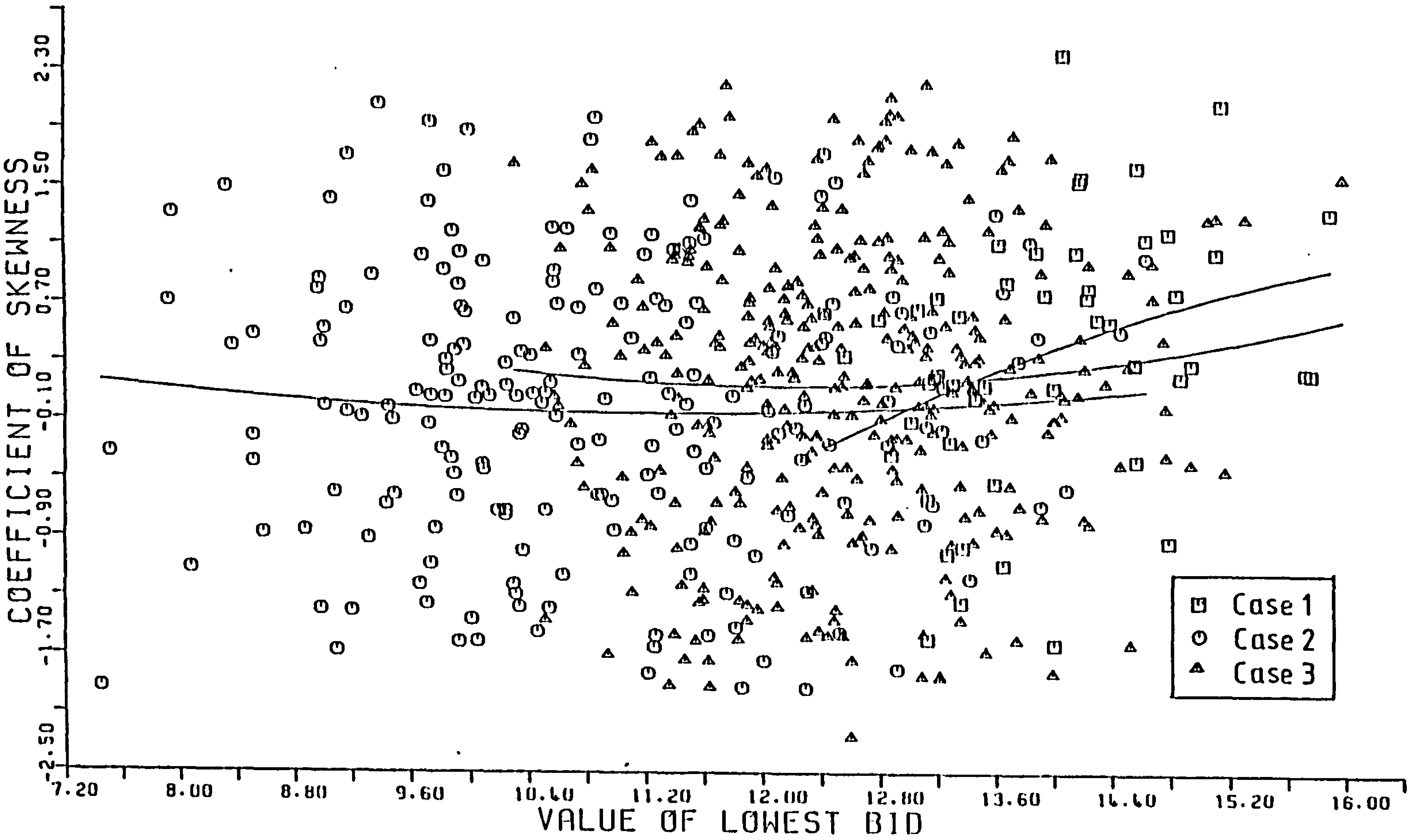
Table 8.11: Results of regression of number of bidders per project against skewness coefficient

Regression		Case	
$Y_{1j} = \alpha+\beta n_j$	1	2	3
Product Moment Corr. Coeff.	0.268	0.111	0.048
α	0.5349	0.2227	0.1850
β	0.0836	0.0347	0.0321
t-test (β=0)	$1.94_{(49)}$	$1.55_{(193)}$	$0.87_{(331)}$

8.4.1.3 The relationship between the coefficient of skewness and project value

Figure 8.12 shows the coefficients of skewness recorded for each project plotted against the log project value (lowest bid) for all cases together with the fitted second degree polynomial regression lines for each case. The plot suggests little correlation to exist between project value and the individual coefficients of skewness. The product moment correlation coefficients and t-tests on the β

Fig 8.12 LOG DATA: LOW BID AGAINST SKEWNESS (ALL CASES)



coefficients of the linear regression generally confirm this view, certainly for Case 2 and 3 (Table 8.12).

Table 8.12: Results of regression of project value on the coefficients of skewness for each project

Case	α	β	Prod.Mom Corr.Coeff.	$t(\beta=0)$
1	0.183	1.432 E-07	0.273	$1.93_{(49)}$
2	0.123	1.158 E-07	0.021	$0.43_{(193)}$
3	0.206	9.591 E-08	0.061	$1.31_{(331)}$

The results for Case 1 do indicate some correlation but the relatively small number of projects involved (51) suggests that the correlation may be spurious.

8.4.1.4 Tests of distributional shape

8.4.1.4.1 Introduction

As no satisfactory evidence was found to suggest that the shape of the distribution of bids for each project was dependent on either the number of bids or project value it was decided to test the assumption that the shape of the distribution of bids for each project was the same for each project. In order to do this it was necessary to find a parametric shape that would reasonably model the distribution of bids for each project.

More formally, it is assumed that "each of a set of observations $x = \{x_1, \ldots, x_n\}$ is independently drawn from a density $p(x_i | \theta, \sigma, \beta)$, where θ and σ are location and scale parameters, and β is a shape parameter that indexes a parametric family that contains the simplifying assumption as a special case β_0. " Spiegelhalter (1983, p401).

In the present context, where there are several sets of observations (one set for each project) for each case, the notation x_j $\{x_{1j}, \ldots ,$ $x_{nj}\}$ is adopted and the density is
$p(x_{ij}|\theta_j,\sigma_j,\beta)$ for the j^{th} project ($j = 1,\ldots,c$).

No global procedure appears to be available to estimate β from the data. It has been suggested that "we can go a long way in the process of approximation if we make the distribution used have the correct values [estimated from the data] of its first four moments" (Pearson, 1963, p109). In determining β this implies the use of the third and fourth moments only and in particular the coefficient of skewness (Y_1) and kurtosis (Y_2), which are independent of any linear transformation of x. Two particular problems exist in this approach to these data. Firstly there are several sets of observations and therefore several estimates of Y_1 and Y_2, that is Y_{11}, Y_{12}, ... , Y_{1c} and Y_{21}, Y_{22}, ..., Y_{2c}. Secondly, each set of observations contains different, and few, values. It is clear therefore that some knowledge is needed of the distribution of small sample estimates of the standardized third and fourth moments of potential models. A procedure will also be required to combine the moments obtained from different sample sizes. In view of these considerations it was decided to limit the range of possible distribution options to those suggested in the literature, that is the Uniform, Normal, Gamma, Weibull and Log-normal distributions (Table 7.2).

The preceding preliminary conclusions on the shape of the distribution of bids strongly suggest a lack of symmetry (positive skewness) to exist and therefore that the Uniform and Normal distributions will not be appropriate. McCaffer (1976a), however, although finding a substantially high average coefficient of skewness for his Belgium building contracts later concluded, after a more sophisticated analysis involving small sampling distribution of the Anderson-Darling statistic, that the data could be regarded as Normally distributed. In addition, both the Uniform and Normal distributions have been frequently adopted in bidding models as particularly suited to further theoretical treatment. It was decided therefore to retain the Uniform and Normal models in the analysis.

Several tests are available to compare the five selected distributions. The ratio of geometric to arithmetic means for instance has been shown to be the uniformly most powerful (UMP) invariant test statistic for the gamma shape against exponential alternatives (Shorack, 1972). Englehardt & Bain (1975) use the maximum likelihood estimate of the exponential distribution to test against various Weibull alternatives, whilst Farewell & Prentice (1977) have parametised the comparative tests for Weibull, Gamma and Log-Normal by embedding in an extended generalized gamma family.

No UMP test for the Normal shape against parametric families appears to exist as yet. The optimal test for the Normal against Cauchy distributions has recently been obtained by Franck (1982); Geary's test statistic, the ratio of mean deviation to standard deviation has been shown by Uthoff (1973) to be asymptotically equivalent to the optimal test between the Normal and Laplace shapes; and Uthoff (1970) has also shown the ratio of range to standard deviation to be the optimal test between the Uniform and Normal shapes.

Shapiro et al (1968) have studied nine statistics for testing the Normal assumption, Shapiro-Wilks 'W', the coefficient of Skewness Y_1, the coefficient of kurtosis Y_2, the Kolmogorov-Smirnov statistic K-S, Cramer Von Mises statistic CM, the weighted CM statistic WCM (Anderson-Darling), a modified K-S statistic D, the chi-square statistic X^2 , and the studentised range μ. The statistics were used in testing 45 alternative distributions in 12 families and 5 sample sizes (10,15,20,30,50). Neither the Lognormal or Weibull distributions were examined. For the Group 5 distributions, that is where $|Y_1| < 0.3$, a typical value for bidding data, "none of the tests showed much sensitivity against the alternatives in this group" (Shapiro et al, 1968, p1366) as no power exceeded 24% at $\alpha = 5\%$. The WCM (Anderson-Darling) test was found to exhibit "surprisingly low power", contrary to popular belief. The size of the samples used was found to be an important factor, sometimes having a dramatic effect on the power of the test.

On the evidence of literature it would appear that the unpredictability of small sample test statistics, certainly for almost symmetrical distributions, precludes the selection of a particularly most powerful statistic and that a battery of such statistics would be needed. It was considered, however, that tests between the fixed shape Gamma, Weibull and Lognormal distributions would best be conducted by the parametised approach of Farewell & Prentice and possibly by a comparison of log likelihood ratios.

Of the five types of probability distributions, only three, the Normal, Lognormal and Uniform distributions can be tested for shape without the need for estimates of parameters. These are usually referred to as parameter-free tests (Kendall & Stuart, 1963). The general Lognormal test involves the log transformation of the data and is discussed in a later section.

8.4.1.4.2 Tests for Normal and Uniform shape

Several tests are available for distribution shape, mostly tests for departure from Normality. Pearson & Hartley (1966, p67) indicate two methods of dealing with the problem in common use:

(i) A Normal curve is fitted to the sample data and the X^2 [or Kolmagorov-Smirnov] test for goodness of fit appled.

(ii) Certain functions of the moments of the sample are calculated and the significance of their departure from the expected value for a Normal population is examined.

The same methods can, of course, be applied for distributions other than normal.

For small samples (eg. n $<$ 14) the test (i) is unlikely to provide any meaningful results.

The most suitable approach would seem to be that adopted by McCaffer and Pettit (1976) which involves calculating the moment statistic for each sample (project) and testing the distribution of that statistic against the known distribution of the sample statistic for a theoretical universe. To do this, it is first necessary to know the distribution of the sample statistic for the various theoretical distributions under consideration.

Five different sample test statistics were considered as possible candidates for this purpose:

1. The sample coefficient of skewness, Y_1
2. The sample coefficient of kurtosis, Y_2
3. Geary's 'a' statistic
4. The sample studentized range W/S
5. Anderson-Darling's A^2 statistic

1. The sample coefficient of skewness, Y_1

The formula for calculating this statistic is given in Appendix A. The 5% and 1% points of the distribution of Y_1 for samples from a Normal population have been tabulated (see Pearson & Hartley, 1966, Table 34B, p207). The points were obtained from a t-distribution approximation (except for n=25 where a Hansmann curve was used) (Pearson, 1963, p106). As yet the exact distribution of Y_1 is unknown for any distribution being considered in this study and even approximations are difficult as the sample size becomes small (McKay 1933a,b and Geary 1947a,b).

2. The sample coefficient of kurtosis, Y_2

The formula for calculating this statistic is given in Appendix A. The approximate upper and lower 5% and 1% points of the distribution of Y_2 for samples from a Normal population have been tabulated for samples of size 50 and upwards (see Pearson & Hartley, 1966, Table 34, p208) by fitting a Pearson type IV curve to the first four moments. Pearson (1963, p106) shows that the third and fourth moments of Y_2 follow a "very strange" trend for sample size $n < 50$ when the problem becomes even worse than the Y_1 statistic. Pearson further adds that the distribution of the reciprocal may be easier to predict but no further information is available.

3. Geary's 'a' Statistic

Geary (1935, 1936) has suggested an alternative statistic which may be used for detecting changes in kurtosis, particularly when samples contain less than 50 observations. This statistic is

$$a = \frac{\text{mean deviation}}{\text{standard deviation}} = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{\{n \sum_{i=1}^{n} (x_i - \bar{x})^2\}^{½}}$$

The exact upper and lower 10, 5, and 1% points of the distribution of a few samples from a Normal population have been tabulated for sample of size n = 11, 16, 21, ... (see Pearson & Hartley, 1963, Table 34c, p207). No details are available either in Geary's original papers or elsewhere of the percentage points for other sample sizes and other parent distributions.

4. The Sample Studentized Range, W/S

McCaffer & Pettit (1976a) have used the studentized range statistic, W/S, to test a set of 350 roads and 185 building contracts. The test was applied to indicate whether the data more closely approximated a Normal or a Uniform parent population.

The statistic is:

$$W/S = \frac{\text{range}}{\text{standard deviation}} = \frac{\max(x_i) - \min(x_i)}{\{(n-1)^{-1} \sum_{i} (x_i - \bar{x})^2\}^{½}}$$

The exact upper and lower 0.0, 0.5, 1.0, 2.5, 5.0 and 10.0% points of the distribution of W/S for samples from a Normal population have been tabulated for samples of size $n > 3$ (see Pearson & Hartley, 1966, Table 29c, p200). McCaffer & Pettit (1976, Table 1b, p837) have also tabulated empirical upper 10% points of the distribution of W/S for samples from a Uniform population by simulation of 1000 samples for n = 3(1)18. No tabulations are readily available for further percentage points or for other population distributions.

5. Anderson-Darling's A^2 Statistic

McCaffer & Pettitt (1976) have used the Anderson-Darling statistic, A^2, to test the assumption that their data were obtained from a Normal population. The statistic is calculated as follows:

$$\text{Let } t_i = \Phi(y_i) \text{ with } \Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{(2\pi)}} e^{-\frac{1}{2}y^2} dy$$

$$\text{where } y_i = \frac{x_i - \bar{x}}{\{(n-1)^{-1} \sum_i (x_i - \bar{x})^2\}^{\frac{1}{2}}}$$

Then

$$A^2 = -\sum_{i=1}^{n} (2i-1) \{\ell n\, z_i + \ell n\, (1 - z_{n-i+1})\}/n - n$$

where z_i = i th smallest of $(t_1, \ldots, t_n)$

The approximate 0.5 (0.5), 0.95, 0.975, 0.99% points of the distribution of A^2 for samples from a Normal population have been tabulated for samples of size n = 4,5,6,.. (see Pettitt, 1975, Table 2). The points were obtained from 10,000 simulated samples for each n and after applying a smoothing technique. No tabulations are readily available for percentage points from other population distributions.

The Anderson-Darling A^2 statistic is closely related to the Cramér-von-Mises W^2 statistic which, according to Pearson (1963), has a small sampling distribution as equally difficult to approximate as the Y_2 statistic discussed above.

Generation of Approximate Sampling Distributions

It is clear that some difficulties exist in determining even approximations of the sampling distributions of the above test statistics. Work on the coefficients of skewness and kurtosis for normal populations has been continuing for over 50 years with limited success. The necessary percentage points for all but the Anderson-Darling A^2 statistic for a normal population require some degree of preliminary analysis and even these will probably involve a lengthy quadrature process to achieve a reasonable approximations.

It was decided at this point therefore to generate the approximate percentage points by simulation (without smoothing) in order to gain an indication of the likely values.

As a major point of interest in the bidding literature centres on the Normal/Uniform assumption, it was decided to approximate the unknown percentage points for each of the five statistics for each of the two distribution. In doing this two alternative approaches are available. Firstly, the percentage points may be estimated by using one long run simulation and alternatively the percentage points may be estimated by taking the average of several small simulations. The point has been resolved by Juritz et al (1983) who conclude that the occurance of bias is smaller in the former case.

The Department of Civil Engineering's prime computer was therefore assigned the task of generating 20,000 random samples of size n = 3(1)14 (n = 4(1)14 for the Y_2 statistic) for all the percentage points required. Each table took about 2 hours to compile (Tables 8.13 to 8.22).

Percentage points (where comparable) from the literature are included as a check against the accuracy of the programs.

Table 8.13: Percentage points of the skewness statistic γ_1
Empirical distribution obtained from 20,000 simulated n size random samples from a normal population N (1, 0.01)

Size of Sample	Percentage Points								
n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
3	-1.652	-1.400	-1.005	-0.521	-0.005	0.535	1.033	1.416	1.651
4	-1.459	-0.961	-0.561	-0.230	0.002	0.231	0.573	0.971	1.445
5	-1.223	-0.774	-0.494	-0.239	0.013	0.264	0.506	0.784	1.232
6	-1.083	-0.712	-0.447	-0.210	0.002	0.208	0.439	0.710	1.084
7	-1.014	-0.643	-0.404	-0.197	0.001	0.192	0.395	0.645	1.006
8	-0.953	-0.608	-0.374	-0.182	0.002	0.184	0.382	0.606	0.944
9	-0.899	-0.590	-0.367	-0.177	-0.004	0.170	0.356	0.581	0.910
10	-0.846	-0.535	-0.323	-0.151	0.008	0.166	0.345	0.549	0.853
11	-0.831	-0.532	-0.326	-0.155	0.002	0.163	0.331	0.530	0.819
12	-0.807	-0.508	-0.315	-0.153	-0.008	0.150	0.311	0.510	0.795
13	-0.775	-0.500	-0.307	-0.146	0.001	0.148	0.308	0.499	0.768
14	-0.754	-0.477	-0.296	-0.140	0.005	0.144	0.295	0.473	0.730

Table 8.14: Percentage points of the skewness statistic γ_1
Empirical distribution obtained from 20,000 simulated n size random sample from a uniform population U (1, 0.01)

Size of Sample	Percentage Points								
n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
3	-1.667	-1.456	-1.080	-0.580	-0.009	0.597	1.099	1.470	1.669
4	-1.482	-0.960	-0.527	-0.205	-0.003	0.186	0.514	0.942	1.444
5	-1.124	-0.688	-0.463	-0.236	-0.006	0.233	0.472	0.707	1.165
6	-0.967	-0.639	-0.384	-0.168	0.011	0.192	0.408	0.664	0.972
7	-0.888	-0.569	-0.348	-0.168	0.007	0.175	0.358	0.579	0.906
8	-0.802	-0.521	-0.323	-0.151	0.001	0.156	0.326	0.525	0.805
9	-0.736	-0.464	-0.284	-0.133	0.006	0.148	0.304	0.485	0.750
10	-0.698	-0.454	-0.276	-0.131	0.004	0.137	0.282	0.449	0.696
11	-0.660	-0.421	-0.269	-0.131	-0.005	0.121	0.256	0.421	0.647
12	-0.626	-0.401	-0.249	-0.116	0.003	0.119	0.247	0.405	0.630
13	-0.595	-0.384	-0.241	-0.122	-0.009	0.106	0.229	0.378	0.587
14	-0.555	-0.359	-0.222	-0.106	0.005	0.114	0.233	0.373	0.570

Table 8.15: Percentage points of the kurtosis statistic γ_2

Empirical distribution obtained from 20,000 simulated n size random samples from a normal population N (1, 0.01)

Size of Sample	Percentage Points								
n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
4	-4.164	-2.596	-1.232	-0.107	0.719	1.325	1.732	2.316	3.083
5	-2.597	-2.011	-1.391	-0.757	-0.112	0.528	1.203	1.863	2.785
6	-1.921	-1.580	-1.217	-0.807	-0.341	0.205	0.781	1.527	2.480
7	-1.725	-1.324	-1.026	-0.724	-0.373	0.071	0.577	1.257	2.230
8	-1.572	-1.208	-0.912	-0.622	-0.330	0.026	0.485	1.126	2.103
9	-1.462	-1.145	-0.861	-0.586	-0.313	0.017	0.413	0.965	1.905
10	-1.381	-1.097	-0.840	-0.583	-0.315	-0.009	0.374	0.884	1.734
11	-1.311	-1.024	-0.789	-0.548	-0.287	-0.001	0.359	0.832	1.691
12	-1.255	-0.988	-0.758	-0.532	-0.276	0.007	0.346	0.790	1.556
13	-1.202	-0.922	-0.693	-0.481	-0.251	0.011	0.328	0.760	1.517
14	-1.164	-0.922	-0.693	-0.481	-0.251	0.011	0.328	0.760	1.517

Table 8.16: Percentage points of the kurtosis statistic γ_2

Empirical distribution obtained from 20,000 simulated in size random samples from a uniform population U (1, 0.01)

Size of Sample	Percentage Points								
n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
4	-4.808	-3.399	-1.983	-0.713	0.360	1.149	1.684	2.338	3.154
5	-2.881	-2.501	-2.064	-1.512	-0.889	-0.161	0.624	1.541	2.585
6	-2.228	-1.870	-1.674	-1.390	-1.039	-0.567	0.029	0.803	1.992
7	-2.102	-1.782	-1.492	-1.236	-1.005	-0.725	-0.325	0.273	1.352
8	-1.941	-1.699	-1.457	-1.222	-0.978	-0.721	-0.411	0.022	0.913
9	-1.821	-1.602	-1.413	-1.217	-1.001	-0.759	-0.475	-0.088	0.618
10	-1.767	-1.560	-1.383	-1.222	-1.047	-0.833	-0.578	-0.218	0.357
11	-1.712	-1.530	-1.369	-1.207	-1.048	-0.867	-0.641	-0.319	0.217
12	-1.676	-1.508	-1.362	-1.217	-1.066	-0.902	-0.696	-0.417	0.081
13	-1.638	-1.477	-1.343	-1.212	-1.079	-0.924	-0.731	-0.469	-0.040
14	-1.622	-1.461	-1.330	-1.206	-1.072	-0.932	-0.750	-0.527	-0.122

Table 8.17: Percentage points of Geary's statistic, a

Empirical distribution obtained from 20,000 simulated n size random samples from a normal population (1, 0.01)

Size of Sample	Percentage points								
n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
3	0.840	0.861	0.880	0.897	0.911	0.922	0.931	0.938	0.942
4	0.789	0.813	0.827	0.840	0.853	0.867	0.901	0.934	0.967
5	0.756	0.780	0.804	0.830	0.854	0.876	0.896	0.916	0.939
6	0.738	0.779	0.807	0.829	0.847	0.864	0.881	0.899	0.924
7	0.740	0.776	0.801	0.820	0.837	0.854	0.870	0.890	0.915
8	0.740	0.774	0.797	0.815	0.832	0.848	0.865	0.884	0.907
9	0.740	0.772	0.794	0.812	0.827	0.843	0.860	0.877	0.899
10	0.742	0.771	0.792	0.810	0.825	0.840	0.855	0.872	0.894
11	0.743(0.7409)	0.772	0.792	0.808	0.823	0.837	0.852	0.869	0.890(0.8899)
12	0.740	0.769	0.789	0.805	0.820	0.834	0.848	0.864	0.884
13	0.743	0.770	0.789	0.805	0.818	0.832	0.845	0.861	0.881
14	0.744	0.770	0.789	0.804	0.818	0.830	0.843	0.858	0.877

Note: figures in parentheses are from Pearson & Hartley (1966, Table 34A, p.207)

Table 8.18: Percentage points of Geary's statisitc, a

Empirical distribution otained from 20,000 simulated n size random samples from a uniform population (1, 0.01)

Size of Sample n	Percentage Points								
	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
3	0.843	0.866	0.885	0.902	0.915	0.926	0.934	0.939	0.942
4	0.798	0.820	0.834	0.847	0.861	0.886	0.920	0.951	0.942
5	0.768	0.794	0.828	0.857	0.879	0.897	0.915	0.933	0.953
6	0.764	0.807	0.834	0.855	0.872	0.887	0.902	0.918	0.941
7	0.778	0.812	0.834	0.851	0.865	0.880	0.897	0.916	0.938
8	0.782	0.812	0.832	0.850	0.866	0.881	0.895	0.911	0.931
9	0.786	0.814	0.835	0.851	0.865	0.879	0.892	0.907	0.927
10	0.789	0.818	0.836	0.851	0.864	0.877	0.890	0.905	0.925
11	0.793	0.819	0.836	0.851	0.864	0.876	0.889	0.903	0.922
12	0.798	0.822	0.838	0.852	0.864	0.876	0.888	0.902	0.919
13	0.800	0.822	0.839	0.852	0.864	0.875	0.887	0.900	0.918
14	0.804	0.825	0.840	0.852	0.864	0.875	0.886	0.898	0.915

Table 8.19: Percentage points of the studentized range statistic W

Empirical distribution obtained from 20,000 simulated n size random samples from a normal population N (1, 0.01)

Size of Sample	Percentage Points								
n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
3	1.783 (1.782)	1.828	1.868	1.902	1.932	1.957	1.975	1.989	1.997 (1.997)
4	2.053 (2.04)	2.122	2.164	2.207	2.247	2.287	2.328	2.368	2.409 (2.409)
5	2.216 (2.22)	2.322	2.388	2.439	2.484	2.531	2.584	2.642	2.712 (2.712)
6	2.365 (2.37)	2.474	2.552	2.618	2.675	2.730	2.790	2.860	2.947 (2.949)
7	2.482 (2.49)	2.595	2.682	2.756	2.825	2.890	2.957	3.035	3.143 (3.143)
8	2.593 (2.59)	2.708	2.796	2.877	2.951	3.026	3.103	3.188	3.300 (3.308)
9	2.678 (2.68)	2.799	2.895	2.981	3.062	3.144	3.226	3.322	3.448 (3.449)
10	2.763 (2.76)	2.885	2.985	3.072	3.157	3.242	3.331	3.439	3.578 (3.57)
11	2.841 (2.84)	2.967	3.071	3.162	3.247	3.334	3.427	3.534	3.685 (3.68)
12	2.910 (2.90)	3.037	3.134	3.230	3.319	3.408	3.505	3.624	3.783 (3.78)
13	2.964 (2.96)	3.099	3.203	3.300	3.391	3.485	3.585	3.709	3.877 (3.87)
14	3.027 (3.02)	3.167	3.269	3.360	3.455	3.548	3.655	3.778	3.957 (3.95)

Note: Figures in parentheses are from Pearson & Hartley (1966, Table 29c, p.200

Table 8.20: *Percentage points of the studentized range statistic W*

Empirical distribution obtained from 20,000 simulated n size random samples from a uniform population U (1, 0.01)

Size of sample n	Percentage Points 10%	20%	30%	40%	50%	60%	70%	80%	90%
3	1.775	1.816	1.854	1.890	1.922	1.949	1.971	1.987	1.997 (1.997)
4	2.006	2.093	2.136	2.174	2.213	2.255	2.300	2.347	2.398 (2.394)
5	2.139	2.245	2.318	2.378	2.427	2.475	2.528	2.589	2.670 (2.666)
6	2.264	2.355	2.431	2.499	2.564	2.628	2.691	2.764	2.865 (2.861)
7	2.359	2.452	2.529	2.600	2.668	2.738	2.817	2.904	3.014 (3.026)
8	2.437	2.536	2.613	2.683	2.753	2.826	2.910	3.008	3.142 (3.317)
9	2.501	2.601	2.677	2.749	2.818	2.892	2.974	3.079	3.220 (3.204)
10	2.556	2.656	2.730	2.801	2.872	2.943	3.028	3.135	3.288 (3.292)
11	2.602	2.703	2.779	2.850	2.921	2.994	3.077	3.180	3.331 (3.336)
12	2.642	2.740	2.818	2.888	2.958	3.030	3.114	3.218	3.371 (3.374)
13	2.675	2.772	2.851	2.924	2.992	3.064	3.150	3.254	3.406 (3.406)
14	2.713	2.815	2.891	2.963	3.030	3.103	3.183	3.284	3.436 (3.480)

Note: Figures in parantheses are from McCaffer & Pettitt (1976, Table 16, p.837)

Table 8.21a: Percentage points of the Anderson-Darling Statistic A^2
Empirical distribution obtained from 20,000 simulated n size
random samples from a normal population N (1, 0.01)

Size of Sample n	Percentage Points 10%	20%	30%	40%	50%	60%	70%	80%	90%
3	0.193	0.202	0.217	0.238	0.267	0.299	0.338	0.383	0.432
4	0.181	0.208	0.235	0.261	0.287	0.319	0.351	0.400	0.484
5	0.181	0.212	0.239	0.266	0.294	0.328	0.368	0.424	0.518
6	0.185	0.216	0.243	0.271	0.302	0.336	0.380	0.437	0.535
7	0.183	0.216	0.245	0.275	0.308	0.343	0.388	0.451	0.554
8	0.185	0.218	0.248	0.278	0.311	0.348	0.397	0.460	0.564
9	0.185	0.218	0.248	0.278	0.313	0.350	0.399	0.462	0.571
10	0.185	0.220	0.251	0.282	0.315	0.355	0.405	0.470	0.580
11	0.184	0.220	0.252	0.283	0.318	0.358	0.405	0.472	0.583
12	0.188	0.223	0.255	0.286	0.321	0.360	0.408	0.473	0.580
13	0.187	0.222	0.255	0.288	0.323	0.363	0.413	0.479	0.590
14	0.188	0.224	0.256	0.288	0.324	0.365	0.415	0.483	0.596

Table 8.21(b): Percentage points of the Anderson-Darling statistic A^2

From Pettitt (1975, Table 2)

	Percentage Points								
Size of Sample n	10.0	20.0	30.0	40.0	50.0	60.0	70.0	80.0	90.0
3	-	-	-	-	-	-	-	-	-
4	0.182	0.209	0.236	0.262	0.289	0.319	0.353	0.400	0.481
5	0.182	0.212	0.240	0.267	0.297	0.330	0.369	0.424	0.516
6	0.183	0.214	0.243	0.272	0.303	0.337	0.380	0.439	0.538
7	0.183	0.216	0.246	0.275	0.307	0.343	0.388	0.450	0.553
8	0.184	0.218	0.248	0.278	0.311	0.348	0.394	0.458	0.564
9	0.185	0.219	0.250	0.280	0.314	0.352	0.398	0.464	0.572
10	0.186	0.220	0.251	0.282	0.316	0.354	0.402	0.469	0.578
11	0.186	0.221	0.252	0.284	0.318	0.357	0.405	0.472	0.584
12	0.187	0.222	0.254	0.285	0.320	0.359	0.408	0.476	0.588
13	0.187	0.223	0.255	0.287	0.322	0.361	0.410	0.478	0.591
14	0.187	0.224	0.255	0.288	0.323	0.362	0.412	0.481	0.594

Table 8.22: Percentage points of the Anderson-Darling statistic A^2
Empirical distribution obtained from 20,000 simulated n size random samples from a uniform population U (1, 0.01)

Size of Sample n	Percentage Points								
	10%	20%	30%	40%	50%	60%	70%	80%	90%
3	0.193	0.204	0.222	0.247	0.277	0.313	0.352	0.395	0.440
4	0.187	0.217	0.247	0.276	0.306	0.335	0.370	0.422	0.505
5	0.192	0.227	0.257	0.285	0.315	0.351	0.394	0.451	0.538
6	0.196	0.232	0.263	0.294	0.327	0.365	0.410	0.470	0.561
7	0.201	0.238	0.270	0.302	0.338	0.377	0.425	0.489	0.592
8	0.204	0.243	0.277	0.311	0.348	0.389	0.439	0.505	0.607
9	0.209	0.248	0.286	0.322	0.361	0.405	0.456	0.524	0.629
10	0.210	0.254	0.291	0.329	0.368	0.413	0.466	0.537	0.649
11	0.215	0.260	0.298	0.337	0.377	0.422	0.478	0.552	0.666
12	0.222	0.268	0.308	0.348	0.391	0.439	0.495	0.571	0.690
13	0.225	0.273	0.315	0.356	0.400	0.449	0.509	0.583	0.704
14	0.229	0.279	0.323	0.364	0.408	0.460	0.519	0.596	0.718

The Joint Assessment of the Shape Assumption of Several Samples

Five methods of combining each statistic were considered:

1. Fishers method
2. Kolmogorov-Smirnov D^- statistic
3. Kolmogorov-Smirnov D^+ statistic
4. Kolmogorov-Smirnov D statistic
5. X^2 test.

1. Fishers Method

Pettitt (1975) has discussed the use of this method when testing for Normality with the Anderson-Darling A^2 statistic. If a_i is the sample value of the statistic A^2 then

$$S = -2 \ln (\prod_{i}^{k} P_i) \qquad P_i = P(A^2 < a_i)$$

has a $X^2{}_{2k}$ degrees of freedom when the null hypothesis of Normality is true.

2. Kolmogorov-Smirnov's D^- Statistic

Pettitt (1975) suggests this alternative let k_1, k_2, ..., k_{10}, where k_j is the number of samples with values of significance probability, P_i, in the range $\{(j - 1)/10, j/10\}$ $j = 1, \ldots, 10$

$$\text{Then } T = \sup_{i}\{i/10 - \sum_{j=1}^{i} k_i/k\}$$

is approximated by the Kolmogorov-Smirnov statistic D^-, calculated from continuous data, with $P(T>x)$ $< P(D^- > x)$.

3. Kolmogorov-Smirnov's D^+ Statistic

McCaffer & Pettitt (1976) use this method in analysing their building data. The formulae are as in 2 above except only *positive* deviations are considered. The distribution of D^+ for categorized data is given by Conover (1972). McCaffer & Pettitt (1976) found the 1, 2.5, 5, 10 and 15 percentage points by simulation.

4. Kolmogorov-Smirnov's D Statistic

This two-sided test again uses the formulae above but for the modulus of all deviations.

5. X^2 Test

Letting $k_1, \ldots, k_{10}$, where k_j is the number of samples with values of significance probability, P_i, in the range $\{(j_i/10) - 0.1, j_i/10\}$ $j = 1, \ldots 10$.

$$\text{Then } T = \frac{(k^j - i/10)^2}{i/10}$$

is approximated by the $X^2_{(9)}$ statistic, providing $i/10 > 5$ (a usual empirical constraint)

Discussion

Method 1 requires the probability P_i to be known for each value. Pettitt suggests interpolation of Table 8.21(b) in his example. Such interpolation would, it was considered, be an over-approximation.

Methods 2 & 3 are both one-sided tests and their choice therefore is dependent on the alternative hypothesis. Not wishing to accept such a restriction at this stage a two-sided test was considered more appropriate.

Method 4 is a two-sided test but is intended for continuous distributions only. An adjustment such as Conover's (1972) or a simulation study was considered but it was decided to confine attention to Method 5, the Chi-Square test, at least until some initial assessment becomes available.

8.4.1.4.3 Test for fixed shape Gamma, Weibull and Lognormal distributions

Unlike the Normal and Uniform distributions, the fixed shape Gamma, Weibull and Lognormal distribution shapes are a function of their parameters. In order to test the distribution shape (under the current assumption that the distribution shapes are common for all projects), it is necessary to first estimate the parameters of the distributions.

The Generalized Gamma distribution which encompasses all three distributions requires the estimation of 3 parameters: a (location), b (shape) and k (a constant) where b = 1 for Weibull and $k \rightarrow \infty$ for Lognormal (see Appendix A).

For bidding data:

$$f(x_j) = \frac{b}{a_j^{bk}\,\Gamma(k)}\, x_j^{bk-1} \exp\left\{ -(x_j/a_j)^b \right\} \quad x > 0$$

The log-likelihood function is then:

$$\ell n\ L = N\ \ell n\ b - N\ \ell n\ \Gamma(k) - bk \sum_{j=1}^{c} n_j\ \ell n\ a_j + (bk-1) \sum_{j=1}^{c} \sum_{i=1}^{c_j} \ell n\ x_{ij}$$

$$- \sum_{j=1}^{c} \sum_{i=1}^{n_j} (x_{ij}/a_j)^b \qquad (1)$$

where x_{ij} is the i^{th} bid for the j^{th} project ($j=1, \ldots, c$; $i=1, \ldots, n_j$)

n_j is the total number of bids for the j^{th} project

c is the total number of projects

$N = \sum_{j=1}^{c} n_j$ is the grand total number of bids

Values of a_j and either b or k can be obtained by solving the following formula obtained from the partial derivities of ℓnL with respect to a_j and b:

$$Nb^{-1} - k\sum_{j=1}^{c} n_j \ \ell n\ a_j + k\sum_{j=1}^{c}\sum_{i=1}^{nj} \ell n\ x_{ij} - \sum_{j=1}^{c}\sum_{i=1}^{nj}$$
$$\{(x_{ij}/a_j)^b\ \ell n\ (x_{ij}/a_j)\} = 0 \qquad (2)$$

where $a_j = (\sum_{i=1}^{nj} x_{ij}^b/k_{nj})^{1/b}$

For computing purposes:

$$a_j = \bar{x}_j \left\{ (kn_j)^{-1}\sum_{i=1}^{nj} (x_{ij}/\bar{x}_j)^b \right\}^{1/b}$$

In testing for a Gamma disribution for instance equation (1) should be maximum when b=1. In testing for a Weibull or Lognormal distribution the maximum likelihood ratio is:

$$MLR = \frac{\ell n\ L_{c1}}{\ell n\ L_{c2}}$$

where

$\ell n\ L_{c1}$ is the maximum log-likelihood function (Eqn.1) at $k = c_1$

$\ell n\ L_{c2}$ is the maximum log-likelihood fuction at $k = c_2$

c_1 is a small value

c_2 is a large value

should have the property that:

for $0 < \ell nL_{c1}$, $0 < \ell nL_{c2}$

MLR $>$ 1.0 for the Weibull distribution

MLR $<$ 1.0 for the Lognormal distribution

and the reverse for $0 > \ell nL_{c1}$, $0 < \ell nb_{c2}$ (this latter case applied with these data).

For the Gamma distribution an estimated value of $b \approx 1$ is anticipated, irrespective of MLR.

Because of the problem of roundoff errors it was found necessary to restrict the computations for values of $c_1 = 1$ and $c_2 = 10$.

The approximate distribution of the MLR test statistics were determined for each of the cases under study as described in the next section.

8.4.1.4.4 The Approximate Disribution of the Test Statistics

It was decided, in the absence of any *a-priori* hypothesis, to include all the tests in a battery as shown in Table 8.23.

An approximation of the distribution of the probability of the X^2 test statistics for for Test 1-10 and the test statistic for Test 11 were obtained by simulation.

The simulation procedure adopted was to generate random values from a specified distribution via the NAG pseudo-random number generator (initializing to a non-repeatable state after each simulation) for each project.

Table 8.23: The Test Battery

Test No.	Description	Test for Distn.	Statistic	Table for % Points
1	Skewness Test	Normal	Y_1	8.13
2	Kurtosis Test	"	Y_2	8.15
3	Geary's Test	"	'a'	8.17
4	Studentized Range Test	"	W/S	8.19
5	Anderson-Darling test	"	A^2	8.21b*
6	Skewness Test	Uniform	Y_1	8.14
7	Kurtosis Test	"	Y_2	8.16
8	Geary's Test	"	'a'	8.18
9	Studentized Range Test	"	W/S	8.20
10	Anderson-Darling Test	"	A^2	8.22
11	Generalized Gamma Test	Weibull/ Log-Normal	LLR LLR	-
12	Generalized Gamma Test	Gamma	b	-

* Table 8.21b was, supplemented by Table 8.21(a) for sample sizes of 3

In order to simulate the data as closely as possible, estimates of the parameters were made for the j^{th} project as follows:

1. Normally disributed bids

The population mean n_j and variance σ_j^2

$$\hat{\mu}_j = \bar{x}_j$$

$$\hat{\sigma}_j^2 = s_j^2$$

2. Uniformly distributed bids

The population minimum α_j and maximum β_j

$$\hat{\alpha}_j = \bar{x}_j - \sqrt{(12s_j^2)}$$

$$\hat{\beta}_j = \bar{x}_j + \sqrt{(12s_j^2)}$$

3. Fixed shape Gamma, Weibull and Lognormal distributions

For the Gamma distribution the estimated population location a_j and the constant k were obtained by solving Eqn (2) with b = 1. It was, unfortunately, not possible to generate pseudo-random values from the Gamma distributions except for integers and half integer values of k. As values of k were estimated from the data (given b = 1) the Gamma simulation and therefore Test 12, had to be abandoned at this stage. For the Weibull and Lognormal distributions the population location a_j and shape b were obtained by solving Eqn (2) with k = 1 for Weibull and k = 10 for Lognormal.

For the Lognormal distributed bids, estimates of the population mean μ_j and variance σ_j^2 were

$$\hat{\mu}_j = \frac{\hat{a}_j\,\Gamma(k + \frac{1}{b})}{\Gamma(k)}$$

$$\hat{\sigma}_j^2 = \hat{a}_j^2 \left[\frac{\Gamma(k + \frac{2}{b})}{\Gamma(k)} - \left\{ \frac{\Gamma(k + \frac{1}{b})^2}{\Gamma(k)} \right\} \right]$$

Bids were generated via the following NAG routines:

Table 8.24: Random Number Generation

Distribution Type	NAG Routine	Parameters
Uniform	G05DAF	$\hat{\alpha}_j$, $\hat{\beta}_j$
Normal	G05DDF	$\hat{\mu}_j$, $\hat{\sigma}_j$
Log-Normal	G05DEF	A_j, B_j
Weibull	G05DPF	$\hat{b}$, $\hat{a}_j^{\hat{b}}$

Where the Lognormal A_j and B_j are

$A_j = \ln (\eta_j^2 + 1)$

$B_j = \ln \hat{\mu}_j - \hat{\sigma}_j^2/2$

$\eta_j^2 = \hat{\sigma}_j^2/\hat{\mu}_j^2$

Each case was simulated 1000 times for each of the four distributions, a total of 12000 iterations. Test statistics 1 to 10 were computed on each occasion a Normal and Uniform distribution was used and test statistic 11 was computed each time a Weibull or Lognormal distribution was used. The 1000 values of each of the 11 test statistics for each case were ordered and assigned probabilities calculated as follows:

$$\Pr (t_i) = \frac{i}{1000} - \frac{1}{2000} \qquad (i = 1, \ldots , 1000)$$

where t_i is the i^{th} lowest test statistic.

The resulting 1, 2½, 5, 95, 97½ and 99% percentage points estimated for each test statistic for each case are given in Tables 8.25 and 8.26.

Table 8.25: Critical percentage points obtained from simulated shape tests

(a) Case 1 - Normal distribution

	Test Statistics at:					
Test	1%	2½%	5%	95%	97½%	99%
1	0.008454	0.019291	0.054539	0.950548	0.967323	0.989106
2	0.010237	0.023545	0.058984	0.955835	0.971699	0.991468
3	0.009723	0.022057	0.048105	0.950548	0.980082	0.994888
4	0.012826	0.028742	0.061746	0.950548	0.980082	0.989106
5	0.008454	0.028742	0.054539	0.950548	0.980082	0.989106
6	0.004791	0.011172	0.022057	0.904842	0.950548	0.980082
7	0.000034	0.000199	0.000700	0.657933	0.779188	0.883171
8	0.000026	0.000109	0.000600	0.612637	0.772760	0.876297
9	0.000001	0.000005	0.000036	0.493241	0.653447	0.734017
10	0.000947	0.005529	0.016854	0.904842	0.967323	0.980082

- Uniform distribution

Test	1%	2½%	5%	95%	97½%	99%
1	0.007344	0.019291	0.042374	0.929683	0.950548	0.980082
2	0.000003	0.000029	0.000123	0.657933	0.779188	0.980082
3	0.000007	0.000127	0.000515	0.694070	0.809752	0.904842
4	0.000001	0.000004	0.000022	0.493241	0.612637	0.772760
5	0.003590	0.009723	0.019291	0.904842	0.929683	0.980082
6	0.002682	0.008454	0.019291	0.904842	0.950548	0.980082
7	0.008879	0.026948	0.058984	0.955835	0.971699	0.991468
8	0.011172	0.022057	0.042374	0.950548	0.967323	0.989106
9	0.011172	0.025193	0.048105	0.950548	0.967323	0.989106
10	0.011172	0.025193	0.054539	0.967323	0.980082	0.989106

(b) Case 2 - Normal distribution

	Test Statistic at					
Test	1%	2½%	5%	95%	97½%	99%
1	0.009998	0.027940	0.046361	0.934858	0.968223	0.990192
2	0.006047	0.023007	0.053655	0.946195	0.970175	0.983535
3	0.017137	0.037947	0.058318	0.955604	0.978372	0.991867
4	0.008961	0.027940	0.051180	0.945790	0.968223	0.991867
5	0.008638	0.016540	0.043382	0.940463	0.968223	0.981226
6	0.001150	0.002501	0.006196	0.872546	0.940463	0.975257
7	0.000000	0.000000	0.000000	0.032247	0.064022	0.121730
8	0.000000	0.000000	0.000000	0.037947	0.082824	0.184626
9	0.000000	0.000000	0.000000	0.002501	0.006431	0.018393
10	0.000046	0.000321	0.001106	0.700892	0.811408	0.945790

- Uniform distribution

Test						
1	0.001294	0.004594	0.013351	0.880457	0.934858	0.981226
2	0.000000	0.000000	0.000000	0.009040	0.019754	0.068652
3	0.000000	0.000000	0.000000	0.043382	0.068505	0.142810
4	0.000000	0.000000	0.000000	0.001106	0.002700	0.008327
5	0.000033	0.000296	0.001063	0.690341	0.811408	0.928977
6	0.002599	0.007186	0.017755	0.902773	0.945790	0.981226
7	0.011023	0.028839	0.051775	0.940402	0.961578	0.990458
8	0.008638	0.022700	0.044849	0.955604	0.975257	0.986182
9	0.010751	0.022700	0.041960	0.945790	0.968223	0.983827
10	0.014865	0.021169	0.052886	0.934858	0.968223	0.990192

(c) Case 3 - Normal distribution

	Test Statistic at					
Test	1%	2½%	5%	95%	97½%	99%
1	0.005791	0.022434	0.055721	0.947984	0.974830	0.990735
2	0.007422	0.026435	0.049846	0.948439	0.969690	0.987608
3	0.010817	0.023849	0.053632	0.947984	0.976707	0.991703
4	0.006458	0.023368	0.045968	0.950904	0.974830	0.993429
5	0.008552	0.026393	0.046868	0.956453	0.972862	0.988578
6	0.000257	0.001151	0.002385	0.819409	0.909625	0.961618
7	0.000000	0.000000	0.000000	0.005033	0.010341	0.032767
8	0.000000	0.000000	0.000000	0.006600	0.018649	0.075286
9	0.000000	0.000000	0.000000	0.000109	0.000499	0.001737
10	0.000003	0.000042	0.000163	0.665235	0.780283	0.888579

- Uniform distribution

1	0.001946	0.004969	0.013084	0.917432	0.953727	0.981798
2	0.000000	0.000000	0.000000	0.000580	0.002099	0.008879
3	0.000000	0.000000	0.000000	0.006892	0.016815	0.045968
4	0.000000	0.000000	0.000000	0.000038	0.000213	0.001074
5	0.000005	0.000051	0.000270	0.658995	0.774503	0.888579
6	0.000777	0.002729	0.008552	0.879577	0.935349	0.972862
7	0.014334	0.029094	0.053383	0.955190	0.980883	0.991468
8	0.011286	0.025347	0.052614	0.938651	0.972862	0.989694
9	0.005306	0.017527	0.053632	0.947984	0.970802	0.991703
10	0.008552	0.023849	0.044216	0.947984	0.972862	0.989694

Table 8.26: Critical percentage points obtained from the Weibull & Lognormal distributions (Test 11)

	Case					
	1		2		3	
% Points	Weibull	Lognormal	Weibull	Lognormal	Weibull	Lognormal
1%	0.99215	0.99951	0.99234	1.00051	0.99350	1.00045
2½%	0.99303	0.99984	0.99300	1.00073	0.99370	1.00068
5%	0.99354	1.00024	0.99320	1.00110	0.99389	1.00078
95%	0.99809	1.00361	0.99604	1.00314	0.99600	1.00227
97½%	0.99859	1.00394	0.99627	1.00334	0.99617	1.00248
99%	0.99894	1.00427	0.99670	1.00360	0.99634	1.00271

8.4.1.4.5 Results of Shape Tests

Having obtained the approximate critical values of the ten test statistics for Uniform and Normal distributions, and the one for the fixed shape Weibull and Lognormal, for each case, the next step was to decide which of the test statistics to apply to the data. There are clearly four hypotheses for each case, that the bids for each project can be adequately modelled by the (1) Uniform distribution $u(\alpha_{j\ell}, \beta_{j\ell})$ (2), Normal distribution $N(\mu_{j\ell}, \sigma_{j\ell}^2)$ (3), Weibull distribution $W(b_\ell, a_{j\ell})$ (4), Log Normal $(A_{j\ell}, B_{j\ell})$ where $\ell = 1.\ 2.\ 3.$ the number of cases studied and the various parameters are estimated by the methods defined previously. A further hypothesis generalises to all cases by stating that the shape β is the same for all cases, ie. $\beta_1 = \beta_2 = \beta_3$.

At first glance it would seem rather unreasonable to apply all the tests available as the chances of the correct hypothesis being rejected would seem to be increased by the imposition of each further test. The degree to which such chances are increased is determined by the degree of independence between tests. In this respect tests 1-10 are not entirely independent. Indeed, a detailed examination of these tests reveals a great deal of similarity. The major differences between the tests are centred on the use of Y_1 and Y_2 statistics which may or may not be correlated depending on the particular functional shape. In many cases, it was judged Y_1 and Y_2 are interrelated and thus the chances of a false rejection of the correct hypothesis would not be substantially reduced by the introduction of further tests.

Test 11 is rather different from Tests 1-10 in that distinctly different parameters are involved and these two sets of tests may well be quite independent of each other. On the other hand these sets are testing for different shapes and it would therefore be entirely satisfactory if, say, normal distribution hypothesis was not rejected by Tests 1-10 and Test 11 did reject both the Weibull and Lognormal distributions. The uses of multiple tests is not without precedent in modelling data, Ali & Giacotto <1982), for instance, have tested the identical distribution hypothesis for stock market prices by using all their tests for alternatives. Hsu (1979) has noted that "when faced with the formality of significance testing, one may find difficulty in determining the overall significance level of the results of multiple tests. In the spirit of data analysis , however, it seems sensible to examine different aspects of the data by a *variety of tests* to help illuminate the nature of the data". Difficulties could, however, occur if Tests 1-10 do not reject either the Uniform or Normal hypothesis and Test 11 did not reject either the Weibull or Log-Normal distributions, the next step must then be to devise a further test to distinguish between the non-rejected hypotheses. An alternative is that all hypotheses are rejected in which case a different model will be needed either by hypothesising another simple parametric distribution or by some simple transformation.

In the event it was decided that all tests should be applied at the 5% level (two-tailed), this being regarded as not particularly severe although the ultimate effect on the decision model was not clear at that time. In addition the final hypothesis, that the distribution shape is the same for all three cases was considered to offer particularly strong evidence of the suitability of a general hypothesis across all data of this kind.

The test statistics obtained from the data are provided in Table 8.27. It can be seen that for all cases the Uniform and fixed shape Weibull and Lognormal hypotheses must be rejected. The Normal distribution appears to fit reasonably well in Cases 1 & 2 (except for Test 6 in Case 1) but not for Case 3. These findings do not confirm with the findings and assumptions of many researchers (see Table 7.3) although it should be recognized that in many cases the researchers are either concerned with the ratio of bids to cost estimates or different industries.

Table 8.27: Shape test statistics obtained from the data

Test	Case 1 Test Statistic	Case 1 Hypo. Rejected	Case 2 Test Statistic	Case 2 Hypo. Rejected	Case 3 Test Statistic	Case 3 Hypo. Rejected
1	0.042374		0.052886		0.000024	U,N
2	0.816537	U	0.341961	U	0.003218	U,N
3	0.734017		0.423196	U	0.007515	N
4	0.809752	U	0.461272	U	0.252199	U
5	0.321175		0.160300		0.085634	
6	0.001998	U,N	0.126959		0.000022	U,N
7	0.319084		0.000001	U	0.000000	U
8	0.734017		0.000000	U	0.000001	U
9	0.292183		0.000018	U	0.000000	U
10	0.455567		0.583886		0.475795	
11	0.99868	W,L	1.00049	W,L	1.00005	W,L

U	Uniform distribution
N	Normal distribution
W	Weibull distribution
L	Log-Normal distribution

8.4.1.5 Transformations

In the event of the rejection of all hypotheses in all cases, two alternative courses of action are available as discussed in the previous section, that is to try another parametric shape or investigate a suitable simple transformation. The difficulty with the former alternative is that the laborious procedure for generating an approximate distribution of the appropriate test statistic is required for each new hypothesis. The transformation option, however, offers a far simpler procedure in that Tests 1-10, which are parameter free may be used without modification. A further advantage of the transformation option is that a transformation may be found which enables the Normal model to be applied. Such a model would offer some considerable benefits in further testing. Barlett's test for equal variances, for instance, is particularly sensitive to the Normal assumption.

Two types of transformations were attempted, power and log transformations:

Power transformations

The data were transformed to $y_{ij} = x_{ij}^{1/p}$ and test statistics 1-10 calculated for each value of $p = 1, 2, \ldots, 10$. The hypothesis rejections are given in Table 8.28. For Case 1 & 2, values of p = 2, 3, 4, & 5 appear to provide values of y_{ij} approximating the Normal distribution. This cannot be said however for Case 3 where all values of p failed to remove the positive skewness denoted by test statistic 1.

Table 8.28: Results of power transformations

Data	Case					
	1		2		3	
	Dist. Test Fails		Dist. Test Fails		Dist. Test Fails	
	Normal	Uniform	Normal	Uniform	Normal	Uniform
$y_{ij} = x_{ij}^{1/p}$						
p = 1	6	2,4,6		2-4,7-9	1-3,6	1,2,4,6-9
2		2,4		2-4,7-9	1,3,6	2-4,6-9
3		2,4		2-4,7-9	1,6	2-4,6-9
4		2-4		2-4,7-9	1	2-4,6-9
5		3,4		2-4,7-9	1	2-4,6-9
6		3,4	10	2-4,7-9	1	2-4,6-9
7		3,4	10	2-4,7-9	1	2-4,6-9
8		3,4	10	2-4,7-9	1	2-4,7-9
9		3,4	10	2-4,7-9	1	2-4,7-9
10		3,4	10	2-4,7-9	1	3,4,7-9

Log transformations

The data were transformed to $y_{ij} = \ell n(x_{ij} - mc_j)$ where values of c_j were obtained by two methods, the average bid value $c_j = \bar{x}_j$ and the lowest bid value $c_j = x_j$ (min) expressed as $c_j = x_{1j}$. The test statistics of values of y_{ij} obtained from $m = 0, 0.1, 0.2, \ldots, 0.8$ were computed and the test fails recorded. Table 8.29 indicates the results obtained. It would appear that for all cases values of $m = 0.1$ to $m = 0.5$ provide a suitable transformation to the Normal distribution for $c_j = \bar{x}_j$ and $m = 0.1$ to 0.4 for $c_j = x_{1j}$.

Table 8.29: Results of log transformations

(a) $y_{ij} = \ell n \; (x_{ij} - m\bar{x}_j$

	Case					
	1		2		3	
	Dist. Test Fails		Dist. Test Fails		Dist. Test Fails	
m	Normal	Uniform	Normal	Uniform	Normal	Uniform
0		4	10	2-4,7-9	1	2-4,7-9
0.1		4		2-4,7-9		2-4,7-9
0.2		3,4		2-4,7-9		2-4,7-9
0.3		3,4		2-4,7-9		2-4,7-9
0.4		3,4		2-4,7-9		2-4,7-9
0.5		3-5		2-4,7-9		2-4,7-9
0.6	8	3-5	-	-	4	2-4,7-9
0.7	8	3-5	-	-	4	2-4,7-9
0.8	-	-	-	-	-	-

(b) $y_{ij} = \ell n(x_{ij} - mx_{1j})$

0		4	10	2-4,7-9	1	2-4,7-9
0.1		4		2-4,7-9		2-4,7-9
0.2		3,4		2-4,7-9		2-4,7-9
0.3		3,4		2-4,7-9		2-4,7-9
0.4		3,4		2-4,7-9		2-4,7-9
0.5		3,4	1	2-4,7-9		2-4,7-9
0.6	5,8	3,5	1,6	2-4,7-9		2-4,7-9
0.7	5,8,9	5	1,4,6	1-4,6-9	4	2-4,7-9
0.8	-	-	1,6	1-4,6-9	-	2-4,7-9

[-] no result available

8.4.2 Spread

8.4.2.1 General

Three commonly used statistics are available for expressing the spread of observations, the variance σ^2, the standard deviation σ, and the coefficient of variation cv = $(100\ \sigma)/\mu$. Estimates of these quantities are provided by the data in the form s^2, s and cv = $(100s)/\bar{x}$ (see Appendix A).

For cases $\ell = 1, 2, 3$ and projects $j = 1, 2, \ldots, c_\ell$

$$\text{av. } \bar{x}_\ell = \frac{1}{c_\ell} \sum_{j=1}^{c_\ell} \bar{x}_{j\ell}$$

$$\text{av. } s_\ell^2 = \frac{1}{N_\ell - C_\ell + 1} \sum_{j=1}^{c_\ell} s_{j\ell}^2 (n_{j\ell} - 1)$$

$$\text{av. } cv_\ell = \frac{1}{c_\ell} \sum_{j=1}^{c_\ell} (100 s_{j\ell}/\bar{x}_{j\ell})$$

where $N_\ell = \sum_{j=1}^{c_\ell} n_{j\ell}$

Table 8.30: Spread statistics obtained from the data

Statistic	Case 1	2	3
No. of Projects:	51	218	373
Av. $\bar{x}_\ell$	1691402	138605	493558
Av. s_ℓ	125003	24135	54378
Av. $cv_\ell = (100s_\ell/\bar{x}_\ell$	6.8	13.5	7.8

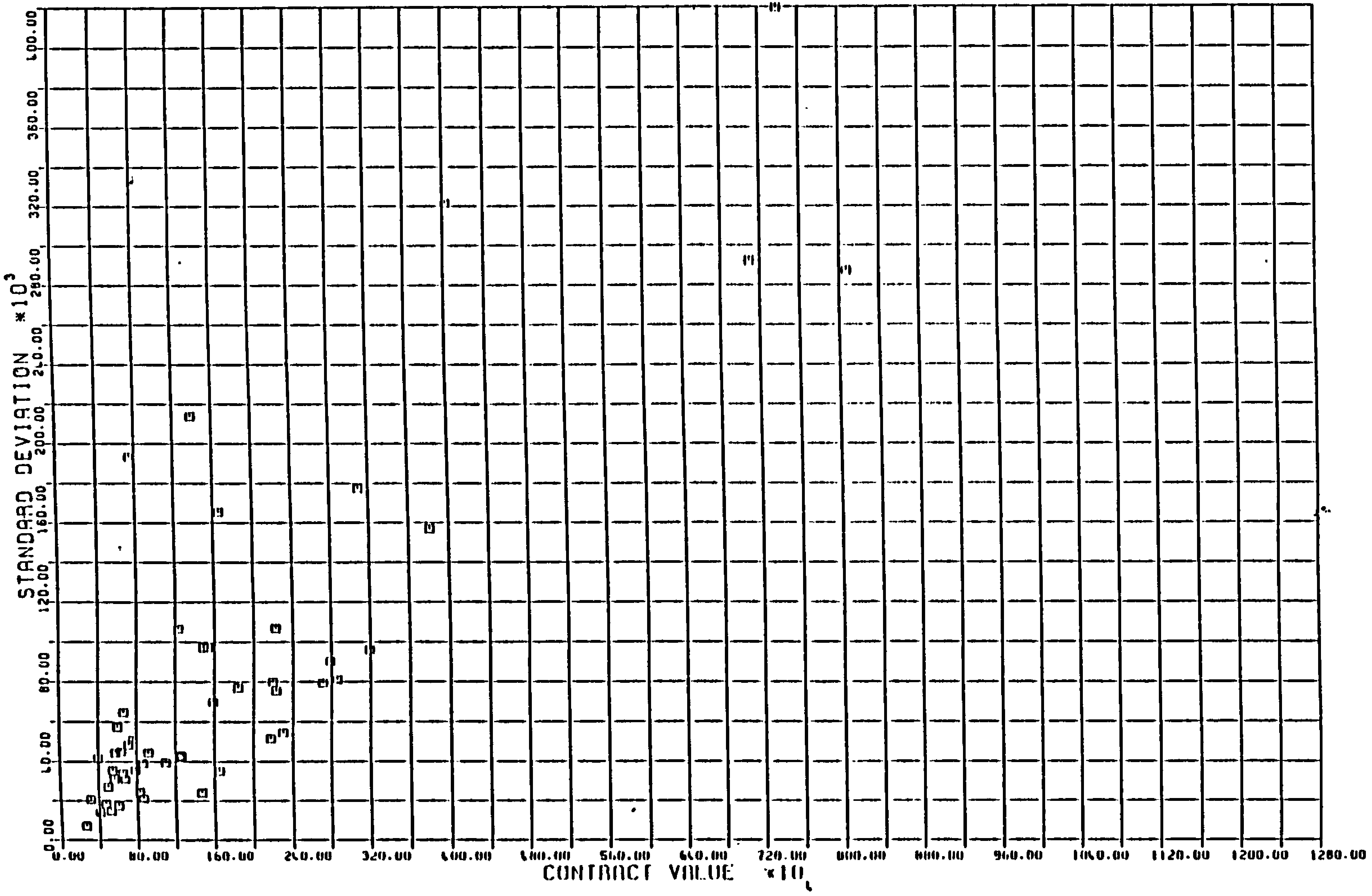
For data of this kind where the standard deviation is expected to be closely related to the value of the project the average cv is in popular use. Table 8.31 provides a comparison of the cv's obtained from these data and those of other researchers.

The assumption that the project standard deviation was correlated with the project value (low bid) was examined by plotting the values. Figures 8.13 to 8.15 show the plots for all cases. A general trend is observable. The product-moment correlation coefficients tend to confirm this (Table 8.32). Tables were also constructed for the log transformations known to provide Normal distributions (Table 8.33).

Table 8.31: Mean coefficients of variation of construction bids

Source	Mean cv	No. of Projects
Fine & Hackemar (1970)	5	"Adequate"
Beeston (1974)	5.2 to 6	"Large"
Grinyer & Whittaker (1973)	6.04	153
Skitmore (1981)	6.4	269 (unweighted)
Barnes (1971)	6.5	159
McCaffer (1976)	6.5	185
AICBOR (1967)	6.8	213
Case 1	6.8	51
McCaffer (1976)	7.5	16 (bridges)
Case 3	7.8	373
McCaffer (1976)	8.4	384 (roads)
Case 2	13.5	218

Fig 8.13 CASE1 - CONTRACT VALUE V STANDARD DEVIATION



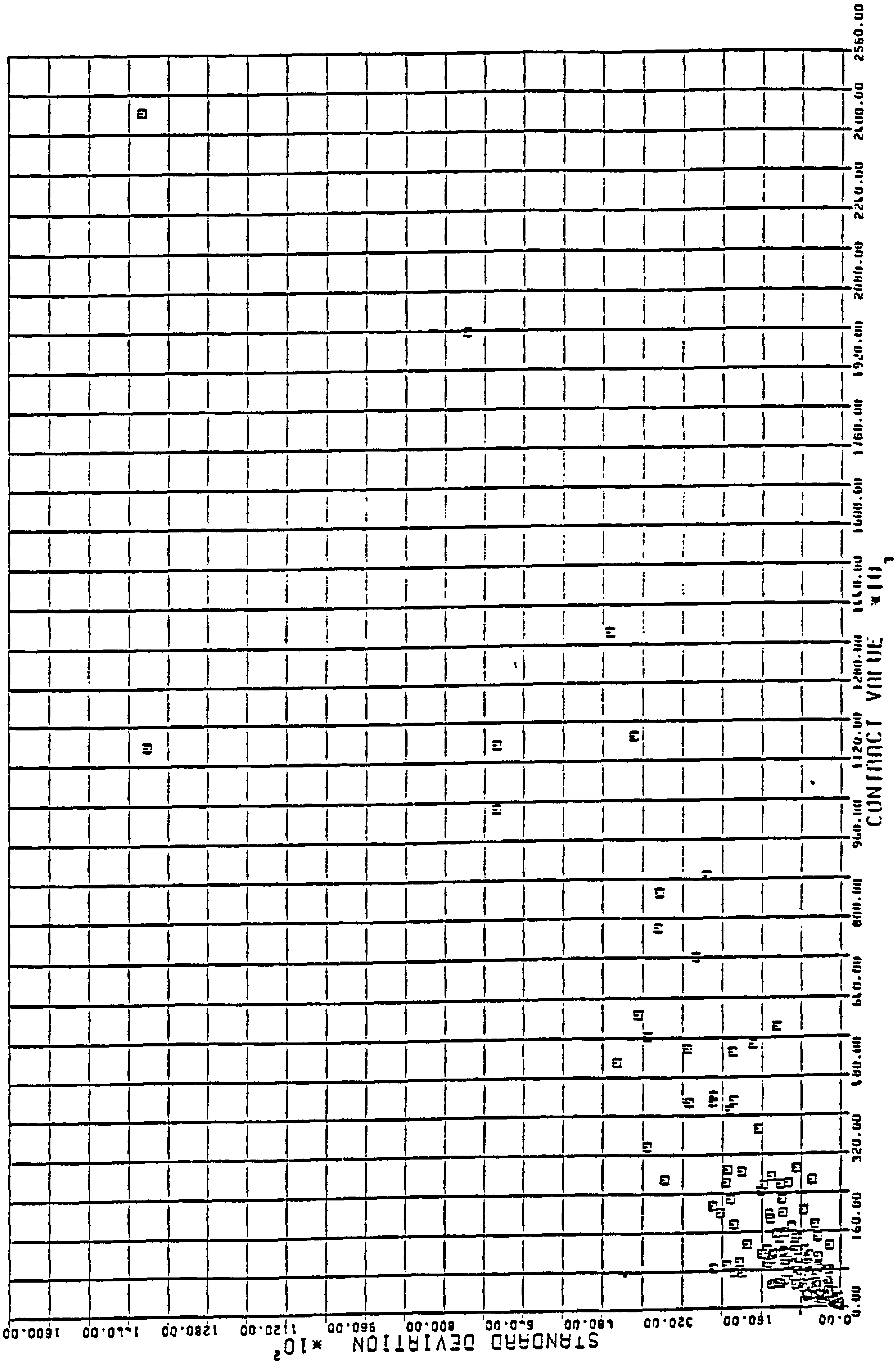


Fig 8.14 CASE2 - CONTRACT VALUE V STANDARD DEVIATION

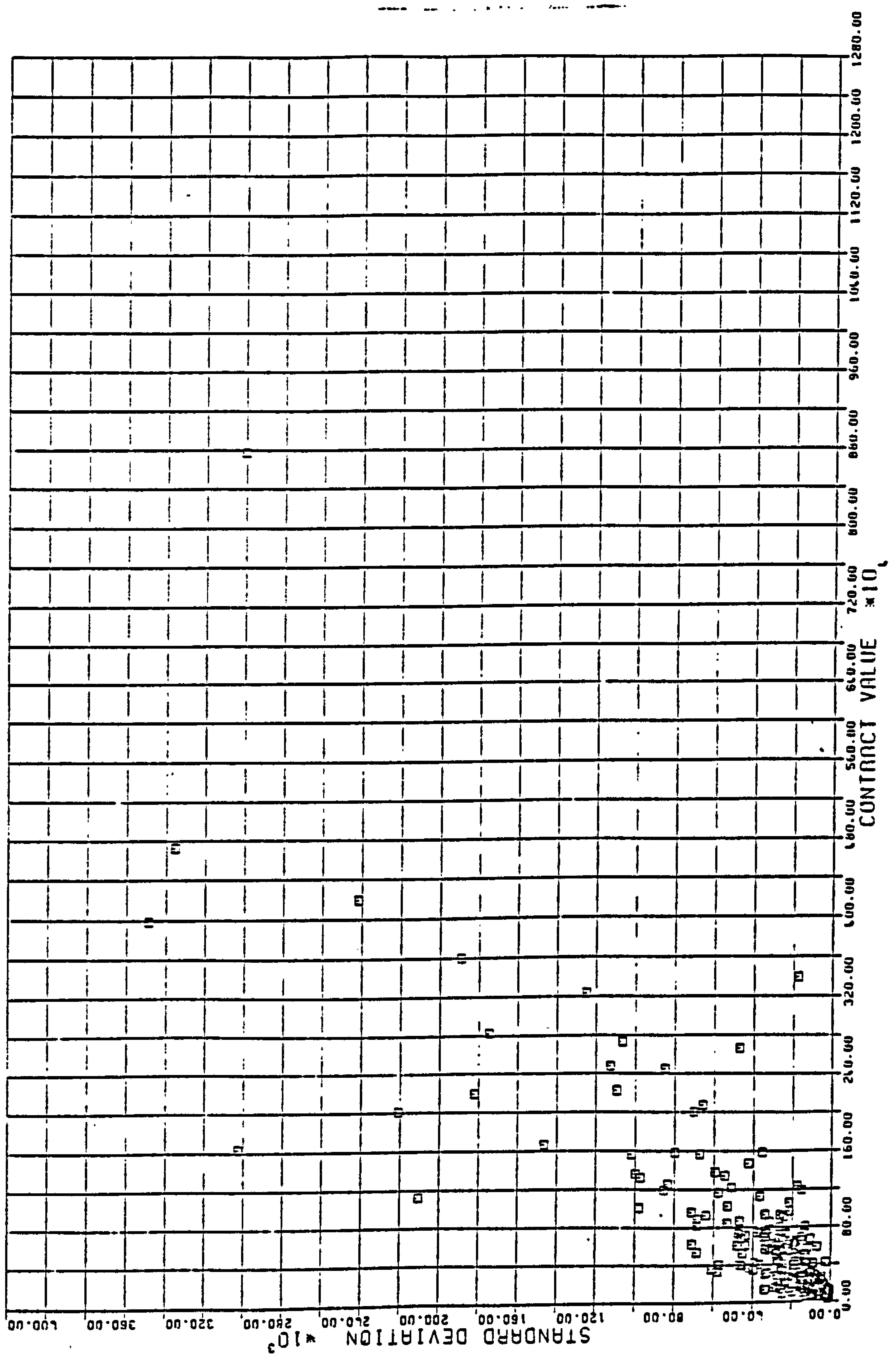


Fig 8.15 CASE3 - CONTRACT VALUE V STANDARD DEVIATION

Table 8.32: Results of regression of project value on the standard deviation of each project

Regression	Case 1	Case 2	Case 3
$\sigma_{xj} = \alpha + \beta x_{ij}$			
Prod.mom.corr.coeff.	0.823	0.869	0.802
α	13748	652	4555
β	0.04355	0.05381	0.04860
$y_{ij} = \ell n\ (x_{ij})$ $\sigma_{ij} = \alpha + \beta y_{ij}$			
Prod.mom.corr.coeff.	-0.248	-0.482	-0.283
α	0.21228	0.41038	0.20688
β	-0.01131	-0.02714	-0.01157

On this evidence the least squares approximation appears to provide reasonable predictions of the standard deviations. A plot of the residuals however suggests that the error term may be greater for larger project values, indicating a power or log transformation to be appropriate.

Estimates s^2 of variance are, however, known to be sensitive to distribution shape. For example, if the measure of kurtosis is Y_2 = 1, the variance s^2 is about 1.5 times as large as it is in a Normal population (Snedecor & Cochran, 1980, p81). A further difficulty is that the regression technique assumes the error term is normally distributed whilst s^2 is of course distributed as χ^2. A better approach therefore was considered to be to attempt some variance stabilizing transformation.

Table 8.33: Results of regression of project value on the standard deviation of each project

$\sigma_{yj} = \alpha + \beta y_{ij}$	Case 1				Case 2				Case 3			
	α	(SE)	β	(SE)	α	(SE)	β	(SE)	α	(SE)	β	(SE)
$y_{ij} = \ell n(x_{ij} - m\bar{x}_j)$												
m = 0	0.21228	0.0882	-0.01131	0.0063	0.41038	0.0364	-0.02714	0.0036	0.20688	0.0254	-0.01157	0.0020
.1	0.23945	0.0968	-0.01293	0.0069	0.45703	0.0399	-0.03059	0.0037	0.23144	0.0280	-0.01310	0.0023
.2	0.27489	0.1075	-0.01508	0.0078	0.51693	0.0443	-0.03510	0.0042	0.24724	0.0298	-0.01395	0.0024
.3	0.32312	0.1210	-0.01809	0.0089	0.59735	0.0500	-0.04130	0.0048	0.30487	0.0351	-0.01783	0.0029
.4	0.39291	0.1339	-0.02255	0.0104	0.71294	0.0587	-0.05060	0.0057	0.36377	0.0405	-0.02178	0.0034
.5	0.50400	0.1640	-0.02995	0.0124	0.90168	0.0695	-0.06640	0.0070	0.45335	0.0481	-0.02881	0.0041
.6	0.71541	0.2030	-0.04475	0.0156	-	-	-	-	0.61014	0.0600	-0.03953	0.0053
$y_{ij} = \ell n(x_{ij} - mx_{1j})$												
m = 0	0.21228	0.0882	-0.01131	0.0063	0.41038	0.0364	-0.02714	0.0036	0.20688	0.0254	-0.01157	0.0020
.1	0.23037	0.0946	-0.01231	0.0068	0.44080	0.0387	-0.02926	0.0036	0.22519	0.0274	-0.01265	0.0022
.2	0.25206	0.1022	-0.01351	0.0074	0.47654	0.0413	-0.03176	0.0030	0.24724	0.0298	-0.01395	0.0024
.3	0.27858	0.1116	-0.01498	0.0082	0.51922	0.0445	-0.03474	0.0042	0.27436	0.0327	-0.01555	0.0027
.4	0.31184	0.1220	-0.01682	0.0091	0.57125	0.0481	-0.03838	0.0048	0.30861	0.0362	-0.01759	0.0030
.5	0.35497	0.1355	-0.01921	0.0102	0.63641	0.0526	-0.04294	0.0052	0.35337	0.0463	-0.02026	0.0035
.6	0.41348	0.1527	-0.02458	0.0117	0.72105	0.0581	-0.04886	0.0059	0.41470	0.0464	-0.02393	0.0040

8.4.2.2 Bartlett's Test

The criterion statistic chosen for testing the successfulness of a variance stabilizing transformation was Bartlett's statistic where

$$M = N\,\ell n\,\{N^{-1}\sum_{j=1}^{c} v_j\, s_j^2\} - \sum_{j=1}^{c} v_j\,\ell n\, s_j^2$$

with parameters $k = c$ and $c_1 = \sum_{j=1}^{c} v_j^{-1} - N^{-1}$

where $v_j = n_j - 1$ and $N = \sum_{j=1}^{c} v_j$

for projects $j = 1, 2, \ldots, c$ (s_j^2 and n_j are defined in Appendix A)

The criterion statistic M/C where $C = 1 + c_1/{}^3(k - 1)$ is closely approximated by χ^2 for $(k-1)$ degrees of freedom.

Bartlett's test has been studied by several people, some comments by Pearson & Hartley (1966) are worthy of mention.

1. Box (1953) has found that discrepancies from asymptotic Normal theory become larger as more variances are compared.

2. From Normal theory, all tests on variances depend on the ratio $Z = \Sigma\ (x_i - \bar{x})^2/\sigma^2$, which is distributed like χ^2 with $(n-1)$ degrees of freedom. This makes the test particularly sensitive to deviations from mesokurtosis.

3. Suppose that c samples of sizes n_j $(j = 1, 2, \ldots, c)$ are drawn from populations each of which have the same variance σ^2 and the same kurtosis coefficient γ_2 then the LR criterion for comparing c Normal variances (Barlett's test) is that $-2\ \ell n\ \ell^*/(1 + \frac{1}{2}\gamma_2)$ itself is distributed asymptotically as χ^2 with $(k-1)$ degrees of freedom. The effects of this correction can be quite extreme. For instance, when $\gamma_2 = -2$ the true probability of exceeding the asymptotic Normal theory critical value for $\alpha = 0.05$ is 0.498 for 30 samples.

4. The lack of robustness in the variance test is so striking that Box (1953) was led to consider ℓ^* as a test statistic for kurtosis and found its sensitivity to be of the same order as the generally used tests for kurtosis!

There appear to be several practical approaches available:

1. Box and Anderson (1953) have suggested the correction : $M' = M/(1 + \frac{1}{2}\gamma_2)$ where γ_2 is the population kurtosis coefficient. The correction however is generally applicable when the population coefficient is known and therefore may be misleading when γ_2 is estimated from the data. In addition, investigations conducted by Nair and Bishop & Nair showed that the correction is not always adequate if some of the r_j are 1. 2 or 3.

2. Cox (1955, p28) has suggested using tests based on sample range for small companies (up to size 7 or 8).

3. Graph the estimated sample variances against some variable, say $\bar{x}_j$, and look for marked deviations (Anscombe, 1955, p29).

4. Divide the sample variances into groups of, say, project values and conduct an analysis of variance (ANOVA). Bartlett & Kendall (1946) have shown the value of the logarithmic transformation for this approach.

The simplest procedure was found to be to restrict the use of Bartlett's test to transformed variables considered to be Normally distributed thereby diminishing the impact of any distortions resulting from the use of non-Normal variables. To overcome the problem of small samples it was decided to conduct a simulation to determine the approximate distribution of the probabilities given by the original Bartlett statistic.

8.4.2.3 Simulation Programme

A simulation programme was devised to ascertain the percentage points of the probabilities associated with Bartlett's statistic for each of the three cases under study. To match the data reasonably closely and in anticipation of future analysis, the logs of the recorded bids were used $y_{ij} = \ln(x_{ij})$ were used. The overall average variance for the case s^2 was obtained and, together with the sample averages $\bar{y}_j$, the bids were simulated from $N(\bar{y}_j, s^2)$ for the $j = 1, 2, \ldots, c$ projects. Bartlett's test statistic was then computed and the probability recorded. The procedure was repeated 1000 times for each case and the critical percentage points noted. The results of the simulations are provided in Table 8.34. The percentage points and the 'probabilities' are, as can be seen, roughly equivalent, approximate values at the 5% level being 0.049, 0.055 and 0.079 for Cases 1, 2 & 3 respectively.

Table 8.34: Simulated variance tests

Critical values of Barlett's probability (log simulation using average variance, normal distribution, 1000 trials)

Test Statistic	Case		
	1	2	3
1 %	0.007647	0.016044	0.018292
2½%	0.016298	0.026412	0.045380
5 %	0.048815	0.055181	0.078796
7½%	0.070782	0.094707	0.112430
10 %	0.086277	0.115414	0.145023
95 %	0.942019	0.956412	0.965519
97½%	0.967351	0.976530	0.981440
99 %	0.987894	0.994715	0.995163

8.4.2.4 Variance stabilizing transformations

An advantage of applying variance stabilizing transformations is that the same transformations "often serves to normalize the distribution to which they apply" (Kendal and Stuart, 1961, p469). It is likely therefore that the reverse will also apply in that known Normalizing transformations may have a variance stabilizing effect.

It was found in the previous section that the transformation
$y_{ij} = \ell n\,(x_{ij} - m\bar{x}_j)$ $m = 0.1, 0.2, 0.3, 0.4, 0.5$ and
$y_{ij} = \ell n\,(x_{ij} - mx_{ij})$ $m = 0.1, 0.2, 0.3, 0.4$
provided a reasonable Normalizing effect in all the cases. These transformations were therefore applied and the 'probability' estimated by Bartlett's statistic compared with that associated with the critical value at the 5% point in Table 8.34. The results of these tests can be found in Table 8.35. These results are quite conclusive in rejecting all the variance stabilizing attempts.

Further, casewise transformations were attempted for the power transformations $y_{ij} = x_{ij}^{1/p}$ (p = 2, 3, ... , for Case 1; p = 1, ... 5 for Case 2) which also resulted in conclusive rejections.

Table 8.35: Results of tests on variance stabilizing transformations

Transformation	'Probability' predicted by Bartlett's statistic					
	Case					
	1		2		3	
$y_{ij}=\ell n(x_{ij}=m\bar{x}_j)$	$\chi^2(50)$	'prob'	$\chi^2(217)$	'prob'	$\chi^2(372)$	'prob'
m = 0.0	175.3	0.000	622.0	0.000	834.0	0.000
0.1	175.4	0.000	623.3	0.000	834.3	0.000
0.2	175.8	0.000	626.2	0.000	835.2	0.000
0.3	176.6	0.000	632.2	0.000	837.6	0.000
0.4	178.5	0.000	645.3	0.000	843.0	0.000
0.5	182.5	0.000	678.2	0.000	855.0	0.000
0.6	194.1	0.000	-	-	888.4	0.000
$y_{ij}=\ell n(x_{ij}-mx_{ij})$						
m = 0.0	175.3	0.000	622.0	0.000	834.0	0.000
0.1	171.0	0.000	607.1	0.000	819.9	0.000
0.2	166.0	0.000	590.0	0.000	803.1	0.000
0.3	160.1	0.000	570.0	0.000	783.0	0.000
0.4	153.1	0.000	546.2	0.000	758.4	0.000
0.5	144.5	0.000	517.4	0.000	727.4	0.000
0.6	133.7	0.000	481.6	0.000	687.1	0.000

A refinement of $c_j = m\bar{x}_j$ and $c_j = mx_{1j}$ in the above transformation was considered by the 3 parameter Lognormal model $\ln(x_{ij}) = y_j \sim N(r_j, \mu_j, \sigma_j^2)$.

Using Aitcheson & Brown's (1963) notation, ie. r_j is the 'location' of the parent variable X (where $y = \ln X$) and μ_j and σ^2 are its parameters.

These methods of estimating r_j, μ_j and σ^2 were considered. These were in Aitcheson & Brown's order of preference:

(i) Cohen's least sample value method
(ii) The method of maximum likelihood
(iii) The method of moments

(i) Cohen's least sample value method

Estimates c_j of r_j are obtained from the equation:

$$\phi(c_j) = \ln(x_{1j} - c_j) - \frac{1}{n_j}\sum_{i=1}^{n_j}\ln(x_{ij} - c_j) - \nu\left[\frac{1}{n_j}\sum_{i=1}^{n_j}\{\ln(x_{ij}-c_j)\}^2 - \frac{1}{n_j^2}\left\{\sum_{i=i}^{n_j}\ln(x_{ij} - c_j)\right\}^2\right] \qquad (1)$$

Where
$$m_j = \frac{1}{n_j}\sum_{i=1}^{n_j}\ln(x_{ij} - c_j) \qquad (2)$$

$$s_j^2 = \frac{1}{n_j}\sum_{i=1}^{n_j}\{\ln(x_{ij} - c_j)\} - m_j^2 \qquad (3)$$

x_{1j} is the lowest value in the j^{th} sample

ν is the N(0,1) quantile of the order n_{ij/n_j}, and

c_j, m_j, and s_j^2 are estimates of r_j, μ_j and σ_j^2 respectively.

Applying Aitcheson & Brown's rule of false position technique a solution to equation (1) was attempted, the results obtained being shown in Table 8.36. As can be seen only a few results were available. All other projects failed to produce a root of equation (1). The problem was probably caused by small sample sizes and the method had to be abandoned.

(ii) Method of maximum likelihood

This method was not recommended by Aitcheson & Brown on theoretical grounds and on some tests done by them with simulated data (where the least sample value method was said to be more reliable). However, it was considered that the method of ML may converge better with these sample sizes.

In this case:

$$\phi\,(c_j) = \sum_{i=1}^{n_j} \frac{1}{x_{ij} - c_j} \left[\frac{1}{n_j} \sum_{i=1}^{n_j} \{\ln\,(x_{ig} - c_j)\}^2 - \frac{1}{n_j} \sum_{i=1}^{n_j} \ln\,(x_{ij} - c_j) - \frac{1}{n_j^2} \left\{ \sum_{i=1}^{n_j} \ln\,(x_{ij} - c_j) \right\}^2 \right] + \sum_{i}^{n_j} \left[\frac{\ln\,(x_{ij} - c_j)}{x_{ij} - c_j} \right] = 0 \qquad (4)$$

and m_j and s_j^2 are obtained from equations (2) and (3) above. First attempts at evaluating equation (4) resulted in most low values of r_j satisfying the equation. The best method was found to be to take c_j as high as possible $c_j \to x_{ij}$ and take incremental reductions until (4) is satisfied.

Table 8.36: Estimates of the location parameter c_j using Cohen's Method

Case 1

Project (j)	c_j
3	1230385
20	7703505
21	878621
32	150505
47	1284251

Case 2

Project (j)	c_j
32	19551
57	22634
108	395101
110	233835
148	40944
150	80949
165	102416
168	11211
193	4480
196	8312
198	1675

Case 3

Project (j)	c_j	Project (j)	c_j
20	204407	310	493965
34	174476	311	263673
60	817300	338	1135994
66	324919	340	301972
112	328319	343	123776
128	126932	344	106893
146	63853	361	6266600
180	378960	364	257940
195	753144	372	339484
231	333831	373	85002
238	424462		
253	324945		
280	2888649		

This method, though better than the least value method, was not considered suitable as some solutions could not be obtained.

(iii) The Method of Moments

By this method estimates are obtained from:

$$s_j^2 = \ell n\ (1 + \mu_j^2)$$

$$m_j = \tfrac{1}{2}\ [\ell n\ \ell z_j - \ell n\ \{\mu_j^2\ (1 + \mu_j^2)\}]$$

$$c_j = \ell_{ij} - e^{m_j}\ (1 + \mu_j^2)^{\frac{1}{2}}$$

Values of μ_j and ℓ being obtained from the moments. However, as Aitcheson & Brown observe, this method is not efficient except for small values of σ^2, which certainly do not exist in the parent populations under study.

A further approach was devised which utilized the homoscedasic assumption, ie. $\ell n\ (x_{ij}) = y_{ij} \sim N(\mu_j, \sigma^2)$ as follows:

Let $y_{ij} = \ell n\ (x_{ij} - c_j)$ where $y_{ij} \ni Y_j \sim N\ (\mu_j, \sigma^2)$ (5)

and $x_{ij} \ni X_j \sim f\ (m_j, s_j^2)$

and $z_{ij} = e^{y_{ij}}$ $z_{ij} \ni Z_j \sim g\ (a_j,\ b_j^2)$ (6)

Then it is known that:

$$\sigma^2 = \ell n\ \left[\frac{b_j^2}{a^2_j} + 1\right]$$

and thus $a_j = b_j\ (e^{\sigma 2} - 1)^{-\frac{1}{2}}$ (7)

But, from (5) and (6)

$$z_{ij} = x_{ij} - c_j$$

so, $a_j = m_j - c_j$

and $b_j^2 = s^2{}_j$

Therefore substituting in (7)

$$c_j = m_j - s^2{}_j (e^{\sigma^2} - 1)^{-\frac{1}{2}} \tag{8}$$

Estimates of m_j and $s^2{}_j$ can be obtained from the data

$$\hat{m}_j = \frac{1}{n_j} \sum_{i=1}^{n_j} x_{ij}$$

$$\hat{s}^2{}_j = \frac{1}{n_{j-1}} \sum_{i=1}^{n_j} (x_{ij} - \hat{m}_j)^2$$

It can be seen therefore that c_j is a function of σ^2, the common variance. The problem now is to find values of σ^2 which will satisfy the Normal assumption.

This was done by a trial and error method involving guessing values of σ^2, solving (8) for c_j, inserting c_j values into $y_{ij} = \ell n\, (x_{ij} - c_j)$ and subjecting the resulting y_{ij} values to Shape tests 1-10 described in the previous section. The values of σ^2 which satisfy all the tests for the normal assumption were found to be $0.05 < \sigma^2{}_1 < 0.9$ for Case 1, $0.056 < \sigma^2{}_2 < 0.062$ for Case 2, and $0.04 < \sigma^2{}_3 < 0.1$ for Case 3. It can be seen therefore that $0.056 < \sigma^2{}_\ell < 0.062$ for all cases $\ell = 1, 2, 3$.

The next step was to attempt to predict the c_j values obtained above from the project values x_{ij} for each case. A plot of the calculated c_j values for each case for $\sigma^2 = 0.60$ is shown in Figure 8.13. A linear regression was performed of x_{ij} on c_j for each case and for the pooled values of all cases. (Table 8.37).

Fig 8.16 ALL CASES:LOWEST BIDS V THRESHOLD VALUES

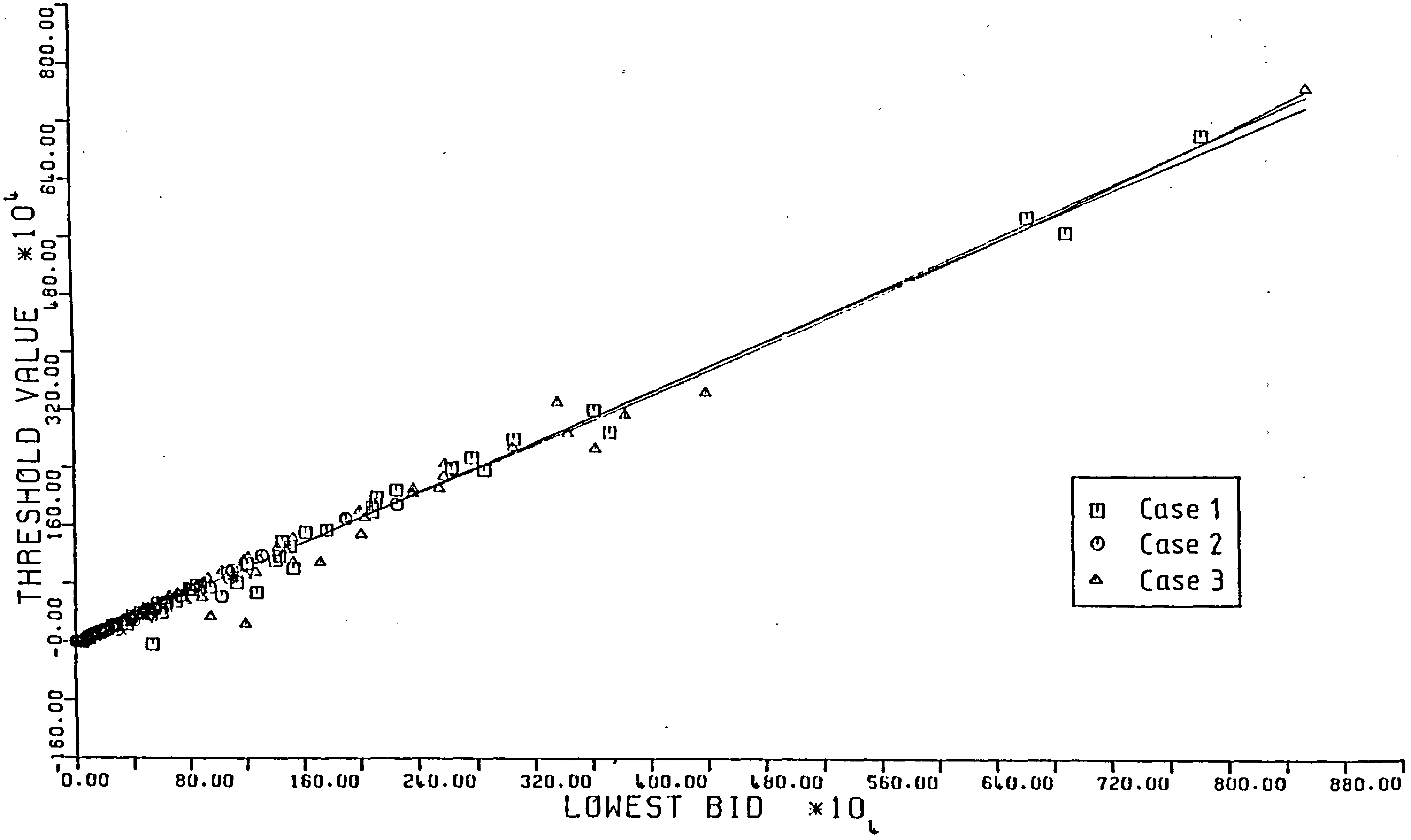


Table 8.37: Results of regression of project values (x_{ij}) on threshold values (c_j) for σ^2 = 0.60

Case	α	se	β	se
1	-43640	28462	0.8785	0.0122
2	-10909	1881	0.8541	0.0060
3	-14006	4694	0.8595	0.0055
1-3 (pooled)	-15517	3244	0.8650	0.0034

The standard errors of α and β suggest the ranges shown in Table 8.38.

Table 8.38: Ranges of α and β

Case	α		β	
	From	To	From	To
1	+13284	-100564	0.8544	0.9029
2	- 7147	- 14671	0.8420	0.8661
3	- 4618	- 23394	0.8486	0.8704
1-3 (pooled)	- 9029	- 22005	0.8581	0.8719

Values for c_j were then calculated from $-9000 < \alpha < -24000$ and $0.858 < \beta < 0.872$ for each case and the resulting $y_{ij} = \ell n\ (x_{ij} - c_j)$ subjected to the tests for Normal shape and homogeneity. No cases were found where the data passed the Normal *and* homogeneity tests.

Futher analyses, not reported here, were conducted of the transformation $y_{ij} = \ell n\ (x_{ij} - c_j)$ for $c_j = a + mx_{ij}$ with limited success for Cases 1 and 3. An additional model $c_j = a + mx_{ij} + bx_{ij}^2$

was also introduced for Case 2 but with no improvement.

8.4.3 Conclusions

Five models of distributional shape of bids for each project were examined, Uniform, Normal, Weibull, Lognormal and Gamma. The evidence suggests that none of these first four shapes model all the data satisfactorily. The transformation $y_{ij} = \ell n\ (x_{ij} - c_j)$ was found to provide values approximating the Normal distribution for $c_j = m\bar{x}_j$ (m = 0.1, 0.2, 0.3, 0.4, 0.5) and $c_j = mx_{1j}$ (m = 0.1, 0.2, 0.3, 0.4).

Estimates of the spread of bids for each project are not readily available. Several variance stabilizing transformations were attempted with little success. The main problem is that the test for homoscedasity (Bartlett's test) is dependent on the values being Normally distributed, this severely limiting the transformations available for these data. It is possible, however, that a non-parametric test for homoscedasity may be of benefit. It would seem therefore, in the absence of any further analyses, that approximate values of spread will have to be obtained by regression on the project values.

CHAPTER 9

The individual bidders

9 THE INDIVIDUAL BIDDERS

9.1 Introduction

The objective of this chapter is to identify simple statistical models of individual bidders *sui generis*. The first section proposes a procedure for estimating the probability that a specified bidder will enter a bid for a project. The second section examines the distribution of bids.

9.2 The Probability that Certain Bidders Bid for a Project

The probability that a bidder i bids for a project j ($i = 1,2,\ldots,r$; $j = 1,2,\ldots,c$) is given by

$$Pr\ (i) = n_i/c$$

where n_i is the number of projects on which bidder i has bid. Therefore, it can be stated, rather naively, that the probability of bidder i entering a bid for project c+1 is $Pr\ (i) = n_i/c$ and the probability of any set of bidders say i, i+1, i+2 is $Pr(i).Pr(i+1).Pr(i+2)$, assuming independence. The independence assumption is, of course, not likely to hold with these data as it is generally believed that the same bidders frequently bid for the same projects. One approach to this is to estimate the covariance matrix of $Pr(i)\ Pr(i+1)$ from the data and further matrices for the higher order covariances $Pr(i)\ Pr(i+1)\ Pr(i+2)\ldots$ It is not likely, however, that sufficient data will be available for this procedure. An alternative approach is to estimate Pr(i) as some function of the project value, a procedure which, as it is generally considered that certain bidders are associated with certain project characteristics, should go some way towards removing interdependencies amongst bidders.

The procedure adopted was to attempt to predict the likelihood of bidder i bidding by regressing the project value x_{ij} on the binary k=1 if a bid was entered and k=0 if no bid was entered for project j. The predicted values of k can then be treated as proxy probabilities for the probability that bidder i bids given a project value of x. The regression results for several of the most frequent bidders in Case 3 are given in Table 9.1. No tests were made to ascertain the distribution of the residuals, but a reasonable assumption is that they are normally distributed, the standard error of the co-efficients can, therefore, be utilised in estimating the necessary distributional parameters.

This procedure can clearly be extended to a MRA involving several predictor variables representing project characteristics, none of which were available in the data studied.

Table 9.1. Results of regression of log project value (lowest bidder) on probability of a specified bidder entering a bid

$\Pr(i \text{ bids}) = \alpha_i + \beta_i y_j$

$y_j = \ln(x_{ij})$

Bidder (i)	No of Bids	$\alpha_{(i)}$	(SE)	$\beta_{(i)}$	(SE)	t-test ($\beta=0$)
55	33	-0.45342	0.17036	0.04363	0.01366	3.19*
115	32	0.24042	0.17010	-0.01245	0.01364	0.91
152	34	-1.21123	0.16133	0.10485	0.01294	8.10*
173	36	0.24485	0.17940	-0.01194	0.01439	0.83
175	51	-0.26855	0.20784	0.03263	0.01667	1.96
268	57	0.82035	0.21601	-0.05374	0.01733	3.10
294	30	-0.52286	0.16235	0.04857	0.01302	3.73

*null hypothesis rejected at 5% level (2 tailed)

9.3. The Distribution of Bids Values Entered by Each Bidder

9.3.1. Introduction

Skitmore (1982) has proposed the model

$$\ln(x_{ij}) = y_{ij} \ni Y_{ij} \sim f(\alpha_i+\beta_j\ ,\ \sigma^2_i) \qquad (1)$$

Where x_{ij} is bidder i's bid for project j ($i = 1,2,\ldots,r$; $j = 1,2,\ldots,c$) and x_{lj} is another bidder l's bid for project j ($l = 1,2,\ldots,c$; $l \neq i$). Then, assuming bids are independent, estimates of the parameters in (1) may be obtained from

$$\bar{y}_i - \bar{y}_l \approx \alpha_i - \alpha_l$$
$$s^2_i \approx \sigma^2_i$$

by solving the two sets of equations:

$$\begin{array}{lcl}
E\,[Y_1] - E\,[Y_1] & = & \bar{z}_{11} \\
E\,[Y_1] - E\,[Y_2] & = & \bar{z}_{12} \\
\vdots \quad\quad \vdots & & \\
E\,[Y_1] - E\,[Y_r] & = & \bar{z}_{1r} \\
\vdots \quad\quad \vdots & & \\
E\,[Y_r] - E\,[Y_r] & = & \bar{z}_{rr}
\end{array} \qquad ,\text{and} \qquad (2)$$

$$\begin{array}{lcl}
\mathrm{Var}\,[Y_1] + \mathrm{Var}\,[Y_1] & = & s^2_{11} \\
\mathrm{Var}\,[Y_1] + \mathrm{Var}\,[Y_2] & = & s^2_{12} \\
\vdots \quad\quad \vdots & & \\
\mathrm{Var}\,[Y_1] + \mathrm{Var}\,[Y_r] & = & s^2_{1r} \\
\vdots \quad\quad \vdots & & \\
\mathrm{Var}\,[Y_r] + \mathrm{Var}\,[Y_r] & = & s^2_{rr}
\end{array} \qquad (3)$$

Where $z_{ilj} = (y_{ij} - y_{lj})$

$$z_{il} = \frac{1}{n_{il}} \sum_{j=1}^{c} \delta_{ilj} \quad z_{ilj}$$

$$s^2_{il} = \frac{1}{(n_{il}-1)} \sum_{j=i}^{c} \delta_{ilj} \, (z_{ilj} - \bar{z}_{il})^2$$

$$n_{il} = \sum_{j=1}^{c} \delta_{ilj}$$

δ_{ilj} = 1 when bidders i and l both enter bids for project j, 0 otherwise

The problem can, theoretically, be solved by the standard regression procedure as follows.

Letting the event that bidder i bids against bidder l be denoted by

$$W = \left(\begin{array}{ccccccc} 11 & 21 & \dots & i1 & \dots & r1 \\ 12 & 22 & \dots & i2 & \dots & r2 \\ . & . & & . & & . \\ . & . & & . & & . \\ . & . & & . & & . \\ 1l & 2l & \dots & il & \dots & rl \\ . & . & & . & & . \\ . & . & & . & & . \\ . & . & & . & & . \\ 1r & 2r & \dots & ir & \dots & rr \end{array} \right)$$

which is indexed by:

$k=1$ for W^{12}, $k=2$ for W^{13}, $k=l-1$ for W^{1l}, $k=r-1$ for W^{1r}

$k=r$ for W^{23}, $k=r+1$ for W^{24}, $k=r+l-3$ for W^{2l}, $k=2r-3$ for W^{2r}

$k=2r-2$ for W^{34}, $k=2r-1$ for W^{35}, $k=2r+l-6$ for W^{3l}, $k=3r-6$ for W^{3r}

.   .   .   .

.   .   .   .

.   .   .   .

$k = \{(i-1)r - \sum_{p=1}^{i-1} p\}+1$ for $W^{i,i+1}$ ... $k = \{(i-1)r - \sum_{p=1}^{i-1} p\}+1$ for W^{il}

$$k = ir - \sum_{p=1}^{i} p \text{ for } W^{ir}$$

$$n = k = \{(r-1)r - \sum_{p=1}^{r} p\} \quad \text{for } W^{r-1,r}$$

Then if

X_{kij} = 1 when the event indexed by k occurs, which includes the lower numbered bidder i, on contract j, and
X_{kij} =-1 when the event indexed by k occurs, which includes the higher numbered bidder i, on contract j, otherwise
X_{kij} = 0

and z_{kj} is the difference in bids between the two bidders when event k occurs on contract j

The normal equations are

$$\begin{array}{ccccccccccc}
b_1X_{111} & + & b_2X_{121} & + & b_3X_{131} & + & \ldots & + & b_rX_{1r1} & = & z_{11} \\
b_1X_{211} & + & b_2X_{221} & + & b_3X_{231} & + & \ldots & + & b_rX_{2r1} & = & z_{21} \\
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
b_1X_{k11} & + & b_2X_{k21} & + & b_3X_{k31} & + & \ldots & + & b_rX_{kr1} & = & z_{k1} \\
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
b_1X_{m11} & + & b_2X_{m21} & + & b_3X_{m31} & + & \ldots & + & b_rX_{mr1} & = & z_{m1} \\
\hline
b_1X_{112} & + & b_2X_{122} & + & b_3X_{132} & + & \ldots & + & b_rX_{1r2} & = & z_{12} \\
b_1X_{212} & + & b_2X_{222} & + & b_3X_{232} & + & \ldots & + & b_rX_{2r2} & = & z_{22} \\
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
b_1X_{k12} & + & b_2X_{k22} & + & b_3X_{k32} & + & \ldots & + & b_rX_{kr2} & = & z_{k2} \\
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
b_1X_{m12} & + & b_2X_{m22} & + & b_3X_{m32} & + & \ldots & + & b_rX_{mr2} & = & z_{m1} \\
\hline
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
\hline
b_1X_{11c} & + & b_2X_{12c} & + & b_3X_{13c} & + & \ldots & + & b_rX_{1rc} & = & z_{1c} \\
b_1X_{21c} & + & b_2X_{22c} & + & b_3X_{23c} & + & \ldots & + & b_rX_{2rc} & = & z_{2c} \\
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
b_1X_{k1c} & + & b_2X_{k2c} & + & b_3X_{k3c} & + & \ldots & + & b_rX_{krc} & = & z_{kc} \\
\vdots & & \vdots & & \vdots & & & & \vdots & & \vdots \\
b_1X_{m1c} & + & b_2X_{m2c} & + & b_3X_{m3c} & + & \ldots & + & b_rX_{mrc} & = & z_{mc}
\end{array}$$

Estimates of $E[Y_1]$, $E[Y_2]$,... will therefore be provided by the vector

$$B = C^{-1}D$$

Where

$$B = \begin{pmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_r \end{pmatrix} \qquad D = \begin{pmatrix} \sum_k^m \sum_j^c X_{k1j}\ z_{kj} \\ \sum_k \sum_j X_{k2j}\ z_{kj} \\ \cdot \\ \cdot \\ \cdot \\ \sum_k \sum_j X_{krj}\ z_{kj} \end{pmatrix}$$

$$C = \begin{pmatrix} \sum_k^m \sum_j^c X^2_{k1j} & \sum_k \sum_j X_{k1j}X_{k2j} & \dots & \sum_k \sum_j X_{k1j}X_{krj} \\ \sum_k \sum_j X_{k2j}X_{k1j} & \sum_k \sum_j X^2_{k2j} & \dots & \sum_k \sum_j X_{k2j}X_{krj} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \sum_k \sum_j X_{krj}X_{k1j} & \sum_k \sum_j X_{krj}X_{k2j} & \dots & \sum_k \sum_j X_{krj}^2 \end{pmatrix} \qquad (4)$$

And the variance of b_1 is estimated by

$$\text{Var } b_1 = \frac{S}{N-r}\ c^{ii}$$

$$\text{Where } S = \sum_k^m \sum_j^c \{ z_{kj} - (b_1X_{k1j} + b_2X_{k2j} + \dots + b_rX_{krj}) \}^2$$

and N is the total number of paired observations.

The major difficulty with this approach is in the sparseness of the matrix system. In each row (in eqn 4) there are r-2 empty cells. Afifi and Elashoff (1966) have reviewed the literature on the problem of handling multivariate data with observations missing for some or all of the variables under study noting that the estimation problems can often be simplified if the missing data follows certain patterns. Hocking & Smith (1968) have used estimates of parameters from one part of a (mulitvariate normal) data structure to insert into the other parts prior to using an iterative procedure. Elman (1982) has considered the use of direct and iterative methods of solving large sparse nonsymmetric systems of linear equations finding difficulties with direct methods due to the factoring process generating many more non-zeros than the coefficient matrix, thereby increasing the computational storage size needed. A further problem encountered was that the number of arithmetic operations could become excessive. His general conclusion was that "... although progress has been made in the development of orderings for the unknowns that decrease the complexity of directness for solving sparse problems ... many large sparse problems cannot be solved by direct methods on present day computers".

Some early tests on the data using matrix methods confirmed Elman's view that direct methods were unsuitable. The extreme sparseness of the data under study, in the proportion cr to c(r-2), produced results severely distorted by computational rounding errors. It was, therefore, considered that an iterative procedure would be more appropriate.

9.3.2 The iterative procedure

The model adopted was, from eqn (1)

$$y_{ij} = \alpha_i + \beta_j + \epsilon_{ij} \qquad (5)$$

Where ϵ_{ij} is $f(o,\sigma^2_i)$

(it was noted that $y_{ij} - y_{lj} = \alpha_i - \alpha_l + \epsilon_{ij} - \epsilon_{il}$, where $\epsilon_{ij} - \epsilon_{il}$ is $f\,(o,\sigma^2_i + \sigma^2_l)$ and that, although appropriate for differences, eqn (5) was preferred as less information is lost).

Assuming $f\,(o,\sigma^2_i)$ is $N\,(o,\sigma^2_i)$

y_{ij} has a pdf $\dfrac{1}{\sigma^2_i} \exp\{ - \dfrac{1}{2\sigma^2_i} (y_{ij} - \alpha_i - \beta_j)^2\}$

The log-likelihood is

$$\ln L = - \sum_{i=1}^{r} (n_i/2) \ln \sigma^2_i - \tfrac{1}{2} \sum_{i=1}^{r} (1/\sigma^2_i) \sum_{j=1}^{c} \delta_{ij}\, (y_{ij} - \alpha_i - \beta_j)^2$$

Where Kroneka's δ_{ij} = 1 if bidder i bids for project j
= 0 if bidder i does not bid for project j

$n_i = \sum_{j=1}^{c} \delta_{ij}$ = number of bids made by bidder i

The MLL over α's, β's and σ^2 is

$$\frac{\delta \ln L}{\delta \beta j} = \sum_{i=1}^{r} \delta_{ij} \, (y_{ij}-\alpha_i-\beta_j)/\sigma^2{}_i = 0$$

$$\Rightarrow \beta_j = \sum_{i=1}^{r} \delta_{ij} \, (y_{ij}-\alpha_i)/n_i \tag{6}$$

$$\frac{\delta \ln L}{\delta \alpha_i} = (1/\sigma^2{}_i) \sum_{j=1}^{c} \delta_{ij} \, (y_{ij}-\alpha_i-\beta_j) = 0$$

$$\Rightarrow \alpha_i = \sum_{j=1}^{c} \delta_{ij} \, (y_{ij}-\beta_j)/n_i \tag{7}$$

$$\frac{\delta \ln L}{\delta \sigma^2{}_i} = \frac{-n_i}{2\sigma^2{}_i} + \frac{1}{2\sigma^{4i}} \sum_{j=1}^{c} \delta_{ij} \, (y_{ij}-\alpha_i-\beta_j)^2$$

$$\Rightarrow \sigma^2{}_i = \frac{1}{n_i} \sum_{j=1}^{c} \delta_{ij} \, (y_{ij}-\alpha_i-\beta_j)^2$$

The procedure used was to initialise all $\alpha_i = 0$ and iterate equations (6) and (7) to convergence. The estimates of $\sigma^2{}_i$ were adjusted for bias by the approximation

$$\hat{\sigma}^2{}_i = \sigma^2{}_i \left\{ \frac{n_i}{(n_i-1)\ (1 - \frac{C-1}{N-r})} \right\}$$

where $N = \sum_{j=1}^{c} n_i$, the total number of observations

For computational purposes it is unnecessary to introduce once only bidders, n_i=1, until after convergence of the iteration procedure. Convergence was taken to have occurred when the change in estimated value of any α_i in consecutive iterations was less than e_α, where e_α is small (the appropriate value of e_α was found, after various trials, to be 0.000001).

The data were transformed by $y_{ij}=\ln(x_{ij})$ and the values of β_j, α_i and $\sigma^2{}_i$ obtained for each Case. The results for Case 1 are given in Table 9.3.

The distribution of the residuals was then inspected to obtain some impression of the nature of $f(0,\sigma^2)$ of ϵ_{ij}.

Table 9.2 Sample of predictions from iterative procedure for Case 1

(a) α_i & σ^2_i values

Bidder (i)	No of bids (n_i)	α_i	σ^2_i
6	2	0.01667	0.00052
8	1	0.00717	
12	1	-0.00871	
20	1	-0.07646	
24	7	-0.04071	0.00099
31	1	-0.03508	
55	20	0.03154	0.00253
60	2	0.06268	0.00292
64	1	-0.03523	
72	1	-0.02970	
73	1	0.00579	
75	2	-0.02239	0.00273
79	4	-0.04906	0.00042
83	2	0.09766	0.00361

(b) β_j values

Project (j)	β_j
1	14.18677
2	13.15789
3	14.09286
4	13.42515
5	12.88743
6	14.58104
7	14.95664
8	15.82349
9	13.70479
10	13.90194

9.3.3 Distribution of aggregated residuals

9.3.3.1 Shape

Figs 9.1 to 9.3 indicate the frequency distribution of the residuals $z_{ij} = y_{ij} - \alpha_i - \beta_j$, $y_{ij} = \ln(x_{ij})$ for each of the three cases. The shape indicated in all the cases suggested that the Normal distribution may be an appropriate model. The cumulative probability plots (Fig 9.4 to 9.6), however, indicate that the Normal model may not be the most appropriate, the data being rather heavy tailed. Superimposition of the Normal curves on the histograms (Fig 9.1 to Fig 9.3) illustrates the position.

The plots suggest that the distribution of residuals may be similar for each Case. This was tested by comparing the frequency distribution of residuals for each Case with the frequency distribution of the pooled residuals for all Cases. On the assumption that the pooled residuals represent the total population of residuals, the chi-square test was applied to test the hypothesis that the residuals for each Case were samples from the total population. The results of this test indicated that the hypothesis should not be rejected, chi-square values of 2.9 (8df), 3.6 (10df) and 4.0 (14df) being recorded for Cases 1, 2 and 3 respectively, after having standardised the residual values by dividing by the estimated standard deviation for each Case.

Having concluded that the distribution of the (standardised) residuals could be considered to be the same for all Cases, the residuals were pooled and some tests applied to determine the shape of the resulting distribution.

9.3.3.2 Normal model

Visual inspection of the frequency distribution of the pooled residuals suggested the Normal model to be a possible approximation. On attempting to fit a Normal distribution of zero mean and unity variance it was immediately apparent that a smaller variance would provide a better visual fit. Several variances were, therefore, attempted (Fig 9.7). Plots of Normal order statistics against the frequency of the

Fig.9.1 Case 1: Distribution of residuals

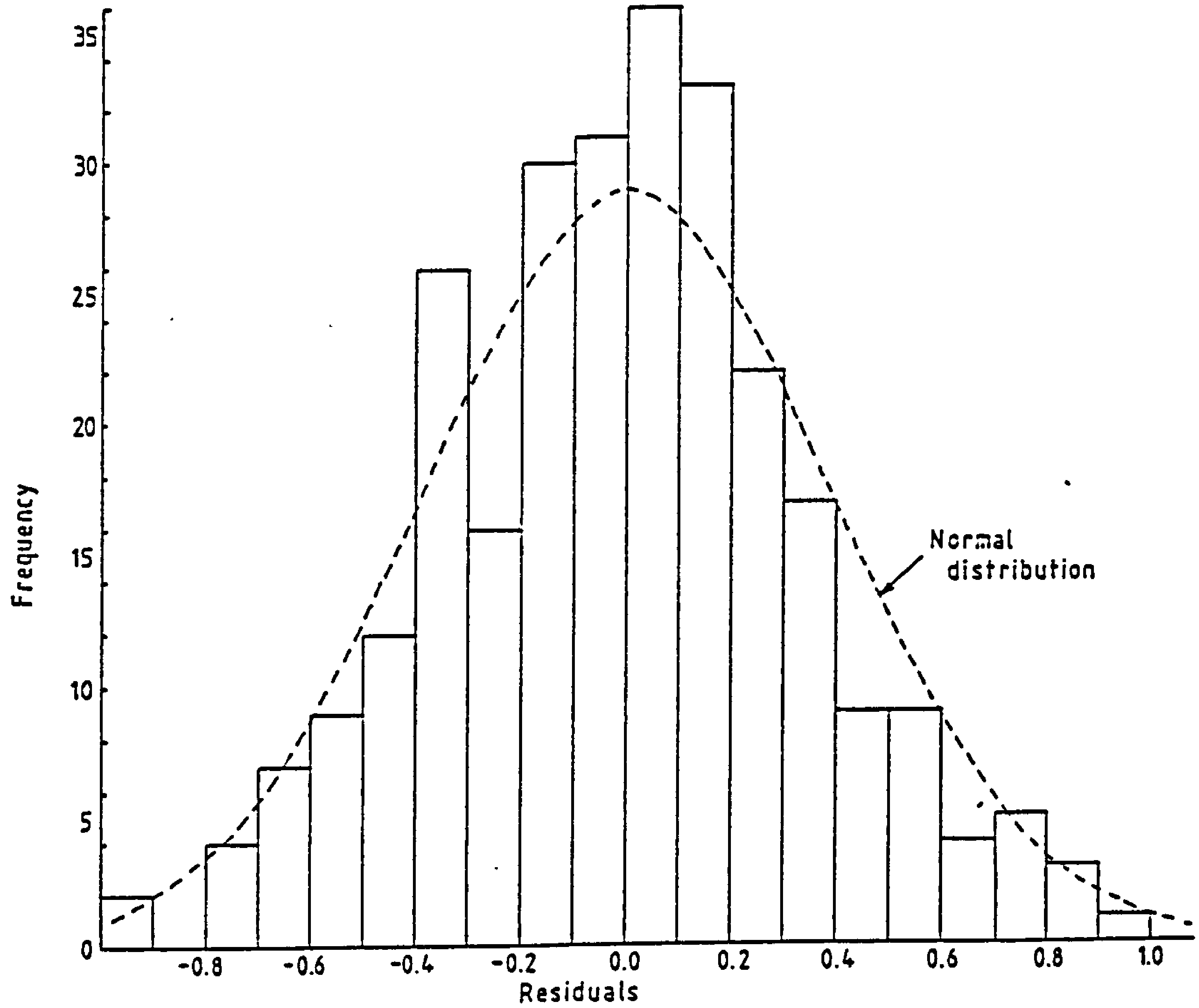


Fig 9.2 Case 2: distribution of residuals

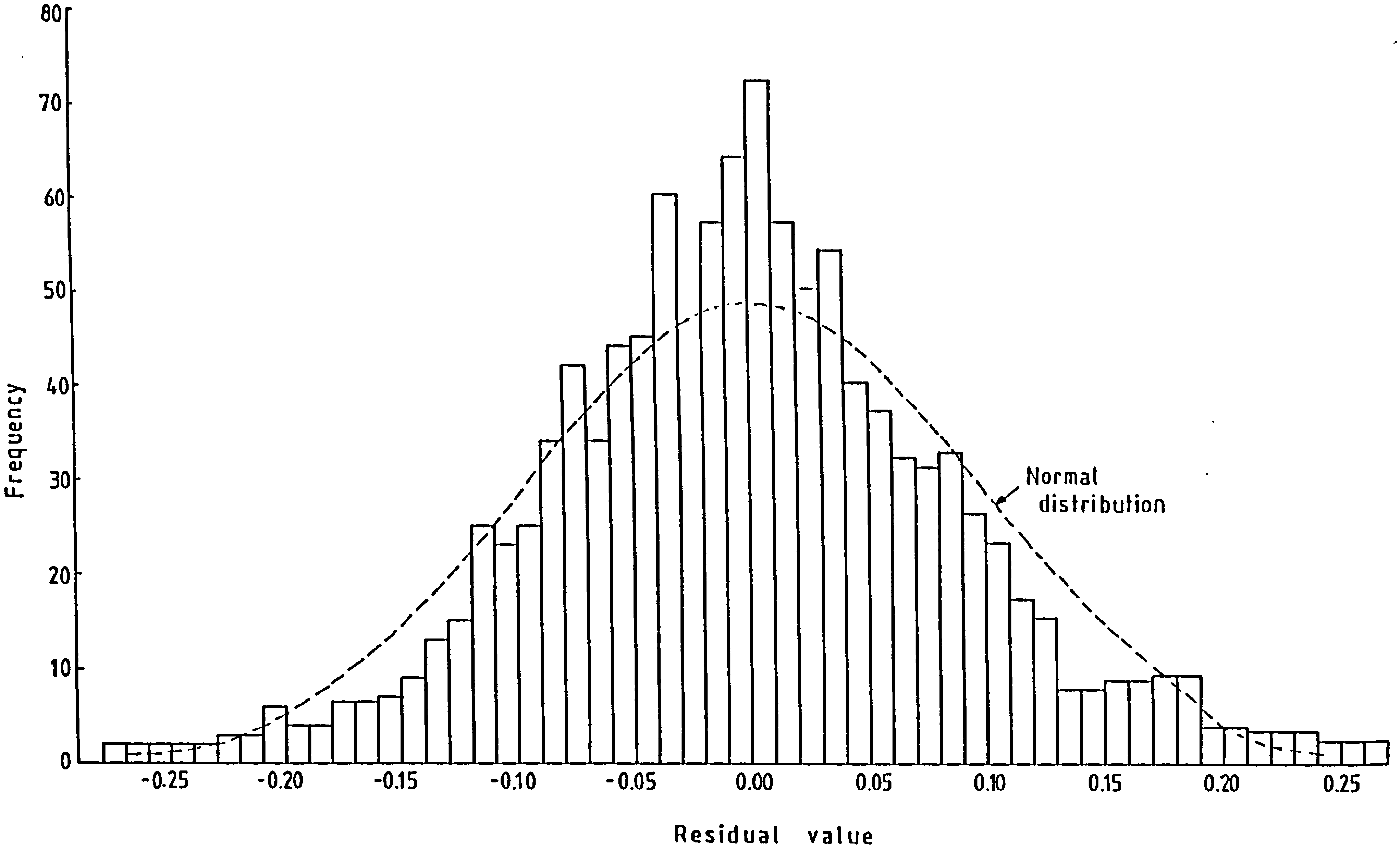


Fig.9.3 Case 3: Distribution of residuals

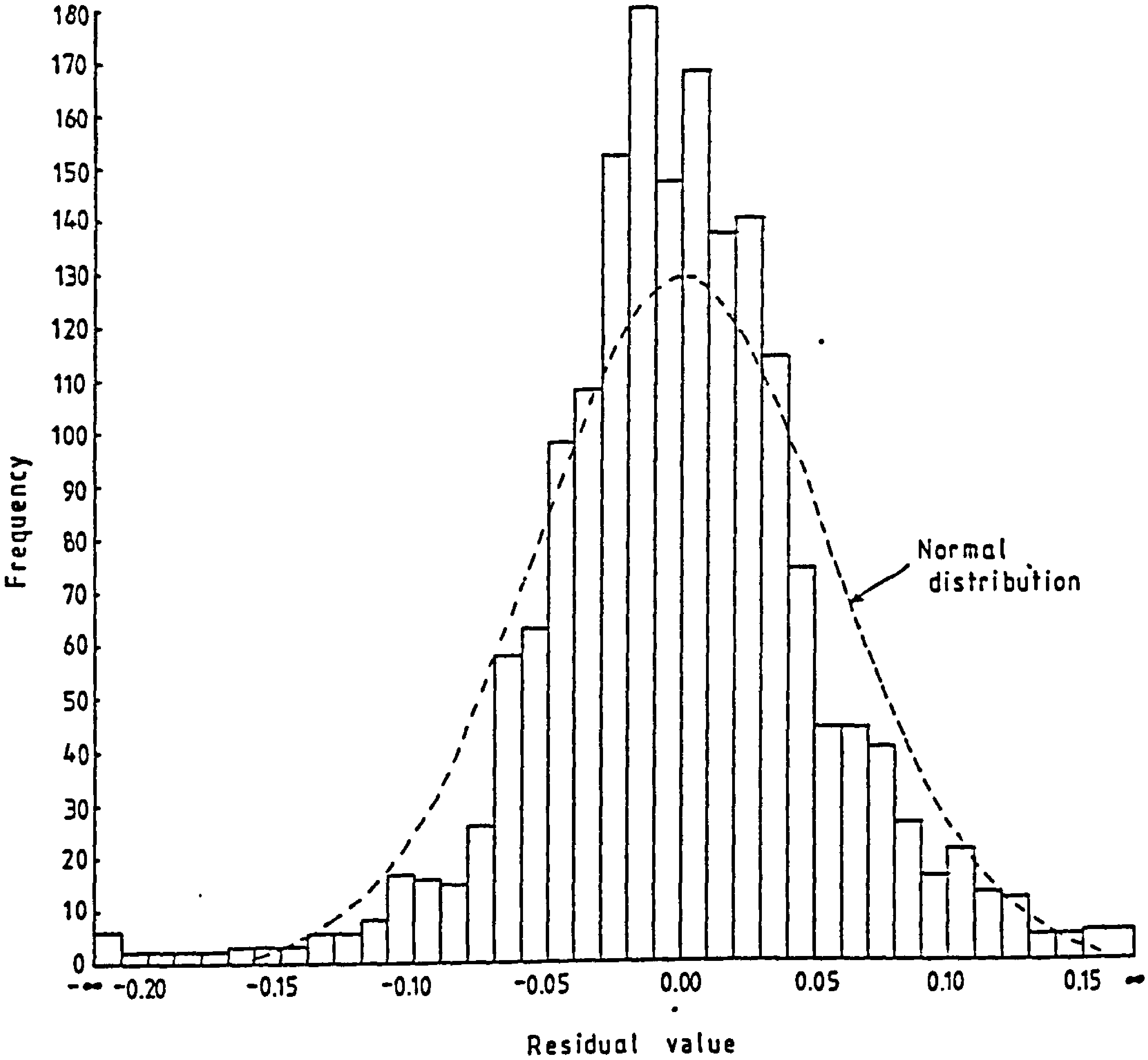
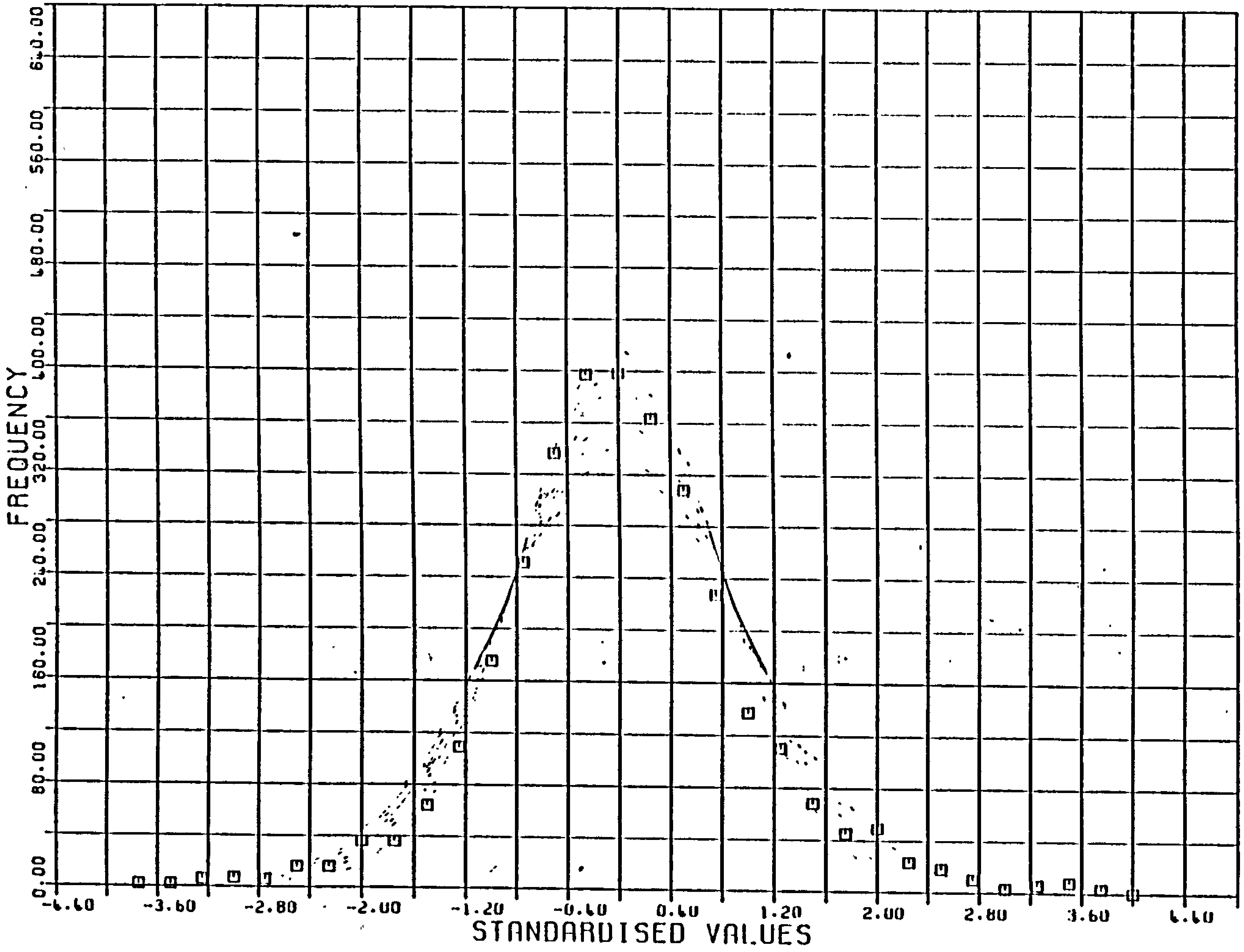


Fig 9.4 Case 1: probability plot of residuals

Fig 9.5 Case 2: probability plot of residuals

Fig 9.6 Case 3: probability plot of residuals

Fig 9.7 DISTRIBUTION OF POOLED RESIDUALS



pooled residuals were made. Fig 9.8 shows the plot against N(0,1) and Fig 9.9 against N(0,0.6). This visual inspection suggested the N(0,0.6) model to be the best approximation. This visual fit was not confirmed, however, by the chi-square and Kolmogorov-Smirnov goodness of fit tests (Table 9.3).

Table 9.3 Goodness of fit tests for various Normal distributions to the standardised pooled residuals

$N(0,\sigma^2)$	$\chi^2_{(23)}$	K-S D Statistic
σ^2 = 1.0	204	0.047
0.9	229	0.063
0.8	322	0.126
0.7	547	0.203
0.6	876	0.298

As the critical values at the 5% significance level are $\chi^2_{(23)} < 35.17$ and $D < 0.024$ none of the Normal distributions attempted were judged to be of a sufficiently good fit. The results for $\sigma^2 = 1.0$ are, of course, not surprising as $\sigma^2 = 1.0$ is the best estimate for standardised values.

The visual closeness of the distribution of the pooled residuals to a Normal distribution suggested that some function of the Normal distribution would be the best approach. Pearson's distributions were first consulted to check the possibility that a relatively simple unique function may suffice.

9.3.3.3. Pearson's distributions

The criterion k was calculated from the formula

$$k = \frac{\beta_1 (\beta_2 + 3)^2}{4 (2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)}$$

Fig.9.8 Plot of Normal order statistics N(0,1) against the frequency of the standardised pooled residuals

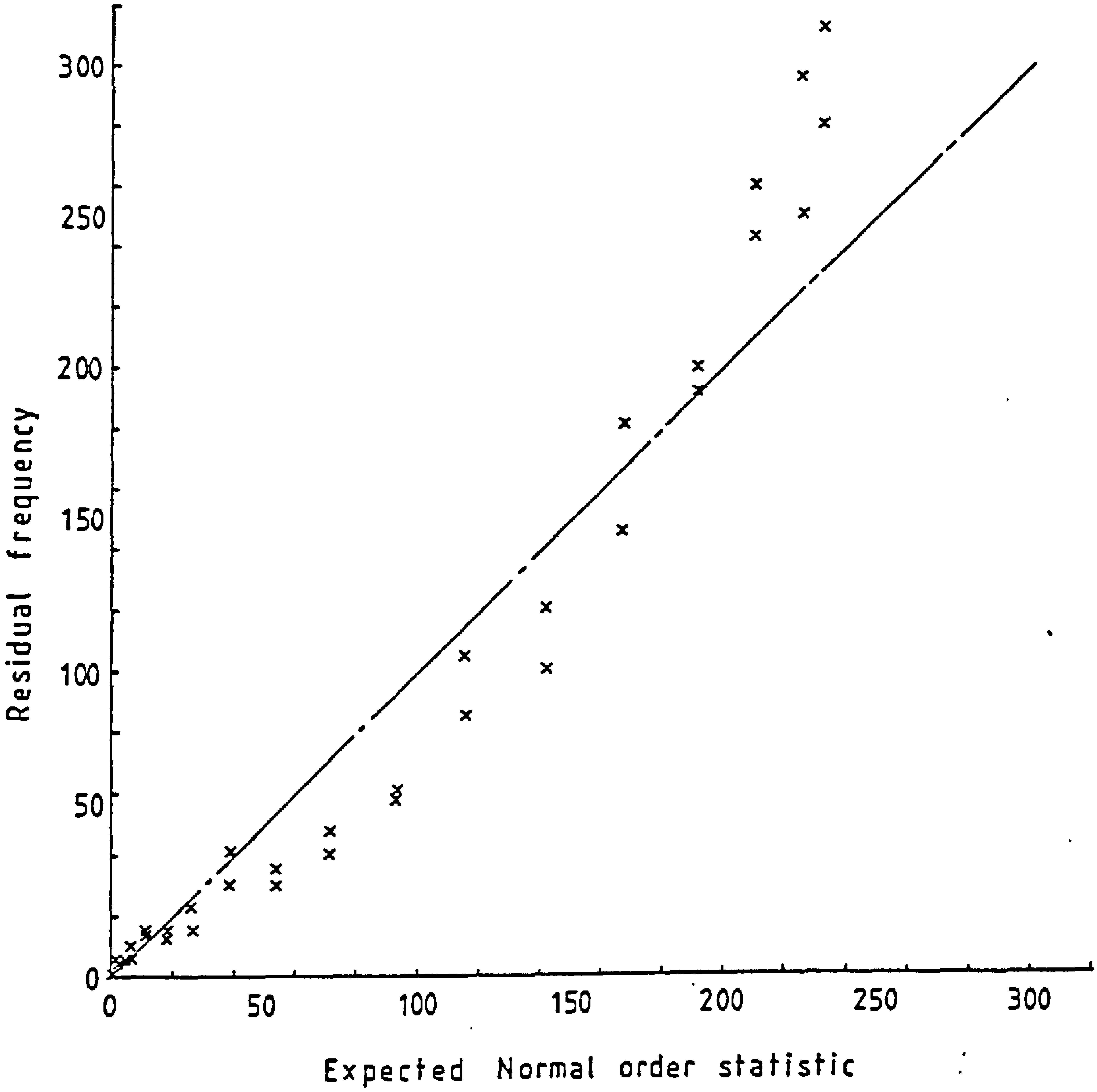
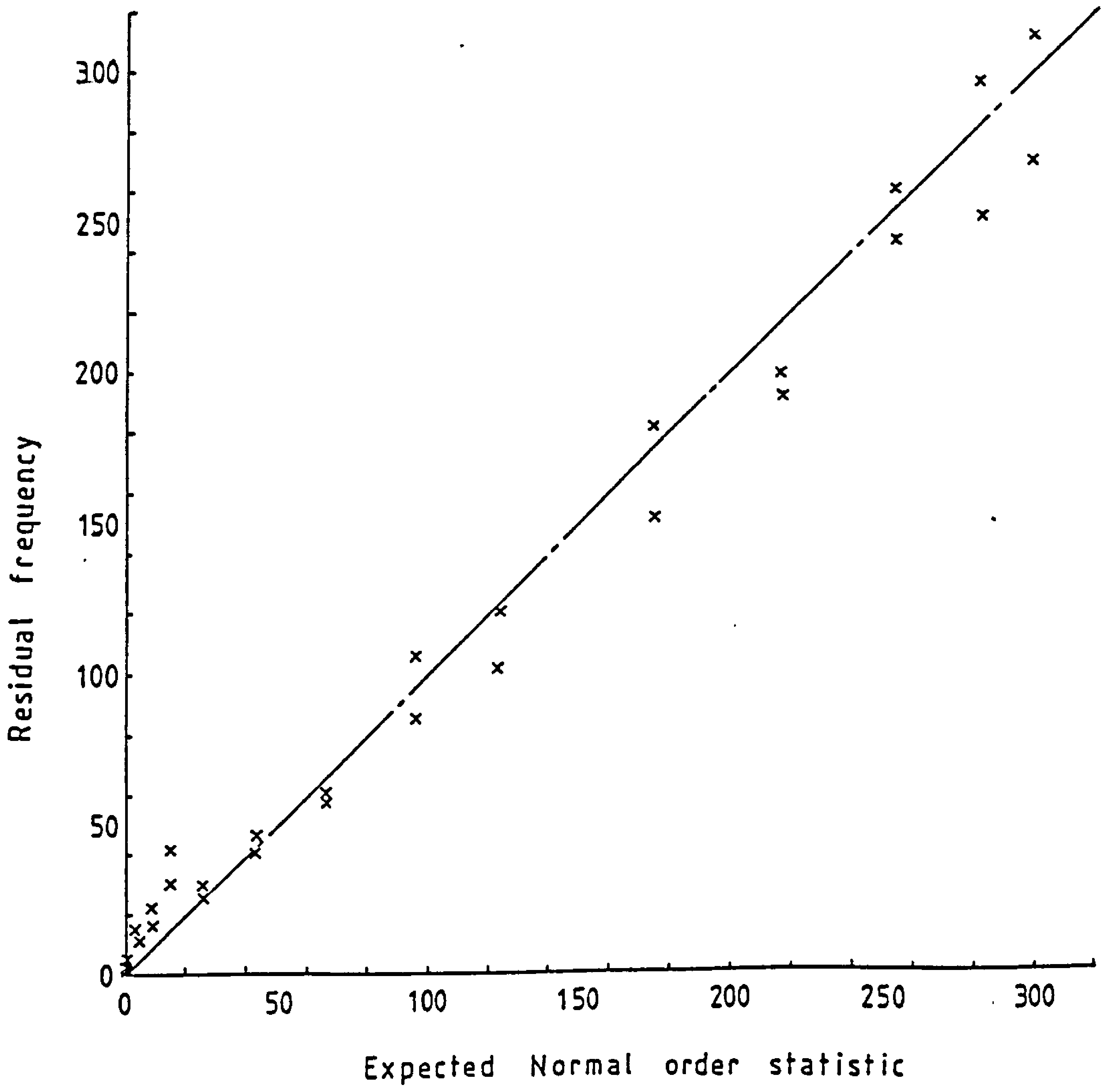


Fig.9.9 Plot of Normal order statistics N(0,0.6) against the frequency of the standardised pooled residuals



and found to be between zero and unity, suggesting the Type IV distribution as the most appropriate. However, as Type IV distribution requires repeated numerical integration of the pdf in its application, it was decided that a computationally more efficient distribution should be adopted. One distribution of this type is the Gram-Charlier Type A series or the Edgeworth expansion.

9.3.3.4 Gram-Charlier series of Type A and Edgeworth's form

Kendall & Stuart (1963,p162) note that the Type A series can encounter difficulties when cumulants above the fourth are included, unless the skewness coefficient is "close enough" to zero. A coefficient of $|\beta_1| \geq 0.25$ will produce a non-unimodal distribution with Edgeworth's form. Similarly, $|\beta_2| \geq 0.50$ produces negative frequencies. The Gram-Charlier series has a wider range of acceptability, but certainly non-unimodal if $|\beta_1| \geq 0.7$.

As the skewness coefficient of the pooled residuals was 0.05748, it was considered that both Gram-Charlier and Edgeworth's form of Type A series would be appropriate for more than the fourth cumulant. Further considerations (see Appendix A) on the similarity between the Gram-Charlier and Edgeworth form indicated that only the Gram-Charlier series need be used.

The first 4 terms of the Gram-Charlier Type A series give a Cdf as follows:

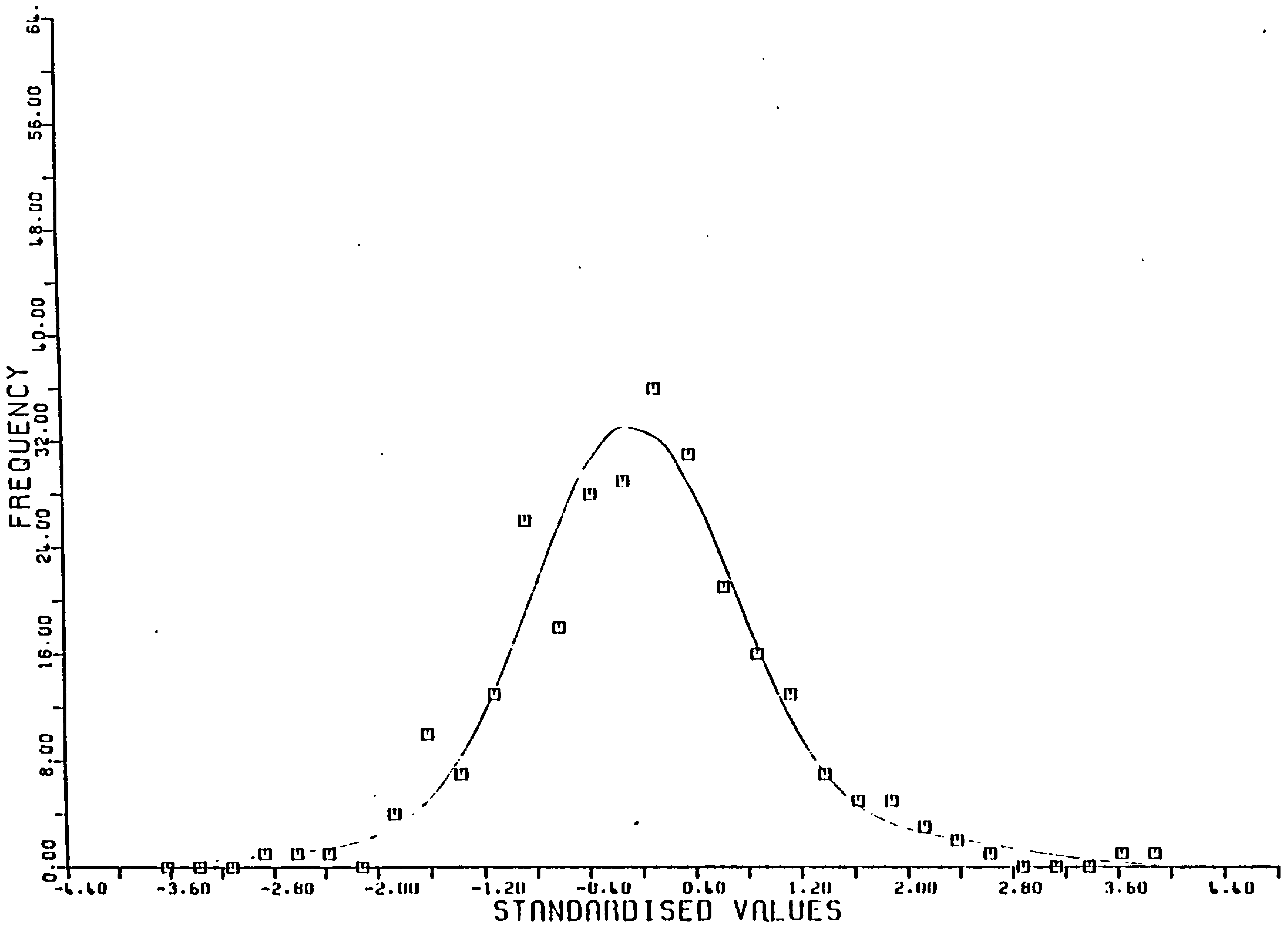
$$\mathrm{Cdf}\ (x) = \{1/(2\pi)^{1/2}\} \int_{-\infty}^{\infty} \exp\ (-\tfrac{1}{2}x^2)\,dx - [\{1/(2\pi)^{1/2}\exp(-\tfrac{1}{2}x^2)\}\ \{(\mu_3/6)(x^2-1) + (1/24)(\mu_4-3)(x^3-3x)\}]$$

where x is standardised.

The population moments μ_3 and μ_4 were estimated from the data as 0.16 and 6.03 respectively.

After a series of trials it was found that a value of 4.5 for the fourth movement produced a more satisfactory fit (Fig 9.10). See Appendix A for estimating moments of population. Tests of goodness of fit resulted in $\chi^2_{(27)} = 32.7$ and Kolmogorov's D = 0.012 - the theoretical distribution not being significantly different from the data

Fig 9.10 DISTRIBUTION OF RESIDUALS (1)



(at 5% significance).A casewise check was made to establish that the model was appropriate in all Cases, by fitting the model to the standardised residuals of each of the three sets of residuals (Figs 9.11 to 9.13).

Table 9.4 provides the results of the goodness of fit tests.

Table 9.4 Gram-Charlier Type A series - goodness of fit tests

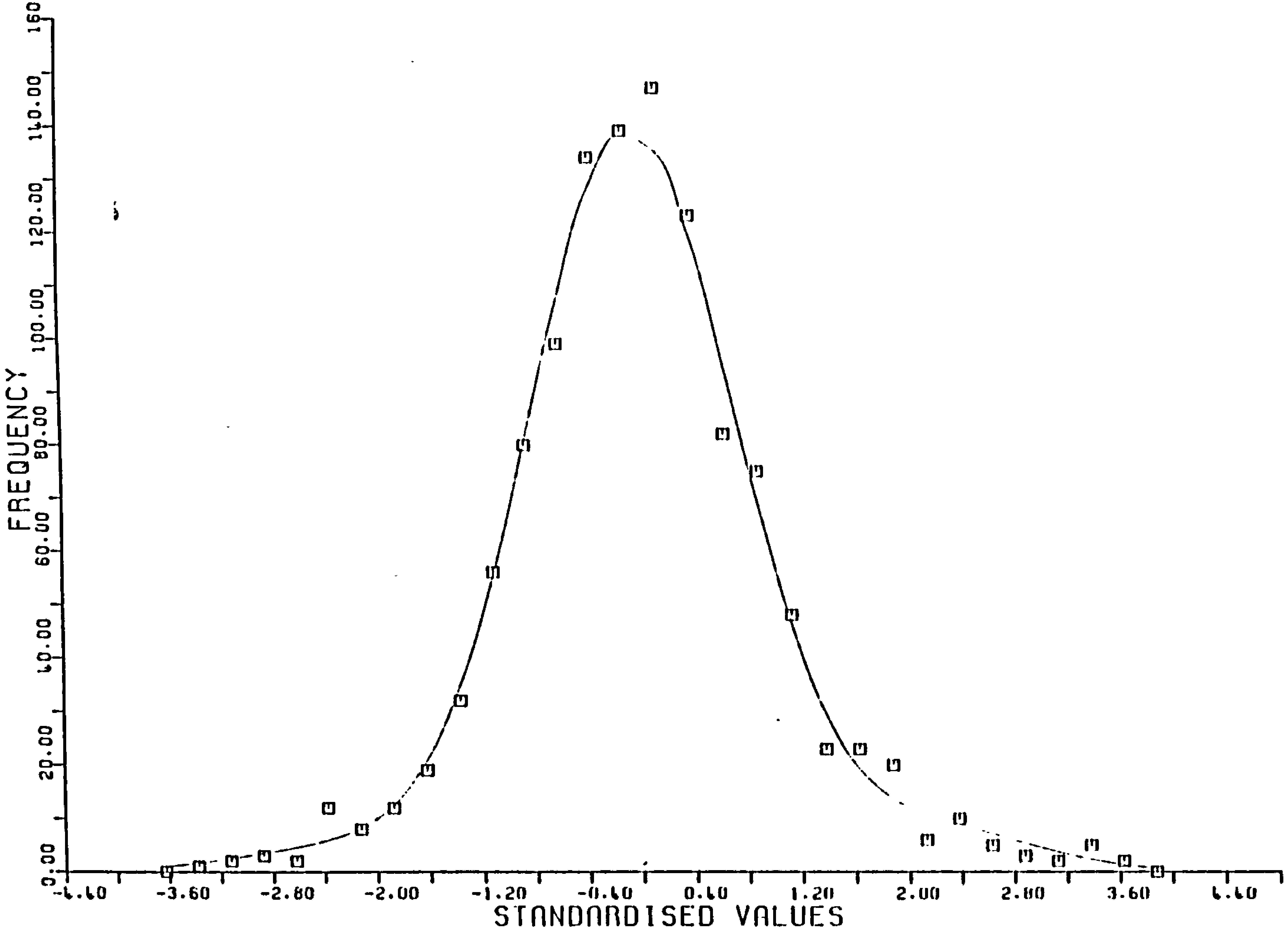
Case	χ^2	Df	Crit Value	Kolmogorov's D	Crit Value
1	13.3	15	25.0	0.035	0.081
2	19.8	23	35.0	0.016	0.040
3	31.5	24	36.0	0.019	0.032
all cases	32.7	27	-	0.012	-

As a final check, a simulation exercise was conducted in which random standardised values were generated for each bidder from a Gram-Charlier Type A distribution for $\mu_3 = 0.16$ and $\mu_4 = 4.5$ (see Appendix A for details). The chi-square and K-S tests both failed to reject the null hypothesis in each and every Case. It was, therefore, concluded that the Gram-Charlier series with parameters $\mu_3 = 0.16$ $\mu_4 = 4.5$ was a reasonable model of the standardised residuals.

9.3.3.5 Discussion

The discovery of high peaked, heavy tailed distributions is not a new phenomenon in empirical studies of data of these kind. Ali & Giacotto (1982), for instance, in their study of stock market prices found that "... the empirical distributions of price changes are usually high peaked with heavy tails when compared with the normal distribution". Studies by Clark (1973) and Hsu et al (1974) of similar data suggest that "... if the price changes are normal but *not* identically distributed, it is likely that the empirical distributions would be highly peaked and heavy tailed compared to the Normal distribution" (Ali & Giacotto,1982,p19) the major differences being attributed to the lack of tenability of the constant scale assumption.

Fig 9.11 DISTRIBUTION OF RESIDUALS (2)



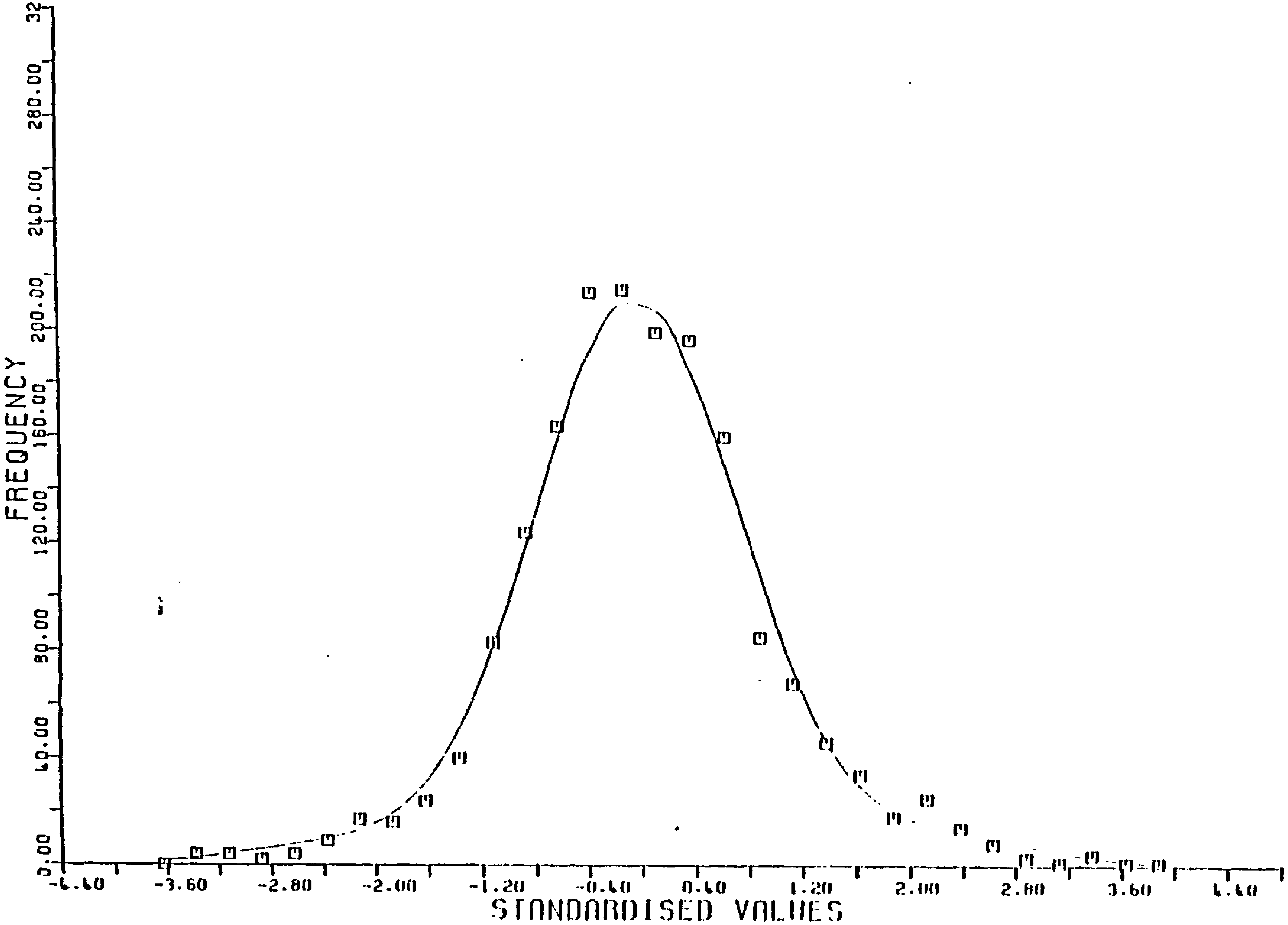
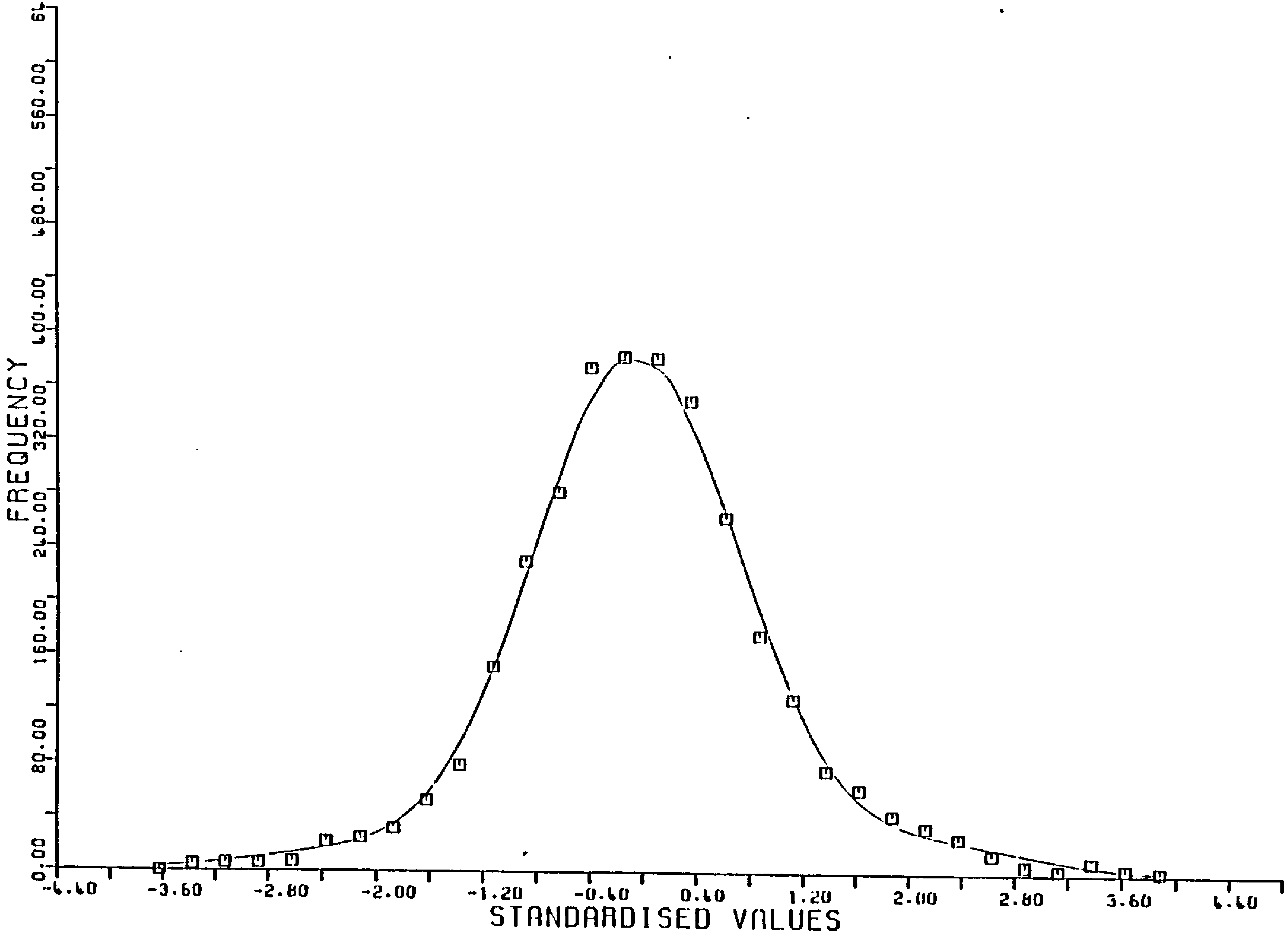


Fig 9.12 DISTRIBUTION OF RESIDAULS (3)

Fig 9.13 DISTRIBUTION OF POOLED RESIDUALS



On this evidence, therefore, it was decided to test the distribution of individual bidders' bids as determined by the distribution of the residuals obtained for each bidder. This was conducted along the lines of the previous Chapter in which the distribution of bids for each project was separately assessed before aggregating the test statistics.

9.3.4 Distribution of individual bidders' bids

9.3.4.1 Shape

The distribution shape under consideration is that of the values $y_{ij}-\hat{\beta}_j$ around $\hat{\alpha}_i$.

It was expected that the distribution of $y_{ij}-\beta_j$ around α_i the estimated values would have been modified by the estimation process, so a simulation study was conducted by generating the appropriate values of y_{ij} from a $N(\mu_j,\sigma^2_j)$ distribution where μ_j and σ^2_j were estimated by

$$\mu_j = \ln(\bar{x}_j) - \tfrac{1}{2}\sigma^2_j$$

$$\sigma^2_j = \ln\{(\bar{x}^2_j/s^2j)-1\}$$

$\bar{x}_j$ and s^2_j being obtained from the raw data. The resulting simulated bids were then subjected to the iterative procedure, values for α_i and β_j computed and thence the values $z_{ij} = y_{ij} - \beta_j$. The z_{ij} values were then subjected to each of the shape tests in a similar manner to the last Chapter, except that tests statistics were derived for *each bidder* i instead of each project j. The probabilities of each resulting statistics were tested for Uniform distribution and the probability of the chi-square statistic obtained for the Case exactly as before. The process was repeated 1000 times for each Case and then again for the Lognormal distribution.

The critical values estimated from these simulations are shown in Table 9.5a. The critical values obtained for tests 6-10 were considered to be of little use at this stage, due to their lack of relationship with the expected values. Tests 1-5, however, were remarkably unaffected by the iterative procedure, the probabilities being very close to those

Table 9.5a Simulated test shape

Critical values (simulation of log values obtained by iteration, average variance, 1000 trials)

Case 1

			Test statistic at			
Test	1%	2½%	5%	95%	97½%	99%
Normal distribution						
1	0.009706	0.023149	0.043157	0.964295	0.964295	0.994833
2	0.009594	0.023754	0.042327	0.939987	0.939987	0.994833
3	0.009706	0.028577	0.052778	0.934318	0.964295	0.994833
4	0.012107	0.035174	0.052778	0.934318	0.964295	0.984058
5	0.012107	0.028577	0.064317	0.934318	0.964295	0.984058
6	0.003923	0.018700	0.043157	0.934318	0.964295	0.994833
7	0.000064	0.000364	0.001945	0.726796	0.811000	0.939987
8	0.000104	0.001212	0.003113	0.845066	0.934318	0.934318
9	0.000038	0.000220	0.000954	0.788728	0.845066	0.894201
10	0.002465	0.012107	0.028577	0.934318	0.964295	0.964295
Uniform distribution						
1	0.009706	0.023149	0.043157	0.934318	0.964295	0.984058
2	0.001945	0.009544	0.031802	0.883825	0.939987	0.976507
3	0.002465	0.012107	0.028577	0.934318	0.964295	0.984058
4	0.003113	0.012107	0.028577	0.934318	0.934318	0.964295
5	0.012107	0.023149	0.043157	0.934318	0.964295	0.984098
6	0.007764	0.023149	0.052778	0.934318	0.964295	0.984058
7	0.002696	0.007207	0.023754	0.883825	0.939987	0.976507
8	0.003923	0.012107	0.028577	0.934318	0.964295	0.984058
9	0.000588	0.003923	0.015065	0.894201	0.964295	0.984058
10	0.004935	0.012107	0.035174	0.934318	0.964295	0.984058

Case 2

	Test statistic at					
Test	1%	2½%	5%	95%	97½%	99%
Normal distribution						
1	0.012055	0.025360	0.045444	0.941662	0.968055	0.985481
2	0.007634	0.024786	0.043348	0.958286	0.976185	0.992385
3	0.012055	0.027091	0.051552	0.941662	0.968055	0.985481
4	0.012914	0.028931	0.048410	0.941662	0.968055	0.985481
5	0.014811	0.025360	0.045444	0.930822	0.974825	0.985481
6	0.001736	0.006892	0.015855	0.906175	0.941662	0.974825
7	0.000000	0.000000	0.000000	0.002416	0.007000	0.026883
8	0.000000	0.000000	0.000000	0.019421	0.048410	0.106543
9	0.000000	0.000000	0.000000	0.000354	0.001388	0.006892
10	0.000058	0.000222	0.000884	0.696088	0.811546	0.930822
Uniform distribution						
1	0.005980	0.012914	0.032966	0.918984	0.941662	0.980619
2	0.000005	0.000034	0.000160	0.587075	0.711896	0.783571
3	0.000022	0.000303	0.001109	0.716158	0.862344	0.918984
4	0.000005	0.000050	0.000222	0.614674	0.755509	0.877806
5	0.004826	0.016969	0.037518	0.951484	0.974825	0.989467
6	0.006892	0.020769	0.035174	0.960281	0.980619	0.989467
7	0.000008	0.000074	0.000417	0.687154	0.783571	0.888457
8	0.000101	0.000327	0.001109	0.793358	0.862344	0.941662
9	0.000000	0.000002	0.000011	0.439086	0.574097	0.755509
10	0.000139	0.000653	0.003616	0.829151	0.906175	0.968055

Table 9.5b Results of tests of distribution shape for each bidder

Transformation	Test results (* no fails) Case 1	2	3
$y_{ij} = \ln(x_{ij} - m\bar{x}_j)$			
m=0.0	*	*	2,3,5
m=0.1	*	*	2,3
m=0.2	*	5	2,3
m=0.3	*	5	2,3,5
m=0.4	*	2,5	2,3
m=0.5	*	-	2,3,5
m=0.6	*	-	2,3,4
$y_{ij} = \ln\ (x_{ij} - mx_j)$			
m=0.0	*	*	2,3,5
m=0.1	*	*	2,3
m=0.2	*	*	2,3
m=0.3	*	*	2,3
m=0.4	*	*	2,3,4,5
m=0.5	*	*	2,5
m=0.6	*	*	*
m=0.7	*	*	*
m=0.8	*	*	*
m=0.9	*	1	*
m=0.95	*	1,5	*
$y_{ij} = x_{ij}{}^{1/p}$			
p=1	2	1,2,3,4,5	1,2,3,4,5
p=2	1	2,5	1,2,3,4,5
p=3	*	*	1,2,3,5
p=4	*	3,4	1,2,3
p=5	*	*	2,3
p=6	*	*	2,3
p=7	*	3	2,3
p=8	*	*	2,3,4

expected. It was decided, therefore, to concentrate on Tests 1-5 to determine the suitability of the Lognormal model.

Table 9.5b gives the results of the shape tests for all Cases, using similar log and power transformations to those used previously.

9.3.4.2 Spread

The possibility was considered that the distribution of Bartlett's test statistic may be affected by the iterative procedure in addition to the different sample sizes caused by the analysis of bidders instead of projects. , A further simulation study was, therefore, conducted by generating Lognormal bid values for each bidder with an equal variance (estimated from the data). The iterative procedure was implemented and the probability of Barlett's statistic computed from the residuals $y_{ij} - \beta_j$ for each bidder. This was repeated 1000 times for Cases 1 and 2, 100 times for Case 3, due to the length of time for each trial (20 minutes for Case 3). The values at the critical percentage points are shown in Table 9.6. The low values obtained for Case 2 were unaccounted for but, as the same computer program was used for all cases, the values were accepted as being correct.

The results of the analysis of the actual Case data are given in Table 9.7 for the various transformations

Table 9.6 Simulated variance tests

Critical values of Bartlett's probability (simulation of log values obtained by iteration, average variance, Normal distribution)

Test Statistics at	CASE 1	2	3
1 %	0.010030	0.001980	0.003340
2½%	0.029890	0.003500	0.019610
5 %	0.051730	0.008940	0.036155
7½%	0.084520	0.016180	0.051650
10 %	0.117130	0.021650	0.066625
95 %	0.953750	0.771060	0.935750
97½%	0.976570	0.828170	0.978470
99 %	0.990170	0.932930	0.993800

Table 9.7 Results of tests for homoscedacity of bidders (Bartlett's test)

Transformation	Case 1 $\chi^2_{(49)}$	 'prob'	 2 $\chi^2_{(127)}$	 'prob'	 3 $\chi^2_{(231)}$	 'prob'
$y_{ij} = \ln(x_{ij} - m\bar{x}_j)$						
m=0.0	60.6	0.124	257.0	0.000	446.0	0.000
m=0.1	60.1	0.133	258.3	0.000	446.0	0.000
m=0.2	59.5	0.145	260.6	0.000	446,5	0.000
m=0.3	58.8	0.159	264.7	0.000	448.0	0.000
m=0.4	58.1	0.176	272.9	0.000	451.9	0.000
m-0.5	57.2	0.197	-	-	462.9	0.000
m=0.6	56.1	0.225	-	-	485.7	0.000
$y_{ij} = \ln(x_{ij} - mx_{1j})$						
m=0.0	60.6	0.124	257.0	0.000	446.0	0.000
m=0.1	59.6	0.143	253.5	0.000	438.7	0.000
m=0.2	58.5	0.166	249.7	0.000	431.0	0.000
m=0.3	51.4	0.192	245.5	0.000	422.5	0.000
m=0.4	56.2	0.223	240.8	0.000	415.2	0.000
m=0.5	55.0	0.259	235.6	0.000	398.2	0.000
m=0.6	53.6	0.300	229.8	0.000	358.4	0.000
m=0.7	52.4	0.344	223.8	0.000	358.4	0.000
m=0.8	51.9	0.361	218.5	0.000	330.9	0.000
m=0.9	52.6	0.335	219.5	0.000	290.3	0.005
m=0.95	42.1	0.745	234.7	0.000	260.5	0.089
$y_{ij} = x_{ij}^{1/p}$						
p=1	125.5	0.050	1400.1	0.000	1733.1	0.000
p=2	76.9	0.007	339.3	0.000	709.9	0.000
p=3	62.1	0.098	204.6	0.000	540.6	0.000
p=4	59.6	0.143	183.8	0.000	474.3	0.000
p=5	58.8	0.160	183.4	0.000	449.4	0.000
p=6	58.5	0.166	187.7	0.000	439.1	0.000
p=7	58.5	0.167	192.9	0.000	434.4	0.000
p=8	58.5	0.166	197.8	0.000	432.2	0.000

9.3.4.3 Location

A simplification of the model is that each bidder has the same location parameter ie $\alpha_1 = \alpha_2 = \dots = \alpha_i = \dots = \alpha_r$. An approximate test chosen was the analysis of variance (ANOVA). The ANOVA relies on the assumption that the data are Normally distributed and the variance are equal, although the test is known to be robust certainly as far as the Normal assumption is concerned (Kendall & Stuart,1963,p465).

The ANOVA, in this case, involves the test statistic

$$F = \frac{SSQ_B/(r-2)}{SSQ_w/(N-c-r+1)}$$

Where

$$SSQ_w = \sum_i \sum_j \delta_{ij} (y_{ij} - \alpha_i - \beta_j)^2$$

$$SSQ_B = \sum_i n_i (\alpha_i - \bar{\alpha})^2$$

$$\bar{\alpha} = \sum_i n_i \alpha_i / \sum_i n_i$$

And F follows the F distribution with N-r-c+1 and r-2 degrees of freedom. An example for Case 1 is given in Table 9.8.

Table 9.8 ANOVA example for Case 1 $\{y_{ij} = \ln(x_{ij})\}$

Source	SSQ	df	Mean Square	F	Prob.
Between bidders	1.1312	91	0.0124	5.263	0.000
Within bidders	0.4134	175	0.0024		
Total	1.5446	266	0.0058		

The results obtained for the various transformations are provided in Table 9.9. Apart from the raw data for Case 3 (which is considered to be highly non Normal and heterogeneous and, therefore, inappropriately tested by the F test), these results clearly indicate that the bidders cannot be modelled as bidding from a distribution with the same location parameters.

Table 9.9 ANOVA results for all Cases

Transformation	Cases					
	1		2		3	
	F	Prob	F	Prob	F	Prob
$y_{ij}=\ln(x_{ij}-m\bar{x}_j)$						
$m = 0.0$	5.26	0.000	4.54	0.000	2.59	0.000
$m = 0.1$	5.29	0.000	4.54	0.000	2.59	0.000
$m = 0.2$	5.35	0.000	4.53	0.000	2.58	0.000
$m = 0.3$	5.42	0.000	4.51	0.000	2.58	0.000
$m = 0.4$	5.57	0.000	4.48	0.000	2.58	0.000
$m = 0.5$	5.67	0.000	4.40	0.000	2.08	0.000
$m = 0.6$	6.30	0.000	-	-	-	-
$y_{ij}=\ln(x_{ij}-mx_{1j})$						
$m = 0.0$	5.26	0.000	4.54	0.000	2.59	0.000
$m = 0.1$	5.20	0.000	4.55	0.000	2.60	0.000
$m = 0.2$	5.13	0.000	4.57	0.000	2.61	0.000
$m = 0.3$	5.04	0.000	4.58	0.000	2.62	0.000
$m = 0.4$	4.93	0.000	4.59	0.000	2.63	0.000
$m = 0.5$	4.80	0.000	4.61	0.000	2.65	0.000
$m = 0.6$	4.62	0.000	4.62	0.000	2.67	0.000
$m = 0.7$	4.40	0.000	4.63	0.000	2.69	0.000
$m = 0.8$	4.08	0.000	4.60	0.000	2.71	0.000
$m = 0.9$	3.58	0.000	4.48	0.000	2.71	0.000
$m = 0.95$	3.18	0.000	4.27	0.000	2.70	0.000
$m = 0.99$	2.57	0.000	3.68	0.000	2.57	0.000
$m = 0.999$	2.58	0.000	3.12	0.000	2.33	0.000
$y_{ij}=x_{ij}^{1/p}$						
$p = 1$	2.58	0.000	1.93	0.000	1.11	0.104
$p = 2$	3.70	0.000	3.32	0.000	1.87	0.000
$p = 3$	4.23	0.000	3.87	0.000	2.16	0.000
$p = 4$	4.50	0.000	4.10	0.000	2.29	0.000
$p = 5$	4.66	0.000	4.22	0.000	2.37	0.000
$p = 6$	4.77	0.000	4.30	0.000	2.41	0.000
$p = 7$	4.84	0.000	4.34	0.000	2.44	0.000
$p = 8$	4.90	0.000	4.38	0.000	2.46	0.000

9.4 Analysis of one bidder

9.4.1 Introduction

The Case 1 data provided in Appendix B consists of a sub-set of material obtained from one construction company, referred to as bidder 304. The total set of data for bidder 304 contains details of all (85) projects in which the company was involved during the period under study. Much of these data are of a confidential nature, details of which are available from the writer, subject to bidder 304's prior consent. A brief summary of these projects is given in Table 9.10. The Table indicates a ratio of 0.192 for projects acquired to the known results, and 0.152 for those obtained in competitive tender, that is Type 1 and 2 bids. (Type 1 bids are those commonly termed genuine or *bone fide* bids, Type 2 are cover prices).

Table 9.10 Project details

Type	Won	Lost	Total
Package deal	-	1	1
Negotiated	2	-	2
Schedule of rates	-	2	2
Aborted	NA	1	1
Type 2 bids	-	21	21
Type 1 bids	11	43	54
Type 1 or 2 bids (undistinguished)	1	3	4
Total	14	71	85

Details were not available of competitors' bids for several projects, although mark-up percentages were more freely available. Table 9.11 summarises the frequency of such projects where the relevant data was available. Table 9.12 indicates the projects won and lost where details of competitors' bids were known. It is interesting to note that no Type 2 bids were responsible for obtaining projects, although discussions with bidder 304 indicated that projects were occasionally obtained on this basis.

Table 9.11 Data available

Type	Mark up known	all competitors' bids known	Low bid only known
Type 2 bids	NA	17	-
Type 1 bids	57	34	7
Type 1 or 2 bids (undistinguished)	--	--	-
Totals	57	51	7

Table 9.12 Competitors' bids known

Type	Won	Lost	Total
Mark ups	10	47	57
Type 2 bids	--	17	17
Type 1 bids	8	26	34

Several analyses were conducted on these data to gain an indication of the modelling implications.

9.4.2 Detecting Type 2 bids

Three methods of detecting Type 2 bids were attempted: the highest bid in each contract, the highest bids relative to a bidder's alpha estimate, and the highest bids relative to a bidder's cost estimate.

9.4.2.1 The highest bid in each project

Bidder 304 recorded the highest bid on 14 projects as shown in Table 9.13.

Table 9.13 Projects in which bidder 304 entered the highest bid

Project no	Bid Type
3	1
5	1
11	1
13	1
17	2
26	1
27	1
34	2
35	2
39	1
40	2
45	2
49	2
51	1

The method, therefore, detected six Type 2 bids out of 14 attempts, a factor of success of 6/14 = .429.

The actual number of Type 2 bids for bidder 304 was 17 out of 51, a factor of 17/51 = .333, and it would therefore seem that the method gives slightly better results than that would be obtained by pure chance.

9.4.2.2 The highest bids relative to a bidder's alpha value

On completion of the iterative procedure described previously, the values of $y_{304j} - \beta_j - \alpha_{304}$ $(j = 1,2,\ldots,c)$ were computed and the 10 highest values recorded, as shown in Table 9.14.

Table 9.14 Highest valued residuals for bidder 304

Project No (j)	$y_{304} - \beta_j - \alpha_{304}$	Bid Type
40	0.146415	2
27	0.103807	1
51	0.088718	1
26	0.062546	1
17	0.057663	2
11	0.053072	1
13	0.047439	1
5	0.045233	1
43	0.038940	2
35	0.036022	2

The success rate here is 4/10 = .4, slightly worse than method 1, and thus not considered to be appropriate.

9.4.2.3 Comparison between bidders

The most frequent competitor to bidder 304 was bidder 55, who entered bids for the same project on 20 occasions. A comparison was made of bidders 304 and 55 of cost estimates and bids, providing the ratios shown in Table 9.15.

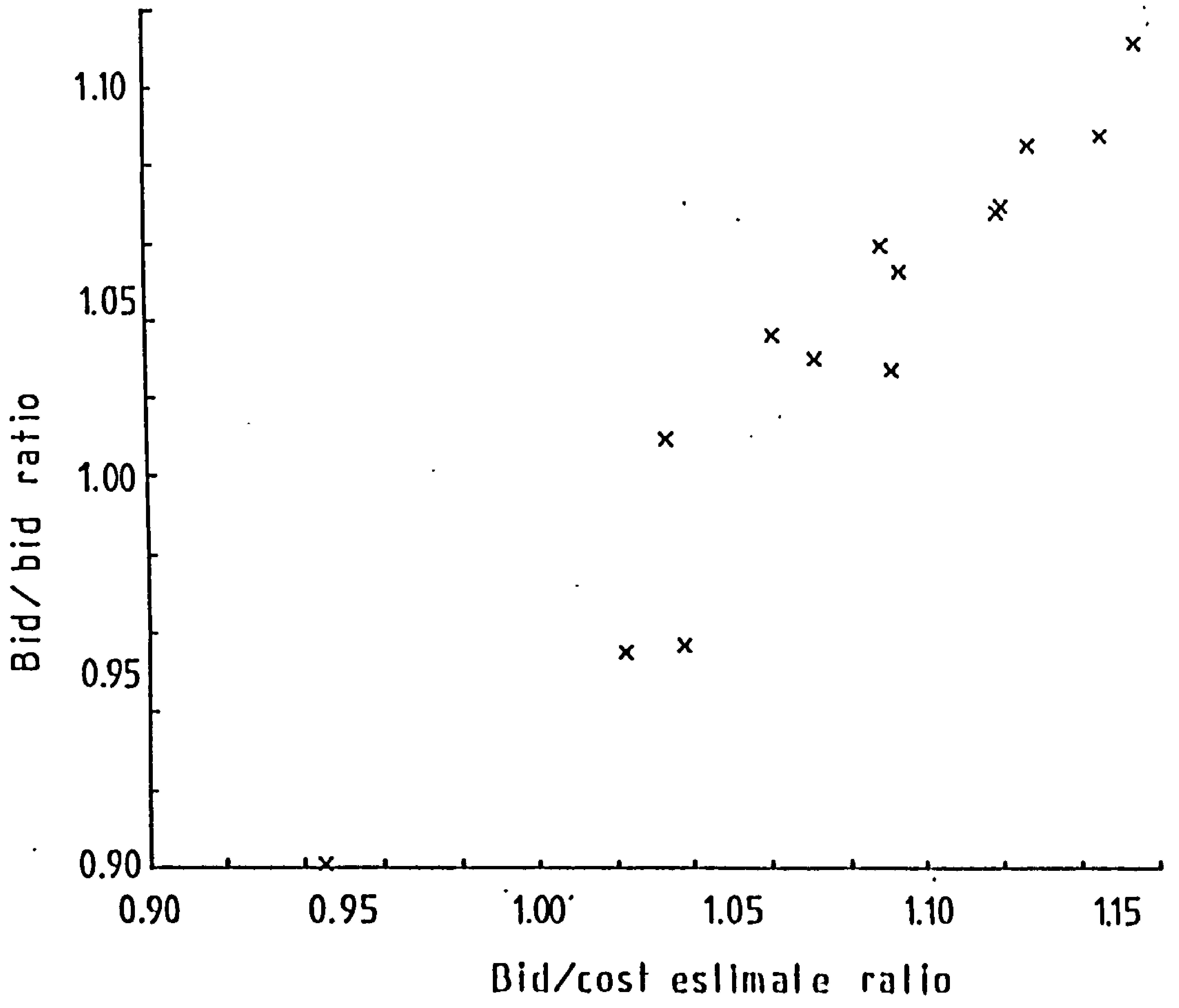
Table 9.15 Comparison of bidders' 304 and 55 estimates and bids

Project No	55 Bids / 304 Estimates	55 Bids / 304 Bids
1	1.092	1.027
5	1.038	0.957
7	1.120	1.067
13	1.022	0.955
15	cover	1.136
18	cover	0.924
19	1.156	1.111
20	1.034	1.009
21	1.072	1.030
25	cover	0.972
26	0.945	0.900
29	cover	0.919
30	1.128	1.084
32	1.147	1.087
33	cover	0.993
42	1.062	1.036
44	1.090	1.058
45	cover	0.979
47	1.094	1.052
48	1.121	1.068

As Fig 9.14 shows, there appears to be a reasonably close correspondence between Bid/Cost Estimate and Bid/Bid ratios to justify the use of the latter ratios as a ranking device for the former. This is illustrated in Table 9.16, where both sets of ratios are rearranged in descending order. Thus the 5 highest ranked ratios are from identical sequence numbers for each ratio group.

Now let us consider bidder 55's attempts to identify cover prices submitted by 304. The obvious cases to look for are those where 304 bids are high relative to 55 estimates. But we do not know 55

Fig 9.14 Bidder 304 and 55 bid/bid and bid/cost estimate ratios



estimates. However, his bids provide a reasonable substitution as we have seen, so let us now present the picture as 55 might see it for *all* cases (Table 9.17)

Table 9.16 Bid/estimate ratios

Ranking	Project number	
	55 Bid / 304 Estimate ratio	55 Bid / 304 Bid ratio
1	19	19
2	32	32
3	30	30
4	48	48
5	7	7
6	47	44
7	1	47
8	44	42
9	21	21
10	42	1
11	5	20
12	20	5
13	13	13
14	26	26

Table 9.17 As bidder 55 might see bidder 304

Project No	304 Bids / 55 Bids ratio	Ranking
1	0.974	10
5	1.045	5
7	0.937	15
13	1.047	4
15	0.880	20
18	1.082	3
19	0.900	19
20	0.991	9
21	0.971	11
25	1.029	6
26	1.111	1
29	1.088	2
30	0.922	17
32	0.920	18
33	1.007	8
42	0.965	12
44	0.945	14
45	1.021	7
47	0.951	13
48	0.936	16

Now the assumption that the highest ratios predict 304's Type 2 bids can be examined (Table 9.18).

Table 9.18 Bidder 55's prediction of bidder 304's Type 2 bids

Project numbers for actual Type 2 bids	Project numbers for predicted Type 2 bids
15	26
18	29*
25	18*
29	13
33	5
45	25*

* successful predictions

In this case the success rate is 3/6 = 0.5 which is slightly better than the previous two methods.

9.4.2.4 Conclusion

Whilst all three methods detect Type 2 bids better than by pure chance, that is better than a rate of 0.333, it was not considered that any of the prediction methods were particularly successful or, consequently, that the presence of Type 2 bids would significantly distort the distributional analyses.

9.4.3 The distribution of mark-up values

The first analysis was to check if mark-up values were different, that is lower, for projects won from those where projects were not won. A total of 57 mark-up values were available, 46 of which were for projects not won. The average percentage mark-up value for the projects won was 5.265 (s = 2.543) whilst for those projects not won a value of 5.648 (s = 2.178) was obtained. There is clearly little difference between these averages, confirmed by the t-test at the 5% significance level.

The distribution of mark-up values was then examined both for the percentage mark-up and the mark-up multiplier for possible parametric models. The frequency distribution of m is shown in Fig 9.15. The chi-square test was applied to test the goodness of fit of Normal models resulting in $\chi^2_{(5)}$ = 5.781, which is well below the critical value at the 5% levels. A further chi-square test indicated that the log of the mark-up multiplier m' = ln (bid/cost estimate) could also be regarded as Normally distributed $\chi^2_{(5)}$ = 3.065. It was concluded, therefore, that Normal and Lognormal models provided a reasonable approximation to the percentage mark-up and the mark-up multiplier respectively.

9.4.4 The distribution of low bid/cost estimate ratios

The data provided a total of 42 low bid/cost estimate ratios. Where bidder 304 entered the lowest bid himself the second lowest bid was taken. The mean value of 0.9896 is similar to the value obtained from data in the literature. The mode is 0.999. The frequency distribution of the ratios is shown in Fig 9.16. The literature suggests that the ratios can be considered to follow the Normal distribution. The chi-square test produced a value of $\chi^2_{(4)}$ = 10.041 which has a probability of less than 0.05 and was taken to provide sufficient evidence to reject the null hypothesis. A similar test of the distribution of log ratios, however, resulted in $\chi^2_{(4)}$ = 7.06, the probability of which is greater than 0,05. It was concluded, therefore, that the Lognormal model provides a reasonable approximation to these data.

9.4.5 The probability of entering the lowest bid (ordinal scale estimation)

An empirical estimation of the probability of bidder 304 entering the lowest value bid was made on the ordinal scale.

$$\Pr (x_{304j} < x_{ij}, \text{ for all } x_i, i \neq 304) = 1 - \frac{R_i}{(n_j+1)}$$

Where R is the rank of bidder 304's bid compared with bids for project j and n_j is the number of bids entered for the project.

Fig.9.15 Frequency of mark-up values

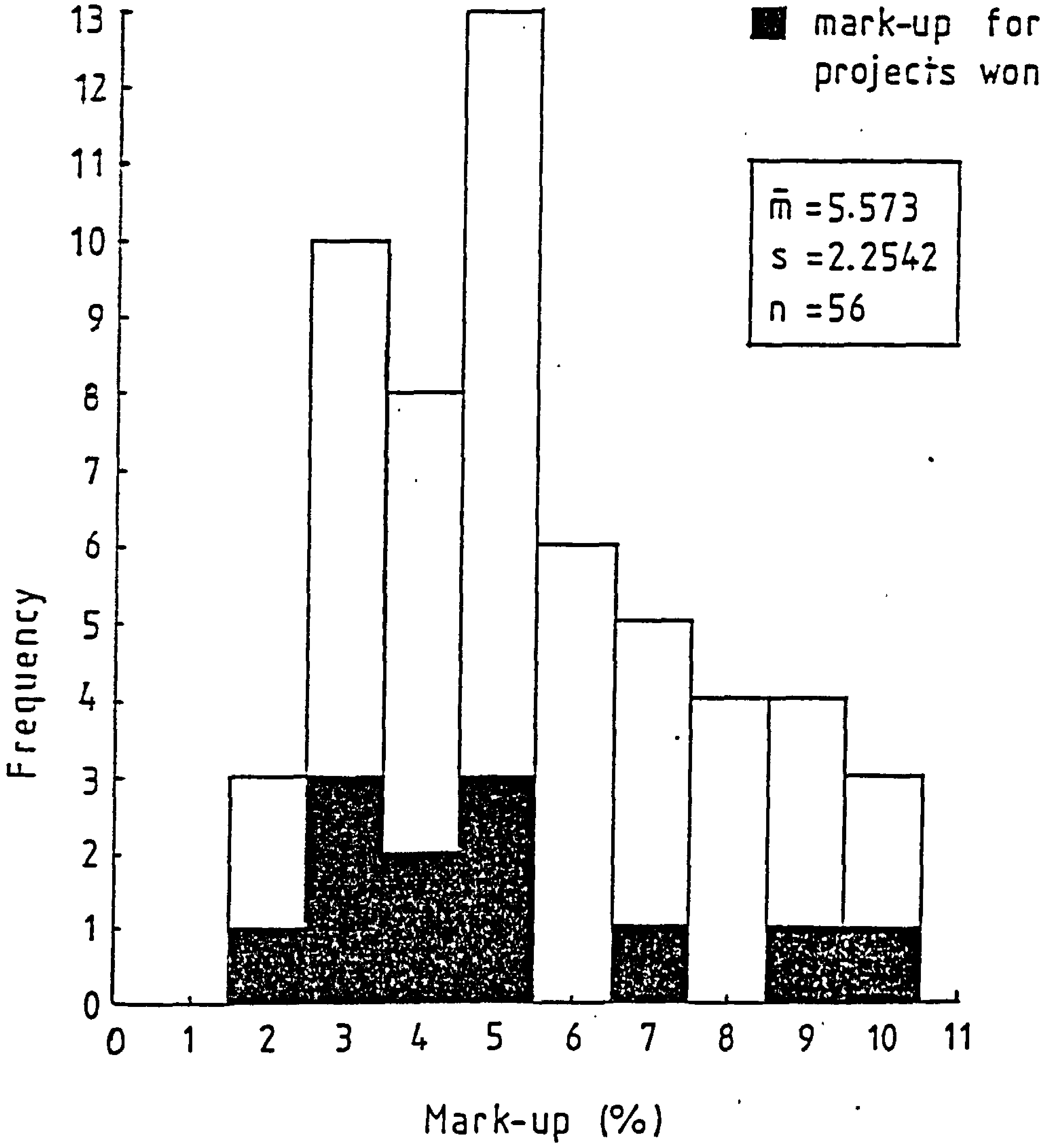
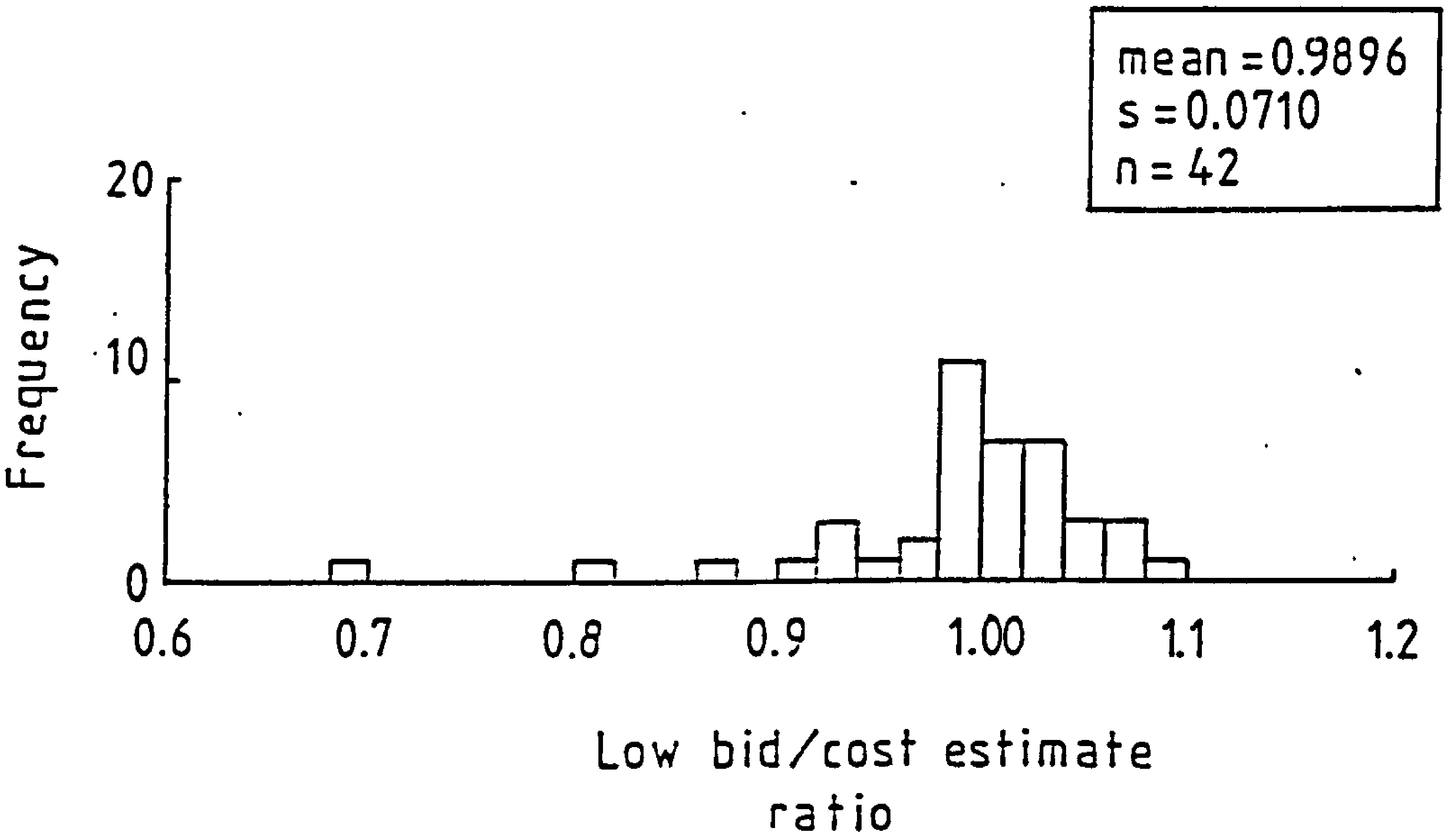


Fig.9.16 Frequency of low bid/cost estimate ratios



Thus, say, a bid which is ranked third for a project receiving a total of 5 bids in all, will be accorded an estimated empirical probability of 1 - 3/(5+1) = 0.5. In this example, the lowest bid (ranked 1) and the highest bid (ranked 5) will be accorded an estimated empirical probability of 0.83 and 0.16 respectively. This, albeit rather crude, statistic was derived from the data available for bidder 304 partitioned into Type 1 and Type 2 bids. The resulting means, 0.490 (s=0.253) and 0.382 (s=0.305) for Types 1 and 2 respectively, were compared by the t-test resulting in a value of 1.42 with 59 degrees of freedom. The indication given by this rough test again confirms difficulty in distinguishing between Type 1 and Type 2 bids.

A plot of the empirical probabilities estimated in this way is provided in Fig 9.17 to ascertain any correlation with mark-up values, Visual inspection suggests that no such correlation exists.

A more detailed analysis of the probability of entering the lowest bid is contained in the next chapter.

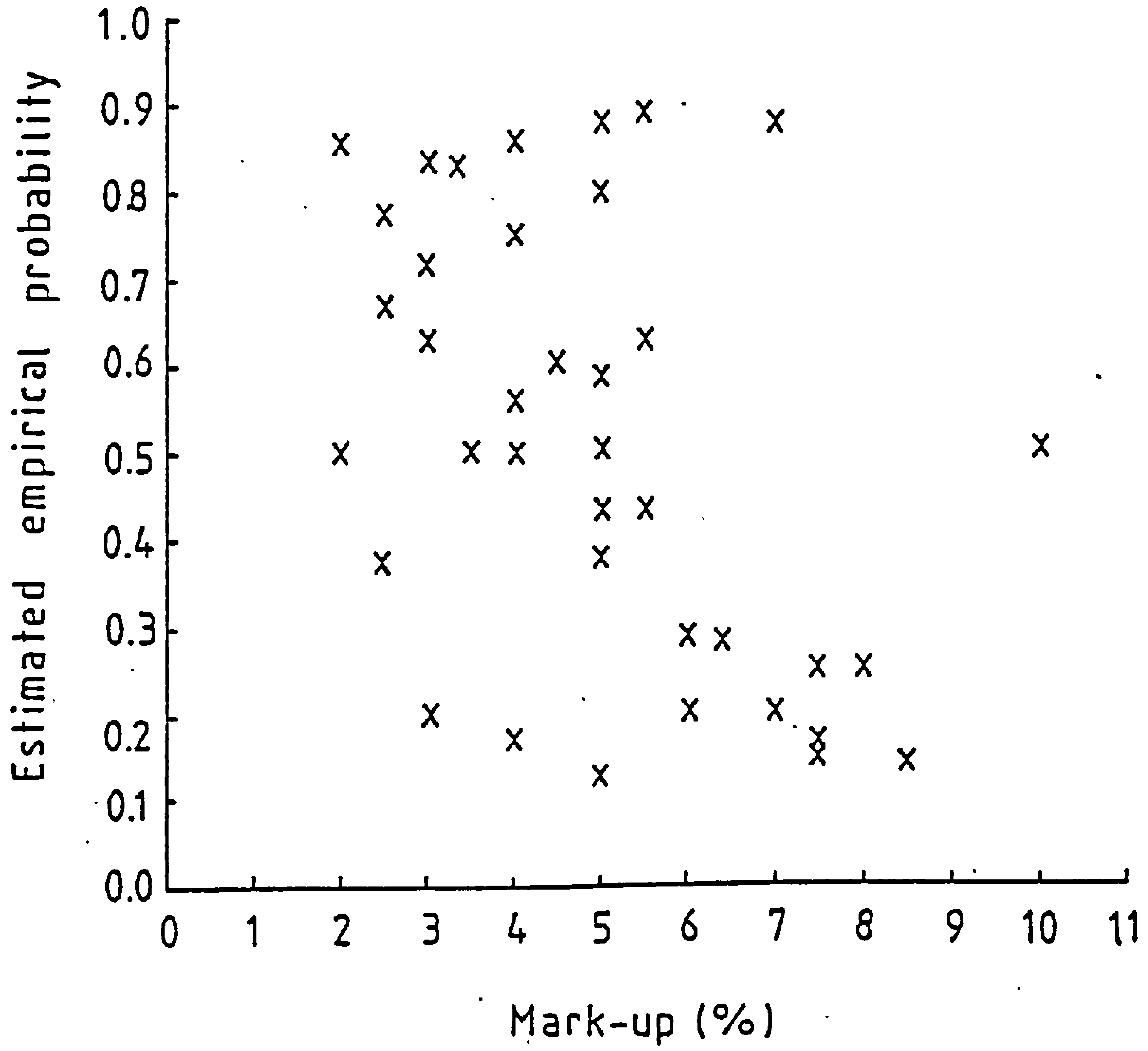
9.5 Summary and Conclusions

A method has been proposed for estimating the probability that a particular bidder will enter a bid for a project of a certain value. The assumption is that the estimates of this probability are Normally distributed, although this assumption was not tested. This technique, it has been suggested, could be extended to a multivariate approach using other project characteristics as predictor variables.

The distribution of bid values entered by each bidder has been examined in relation to a linear model (1) in which the project size is isolated. Some approaches to estimating the parameters of this model have been proposed and, in view of the sparsity of the implied matrix, an iterative procedure proposed. This procedure was applied to the data to obtain the required parameter estimates.

Inspection of the aggregated residuals resulting from fitting the linear model indicates that all three Cases are similarly shaped, a distribution fom a Gram-Charlier Type A series being fitted. Reference to the literature suggests that a highly peaked, heavy tailed

Fig.9.17 Plot of mark-up against estimated empirical probability of entering the lowest bid



distribution of this kind may reflect differences in distributions for individual bidders and a further analysis was conducted to test for the existence of such differences. A procedure similar to that used in the previous Chapter was employed to determine the suitability of the normal model for individual bidder's bids. Such a model was found to be a reasonable approximation of suitably transformed bids in all three Cases (Table 9.5). Tests for variance stabilisation after similar transformations revealed that some degree of homoscedacity could be achieved (Table 9.7).

Comparisons of Tables 9.5b and 9.7 indicates some quite pronounced differences between the three Cases. Case 1 bidders appear to be modelled well by the Normal distribution for most of the transformations applied, the variances σ^2_i being generally common amongst bidders. Case 2 bidders, however, seem to fit the Normal model better by the $y_{ij} = \ln(x_{1j} - mx_{ij})$ transformation, for values of $0 \leq m \leq 0.8$ but with no commensurate commonality of variance. Case 3 bidders, on the other hand, appear to be approximated by the Normal distribution after the $y_{ij} = \ln (x_{ij} - x_{1j})$ transformation for $m \geq 0.6$, with common variances where $m \approx 0.95$.

Tests for commonality of the location parameter indicated that, with the transformation attempted, no such commonality was found to exist.

In examining the data obtained for Case 1 it was seen that bidder 304 entered Type 2 bids on one third of occasions. Several approaches to detecting the presence of Type 2 bids were attempted but none succeeded in identifying more than one half of these bids. The conclusion reached was that, as such bids were difficult to detect, their presence was unlikely to have any major influence on the analysis of distributions.

The distribution of bidder 304's mark-up values was found to be adequately modelled by the Normal distribution (for percentage mark-up values) and the Lognormal distribution (for the mark-up multiplier values).

The distribution of low bid/cost estimate ratios was not found to follow the Normal distribution, the Lognormal model was, however, found to provide an adequate approximation. It was also noted that the mean of this sample of ratios, 0.9896, was similar to other recorded instances of this statistic.

Finally, a crude approach to determining the probability of entering the lowest bid was advanced in which an estimate of the empirical probability was derived from the ordered bids. This estimated empirical probability was then compared with Type 1 and 2 bids and mark-up values for any indication of possible trends. Little evidence of any such trends was detected.

Table 9.19 summarises the various models and applications, together with the parameter estimates, found to be appropriate for the aspects of the bidding model examined in Chapters 8 and 9.

The following Chapter considers some implications of these conclusions on the project selection/bidding model outlined in the earlier Chapters.

Table 9.19 Summary of models successfully fitted

(1) Project value $Pr(x_{1j})$

(i) Model: $w_j \sim Expn(\lambda)$
Transformation: $w_j = x_{1j}/\bar{x}_j$
Parameter estimate:
Case 1 $\lambda=1$

(ii) Model: $y_j \sim N(\mu, \sigma^2)$
Transformation: $y_j = \ln(x_{1j})$
Parameter estimates:
Case 1 $\mu=13.945$ *Case 3* $\mu=12.421$
$\sigma= 0.834$ $\sigma= 1.066$

(iii) Model: $z_j \sim N(\mu, \sigma^2)$
Transformation: $z_j = \ln\{\ln(x_{1j})\}$
Parameter estimates:
Case 1 $\mu= 2.633$ *Case 2* $\mu=2.364$ *Case 3* $\mu=2.516$
$\sigma= 0.059$ $\sigma=0.137$ $\sigma=0.086$

(2) Number of bidders $Pr(n_j)$

(i) Model: $n_j = \beta y_j$
Transformation: $y_j = \ln(x_{1j})$
Parameter estimates:
Case 1 $\beta=0.4472$ *Case 2* $\beta=0.5321$
SE=0.0143 SE=0.0125

(ii) Model: $n_j = \alpha + \beta y_j$
Transformation: $y_j = \ln(x_{1j})$
Parameter estimates:
Case 1 $\alpha=-0.4129$ *Case 2* $\alpha=-2.4455$ *Case 3* $\alpha=-5.2745$
SE= 3.3737 SE= 0.9931 SE= 1.0483
$\beta= 0.4676$ $\beta= 0.7559$ $\beta= 0.8380$
SE= 0.2415 SE= 0.0917 SE= 0.0841

(3) Specified bidder enters a bid $Pr(i)$

(i) Model: $i = \alpha_i + \beta_i y_j$
Transformation: $y_j = \ln(x_{1j})$
Parameter estimates:
calculated for each bidder eg. *Case 1* $\alpha_{55}=-0.4534$
$SE_{55}= 0.1704$
$\beta_{55}= 0.0436$
$SE_{55}= 0.1366$

(4) Bid values $Pr(x_{ij})$

(i) Model: $v_{ij(p)} \sim N(.,.)$
Transformation: $v_{ij(p)} = x_{ij}^{1/p}$
Parameter estimates:
Case 1 $2 \leqslant p \leqslant 10$ *Case 2* $0 \leqslant p \leqslant 5$

(ii) Model: $y_{ij(m)} \sim N(.,\sigma^2_{j(m)})$
Transformation: $y_{ij(m)}=\ln(x_{ij}-m\bar{x}_j)$
Parameter estimates:

Case 1 $0.0 \leqslant m \leqslant 0.5$
$\sigma_{j(0.0)}=0.21228-0.0113y_{j(0.0)}$
$\sigma_{j(0.1)}=0.23945-0.0129y_{j(0.1)}$
$\sigma_{j(0.2)}=0.27489-0.0151y_{j(0.2)}$
$\sigma_{j(0.3)}=0.32312-0.0181y_{j(0.3)}$
$\sigma_{j(0.4)}=0.39291-0.0226y_{j(0.4)}$
$\sigma_{j(0.5)}=0.50400-0.0300y_{j(0.5)}$

Case 2 $0.1 \leqslant m \leqslant 0.5$
$\sigma_{j(0.1)}=0.45703-0.0306y_{j(0.1)}$
$\sigma_{j(0.2)}=0.51693-0.0351y_{j(0.2)}$
$\sigma_{j(0.3)}=0.59735-0.0413y_{j(0.3)}$
$\sigma_{j(0.4)}=0.71294-0.0506y_{j(0.4)}$
$\sigma_{j(0.5)}=0.90168-0.0664y_{j(0.5)}$

Case 3 $0.1 \leqslant m \leqslant 0.5$
$\sigma_{j(0.1)}=0.23144-0.0131y_{j(0.1)}$
$\sigma_{j(0.2)}=0.24724-0.0140y_{j(0.2)}$
$\sigma_{j(0.3)}=0.30487-0.0178y_{j(0.3)}$
$\sigma_{j(0.3)}=0.36377-0.0218y_{j(0.4)}$
$\sigma_{j(0.5)}=0.45335-0.02881_{j(0.5)}$

(iii) Model: $y_{ij(m)} \sim N(.,\sigma^2_{j(m)})$
Transformation: $y_{ij(m)}=\ln(x_{ij}-mx_{1j})$
Parameter estimates:

Case 1 $0.0 \leqslant m \leqslant 0.5$
$\sigma_{j(0.0)}=0.21228-0.0113y_{j(0.0)}$
$\sigma_{j(0.1)}=0.23037-0.0123y_{j(0.1)}$
$\sigma_{j(0.2)}=0.25206-0.0135y_{j(0.2)}$
$\sigma_{j(0.3)}=0.27858-0.0150y_{j(0.3)}$
$\sigma_{j(0.4)}=0.31184-0.0168y_{j(0.4)}$
$\sigma_{j(0.5)}=0.41348-0.0246y_{j(0.5)}$

Case 2 $0.1 \leqslant m \leqslant 0.6$
$\sigma_{j(0.1)}=0.44080-0.0293y_{j(0.1)}$
$\sigma_{j(0.2)}=0.47654-0.0318y_{j(0.2)}$
$\sigma_{j(0.3)}=0.51922-0.0347y_{j(0.3)}$
$\sigma_{j(0.4)}=0.57125-0.0384y_{j(0.4)}$
$\sigma_{j(0.5)}=0.63641-0.0429y_{j(0.5)}$
$\sigma_{j(0.6)}=0.72105-0.0489y_{j(0.6)}$

Case 3 $0.1 \leqslant m \leqslant 0.6$
$\sigma_{j(0.1)}=0.22519-0.0127y_{j(0.1)}$
$\sigma_{j(0.2)}=0.24724-0.0140y_{j(0.2)}$
$\sigma_{j(0.3)}=0.27436-0.0156y_{j(0.3)}$
$\sigma_{j(0.4)}=0.30861-0.0176y_{j(0.4)}$
$\sigma_{j(0.5)}=0.35337-0.0203y_{j(0.5)}$
$\sigma_{j(0.6)}=0.41470-0.0239y_{j(0.6)}$

(iv) Model: $v_{ij(p)}=\alpha_{i(p)}+\beta_{j(p)}+e_{(p)}$, where $e_{(p)} \sim N(0,\sigma^2_{i(p)})$
Transformation: $v_{ij}=x_{ij}^{1/p}$
Parameter estimates:
Case 1 $3 \leqslant p \leqslant 8$ *Case 2* $p=3,5,6,8$
α_i and β_j estimated by iterative procedure

(v) Model: $y_{ij(m)}=\alpha_{i(m)}+\beta_{j(m)}+e_{(m)}$, where $e_{(m)} \sim N(0,\sigma^2_{i(m)})$
Transformation: $y_{ij}=\ln(x_{ij}-m\bar{x}_j)$
Parameter estimates:
Case 1 $0 \leqslant m \leqslant 0.6$ *Case 2* $0 \leqslant m \leqslant 0.1$
α_i and β_j estimated by iterative procedure

(vi) Model: $y_{ij(m)}=\alpha_{i(m)}+\beta_{j(m)}+e_{(m)}$, where $e_{(m)} \sim N(0,\sigma^2_{i(m)})$
Transformation: $y_{ij}=\ln(x_{ij}-mx_{1j})$
Parameter estimates:
Case 1 $0 \leqslant m \leqslant 0.95$ *Case 2* $0 \leqslant m \leqslant 0.8$ *Case 3* $0.6 \leqslant m \leqslant 0.9$
α_i and β_j estimated by iterative procedure

(5) Mark-up values $Pr(b_j)$

(i) **Model:** $b_j \sim \Lambda(\mu,\sigma^2)$
Transformation: $b_j=100(xj/c_j)-100$
Parameter estimates:
Case 1 $\mu=5.573$
$\sigma=2.2542$

(ii) **Model:** $b_j \sim N(\mu,\sigma^2)$
Transformation: $b_j=\ln(x_j/c_j)$
Parameter estimates:
Case 1 $\mu=0.05022$
$\sigma=0.01748$

(6) Low bid/cost estimate ratio $Pr(R_j)$
(i) **Model:** $R_j=\Lambda(\mu,\sigma^2)$
Transformation: $R_j=x_{1j}/c_j$
Parameter estimates:
Case 1 $\mu=0.9896$
$\sigma=0.0710$

CHAPTER 10

On estimating the probability of entering the lowest bid

10 ON ESTIMATING THE PROBABILITY OF ENTERING THE LOWEST BID

10.1 Introduction

A major aspect of the project selection and bidding decision is that some prior knowledge is needed of future events. The effect of statistical modelling is to enable an indication of the likelihood or probability of occurrence of future events to be estimated. One of the most important of these future events is whether a project will be obtained, which, in many cases, is determined by entering the lowest bid. This Chapter examines the application of models identified in the previous two Chapters in deriving the probability of entering the lowest bid.

10.2 General proposition

The general proposition is that *a bid can be adequately modelled as being a random value from a probability distribution unique to the bidder at the time of bidding.*

Some comments are particularly relevant:

(1) The bid is a value entered supposedly simultaneously with bids from other competitors for certain rights, eg. to obtain property in return for the value of the bid, or to deliver services in return for the value of the bid.

(2) Philosophically, there can be no certainty in attributing causal rules to phenomena, the best hope being to devise a model exhibiting similar behavioural characteristics. The adequacy of the model depends on the circumstances. One method of measuring the degree of adequacy might be to examine the cost consequences of differences between the behaviour of the model and that of the 'real world'.

(3) A random value is regarded in the strict statistical sense, sometimes termed the 'noise' in the system.

(4) Associated with random values is the statistical concept of

the probability of a value occuring. Many theoretical probability distributions have been studied and, in some cases, their properties well defined.

(5) Each bidder has his own unique probability distribution, different from other bidders.

(6) Changes in bidding behaviour take place over time.

(7) A major difficulty is in predicting the appropriate probability distributions from the information available.

10.3 Probability of entering the lowest bid

Let x_i represent a possible bid by the ith bidder for a project then, if the competition is modelled as the joint distribution of two or more variables, the probability that x_1 enters the lowest bid is given by

$$Pr\ (x_1 < x_i \text{ for all } i,\ i \neq 1) =$$

$$\int_{x_1=-\infty}^{\infty} \int_{x_2=x_1}^{\infty} \int_{x_3=x_1}^{\infty} \cdots \int_{x_n=x_1}^{\infty} f(x_1, x_2, x_3, \ldots, x_n)\ dx_n \ldots dx_3\ dx_2\ dx_1 \qquad (1)$$

If the above, for instance, is a multivariate normal distribution then

$$f\ (x_1, x_2, x_3, \ldots, x_n) =$$

$$dF = (2\pi)^{-\frac{1}{2}(n)}\ |V^{-1}|\ \exp\{-\tfrac{1}{2}\ (x - \mu_x)\ V^{-1}\ (x - \mu_x)\}\ \Pi dx \qquad (2)$$

(Kendall & Stuart, 1963, vol. 1. p349)

Where x is the vector of $x_1, x_2, x_3, \ldots, x_n$
μ_x is the vector of means for $x_1, x_2, x_3, \ldots, x_n$
V is the variance/co-variance matrix eg.

$$V = \begin{pmatrix} \sigma^2_1 & p\sigma_1\sigma_2 \\ p\sigma_1\sigma_2 & \sigma^2_2 \end{pmatrix} \quad \text{for the bivariate normal distribution}$$

Now assuming that the non-diagonal elements of V are zero, ie the variables are *independent* if normally distributed, it follows from (1) that

$\Pr(x_1 < x_i \text{ for all } i,\ i \neq 1) =$

$$\int_{-\infty}^{\infty} f_1(x_1) . \left\{ \prod_{i=2}^{n} \int_{x_i = x_1}^{\infty} f_i(x_i) dx_i \right\} dx_i \qquad (3)$$

In the case of the uniform (rectangular) distribution, it follows that

$$\Pr(x_1 < x_i,\ i \neq 1) = \int_{a_1}^{b_1} (b_1 - a_1)^{-1} . \left\{ \prod_{i=2}^{n} \int_{x_i = x_1}^{b_i} (b_i - a_i)^{-1} dx_i \right\} dx_1 \qquad (4)$$

Similarly, in the case of the Normal distribution

$\Pr(x_1 < x_i,\ i \neq 1) =$

$$\int_{-\infty}^{\infty} \{(2\pi)^{1/2}\}^{-1} \exp(-\tfrac{1}{2} x^2_1) . \left\{ \prod_{i=2}^{n} \int_{x_i = (\sigma_1 x_1 + \mu_1 - \mu_i) \sigma_i^{-1}}^{\infty} \{(2\pi)^{1/2}\}^{-1} \exp(-\tfrac{1}{2} x^2_i)\ dx_i \right\} dx_1 \qquad (5)$$

written in standard form.

And for the special case where $f_1(x_1) = f_2(x_2) = \ldots = f_n(x_n)$

$$\Pr(x_1 < x_i,\ i \neq 1) = n^{-1} \qquad (6)$$

10.4 Identity of bidders known

The analysis of bids in Chapters 8 and 9 indicates that eqn (6) is likely to be an oversimplification. The transformation $y_{ij} = \ln(x_{ij} - m x_{1j})$ ($\leq 0.6 \leq m \leq 0.8$) has been found to have a normalising effect (Table 9.19, Model 4vi), which suggests eqn (5) to be the most appropriate. On the assumption that the distributions successfully fitted to the data would apply to future events, the transformation $y_{ij} = \ln(x_{ij} - 0.8 x_{1j})$ was performed, the iterative procedure described in Chapter 9 applied and estimates α_i and s^2_i obtained. Once only bidders ($n_i = 1$) were assigned an 'average' variance

$$s^2_i = \sum_j \sum_i \delta_{ij} (y_{ij} - \beta_j - \alpha_i)^2 / (N - c - r + 1)$$

The resulting estimates are reproduced in Appendix C.

The estimated expected probability E[p] of entering the lowest bid can now be obtained by inserting the estimates of μ_i and σ^2_i into eqn (5) . It is clear that estimates of μ_i and σ^2_i, though efficient and largely unbiased, may not be very precise. A simulation study was, therefore, conducted to gain an indication of the distribution of P. This was conducted on the bidders involved in the first project in Case 1, bidders 55, 73, 134, 150, 154 and 304. The estimates α and s^2 for μ and σ^2 for these bidders are, from Appendix C, 0.05083 (0.03510), -0.06594 (0.03277), -0.01402 (0.02540), -0.11644 (0.01256), -0.03027 (0.05618) and 0.00000 (0.03204). (Bidder 304's μ is fixed at zero, the estimate of μ for the other bidders being relative to this, only the *difference* between μ's being required by eqn (5). New estimates of μ_i and σ^2_i were generated from the α_i and s^2_i values in accordance with their approximate sampling distributions.

$\mu_i \sim N\ (\alpha_i,\ s^2_i/n_i)$

$\sigma^2_i\ (n_i-1)/s^2_i \sim \chi^2\ (n_i-1) \qquad n_i > 1$

$\sigma^2_i\ (N-c-r+1)/s^2_i \sim \chi^2\ (N-c-r+1) \qquad n_i = 1$

The probability of bidder 304 entering the lowest bid against the five competitors was computed from eqn (5) and repeated 100 times with different values of μ_i and σ^2_i. The resulting frequency distribution of P is shown in Fig 10.1.

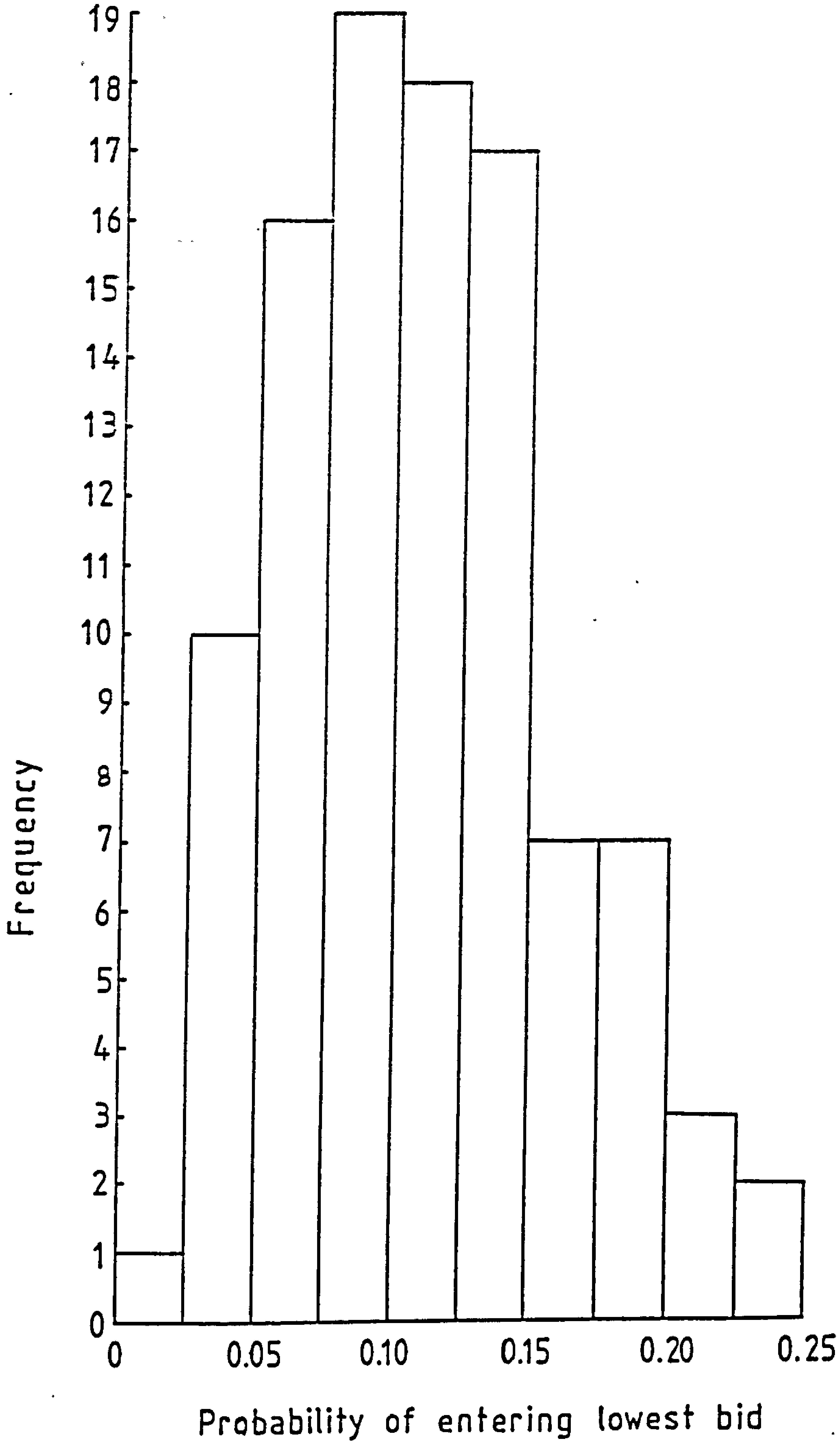
The results confirm the suspicion that estimates of P may not be very precise with values ranging from 0 to 0.25.

10.5 Identity of bidders not known

In the absence of knowledge of the identity of competitors on a future project, two possible approaches were considered. One approach is to model all competitors as equal, Friedman's 'average bidder'. This approach, however, was considered to be an oversimplification as a result of the analysis in Chapter 9.

The alternative is to attempt to predict the identity of competitors. This approach was adopted by utilising the results of the regression

Fig 10.1 Frequency of probability of bidder 304 entering lowest bid against bidders 55, 73, 134, 150, 154



analysis described in Chapter 9. The regression coefficients and standard errors were obtained for all bidders (Appendix D).

A simulation study was conducted, as described in the previous section, with the additional feature of the identity of the competitors being predicted by the regression formula (Table 9.19, Model 31). The procedure adopted was to compute the probability of each bidder entering a bid

$pr(i) = a_i + b_i y_j$

where a_i and b_i were obtained from Appendix D and $y_i = \ln(x_{ij})$ (x_{ij} = 1454515 in this example).

In order to accommodate the imprecision of the estimates of a_i and b_i the standard error of the estimates was utilised in generating new estimates of a_i and b_i for each iteration in accordance with their approximate sampling distributions

$$a_i \sim N(a_i, SE^2_{ai})$$
$$b_i \sim N(b_i, SE^2_{bi})$$

The five bidders with the highest values of pr(i) were selected and the procedure outlined in section 10.4 repeated. The resulting frequency of probabilities is given in Fig 10.2.

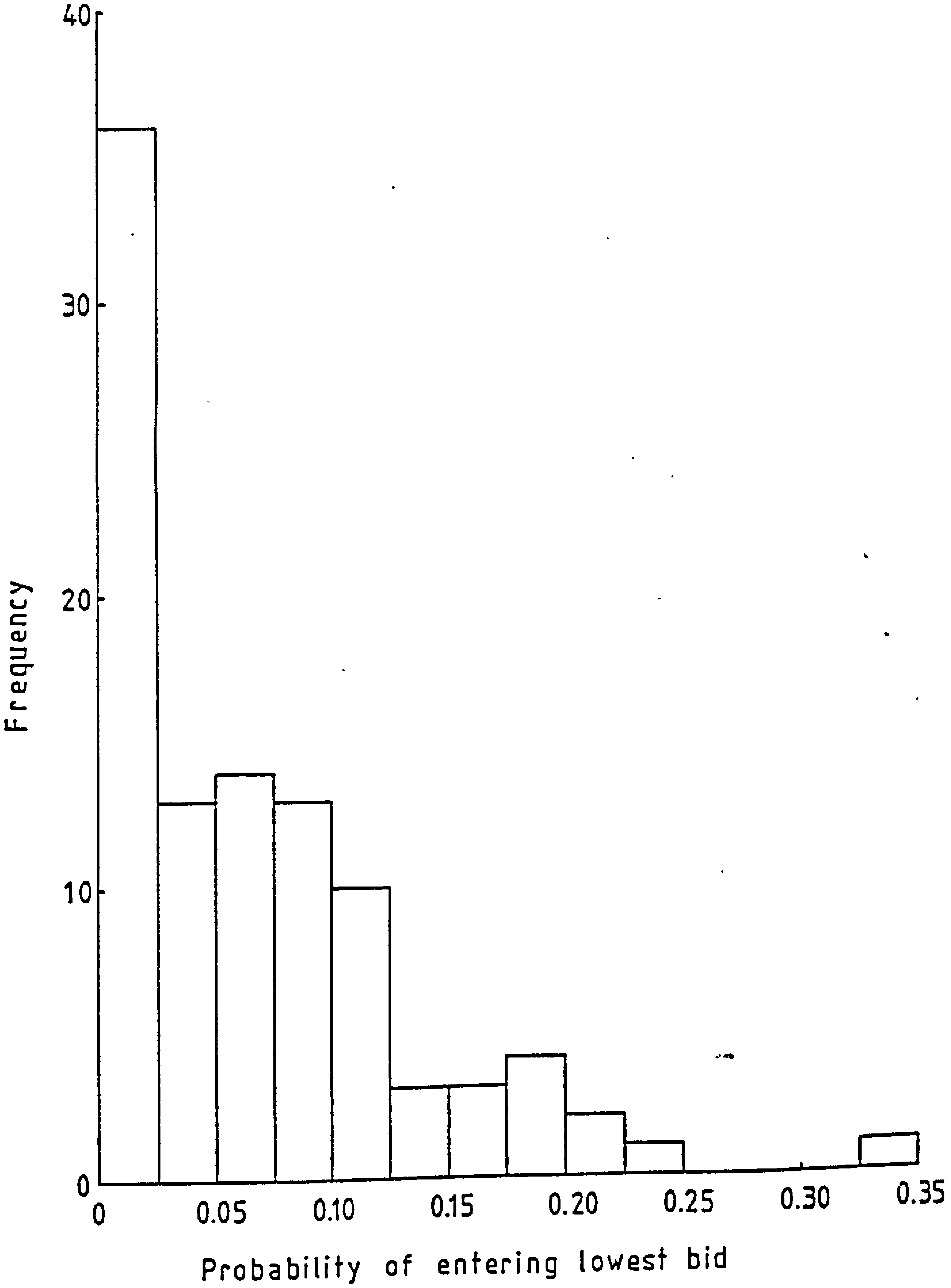
The results in this case indicate a somewhat greater spread of probability estimates together with many more low values of P. The explanation of the increased frequency of low P values is due to occurrence of bidders with substantially lower μ values and/or greater σ^2 values.

10.6 Number of bidders not known

From the analysis in Chapter 8 it would seem that some estimate of the number of bidders competing for a project can be made based on the project value. In Case 1, for instance, the number of bidders is predicted by

$$n_j = -0.4129 = 0.4676\ y_j$$

Fig 10.2 Frequency of probability of bidder 304 entering lowest bid against 5 unspecified competitors



where $y_j = \ln(x_{ij})$, the standard error of the coefficients being 3.3737 and 0.2415 respectively, n_j being rounded to the nearest integer (Table 9.19, Model 2ii).

The simulation described in section 10.5 was repeated but, with the number of bidders being obtained by simulation of the coefficients in accordance with their approximate sampling distributions. A total of 150 iterations were made and, in 22 cases, the number of bidders was predicted to be one or less. The frequency distribution of the resulting 128 probability estimates is given in Fig 10.3.

The results indicate a wider spread of P estimates and a considerable number below 0.025.

10.7 Project value not known

Where the project value is not known even approximately, a simulation procedure can be invoked utilising the results of the analysis in Chapter 8.

For Case 1, a prediction of project value may be obtained from the result

$$z_j \sim N(2.633, 0.059^2)$$

where $z_j = \ln\{\ln(x_{ij})\}$ (Table 9.19, Model 1iii)

Once again, a simulation study was conducted to generate project values and, hence, the probability of bidder 304 entering the lowest bid. The results for 50 iterations are given in Table 10.1. On 7 occasions the number of bidders predicted was one or less. The probabilities of entering the lowest bid are shown for the remaining cases, together with an indication of expected income ie project value x probability of entering the lowest bid.

The major criticism is that the number of bidders generated by the model appears to be rather wild. The reason for this may be due to the insufficiency of data in the Case 1 analysis.

The procedure was repeated for Cases 2 and 3, for bidders 2 and 3 respectively, using Models 4vi, 3i, 2ii and 1iii (Table 9.19) for the

Fig 10.3 Frequency of probability bidder 304 entering lowest bid against an unspecified number of competitors

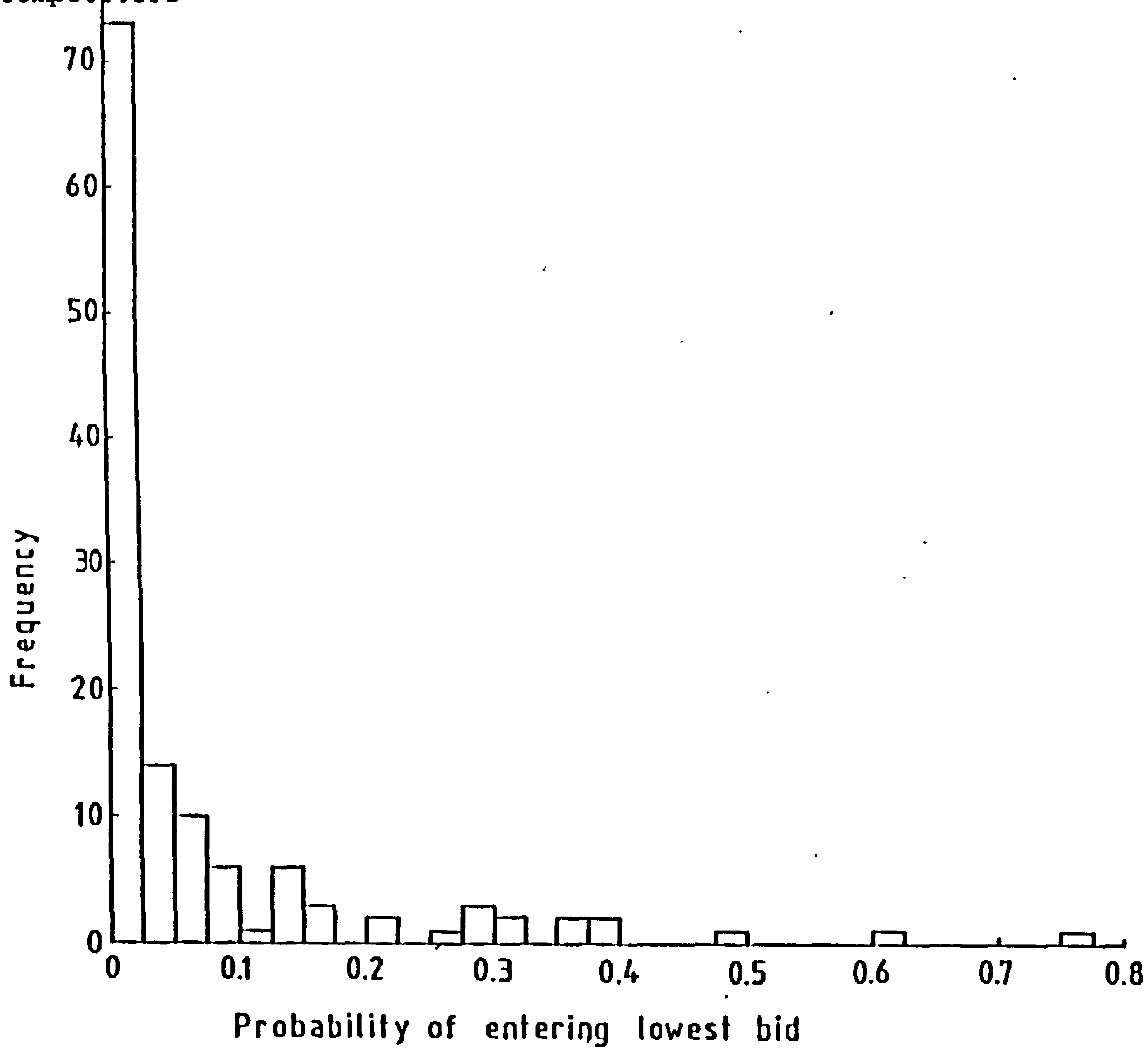


Table 10.1 Results of simulation of bidder 304's bidding for projects of unspecified value

```
PROJ.NO    VALUE     BDRG    PROB(W)        EXP.INCOME
  1        640539.    16     0.01993           12771.
  2        623578.     3     0.18341          113859.
  3       1380944.     5     0.00000               1.
  4        771507.     6     0.02412           18304.
  5       3140557.     7     0.00577           18277.
  6       1411522.     4     0.15773          277114.
  7        157856.     4     0.47024           74111.
  8       1552533.     7     0.07433          114715.
  9        557413.    14     0.00017              89.
 10        674256.     7     0.05710           38497.
 11        444537.     3     0.07015           31180.
 12       1994758.     1     0.75174         1499720.
 13       1657455.     7     0.05344           88600.
 14       1023457.     7     0.01420           14536.
 15       1551634.     3     0.45891          712012.
 16        751177.     4     0.06557           75171.
 17       5537110.    11     0.00000               1.
 18       3167963.     2     0.17559          557214.
 19                    0
 20       4154271.     9     0.02117           87959.
 21       1455821.     9     0.00000               0.
 22        178057.     3     0.44105           78599.
 23        854573.     7     0.04035           34484.
 24       1345463.     6     0.03862           51979.
 25                    1
 26        934628.     3     0.04046           37837.
 27                   -1
 28        344371.     3     0.15158           52201.
 29       1274148.     6     0.16927          217061.
 30        666912.     7     0.02003           13350.
 31        798433.     8     0.00000               0.
 32       1600576.     5     0.00023             366.
 33       3916174.    13     0.00000               0.
 34       1106757.    12     0.00000               1.
 35                   -4
 36       1600034.    12     0.00000               1.
 37       1467414.     6     0.11966          175833.
 38        464145.     7     0.15706           72978.
 39                    1
 40       5462170.     1     0.68577         3747857.
 41                   -1
 42       1413387.     3     0.03296           46548.
 43        671539.     9     0.00000               0.
 44        493523.    13     0.00410            2024.
 45        539473.    15     0.00000               0.
 46        925744.    18     0.00013             121.
 47        360699.     7     0.02347            8467.
 48        779284.     7     0.00641            4996.
 49                    1
 50        528232.     7     0.01684            8895.
```

appropriate Case. Only the simulations generating the predicted presence of the reference bidder were recorded (Tables 10.2 and 10.3). The Case 2 simulation of 500 iterations produced only one project with one or less bidders and Case 3 with 500 iterations produced only four such projects.

10.8 The probability of entering the lowest bid for a given mark-up

Considering a bid x to comprise a cost estimate c and a mark-up multiplier m, then letting $mg(c) = f_1(x_1)$, it follows from eqn(2) that

$$Pr(mc<x_i) = \int_{-\infty}^{\infty} mg(c).\left\{\prod_{i=2}^{n}\int_{x_i=mc}^{\infty} f_i(x_i)\ dx_i\right\} dc \qquad (6)$$

and therefore, for the Uniform distribution

$$Pr(mc<x_i) = \int_{a_c}^{b_c} m(b_c-a_c)^{-1}.\left\{\prod_{i=2}^{n}\int_{x_i=mc}^{b_2} (b_i-a_i)^{-1}\ dx_i\right\} dc \qquad (7)$$

and for the Normal distribution

$Pr(mc<x_i) =$

$$\int_{-\infty}^{\infty} m\{(2\pi)^{½}\}^{-1}.\exp(-½c^2).\left\{\prod_{i=2}^{n}\int_{x_i=(m\sigma_c+m\mu_c-\mu_i)\sigma_i^{-1}}^{\infty} \{(2\pi)^{½}\}^{-1}.\exp(-½x^2_i)\ dx_i\right\} dc \qquad (8)$$

written in standard form

A special case is where

$f_2(x_2) = f_3(x_3) = \ldots = f_n(x_n)$

which provides the general result, from eqn(6)

$$Pr(mc<x_i) = \int_{-\infty}^{\infty} mg(c).\left\{\int_{x_i=mc}^{\infty} f_n(x_n)\ dx_n\right\}^{n-1} dc \qquad (9)$$

for the Uniform distribution, from eqn(7) and (9)

$$Pr(mc<x_i) = \int_{a_c}^{b_c} m(b_c-a_c)^{-1}.\left\{\int_{x_n=mc}^{b_n} (b_n-a_n)^{-1}\ dx_n\right\}^{n-1} dc \qquad (10)$$

Table 10.2 Results of simulation of bidder 2's bidding (Case 2)

PROJ.NO	VALUE	BDRS	PROB(W)	EXP.INCOME
7	5180.	5	0.22293	1155.
42	163884.	10	0.01755	2876.
44	18587.	7	0.15452	2873.
49	74009.	9	0.00748	554.
58	1070665.	8	0.08445	90417.
80	48861.	4	0.88086	43039.
81	133590.	5	0.01249	1669.
85	225739.	7	0.26341	60170.
107	9523.	4	0.44222	4211.
133	44155.	7	0.00020	9.
138	100668.	6	0.01408	1417.
142	143904.	9	0.01599	2301.
146	33205.	7	0.17537	5823.
155	100730.	8	0.04272	4303.
169	299513.	7	0.10509	31474.
171	65999.	6	0.00051	34.
200	81413.	7	0.03097	2521.
217	885307.	10	0.00008	67.
220	21357.	5	0.01187	254.
303	88260.	8	0.04921	4343.
307	59395.	8	0.37627	22356.
309	13784.	7	0.26988	3718.
320	204800.	9	0.64502	132100.
328	15089.	5	0.05275	796.
349	51704.	5	0.05870	3035.
384	64074.	6	0.41281	26450.
437	250896.	9	0.00956	2398.
478	27493.	6	0.19083	5247.
493	1708655.	8	0.02693	46011.
540	9025.	5	0.69293	6253.
541	33945.	5	0.01738	590.
554	37090.	5	0.10932	4055.
563	210803.	7	0.32537	68590.
583	78444.	9	0.11156	8751.
584	73799.	7	0.00444	328.
603		1		
605	71192.	7	0.10668	7261.
620	46301.	6	0.40725	18856.
628	143770.	8	0.88216	126828.
631	793339.	4	0.25710	203969.
653	149708.	9	0.00000	0.
687	18856.	5	0.09206	1736.
701	1594516.	11	0.00423	6741.
713	107243.	7	0.16153	17323.
718	56034.	7	0.18365	10290.
723	24284.	5	0.55616	13506.
738	22457.	7	0.00001	0.
751	21930.	7	0.04876	1069.
756	740642.	6	0.98324	726095.
759	60231.	6	0.10674	6429.
770	13103.	6	0.04304	564.
773	41903.	7	0.00069	29.
776	216277.	7	0.02987	6461.
780	955633.	6	0.16196	154773.
789	41929.	8	0.34150	14319.

Table 10.3 Results of simulation of bidder 3's bids (Case 3)

PROJ.NO	VALUE	FIRS	PROB(W)	EXP.INCOME
3	313614.	7	0.35347	110853.
5	786000.	5	0.10053	79033.
11	184694.	6	0.09771	18046.
46	1209710.	7	0.39661	479784.
74		1		
89	171533.	7	0.18773	32203.
99	150137.	7	0.01037	1558.
111	754436.	5	0.08154	15459.
116	94535.	7	0.04831	4567.
129	1013501.	7	0.00237	2406.
136	117401.	3	0.73493	86282.
154	3451997.	9	0.10769	371584.
184	235163.	5	0.16777	39461.
213	403745.	7	0.09653	38977.
214	1888860.	4	0.20739	391734.
258	176513.	7	0.20954	36987.
277	240096.	5	0.70768	169911.
290		1		
304	148508.	6	0.26334	39214.
313	6780011.	9	0.19494	1321724.
314	179376.	7	0.01301	2340.
316	559275.	6	0.30441	170249.
336	988543.	7	0.07154	70642.
343		1		
372	124221.	3	0.29027	36057.
392	1143378.	5	0.17983	205617.
395	2875010.	9	0.17612	506337.
417	158377.	6	0.02855	4522.
443	489135.	5	0.04064	19881.
447	44449.	5	0.56726	25214.
452	501554.	6	0.12546	62926.
482		1		
485	200304.	7	0.29816	59723.
497	1168484.	6	0.10660	124557.

and for the Normal distribution, from eqn(8) and (9)

$Pr(mc<x_i) =$

$$\int_{-\infty}^{\infty} m\{(2\pi)^{½}\}^{-1}.\exp(-½c^2).\left\{\int_{x_n=(m\sigma_c+m\mu_c-\mu_n)\sigma_n^{-1}}^{\infty} \{(2\pi)^{½}\}^{-1}.\exp(-½x^2_n)\ dx_n\right\}^{n-1} dc \qquad (11)$$

Analysis of the data indicates that a similar form to eqn(8) is desirable. Eqn(8), however, assumes that the cost estimate can be considered to be adequately modelled by the Normal distribution. This is, of course, not necessarily the case for, if the *bid* is assumed to be three parameter Lognormally distributed with threshold parameter r, then it is highly unlikely that the *cost estimate* will also be similarly distributed except in rather special circumstances (r=0 for instance).

Several approaches appear to be available

(1) Include the cost estimate as another bidder in the iterative procedure described in Chapter 9. This will provide estimates of μ and σ^2. An indication of the appropriate distribution model may be obtained by fitting candidate parametric distributions to the ensuing residuals. One of the Pearson or Gram-Charlier forms may be appropriate.

(2) Utilise the probability distribution of the mark-up in some way, eg. by simulation of

$$\ln(c_j-0.8x_{1j}) = \ln[\{\exp(\alpha_i+\beta_i)-0.8x_{1j}(m_j-1)\}/m_j]$$

where α_i and m_j are random variables with assumed distributions and estimated parameters (Table 9.19, Model 5ii, $M\sim\Lambda(0.05022,0.01748^2)$)

(3) Utilise the knowledge that $E[x_{1j}/c_j] \approx 1$ or the ratio distribution generally (Table 9.19, Model 6i).

10.9 Conclusions

This Chapter has provided an introduction to an application of the models fitted to the data analysed in the previous two Chapters. This application, estimating the probability of entering the lowest bid for future projects, is clearly in its infancy insofar as the multivariate approach adopted in this research is concerned. The present indications are, however, that the estimates of probability are not very precise. The implications of imprecise probability estimates derived in this manner is considered to be worthy of further study. Additional work is also needed in devising a suitable model of the cost estimate distribution and hence the distribution of expenditure, income and profit.

CHAPTER 11

Summary and conclusions

11 SUMMARY AND CONCLUSIONS

The thesis has examined the construction organisation's project selection and bidding decision to identify suitable models for conceptualising and formulating the problem.

Chapter 1 introduced the subject of decisions and their relationship with objectives indicating "luck" and "foresight", in the face of some degree of "uncertainty", to be key elements, demanding supporting knowledge of available decision options and outcomes.

Chapter 2 examined the types of decisions made by construction organisations and the decision choice process involving the identification, evaluation and selection of options. Corporate decision systems and their use in construction companies were discussed. Some reasons for the lack of use of such systems were identified, including problems associated with the level of managerial ability, the co-ordination of aims and objectives, communication, accuracy of forecasts, capital policies, and political or economic uncertainty overseas. The special characteristics of the industry and the complexity of the construction process, together with the time and informational constraints, appear to be further factors. The relevance of existing problem solving techniques was also regarded as an important issue.

The scope of a decision system was considered to involve internal and external environments and an array of both economic and social goals over a period of time. Plans, it would seem, need to be made ahead of environmental changes and on a contingent basis to suit the decision-maker. This requires some indication of future events which may be gained by a device such as Ansoff's "weak signals" or simulation studies.

The practical needs of a decision system are centred on the tension between risk and cost in striving for simplicity and, at the same time, versatility in accommodating the potential preferences of the decision-maker.

In designing such a system, the accent is on providing suitable management informational support to provide the necessary sensitive system of indicators. This requires information from both internal and external sources in the appropriate manner and time, covering relevant aspects of the general environment, economic, technical and political factors of production, competition and future demand. Difficulties were anticipated in determining the exact nature of informational needs by analysis of the current process. A more suitable approach was considered to be through the development of a conceptual model which reflects the pivotal factors of the problem. No such model, however, appeared to be available in the present context. A basic model was, therefore, proposed (Fig 2.1) in which options are identified, evaluated and selected on an incremental basis by comparison with the previous 'best' selection.

Chapter 3 extended the basic model into the project selection decision by considering deterministic aspects of the problem. The decision environment was defined and divided into the project generating and the decision outcome environments. The outcome environment was further divided into aspirational (people) and non-aspirational (money and property) aspects. Maslow's needs/drives hierachy was tentatively proposed as a means of determining effects of decisions on the state of the aspirational environment (development states) and some measures of non-aspirational states noted. The resolution of the conflict between different aspects of the outcome environment was discussed in terms of resource control, in which the type of organisational structure is seen to be a major factor.

The relationship between project characteristics and the outcome environment was examined and four major and interrelated factors - type of work, client, location and competitors - identified as accounting for over 97% of reasons underlying the project selection decision.

The criteria for project selection were examined in terms of company objectives, suggesting that multiple and conflicting objectives often exist. Some proposals were considered for formulating and solving multiple attribute problems of this nature.

Chapter 4 introduced time-dependent aspects of the decision. These were considered to be implicated in the causal relationship between the outcome environment and projects together with the dimensional effect generally. The effect of time was seen to redefine the problem from that of simultaneous to sequential selection. The effect of a decision on further project opportunities was discussed in terms of marketing. The implications for evaluation and selection are that knowledge is needed of states at any moment in time.

Gottinger's 'sequential machine' model has been applied to the decision system as a means of accommodating the dynamical complexities introduced by the time considerations, a possible configuration being outlined in Fig 4.2.

Chapter 5 introduced the aspect of imperfect knowledge and the considerable uncertainties that inure the decision environment. The subject was addressed in terms of the relationship between the prototype (real world) and the model (perception of the real world). Various approaches were outlined in which changes in the prototype can be modelled and predicted by the use of proxy measures. The volume and type of project opportunities was considered to be related to political, economic, social and technological factors. Further information appears to be available directly through market intelligence activities.

Predicting events in the outcome environment has been discussed, based on information of project characteristics and the nature of tasks and performance. The prediction process itself has been examined and several approaches to anticipating its actions and accuracy considered.

Four separate studies were consulted to identify project selection methods for non-deterministic models with multiple criteria, indicating that some aspects of the problem may be dealt with probabilistically but that sensitivity tests together with the decision-maker's subjective judgement should also be employed.

A final conceptual model was provided in Fig 5.4, which incorporates the major features of the decision. This was conceived as consisting of several machines, by Gottinger's definition, representing the project generation, decision and outcome events. Such a model typifies a system of "intermediate complexity", suggesting that the model should be aimed at improvements rather than optimality, sensitivity analysis to formal hypothesis testing, an interaction between humans and machines, and the system should be integrated with other systems. Computer simulation was also associated with systems of this type.

Chapter 6 completed the specification of the project decision system by defining the various options available to the decision-maker. Several general decision strategies were considered as possible option identification rules. Low and high risk exogenous factors and strategies were identified as two general groupings,low risk strategies being commonly adopted except in unusual circumstances.

Consideration of future decisions led to the proposal of a nested set of decision machines with a facility to simulate future events.

The final version of the complete system was then examined in terms of computational load. The major difficulty was found to be in the size of the set of potential option combinations and some possible strategies identified to alleviate the situation.

Chapter 7 continued the examination of simplifying models by considering statistical approaches to some aspects of the problem. Models of construction demand and the occurrence of project characteristics were discussed together with such aspects of the outcome environment as cost and estimated cost. Various likely probability distribution types and parameters were introduced and these are summarised in Table 7.2. Income and cash flow were examined similarly.

Statistical models of collective competitors' bidding behaviour were considered (Table 7.3) together with the relationship with other factors such as the state of the market. The distribution of lowest bids was examined in relation to cost estimates and a close similarity noted between the expected value of the cost estimate and lowest bid.

Models of individual bidding behaviour were found to be usually derived from bid/cost estimate ratios, although multivariate methods have been proposed. The independence assumption was also noted.

Several models were found to incorporate features of the project generating environment in order to utilise error trends. These include game theoretic approaches, where each bidder is assumed to adopt similar (non-random) strategies. The degrading effects of time on the data was also considered.

Collusion was not generally thought to be prevalent in the construction industry and it was considered that non-serious and unrealistic bids should not be removed from any analysis without any reliable means of detection.

Data limitations were considered to be a major problem in attempting to fit models to bidding data by univariate analysis. Two approaches were considered to be feasible, using the collective competitor model or a multivariable procedure.

Project acquisition was considered and some work on excluding suicidally low bids noted, together with Simmonds' proposals for accomodating non-price features.

There appears to be a great deal of support for the view that the likelihood of entering the lowest bid for a project is largely determined by chance, the popular view being that , for each marginal change in mark-up there is a corresponding change in the probability of success. An expression of this conditional probability has been included.

Chapters 8 and 9 describe an analysis of three sets (Cases) of bidding data for indications of suitable models. Several parametric distributions were posited for the frequency of project value (lowest bid), the Log-lognormal model being found the most appropriate for all three Cases. Friedman's suggestion that the number of bidders followed a Poisson distribution was not found to be appropriate, a regression prediction on log project value with Normally distributed errors being preferred.

The distribution of bids was found to be generally positively skewed, coincident with other work in the field. No significant relationship was found between the number of bidders or project values and the coefficient of skewness.

Several tests were considered to evaluate models of distributional shape of bids. Due to the small sizes of the samples involved, the percentage points of the distribution of the sample statistics for each test had to be estimated by simulation (Tables 8.13 to 8.22). Further simulations were carried out on the data structure for each Case to ascertain the critical values (Tables 9.25 to 9.26). The results of these tests suggested that neither the Normal, Uniform, Weibull or fixed shape Lognormal models would be appropriate in all Cases. Various power and log transformations were applied with some success.

Several power and log variance stabilising transformations were attempted and tested by a version of Bartlett's test. An attempted three parameter log transformation was also made by estimating the threshold value in the manner prescribed by Aitcheson & Brown. None of the recommended methods were found acceptable but a further approach utilising the homoscedasic assumption achieved a satisfactory solution. It was not found possible, however, to predict the threshold values from the project value. It was concluded, therefore, that the spread parameter would have to be predicted by a simple regression on project value.

A method of predicting the identity of the bidders was proposed, in which the 'probability' of a bidder entering a bid is estimated by a regression of (log) project value.

The distribution of bids entered by each bidder was investigated by a multivariate analysis. The standard regression method was found to be inappropriate due to the large sparse matrices involved. An iterative procedure was devised and the distribution of bids analysed indirectly through the residuals obtained after fitting the model to the log data. The distribution of the pooled residuals was not found to follow a Normal distribution, but the Gram-Charlier Type A series provided a reasonable approximation with μ_3 = 0.16 and μ_4 = 4.5 in all three Cases.

Some consultation with the literature suggested high peaked, heavy tailed distributions of this kind to be associated with the aggregation of values from different distributions. It was, therefore, decided to test the distribution of individual bidders' bids. This was done in a similar manner to that described for the distribution of project bids, by simulating bids for individual bidders and tabulating the critical values of the statistics for both the shape and variance (Bartlett's) tests (Table 9.5a and 9.6). The results of these tests indicated marked differences between the Cases, although the transformation $y_{ij} = \ln(x_{ij} - mx_{1j})$ $(0.6 \leq m \leq 0.8)$ did appear to produce a Normal distribution in all Cases. Tests for the homoscedasic assumption met with little success and tests for equal location parameters also failed generally for the power and log transformations applied.

Analyses of details obtained from an individual bidder supported the view that Type 2 (cover) bids are very difficult to detect, the best of the methods applied only detecting three in six attempts. The distribution of the mark-up values were considered to be Normal and Lognormal for the percentage and multiplier mark-up values respectively. The distribution of low bid/cost estimate ratios was found to be more appropriately modelled by the Lognormal distribution than the Normal distributions.

A summary of the models successfully fitted to the data was provided in Table 9.19. These models are concerned with six variables: (1) project value; (2) the number of bids entered for a project; (3) the probability that a specified bidder enters a bid; (4) the bid values: (5) the mark-up value; and (6) low bid/cost estimate ratios.

The Exponential model was found to fit the distribution of project values for Cases 1 and 3 data and the Lognormal model for all Cases.

The numbers of bidders were predicted by the standard regession of log project value (forced through the origin) for Cases 1 and 2, and for all Cases when not forced through the origin.

The probability that a specified bidder enters a bid was predicted by the standard regression on log project value (parameter estimates for all bidders are contained in Appendix D).

The distribution of bids for all (unidentified) bidders followed a Normal distribution after a suitable power transformation for Cases 1 and 2, and a three parameter Lognormal distribution for all three Cases. The distribution of bids for identified bidders was found to be Normal for Cases 1 and 2 after a suitable power transformation, three parameter Lognormal for Cases 1 and 2 with the threshold parameter estimated from the mean bid, and three parameter Lognormal for all Cases with the threshold parameter estimated from the lowest bid.

The Normal and Lognormal model was found to fit the distribution of mark-up values expressed as a percentage and ratios of bid to cost estimates respectively.

The Lognormal model was found to fit the ratios of low bid to cost estimates

Chapter 10 examined the application of the models identified in the empirical analysis in deriving the probability of entering the lowest bid. The general proposition is made that a bid can be adequately modelled as being a random value from a probability distribution unique to the bidder at the time of bidding.

The theoretical probability of entering the lowest bid was advanced and empirical estimates obtained in a series of examples. The examples considered the cases where (1) the identity of the bidders is known, (2) the number of bidders is known (3) the project value is known, (4) none of the above is known. The distribution of probabilities was found to be rather imprecise.

Some final considerations concerned the probability of entering the lowest bid for a given mark-up. The theory was advanced and some approaches proposed for modelling the distribution of cost estimates.

Suggestions for further research

The first part of the research proposed a conceptual model of the project selection and bidding environment. As the model is the first to encapsulate all the factors involved it is necessarily of a *post hoc* nature. The next step would be to examine the validity of the model by empirical analysis by means of a structured questionnaire and case studies. In addition to verification and modification where necessary, an important contribution would be to estimate the strengths of relationships between factors. The dynamical nature of the situation under study suggests that a form of causal analysis may be appropriate.

A natural development on completion of such an analysis would be then to construct a practical decision system for testing and development in a 'live' situation.

Statistical modelling of factors is, despite over 30 years of research, clearly still in its infancy. A substantial amount of further empirical analysis is yet needed to establish the reliablity of such models. Some of the apparently important factors identified in the first part of the research, such as project characteristics and the project generating environment generally, have not received any attention at all.

Two fundamental approaches appear to be available. One is to start with the assumption that the problem is purely deterministic, develop solution techniques, and gradually relax the assumption by introducing appropriate random variables. The alternative approach would be to consider the whole problem as probabilistic and gradually introduce deterministic or partially deterministic decision or explanatory variables.The first of these approaches is exemplified in Chapters 2 to 6 of this thesis as a convenient means of analysing the literature. The second approach has effectively been adopted in the subsequent statistical analyses of some empirical data. The intention in both parts of the research has been to establish a sound foundation for both approaches.

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APPENDIX A

Statistical formulae

A STATISTICAL FORMULAE

1 Calculation of mean, variance and coefficients of skewness and kurtosis

For a sample of values of x_i $(i=1,2, \ldots ,n)$, the mean, variance coefficients of skewness and kurtosis ($\bar{x}$, s^2, γ_1, γ_2 respectively) of the population were estimated as follows:

$$\bar{x} = (1/n) \sum_{i=1}^{n} x_i$$

$$s^2 = n/(n-1).m_2$$

$$\gamma_1 = \mu_3/s^{3/2}$$

$$\gamma_2 = (\mu_4/s^2) - 3$$

where

$$\mu_3 = \frac{n}{(n-1)(n-2)}.m_3$$

$$\mu_4 = \frac{n^2}{(n-1)(n-2)(n-3)}. \{(n+1)m_4 - 3(n-1)m^{3}_{2}\} + 3s^2$$

and

$$m_r = \sum_{i=1}^{n} (x_i - x)^r$$

2. Probability distributions

(a) The Uniform (rectangular) distribution. $U(\alpha,\beta)$

$f(x) = (\beta-\alpha)^{-1}$ $\qquad\alpha<x<\beta$

$F(x) = (x-\alpha)/(\beta-\alpha)$

$E[x] = (\beta+\alpha)/2$

$Var[x] = (\beta-\alpha)^2/12$

$\gamma_1 = 0$

$\gamma_2 = -1.2$

α is estimated by x {min}

β is estimated by x {max}

Random number generator, NAG: G05DAF (parameters α & β)

(b) The Normal (Gaussian) distribution. $N(\mu,\sigma^2)$

$f(x) = \{\sigma(2\pi)^{1/2}\}^{-1}.\exp\{-(x-\mu)^2/2\sigma^2\}$ $\qquad -\infty<x<\infty,\ \sigma>0$

$F(x)$ is intractable

$F(z)$, the standard Normal deviate, where $z = (x-x)/\sigma$, is available in tabular form or NAG:S15ABF

$E[x] = \mu$

$Var[x] = \sigma^2$

$\gamma_1 = 0$

$\gamma_2 = 0$

Random number generator, NAG: G05DDF (parameters μ & σ)

(c) The Exponential distribution. $Expn(\lambda)$

$f(x) = \lambda e^{-\lambda x}$ $\qquad 0\le x\le\infty,\ \lambda\ge 0$

$F(x) = 1-e^{-\lambda x}$

$E[x] = \lambda^{-1}$

$\mathrm{Var}[x] = \lambda^{-2}$

$\gamma_1 = 2$

(d) The Poisson distribution. $P(\lambda)$

$f(x) = (\lambda^x e^{-\lambda})/x!$ $\qquad x=0,1,2,\ldots$ $\quad \lambda \geq 0$

$F(x) = \sum_{r=0}^{x} (\lambda^r e^{-\lambda})/r! = \chi^2_{2(x+1)}=2\lambda$

$E[x] = \lambda$

$\mathrm{Var}[x] = \lambda$

$\gamma_1 = \lambda^{-1/2}$

(e) The Gamma distribution. $G(\alpha,\lambda)$

$f(x) = \lambda(\lambda x)^{\alpha-1}e^{-\lambda x}/\Gamma(\alpha)$ $\qquad 0 \geq x \geq \infty,\ \alpha>0,\ \lambda>0$

$F(x)$, no form found, the approach taken was to evaluate

$\int_0^x f(x)dx$ by numerical integration

$E[x] = \alpha/\lambda$

$\mathrm{Var}[x] = \alpha/\lambda^2$

$\gamma_1 = 2\alpha^{-1/2}$

α is estimated by $\bar{x}^2/s^2_x$

λ is estimated by $\bar{x}/s^2_x$

(f) The Beta distribution of the first kind. $BT(a,b,\alpha,\beta)$

$f_x(x) = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} (x-a)^{\alpha-1}(b-x)^{\beta-1}$ $\qquad a \leq x \leq b,\ \alpha,\beta>0$

$F_x(x) = F_y\{(x-a)/(b-a)\}$ obtained by NAG:G01BDF

$E[x] = a+\alpha(\alpha+\beta)^{-1}(b-a)$

$\mathrm{Var}[x] = (b-a)^2\alpha\beta(\alpha+\beta)^{-2}(\alpha+\beta+1)^{-1}$

$\gamma_1 = \dfrac{2(\beta-\alpha)(\alpha+\beta+1)^{1/2}}{(\alpha+\beta+2)(\alpha\beta)^{1/2}}$

Estimates of a, b, α and β were obtained by letting

$a = x_i$ {min}
$b = x_i$ {max}

and solving

$x = a+\alpha(\alpha+\beta)^{-1}(b-a)$, and
$s_x^2 = (b-a)^2\alpha\beta(\alpha+\beta)^{-2}(\alpha+\beta+1)^{-1}$

(g) The Lognormal distribution $\Lambda(\mu,\sigma^2)$

$f(x) = \ln x \cap N(\mu,\sigma^2)$ $\quad 0 \leq x \leq \infty$

$F(x) = \ln x \cap N(\mu,\sigma^2)$ - see Normal distribution

$E[x] = \exp(\mu+\frac{1}{2}\sigma^2)$

$Var(x) = \exp(\sigma^2)\{\exp(\sigma^2-1)\}\exp(2\mu)$

$\gamma_1 = \{\exp(\sigma^2)+2\}\{\exp(\sigma^2)-1\}^{½}$

Random number generator, NAG: G05DDF (parameters α, β)

where $\alpha = \ln(\tau^2+1)$
$\beta = \ln(\mu)-\sigma^2/2$
$\tau^2 = \sigma^2/\mu^2$

(h) The Weibull distribution. $W(m,\lambda)$

$f(x) = \lambda m x^{m-1}\exp(-\lambda x^m)$ $\quad x \geq 0$

$F(x) = 1-\exp(-\lambda x^m)$

Estimates of λ and m were obtained by solving

$$nm^{-1}+\sum_{i=1}^{n}\ln x_i - \lambda^m \ln\lambda \sum_{i=1}^{n} x^m_i = 0$$

where $\lambda = \{n(m\sum_i x^m_i)^{-1}\}^{1/m}$

Random number generator, NAG: G05DPF (parameters m, λ^m)

(i) The Generalized Gamma distribution. $G(a,b,k)$

$f(x) = b\{a^{bk}\Gamma(k)\}^{-1}x^{bk-1}.\exp\{-(x/a)^b\}$ $\quad x>0$

Stacey(1962)

This distribution subsumes several others for

$G(a, b=1, k=1)$ is the Exponential distribution
$G(a, b, k=1)$ is the Weibull distribution
$G(a, b=1, k)$ is the Gamma distribution
$G(a, b, k\to\infty)$ is the Lognormal distribution
amongst others (Stacey & Milhram, 1965)

The log-likelihood function is

$$\ln L = n\ln b - n\ln\Gamma(k) - bkn \ln a + (bk-1) \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} (x_i/a)^b$$

The partial derivities with respect to a and b are

$$\frac{\delta(\ln L)}{\delta b} = nb^{-1} - kn \ln a + k \sum_i \ln x_i - \sum_i \{(x_i/a)^b \ln(x_i/a)\}$$

$$\frac{\delta(\ln L)}{\delta a} = ba^{1-b} \sum_i x^b{}_i - bkna^{-1}$$

Maximum likelihood estimates of a and b are therefore obtained by solving (for a given value of k)

$$nb^{-1} - kn \ln a + k \sum_i \ln x_i - \sum_i \{(x_i/a)^b \ln(x_i/a)\} = 0$$

where $a = (\sum_i x^b{}_i/kn)^{1/b}$

As $\Sigma\, x^b{}_i$ can be very large, for computational purposes

$$a = \bar{x}\{(kn)^{-1} \sum (x_i/\bar{x})^b\}^{1/b}$$

Estimate of k were then obtained by observing the value of k which maximises the log-likelihood function above.

(j) The Gram-Charlier Series of Type A

The series gives an expansion of $(2\pi)^{-½}.\exp(-½x^2)$ as follows:-

$$\text{pdf}(x) = (2\pi)^{-½}.\exp(-½x^2) \sum_{r=0}^{6} C_r H_r$$ for the first seven degrees

where $C_0 = 1$
$C_1 = 0$
$C_2 = ½(\mu_2-1)$
$C_3 = (1/6)\mu_3$
$C_4 = (1/24)(\mu_4-6\mu_2+3)$
$C_5 = (1/120)(\mu_5-10\mu_3)$
$C_6 = (1/720)(\mu_6-15\mu_4+45\mu_2-15)$

and

$H_0 = 1$
$H_1 = x$
$H_2 = x^2-1$
$H_3 = x^3-3x$
$H_4 = x^4-6x^2+3$
$H_5 = x^5-10x^3+15x$
$H_6 = x^6-15x^4+45x^2-15$

and $\mu_2, \mu_3, \ldots, \mu_r$ are the r^th^ moments about the mean in the data, $\mu^r = n^{-1}\, \Sigma\, (x-\mu_1)^r$, where

n = the number of data points
$\mu_1 = n^{-1}\, \Sigma\, x$

The $\mathrm{Cdf}(x) = \Phi(x) - \{(2\pi)^{-1/2}\}.\exp(-\tfrac{1}{2}x^2).\ \Sigma\ C_r H_{r-1}$

where $\Phi(x) = \{(2\pi)^{-1/2}\} \int_{-\infty}^{x} \exp(-\tfrac{1}{2}t^2)\ dt$

Random number generator:

Considering the standardised values

$$\Pr(x) = \Phi(x) - [\{(2\pi)^{-1/2}\}.\exp(-\tfrac{1}{2}x^2).\{(\mu_3/6)(x^2-1)+ \\ +(1/24)(\mu_4-3)(x^3-3x)\}]$$

For a given probability, $\Pr(x)$, the value of x can be obtained by solving the transcendental equation by a Newton-Raphson procedure

$$\Phi(x) - [\{(2\pi)^{-1/2}\}.\exp(-\tfrac{1}{2}x^2).\{(\mu_3/6)(x^2-1)+(1/24)(\mu_4-3)(x^3-3x)\}]- \\ -\Pr(x)=0$$

Thus by generating a random value for $\Pr(x)$ from a Uniform distribution (range 0 to 1), an appropriate value of x can be computed.

Although appearing rather laborious, values of x were found to be generated at the rate of approximately 100 per second!

(k) Edgeworth's form of the Type A Series

$$\mathrm{Pdf}(x)=\{(2\pi)^{-1/2}\}.\exp(-\tfrac{1}{2}x^2).[1+(k_3/6)H_3+(k_4/24)H_4+(k_5/120)H_5+ \\ +\{(k_6+10k^2{}_3)/720\}H_3]$$

where k_3, k_4, k_5 and k_6 are the 3rd, 4th, 5th and 6th cumulants of the data respectively, and the data is standardised. In this case the $\mathrm{Cdf}(x)$ is given by:-

$$\mathrm{Cdf}(x) = \Phi(x) = \{(2\pi)^{-1/2})\}.\exp(-\tfrac{1}{2}x^2).[0+(k_3/6)H_2+(k_4/24)H_3+ \\ +(k_5/120)H_4+\{(k_6+10k^2)/720\}H_5]$$

where $\Phi(x) = \int_{-\infty}^{x} \{(2\pi)^{-1/2}\}.\exp(-\tfrac{1}{2}t)\ dt$

Note: for moments about the mean

$k_2 = \mu_2$
$k_3 = \mu_3$
$k_4 = \mu_4 - 3\mu^2{}_2$
$k_5 = \mu_5 - 10\mu_3\mu_2$
$k_6 = \mu_6 - 15\mu_4\mu_2 - 10\mu^2{}_3 + 30\mu^3{}_2$

Comparison with the Gram-Charlier Type A Series

The Gram-Charlier Series is

$$\alpha(x) \sum_{r=0}^{6} C_r H_r$$

where $\alpha(x) = \{(2\pi)^{-\frac{1}{2}}\}.\exp(-\frac{1}{2}x^2)$

and

$C_0 = 1$
$C_1 = 0$
$C_2 = \frac{1}{2}(\mu_2-1)$
$C_3 = \mu_3/6$
$C_4 = (\mu_4-6_2+3)/24$
$C_5 = (\mu_5-10\mu_3/120$
$C_6 = (\mu_6-15\mu_4+45\mu_2-15)/720$

The Edgeworth form is

$$\alpha(x) \sum_{r=0}^{6} C_r H_r$$

where $\alpha(x)$ and H_r are identical to the Gram-Charlier Series (in standard form), but

$C_0 = 1$
$C_1 = 0$
$C_2 = 0$
$C_3 = k_3/6$
$C_4 = k_4/24$
$C_5 = k_5/120$
$C_6 = (k_6+10k^2{}_3)/720$

Expressing the Gram-Charlier Series in terms of k (and standardising) we obtain

$C_0 = 1$
$C_1 = 0$
$C_2 = 0$
$C_3 = \mu_3/6 = k_3/6$
$C_4 = (\mu_4-6\mu_2+3)/24 \rightarrow k_4/24$
$C_5 = (\mu_5-10\mu_3)/120 \rightarrow k_3/120$
$C_6 = (\mu_6-15\mu_4+45\mu_2-15)/720 \rightarrow (k_6+10k^2{}_3)/720$

Therefore, for the first seven terms, the formulae are identical.

APPENDIX B

Bidding data

B BIDDING DATA

Case 1

Case 1 data were donated by a construction company operating in the London area. The data covered all the company's bidding activities during a twelve month period in the early 1980's for a total of 86 projects. Due to the confidential nature of the data it has not been possible to reproduce all the information available in this thesis. Of the data that has been reproduced, certain minor changes have been made to obscure the identity of the company involved. The project numbers, for instance, are not reproduced in chronological order. Similarly, the codes given to the various bidders are not in alphabetical order. The bids themselves, however, remain intact.

Details of the type of projects were available but not used in the analysis. Some of the data were incomplete, that is the value of some bids or the identity of bidders were not known by the company. In several cases it was possible to supplement these data from the Case 3 source.

The resulting number of projects for which a full set of bids, together with the identity of the bidder, were available for analysis totalled 51.

The number of occasions that the same two bidders were in competition with one another is given in the following table.

Number of occasions met	Number of pairs
1	438
2	94
3	20
4	11
5	4
6	3
7	3
8	3
9	1
10	2
12	2
20	1

The most frequent competitors were bidders 55 and 304, who met on 20 occasions.

Details were also available of bidder 304's cost estimates for 34 of the projects.

Project number	Cost estimate
1	1386652
2	505291
3	1271146
5	389214
6	2058210
7	2919754
8	7035339
10	1012702
11	1811845
12	1053099
13	652341
19	2884614
20	7646123
21	3705840
22	580203
23	1558574
24	1179413
26	515061
27	1770389
28	2062491
30	2538005
32	530190
35	830407
36	754737
37	7067819
38	550787
41	1530976
42	3641105
44	2187217
46	2787585
47	1381542
48	751767
50	351803
51	645858

A total of 93 bidders entered 318 bids for the 51 projects. A full data listing follows.

```
PROJ    BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID  PAGE    1


PROJ    BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR   BID     BDR
  1  1454515.   150 1514865.   55 1475398.  304 1460775.  134 1447867.  154 1457977.   73
  2   535608.   304  502042.  291  529744.  154  516376.  157
  3  1333142.    75 1331156.  217 1366863.  304 1266892.  281 1276787.  115 1277652.   93 1065545.  360
  4   696743.   304  696972.  292  701062.  237  637015.   79  697826.  361  637015.  280
  5   404110.    55  422297.  304  413224.   97  309196.  117  417489.  362  309848.  157
  6  2116877.   134 2169966.   99 2187991.  293 2161120.  304 2198655.  221 2296100.  137 2145611.    8 2153344.  117 2133608.  294
  7  3065742.   304 3119689.  150 3141641.  170 3153000.  134 3249927.  191 3269760.   55 3335993.  187
  8  7925257.   221 7351929.  304 7374650.  247 6900000.   20
  9   871520.   118  899935.  137  902378.  304  914393.  291  950737.   83  996483.  221
 10  1063337.   304 1154023.  251 1102272.  173 1079657.  201
 11  1759614.   154 1792123.  281 1838532.  157 1918066.  170 1947733.  304 1784215.  308
 12  1126816.   304 1146398.  201 1169795.  154 1227296.   24 1312527.  280 1399472.  221
 13   698005.   304  625501.  268  630208.  308  666545.   55
 14   588810.   364  584833.  365  639229.   79  646341.  145  682802.  304  691474.  154
 15  1429218.   303 1493849.  291 1511033.  304 1521628.   12 1526377.  366 1717715.   55
 16   842319.     6  870894.  304  883617.  185
 17   284947.   367  292692.  356  294694.  368  303700.  152  307202.   85  313203.  134  315727.  369  333597.  118  334353.  370
      348969.   304
 18   461444.   150  483862.  304  482241.  308  447021.   55  493417.  154  455480.  311
 19  2858191.   280 2947007.  371 2950723.  134 2999999.  304 3093507.   60 3099520.    6 3278229.  266 3325198.  170 3333793.   55
 20  7831865.   276 7837276.  304 7859122.  256 7904172.   55 8047230.  152 8145323.  293 8279564.  117 8657685.  134
 21  3971051.    55 3854074.  304 4724785.  372 3955009.  154 3944772.  373 3731545.   79 4001100.  237
 22   573485.   292  596737.  291  597730.  134  613520.  201  615015.  304  621223.  170
 23  1610942.   304 1623447.  163 1646286.  173 1663742.  268 1700000.  152
 24  1196036.    64 1199328.  374 1206837.  107 1226509.  304 1262082.  291 1271000.  152 1295954.  170 1302161.  254
 25  2636397.   137 2654728.  150 2673906.  107 2685127.   55 2762123.  304 2845567.  152
 26   469663.    24  476784.  268  485870.  286  486485.   55  504026.  122  529460.  263  540014.  304
 27  1526553.   201 1533719.  152 1698797.  148 1876612.  304
 28  2106139.   201 2175928.  304 2210065.  308 2223710.  280 2255246.  221 2296623.  117 2331830.  266
 29   499888.   102  559596.   55  592026.  217  602042.  170  608957.  304  619065.  134
 30  2639525.   304 2842407.  308 2874130.  280 2861665.   55 2736300.  152 2770720.  256
 31   732572.   304  599429.  365  623906.  145  691759.   79  744332.  154  607065.  364
 32   546641.   134  539565.  268  608742.   55  538382.   24  599934.  170  559351.  304
 33   792966.   221  811788.   99  819971.  308  847621.   55  847892.  137  853793.  304
 34  2085151.   152 2130217.  107 2150583.  280 2203956.  115 2219653.  137 2241687.  154 2325900.  304
 35   821617.   268  844579.  115  848459.  303  871927.  304  872215.  106  935765.  375
 36   792474.   304  747374.   24  778559.  217  743788.  252  808345.  268  835465.  170
 37  7279854.   304 7650271.   60 7029448.  308 6631664.  150 7089879.  193 7230120.  170 6986341.  247 7143710.  191 6794553.  266
 38   592096.   304  573997.  150  518613.  217  508985.  121  544480.  376
 39   538600.   348  567031.  377  621365.  378  699839.  268  825451.   72  991468.  190 1001254.  304
 40  2087946.   154 2104017.  276 2183122.  186 2205359.  304 2212382.  280 2267987.  112 2332476.  221 2400000.  294
 41  1503739.   247 1536654.   24 1576905.  304 1583595.  154 1616432.  294 1704995.  157
 42  3624453.   191 3694803.  221 3732133.  304 3751115.  170 3773967.  193 3866339.   55 3922937.  134 4122448.  281
 43   629164.   157  695284.  173  723315.  311  729305.  266  743578.  304  768189.  379
 44  2252833.   304 2264310.   24 2274380.  112 2323385.  191 2384494.   55
 45  1202916.   163 1248733.   55 1291365.  221 1294986.  304
 46  2968891.   217 2772626.  280 2822857.  186 2972189.  134 2821600.  276 2857275.  304 2793000.  221
 47  1398400.   286 1401500.  152 1427140.  237 1436804.  304 1453070.  301 1511643.   55 1591986.  371 1665760.   83
 48   698161.   294  709676.   24  758565.  291  789355.  304  797926.  134  842684.   55  751677.  252
 49   248733.    31  251007.  291  251415.  252  261286.  380  264933.  304
 50   358840.   293  362370.  217  386983.  304  421797.  381  456272.  154
 51   527692.   317  570874.  311  588854.   75  609221.  173  636451.  308  694297.  304
```

Case 2

Case 2 data were donated by Lancashire County Council for project bids over approximately four years prior to July 1982. Details for 258 projects were provided in precoded format. In some cases, codes were missing or no tender had been received. In other cases the codes or bids were illegible. The resulting number of projects for which a full set of bids, together with the identity of the bidder, were available for analysis totalled 218.

The number of occasions that the same two bidders were in competition with each other is given in the following table.

Number of occasions met	Number of pairs
1	1224
2	264
3	99
4	62
5	34
6	25
7	15
8	16
9	9
10	6
11	5
12	5
13	8
14	4
17	4
19	2
20	1
21	1
26	1

A total of 187 bidders entered 1235 bids for the 218 projects. A full data listing follows.

Lancashire
County
Council

Roger Booth DipArch(Dist) RIBA FRSA
County Architect

PO Box 26 County Hall Preston PR1 8RE

Mr. R.M. Skitmore,
25 Meadway,
Penwortham,
Nr. Preston.

Please ask for	Preston(0772)	Your ref	Our ref	Date
Mr. H. Edwards	263153		Q/HE/JM	19 July, 1982

Dear Martin,

I enclose with this letter some 258 copies of tenders received for County Council projects to assist you with the Paper that you are preparing on Contractors' success or otherwise in tendering.

As I explained to you on the telephone, I wish to preserve the confidentiality of the tendering procedure and this has been achieved by giving every firm that has submitted a tender a number and recording that number against the respective tender list for which that firm quoted.

I hope this will assist you in carrying out your analysis. The period of time for which these tenders cover is approximately four years and relates to capital works as opposed to maintenance contracts.

I tried to contact you by telephone, but I appreciate you may now be away on leave, hence the purpose of forwarding these schedules to you. Should you require any further assistance, no doubt you can let me know.

Kind regards.

Yours sincerely,

H. Edwards

All replies to be addressed to the County Architect

BUILDING INNOVATION AWARD 1976

Sample data proforma for Case 2 data (project 3)

8 COST FLUCTUATING				FIRM PRICE			
		Clause 23 (j) Deleted				Clause 23 (j) Deleted	
405,525	59	154					
498,334	00	27					
505,381	00	8					
508,614	00	204					
511,132	00	58					
515,515	00	30					
524,620	00	62					
548,291	00	95					
569-655	00	252					
No data [illegible]		18					

```
PROJ       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  PAGE    1

PROJ       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR
   1    13064.    4    13834.    7    14487.  111    14677.  207    15240.  133    15581.   46    16483.   25    17000.   37
   2    34000.   55    35996.   62    39877.    2    40361.  101    40850.    8    42114.  105    44330.  103    49371.   58
   3   405525.  154   498334.   27   505381.    8   508617.  204   511732.   50   515515.   30   524620.   62   548291.   95   569655.  252
   4    16823.   46    17976.   25    18502.   95
   5     1833.  203     2019.   29     2577.   35     2625.  256     3010.  254
   6    73826.  207    81000.  133    82382.   25    88000.   69    89755.   46    99386.   34    99973.   30   100429.  184   101204.   95
       109204.  127
   7     7975.  121     8260.   81     8660.  126
   8     5300.  146     6024.   70     6263.  106     6570.   61     6830.   87     7278.   81     7350.  147     7420.  139
   9   133269.   18   137506.   62   145683.    8   147301.   95   148902.  124   149506.   58   161553.   46
  10   217396.   26   223869.   68   229270.  252   236295.   27   238082.   95   240000.  106   242396.  168   255946.   61   256410.  146
       261142.    2   269576.   41
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215     28434.   26   28903.   89   17381.  153   17653.  154   19940.  155   32110.    8   23409.   25   29934.  156
216     16038.  151   17284.  152   39114.   84   39030.   22   40170.   76   41905.  121   42787.   14   43827.   66
217     37132.  111   38814.   27   11813.  194   12942.    4   14454.  219
218      9550.  218   10684.  121
```

Case 3

Case 3 data were obtained from the records of a bidding information agency in the London area. The agency held details of most bids for most projects in the London area in card form. A period of one week was spent copying a sample of project data for the period November 1976 to February 1977. The bids and associated bidders' names were recorded and the names later encoded for analysis. The resulting number of projects for which a full set of bids, together with the identity of the bidders, were available for analysis totalled 373.

The number of occasions that the same two bidders were in competition with each other is given in the following table.

Number of occasions met	Number of pairs
1	2817
2	473
3	132
4	68
5	21
6	8
7	7
8	1
9	1
11	1

A total of 356 bidders entered 1915 bids for the 373 projects. A full data listing follows.

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PROJ       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  BDR       BID  PAGE     1


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   2  1469827.   99  1513390.  184  1528073.  221  1415104.  154  1464759.  152  1441800.  151
   3    42197.  276    39458.  104    45272.  249
   4    56216.   55    49925.   60    43891.  172
   5  1010053.   55  1081862.  187  1040854.  117  1068857.  152   970500.  207   944625.  201  1067806.  169
   6    81420.   46    81710.  266    87271.  154
   7   149246.  106   168674.  183   157642.  223
   8    62955.  188    66785.  173
   9   478442.  276   448232.  159   449608.  163   403794.  173   427560.  245   432908.  217   426586.   76   481607.    3
  10    26322.  276    25278.   55
  11   280818.  247   300623.  187   305476.  217   322333.   75   302855.  194   338082.  277
  12   189995.  175   207963.  217   198668.  116   198952.  173
  13   504426.   94   439439.  155   440399.   86   438147.  173   423365.  140   431486.  235   475748.  170   472721.  186
  14   118680.  276   118326.  185
  15   112590.  159    84784.  216
  16  5277951.  107  4650254.  137  4547327.  117  4399010.  152  4758170.  230
  17   882481.  150  1021654.  262   859457.   53   920000.  297   954236.  294
  18   958649.  280   931009.  175   928600.   54   907097.  292   946000.   40   886000.  179
  19   173986.   99   154077.   59   167771.  293   149680.  119   157794.    3   155858.  272
  20   216015.   94   220000.  291   232000.   15   229877.  237   215000.    2   215000.  142
  21   177570.  118   170631.  173
  22  1595440.  150  1479927.  230  1586700.   27  1573500.  202  1635000.  152  1635000.  264  1472000.  276
  23    76831.  268    74518.  185
  24   152995.  175   153919.  267    95000.  190
  25    79638.  150    73809.  134    62705.  247
  26   118854.  268   120529.  173   121511.  163   122154.  301   104871.  139   129865.  102
  27  2918605.  221  2802378.  164  2891276.  163  2739309.  117  2791260.  196  3180600.  152  2690000.  297  2550000.  201  2861219.    3
  28   235661.  173   221560.  111
  29   698896.  175   679323.  260   687921.  291   629210.  252   669621.  115   703293.  292
  30  1100894.  303  1171046.  221  1142245.   99  1272945.  241  1200539.   19  1159769.   56  1116607.  102
  31    55474.  186    48972.  304    47681.  146
  32   497884.  219   515144.   93   530564.   12   483161.  292   518913.  240
  33   107105.  276   112132.   11   120589.  292   135051.  262   109777.   53
  34   226611.  276   250890.   55   224601.   11   226758.  113   240914.  294   223736.  222   245249.   53   237590.  223
  35   534981.  164   537098.  247   479699.   94   482113.  252   528785.  291   497870.  118   476044.   95
  36  1699754.  293  1189809.  281  1707334.  157  1856546.  154
  37   131937.  234    89881.  175   106900.  187    90713.  117
  38   135644.  193   127550.  118   118466.  159   119849.  267   133620.  145   157711.    9   150867.  112
  39   268506.   48   249770.  246   283456.  268   253430.  292
  40   230190.  148   227829.  260   221016.  180
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  42    56336.  225    61828.  141    62297.  104
  43   226793.  116   210962.  102   191122.  304   182023.  121   183797.   56   204792.  183   220501.  151   174167.  258   199273.  178
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  47   125276.  118   129525.  175   118811.  266
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124    604348.  193  580055.  151  548264.   62  557532.  313  549797.   56
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127     56215.  175   51631.  197   36690.  306
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 188   1020578.  164   1503477.  163   1220257.  308    943004.  293    988000.  254   1045750.  261
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 208    216784.  163    226557.  164    223804.  139    223785.  313    216907.   37    214812.  156    213825.   76    245626.  171    238332.  279
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 209    602164.  150    540000.  181    540000.  182    532071.  254    520000.   25    540562.    8
 210    678589.  150    705802.  175    681378.   55    663960.  256    691050.  154
 211    297426.   55    340000.  150    339791.   80
 212    149900.  134    126642.  217    109274.   86    122482.  157    125000.  214
 213     87063.  163     94804.  268     90927.  127
 214    526776.  276    573000.  235    559816.  175    517772.   53    514031.  113    545893.  294    503946.   78
 215    186726.  134    179981.  291    185160.  118
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 217    374040.  184  409400.  310  422258.  284  394297.  313  390262.   79  453000.  294
 218     40500.   84   53745.   46   46650.   35
 219    129894.  163  113984.   84  125687.  114  121540.  117  121126.    6  120275.   60
 220    197177.  185  168814.  104  217578.  291  177350.  301  204806.  200
 221    736330.  198  782950.  303  736610.   11  701891.  113  757824.  294  685610.  314  717468.   74
 222    227296.   11  234749.  294  222874.  312
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 225    581291.  150  649812.  193  570930.  268  595000.   19  617550.  152  593620.   76  619757.  112
 226    237693.   11  236914.  113  247736.   53
 227     55764.  284   59967.  269
 228    376496.  164  302834.  316  384483.  198  353950.   52  372636.  119  355719.  272  356767.  279  370982.  222  340332.   50
 229    915369.  251  992221.  280 1159294.  118 1030000.  124  887000.  220 1043876.  129
 230     96854.   46   90057.   90   96984.  104  105790.  151   96086.   93  109462.    9
 231    371918.  173  384107.  175  365279.  217  360840.  148  371782.   54  362051.  292  360275.   36
 232    177759.  173  179021.  307  214944.  200  159635.  224  148550.   91
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 234    272185.  276  421514.  169  209414.   72  332873.  212  310694.   76
 235    257979.  175  241991.  279  233833.  272  249964.  222  261148.   41  250654.  248
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 241    229454.  173  221706.  235  227797.   56  216244.    1  218716.   62  219000.   77
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 244    157761.  307  184853.  292
 245   3932402.  280 3754395.  303 3631000.  228 4624473.  186 4800000.  152 3887827.  151
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 257    379854.  276  376965.   54  453835.   26  383565.  294
 258    171508.  252  196656.  310  168241.  313
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 260    497398.  252  494010.  118  555383.  277  519000.  302
 261   1416709.   55 1259204.  266 1295306.  256 1470985.  152
 262     96879.   92   92636.  106  113883.  168  102188.  145   91165.  209  111285.  115   92086.  275  108919.   51
 263    442604.  303  407800.   68  436351.   36  436780.    3  113241.  292  447066.   62
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 265    338744.  116  324549.  106  353552.  145  333000.  153  335423.  282  330524.  103  339892.  115  363750.   12
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 268    682560.  168  670633.  276  653123.  227  664259.  292  664789.  115
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274     87960.  260   99444.  288   71018.  157
275    410273.  260  416131.  115  410134.  145  412876.  311
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278    120743.  116  109469.  291  112219.  250
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280   3618787.  134 3474226.  187 3449755.  175 3657262.  152 3442183.  170 3935000.  169
281    273442.  268  257739.  156  283200.  109  270714.   37  296333.  313  244831.  258  288368.  122  280763.  233  270583.   76
       264843.   67
282    521824.  221  541306.  150  546262.  247  875657.   60  607803.   10
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284    634536.  150  665816.   70  661725.  164  638082.  304  685875.  152  648834.  154  671368.  294  644665.  251
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289     72901.  268   68169.   26
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292    218354.   94  255000.  101  249000.  262
293    405059.   94  383000.  174  389460.  152  409107.  230  383211.  320
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297    103374.  173  105022.  188   85200.   84   97224.  268  103900.  200  102102.  324
298    129875.  141  113450.   23  121937.  194  137820.  325
299    930827.  164  895244.  304  958865.   60  923500.  297
300    163728.  164  172400.  175  176850.  187  161300.  117
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302     51163.  185   50846.  104   53278.    6
303     84079.  185   78350.   95   75600.  292   72265.  157
304    522073.  163  470000.  156  447000.   41  454685.  272  483108.   71  496411.  262  498700.  183  440197.  279  455433.  326
305    318460.  159  292447.  268  286481.  301  277922.   54  293810.  151  309710.  157  265196.  327
306     70962.  268   69777.  267   66427.  106
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310    499734.  217  509260.  328  502931.   79  498947.  275  655553.  190
311    281109.  164  329960.  303  208791.  322  296106.  329  283619.   69  282919.  321  290245.   37  295582.  330  314222.  214
312    126109.  300  162050.  163  111540.  198
313    106756.   86  105519.  304
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317    159465.  118  135959.   86  122230.  106  136500.  135  127257.  151  127969.  115  137075.  224  136411.  176
318   2418064.  150 2306974.  310 2458150.  187 2365000.   19 2660298.    7 2550794.  154
319    262382.   16  250209.  310  276138.  184  235078.  252  258530.  187  260349.  291  235000.  275
320    307822.  266  312875.  193  385209.  170  316453.  154  384000.  254
321   1585460.  164 1524492.  266 1564234.  163 1598025.  276 1599022.    6 1628178.  294
322    196716.  164  203286.  163  204724.  139  215396.  268  198071.  270  191196.  156
323    605450.   46  617597.  134  546000.  228  612317.    3  595291.   33
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326    135000.  173  124273.   36  127787.  102  127742.  304  117210.  257
327    304223.  175  311059.  106  266066.  236  320680.  183  312460.   89  291077.  294
328    155632.  175  148190.  235  179121.  331  152233.  144
329    319265.  175  303161.  235  347824.  331  296637.  144
330    883120.  247  819948.  241  880171.  221  841213.  235  751630.  262
331   4297728.   99 4158886.  266 3839611.   20 4140609.  150 4499642.  135
332    176333.   55  188724.  246
333    688610.   55  768000.  297  760000.  152  728000.  332
334    920000.  333  911000.  297  980082.  332  908000.  152
335    259987.   55  250737.  124  255000.  235  252327.  332  228751.  334  268660.  186
336   1316208.   55 1346323.  186 1260000.  152 1373529.  185 1370660.  169 1316077.  256 1431436.  335
337    511392.   55  478559.  322  507570.  336  499800.  337  484000.  257  497000.  316  507544.   77  480000.  338
338   1298891.  164 1149736.   20 1136000.  152 1136000.   53 1140000.  339 1330000.  340 1140000.  341
339    264900.  134  283335.  342  289475.  111  277129.  343  277777.   53  270787.   78  282574.  344
340    336743.   94  335000.  174  414333.  247  355262.  252  356187.  291  351440.  290  347417.  345  341982.  120  375870.   81
341    149326.  221  150242.  115  145694.   79
342    260485.  134  257673.  159  242825.  193  228628.   24
343    131428.  309  129908.  184  149483.  135  130980.  168  149894.  115  135910.  145  200421.  311
344    161150.  185  167480.  135  157987.  104  150252.  347  184000.   62
345    117547.  174  115689.   15
346    570678.   94  544868.  145  566022.  348  569807.  349
347    253569.  159  267421.  291  252444.  163  249797.   56  304000.  350  234150.  351
348    127367.   86  134220.  118  112971.  135  119860.  106  129867.  115  111919.  352  121435.  151  137421.  224
349    295171.  268  404930.   55  411650.  353  394680.  183  391364.  151  417807.  124
350   1946410.  276 2129704.  266 1715250.  191 2347613.  154 1054934.  183 2040253.  125
351    767942.  221  734678.  175  721308.  354  805930.  294  751960.  154  737847.  115  802896.   12
352    228672.   94  214105.  184  235986.   73  221220.  252  219530.  134  218568.  355  240000.  291  229015.  356
353    770241.  221  698472.  252  722633.  134  731321.  187  769143.   62  689690.  195
354    308008.  155  348802.  221  341531.  217  300934.  357  329965.  292  349509.  151
355    242247.   86  245585.  268  261377.  291  250617.   73
356    172401.  106  177896.  159  165044.   79  169797.   56  181220.  324  166051.  115
357    487468.  134  512700.  154  500500.   71
358    435353.  193  418347.  186  423502.  134  412895.  115  389990.  105
359     92184.  134   90702.   24   93646.  260   81823.  215   80407.   79   90500.  240
360    560349.  221  559667.  300  591546.  263  583940.   83  545592.    3  519756.  207  613322.  294
361   8659824.   55 8817910.  256 9273057.  152 8555000.  297 8553300.  169
362    162204.   35  152131.   84  169970.  100  153908.  157  162400.  125
363    307667.  139  297604.  163  279742.  259  279479.  323  283159.   77  314668.  122
364    279421.  118  265361.  157  264407.  292  264494.  278
365    715947.   94  697097.   59  679468.  137  686272.  252  509653.   31  712770.  110  633342.  346
366    873367.  280  866843.  266  793510.  117  824797.  170  826036.  294  940947.  125  883372.  256  897000.  292
367    318461.  163  320626.  173  299832.  301  313000.  102  331000.  122  332000.    3  331695.  290
368    141700.  163  178852.  281  157796.  358  157800.  359
369     53476.   84   48159.  104   45914.  173   57639.  183   57396.  295
370    236300.   84  226000.   77  235097.  216  246504.  179  216879.  199  226268.  286
371    108726.   24  107811.   55  102363.  112
372    398695.  221  383467.   15  392330.  254  377433.  230  433642.  294  379514.  117  307999.   10
373    187359.   90  172037.  301  172827.  104  181912.   46  172500.   82  178049.  194
```

APPENDIX C

Estimates of α_i and σ^2_i obtained after the transformation

$$y_{ij} = \ln(x_{ij} - 0.8x_{1j})$$

CASE 1

```
BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA  BDR   ALPHA

  6 -0.01314   8 -0.05293  12 -0.08647  20 -0.41572  24 -0.23905  31 -0.23550  55  0.05003  60  0.16671  64 -0.24196  72 -0.10716
 73 -0.06594  75 -0.12466  79 -0.37051  83  0.29131  85 -0.24139  93 -0.30052  97  0.11160  99 -0.06457 102 -0.66977 106 -0.02611
107 -0.20752 112 -0.03695 115 -0.14500 117 -0.02605 118 -0.08141 121 -0.47774 122 -0.00399 131 -0.01402 137  0.00730 145 -0.42089
148  0.06744 150 -0.11644 152 -0.13256 154 -0.03027 157 -0.11616 163 -0.22697 170  0.09701 173 -0.00641 185  0.13434 186 -0.13093
187 -0.03633 190  0.24403 191 -0.01960 193 -0.00043 201 -0.22011 217 -0.13567 221  0.05720 237 -0.00634 247 -0.17090 251  0.17731
252 -0.21708 254  0.12521 256 -0.13484 263  0.17693 266  0.07721 268 -0.22907 276 -0.21164 280 -0.10700 281 -0.01603 286 -0.19979
291 -0.10893 292 -0.11758 293 -0.13098 294 -0.09243 301 -0.06445 303 -0.25191 304  0.00000 308 -0.04808 311 -0.15586 317 -0.60861
348 -1.40543 356 -0.44465 360  0.96263 361  0.05261 362  0.15270 364 -0.67747 365 -0.72506 366 -0.07399 367 -0.57208 368 -0.41419
369 -0.14023 370  0.05222 371  0.06693 372  0.59404 373 -0.00016 374 -0.23029 375  0.23293 376 -0.17861 377 -1.17120 378 -0.03539
379  0.24578 380 -0.01029 381  0.24444
```

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BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ  BDR   SIGSQ

  6  0.01324   8  0.03277  12  0.03277  20  0.03277  24  0.01457  31  0.03277  55  0.03310  60  0.03426  64  0.03277  72  0.03277
 73  0.03277  75  0.02934  79  0.00679  83  0.04508  85  0.03277  93  0.03277  97  0.03277  99  0.00111 102  0.03277 106  0.03277
107  0.03277 112  0.00732 115  0.00780 117  0.01632 118  0.04412 121  0.03277 122  0.03277 134  0.02510 137  0.02478 145  0.05065
148  0.03277 150  0.01256 152  0.04124 154  0.05618 157  0.07796 163  0.00130 170  0.01533 173  0.00379 185  0.03277 186  0.00000
187  0.04719 190  0.03277 191  0.01734 193  0.00023 201  0.03649 217  0.04650 221  0.04705 237  0.01051 247  0.02919 251  0.03277
252  0.01721 254  0.03277 256  0.00707 263  0.03277 266  0.04246 268  0.03246 276  0.00903 280  0.03960 281  0.11764 286  0.00480
291  0.01078 292  0.07003 293  0.05864 294  0.09743 301  0.03277 303  0.03207 304  0.03204 308  0.01981 311  0.05558 317  0.03277
348  0.03277 356  0.03277 360  0.03277 361  0.03277 362  0.03277 364  0.00016 365  0.00124 366  0.03277 367  0.03277 368  0.03277
369  0.03277 370  0.03277 371  0.12006 372  0.03277 373  0.03277 374  0.03277 375  0.03277 376  0.03277 377  0.03277 378  0.03277
379  0.03277 380  0.03277 381  0.03277
```

CASE 2

```
BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA   BDR   ALPHA

  1 -0.24437    2  0.00000    3  0.48331    4 -0.45775    5 -0.19329    6 -0.05504    7 -0.51149    8 -0.27501    9  0.01542   10 -0.04632
 11 -0.06774   12  0.08193   13 -1.75342   14 -0.03335   15  0.09204   16  0.09691   17  0.60872   18 -0.23734   19 -0.49020   20  0.25693
 21 -0.32140   22 -0.17003   24  0.05972   25 -0.11302   26 -0.16913   27  0.06245   28  0.27606   29 -0.07204   30 -0.04260   31  0.44704
 33  0.14453   34 -0.02424   35  0.01824   36  0.24770   37 -0.33151   38 -0.39764   39 -0.52670   41  0.16290   43 -0.01104   44  0.33560
 45  0.47060   46 -0.05799   47 -0.11173   49  0.15289   50 -0.44415   51 -0.14002   52  0.56406   53  0.10172   54  0.06070   55 -0.36877
 57 -0.02596   58  0.10732   60  0.04244   61 -0.13371   62 -0.13053   63  0.00107   64  0.10265   65 -0.34289   66 -0.19524   67  0.31312
 68 -0.20205   69 -0.15784   70 -0.24159   71  0.04182   72  0.26141   73 -0.16110   74  0.17461   75  0.20782   76 -0.15016   78  0.04686
 80 -0.02023   81 -0.37074   83 -0.20056   84  0.20460   85  0.05131   86  0.40313   87 -0.06411   88  0.25311   89 -0.24067   90  0.23019
 91 -0.47371   92 -0.57202   93  0.07579   94 -0.31865   95  0.11071   96 -0.07697   97  0.31602   98  0.17032   99 -0.11682  101 -0.07049
102  0.09710  103  0.07757  105  0.25257  106 -0.01904  108  0.30701  109 -0.00375  110 -0.12231  111 -0.13625  112  0.15360  113  0.11207
114  0.43095  115 -0.37180  116  0.01021  117  0.09503  119  0.16409  120 -0.20560  121 -0.54787  124 -0.06699  125 -0.46701  126 -0.43566
127  0.20581  128  0.13773  129 -0.01535  130 -0.12167  131 -0.02555  132  0.06739  133 -0.39707  134  0.05146  135 -0.10617  137 -0.01441
138 -1.07217  139 -0.09570  140  0.30702  141 -0.22125  142  0.24122  144  0.16460  145  0.03163  146 -0.11267  147 -0.00929  148  0.35973
149  0.04326  150  0.02446  151 -1.20008  152 -0.00069  153 -0.85915  154 -0.50200  155 -0.41276  156  0.46308  158  0.72504  161  0.20861
163 -1.50231  164 -1.21334  166 -0.04980  167 -0.64495  168 -0.09500  174 -0.56179  175 -0.44422  176  0.07130  177  0.30990  179  0.47927
183 -0.96386  184  0.09544  185  0.11727  186 -1.71207  190 -0.41351  191  0.27586  194 -0.03373  196 -0.10514  198 -1.20733  199 -1.05050
203 -0.48252  204  0.19782  205 -0.00132  206 -0.47176  207 -0.25833  208  0.05097  209 -1.20566  210 -0.77663  216 -0.03292  217  0.14443
218 -1.10147  219  0.17041  221  1.00936  226  0.53774  228 -0.56101  229 -0.32959  230  0.01320  231  0.36159  232  0.45580  238 -0.39060
239  0.04244  243 -0.68349  247 -0.62190  251 -0.79071  252  0.03771  254  0.70075  256  0.65305
```

```
BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ   BDR   SIGSQ

  1  0.05607    2  0.12046    3  0.12011    4  0.28255    5  0.12011    6  0.04828    7  0.08595    8  0.13049    9  0.00550   10  0.10166
 11  0.10038   12  0.00360   13  0.12011   14  0.08880   15  0.00260   16  0.05120   17  0.02404   18  0.07719   19  0.05452   20  0.06337
 21  0.08267   22  0.11833   24  0.12011   25  0.06163   26  0.16114   27  0.09426   28  0.03334   29  0.14274   30  0.06773   31  0.12011
 33  0.02427   34  0.03664   35  0.10049   36  0.06144   37  0.21025   38  0.12011   39  0.07340   41  0.02249   43  0.00510   44  0.04840
 45  0.12011   46  0.11665   47  0.12054   49  0.00113   50  0.20170   51  0.00135   52  0.12011   53  0.00726   54  0.30870   55  0.16943
 57  0.12011   58  0.10168   60  0.25921   61  0.12416   62  0.13771   63  0.01132   64  0.12011   65  0.25751   66  0.09092   67  0.12011
 68  0.20065   69  0.06319   70  0.42260   71  0.08796   72  0.12011   73  0.27063   74  0.09042   75  0.01860   76  0.04273   78  0.01960
 80  0.52977   81  0.22424   83  0.12011   84  0.10855   85  0.12011   86  0.12011   87  0.12098   88  0.12011   89  0.21370   90  0.00013
 91  0.19066   92  0.16391   93  0.00578   94  0.12011   95  0.04192   96  0.25965   97  0.12011   98  0.16173   99  0.04067  101  0.00529
102  0.01364  103  0.11406  105  0.05192  106  0.05645  108  0.12011  109  0.54253  110  0.04695  111  0.12945  112  0.01472  113  0.01183
114  0.12011  115  0.11042  116  0.31920  117  0.12011  119  0.12011  120  0.12011  121  0.12600  124  0.00153  125  0.12011  126  0.11213
127  0.04487  128  0.12011  129  0.06332  130  0.05073  131  0.03604  132  0.12011  133  0.17592  134  0.05235  135  0.01756  137  0.14082
138  0.12011  139  0.08236  140  0.10083  141  0.22981  142  0.03160  144  0.12011  145  1.16798  146  0.20962  147  0.20498  148  0.23014
149  0.01755  150  0.04344  151  0.12011  152  0.12011  153  0.12011  154  0.10093  155  0.12011  156  0.12011  158  0.13087  161  0.12011
163  0.00001  164  0.12011  166  0.27899  167  0.12011  168  0.01830  174  0.04311  175  0.07790  176  0.06797  177  0.12011  179  0.12011
183  0.18333  184  0.15002  185  0.03725  186  0.12011  190  0.13755  191  0.12011  194  0.42539  196  0.03114  198  0.12011  199  0.12011
203  0.11218  204  0.23669  205  0.12011  206  0.22459  207  0.14706  208  0.20732  209  0.12011  210  0.12011  216  0.12011  217  0.12011
218  0.12011  219  0.12011  221  0.12011  226  1.13509  228  0.12011  229  0.12011  230  0.12011  231  0.12011  232  0.12011  238  0.35543
239  0.12011  243  0.27612  247  0.12011  251  0.12011  252  0.07541  254  0.14590  256  0.12011
```

CASE 3

DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA	DOR	ALPHA
1	0.03031	2	-0.08142	3	0.00000	4	0.44267	6	0.03002	7	0.00703	8	-0.02671	9	0.59347	10	0.00073	11	-0.27789
12	0.19625	13	-0.15449	14	-0.46129	15	0.15974	16	-0.09536	17	0.44125	18	-0.23930	19	0.00231	20	-0.22301	21	0.42491
22	-0.02424	23	-0.27055	24	-0.11492	25	-0.21267	26	-0.01547	27	0.07561	28	-1.02776	29	-0.11079	30	0.24459	31	-0.50141
32	-0.61613	33	-0.11859	34	0.28464	35	0.01479	36	0.00389	37	-0.12174	38	-0.32145	39	-0.31394	40	0.11912	41	-0.13690
42	-0.05430	43	-0.64777	44	0.14447	45	0.09565	46	0.00602	47	-0.32692	48	0.01210	49	0.02567	50	-0.27142	51	0.12435
52	0.11406	53	-0.24442	54	-0.09625	55	-0.01542	56	-0.13045	57	-0.05186	58	-0.12492	59	-0.17307	60	0.10380	61	0.06768
62	0.10327	63	-0.03274	64	-0.19948	65	0.09069	66	0.17455	67	-0.24675	68	-0.25352	69	-0.19438	70	0.05022	71	0.04597
72	-0.36536	73	0.05315	74	-0.63456	75	0.06445	76	-0.17618	77	0.13692	78	-0.16819	79	-0.22798	80	0.29354	81	0.10101
82	-0.15777	83	0.06052	84	-0.11264	85	0.22223	86	-0.13417	87	0.03221	88	0.12001	89	-0.00533	90	0.06104	91	-0.40002
92	-0.23667	93	0.06728	94	-0.02783	95	-0.08861	96	-0.06694	97	0.25709	98	-0.35431	99	0.13440	100	-0.47497	101	-0.14771
102	-0.01163	103	0.05853	104	-0.08056	105	-0.17746	106	-0.08275	107	0.06203	108	0.42510	109	-0.02206	110	-0.09700	111	-0.03379
112	0.28023	113	-0.32691	114	0.04542	115	0.01476	116	0.06368	117	-0.14404	118	0.15140	119	-0.23978	120	-0.20349	121	-0.42190
122	0.02245	123	0.92993	124	0.07920	125	0.09677	126	-0.23086	127	-0.00211	128	0.05165	129	-0.06280	130	-0.41704	131	-0.22024
132	-0.02457	133	-0.26742	134	-0.01099	135	0.22168	136	-0.33375	137	-0.15913	138	-0.14137	139	0.15432	140	0.15511	141	0.05152
142	-0.12469	143	0.16640	144	-0.11211	145	-0.00270	146	-0.37562	147	-0.34501	148	-0.12909	149	-0.60241	150	0.06626	151	0.08326
152	-0.02319	153	-0.14219	154	0.06654	155	-0.08692	156	-0.06947	157	-0.10600	158	0.14021	159	0.00445	160	-0.31429	161	-0.32526
162	-0.22223	163	-0.07605	164	-0.03532	165	-0.02076	166	0.34197	167	0.08395	168	0.15371	169	0.24276	170	0.01243	171	0.27027
172	-0.65422	173	-0.00746	174	-0.22253	175	0.08632	176	0.15308	177	-0.20573	178	-0.00252	179	0.05477	180	-0.01908	181	0.11713
182	0.11250	183	0.10497	184	-0.10777	185	-0.07306	186	0.07056	187	0.00721	188	0.14677	189	0.29035	190	-0.05354	191	-0.38766
192	-0.37872	193	0.11983	194	-0.09775	195	-0.04087	196	-0.09685	197	0.10774	198	-0.40139	199	-0.25149	200	0.25696	201	-0.23195
202	0.04281	203	-0.37651	204	-0.22623	205	0.47963	206	-0.10178	207	-0.24354	208	-0.67275	209	-0.50080	210	0.08299	211	0.52020
212	0.12066	213	-0.06364	214	-0.03223	215	-0.18015	216	-0.06403	217	-0.05630	218	-0.30703	219	0.03100	220	-0.10539	221	-0.00752
222	-0.17508	223	-0.04446	224	0.13031	225	-0.05343	226	0.22726	227	-0.10514	228	-0.28419	229	0.04538	230	-0.11082	231	-0.02114
232	-0.05983	233	-0.08113	234	0.54341	235	0.04717	236	0.04513	237	0.03053	238	-0.07782	239	-0.10840	240	0.24615	241	0.15061
242	-0.47050	243	0.06006	244	-0.04601	245	-0.56526	246	-0.21671	247	0.10875	248	0.01013	249	0.54026	250	-0.10365	251	0.07139
252	-0.10042	253	-0.54879	254	-0.04796	255	0.02332	256	-0.10111	257	-0.42396	258	-0.61290	259	-0.10578	260	-0.13604	261	-0.18100
262	0.03634	263	-0.03501	264	0.12040	265	0.95713	266	-0.13936	267	0.10106	268	-0.01009	269	0.42801	270	-0.10923	271	-0.54097
272	-0.30793	273	0.70265	275	-0.42940	276	-0.06295	277	0.44086	278	-0.03775	279	-0.31295	280	0.06145	281	0.10750	282	0.21659
283	-0.07677	284	-0.10160	285	0.26134	286	-0.06209	287	-0.12737	288	0.50520	290	0.01259	291	0.11163	292	0.03605	293	-0.14644
294	0.00345	295	0.13338	296	-0.21692	297	-0.11819	298	0.14251	299	0.26593	300	-0.15743	301	-0.06453	302	0.22604	303	0.01214
304	-0.10635	305	-0.18091	306	-0.42475	307	-0.37661	308	0.01720	309	-0.19458	310	0.01060	311	0.25524	312	-0.28152	313	-0.08749
314	-0.22184	315	-0.33434	316	0.11246	317	-0.75523	318	0.04142	319	-0.01978	320	-0.25171	321	0.53531	322	-0.31421	323	-0.32421
324	0.23693	325	0.34729	326	-0.23299	327	-0.46072	328	-0.34176	329	-0.17427	330	-0.18166	331	0.54653	332	0.03357	333	-0.08061
334	-0.43640	335	0.29845	336	0.04613	337	-0.10977	338	-0.29593	339	0.26176	340	0.33987	341	-0.26174	342	0.00054	343	-0.09237
344	-0.00217	345	-0.12576	346	-0.36160	347	-0.10828	348	0.05617	349	0.08405	350	0.54513	351	-0.36740	352	-0.41073	353	0.31274
354	-0.20310	355	-0.15628	356	0.04334	357	-0.34128	358	0.07161	359	0.07170								

DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ	DUR	SIGSQ
1	0.08277	2	0.01950	3	0.03534	4	0.05220	6	0.02632	7	0.20127	8	0.00845	9	0.03107	10	0.14629	11	0.00590
12	0.01437	13	0.05220	14	0.05220	15	0.07571	16	0.02316	17	0.05220	18	0.05220	19	0.02067	20	0.04577	21	0.05220
22	0.12066	23	0.02534	24	0.02742	25	0.05220	26	0.06024	27	0.05220	28	0.05220	29	0.05220	30	0.05220	31	0.08076
32	0.05220	33	0.01233	34	0.05228	35	0.03402	36	0.06304	37	0.04691	38	0.01260	39	0.05228	40	0.03009	41	0.00632
42	0.01516	43	0.05228	44	0.05228	45	0.06670	46	0.07571	47	0.02310	48	0.03640	49	0.05220	50	0.03160	51	0.00572
52	0.54118	53	0.03023	54	0.07420	55	0.03172	56	0.03291	57	0.02591	58	0.01793	59	0.02940	60	0.07180	61	0.05228
62	0.07032	63	0.05220	64	0.05220	65	0.05228	66	0.00720	67	0.01242	68	0.05220	69	0.00366	70	0.05220	71	0.00453
72	0.05228	73	0.02455	74	0.11515	75	0.09157	76	0.03272	77	0.00704	78	0.03015	79	0.02515	80	0.05228	81	0.01658
82	0.00808	83	0.00250	84	0.11051	85	0.05228	86	0.07292	87	0.05228	88	0.09316	89	0.00237	90	0.07017	91	0.12697
92	0.08099	93	0.02631	94	0.05151	95	0.02227	96	0.05220	97	0.00607	98	0.05220	99	0.05524	100	0.05220	101	0.05220
102	0.02982	103	0.02005	104	0.05927	105	0.00232	106	0.01029	107	0.05220	108	0.10014	109	0.05220	110	0.00105	111	0.04092
112	0.16750	113	0.00035	114	0.00145	115	0.02419	116	0.01538	117	0.01274	118	0.06170	119	0.04779	120	0.00012	121	0.05220
122	0.01441	123	0.05228	124	0.04151	125	0.01097	126	0.05220	127	0.05220	128	0.05220	129	0.05909	130	0.05228	131	0.03766
132	0.11291	133	0.05129	134	0.06949	135	0.15093	136	0.05220	137	0.05626	138	0.05220	139	0.05140	140	0.32175	141	0.03674
142	0.00107	143	0.05228	144	0.00789	145	0.07262	146	0.05220	147	0.05220	148	0.02217	149	0.05220	150	0.04635	151	0.03512
152	0.04905	153	0.02789	154	0.05146	155	0.05429	156	0.12316	157	0.06402	158	0.05220	159	0.06399	160	0.05220	161	0.05228
162	0.01730	163	0.07109	164	0.03113	165	0.05220	166	0.15781	167	0.05220	168	0.10515	169	0.07867	170	0.05953	171	0.05220
172	0.05228	173	0.03559	174	0.00833	175	0.03490	176	0.06737	177	0.05229	178	0.05220	179	0.06307	180	0.04430	181	0.04222
182	0.05954	183	0.04124	184	0.07606	185	0.09000	186	0.07594	187	0.02644	188	0.06655	189	0.05220	190	0.71747	191	0.13201
192	0.05220	193	0.03771	194	0.01675	195	0.05430	196	0.05228	197	0.04523	198	0.11670	199	0.03462	200	0.00017	201	0.12366
202	0.05228	203	0.05220	204	0.05220	205	0.05220	206	0.05228	207	0.01079	208	0.05220	209	0.05220	210	0.05220	211	0.05228
212	0.03769	213	0.03035	214	0.01059	215	0.05001	216	0.12011	217	0.03076	218	0.05220	219	0.02054	220	0.05228	221	0.03457
222	0.02642	223	0.01707	224	0.00476	225	0.05357	226	0.05220	227	0.00047	228	0.04272	229	0.05220	230	0.02711	231	0.05228
232	0.05220	233	0.01163	234	0.28020	235	0.02773	236	0.16016	237	0.01739	238	0.05230	239	0.03210	240	0.00509	241	0.03140
242	0.05220	243	0.02797	244	0.03047	245	0.30007	246	0.07303	247	0.00417	248	0.05220	249	0.05006	250	0.09536	251	0.38735
252	0.02771	253	0.05220	254	0.04674	255	0.01747	256	0.03074	257	0.05229	258	0.00940	259	0.05950	260	0.16118	261	0.05228
262	0.10500	263	0.03714	264	0.00577	265	0.05220	266	0.04174	267	0.12325	268	0.07039	269	0.02081	270	0.05355	271	0.05228
272	0.02020	273	0.05228	275	0.00179	276	0.06090	277	0.00217	278	0.00075	279	0.05140	280	0.03460	281	0.24251	282	0.05071
283	0.05220	284	0.02733	285	0.05220	286	0.00952	287	0.03251	288	0.05220	289	0.02554	291	0.03566	292	0.03746	293	0.09276
294	0.03642	295	0.09763	296	0.00185	297	0.09593	298	0.05229	299	0.11213	300	0.00494	301	0.00317	302	0.05228	303	0.03344
304	0.02574	305	0.05220	306	0.07200	307	0.09310	308	0.05704	309	0.05609	310	0.04474	311	0.07662	312	0.04433	313	0.01697
314	0.26047	315	0.05220	316	0.01026	317	0.05228	318	0.05220	319	0.07407	320	0.05228	321	0.06104	322	0.00146	323	0.00084
324	0.00676	325	0.05228	326	0.05228	327	0.05228	328	0.05228	329	0.05220	330	0.05220	331	0.00513	332	0.02212	333	0.05220
334	0.05220	335	0.05220	336	0.05220	337	0.05220	338	0.05228	339	0.05229	340	0.05228	341	0.05220	342	0.05228	343	0.05228
344	0.05220	345	0.05228	346	0.05220	347	0.05220	348	0.05228	349	0.05220	350	0.05220	351	0.05220	352	0.05220	353	0.05220
354	0.05228	355	0.05220	356	0.05220	357	0.05224	358	0.05220	359	0.05220								

APPENDIX D

Estimates of regression coefficients $\alpha_i + \beta_i$ for prediction of 'probability' a specified bidder enters a bid

CASE 1

BDR	N.BIDS	ALPHA	SE	BETA	SE
6	2	-0.30926	0.46501	0.01782	0.03350
8	1	-0.22733	0.33334	0.01785	0.02386
12	1	-0.07164	0.33500	0.00654	0.02398
20	1	-0.70365	0.31888	0.05186	0.02283
24	7	0.77929	0.82700	-0.04604	0.05920
31	1	0.63026	0.32367	-0.04379	0.02317
55	20	-0.95621	1.16457	0.09669	0.08337
60	2	-1.03759	0.44353	0.07722	0.03174
64	1	-0.00014	0.33525	0.00142	0.02400
72	1	0.32012	0.33249	-0.02155	0.02380
73	1	-0.07685	0.33497	0.00692	0.02395
75	2	0.30508	0.46782	-0.01906	0.03349
79	4	0.35957	0.64885	-0.02016	0.04645
83	2	0.06403	0.44935	-0.00178	0.03360
85	1	0.37547	0.33365	-0.03958	0.02381
93	1	-0.02325	0.33520	0.00307	0.02379
97	1	0.45054	0.32954	-0.03090	0.02359
99	2	-0.06449	0.46713	0.00744	0.03358
102	1	0.35006	0.33191	-0.02370	0.02376
106	1	0.15059	0.33473	-0.00939	0.02396
107	1	-0.22327	0.33345	0.01742	0.02387
113	2	-0.47813	0.46349	0.03710	0.03318
115	3	-0.09593	0.56853	0.01110	0.04070
117	4	-0.76059	0.65391	0.06017	0.04574
118	2	0.70762	0.45966	-0.04757	0.03290
121	1	0.34282	0.33205	-0.02318	0.02377
122	1	0.37510	0.33138	-0.02549	0.02372
134	12	-0.81451	1.01463	0.07528	0.07363
137	5	-0.47828	0.71431	0.04133	0.05113
145	2	0.56432	0.46331	-0.03765	0.03317
148	1	-0.09809	0.33484	0.00844	0.02397
150	6	-0.72227	0.76974	0.06023	0.05510
152	9	-1.31825	0.89665	0.10718	0.06419
154	12	0.58641	1.02446	-0.02518	0.07533
157	5	0.80942	0.71181	-0.05101	0.05095
163	2	-0.12214	0.46879	0.01157	0.03356
170	10	-0.86027	0.94806	0.07575	0.06787
173	4	0.51343	0.64710	-0.03119	0.04632
185	1	0.14060	0.33481	-0.00868	0.02397
186	2	-0.56147	0.46142	0.04307	0.03303
187	3	-0.69558	0.55062	0.05410	0.03999
190	1	0.32012	0.33249	-0.02155	0.02380
191	4	-1.76526	0.59414	0.13221	0.04253
193	2	-1.13294	0.43836	0.08405	0.03138
201	5	0.04039	0.71900	0.00413	0.05147
217	6	1.00565	0.76864	-0.06368	0.05502
221	10	-1.85385	0.91412	0.14700	0.06544
237	3	-0.26754	0.56703	0.02340	0.04059
247	3	-1.48343	0.52440	0.11059	0.03754
251	1	0.04707	0.33524	-0.00197	0.02400
252	3	1.03675	0.55147	-0.07013	0.03948
254	1	-0.00014	0.33525	0.00142	0.02400
256	2	-1.07242	0.44158	0.07971	0.03161

263	1	0.37510	0.33138	-0.02549	0.02372
266	4	-1.00716	0.63126	0.07785	0.04519
268	7	1.49700	0.80902	-0.09751	0.05791
276	3	-1.31598	0.55385	0.09858	0.03822
280	8	-1.40377	0.85054	0.11191	0.06088
281	3	-0.62255	0.56051	0.04893	0.04012
286	2	0.31230	0.46773	-0.01758	0.03348
291	7	1.54460	0.80735	-0.10097	0.05779
292	2	0.54717	0.46370	-0.03642	0.03319
293	3	-0.50070	0.56227	0.04012	0.04032
294	4	-0.32723	0.64747	0.02723	0.04635
301	1	-0.06289	0.33505	0.00592	0.02398
303	2	0.07295	0.46933	-0.00235	0.03360
308	8	-0.23759	0.87754	0.07045	0.06282
311	3	0.98099	0.55344	-0.06613	0.03962
317	1	0.32833	0.33234	-0.02214	0.02379
348	1	0.32012	0.33249	-0.02155	0.02380
356	1	0.57567	0.32565	-0.03788	0.02331
360	1	-0.02235	0.33520	0.00207	0.02379
361	1	0.25524	0.33360	-0.01668	0.02385
362	1	0.45054	0.32954	-0.03090	0.02359
364	2	0.56422	0.46331	-0.03765	0.03317
365	2	0.56422	0.46331	-0.03765	0.03317
366	1	-0.07164	0.33500	0.00654	0.02398
367	1	0.57569	0.32568	-0.03788	0.02331
368	1	0.57569	0.32568	-0.03986	0.02331
369	1	0.57569	0.32568	-0.03788	0.02331
370	1	0.57569	0.32568	-0.03988	0.02331
371	2	-0.41275	0.46489	0.03241	0.03328
372	1	-0.45690	0.32825	0.03417	0.02350
373	1	-0.45690	0.32825	0.03417	0.02350
374	1	-0.00014	0.33525	0.00142	0.02400
375	1	0.15039	0.33475	-0.00739	0.02376
376	1	0.34282	0.33205	-0.02318	0.02377
377	1	0.32012	0.33249	-0.02155	0.02380
378	1	0.32012	0.33249	-0.02155	0.02380
379	1	0.25773	0.33352	-0.01708	0.02387
380	1	0.63026	0.32367	-0.04379	0.02317
381	1	0.48314	0.32863	-0.03324	0.02352

CASE 2

BDR	N.BIDS	ALPHA	SE	BETA	SE
1	2	0.00493	0.04441	0.00003	0.00356
2	2	-0.00203	0.04441	0.00060	0.00356
3	16	-0.22104	0.12241	0.02123	0.00782
4	1	0.03610	0.03139	-0.00269	0.00252
6	11	0.11696	0.10277	-0.00704	0.00824
7	2	-0.03127	0.04437	0.00395	0.00356
8	2	-0.06319	0.04427	0.00344	0.00355
9	2	0.05660	0.04433	-0.00413	0.00356
10	3	0.00091	0.05431	0.00037	0.00436
11	7	-0.00653	0.08250	0.00332	0.00662
12	6	-0.05356	0.07630	0.00342	0.00612
13	1	0.03610	0.03139	-0.00269	0.00252
14	1	0.03809	0.03141	-0.00203	0.00252
15	4	0.01378	0.06263	-0.00026	0.00502
16	6	0.15325	0.07616	-0.01104	0.00611
17	1	0.04495	0.03137	-0.00340	0.00252
18	1	0.05368	0.03137	-0.00270	0.00252
19	7	-0.13030	0.08215	0.01303	0.00659
20	6	-0.20041	0.07566	0.01743	0.00607
21	1	0.03305	0.03140	-0.00345	0.00252
22	2	-0.05961	0.04428	0.00323	0.00355
23	4	-0.01156	0.06262	0.00214	0.00502
24	10	-0.09245	0.09216	-0.00327	0.00787
25	1	-0.01708	0.03142	0.00175	0.00252
26	6	0.02140	0.07650	-0.00043	0.00614
27	1	-0.04966	0.03132	0.00431	0.00251
28	1	0.03136	0.03141	-0.00233	0.00252
29	1	0.00647	0.03144	0.00074	0.00252
30	1	-0.01133	0.03143	0.00117	0.00252
31	2	0.02066	0.04440	-0.00123	0.00356
32	1	0.03809	0.03141	-0.00203	0.00252
33	2	-0.02633	0.04438	0.00154	0.00356
34	1	0.03333	0.03140	-0.00239	0.00252
35	6	0.03358	0.07643	-0.00243	0.00613
36	7	0.02037	0.08232	-0.00013	0.00662
37	11	0.05752	0.10256	-0.00242	0.00825
38	2	-0.00323	0.04440	0.00083	0.00356
39	1	0.04343	0.03137	-0.00338	0.00252
40	4	-0.10904	0.06233	0.00964	0.00500
41	2	-0.00976	0.04440	0.00133	0.00356
42	3	-0.04343	0.05433	0.00414	0.00435
43	1	0.02336	0.03143	-0.00167	0.00252
44	1	-0.00366	0.03144	0.00031	0.00252
45	2	0.05900	0.04422	-0.00452	0.00355
46	10	0.26162	0.09746	-0.01890	0.00782
47	3	0.08827	0.05420	-0.00668	0.00435
48	3	-0.00091	0.05431	0.00073	0.00436
49	1	0.01208	0.03144	-0.00076	0.00252
50	3	-0.01779	0.05429	0.00224	0.00436
51	2	0.02319	0.04440	-0.00144	0.00356
52	2	-0.01793	0.04439	0.00155	0.00356
53	13	-0.09808	0.11131	0.01070	0.00893
54	8	-0.02373	0.08804	0.00460	0.00706

BDR	N.BIDS	ALPHA	SE	BETA	SE
55	33	-0.45342	0.17036	0.04363	0.01366
56	12	0.02220	0.10730	0.00072	0.00861
57	3	0.01482	0.05431	-0.00055	0.00436
58	2	0.06487	0.04430	-0.00479	0.00355
59	3	-0.02305	0.05429	0.00250	0.00435
60	14	0.11158	0.11551	-0.00399	0.00927
61	1	0.03233	0.03140	-0.00239	0.00252
62	10	-0.07523	0.09308	0.00821	0.00787
63	1	0.00777	0.03144	-0.00057	0.00252
64	1	-0.05072	0.03132	0.00430	0.00251
65	1	0.01118	0.03144	-0.00068	0.00252
66	3	0.07172	0.05421	-0.00513	0.00435
67	4	0.06341	0.06257	-0.00440	0.00502
68	1	-0.01193	0.03143	0.00112	0.00252
69	2	-0.04441	0.04433	0.00401	0.00356
70	1	0.00518	0.03144	-0.00020	0.00252
71	2	-0.03136	0.04437	0.00296	0.00356
72	1	-0.00005	0.03144	0.00022	0.00252
73	4	0.03244	0.06262	-0.00175	0.00502
74	3	0.03237	0.05430	-0.00196	0.00436
75	5	0.04971	0.06970	-0.00292	0.00561
76	12	-0.01937	0.10725	0.00417	0.00860
77	6	0.02109	0.07650	-0.00040	0.00614
78	5	-0.04234	0.06987	0.00450	0.00560
79	10	0.00178	0.09321	0.00201	0.00788
80	1	-0.00265	0.03144	0.00043	0.00252
81	2	0.00781	0.04441	-0.00020	0.00356
82	5	-0.02118	0.06971	0.00278	0.00561
83	3	-0.05320	0.05422	0.00493	0.00435
84	9	0.27903	0.09236	-0.02057	0.00741
85	1	-0.02846	0.03140	0.00251	0.00252
86	10	0.10277	0.09214	-0.00611	0.00787
87	1	-0.02411	0.03141	0.00216	0.00252
88	4	0.01320	0.06265	-0.00020	0.00502
89	3	-0.05276	0.05427	0.00456	0.00435
90	3	0.09233	0.05411	-0.00727	0.00434
91	2	0.02210	0.04440	-0.00135	0.00356
92	3	0.18833	0.05349	-0.01455	0.00429
93	4	-0.01200	0.06262	0.00183	0.00502
94	11	-0.12704	0.10255	0.01260	0.00823
95	7	-0.02834	0.08248	0.00379	0.00662
96	1	0.03305	0.03140	-0.00245	0.00252
97	2	-0.08710	0.04414	0.00744	0.00354
98	1	0.03004	0.03141	-0.00220	0.00252
99	14	-0.43326	0.11294	0.03790	0.00906
100	1	0.01732	0.03143	-0.00118	0.00252
101	1	0.03775	0.03139	-0.00282	0.00252
102	12	0.13004	0.10718	-0.00788	0.00860
103	3	0.01081	0.05431	-0.00022	0.00436
104	11	0.37209	0.10174	-0.02356	0.00816
105	3	-0.00979	0.05431	0.00144	0.00436
106	17	0.37876	0.12563	-0.02682	0.01008
107	1	-0.00792	0.03144	0.00085	0.00252
108	2	-0.02809	0.04437	0.00269	0.00356
109	1	0.00306	0.03144	-0.00003	0.00252

BDR	N.BIDS	ALPHA	SE	BETA	SE
110	2	0.03047	0.04439	-0.00202	0.00356
111	2	0.00675	0.04441	-0.00011	0.00356
112	9	0.05493	0.09329	-0.00248	0.00748
113	4	-0.03562	0.06258	0.00373	0.00502
114	2	0.02989	0.04439	-0.00197	0.00356
115	32	0.24042	0.17010	-0.01245	0.01364
116	8	0.11339	0.08796	-0.00740	0.00706
117	13	-0.32268	0.10976	0.02878	0.00882
118	26	0.01300	0.15482	0.00457	0.01242
119	4	0.00803	0.06263	0.00022	0.00502
120	2	0.02194	0.04440	-0.00133	0.00356
121	1	0.01307	0.03144	-0.00084	0.00252
122	12	-0.07527	0.10715	0.00565	0.00859
123	1	0.05623	0.03132	-0.00432	0.00251
124	11	-0.23212	0.10197	0.02106	0.00818
125	6	-0.05436	0.07641	0.00567	0.00613
126	1	0.03728	0.03139	-0.00279	0.00252
127	1	0.03345	0.03140	-0.00248	0.00252
128	1	0.01396	0.03144	-0.00091	0.00252
129	3	-0.14970	0.05369	0.01272	0.00431
130	1	0.03868	0.03139	-0.00290	0.00252
131	3	0.08726	0.05416	-0.00638	0.00434
132	6	-0.17101	0.07588	0.01506	0.00609
133	2	-0.00122	0.04441	0.00053	0.00356
134	21	-0.05572	0.14004	0.00903	0.01123
135	12	0.01551	0.10729	0.00134	0.00861
136	1	-0.00014	0.03144	0.00023	0.00252
137	12	-0.23279	0.10627	0.02294	0.00852
138	1	-0.01687	0.03143	0.00157	0.00252
139	8	0.11210	0.08797	-0.00730	0.00706
140	2	0.01337	0.04441	-0.00064	0.00356
141	6	0.11904	0.07631	-0.00829	0.00612
142	2	-0.02413	0.04438	0.00237	0.00356
143	1	0.01321	0.03144	-0.00086	0.00252
144	5	-0.00297	0.06992	0.00132	0.00561
145	9	0.09315	0.09323	-0.00576	0.00748
146	1	0.05115	0.03134	-0.00390	0.00251
147	1	0.02809	0.03141	-0.00203	0.00252
148	6	-0.01636	0.07648	0.00260	0.00613
149	1	0.00414	0.03144	-0.00017	0.00252
150	12	-0.51048	0.14015	0.04585	0.01124
151	20	-0.04441	0.13698	0.00789	0.01098
152	14	-1.21133	0.16135	0.10405	0.01294
153	2	-0.01314	0.04440	0.00149	0.00356
154	27	0.47463	0.15497	0.04104	0.01243
155	9	-0.05236	0.09322	0.00616	0.00748
156	6	0.03676	0.07649	-0.00166	0.00614
157	12	0.15553	0.10721	-0.00993	0.00859
158	1	0.03728	0.03139	-0.00279	0.00252
159	16	0.20601	0.12292	-0.01313	0.00986
160	1	-0.02203	0.03142	0.00179	0.00252
161	1	-0.02558	0.03141	0.00223	0.00252
162	5	-0.17082	0.06927	0.01483	0.00556
163	20	0.10647	0.16022	-0.00252	0.01285
164	29	-0.32992	0.16144	0.03282	0.01295

SDR	N.BIDS	ALPHA	SE	BETA	SE
165	1	-0.01687	0.03143	0.00157	0.00252
166	2	0.08540	0.04421	-0.00644	0.00355
167	1	-0.04582	0.03134	0.00270	0.00251
168	5	0.15519	0.06934	-0.01142	0.00558
169	6	-0.25771	0.07516	0.02206	0.00603
170	8	-0.19083	0.05740	0.01709	0.00701
171	1	0.00735	0.03144	-0.00042	0.00252
172	1	0.05559	0.03133	-0.00410	0.00251
173	56	0.24185	0.17910	-0.01174	0.01439
174	5	-0.03215	0.06972	0.00158	0.00561
175	51	-0.36255	0.20754	0.03263	0.01667
176	7	-0.00817	0.08230	0.00217	0.00632
177	1	0.05545	0.03133	-0.00435	0.00251
178	1	0.01137	0.03141	-0.00084	0.00252
179	3	-0.05725	0.05103	0.00064	0.00433
180	7	0.20018	0.08172	-0.01523	0.00657
181	3	-0.01225	0.04440	0.00145	0.00356
182	2	-0.01116	0.04439	0.00173	0.00356
183	11	-0.04672	0.10250	0.00614	0.00823
184	13	0.05052	0.10726	-0.00456	0.00860
185	10	0.17251	0.09773	-0.01173	0.00785
186	17	-0.31427	0.12610	0.02592	0.01011
187	15	-0.27119	0.11850	0.02571	0.00949
188	6	0.15139	0.07617	-0.01087	0.00611
189	1	0.05349	0.03131	-0.00448	0.00251
190	3	0.00237	0.05431	0.00046	0.00436
191	2	-0.11781	0.04394	0.00792	0.00352
192	1	0.01130	0.03144	-0.00080	0.00252
193	17	-0.11232	0.12443	0.01545	0.01004
194	9	0.10750	0.09133	-0.00639	0.00748
195	6	0.11130	0.07015	0.01130	0.00611
196	1	-0.08551	0.03124	0.00551	0.00251
197	3	0.09259	0.05412	-0.00713	0.00434
198	5	-0.02173	0.06925	0.00348	0.00561
199	2	0.02710	0.04409	-0.00175	0.00354
200	2	0.04508	0.04423	-0.00344	0.00354
201	4	-0.03179	0.05245	0.00743	0.00501
202	1	-0.04763	0.03122	0.00421	0.00251
203	1	-0.14503	0.03134	0.00290	0.00251
204	1	0.02712	0.03142	-0.00177	0.00252
205	1	-0.00264	0.03144	0.00031	0.00252
206	1	-0.07755	0.03141	0.00228	0.00252
207	2	-0.04230	0.04434	0.00194	0.00356
208	1	0.04343	0.03127	-0.00328	0.00252
209	1	0.07110	0.03141	-0.00237	0.00252
210	1	-0.01183	0.03147	0.00117	0.00257
211	1	0.01307	0.03144	-0.00084	0.00252
212	4	-0.01134	0.06262	0.00190	0.00502
213	1	-0.03134	0.04137	0.00355	0.00356
214	6	0.00094	0.07680	0.00127	0.00614
215	6	0.00234	0.07450	0.00099	0.00614
216	5	0.05018	0.05125	-0.00104	0.00435
217	14	0.02303	0.11557	-0.00013	0.00937
218	1	-0.02222	0.03141	0.00209	0.00252
219	8	0.00724	0.06609	0.00109	0.00707

.BDR	N.BIDS	ALPHA	SE	BETA	SE
220	1	-0.02293	0.03141	0.00206	0.00252
221	27	-0.31784	0.16152	0.03155	0.01276
222	4	0.00000	0.06263	0.00086	0.00502
223	5	-0.06121	0.06982	0.00601	0.00560
224	7	0.05478	0.08214	-0.00231	0.00641
225	4	0.10046	0.06246	-0.00722	0.00501
226	1	-0.01630	0.03143	0.00153	0.00252
227	2	0.00226	0.04441	0.00017	0.00356
228	7	-0.12443	0.08218	0.01155	0.00659
229	1	-0.02411	0.03141	0.00216	0.00252
230	4	-0.15125	0.06206	0.01304	0.00498
231	1	-0.00366	0.03144	0.00031	0.00252
232	1	-0.02731	0.03140	0.00241	0.00252
233	5	0.02644	0.06973	-0.00205	0.00561
234	2	0.05765	0.04432	-0.00457	0.00355
235	15	-0.07936	0.11930	0.00763	0.00957
236	7	-0.01599	0.08250	0.00230	0.00662
237	6	-0.15660	0.07597	0.01390	0.00609
238	1	-0.05575	0.03128	0.00495	0.00251
239	2	-0.00140	0.04441	0.00034	0.00356
240	2	0.01887	0.04410	-0.00159	0.00356
241	3	-0.05171	0.05301	0.00633	[illegible]
242	1	-0.00524	0.03144	0.00064	0.00252
243	2	0.05569	0.04430	-0.00436	0.00355
244	2	0.01245	0.04410	-0.00105	0.00356
245	2	0.07704	0.04429	-0.00175	0.00356
246	4	0.04667	0.06260	-0.00259	0.00502
247	14	-0.29067	0.11430	0.02612	0.00917
248	1	0.00441	0.03144	0.00014	0.00252
249	2	0.06643	0.04429	-0.00491	0.00355
250	3	0.10731	0.05404	-0.00215	0.00434
251	2	-0.06537	0.04425	0.00569	0.00355
252	15	-0.02273	0.11944	0.00346	0.00958
253	1	0.03004	0.03141	-0.00220	0.00252
254	9	-0.05537	0.09312	0.00706	0.00747
255	2	0.03425	0.04425	-0.00257	0.00356
256	9	-0.29605	0.09150	0.02578	0.00734
257	1	0.02471	0.03142	-0.00177	0.00252
258	2	0.01214	0.04440	-0.00087	0.00356
259	3	0.00259	0.05431	0.00021	0.00436
260	6	0.07779	0.07643	-0.00516	0.00613
261	1	-0.03257	0.03138	0.00314	0.00252
262	10	-0.12756	0.09759	0.01243	0.00785
263	5	0.05988	0.06989	-0.00374	0.00561
264	3	-0.11038	0.05396	0.00953	0.00433
265	1	-0.00417	0.03144	0.00035	0.00252
266	9	-0.19461	0.09261	0.01761	0.00743
267	4	0.16132	0.06214	-0.01215	0.00498
268	57	0.22135	0.21601	-0.05274	0.01733
269	2	0.07529	0.04424	-0.00597	0.00355
270	4	-0.02250	0.06261	0.00276	0.00503
271	1	-0.01687	0.03143	0.00137	0.00252
272	6	-0.01403	0.07848	0.00142	0.00613
273	1	0.00414	0.03144	-0.00012	0.00252
274	3	0.01551	0.05431	-0.00094	0.00436

BDR	N.BIDS	ALPHA	SE	BETA	SE
276	35	0.03004	0.15204	0.00378	0.01220
277	2	-0.01834	0.04419	0.00192	0.00356
278	2	-0.00530	0.04140	0.00095	0.00356
279	6	-0.03461	0.07645	0.00434	0.00613
280	13	-0.47389	0.10833	0.04094	0.00849
281	10	0.13463	0.09806	-0.00369	0.00787
282	3	0.00354	0.05431	0.00036	0.00436
283	1	-0.00601	0.03144	0.00070	0.00252
284	7	0.16083	0.08212	-0.01144	0.00659
285	1	-0.01515	0.03145	0.00144	0.00252
286	4	0.00573	0.06243	0.00040	0.00502
287	2	-0.03350	0.04430	0.00474	0.00355
288	1	0.03744	0.03138	-0.00196	0.00252
290	4	-0.05755	0.06233	0.00550	0.00502
291	25	-0.12239	0.15153	0.01546	0.01229
292	22	-0.14597	0.14575	0.01533	0.01169
293	7	-0.12008	0.08220	0.01118	0.00659
294	30	-0.52236	0.16233	0.04837	0.01302
295	2	0.07552	0.04425	-0.00567	0.00355
296	2	0.00564	0.04441	-0.00002	0.00356
297	12	-0.58767	0.10667	0.05038	0.00856
298	1	-0.03237	0.03144	0.00045	0.00252
299	2	0.04834	0.04435	-0.00350	0.00356
300	8	0.00134	0.08839	0.00158	0.00707
301	6	0.08079	0.07643	0.00221	0.00613
302	1	-0.01757	0.03143	0.00123	0.00252
303	17	-0.59236	0.12235	0.05160	0.00981
304	16	-0.30106	0.11855	0.01964	0.00953
305	1	-0.03903	0.03137	0.00336	0.00252
306	6	-0.03370	0.07647	0.00320	0.00613
307	6	0.06943	0.07645	-0.00139	0.006.3
308	7	-0.14734	0.08206	0.01317	0.00658
309	6	0.25336	0.07549	-0.01710	0.00606
310	11	-0.09940	0.10145	0.01033	0.00837
311	13	0.17306	0.10735	-0.01136	0.00859
312	3	0.03737	0.05433	-0.00136	0.00436
313	11	-0.08267	0.10275	0.00790	0.00824
314	2	-0.06776	0.04424	0.00589	0.00355
315	1	-0.04231	0.03134	0.00362	0.00252
316	3	-0.01186	0.05430	0.00140	0.00436
317	1	0.01354	0.03142	-0.00136	0.00252
318	1	-0.01085	0.03143	0.00107	0.00251
319	2	0.00370	0.04140	0.00087	0.00376
320	1	-0.01007	0.03144	0.00103	0.00252
321	2	0.00860	0.04441	0.00036	0.00356
322	3	-0.00803	0.05431	0.00129	0.00436
323	2	0.00578	0.04441	-0.00027	0.00356
324	3	0.04011	0.05417	-0.00359	0.00435
325	1	0.02567	0.03142	-0.00153	0.00252
326	1	-0.01418	0.03143	0.00136	0.00252
327	1	0.00073	0.03141	0.00016	0.00252
328	1	-0.01786	0.03142	0.00145	0.00252
329	1	-0.00100	0.03144	0.00030	0.00252
330	1	-0.00105	0.03144	0.00020	0.00252
331	2	0.01524	0.04440	0.00080	0.00356

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BDR  N.SIDE    ALPHA       SE        BETA       SE


332     3    -0.05773   0.05471   0.00550   0.00435
333     1    -0.03548   0.03135   0.00107   0.00257
334     1     0.00206   0.03134  -0.00019   0.00252
335     1    -0.04509   0.03134   0.00793   0.00211
336     1    -0.01444   0.03143   0.00156   0.00252
337     1    -0.01384   0.03147   0.00156   0.00252
338     1     0.01444   0.03143   0.00156   0.00252
339     1    -0.04104   0.03133   0.00540   0.00211
340     1    -0.01334   0.03133   0.00540   0.00253
341     1    -0.04204   0.03135   0.00540   0.00253
342     1     0.00075   0.03144   0.00016   0.00253
343     1     0.00075   0.03143   0.00016   0.00253
344     1     0.00075   0.03144   0.00016   0.00252
345     1    -0.00325   0.03140   0.00071   0.00251
346     1    -0.02177   0.03141   0.00205   0.00253
347     1     0.01574   0.03144  -0.00107   0.00251
348     1    -0.02045   0.03143   0.00156   0.00252
349     1    -0.02045   0.03142   0.00134   0.00251
350     1     0.00457   0.03144   0.00014   0.00252
351     1     0.00137   0.03144  -0.00014   0.00251
352     1     0.03107   0.03142  -0.00188   0.00253
353     1    -0.02245   0.03144   0.00011   0.00251
354     1    -0.03369   0.03140   0.00153   0.00253
355     1     0.00701   0.03144  -0.00035   0.00251
356     1     0.00701   0.03144  -0.00035   0.00252
357     1    -0.00563   0.03144   0.00011   0.00253
358     1     0.01714   0.03142  -0.00132   0.00253
359     1     0.01124   0.03142  -0.00155   0.00252
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CASE 3

DIR	N.BIDS	ALPHA	SE	BETA	SE
1	2	0.00495	0.04441	0.00033	0.00356
2	3	-0.00203	0.04441	0.00040	0.00356
3	16	-0.22104	0.12344	0.02125	0.00903
4	1	0.03610	0.03139	-0.00369	0.00252
6	11	0.11696	0.10277	-0.00704	0.00824
7	3	-0.05127	0.04437	0.00195	0.00356
8	3	-0.06219	0.04437	0.00344	0.00355
9	2	0.05660	0.04433	-0.00113	0.00356
10	3	0.00391	0.05431	0.00037	0.00434
11	7	-0.00053	0.08250	0.00332	0.00662
12	6	-0.05036	0.07530	0.00843	0.00612
13	1	0.03610	0.03139	-0.00369	0.00252
14	1	0.03309	0.03141	-0.00203	0.00251
15	4	0.01378	0.06263	-0.00026	0.00503
16	6	0.15033	0.07616	-0.01104	0.00611
17	1	0.04475	0.03137	-0.00340	0.00252
18	1	0.03368	0.03139	-0.00270	0.00252
19	7	-0.13050	0.08215	0.01203	0.00659
20	6	-0.20041	0.07566	0.01745	0.00607
21	1	0.03105	0.03140	-0.00345	0.00252
22	3	-0.35761	0.04433	0.00523	0.00355
23	4	-0.01356	0.06263	0.00214	0.00503
24	10	0.09148	0.09016	-0.00537	0.00787
25	1	-0.01708	0.03142	0.00175	0.00252
26	6	0.03140	0.07650	-0.00043	0.00614
27	1	-0.04766	0.03132	0.00421	0.00251
28	1	0.03136	0.03141	-0.00155	0.00252
29	1	-0.00647	0.03144	0.00074	0.00252
30	1	-0.01155	0.03143	0.00117	0.00253
31	3	0.03066	0.04440	-0.00135	0.00356
32	1	0.02307	0.03141	-0.00203	0.00251
33	3	-0.03623	0.04438	0.00154	0.00356
34	1	0.03111	0.03143	-0.00339	0.00252
35	6	0.02053	0.07642	-0.00243	0.00613
36	6	0.02139	0.08111	-0.00013	0.00611
37	11	0.05753	0.10326	-0.00342	0.00825
38	3	-0.00333	0.04343	0.00083	0.00356
39	1	0.04347	0.03137	0.00178	0.00252
40	4	-0.10704	0.06252	0.00961	0.00500
41	3	0.00976	0.04440	0.00123	0.00356
42	3	-0.04341	0.05420	0.00414	0.00425
43	1	0.03136	0.03142	-0.00167	0.00253
44	1	-0.00366	0.03144	0.00031	0.00252
45	2	0.05900	0.04431	-0.00132	0.00355
46	10	0.26161	0.09746	-0.01070	0.00782
47	3	0.05527	0.04420	-0.00668	0.00355
48	3	-0.00091	0.05431	0.00072	0.00436
49	1	0.01108	0.03144	-0.00076	0.00252
50	3	-0.01779	0.05429	0.00224	0.00436
51	3	0.03119	0.04440	-0.00144	0.00356
52	3	-0.01795	0.04439	0.00133	0.00356
53	13	0.09808	0.11131	0.01070	0.00873
54	3	-0.03873	0.05504	0.00460	0.00706

BDR	N.BIDS	ALPHA	SE	BETA	SE
55	53	-0.48342	0.17036	0.04263	0.01366
56	32	0.03320	0.10730	0.00073	0.00841
57	3	0.01482	0.05431	-0.00055	0.00436
58	2	0.06497	0.04430	-0.00479	0.00355
59	3	-0.02305	0.05429	0.00250	0.00435
60	14	0.11150	0.11581	-0.00397	0.00927
61	1	0.03233	0.03140	-0.00259	0.00252
62	10	-0.07811	0.09808	0.00811	0.00787
63	1	0.00977	0.03144	-0.00057	0.00252
64	1	-0.05072	0.03132	0.00430	0.00251
65	1	0.01118	0.03134	-0.00068	0.00252
66	3	0.07172	0.05421	-0.00813	0.00415
67	4	0.02544	0.06257	-0.00140	0.00502
68	1	-0.01193	0.03143	0.00113	0.00252
69	2	-0.04441	0.04433	0.00401	0.00356
70	1	0.00218	0.03141	-0.00020	0.00251
71	2	-0.03136	0.04437	0.00276	0.00356
72	1	-0.00005	0.03144	0.00011	0.00252
73	4	0.03244	0.06262	-0.00175	0.00502
74	3	0.03239	0.05430	-0.00196	0.00436
75	5	0.04971	0.06990	-0.00373	0.00561
76	11	-0.01757	0.10726	0.00417	0.00860
77	6	0.02109	0.07650	-0.00140	0.00614
78	5	-0.04234	0.06927	0.00450	0.00560
79	10	0.00178	0.09821	0.00201	0.00768
80	1	-0.00266	0.03144	0.00043	0.00252
81	2	0.00751	0.04441	-0.00020	0.00356
82	5	-0.02118	0.06991	0.00275	0.00561
83	3	-0.05530	0.05422	0.00477	0.00435
84	9	0.27903	0.09236	-0.02033	0.00741
85	1	-0.02346	0.03140	0.00251	0.00252
86	10	0.10577	0.09814	-0.00811	0.00787
87	1	-0.02111	0.03141	0.00214	0.00252
88	4	0.01320	0.06243	-0.00020	0.00501
89	3	-0.05236	0.05432	0.00426	0.00435
90	3	0.09333	0.05411	-0.00727	0.00434
91	2	0.03310	0.04440	-0.00135	0.00356
92	3	0.18853	0.05319	-0.01455	0.00427
93	4	-0.01300	0.06262	0.00193	0.00502
94	11	-0.11704	0.10235	0.01260	0.00813
95	7	-0.02534	0.08345	0.00379	0.00662
96	1	0.03103	0.03140	-0.00245	0.00252
97	3	-0.02710	0.04114	0.00114	0.00254
98	1	0.17634	0.03141	-0.01130	0.00251
99	14	-0.45336	0.11194	0.03770	0.00926
100	1	0.01733	0.03143	-0.00113	0.00251
101	1	0.03775	0.03139	-0.00252	0.00252
102	11	0.13004	0.10713	-0.00783	0.00860
103	3	0.01081	0.05431	-0.00033	0.00436
104	11	0.11109	0.10174	-0.02256	0.00816
105	3	-0.00775	0.05431	0.00144	0.00436
106	17	0.27876	0.12363	-0.02382	0.01008
107	1	-0.00752	0.03144	0.00065	0.00252
108	2	-0.02809	0.04427	0.00269	0.00356
109	1	0.00306	0.03144	0.00003	0.00252

BDR	N.BINS	ALPHA	SL	BETA	SE
110	2	0.03047	0.04439	-0.00202	0.00356
111	2	0.00675	0.04441	-0.00011	0.00356
112	9	0.05498	0.09329	-0.00248	0.00748
113	4	-0.05562	0.06255	0.00373	0.00302
114	2	0.02789	0.04439	-0.00197	0.00356
115	32	0.24043	0.17010	-0.01245	0.01364
116	8	0.11339	0.08796	-0.00740	0.00706
117	12	-0.32263	0.10776	0.02078	0.00682
118	26	0.01300	0.15482	0.00457	0.01242
119	4	0.00603	0.06262	0.00022	0.00302
120	2	0.02194	0.04440	-0.00133	0.00356
121	1	0.01307	0.03144	-0.00084	0.00232
122	12	-0.07327	0.10715	0.00865	0.00839
123	1	0.05635	0.03132	-0.00432	0.00231
124	11	-0.33212	0.10197	0.02106	0.00818
125	6	-0.05436	0.07641	0.00867	0.00613
126	1	0.03728	0.03139	-0.00279	0.00252
127	1	0.03345	0.03140	-0.00248	0.00232
128	1	0.01396	0.03144	-0.00091	0.00252
129	3	-0.14970	0.05369	0.01572	0.00431
130	1	0.03868	0.03179	-0.00290	0.00252
131	3	0.08726	0.05416	-0.00633	0.00434
132	6	-0.17101	0.07588	0.01506	0.00609
133	2	-0.00122	0.04441	0.00033	0.00356
134	21	-0.05573	0.14004	0.00703	0.01123
135	12	0.01551	0.10729	0.00134	0.00861
136	1	-0.00014	0.03144	0.00023	0.00252
137	12	-0.33279	0.10627	0.02394	0.00252
138	1	-0.01657	0.03143	0.00157	0.00252
139	8	0.11210	0.05797	-0.00730	0.00706
140	2	0.01337	0.04441	-0.00064	0.00356
141	6	0.11904	0.07631	-0.00329	0.00612
142	2	-0.02413	0.04438	0.00237	0.00356
143	1	0.01331	0.03144	-0.00056	0.00232
144	5	-0.00397	0.06792	0.00132	0.00561
145	9	0.09315	0.09323	-0.00396	0.00748
146	1	0.05115	0.03134	-0.00390	0.00251
147	1	0.07509	0.03141	-0.00103	0.00232
148	6	-0.01626	0.07648	0.00260	0.00613
149	1	0.00414	0.03144	-0.00012	0.00232
150	12	-0.51048	0.14015	0.04385	0.01124
151	20	-0.04441	0.13503	0.00707	0.01098
152	34	-1.21123	0.16133	0.10485	0.01274
153	2	-0.01314	0.04440	0.00149	0.00356
154	27	-0.47463	0.15497	0.04404	0.01243
155	9	-0.05236	0.09323	0.00616	0.00748
156	6	0.03676	0.07649	-0.00166	0.00614
157	12	0.15553	0.10711	-0.00773	0.00539
158	1	0.03728	0.03139	-0.00279	0.00252
159	16	0.20601	0.11292	-0.01213	0.00936
160	1	-0.02203	0.03143	0.00199	0.00252
161	1	-0.02555	0.03141	0.00228	0.00232
162	5	-0.17082	0.06937	0.01483	0.00536
163	28	0.10647	0.16022	-0.00232	0.01203
164	19	-0.32972	0.16144	0.03382	0.01295

BDR	N.BIDS	ALPHA	SE	BETA	SE
165	1	-0.01687	0.03143	0.00157	0.00252
166	1	0.08340	0.04421	-0.00644	0.00355
167	1	-0.04352	0.03134	0.00190	0.00251
168	5	0.15519	0.06934	-0.01142	0.00558
169	6	-0.25791	0.07516	0.02206	0.00603
170	8	-0.19083	0.08740	0.01709	0.00701
171	1	0.00785	0.03144	-0.00042	0.00252
172	1	0.05159	0.03133	-0.00410	0.00251
173	56	0.24485	0.17940	-0.01174	0.01439
174	5	-0.00618	0.06972	0.00158	0.00561
175	51	-0.26255	0.20784	0.03263	0.01667
176	7	-0.00817	0.08230	0.00217	0.00662
177	1	0.05245	0.03132	-0.00135	0.00251
178	1	0.01307	0.03144	-0.00034	0.00252
179	3	-0.09725	0.05403	0.00064	0.00433
180	7	0.20818	0.08172	-0.01522	0.00657
181	2	-0.01265	0.04440	0.00145	0.00356
182	2	-0.01616	0.04439	0.00172	0.00356
183	11	-0.04472	0.10280	0.00214	0.00825
184	12	0.08382	0.10725	-0.00456	0.00860
185	10	0.17252	0.09775	-0.01173	0.00785
186	17	-0.21425	0.13210	0.02092	0.01061
187	15	-0.27719	0.11830	0.02571	0.00949
188	6	0.15129	0.07617	-0.01087	0.00611
189	1	0.05239	0.03131	0.00148	0.00251
190	3	0.00229	0.05431	0.00046	0.00436
191	2	-0.11782	0.04374	0.00772	0.00352
192	1	0.01260	0.03144	-0.00080	0.00252
193	17	-0.14632	0.13245	0.01545	0.01014
194	9	0.10720	0.09222	-0.00257	0.00740
195	6	-0.12450	0.07615	0.01130	0.00611
196	1	-0.05531	0.03134	0.00151	0.00251
197	3	0.09659	0.05412	-0.00713	0.00434
198	5	-0.02978	0.06989	0.00348	0.00561
199	2	0.02710	0.04437	-0.00173	0.00356
200	2	0.04105	0.04435	-0.00344	0.00356
201	4	0.00159	0.06245	0.00743	0.00501
202	1	-0.04966	0.03135	0.00421	0.00251
203	1	-0.04523	0.03134	0.00390	0.00251
204	1	0.02712	0.03142	-0.00197	0.00252
205	1	-0.00366	0.03144	0.00051	0.00252
206	1	-0.02359	0.03141	0.00213	0.00252
207	2	-0.04170	0.04434	0.00394	0.00356
208	1	0.04242	0.03137	-0.00328	0.00252
209	1	0.03210	0.03141	-0.00257	0.00252
210	1	-0.01183	0.03143	0.00117	0.00252
211	1	0.01307	0.03144	0.00094	0.00252
212	4	-0.01284	0.06263	0.00170	0.00502
213	2	-0.03134	0.04437	0.00255	0.00356
214	6	0.00374	0.07650	0.00132	0.00614
215	6	0.00254	0.07650	0.00097	0.00614
216	5	0.05318	0.06923	-0.00401	0.00555
217	14	0.03903	0.11557	-0.00013	0.00927
218	1	-0.02222	0.03141	0.00109	0.00252
219	8	0.00756	0.08809	0.00109	0.00707

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BDR  N.BIDS   ALPHA        SE        BETA        SE


220     1   -0.02292   0.03141    0.00206   0.00252
221    29   -0.31784   0.16152    0.03185   0.01276
222     4    0.00000   0.06263    0.00086   0.00502
223     5   -0.06121   0.06987    0.00601   0.00560
224     7    0.05478   0.05244   -0.00331   0.00661
225     4    0.10046   0.06246   -0.00722   0.00501
226     1   -0.01630   0.03143    0.00153   0.00252
227     2    0.00326   0.04441    0.00017   0.00356
228     7   -0.12442   0.08218    0.01153   0.00659
229     1   -0.02411   0.03141    0.00216   0.00252
230     4   -0.15135   0.06206    0.01304   0.00498
231     1   -0.03366   0.03144    0.00351   0.00252
232     1   -0.02731   0.03140    0.00241   0.00252
233     5    0.02644   0.06993   -0.00103   0.00561
234     2    0.05765   0.04433   -0.00437   0.00355
235    15   -0.07936   0.11930    0.00963   0.00937
236     7   -0.01379   0.08250    0.00280   0.00662
237     6   -0.15620   0.07597    0.01390   0.00609
238     1   -0.02875   0.03128    0.00495   0.00251
239     2   -0.00140   0.04441    0.00034   0.00356
240     2    0.01357   0.04410    0.00109   0.00356
241     5   -0.05371   0.08801    0.00603   0.00706
242     1    0.00334   0.03144    0.00064   0.00252
243     2    0.06569   0.04430   -0.00436   0.00355
244     2    0.01845   0.04410   -0.00105   0.00356
245     2    0.02704   0.04439   -0.00175   0.00356
246     4    0.04667   0.06260   -0.00289   0.00502
247    14   -0.29067   0.11430    0.02643   0.00917
248     1    0.00141   0.03144   -0.00014   0.00252
249     2    0.06643   0.04427   -0.00472   0.00355
250     3    0.10934   0.05406   -0.00815   0.00434
251     2   -0.06527   0.04425    0.00569   0.00355
252    15   -0.00273   0.11944    0.00346   0.00958
253     1    0.03004   0.03141   -0.00230   0.00252
254     9   -0.08837   0.09113    0.00906   0.00747
255     2    0.03428   0.04435   -0.00233   0.00356
256     9   -0.29605   0.09180    0.02576   0.00736
257     1    0.02471   0.03142   -0.00177   0.00252
258     2    0.01614   0.04440   -0.00097   0.00356
259     3    0.00329   0.05431    0.00021   0.00436
260     6    0.07979   0.07643   -0.00513   0.00613
261     1   -0.03657   0.03133    0.00316   0.00252
262    10   -0.12756   0.09789    0.01243   0.00785
263     5    0.05988   0.06989   -0.00674   0.00561
264     3   -0.11038   0.05396    0.00953   0.00433
265     1   -0.00417   0.03144    0.00035   0.00252
266     9   -0.19461   0.09261    0.01761   0.00743
267     4    0.16162   0.06214   -0.01215   0.00498
268    57    0.52035   0.21601   -0.05374   0.01733
269     2    0.07829   0.04434   -0.00587   0.00355
270     4   -0.02250   0.06261    0.00276   0.00502
271     1   -0.01687   0.03143    0.00157   0.00252
272     6   -0.01405   0.07648    0.00242   0.00613
273     1    0.00414   0.03144   -0.00012   0.00252
275     3    0.01651   0.05431   -0.00084   0.00436
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BDR	N.BIDS	ALPHA	SE	BETA	SE
276	25	0.02006	0.15204	0.00378	0.01220
277	2	-0.01534	0.04439	0.00172	0.00256
278	2	-0.00520	0.04440	0.00085	0.00256
279	6	-0.03561	0.07645	0.00424	0.00613
280	13	-0.47569	0.10055	0.04074	0.00809
281	10	0.13463	0.09806	-0.00868	0.00787
282	3	0.00534	0.05431	0.00036	0.00436
283	1	-0.00601	0.03144	0.00070	0.00252
284	7	0.16083	0.08218	-0.01144	0.00659
285	1	-0.01515	0.03143	0.00144	0.00252
286	4	0.00573	0.06263	0.00040	0.00502
287	2	-0.05550	0.04420	0.00474	0.00355
288	1	0.03744	0.03138	-0.00276	0.00252
290	4	-0.03755	0.06255	0.00550	0.00502
291	26	-0.12259	0.15152	0.01546	0.01229
292	33	-0.16597	0.14579	0.01531	0.01149
293	7	-0.12008	0.08220	0.01118	0.00659
294	30	-0.52256	0.16235	0.04837	0.01302
295	2	0.07582	0.04425	-0.00567	0.00355
296	2	0.00524	0.04441	-0.00002	0.00356
297	13	0.58967	0.10467	0.05078	0.00876
298	1	-0.00159	0.03141	0.00045	0.00252
299	2	0.04284	0.04435	-0.00350	0.00356
300	3	0.00134	0.05509	0.00155	0.00707
301	6	0.06079	0.07443	-0.00521	0.00613
302	1	-0.01757	0.03143	0.00122	0.00252
303	17	-0.59556	0.12255	0.05160	0.00981
304	16	-0.20156	0.12755	0.01964	0.00953
305	1	0.03902	0.03137	0.00236	0.00252
306	6	-0.02170	0.07647	0.00220	0.00613
307	6	0.06943	0.07643	0.00429	0.00613
308	7	-0.14134	0.08206	0.01557	0.00655
309	6	0.25536	0.07547	-0.01910	0.00626
310	11	-0.09740	0.10265	0.01078	0.00833
311	12	0.17206	0.10705	-0.01136	0.00859
312	3	0.07727	0.05430	-0.00536	0.00436
313	11	-0.06567	0.10275	0.00790	0.00824
314	2	-0.02776	0.04424	0.00237	0.00355
315	1	-0.04231	0.03136	0.00362	0.00252
316	3	-0.01136	0.05430	0.00160	0.00436
317	1	0.02584	0.03142	-0.00156	0.00252
318	1	-0.01053	0.03143	0.00107	0.00252
319	2	0.00270	0.04440	0.00079	0.00356
320	1	-0.01035	0.03144	0.00107	0.00252
321	2	0.00340	0.04441	-0.00026	0.00356
322	3	-0.03855	0.05431	0.00127	0.00436
323	2	0.00375	0.04431	0.00027	0.00356
324	3	0.04611	0.05429	-0.00239	0.00435
325	1	0.02367	0.03142	-0.00185	0.00252
326	1	-0.01118	0.03143	0.00136	0.00252
327	1	0.00072	0.03144	0.00016	0.00252
328	1	-0.01706	0.03142	0.00135	0.00252
329	1	-0.00100	0.03144	0.00030	0.00252
330	1	-0.00101	0.03144	0.00050	0.00252
331	2	0.01324	0.04440	-0.00080	0.00356

BDR	N.BIDS	ALPHA	SE	BETA	SE
332	3	-0.03773	0.05421	0.00330	0.00435
333	1	-0.03546	0.03139	0.00307	0.00232
334	1	0.00506	0.03144	-0.00019	0.00232
335	1	-0.04509	0.03134	0.00333	0.00231
336	1	-0.01664	0.03143	0.00156	0.00232
337	1	-0.01664	0.03143	0.00156	0.00232
338	1	-0.01664	0.03143	-0.00156	0.00232
339	1	-0.04204	0.03136	0.00360	0.00232
340	1	-0.04204	0.03136	0.00360	0.00232
341	1	-0.04204	0.03136	0.00360	0.00232
342	1	0.00075	0.03144	0.00016	0.00232
343	1	0.00075	0.03144	0.00016	0.00232
344	1	0.00075	0.03144	0.00016	0.00232
345	1	-0.00615	0.03144	0.00071	0.00232
346	1	-0.02277	0.03141	0.00205	0.00232
347	1	0.01594	0.03144	-0.00107	0.00232
348	1	-0.02045	0.03142	0.00186	0.00232
349	1	-0.02045	0.03142	0.00186	0.00232
350	1	0.00437	0.03144	-0.00014	0.00232
351	1	0.00437	0.03144	-0.00014	0.00232
352	1	0.02607	0.03142	-0.00188	0.00232
353	1	-0.00243	0.03144	0.00041	0.00232
354	1	-0.03649	0.03140	0.00253	0.00232
355	1	0.00701	0.03144	-0.00035	0.00232
356	1	0.00701	0.03144	-0.00035	0.00232
357	1	-0.00368	0.03144	0.00031	0.00232
358	1	0.01914	0.03143	-0.00132	0.00232
359	1	0.01914	0.03143	-0.00132	0.00232