APPLICATIONS OF DELAY TIME THEORY TO MAINTENANCE PRACTICE OF COMPLEX PLANT

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ABSTRACT

This thesis is concerned with investigating and understanding the role and consequence of different modelling options and parameter estimation options for modelling a complex plant. As systems become more complicated and required new technologies and methodologies, more sophisticated maintenance models and control policies are need to solve the maintenance problems. The initial chapter introduces the review of previous work on a single component system and multi-component system. Although in recent years there has been a shift in the maintenance literature from consideration of single items to systems composed of several components, so far only a few papers have tackled the modelling of actual multi-component plant. In the third chapter, delay time concept and analysis technique have been presented. Of particularly importance are parameter estimation methods, namely the objective method and the subjective methods. In the fourth chapter the component PM model and the system PM model for downtimes and costs based upon various PM policies are discussed. The key options within maintenance modelling are to determine regular PM/inspection periods for the system modelled as a whole, and to determine the periods for the plant as a set of separate component models. An extension to the downtime model is presented for the case when the downtime due to failures within system is not small, and impacts upon the estimate of the number of failures arising over a specified time zone. In the following chapter, we address parameter estimation methods using simulated data, and assess the ability of estimation techniques to capture the true parameter values. Particular attention is paid to the problem arising during the parameter estimating process because of the inadequate recording of PM data and implied correlation between model parameters. Finally, a case study is presented of maintenance modelling of production plant in a local company with view to improving current practice. The model developed is based upon the delay time concept where because of an absence of PM data, using the results of earlier chapters, the process parameters and the delay time distribution were estimated from failure data only using the method of maximum likelihood. The modelling was repeated based upon subjective assessments of parameter, and considerable consistency with the objectively based case obtained. For the plant study, modelling indicated the current PM inspection program was ineffective. A snap-shot approach is then applied to assess other ways of reducing the downtime, and the possibility of improving the PM inspection practice. This leads to readily adapted improvements.

Chapter 1

Introduction

1.1 The Optimisation of Maintenance

During the last decade the importance of the maintenance function to technical systems has been increasingly realised. Until three or four decades ago, maintenance was simply regarded as an unavoidable and difficult to control part of production. As systems become more complicated, automated and required new technologies and methodologies, more sophisticated maintenance models and control policies are needed to solve the maintenance problems. This thesis is concerned with investigating and understanding the role and consequence of different modelling options for modelling a complex plant.

One of the aims of research in maintenance management is to provide decision-making tools for the maintenance manager. Operations Research / Management Science techniques are among the tools which can help maintenance decision making. They allow subjective decisions to be replaced by objective decisions, taking into account accurately formulated objective functions and a complex set of constraints. OR / MS techniques have long been used and appreciated in areas like production and inventory management (Pintelon and Gelders, 1992).

Preventive maintenance optimisation models are mathematical models which aim to balance quantitatively the costs and benefits of preventive maintenance in order to determine optimum policies. The models can deal with many aspects of the maintenance process, such as determining the right type of maintenance, the optimum frequency of execution, the best way of planning and scheduling, the best combination of maintenance activities across plants, and the optimum design of maintenance facilities. Thus in order to fulfil the maintenance objective, the industrial organization needs management skills to integrate people, policies, equipment and practice. It also needs adequate engineering and technological skills in order to provide the best possible preventive maintenance, repair and overhaul for increasingly automatic production equipment.

There are many ways in which maintenance optimisation models can be applied in maintenance areas. One of the main problems associated with maintenance is the determination of inspection frequencies. Inspection is one of the key functions within PM. The basic purpose of an inspection is to reveal the true state or condition of a system. As a result of an inspection, a repair or replacement action may be performed to prevent further deterioration or failures of the system. Thus the development of models for inspection decisions is mainly centred around determining the optimal inspection interval which optimises the criteria of interest such as total expected cost or downtime per unit time.

Of the first maintenance models which appeared in the sixties, many dealt with the problem of finding optimal inspection policies for systems which are subject to failures. In the beginning, the maintenance models were relatively simple in that they considered a single component only, McCall (1965), Pierskalla and Voelker (1976), Sherif and Smith (1981), and Valdez-flores and Feldman (1989). However, in recent years there has been a shift in the reliability and maintenance literature from the consideration of single units to modelling systems composed of several units, see Thomas (1986) and Cho and Parlar (1991). Some maintenance models are concerned with optimal maintenance policies for a system consisting of several units within a machine, or many pieces of equipment, which may or may not depend on each other in terms of economic or stochastic dependency. Stochastic dependence implies that each component's condition probability depends on the other components'. Economic dependence means that set-up costs can possibly be saved when several components are jointly maintained instead of separately, and vice versa. Since combining such forms of dependence makes the models very complicated, most maintenance models consider only one of these dependencies.

In practice, problems arise because there may be many relationships which exist between components to be maintained. Modelling these relations directly and non-selectively yields large models, which are difficult to analyse as they suffer from the curse of dimensionality. Dekker (1995) recommends a decomposition approach for this problem. In such an approach one applies simple models for individual components and uses the outcomes as input in a comprehensive model. Other problems are encountered in the implementation of maintenance policies for individual components. It can be profitable to combine maintenance activities, thereby saving common preparation work. Therefore, for modelling a complex plant, the system modelled as a whole and as a set of component models will be contrasted.

In maintenance modelling, most of maintenance models assume the availability of data. If the maintenance records of failures and recorded findings at maintenance interventions, such as inspections, are available and sufficient in quantity and quality, the delay time distribution can be estimated by the statistical method, called objective method, see Baker and Wang (1992, 1993), Christer and Wang (1995), and Christer *et al* (1995). However enough suitable and correct data is not always present. Since in delay time modelling it is essential to obtain the estimates of the delay time and initial point distribution, to overcome the problem when there is a lack of data, a method for estimating delay time parameters called subjective estimation, suggested by Christer (1982), has been developed using opinions of experts for estimation of the delay time parameters. For obtaining an estimate of delay time $f(h)$, we use a revised parameter estimation method (see Wang, 1997) which is based upon the analysis of historic data of failures and the delay time concept where the distribution of the delay time was estimated from the subjective data obtained from the expert.

Most production systems have numerous failure types or modes. Therefore, when one is optimising all maintenance for a given system, there is a natural tendency to consider every failure type or mode. Here we use the data which recorded by machine operator as a failure mode. The distribution of the delay time has been estimated from the subjective data for each failure mode.

1.2 Overview of the Thesis

In Chapter 2 we review the existing literature on maintenance models relating to the developments and applications of modelling. We distinguish between a single component maintenance model and a multi-component maintenance model. These models deal with the problem of finding optimal inspection policies for systems which are subject to failures. Maintenance models for single components can be useful for modelling the maintenance of individual components that are part of more complex systems. Due partly to improvements in analytic techniques, the work on policies for maintenance and replacement of deteriorating components has recently been extended to consider systems comprised of several components which are dependent on or independent of one another.

In Chapter 3 the concept and developments of the delay time modelling are presented. A technique called delay time analysis has been initially developed for modelling inspection policies for industrial inspection maintenance when the equipment is regularly inspected. In maintenance modelling the successful use of the delay time concept depends upon how well the underlying delay time distribution can be estimated from available information sources. One of the key issues in the delay time modelling is the estimation of the delay time parameters which are usually the rate of occurrence of defects, the distribution of underlying delay time h of a defect, and the probability of identifying and removing a defect at PM. Two basic approaches to solve the associated estimation problems, namely subjective and objective methods, have been presented using the information obtainable from maintenance engineers who repair the machine.

Chapter 4 describes an investigation of PM modelling which is concern with the downtime and cost aspects of various maintenance policies. The component PM model and system PM model are presented in the case where downtime due to failure is relatively small. The value of the approach is that it looks at the maintenance schedule problem in its entirety, with account taken of the specification of equipment, inspection plan and the nature of the plant. By developing two models, that is the system model and a collection of sub-systems models, the effectiveness of maintenance scheduling for a system can be analysed. We revise the downtime model to embrace the case when the downtime due to failures of system is not very small. This can change the failure process over the PM period $(0, T)$. The actual operating time over the calendar time $(0, T)$ of the system is obtained, and the downtime models are then extended to be based upon the actual operating time.

The parameter estimation options with and without PM information are presented in Chapter 5. To investigate and verify parameter estimation methods a simulation study is undertaken. We test parameter estimation methods in terms of the ability to recapture known parameters, and use simulated data to check the consequences of different volumes and types of data upon the accuracy of parameter estimates for maintenance models. The given maintenance record data includes the failures times, or number of failures per day, and the number of defects identified at PM. The importance and value of having data on PM activities and inspection results is highlighted.

In Chapter 6 we present two modelling studies of preventive maintenance (PM) policy of production plant in a local company with a view to improving current practice. An objective data based model is developed based upon the delay time concept where, because of an absence of PM data, the process parameters and the delay time distribution were estimated from failure data only using the maximum likelihood. Particular attention is paid to the problem arising during the parameter estimating process because of the inadequate recording of PM data and the implied correlation between model parameters. The case of data deficiency explored in the study is important because it is a relatively general situation in practice. A subjective data based method carried out at the same company and the same plant parallels the objective data based study. The two studies of the same problem provide a rare opportunity to compare the model formats and parameter values resulting from the two approaches and to consider the degree of consistency between the subsequent decision consequences of the two methods. The consistency is reassuring. In addition, to reduce the further downtime in this case study, a snap-shot type of surveys technique is undertaken and presented.

We conclude the thesis in Chapter 7 with discussions and final remarks for further research.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Opportunities for the application of Operational Research modelling to problems of plant maintenance management are numerous. Whenever a management decision arises, the potential for OR modelling exists. As systems become more complicated and required new technologies and methodologies, more sophisticated maintenance models and control policies are needed to solve the maintenance problems. Thus, it is worth noting that one expects attention given to the area of maintenance modelling, in general, and industrial maintenance modelling in particular, to grow over the next decade.

Over the last few decades, numerous papers have appeared in the literature which deal with the problem of finding optimal inspection policies for systems which are subject to failures. This phenomenon is indicated in various surveys of maintenance models by McCall (1965), Pierskalla and Voelker (1976), Sherif and Smith (1981), Sherif (1982), Thomas (1986), Kralj and Petrovic (1988), Valdez-flores and Feldman (1989), Cho and Parlar (1991), Thomas *at al.*(1991) and Scarf (1997). Valdez-Flores and Feldman (1989) used a classification scheme that catagorizes single-unit maintenance models into four topical categories: inspection models, minimal repair models, shock models and other replacement models.

Although a system may consists of several components, it is sometimes practical to consider the system as a single unit that behaves in such a way that individual components do not directly affect the reliability of the system. Another important reason to consider single unit systems is because in practice there are many instances in which it is difficult to obtain reliability data for smaller components; whereas data for the stochastic behaviour of the entire system is available or easier to obtain. Cho and Parlar (1991) also used classification scheme used by Barlow and Proschan (1965) to some extent as well as by MaCall (1965). They surveyed and categorized multi-unit maintenance models into the five topical categories: machine interference / repair models, group / block / canibalization / opportunistic maintenance models, inventory and maintenance models, other maintenance and replacement models, and inspection / maintenance (preparedness maintenance) models. Multi-unit maintenance models are concerned with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other (i.e., economic / stochastic dependency). Stochastic dependence means that each component's condition probability depends on the other components', whereas economic dependence implies an opportunity for a group replacement of several components provided that a joint replacement of several components costs less than separate replacements of the individual components (Sethi, 1977). If all units in the system are economically and stochastically independent of one another, a maintenance policy for the single unit models may be applied to the multi-unit maintenance problems. On the other hand, if any units in the system are economically or stochastically dependent upon each other, then an optimal decision on the repair or replacement of one unit is not necessarily optimal for the whole system. A decision must be made to improve the whole system, rather than any subsystem. Thus, in recent years there has been a shift in the reliability and maintenance literature from consideration of single units to systems composed of several units (Thomas, 1986).

In this Chapter, single component and multi-component systems maintenance models for inspection will be presented.

2.2 Single Component Maintenance Models

2.2.1 Basic Maintenance Model

Maintenance can be defined as the combination of all technical and associated administrative actions intended to retain an item or system in, or restore it to, a state in which it can perform its required function (British Standards BS 3811, 1984). Most plant including machinery, electronic system, components, vehicles, and buildings wear out and fail. The time at which equipment fails however is not known in advance, and it is likely to fail when in operation. These operating failures can be quite expensive not only in repairing or replacing the item, but also because of the disruption and delay to the operation of the system. Thus such items are often subject to a replacement, maintenance, or inspection policy. Preventive maintenance was advocated as a means to reduce failures, unplanned downtime and even operating cost.

Maintenance involves planned and unplanned actions carried out to retain a system in or restore it to an acceptable condition. Maintenance models usually assume that the condition of the system is completely unknown unless an inspection is performed. Every inspection is normally assumed to be perfect in the sense that it reveals the true condition of the system without error. In the absence of repairs or replacement actions, the system evolves as a stochastic process. Optimal maintenance policies aim to minimize the total expected cost or downtime per unit time for the most effective use of systems.

Since the 1965 survey on maintenance by McCall (1965), different authors have produced many interesting and significant results for variations of inspection models. The different models developed depend on the assumptions made regarding the time horizon, the amount of information available, the nature of the cost functions, the objective of the models, the system's constraints, etc. However, many inspection models follow the premise of the basic model presented by Barlow and Hunter (1960) (see also Barlow *et al* (1963)). The model assumptions are generally as follows

(a) the state of the system is known only by inspection.
(b) inspections do not degrade the system and take negligible time.
(c) inspections are perfect in that any failure within the system will be identified.
(d) inspection ceases upon discovery of failure and repair or replacement takes place which returns the system to as good as new.
(e) the system cannot fail while being inspected.
(f) each inspection cost is c_1 per unit time.
(g) the cost of leaving an undetected failure is c_2 per unit time.
(h) the failure time distribution, $F(t)$ of system is known.

Hence the total cost per inspection cycle is given by

$$C(t,\mathrm{x}) = c_1 n + c_2(x_n - t), \tag{2.1}$$

where t is the time to failure, $\mathrm{x} = (x_1, x_2, \ldots)$ is the sequence of inspection times with $x_1 < x_2 < x_3 \ldots$, and n is the inspection which detects the failure occurring at time t, that is, $x_{n-1} < t < x_n$. The optimal inspection policy x^* is the one that minimizes the expected total costs due to inspection and the cost due to leaving the system in the failed state until it is detected, $E[C(t, \mathrm{x})]$, where

$$E[C(t,\mathrm{x})] = \sum_{n=0}^{\infty} \int_{x_n}^{x_{n+1}} [c_1(n+1) + c_2(x_{n+1} - t) f(t)]dt, \tag{2.2}$$

and satisfies the recurrence relation

$$x_{n+1} - x_n = \frac{F(x_n) - F(x_{n-1})}{f(x_n)} - \frac{c_1}{c_2}, \qquad n = 1, 2, \ldots, \tag{2.3}$$

where $f(\bullet)$ and $F(\bullet)$ are the failure time probability and cumulative distribution functions, respectively. When $f(x_n) = 0$, $x_{n+1} - x_n = \infty$ so that no more checks are scheduled. The sequence is determined recursively once x_1 is chosen.

A difficulty with the model is the computation of the optimal inspection procedures numerically, because the computations are repeated until the procedures are determined to the required degree by iterating upon the first inspection time, x_1. Munford and Shahani (1972) established a more convenient way than the basic model. They suggested that an inspection policy $\mathbf{x}_p$ which has a meaningful single parameter p. To decide inspection policy $\mathbf{x}_p$, they define the probability of a transition from state 0 to state 1 during the interval (x_{i-1}, x_i) given that the system was in state 0 at time x_{i-1}, which is given by

$$\frac{F(x_i)-F(x_{i-1})}{1-F(x_{i-1})}, \qquad \text{for } i = 1, 2, 3, \ldots, \tag{2.4}$$

where $0 < p < 1$, $x_0 = 0$ and $F(0) = 0$. And also defining an inspection policy X to be an $\mathbf{x}_p$ policy if for all i, and for a constant p in the interval (0, 1):

$$\frac{F(x_i)-F(x_{i-1})}{1-F(x_{i-1})} = p. \tag{2.5}$$

The above equation (2.5) may be rewritten as:

$$F(x_i) = p + (1-p)F(x_{i-1}), \tag{2.6}$$

and

$$\begin{aligned} F(x_i) &= 1-(1-p)^i \\ &= 1-q^i, \end{aligned} \tag{2.7}$$

where $q = 1 - p$ Thus for a given p, x_i can be found from

$$x_i = F^{-1}(1-q^i). \tag{2.8}$$

To choose an optimal p, let a random variable I denote the number of inspections necessary for the detection of state 1. We have

$$\Pr(I = i) = q^{i-1}p, \qquad \text{for } i = 1, 2, 3, \ldots, \tag{2.9}$$

so that

$$E(I) = \sum_{i=1}^{\infty} iq^{i-1}p = \frac{1}{p}. \tag{2.10}$$

If the transition occurs at time t and it is detected by an inspection at time x_i, then $(x_i - t)$ is the time for which the system was left in service in state 1. The mean time for which the system will be left in service in state 1 is

$$\begin{aligned} \tau &= \sum_{i=1}^{\infty} \int_{x_{i-1}}^{x_i} (x_i - t) f(t)dt \\ &= \sum_{i=1}^{\infty} x_i q^{i-1} p - E(T), \end{aligned} \tag{2.11}$$

where $E(T) = \int_0^{\infty} tf(t)dt.$

Clearely, τ is a function of p, that is

$$\tau = \tau(p). \tag{2.12}$$

So the total expected cost until a failure is detected is given by where $q = 1 - p$ Thus for a given p, x_i can be found from

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Clearely, τ is a function of p, that is

$$\tau = \tau(p). \tag{2.12}$$

So the total expected cost until a failure is detected is given by

$$E(C) = c_1 / p + c_2 (\sum_{i=1}^{\infty} x_i (1-p)^{i-1} p - E(T)), \tag{2.13}$$

where $x_i = F^{-1}(1-(1-p)^i)$.

The optimal p can be chosen such that $E(C)$ is minimized. In some cost comparisons in the particular case of the basic model, Munford and Shahani (1972) have indicated that it is reasonable to suppose that $\mathbf{x}_p$ is nearly optimal. Later, this inspection policy was used for the Weibull failure distribution case in Munford and Shahani (1973). Their optimal policy shows that if the system has an increasing failure rate, the between successive inspection times will decrease. Similarly it can be seen that if the failure rate function is decreasing, the inspection intervals form an increasing sequence. In another development, Keller (1974) introduced a continuos density $n(t)$ of the number of inspections per unit of time. He let $F(t)$ be the probability of failure occurring between the initial time 0 and the time t. Then $F'(t)$ is the probability of failing per unit time. The interval between tests is $n(t)^{-1}$, and the average loss due to elapsed time T between the occurrence of failure during this interval and its detection is $n(t)\int_0^{n(t)} L(T)dT$. When n is large, this average loss is $L[1/2n(t)] + O(n^{-2})$. When L is linear, the average loss is $L(1/2n)$ for any value of n. Thus if failure occurs at time t its average cost is

$$c\int_0^t n(\tau)d\tau + L[1/2n(t)]. \tag{2.14}$$

Then the expected cost E up to detection of the first failure is given by

$$E = \int_0^{\infty} \{c\int_0^t n(\tau)d\tau + L[1/2n(t)]\}F'(t)dt. \tag{2.15}$$

Therefore we can find that function $n(\mathrm{t})$ which minimizes E.

Nakagawa and Yasui (1979) asymptotically calculated the optimum checking time which minizes the expected total cost until detection of a failed unit for periodic inspection times. They also suggested an approximate calculation of optimal checking procedures which computes successive inspection times backwards in Nakagawa and Yasui (1980). This computation was reported to be much easier than that of Barlow and Proschan (1965) and approximation can be fairly good for a Weibull failure distribution.

Zuckerman (1980) considered a maintenance model in which the state of the system can be determined only by inspection. The decision variables include the inspection interval and the scheduling of preventive replacements. The problem is to specify a replacement rule which minimizes the long-run average cost per unit time. This model assumes that (a) the cost rate per unit time associated with the inspection process is a monotonically non-increasing function of the time interval between two successive inspections, (b) a failure is discovered only by inspection, (c) upon detection of failure, the system is replaced by a new identical one, (d) the costs incurred include cost of inspection, operating costs, failure cost and a cost associated with planned replacements, (e) inspections and replacements are instantaneous, (f) the state of space of damage process is an arbitrary state space and (g) an optimal policy is a control limit policy, provided some conditions are satisfied. He notes that the level of difficulty in expressing the objective function, $r(\bullet)$ explicitly depends heavily on the structure of the survival function of the system and the distribution function, $F(\bullet)$ of the magnitude of the shocks.

Luss (1983) generalizes the basic model of Barlow *et al* (1963). Inspection policy models considered are stochastically failing systems in which failures are detected by inspection only. He presents a dynamic programming algorithm that maximizes the expected profit between two successive repairs. He notes that this model is especially suitable if the cost of testing each product is high relative to its value, or if the test itself is destructive.

Kaio and Osaki (1984) have developed keller's method using the smooth density, called inspection density, and obtained the more analytically exact nearly optimum inspection policy. In 1987, they also numerically compared the optimum inspection policy by Barlow *et al* (1963, 1965) with nearly optimum ones by, Munford and Shahani (1972), Nakagawa and Yasui (1980) and Kaio and Osaki (1984), assuming a gamma lifetime distribution. From the result comparison, they conclude that there are no positive differences among the optimum and nearly optimum inspection policies. They also note that the inspection policy is one of the most important policies applicable in the practical systems.

In addition to the computational difficulty of the Barlow *et al.* basic model, and despite its simplicity, the model also seems to suffer practical drawbacks. First, it is difficult, in practice, to obtain the estimate of c_2 which is the penalty cost due to leaving the system in a failed state per unit time. In this respect, Munford (1990) later treated the value of c_2 as proportional to the duration of the inspection interval containing the failure. Second, since the model assumes that a failure at time t can only be detected at time x_n, where $x_{n-1} < t \leq x_n$, this implies that a failure can only be attended at the next inspection time. However, it is usually uncommon, in practice, to leave the failed system until the next inspection epoch.

2.2.2 Modified Inspection Model

In the maintenance of a system that deteriorates stochastically, there exists the problem of determining the sequence of actions, such as replacements, repairs, inspections, etc., that in some sense minimizes the total cost of operation. If the state of the system is always known, the optimum sequence of actions is often apparent or easily calculated. However, it may not be practical or possible to determine the system state exactly, that is the observables need not coincide with the states. Therefore, in addition to two-state (good and failed states) inspection models such as Barlow and Proschan (1965), many inspection models available in the literature adopt a Markovian approach where the working condition of a system is represented by several degradation states which usually denoted as $0, 1, ..., n, n + 1,$ where 0 represents the good state, 1, ..., n represent the degraded states, and $n + 1$ represent the failed state (Derman, 1963). An inspection reveals that the system is in one of the several degradation states. In this respect, Eckles (1968) also considers a system which is characterized by a discrete-parameter, non-stationary, finite-state Markov process. A maintenance policy is defined as a rule for choosing an action at each time, based on the information (observables) available at that time. Costs dependent on the action selected, the state of the system, and the age, are assigned to each possible transition. It is shown that the action that minimizes the discounted value of expected immediate and future costs is determined by the system's age and the posterior distribution over the states. He presents a technique for the calculation of optimum maintenance policies using a dynamic programming method.

Luss (1976) considers a model in which inspections may reveal, in addition to a malfunction, intermediate states of the system that represent varying degrees of deterioration. Maintenance policies, dependent on the system's state at inspection times, are determined to minimize the expected cost per time unit. The costs incurred include costs of inspections, state occupancy costs, costs of preventive repairs, and the costs for repairing a failed system either at an inspection event or immediately after the occurrence of a malfunction. He uses a Markovian model in which the holding times in the various states are exponentially distributed. A similar model is also presented by Sengupta (1981). However, he assumes that the replacement cost can be an increasing function of the degradation states and allows a delayed replacement action. He shows that the policy that minimizes the long-run expected cost per unit time calls for inspection and delayed replacement intervals that are decreasing in the degradation state. He also shows that the optimal solution is a control limit policy when replacements are made at inspection times.

Abdel-Hameed (1987) determines the optimal inspection policy of a system subject to deterioration. The deterioration is assumed to be an increasing pure jump Markov process. Examples of pure jump processes are: (1) compound Poisson processes with positive jumps, (2) gamma processes, (3) Pure-berth processes and (4) stable processes. He finds the optimal inspection period that minimizes the long-run expected cost per unit time.

Luss and Kander (1974), consider the case when the duration for inspection and repair is non-negligible, and the system can fail while being inspected. Costs introducted here are the cost of checking and of unknown lost time. Optimal inspection policies are calculated for three different objective functions: expected loss per cycle, per unit time and per unit of good time. The loss functions are obtained and solved by both differentiation, which leads to efficient algorithms for IFR (Increasing Failure Rate) distributions, and by dynamic programming, which can be used for any failure rate. The optimal policy is again a sequence of checking times minimizing the loss per life cycle or alternatively per time unit.

Variants to the Barlow *et al.* basic model include a model propose by Beichlet (1981) which considers a model for proper scheduling of inspection where system failures can be detected only by checking. Cases of replacement and no replacement of a failed system are analyzed. On condition that no or only partial information on the failure time distribution of a system (only the expected system's lifetime is assumed to be known) was available, minimax inspection strategies are obtained with respect to a cost criterion.

Inspection is also usually assumed to be perfect. This assumption may not be valid in some situations. Anderson and Friedman (1977, 1978) present a model which involves the imperfect inspection case. They determine the optimal inspection times by reducing the stochastic problem to a free boundary problem in analysis, which is then solved using iterative procedures. Another aspect to consider is that an inspection may also pose a hazard to the system to be checked. Wattanapanom and Shaw (1979) introduced a model in which the ith inspection increases the remaining failure rate without changing the form of the conditional lifetime distribution. The loss function to minimize in their model is given as

$$L = E[c_1 N + c_2 d], \tag{2.16}$$

where N is the number of inspections until the first one after the failure, and d is the time between the failure time and the subsequent detection time t_n. They also give algorithms for finding optimal inspection times when the conditional distributions are exponential or uniform. A similar concept is presented by Butler (1979), and Chou and Butler (1983) who studied hazardous inspection models for ageing systems. They found optimal policies that minimize the expected lifetime of the system under inspection. Their model assumed that each inspection either causes immediate failure or else increase the failure rate. The models are an extension to a model by Barlow and Hunter (1961) where the objective is to determine a sequence of inspection times that will minimize the mean cost of testing plus the mean cost of an undetected failure when inspections are perfect. Another aspect to be considered is that if the inspection contains some elements of preventive maintenance, it may add some benefit to the system to be inspected. Baker (1991) considers this problem and presents a model to test the effect of such preventive maintenance.

2.3 Multi-Component Maintenance Models

2.3.1 Simple Inspection Models

The work on policies for maintenance and replacement of deteriorating items (components) has recently been extended to consider systems comprised of several items (components) which are dependent on one another (Thomas, 1986). When defining a component, Redmond (1997) defines a component (item) as a device that can essentially fail in one failure mode when in operation. Examples are light bulbs, valves and fuses. Clearly, a component for one use may be a system for another depending upon the level of disaggrigation involved. A system is a deterministic entity comprising an interconnected or interacting collection of discrete components (Villemeur, 1992). The word 'deterministic' implies that the considered system can be identified, which is obviously necessary. It is noted that this definition indicates that the system is made of interacting components: it is assumed that it is not simply the sum of its subsystem or components. Moreover, if the physical nature of a subsystem or component is altered as a result of a failure, the system itself is modified. Thus, more precisely, a system with a failed component becomes a new system that may be entirely different from the previous one.

Since the general inspection model by Barlow and Proschan (1965), many papers have been written about various inspection policies relating to maintenance/replacement models for single component systems. Now we investigate the literature which considers systems consisting of several components, so called multi-components system, which may or may not depend on each other.

To this end, Butterworth (1972) presents two faults testing models for a k out of n system, which have different objectives. The fundamental assumptions are as follow :

(1) a system is composed of n component.
(2) systems considered are the k-out-of-n type, which works if and only if any k or more of its components work $n \geq k$.
(3) inspection is perfect and takes a time t_i.
(4) components can be individually inspected.
(5) components function or fail independently of each other.

The model is to find the state of the system with the minimum expected inspection time used. He gives the two cases of inspection model. The first model is to determine the state of each component if the system has failed, while the second has the objective of just determining the state of the system itself. He also separated the test procedures into sequential ones, which pre-specify the sequence of tests and non-sequential ones, where the results of earlier tests may decide the order of later ones. Obviously sequential tests are easier to implement. Butterworth showed that in order just to determine the state of the system the optimal policy is a sequential procedure, but if one wants to determine the state of all the units that make up the system the optimal procedure is non-sequential.

Anbar (1976) also considered a multi-component system consisting of N independent components which fail exponentially with an unknown parameter. He presents an adaptive sequential inspection policy which is asymptotically optimal. The components are working in parallel with identical failure distribution. The variables of the lifetime of components, T_i, are independent and identically distributed according to the exponential distribution with (unknown) parameter θ which is given by

$$P(T_i \le t) = F(t) = 1 - e^{-\theta t} \qquad \text{for } t > 0,\ \theta > 0. \tag{2.17}$$

When inspection takes place, all components are inspected and failing components are replaced by new ones. He assumed simultaneous inspections for all components and negligible inspection/replacement times. A failing component costs c_0 per component of idle time. Replacement of a failing component costs c_1 and inspection costs c_2 per component. If $N(t)$ is the number of failures which occur during that time, the expected cost per unit of time during that period is given by

$$c(t) = \frac{1}{t} E\left[c_0 \sum_{i=1}^{n} (t - T_i)^+ + c_1 N(t) + n c_2 \right], \tag{2.18}$$

where $(\bullet)^+ = \begin{cases} (\bullet) & \text{if } (\bullet) > 0 \\ 0 & \text{otherwise.} \end{cases}$

Thus

$$c(t) = \frac{n}{t}[c_0 E(t - T_i)^+ + c_1 F(t) + c_2], \tag{2.19}$$

where T is a random variable distributed exponentially with parameter θ and $F(t)$ is given by (2.18)

$$E(t - T) = \int_0^t (t - u) dF(u) = tF(t) - \int_0^t u dF(u)$$

$$= tF(t) - \theta \int_0^t u e^{-\theta u} du = t - \frac{F(t)}{\theta}.$$

Therefore

$$c(t) = \frac{n}{t}\left[c_0 (t - \frac{F(t)}{\theta}) + c_1 F(t) + c_2 \right]. \tag{2.20}$$

Since $c(t) \to \infty$ as $t \to 0$ and $c(t) \to nc_0$ as $t \to \infty$, there exists a value τ (possibly infinite) for which $c(t)$ is minimized. By differentiating equation (2.20), it can be seen that a necessary condition for τ to minimize $c(t)$ is that it is the solution of the equation

$$(1 + \theta\tau)e^{-\theta\tau} = 1 - \frac{c_2}{(c_0 / \theta) - c_1}. \tag{2.21}$$

The decision problem is to determine an optimal time interval τ^* between successive inspections which minimizes the expected cost per unit time. When the value of θ is unknown, the optimal value, τ cannot be computed. Anbar suggested an adaptive sequential approach where τ is estimated sequentially at the time of each inspection.

Kander (1978) also considered a model for a unit made up of N sub-units. This unit is capable of rendering its mission as long as one sub-unit has not failed. He assumed that at any given time, one sub-unit only is subject to possible random (Poisson) failure. After such failure has taken place, a further sub-unit is moved up, and carries out the function of the unit until it fails, too. This is repeated until the failure of the last sub-unit. Inspection discloses which particular sub-unit is carrying out the unit function. In this model, inspection reveals the number of failed units. Another case example is given in which a structure exhibits parallel redundancy. He presents an inspection model where stochastic failure is detected by inspections carried out intermittently. A multi-level quality system is described by a semi-markov process. Kander assumed that the system can move from N, the perfectly good state, only downwards to *N-1*, *N-2*,... until it reaches 0, the failed state. No ageing takes place during the stay in any state.

The reliability of a system is often the major criterion in developing models for determining the optimum maintenance policies. On the other hand, several cost factors have been incorporated to develop the optimum preventive maintenance policies. Okumoto and Elsayed (1983) present an optimum maintenance policy for a group of machines subject to stochastic failures where the repair cost and production loss due to the breakdown of machines are jointly minimized. They develop the simple nomograph to obtain the optimum maintenance schedule and the minimum cost per unit time. In a similar point of view, Assaf and Shanthikumar (1987) consider a group inspection policy for a set of N machines subjected to stochastic failures under continuous and periodic inspections. The times till failure are assumed as independent having an exponential distribution. A failed machine can be repaired at any time, and a repaired machine is considered as good as new. Costs incurred are the repair cost, which is composed of a fixed overhead cost per repair, C_0, and a repair cost per machine, C_1, and a penalty costs associated with production losses for n failing machines during a time interval of length h, that is nhC_2. It is also assumed in the model that the number of failed machines in the system is unknown unless an inspection is carried out. Upon an inspection, a decision must be made on whether to repair the failed machines or not, based on the number of failed machines in the system.

When multi-component system fail, there are two extra problems which do not occur in the single component case (Thomas, 1986). The first problem is the detection of units which have failed, that is to determine an inspection sequence to check the individual units in the system for faults. Secondly, if more than one unit has failed, then one must decide which unit must be repaired first in order to give the system the best possible characteristics.

An improper sequence of inspections performed in a multi-component system, could be costly due to unnecessary time and work involved. In this respect, Butler and Lieberman (1984) developed a program to implement the heuristic policy of testing the unit so that those with the highest failure probability conditioned on the fact that the system has failed are tested first. They argue that this is a good, if not optimal, procedure for detecting the units whose failure was critical. Salloum and Breuer (1984) gave a non-sequential, polynomial time, algorithm for determining the state of a system with symmetric units.

2.3.2 Imperfect Maintenance Models

One important research area in reliability engineering is the study of various maintenance policies in order to prevent the occurrence of system failure and improve system availability. Proper maintenance techniques have been emphasized in recent years due to increased safety and reliability requirements of systems, increased complexity, and rising costs of material and labor (Sherif and Smith, 1981). Various treatment methods and optimal policies assuming imperfect maintenance are discussed and summarized by Pham and Wang (1996). So far, we have mainly reviewed perfect maintenance models. These models include perfect repair or perfect maintenance which restores the system operating condition to as good as new. That is, upon perfect maintenance, a system has the same lifetime distribution and failure rate function as if brand new. In practice, the maintenance of a deteriorating system is often imperfect: the system after maintenance will not be as good as new.

In this respect, Shaked and Shanthikumar (1986) introduce the multivariate imperfect repair concept. They consider a system whose components have dependent lifetimes and are subject to imperfect repairs respectively until they are replaced. For each component the repair is imperfect according to the (p, q) rule, i.e., at failure the repair is perfect with probability p and minimal with probability q. Assuming that n components of the system start to function at the same time 0, and no more than one component can fail at a time, they establish the joint distribution of the times to next failure of the functioning components after a minimal repair or perfect repair. They also derive the joint density of the resulting lifetimes of the components and other probabilistic quantities of interest, from which the distribution of the lifetime of the system can be obtained. For a series system Zhao (1994) presents a series system availability model in which either minimal repair or perfect repair of all components can be modelled. He assumes that the repaired component might not be as good as new and its lifetime may follow any distribution which can be different after repair from that of the original, and obtains the limiting availability and mean system down and up time. In this model of a series system, repair time is not negligible and thus it is more practical.

In maintenance, one can often detect failures in a system by inspection, such as the failure of units in a storage. In a perfect inspection model, all failures can be detected at the time of inspection. In many practical situations, one can not detect all the failures upon inspection. The models treated so far in perfect inspection usually assume that (1) the state of the system is identified completely and (2), the state of the system can be observed only through costly inspections. Ohnishi, Kawai and Mine (1986) on the other hand, consider that the system is monitored incompletely by a certain mechanism which gives the decision maker some information about the exact state of the system. They describe a system having the following properties.

(1) The deterioration levels of the system are classified into a finite number of states 0, 1, ..., N. The state numbers are order to reflect the degree of the deterioration.

(2) The state of the system undergoes deterioration according to a stationary discrete-time Markov chain having a known transition law. P_{ij} denote the 1-step transition probability from state i to state j.

(3) At each time period, the state of the system is monitored incompletely by some monitoring mechanism. The outcome of the monitoring is classified into finite levels 0, 1, ..., M . The probabilistic relation between the state of the system and the outcome of the monitoring is prescribed by :

   $q_{i\theta} \cong \Pr\{$the outcome of the monitoring is level $\theta \mid$ the system is in state i $\}$,

   where $i = 0, 1, \ldots, N$ and $\theta = 0, 1, \ldots, M$.

(4) At any given time period, the decision-maker selects only one of the following three actions.

a_C : the action to continue the system operation with incomplete monitoring,

a_I : the action to operate the system with inspection,

a_R : the action to replace the system by a new one.

The cost structure considered is as follows.

L_i : the operating cost per period in state *i*,

C_i : the replacement cost for the system in state *i*,

K (>0) : the inspection cost.

The objective is to obtain an optimal inspection and replacement policy minimizing the expected total discounted cost over an infinite horizon and formulated as a partially observable Markov decision process. Under some reasonable conditions reflecting the practical meaning of the deterioration, they present an optimal inspection and replacement problem to minimize the expected total discounted cost over at infinite horizon. In this paper, the model assumed that the transition probability of the deteriorating process of the system and the probabilistic relation between the system and the monitoring mechanism are completely known. However, this assumption does not always hold in real situations. A similar case model is presented by Devooght, Dubus and Smidts (1990). They consider complex systems regularly inspected to maintain a high availability level which may be described by a finite state Markov chain, and include human errors. Inspection and repair are imperfect due to incomplete information and / or instrumentation failure. They develop a dynamic programming algorithm based on the use of importance parameters for components to obtain sub-optimal inspection policies.

Özekici and Pliska (1991) consider an inspection model where the information is imperfect in the sense that both false positives and false negatives are allowed. A corrective action is carried out when a true positive is observed, thereby reducing the chance of system failure. All costs incurred in this model are that of inspections, false positives, the corrective action and failure. They use dynamic programming to compute the optimal inspection schedule. They also show the model, which is suited for medical screening, is applied to the problems of post-operative periumbilical pruritis and breast cancer. Srivastava and Wu (1993) also examined an imperfect inspection model. The standard notation is given by

ϕ : constant failure rate for each unit

T : fixed interval between inspections

p : failure probability between successive inspections: $\exp f(\phi T)$

β : Pr{detecting a failure at inspection}

k : inspection times, $k = 1, \ldots, N$

n : number of units in the system

X_k, Y_k : number of [detected, undetected] failures at inspection k; that number is [observable, unobservable]

q : binomial probability related to y_k

Z_{1k}, Z_{2k} : number of failures undetected at inspection k that later fail and are [detected, undetected] at inspection $k+1$

Z_{3k} : number of failures undetected at inspection k that are detected at inspection $k+1$

trinm($r_1, r_2; p_1, p_2, n$): $[n!/(r_1! \cdot r_2! \cdot (n-r_1-r_2)!]p_1^{r_1} \cdot p_2^{r_2} \cdot (1-p_1-p_2)^{n-r_1-r_2}$: trinomial pmf

They develop the probability model to be assumed as follows:

(1) The system consists of n components which have IID (independent and identically distributed) exponential life distributions with rate ϕ.

(2) The system is inspected periodically at times $\{T, 2T, \ldots,\}$. Inspection duration is negligible, $p = \exp f(\phi T)$.

(3) When a failed component is detected, it is replaced by a new one.

(4) a. Pr{detecting a failure} = β, $0 < \beta \leq 1$. The number of detected failures at the inspection k, X_k, depends on the number of undetected failures, Y_{k-1}, after inspection k-1.

b. Pr{a good component has failed} = 0.

c. The inspection process is otherwise benign.

They note that there is a difficulty with the model since X_k itself is not a Markov chain. To overcome this, they consider the 2-dimensional process $\{(X_k, Y_k)\}$, which is a Markov chain whose joint distribution depends only on Y_{k-1}. They derive the Markov relationship between the random variables, (X_{k+1}, Y_{k+1}) and Y_k. After inspection k and replacement;

- the number of undetected failures is Y_k,
- the number of operative components is $n - Y_k$.

At inspection $k + 1$, Z_{3k} of the undetected (at inspection k) failures are detected; the probability mass function, pmf$\{Z_{3k}\}$ = binm(Z_{3k}; β, Y_k) which is $\binom{Y_k}{\beta} \cdot \beta^{Z_{3k}} \cdot (1-\beta)^{Y_k - Z_{3k}}$. The number of the undetected failures out of Y_k remaining in the system is $Y_k - Z_{3k}$.. On the other hand, at inspection $k + 1$, some of those operative components are failed but not all of them are necessarily detected. For given Y_k,

$$\text{pmf}\{Z_{1k}, Z_{2k}\} = \text{trinm}(Z_{1k}, Z_{2k}; \beta p, \bar{\beta} p, n-Y_k) \tag{2.22}$$

By combing the above arguments, we obtain:

$$(X_{k+1}, Y_{k+1}) = (Z_{1k} + Z_{3k}, Z_{2k}+Y_k - Z_{3k}), k = 1,2,\ldots . \tag{2.23}$$

pmf$\{Z_{1k}, Z_{2k}\}$ = trinm(Z_{1k}, Z_{2k}; $\beta p, \bar{\beta} p, n-Y_k$), pmf$\{Z_{3k}\}$ = binm(Z_{3k}; β, Y_k), and, (Z_{1k}, Z_{2k}) and Z_{3k} are conditionally (given Y_k) independent. Therefore, at inspection 1, since $X_0 = n$, $Y_0 = 0$,

$$\text{pmf}\{X_1, Y_1\} = \text{trinm}(X_1, Y_1; \beta p, \bar{\beta} p, n) \tag{2.24}$$

They are interested in the steady-state behaviour of the system. Since (X_k, Y_k) is a finite-state Markov chain, its steady-state distribution exists and can be obtained. They note that the possibility of imperfect inspection makes the inference about the parameters very difficult. They also show the approximation of the original likelihood by combining the complementary variable method with the likelihood filtering method when components of life distribution is exponential.

Surprisingly, perhaps, it is hard to find papers indicating the application of such a massive set of inspection models to real-world problems. In the literature on maintenance modelling, multi-component system models have been addressed by many authors; see the survey by Thomas (1986) and Cho and Parlar (1991). However only a few tackled real problem, and most of these model the problem using the delay time concept (see Baker and Christer, 1994). The delay time approach to modelling is different in both nature and evident applicability to the majority of the maintenance literature discussed above. We will consider here, some cases briefly relating to a delay time concept for system model which compose of many components. In the papers by Christer (1982), Christer and Waller (1984a, b), Christer (1988), Christer and Redmond (1990, 1992), Chilcott and Christer (1991), Christer and Desa (1992), Christer and Wang (1995), the models assume that the number of component is large and that only failed components are repaired or replaced at failures. Defects are assumed to arise as a stochastic process with each defect having a delay time period before causing a breakdown. Inspections are assumed to be perfect or imperfect in that all defects presented at an inspection may or may not be detected. In Chapter 3, delay time models are discussed in detail.

2.4 Summary of Literature Review

This Chapter has investigated numerous models in the literature which deal with the problem of finding optimal inspection policies for systems which are subject to failures. Since the general inspection model by Barlow and Proschan (1965), many papers have been written about various inspection policies relating to maintenance/replacement models. Most of these models deal with systems that can be considered as a single component. Not as many models have considered a multi-component system consisting of independent, or dependent components. There is an important reason to consider single component systems, namely that in practice there are many instances in which it is difficult to obtain reliability data for smaller sub-components; although data for the stochastic behaviour of the entire system is available or easier to obtain. Maintenance models for single component systems can also be useful for modelling the maintenance of individual components that are part of more complex systems.

However, in recent years there has been a shift in the maintenance literature from consideration of single items to systems composed of several components. This is partly due to improvements in analytic techniques, which allow more complex systems to be investigated, but also because of the realisation that the interactions between the components in a system are one of the major factors in the system's reliability and should be taken into account in any maintenance or replacement policy. Although many authors are interested in multi-component systems, so far only a few papers have deal with inspection models for multi-component systems, and even fewer have been used to model actual plant. Because of its importance in this respect, the next Chapter which introduces delay time theory, will be developed in sufficient detail to indicate the nature of its contribution to maintenance modelling.

Chapter 3

The Theory of Delay Time

3.1 Introduction

A methodology is advanced for modelling the problem of planned preventive maintenance, thereby relating plant performance to maintenance activity. There are many models of preventive maintenance in the literature, including the Delay-Time Model (DTM) along with others cited in previous Chapters. This Chapter focuses on a Delay Time Theory, which is a developing concept for maintenance and inspection modelling. The concept was first introduced and applied in the context of building maintenance by Christer (1982), following the first mention in the appendix to Christer (1976). In this Chapter, a review of the concept and developments in delay time modeling (DTM) theory is presented.

3.2 The Origin of Delay Time Concept

The origin of the delay time concept proposed by Christer (1976) has been extensively used in maintenance modelling, since it offers a means of modelling the consequences of alternative maintenance and inspection practices of repairable machines. With such a model, the best inspection practice can be identified. In its simplest form, the delay time of a defect is the time period from when a defect is first observable to when a repair would be essential (breakdown) in the absence of corrective action. Applied studies of the delay time concept were initiated by Christer & Waller (1984a, b).

The central concept of this delay time is the delay time h of a defect, which is the time lapse from when a defect could first be noticed until the time its repair can be delayed no longer because of unacceptable consequences. A repair may, therefore, be undertaken any time within this period. This means that a failure does not normally occur instantaneously, but is proceeded by a period as a detectable defect for some time prior to actual failure. The delay time concept defines a two stage failure process in which in the first stage a fault becomes visible, and in the second stage this visible fault causes the eventual breakdown (failure) of the machine, see Figure 3.1.

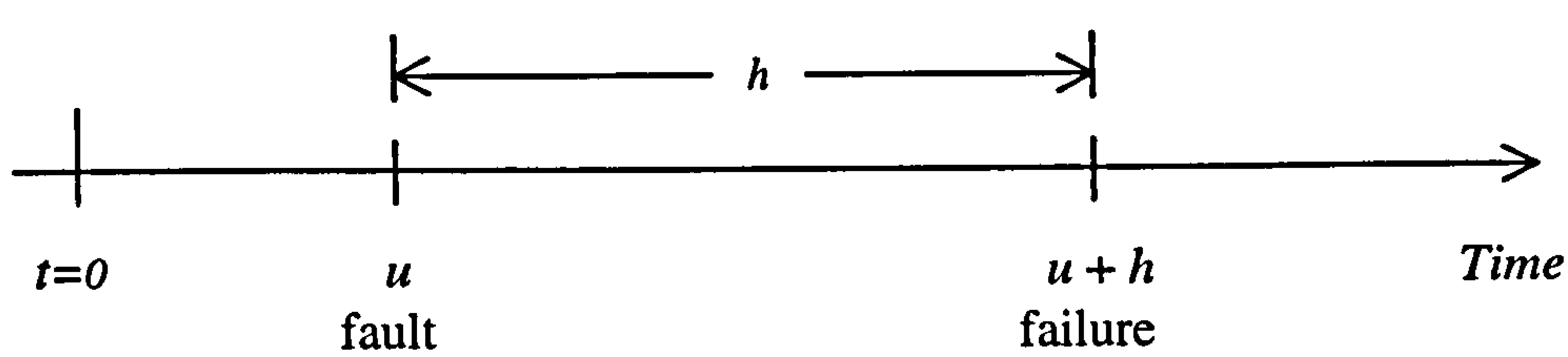


Figure 3.1. The delay time concept

In the case of a repairable component, a fault first initiates at time u from new with probability density function, *pdf* $g(u)$, and gradually develops into a failure after a time period of h with *pdf* $f(h)$. Once these two distributions are known, it is possible to model the reliability, operating cost and availability functions as described in the papers of Christer, Wang, Baker and Sharp (1995). In the case of multi-component or complex plant, the arrival pattern of defects within the system is modelled by an instantaneous arrival rate parameter $\lambda(u)$ at time u. If $\lambda(u)$ is constant, the model is a Homogeneous Poisson Process type (HPP), otherwise it is of a Non-Homogeneous Poisson Process type (NHPP).

The delay time concept itself can readily be understood by engineers. It is this concept which governs the consequence of inspection and preventive maintenance. The reason is simply that periodic preventive maintenance incorporating inspection checks may increase system availability by identifying and replacing or repairing faulty components before they cause a breakdown.

3.3 The Development of Delay Time Model

Since the first mention of the delay time concept in the appendix to Christer (1976), a considerable number of further developments, extension and application of basic delay time theory and modelling has been produced. The delay time model (DTM) was introduced in Christer (1982) in the context of building maintenance. The model was of the Homogenous Poisson process (HPP) type. Subjective and objective information were both to be used.

In 1984 the DTM was applied to problems of complex industrial plant maintenance, where defects were assumed to arise at a constant rate. In Christer and Waller (1984a) the DTM was extended to cater for imperfect inspection, a HPP of defect origination epochs over the interval between inspections, and two cost models which maintenance performed simultaneously for all defects or sequentially. In a related case study paper Christer and Waller (1984b) the delay time analysis and snap-shot modelling were used to derive an optimum-cost maintenance policy at the Pedigree Petfoods canning line. Snap-shot modelling was proposed by Christer and Whitelaw (1983) to define the nature of a maintenance problem and to provide an analysis of defects by classification into machine types where planned maintenance could be an effective option. In another case study, Christer and Waller (1984c), snap-shot analysis and the DTM were applied to modelling preventive maintenance for a vehicle fleet of tractor units operated by Hiram Walker Ltd. This study identified some peculiarities of practice which the model was extended to cope with, *e.g.* some defects were found by drivers, who returned the vehicle for repair at once, and the text scheduled maintenance was brought forwarded to coincide with the repair. This paper also first mentions the observation that vehicle repair costs for consecutive year may be positively correlated.

The notions of inspection and condition monitoring of specific items of plant are very similar in concept, and preliminary studies have been undertaken to investigate the applicability of delay time analysis and delay time models to condition monitoring problems, be they discrete monitoring processes or continuous monitoring processes. In 1987 a perfect-inspection model for a single component as a discrete monitoring process appeared, Christer (1987), and component reliability as a function of inspection interval was calculated using a recursive formula. The model is then expanded to consider the reliability of n components in a parallel system. Similarly to condition-based inspection models, Christer (1988) proposed using the delay time concept for modelling the task of inspecting major civil-engineering structures. A system model is assumed with expected repair costs varying over the delay time period. Here further developments of the pooled component data model are discussed.

Christer and Redmond (1990) discuss an unavoidable bias that arises when estimating delay time from censored component data. A mechanism for correcting this bias when estimating a delay time distribution is proposed, based upon maximum likelihood considerations. This model also is considered as a pooled component data model for revising subjective estimates of delay time.

Chilcott and Christer (1991) used DTM to model the maintenance practices for coal face machinery within British coal. Here the delay time parameters were also estimated based on the subjective method which were then used to model the effectiveness of condition-based monitoring in reducing downtime. In Christer (1992) the DTM for the component-tracking case was discussed from the viewpoint of 0-1 condition-monitoring, and the asymptotic cost per unit time of irregular inspection policies was derived. This cost was used in Christer and Wang (1992), which considered a 0-1 condition model with regularly spaced inspections for a linear pattern of wear characteristic. In this model though not strictly a delay time model, a positive correlation between u and h is induced by the chosen linear form of wear variability within components.

Cerone (1991) later shows that evaluation of the reliability at the inspection points together with the knowledge of the smoothness and monotonicity of the function over an inspection interval simplifies the calculations considerably. He used the reliability function formulated in Christer (1987). Pellegrin (1991) derived a graphical procedure for finding the optimum interval between inspections using a delay time model, which allows the various factors relevant to decision-making to be emphasized.

In all case-related models developed to this point, model parameters have been estimated mainly from and based upon subjective data. Baker and Wang (1992) show that it is possible to estimate model parameters purely from objective data, which is PM data and times of failures. Model parameters are fitted by the method of maximum likelihood, and the Akaike Information Criterion(AIC) was first used to choose the best parameterisation for the delay time model. In a later paper, several extensions to basic model were derived by Baker and Wang (1993), which relax model assumptions and are designed to cope with complexities of real-world situations. They also show the results of fitting the model to data.

Christer and Redmond (1992) also considered more formal methods for revising delay time models of industrial maintenance practice. They develop the model updating techniques when the *pdf* of delay time has been subjectively derived. The scope for updating the prior model through modifying the perceived degree of perfectness of inspection is also discussed.

Christer and Desa (1992) considered the maintenance based availability modelling of bus transport in Malaysia. Initial estimates based on carefully collected subjective data proved to be insufficiently close enough to the *status quo* point to require a revision of the prior distributions and model in this case.

Baker and Christer (1994) discuss the development of delay time analysis as a means of modelling engineering aspects of maintenance problems. This review outlines the current state of knowledge and research in this area, and future trends in modelling application of delay time analysis and research predicted. They note that some, but not all, of the necessary steps required to expand the applicability of delay time analysis to enable engineering aspects of maintenance to be modelled for a wider class of problems are presented.

In 1995, a practical case study carried out by Christer *et al* (1995) to model maintenance practice of a complex production plant. A key machine in the plant, namely an industrial press, is used to illustrate the modelling process and management reaction. Here delay time parameters were estimated using the objective data, namely, the maintenance record data of failures and defects found at PM. The criterion of interest was to minimize total downtime over a PM interval. Using the same case study, the subjective method was also used to estimate delay time model parameters Christer *et al* (1998a). They repeated the modelling task adopting a subjective delay time parameter estimation technique, Christer and Waller (1984a). To compare the model formats and parameter values resulting from the two methods, they tested and validated PM modelling of the same industrial press using the subjective data based delay time technique. In order to remove the sampling bias within the initial subjective estimate of delay time, they used the least square method and the concept of minimum mean-square-error (MMSE) estimation to revise model parameters. It is noted that both modelling techniques lead to very similar results and recommendations.

In similar case study in Leung and Christer (1995), the model has been explored for application to model the reliability of pumping systems for the water supply in some 4,000 high rise housing in Hong Kong. The inspections are assumed to be perfect and non-detrimental. After inspection, the component is assumed returned to the as new condition.

In Christer and Wang (1995), the inspection model was constructed utilizing the delay time concept applied to a multi-component system subject to both planned inspection and opportunity inspection at failure. Defects were assumed to arise according to a non-homogeneous Poisson process (NHPP), and inspections assumed to be perfect in that all defects presented at an inspection are found. They argued that the NHPP is an appropriate model for repairable system analysis because it can provide at least a good first-order model to the real-world problems. Based upon the established delay time model, data on inspection of infusion pumps collected from a local hospital are used to illustrate the modelling, which also obtained sensible answers and confirmed the legitimacy of current practice.

Christer and Lee (1997) extended earlier work applying the delay time concept to the modelling the reliability of equipment where it is subject to regular inspection over a finite mission period, and thereby the modelling of the operational effectiveness of at sea inspections for critical ship equipment. They present a methodology for determining the consequence of different inspection options over a mission period.

Baker *et al.* (1997) present a delay time model for a complex repairable system in which defects arise according to a non-homogeneous Poisson process. In order to estimate model parameters, the likelihood function is derived for general failure and delay time distributions, and for a NHPP of defect arrival, all under imperfect inspection, and a general approach for estimating the optimum inspection interval is discussed. Christer *et al.* (1997) also developed a stochastic model of the behaviour of plant under a service maintenance system of the finishing mills roll change equipment at Llanwern works of British Steel. To assist in improving plant availability, the delay time modelling technique has been used. Particularly, the incomplete repairs or rectifications at roll changes have been modelled by introducing a probability P_r that represents the probability of a complete repair or rectification of a failure at a roll change. This is one of the most complex pieces of plant studied to date, inspection were not perfect, PM was not fault free, failures were not always repaired, and no PM data was recorded.

Redmond *et al.* (1997) developed a delay time based the maintenance modelling of deteriorating concrete components. The model extended to a four phase classification for a components condition; namely defect free, cracking, spalling and failure. Therefore, the delay time was split into phases, namely cracking and spalling, which were represented as h, and v, with *pdfs* $f(h)$ and $w(v)$, respectively. A method to estimate parameters with the censored data based on maximum likelihood has been proposed.

Choi (1997) presents a typical semi-Markov inspection model based upon the delay time concept for a complex repairable system that may fail during the course of its service lifetime. He discusses a case study of the semi-Markov inspection model and the delay time model and results are compared for a component and for a complex repairable system. He shows that the simulation model and semi-Markov model are nearly consistent with the delay time model. In this study, he also notes that the delay time model provides a means of modelling the behaviour of the system and predicting such useful quantities as a reliability, cost or downtime under various inspection policies.

In Christer, Wang and Choi (1998), a delay time model is developed and applied to model and optimise preventive maintenance (PM). This paper reports on a case study of a delay time modelling of maintenance applied to a subsystem of a complex machine used in commercial vehicle break lining manufacturing. In this study, the modelling has been developed to cope with the possible incomplete response at PM, which assumes the recognised defects that are attended to are chosen randomly. They also discuss the problems discovered concerning parameter estimation given inadequate data collected at PMs.

3.4 Basic Delay Time Model

The central concept of delay time has been shown in section 3.2. Again, the basic idea rests on an observation that a failure does not usually occur instantaneously, but is preceded by a detectable defect for some time prior to actual failure, the delay time. Specifically, the delay time h is a property of a defect and defined as the time laps from the moment when a defect could first reasonably be noticed until the moment when a consequential repair can be delayed no longer because of unacceptable consequences. Therefore, delay time is the period between a fault first arising as an identifiable defect, to a subsequent failure if unattended. The moment a defect could first be identified called the initial point, is denoted by time u, If the distribution of delay time (h) is known, and for a component or complex plant the respective distribution of initial point (u) or rate of arrival of defects, $\lambda(u)$, are known, the failure behaviour of the plant can in theory be determined under any specified maintenance policy.

To introduce delay time modelling, consider a particular item of plant which is associated with failures characterized by a delay time h with *pdf* $f(\bullet)$ and *cdf* $F(\bullet)$. Let a plant inspection be undertaken on a regular basis, with period T, and suppose for now that this inspection is perfect in that, if a defect is present at the time of inspection, it will be identified. Between inspections, defects can arise at a time u, say, and subsequently lead to a failure after time h if $h < T - u$, and be identified at an inspection if $h > T - u$, see Figure 3.2.

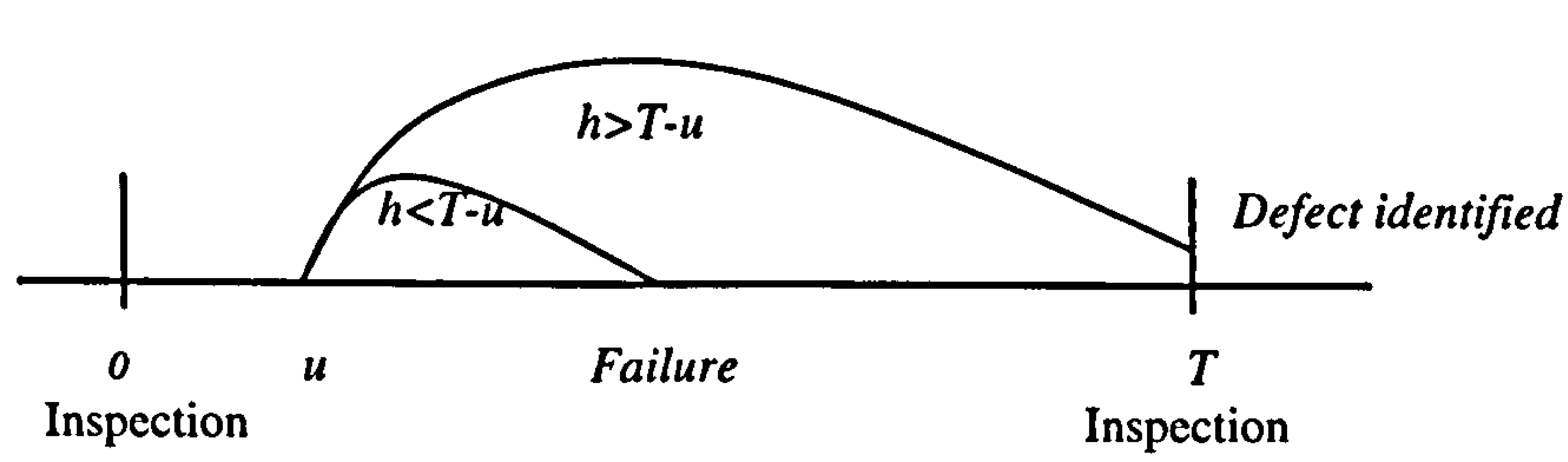


Figure 3.2. Delay time process of basic model

Suppose, for example, the following assumptions are valid for an inspection process;

(1) An inspection takes place every T time units, costs C_i units and requires d_i time units, where $d_i << T$.

(2) Inspections are perfect in that any defect present within the plant will be identified.

(3) Defects identified at an inspection will be repaired within the inspection period at an average cost of C_r.

(4) Failures are repaired as soon as they arise, incurring on average d_f downtime and C_f cost, where $C_f > C_r$ and $d_f << T$.

(5) The initial time u of a defect is uniformly distributed over $(0, T)$, and is independent of h, with defects arise at a rate of k per unit time.

(6) The probability density function(*pdf*) of delay time h, $f(h)$, is known.

The assumption $d_i << T$ in (1) and the assumption (3) may at first seem to be contradictory, but the assumption (3) would seem to be reasonable if sufficient maintenance staff were available to perform repairs simultaneously. The assumption (5) provides an estimate of expected number of defects arising in the period T, namely $K(T)$. This ignores the downtime due to failures, during which no defects would arise since the machinery is idle. However, if this downtime is small compared with T, as indicated in the assumption (4), $d_f << T$, then the error will also be small.

Under the above assumptions, firstly, we determine the form of function $b(T)$. Suppose that a defect arising within the period $(0, T)$ has a delay time in the interval$(h, h+dh)$. The probability that the delay time lies in this interval is $f(h)dh$. The defect will be repaired as a breakdown repair if the defect arises in period $(0, T-h)$ (see Figure 3.3.), otherwise as an inspection repair. The probability of a defect arising before $T-h$, given that a defect will arise, is $(T-h) / T$.

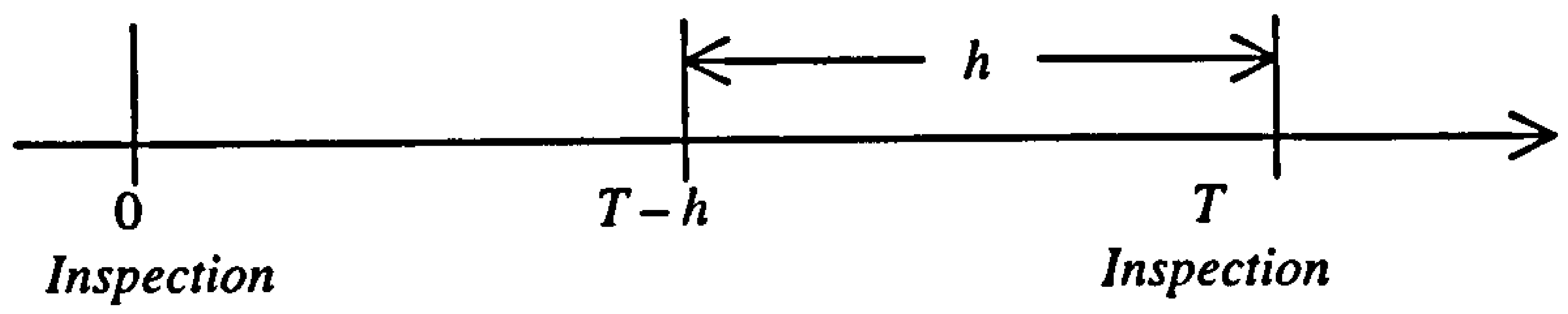


Figure 3.3. Inspection process of delay time model

We have, therefore, that the probability that a defect with delay time in the interval $(h, h+dh)$ is repaired as a failure is

$(T - h) / T f(h)\, dh.$

Then, integrating over all possible delay times, we have that the probability of a defect arising as a failure, $b(T)$, when operating an inspection policy over period $(0, T)$; is given by

$$b(T) = \int_0^T \frac{T-h}{T} f(h)dh. \tag{3.1}$$

Since defect arise at rate of k per unit time, expected number of defects arising over $(0,T)$, $K(T)$ is

$$K(T) = kT, \tag{3.2}$$

and then the expected number of defects arising over $(0, T)$, $B(T)$ is given by

$$B(T) = K(T)b(T) = kTb(T). \tag{3.3}$$

Once $B(T)$ is determined, the expected downtime per unit time $D(T)$, and the expected cost per unit time $C(T)$, may be obtained. Under the assumptions of the case being modelled, if there is no additional expected downtime due to repairing defects identified at an inspection, the total expected downtime per unit time is

$$D(T)=\frac{1}{T+d_i}[B(T)d_f+d_i]. \tag{3.4}$$

Similarly, under the above assumptions, a model of expected cost per unit time as a function of the inspection period T may be obtained directly. The total expected cost over an inspection cycle consists of cost of attending to failures, the expected cost of rectifying defects identified at inspection, and the cost of the inspection itself. Then the total expected cost per unit time over a full inspection cycle length $T+d_i$ is

$$\begin{aligned} C(T) &= \frac{B(T)C_f+[K(T)-B(T)]C_r+C_i}{T+d_i} \\ &= \frac{K(T)[(C_f-C_r)b(T)+C_r]+C_i}{T+d_i}, \end{aligned} \tag{3.5}$$

where the average failure, inspection and inspection repair cost are C_f, C_i and C_r, respectively.

Here, the decision variable T of downtime and cost model would be selected to minimize $D(T)$ and $C(T)$. Therefore, once the parameters which describe the rate of arrival of the initial point u and the delay time distribution $f(h)$ are available, delay time based maintenance models such as those in equations (3.4) and (3.5), may be formulated. So far, the above basic model of delay time has been shown as developed by Christer and Waller (1984a). These equations represent the fundamental form in delay time maintenance modelling, and may be modified according to need.

3.5 Variants to the Basic Delay Time Model

3.5.1 Non-Homogeneous Defect Arrivals Case

For the basic delay time model, assumption (5) may be required to be relaxed for a complex plant, namely the uniform initiation of defects after an inspection. Christer and Waller (1984a) showed that if inspections are perfect and $\lambda(u)$ is the instantaneous rate of occurrence of a defect at time u, the number of defects arriving in the interval (u, u + du) is $\lambda(u)du$. Then the expected number of defects arising over (0, T) is given by

$$K(T) = \int_0^T \lambda(u)du. \tag{3.6}$$

A defect arising in $(u, u + du)$ with a delay time $h < T - u$ will arise as a failure. Therefore the expected number of failures due to defects arising in $(u, u + du)$ is

$$\lambda(u)du \cdot \int_0^{T-u} f(h)dh = F(T-u)\lambda(u)du, \tag{3.7}$$

where $F(x) = \int_0^x f(h)dh.$

Accordingly, the expected number of failures arising over period $(0, T)$ is

$$B(T) = \int_0^T F(T-u)\lambda(u)du. \tag{3.8}$$

It is noted that in the special case of HPP when $\lambda(u)$ is constant, the defects are uniformly distributed over $(0, T)$, and equation (3.8) reduces to equation (3.1) as required. Since the expected number of inspection repairs arising in $(0, T)$ is $K(T) - B(T)$, the total expected downtime per unit time is given by

$$D(T) = \frac{1}{(T + d_i)}[B(T)d_f + d_i] \tag{3.9}$$

and the expected cost per unit time over a full inspection cycle of length $T + d_i$ is

$$C(T) = \frac{B(T)C_f + [K(T) - B(T)]C_r + C_i}{T + d_i}. \tag{3.10}$$

3.5.2 Imperfect Inspection Policy Case

In real-world situation, one of the most suspect assumptions of the above basic model is the requirement for perfect inspections. Inspections may not reveal all defects present in a system, especially for large complex systems. The quality of inspections depend on inspection practices imposed includes inspection techniques used, inspection training and the nature of any supervision. It has been assumed that inspection are perfect in that any defect present will be identified in the basic model. Therefore, in the case of imperfect inspection, it is necessary to introduce a probability r that a specific defect will be identified at an inspection, and a corresponding probability $(1 - r)$ that it will not (Christer and Waller, 1984a). Here we assume a defect is what an engineer assumes to be a defect. Although there is a problem of engineers wrongly assuming a fault, engineering actions and the following modelling are based upon these 'faults'.

Assuming still that the initial point u is uniformly distributed along $(0, T)$, the only change this will produce to the above models is through a modified form of $b(T)$, the probability of a defect arising as a failure. Under these circumstances, to find the new form of $b(T)$, consider a defect which first arises at time u after an inspection at point 0 (see Figure 3.4). If this defect is subsequently identified at an inspection, it could be the inspection at T if $h > (T - u)$ or at $2T$ if $h > (2T - u)$, or failing this , at inspection at $3T$ if $h > (3T - u)$, and so on.

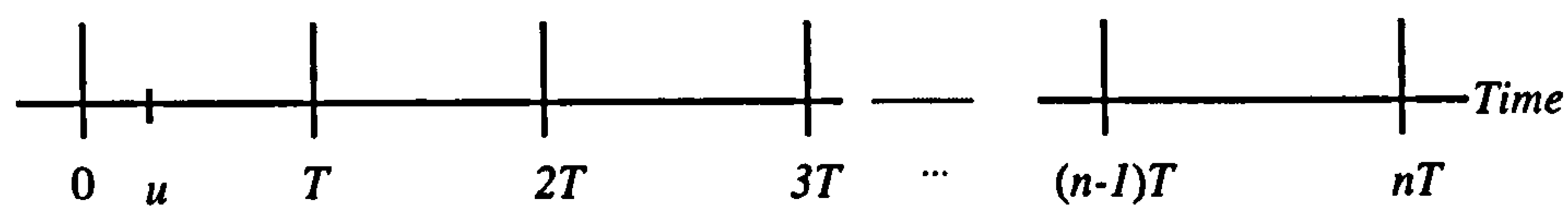


Figure 3.4. Example of inspection process of a defect arising at *u*

Therefore, the probability of defect identified at T for the defect arising at point u is

$$\begin{aligned} &\textit{Prob.(defect identified at T)} \\ &\quad = \textit{Prob.(being identified at T)} \times \textit{Prob.(not resulting in a failure before T)} \\ &\quad = rR(T - u), \end{aligned} \tag{3.11}$$

where $R(x) = \int_0^{\infty} f(h)dh$.

Similarly,

$$\begin{aligned} \textit{Prob.(being identified at 2T)} &= \textit{Prob.(not being identified at T but identified at 2T)} \\ &\quad \times \textit{Prob.(not resulting in a failure before 2T)} \\ &= r(1-r)R(2T - u). \end{aligned} \tag{3.12}$$

Generally, the probability that a defect initiated at point u will be identified at the inspection at nT is

$$r(1-r)^{n-1}R(nT-u), \qquad n = 1, 2, \ldots \tag{3.13}$$

Therefore, we have that the probability the defect at u will be identified at an inspection is given by

$$\sum_{n=1}^{\infty} r(1-r)^{n-1} R(nT-u). \tag{3.14}$$

Summing over all possible u, which is uniformly distributed over T, we have for the probability that a defect arises a failure, $b(T)$,

$$b(T) = 1 - \int_{u=0}^{T} \sum_{n=1}^{\infty} \frac{r}{T} (1-r)^{n-1} R(nT-u)du, \tag{3.15}$$

where $R(\bullet) = 1 - F(\bullet)$, and $F(\bullet)$ is the *cdf.* of delay time. In equation (3.15), for $r = 0$ or 1, $b(T)$ corresponds respectively to the breakdown maintenance case and to the basic inspection model with perfect inspection. This is equivalent to saying that a perfect inspection is carried out with probability r, and that with probability $1 - r$ the inspection is 'omitted'. In this thesis we assume that r is constant. Other models are also possible. For example, if $0 \leq u \leq t \leq h$, the probability of a defect identified at the inspection, r is $r(t-u)$. However, the value of $r(t-u)$ may not be easy to estimate in practice.

In imperfect inspection modelling which is case of $r \neq 1$, $b(T)$ changes in form, but the criteria functions such as $D(T)$ and $C(T)$, given in equations (3.4) and (3.5), remain the same. Christer and Waller (1984c) present an application of delay time analysis modelling of planned maintenance for a vehicle fleet using this imperfect inspection formulation for $b(T)$, with $r \neq 1$.

3.5.3 The Modification of Downtime Model

Consider now the case where assumption (3) of the above basic model is invalid. That is, there are insufficient maintenance staffs available to complete all identical repairs at an inspection. It means that additional time is required to perform inspection repairs subsequent to inspection, with each inspection repair causing additional downtime. Then the equation (3.4) of the basic downtime model, $D(T)$ would be modified as follows (Christer and Waller, 1984a). If d_r is the expected downtime due to an inspection repair, then the total downtime at an inspection is $d_i + kTd_r[1 - b(T)]$, and the expected length of the full cycle is $T + d_i + kTd_r[1 - b(T)]$. Therefore, the total expected downtime over a full inspection cycle length is given by

$$D(T) = \frac{1}{T + d_i + kTd_r[1-b(T)]}\{kTd_f b(T) + d_i + kTd_r(1-b(T))\}$$

$$= \frac{1}{T + d_i + kTd_r[1-b(T)]}[kT\{d_f b(T) + d_r(1-b(T))\} + d_i]. \qquad (3.16)$$

3.6 The Parameter Estimation and Revising of the Delay time Models

3.6.1 A Subjective Parameter Estimation Method

In maintenance modelling, the successful use of the delay time concept depends upon how well the underlying delay time distribution can be estimated from available information sources. If the maintenance records of failures and recorded findings at maintenance interventions such as inspections are available, and sufficient in quantity and quality, the delay time distribution can be estimated objectively by statistical method, see Baker and Wang (1992, 1993), Christer and Wang (1995), and Christer *et al.* (1995). If however , such a data set does not exist, the alternative is to use expert judgment for obtaining the delay time distribution. Christer (1982) developed a method for the use of the subjective opinions of experts in estimation of the delay time distribution (see also Christer and Waller, 1984b, c). This method was also used by Christer and Desa (1992) and Christer, Wang, Baker, and Sharp (1998).

For obtaining an estimate of delay time $f(h)$, the following questions may be asked of maintenance engineers at a failure repair or at an inspection. That is

(a) How long ago could the defect have first been noticed by an inspection or operator (= *HLA*)?
(b) If the repair is not carried out now, how much longer could it be delayed before a failure repair is essential (= *HML*)?

The delay time for each defect is estimated by $h = HML + HLA$ (see Figure 3.5). Given a sufficient accumulation of such measures, $\{h = HML + HLA\}$, the delay time distribution, $f(h)$, may be estimated.

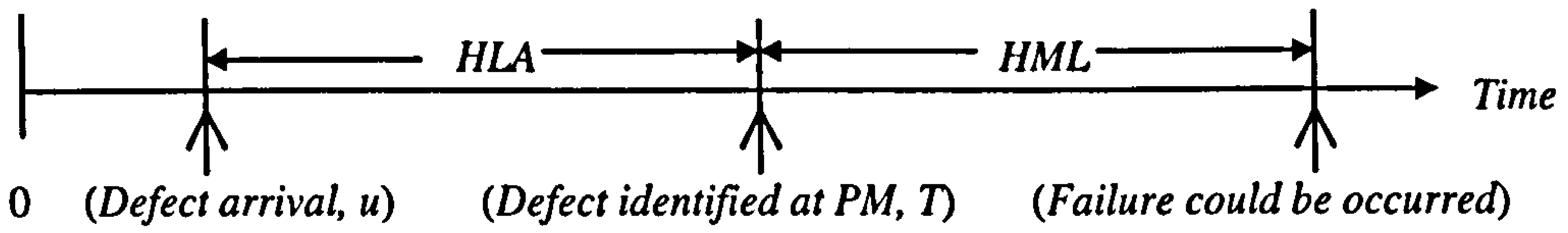


Figure 3.5. Estimating delay time parameter for defect at a PM

At any point in time T when a defect is being attended to, having an estimate of HLA provides at once an estimate of initial point u, namely $u = T - HLA$. It is the set of such estimates that enables the distribution of the initial points u to be estimated for a component. The method of estimating the distribution parameters of delay time h and initial point u in this way is known as the subjective parameter estimation method (or subjective method). It is note that, in adapting the subjective method to estimate the delay time, the definition of defect and failure are important. Here, defects and failures are taken as what the organization deemed them to be, that is, the companies operating definitions and practice is adopted.

Wang (1997) recently also proposed a method for obtaining a subjective estimate of the delay time distribution. He presents a revised procedure and method for obtaining the subjective delay time distribution. An alternative approach is to ask experts or maintenance engineers to estimate the required probability that the mean delay time of a chosen failure type will lie in a specified time interval. He gives an example of questionnaire designed in an appendix.

3.6.2 Revising the Subjectively Estimated Parameter

One of the interesting aspects of delay time modelling is that it can use a synthesis of subjectively derived data, represented by $F(h)$, to model a maintenance situation where the variable of interest can be the expected number of failures over $(0, T)$, $B(T)$, the expected downtime per unit time, $D(T)$, or the expected cost per unit time, $C(T)$. If there is a current policy of inspecting the system at period T_0, say, then one would expect that the relationships such as the following to hold,

$$D_0 = D(T_0), \tag{3.17}$$

$$C_0 = C(T_0), \tag{3.18}$$

where D_0 and C_0 are the currently observed the mean downtime and cost per unit time. Alternatively, if under policy T_0 the probability of a defect arising as a failure is B_0, then one would expect from equation (3.1) or (3.15) that

$$B_0 = B(T_0), \tag{3.19}$$

where B_0 is the current observed the mean number of failures. The left-hand side is an observation of practice, and the right-hand side is a function based on an aggregate of subjective opinions. However, the chance of any of above relationships being satisfied is remote. In common with any process of decision analysis entailing subjective assessments, it is to be expected that some revision will need to be made to prior distributions or perhaps to the prior model.

The problem is simply stated in Figure 3.6.

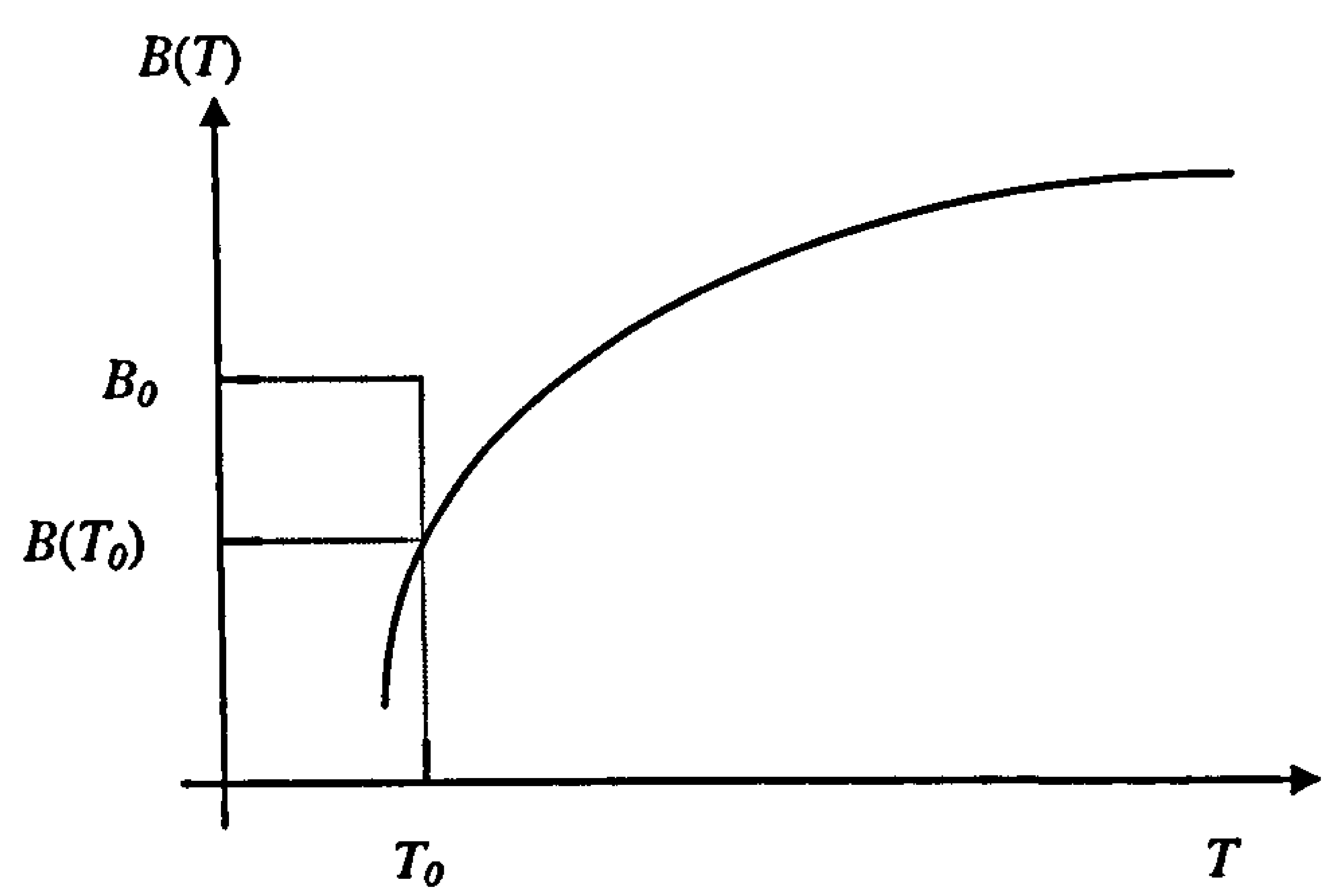


Figure 3.6. Number of failure of prior model and observed

The prior estimate $B(\cdot)$ has been obtained based upon a prior cumulative subjective data led distribution function $F(\cdot)$ and an initial understanding of the problem represented via the assumptions. If T_0 and B_0 denote the current practice, the curve $T \rightarrow B(T)$ needs to be modified to pass through the *status quo* point (T_0, B_0). In this anticipated problem, a method has been developed by Christer and Redmond (1990) to formally revise or update the prior delay time distribution, $f(h)$, using the known B_0, D_0 and C_0 (see also Christer and Redmond, 1992). This is done by a shear transformation of each estimate of delay time h to h', such as $z = \alpha h + \gamma$, where α and γ are the unknown parameter to be determined such that the afore-mentioned relationships hold. The argument for this type of transformation is the tendency for estimators to systematically underestimate delay time which, therefore, need to be extended.

For updating delay time, first it is assumed that the gradient between h' and h is positive, *i.e.*,

$$\alpha > 0, \tag{3.20}$$

otherwise large estimate of h would transform to small actual values h' and vice-versa. Secondly, h' must be nonnegative, so if h_0 is the smallest value of h for which the prior distribution satisfies $F(h) > 0$ for $h > h_0$, then we require,

$$\alpha h_0 + \gamma \geq 0, \tag{3.21}$$

where $h_0 \geq 0$. Therefore, the parametric form for the distribution function of h' is given by,

$$F(\frac{h'-\gamma}{\alpha}), \tag{3.22}$$

where the prior $F(h)$ is assumed zero for $h < 0$, which implies (3.22) is zero when

$$0 \leq h' \leq \alpha h_0 + \gamma. \tag{3.23}$$

If we let the initial model $B(T)$ be constructed with an estimate β_0 for β, and let $B(T;\ \alpha,\ \gamma)$ be the updated parametric form for $B(T)$ when replacing $F(h)$ by the transformed expression (3.22). Therefore, the option of using the transformation $z = \alpha h + \gamma$, in addition to the variable β, is satisfied by the status quo conditions, that is,

$$B(T_0;\alpha,\gamma) = B_0. \tag{3.24}$$

Christer and Redmond (1990) discuss a problem which can be arise from using the subjective method which is due to possible sampling bias. At failures, the delay time estimates obtained are $\{h\} = \{HLA\}$ (since $HML = 0$, by definition). An estimation of $f(h)$ based only upon failure estimates $\{HLA\}$ will generally underestimate h. On the other hand, the delay time estimates based upon inspection repair data only, $\{h\} = \{HLA + HML\}$, will produce an estimate of *pdf* of h which overestimates h. It is note that both the two subjective estimates of delay time are intrinsically biased, and suggest a maximum-likelihood estimation of the subjective estimates to overcome this.

Suppose that there are two data set of estimates, which are failure delay time $\{\hat{h}_j^{(1)}; j = 1, 2, \ldots, n\}$, and inspection repair delay time $\{\hat{h}_k^{(2)}; k = 1, 2, \ldots, m\}$. So far, the practice has been to produce a combined set $\{\hat{h}_j^{(i)}\}$, $i = 1, 2$; $j = 1, \ldots, m_i$ of delay time estimates from which to establish $F(\bullet)$. Here let the prior distribution for the delay time be $F(\bullet, \gamma)$, where γ denotes the distribution parameters. Accepting this distribution, we have for the distributions of delay time of observation, spanning an inspection epoch T and arising as a failure before T, the expressions (see Christer and Redmond, 1990)

$$F_{T,i}(\xi,\gamma) = \begin{cases} \int_0^T \frac{q(u)\{F(\xi,\gamma) - F(T-u,\gamma)\}}{1-b(T)} du & \text{for } \xi \geq T, \\ \int_{T-\xi}^T \frac{q(u)\{F(\xi,\gamma) - F(T-u,\gamma)\}}{1-b(T)} du & \text{for } \xi < T, \end{cases} \tag{3.25}$$

and

$$F_{T,f}(\xi,\gamma) = \begin{cases} \frac{1}{b(T)}\{F(\xi,\gamma)Q(T-\xi) + \int_{T-\xi}^T q(u)F(T-u,\gamma)du\} & \text{for } \xi < T, \\ 1 & \text{for } \xi \geq T. \end{cases} \tag{3.26}$$

The choice of the parameter γ is made by utilizing the maximum likelihood principle in the light of the observations $\{\hat{h}_j^{(1)}\}$ and $\{\hat{h}_k^{(2)}\}$, that is

$$L=\prod_{j=1}^{n} f_{T,f}(\hat{h}_j^{(1)},\gamma)\prod_{k=1}^{m} f_{T,i}(\hat{h}_k^{(2)},\gamma), \qquad (3.27)$$

By taking the logarithm, equation (3.27), the maximum log-likelihood, is

$$Log\ L=\max_{\gamma}\{\sum_{j=1}^{n}\log f_{T,f}(\hat{h}_j^{(1)},\gamma)+\sum_{k=1}^{m}\log f_{T,i}(\hat{h}_k^{(2)},\gamma)\}, \qquad (3.28)$$

where f denotes the density function of F:

$$f_{T,a}(\xi,\gamma)=(\partial/\partial\xi)F_{T,a}(\xi,\gamma) \qquad (a=i,f). \qquad (3.29)$$

This optimization process provides an appropriate fit to the parameter to enable $F(x,\gamma)$ to be defined. Of course, some form of updating adjustment will still be needed, possibly associated with an iteration between correcting for bias, process (3.26), and model and distribution adjustments to the status quo. The main point of the above discussion is that there are methods for estimating and correcting a subjectively derived delay time distribution and model.

3.6.3 The Objective Parameter Estimation Method

Given a repairable system that breaks down from time to time during its service life, and also regularly undergoes inspections, it is desirable to make predictions of the optimum period Δ^* between inspections. Hitherto, model parameters have been estimated mainly from subjective data. If there are objective data available, Baker and Wang (1992, 1993) have recently introduced a method, now known as the objective estimation method. It is both theoretically and practically possible to estimate the delay time distribution from objective data, that is, data from maintenance records of failures and defects found at inspections or PM. Essentially, the data should include a history of failure times, and the results of PMs or inspections which may be positive (defect found) or negative (no defect found).

In the paper by Baker and Wang (1992,1993), the objective method was initially designed for a single component subject to failures and inspections at PM, or a system with a few key components. As will be highlighted in the case study later, the objective method has been extended for estimating parameters of complex plant. For now we comment on the method for component model parameter estimation. It is considered that PM is a perfect inspection process with replacement of visibly defective components for a single component machine. The possible events that can contribute to the likelihood are defined as:

N : Inspection and no defect found (negative inspection),
Y : Inspection and defect found (positive inspection),
B : failure,
E : End of observation period.

In addition, the following notation introduced by Baker and Wang (1992) is useful (see Figure 3.7).

S : Start of observation period (= R),
R : Replacement on a failure (B, or Y),
X : Denotes any event.

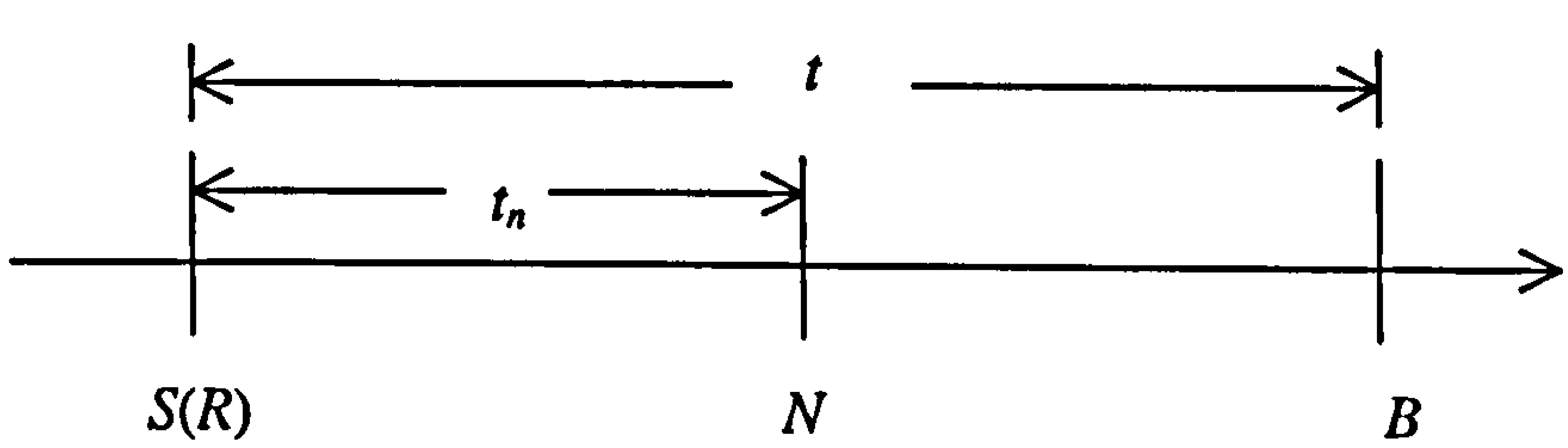


Figure 3.7. Example of notation used

Baker and Wang (1992) establish the likelihood of observing a sequence $X_1, X_2, \ldots, X_n$ of events of types B, E, Y, and N by utilizing the expression,

$$L = P_{X_1} \times P_{X_2|X_1} \times P_{X_3|X_1X_2} \times \cdots \times P_{X_n|X_1 \cdots X_{n-1}}, \tag{3.30}$$

where P_{X_1} denotes the probability of an event X_1, $P_{X_2|X_1}$ means the probability of event X_2 given that event X_1 has occurred, and so on. Since, after a replacement R, the likelihood does not depend on any event previous to R, the likelihood can be written as the product of terms conditional on events $RX_1X_2 \ldots$ starting with the last renewal. Further, since inspections are assumed to be perfect, we have

$$P_{X|RN_1 \cdots N_n} = P_{X|RN_n}. \tag{3.31}$$

Under this concept, three key probabilities can be considered for the described system.

(1) $P_{NB/R}(t_n, t)dt$ is the probability of a sequence of negative inspections of which the last occurs at time t_n from last renewal, and a failure at a time between t and $t + dt$ from last renewal. $P_{NB/R}(t_n, t)$ is given by

$$P_{NB|R}(t_n, t) = \int_{t_n}^{t} g(u) f(t - u) du, \tag{3.32}$$

where $g(u)$ is the *pdf* of initial point u, and $f(h)$ is the *pdf* of delay time h.

(2) $P_{NE/R}(t_n, t)$ is the probability of a sequence of negative inspections of which the last occurs at time t_n from last renewal, and no breakdown before observation ceases at time t from last renewal. This probability is given by

$$P_{NE|R}(t_n,t) = 1 - G(t) + \int_{t_n}^{t} g(u)(1 - F(t-u))du\,, \qquad (3.33)$$

where $G(\bullet)$ is the *cdf* of initial point u, and $F(\bullet)$ is the *cdf* of delay time h.

(3) $P_{NY/R}(t_n,t)$ is the probability of a sequence of negative inspections of which the last occurs at time t_n, followed by a positive inspection at time t from last renewal. This probability is given by

$$P_{NY|R}(t_n,t) = \int_{t_n}^{t} g(u)(1 - F(t-u))du\,. \qquad (3.34)$$

The model key assumptions, which may be characterized as follow;

(1) The time to the initial point of a defect and the subsequent time to failure of the component are independent.

(2) The distributions of initial point u and delay time h are modelled as Exponential or Weibull.

(3) Inspections are perfect.

(4) Repair times are negligible.

(5) Repairs are taken as replacements, so that the faulty component is restored to as new condition.

Based on the above probability definitions and assumptions, Baker and Wang (1992) developed the likelihood function L of observing a sequence of events of

(a) failures at time t_i^B $(i = 1,2,\ldots,n_B)$,

(b) no failure before observation ceases at time t_j^E $(j = 1,2,\ldots,n_E)$,

(c) positive inspections at time t_k^Y $(k = 1,2,\ldots,n_Y)$, as

$$L = \prod_{i=1}^{n_B} P_{NB/R}(t_i^{B^*}, t_i^B) \prod_{j=1}^{n_E} P_{NE/R}(t_j^{E^*}, t_j^E) \prod_{k=1}^{n_Y} P_{NY/R}(t_k^{Y^*}, t_k^Y)\,, \qquad (3.35)$$

where $t_i^{B^*}$ is the time of the latest negative inspection, (or, failing that, the latest renewal) such that $t_i^{B^*} < t$, and similarly for $t_j^{E^*}$ and $t_k^{Y^*}$. By maximizing the likelihood L in equation (3.35), estimates of parameters of the underlying initial point distribution, $q(u)$, and the delay time distribution, $f(h)$, can be obtained.

A further development of the objective method for estimating delay time parameter is given in Christer, Wang, Baker, and Sharp (1995). This study is the first time the objective approach to delay time modelling has been applied in a case study. They develop a model which is different from previous delay time models in that it models the defect-initiating process as a stochastic process, and it is based upon interval data. They present a study carried out for a copper products manufacturing company, developing and applying the delay-time modelling technique to model and thus optimize preventive maintenance (PM) of an industrial press. The data available within the plant included the dates and downtimes occurred due to both PM and failures, the nature of the occurrence, and the number of faults found at PM.

To estimate the parameters of the defect arrival process and the delay-time distribution, Christer Wang, Baker, and Sharp (1995) considered the following assumptions:

(1) Defects arise according to a homogeneous Poisson process with rate λ.
(2) Defects are assumed to arise independently of each other.
(3) The delay time h of a random defect is independent of its time origin and has *pdf* $f(h)$ and *cdf* $F(h)$.
(4) Inspections carried out at PM are assumed to be imperfect in that they can only identify a defect present with probability r. Probabilities of detection of a defect at successive inspections are independent.
(5) All identified defects are rectified by repairs or replacements during the PM period. This does not influence the development of undetected defects.
(6) Failures are identified immediately, and repairs or replacements are made as soon as possible.

Further notation is defined as:

t_i : epoch of the ith PM from new, $i = 1,2,\ldots$

t : failure time from new

λ : rate of occurrence of defects

h : delay time of a defect with *pdf* $f(h)$ and *cdf* $F(h)$

r : probability of detecting a defect if it is there.

Under the above assumptions and notation as defined, we have that the probability of a failure in $(t, t+\Delta t)$ resulting from a defect arising at time y, is given by

$$P(t,t+\Delta t|y) = \begin{cases} (1-r)^{n-i+1}(F(t+\Delta t-y)-F(t-y)) & \text{for } t_{i-1} < y \le t_i,\ i=1,2,\ldots,n \\ F(t+\Delta t-y)-F(t-y) & \text{for } t_n < y \le t \\ F(t+\Delta t-y) & \text{for } t < y \le t+\Delta t \\ 0 & \text{for } t+\Delta t < y\ . \end{cases} \tag{3.36}$$

Then, for $t_n < t \le t_{n+1}$, the expected number of failures over $(t, t+\Delta t)$, $EN_f(t,t+\Delta t)$, is

$$\begin{aligned} EN_f(t,t+\Delta t) &= \lambda\int_0^\infty P(t,t+\Delta t|y)dy \\ &= \lambda\sum_{i=1}^{n}(1-r)^{n-i+1}\int_{t_{i-1}}^{t_i}(F(t+\Delta t-y)-F(t-y))dy \\ &+\lambda\int_{t_n}^{t}(F(t+\Delta t-y)-F(t-y))dy+\lambda\int_t^{t+\Delta t}F(t+\Delta t-y)dy. \end{aligned} \tag{3.37}$$

Changing the integral variable and rearranging the integral sequence, and after some manipulation, we have

$$\begin{aligned} EN_f(t,t+\Delta t) &= \lambda\int_t^{t+\Delta t}\sum_{i=1}^{n}(1-r)^{n-i+1}(F(x-t_{i-1})-F(x-t_i))dx \\ &+\lambda\int_t^{t+\Delta t}F(x-t_n)dx. \end{aligned} \tag{3.38}$$

Also, the mean number of defects found at PM time t_{n+1}, $EN_p(t_{n+1})$, is given by

$$EN_p(t_{n+1}) = \lambda\sum_{i=1}^{n}(1-r)^{n-i+1} r\int_{t_{i-1}}^{t_i}(1-F(t_{n+1}-y))dy$$

$$+\lambda r\int_{t_n}^{t_{n+1}}(1-F(t_{n+1}-y))dy. \qquad (3.39)$$

Since the defect arrival process is assumed to arise according to a Poisson process, as a generalization of proposition 3.3.2 in Ross (1983), the number of failures in $(t, t+\Delta t)$ follows a Poisson distribution with mean $EN_f(t,t+\Delta t)$ and the number of defects found at PM follows a Poisson distribution with mean $EN_p(t_{n+1})$. Therefore, the probability of m failures over $(t, t+\Delta t)$, where $t_n < t \le t_{n+1}$, is given by

$$P(m\ failures\ in\ (t,t+\Delta t)) = \frac{(EN_f(t,t+\Delta t))^m e^{-EN_f(t,t+\Delta t)}}{m!}, \qquad (3.40)$$

and the probability of n defects found at t_{n+1} is

$$P(n\ defects\ at\ t_{n+1}) = \frac{(EN_p(t_{n+1}))^n e^{-EN_p(t_{n+1})}}{n!}. \qquad (3.41)$$

As previously indicated, the data assumed to be available are the number of failures in each working day and the number of defects identified at PM times. To formulate the likelihood function of the observed event, suppose first that n_i defects have been observed at the ith PM time (i=1, 2,..., l). The PM interval (T_{i-1},T_i) is now divided into k nonoverlapping subintervals of equal length Δt, namely

$$I_j^i = (t_{i-1}+(j-1)\Delta t, t_{i-1}+j\Delta t), \qquad j = 1, 2, \ldots, k, \qquad (3.42)$$

where $t_{i-1}+k\Delta t = t_i$.

Let m_{ij} denotes the number of failures occurring in I_j^i over (t_{i-1}, t_i). Since we have assumed that all defects are independent of each other, it follows that a defects resulting in a failure will not have any influence on a defect which is found at PM, i.e. the number of failures since the last PM and the number of faults found at PM are also independent. This being so, the likelihood is simply

$$L = \prod_{i=1}^{l} \{P(n_i \ \ defects \ \ at \ \ t_i) \prod_{j=1}^{k} P(m_{ij} \ \ failures \ \ in \ \ I_j^i)\}. \tag{3.43}$$

Therefore, once the form of $f(h)$ has been specified, it is possible to obtain maximum-likelihood estimates for any unknown parameters. If there is no PM data available, the likelihood function is given by

$$L = \prod_{i=1}^{l} \prod_{j=1}^{k} P(m_{ij} \ failures \ in \ I_j^i). \tag{3.44}$$

After modelling parameters estimated via the maximum likelihood process, the estimated distributions can be compared to the corresponding sample distributions and the appropriate statistical tests-of-fit carried out. If statistical test fail, then the proposed models for u and h, for example Weibull distributions, would need to be revised. In this case, subjective measures of u and h may help to decide on appropriate models. Since the objective method utilizes the observational information, it could be demonstrated with a simulated data set featuring the number of defect identified at PM and times of failures, or number of failures per working day. The simulated data generated subject to a PM policy may be used to test whether the maximum likelihood method can recover the known model parameters. It will be discussed in detail in Chapter 5.

3.7 Discussion

This chapter has presented an overview of the delay time modelling. Over the past ten years, delay time modelling has undergone considerable development and is increasingly being accepted as an important concept for the practical modelling of maintenance of components and systems.

The key factor in delay time modelling is the estimation of modelling parameters, which include the initiation time and delay time. To estimate the parameter, two methods, namely subjective method and objective method, have been developed using the information obtainable from maintenance engineers or historical maintenance records. Both methods are beneficial to model maintenance problems. In the particular situation where objective maintenance data are not available, subjectively derived data can be reliably used as the basis for modelling. Otherwise, if there is only a small sample of objective data, a fusion of both methods would be beneficial due to an increased of data information.

Baker and Wang (1992) and Christer Wang, Baker, and Sharp (1995) have developed parameter estimation techniques using objective method for a single component system (or a system with only a few key components) and a system with a large number of components, respectively. Baker and Wang (1992) note that the individual behavior of each component can be modelled separately and a simple model constructed. As far as modelling a system is concerned, the restriction to a small number of components may not be restrictive, since a component can itself be a system. In Chapter 4, a combination of separate component models to form a system will be discussed and compared with an alternative multi-component system model.

Chapter 4

Modelling of Preventive Maintenance

4.1 Introduction

Single component models and multi-component models of preventive maintenance have been discussed in Chapter 2. In chapter 3, we discussed the delay time theory of modelling and parameter estimation. In this chapter, we develop further the discussion on the system PM models and component PM models.

Preventive maintenance techniques have been emphasized within industry over the past three decades due to complexity of systems, increased quality requirements and rising costs of material and labour. The main management issues for maintenance are to decide how best to cope operationally, that is to decide what to do and how to organize available resources to meet with the demand that exists. For instance, it might be advantageous to have an PM policy to group defects in time, and to some extent also by type, or to undertake some form of planned maintenance to reduce the cost consequences of defects in addition to grouping working activities in time. Therefore, a considerable decision aid for management would be a mechanism or model for identifying beforehand the envisaged consequences of different options and of different periods of operating.

Generally, maintenance encompasses planned and unplanned actions carried out to retain a system in or restore it to an acceptable condition. The aim of modelling planned preventive maintenance is to determine the optimal PM frequency or interval, that is, to minimize downtime, say, while providing for the most effective use of systems in order to secure the desired results in a cost effective way. PM policies have been called the most difficult maintenance operations to model (Christer and

Waller 1984b). The usual approach to the modelling of planned preventive maintenance operations is to build an expression for the profit, cost or availability rate. Denoting a policy by the PM interval or frequency, then the cost rate is calculated by dividing the total cost over the PM cycle by the cycle length. If one has a model of this cost, then the model may be optimized to yield an optimal PM interval.

In this chapter, we present the modelling of various PM policies based on downtimes and costs, and consider a practical approach to obtain expressions for the downtime of a system. In our modelling based upon the delay time concept, we focus on structures, or equipments, which perform specific functions on a continuous basis and which consist of one or more components subject to gradual deterioration. To evaluate the proper PM policies we consider downtimes and costs for different PM policies. Preventive maintenance may be called for in order to avoid high failure costs, but too much preventive maintenance is itself costly. Preventive repair may be undertaken when the actual condition has become bad enough or when it is profitable on economic grounds. Repair here may be assumed to be replacement because we consider the result of a repair is equivalent to starting with a new, identical component.

The problem is to determine a stationary repair strategy for the system as a whole, so as to minimize average system maintenance cost per unit of time in the long run. In doing so we take into account reductions in repair costs if repair of several components is coordinated. Another problem presented in this chapter is to allow for the actual system operating time for a system when the downtime of failure is not very small compared with the PM cycle length.

4.2 Nature of Maintenance Practice

The primary function of maintenance is to control the condition of equipment. Some of the problems associated with this include the determination of inspection frequencies and level, overhaul and repair practice, replacement times, manpower sizes, composition of machines in a workshop, spares provisioning rules and scheduling start time for constituent jobs of a maintenance project (Jardine, 1984). As indicated these many control actions are open to the maintenance manager. The effect of these actions cannot be looked at solely from their effect on the maintenance, since actions may seriously affect other units within a system, such as a production unit.

To illustrate the possible interactions of the maintenance function, consider the effect of decision to perform repairs only, and not do any preventive maintenance. This decision may well reduce the maintenance cost required by the maintenance department, but it may also cause considerable production downtime. Thus, mathematical models can be used to assist the maintenance manager balance these issues.

This chapter is more concerned with determining inspection intervals (or schedules), *i.e.* the points in time at which the inspection action should take place. Since the basic purpose of an inspection is to determine the state of equipment, some further maintenance action may be taken after the inspection, depending on the observed state. When the inspection should take place ought to be influenced by the costs of the inspection and the benefits of the inspection, such as detection and repair of minor defects before major breakdown occurs.

Figure 4.1 illustrates the type of approach taken using a mathematical model to determined the optimal frequency for inspecting equipment, that is balancing the maintenance cost against reduction in downtime. It is assumed in the figure that the objective is to minimize the expected cost per unit time from operating the equipment. This cost includes the total cost per unit time of inspection and maintenance, and the cost of failures, Figure 4.1. This figure assumes a steady state situation.

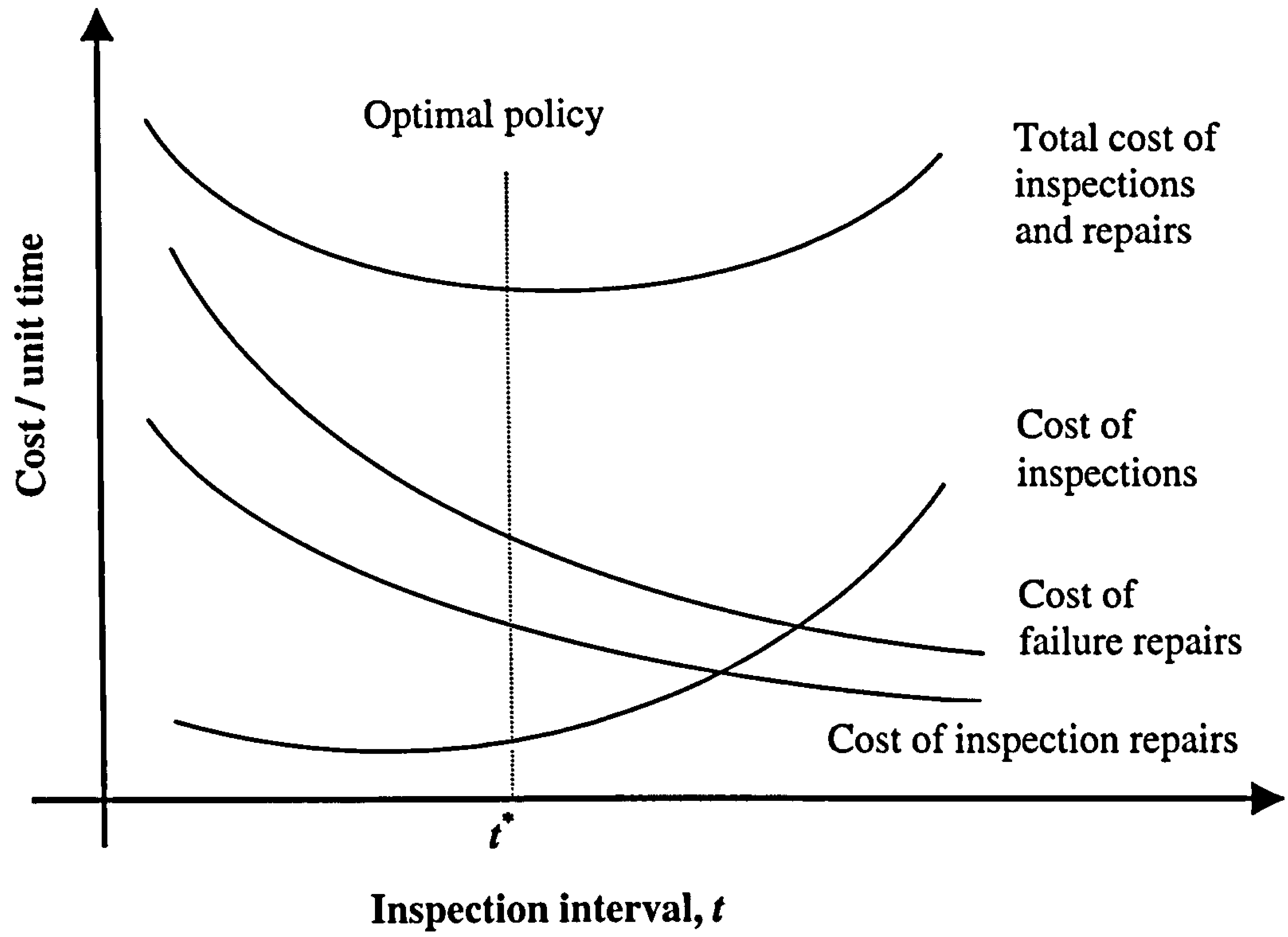


Figure 4.1. Cost curves of preventive maintenance

In maintenance, one can often detect failures in a system only by inspections, such as the failure of units in storage. In a perfect inspection model, all failures can be detected at the times of inspection. However, in many practical problems, one can not detect all the failures upon inspection. For example, the time allotted for inspection might not be enough to detect defects, not to mention the chance of misjudgment. This implies an imperfect inspection.

If actions on different components are combined, the set-up cost may be charged only once because of economic dependence. Although our model allows in principle for more levels of set-up costs, we shall define only two levels: a system set-up cost (e.g. of transport to the plant, administration, and handling), and a "component type" set-up cost per type of component (e.g. special equipment or skills required). If the repair of components of different types is coordinated, then the system set-up cost is reduced. If the repair of a set of components of the same type (which should be more or less identical, so that it is possible to consider one of them as representative) is coordinated, then both the set-up cost of a particular component type and the system set-up cost could be the same. Failure may involve a damage cost charged once per failure. The operating cost of a component functioning in working order may depend on its actual condition and may include the cost of having perfect information about the condition. We do not consider here any stochastic dependencies between components.

Here we recall the concept of system and note the definition 'A system is a set of discrete elements or components which are interconnected to perform one or more functions (Villemeur, 1992). Other terms such as device, socket, unit, part, subsystem and equipment can be found in the literature with definitions varying according to the industrial field they belong to. Therefore, for the sake of clarity, we will use the notion of part, component, and system in the thesis. From the definition above, a system consists of a set of components, and components are associated with parts. The part is an item which is not subject to disassembly. We assume that a system which, after failure to perform at least one of its required functions, can be restored to performing all of its required functions by some methods. We also consider that all components in the system are independent of one another, and an opportunity for a group repair of several components may be considered.

In practical, it is often more difficult to obtain maintenance data for parts than for components or a system. Specific parts normally have relatively few failures over a data collection period, and it is more complicated to identify the exact cause of failures than for lager units. Therefore, in our modelling case we will compare a systems model to a component set model.

This being so, the main management issues relate to deciding how best to cope operationally, that is how to organize available resources to meet with the demand that exists. For instance, it might be advantageous to have an inspection policy to group defects in time and to some extent by type, or to undertake some form of planned maintenance to reduce the cost consequences of defects in addition to grouping working activities in time. Again, a considerable decision aid for management would be a mechanism or model for identifying beforehand the envisaged consequences of different options and different periods of operating.

4.3 Modelling of Preventive Maintenance

4.3.1 General System Description for Modelling

Consider a system with m independently repairable components 1, ..., m. Each components is subject to the instantaneous constant rate of defect occurrence λ_i, i = 1, ..., m. and has a delay time distribution $f_i(h)$, (see Figure 4.2). It is noted that this configuration is termed series independency. It is assumed that if any one component fails, the system will not operate.

Since the individual components for a system may act independently, all components have different defect arrival rates and delay time distributions. Therefore, each component may give rise to different expected downtimes over a time epoch. For the series system, the inspection policy which minimizes the expected total average cost or downtime can be derived.

For modelling purpose, consider the general case of an inspection policy, which may be characterized by the following assumptions.

(1) The condition of the system can be observed by inspections only and a failure may be observed immediately at its occurrence.
(2) The component is repaired immediately upon failure, or at an inspection if a defect is identified, and no opportunistic further inspection of other component takes place.
(3) Inspections are perfect in that any defect present within the system will be identified.

(4) All defect identified at component inspection are repaired.

(5) The component is as good as new after repair.

(6) Defects are independent of each other and arise during use as a HPP with rate of occurrence of defects λ_j for component j and λ for the overall system of components where $\lambda = \sum_{j=1}^{m} \lambda_j$ and m is the total number of components.

(7) The total downtime due to failures is small compared the total operating time, and there is minimal error in assuming rates λ_j are per unit time and not per unit operating time.

(8) The delay time h of a defect is independent of the time of origin, and all defects for component j share a common delay time *pdf* $f_j(h)$ and *cdf* $F_j(h)$.

For this system, the PM policy which minimizes the expected total downtime and cost can be derived.

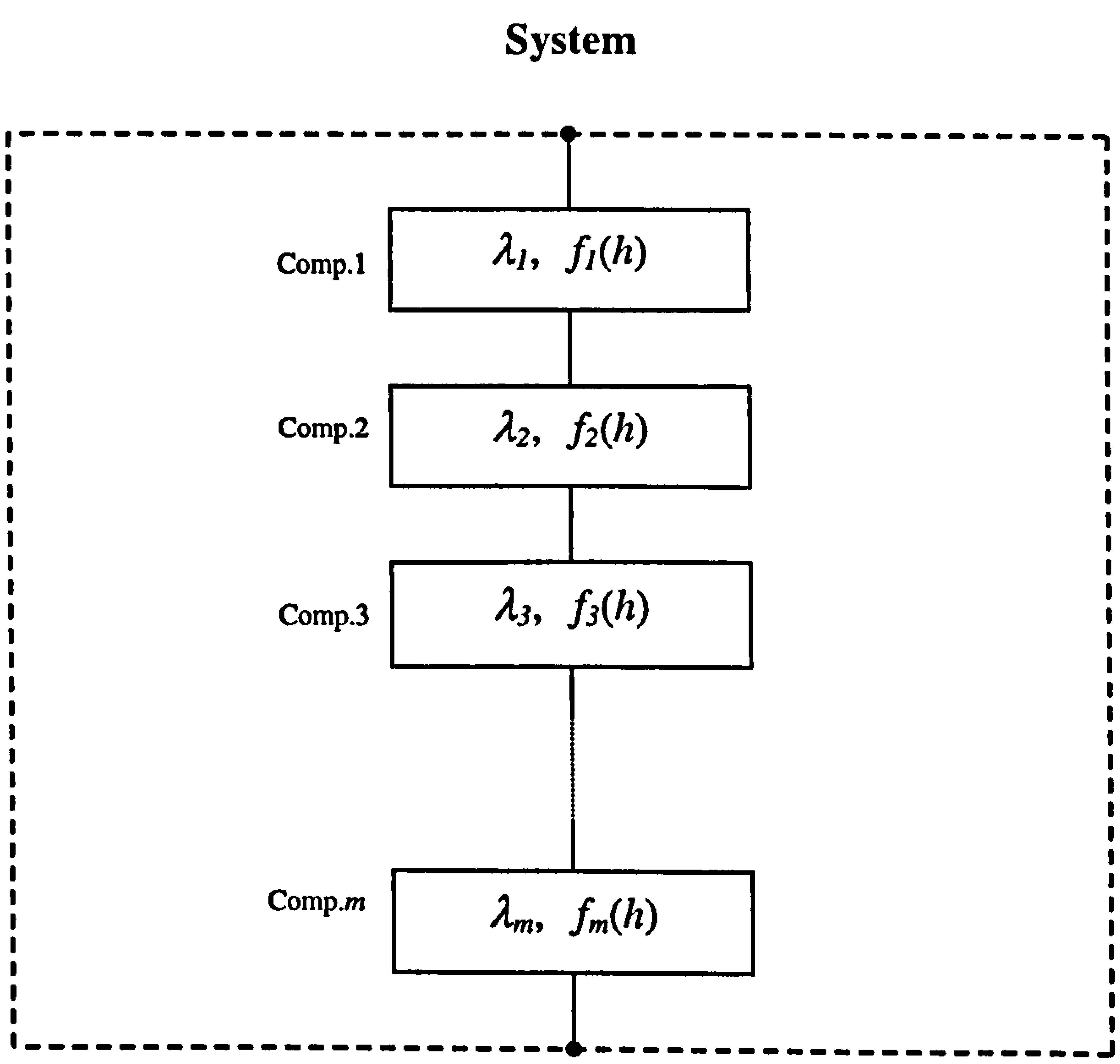


Figure 4.2. The configure of system which has several components

4.3.2 Component PM Model

Let T be the inspection interval, $F_j(h)$ be the *cdf.* of delay time of the component j and d_{pj} be the average inspection duration for checking the component j and repairing if any defects are found. Also, let C_{pj} denote the average inspection cost and C_{dj} denote the average repair cost per defect of component j at an inspection, d_{fj} denotes the average downtime per failure of component j and C_{fj} denote the average repair costs for a failure of component j. T, d_{fj} and d_{pj} are assumed to be expressed in the same units.

Now we have from Christer and Wang (1995) that the expected number of failures of component j over $(0, T)$ is given by

$$EN_{fj}(T) = \int_0^T \lambda_j F_j(h)dh, \tag{4.1}$$

where λ_j is the rate of occurrence of defects for component j and $F_j(u)$ is the *cdf.* of delay time of component j. Therefore, the expected total downtime of component j per unit time over an inspection period is

$$ED_j(T) = \frac{d_{pj} + d_{fj} EN_{fj}(T)}{T + d_{pj}}. \tag{4.2}$$

It follows that the total expected downtime per unit time of all components inspected on the inspection period T in an uncoordinated fashion, $ED_c(T)$, is given by

$$ED_c(T) = \sum_{j=1}^{m} ED_j(T). \tag{4.3}$$

If all component inspections were undertaken at the same time, there may be a reduction in total downtime. This is considered further later.

Similarly, the expected total cost of each component j per unit time over a full cycle of length $T + d_{pj}$ is

$$EC_j(T) = \frac{EN_{fj}(T)C_{fj} + (EN_{dj}(T) - EN_{fj}(T))C_{dj} + C_{pj}}{T + d_{pj}}, \tag{4.4}$$

where $EN_{fj}(T)$ is the expected number of failures over $(0, T)$ for component j, and $EN_{dj}(T)$ is the expected number of defect arising over $(0, T)$ for component j, namely, $\lambda_j T$. It follows therefore that the total expected cost of per unit time of all components inspected in an uncoordinated fashion on the inspection period T, $EC_c(T)$ is

$$EC_c(T) = \sum_{j=1}^{m} EC_j(T), \tag{4.5}$$

where m is the total number of components.

4.3.3 System PM Model

In delay time modelling applications, all defects arising in a component may be assumed to follow a common delay time distribution. The same delay time distribution may be assumed for different components. This is, however, normally only an approximation since individual components may act differently and, therefore, be represented by a different delay time distribution. However, if we assume $d_{fj} \approx d_{fk}$ for $j \neq k$, then a combined delay time distribution representing all failure types can be used where the combined delay time distribution is obtained by taking the weighted average of the delay time distribution of individual components using weight λ_j.

As discussed in previous chapter in section 3.6.3, parameter values are mainly derived from historical data. In many cases the period of observation of a system is short relative to the number of failures it experiences, so the effects of either synchronous or asynchronous sampling cannot be ignored in a completely rigorous analysis (Cox and Lewis, 1966). In practical situations, data which include date and times of downtime occurrence due to PM and failures is only available within limited time windows. If the downtimes associated with each failure can be provided, we can drive the average downtime over the operating time.

An estimate of the average downtime of all components may be estimated from historical data. The total number of failures in the data collection period and the total downtime can provide an estimate of the mean downtime due to failure for either each component or for the system. If data is not available, it may be estimated subjectively.

Let

d_f denotes the average downtime of all components, namely,

$$d_f = \sum_{j=1}^{m} \frac{d_{fj} \times \textit{No. of failures of component } j}{\textit{Total no. of failures of all components}}, \tag{4.6}$$

where m is the total number of components.

Since $F(h)$ denotes the combined *cdf.* of the delay time of all failure types, and $EN_f(T)$ is the expected number of failures of all components, we have

$$EN_f(T) = \int_0^T \lambda F(h)dh, \tag{4.7}$$

where $\lambda = \sum_{j=1}^{m} \lambda_j$.

Therefore, the expected total downtime of all components per unit time over an inspection period, T, is

$$ED_s(T) = \frac{d_p + d_f \int_0^T \lambda F(h)dh}{T + d_p}, \tag{4.8}$$

where $d_p = \sum_{j=1}^{m} d_{pj} \times a_{dp}$ and m is the total number of components, a_{dp} is a reduction factor representing the reduction in time achieved by group inspection. For instance, if $a_{dp} = 1$, there is no real reduction, and the model is equivalent to equation (4.3). However, if $a_{dp} = 1/m$, all components are attended to in parallel in the average time for one component. One might expect $a_{dp} \geq \frac{1}{[\max_j d_{pj}]}$.

To find the *cdf* $F(h)$ of a system, we recall that the instantaneous rate of occurrence of defects and the delay time of component j are λ_j and *cdf* $F_j(h)$, respectively from assumptions (6) and (8). For example, if we have a system which consists of two components, then the component 1 has λ_1, $F_1(h)$ and component 2 has λ_2, $F_2(h)$. Therefore, for the system λ, $F(h)$, we have

$$F(h) = [\frac{\lambda_1 F_1(h) + \lambda_2 F_2(h)}{\lambda_1 + \lambda_2}] \quad \text{and} \quad \lambda = \lambda_1 + \lambda_2. \tag{4.9}$$

Similarly, for m components system, the delay time for the system is given by

$$F(h) = \left\{ \frac{\sum_{j=1}^{m} \lambda_j F_j(h)}{\sum_{j=1}^{m} \lambda_j} \right\}. \tag{4.10}$$

For the inspection cost of the system, $C_p = \sum_{j=1}^{m} C_{pj} \times a_{dc}$, where a_{dc} is as before a reduction factor reflecting the reduction in cost by grouping. If $a_{dc} = 1/m$, then the failure and the inspection repair cost of the system are $C_f = \sum_{j=1}^{m} C_{fj} / m$, and $C_d = \sum_{j=1}^{m} C_{dj} / m$, respectively. Eitherway, the expected cost per unit time resulting from maintaining the unit on an inspection system of period T is $EC_s(T)$. That is, the expected total cost per unit time over PM interval $(0, T)$ for system is

$$EC_s(T) = \frac{EN_f(T)C_f + (EN_d(T) - EN_f(T))C_d + C_p}{T + d_p}, \tag{4.11}$$

where $EN_f(T)$ is the expected number of failures of system and $EN_d(T)$ is the expected number of defects arising over $(0, T)$. Since we assume the instantaneous rate of defect occurrence within system after inspection is constant, namely λ, the expected number of defect arising over $(0, T)$ is $EN_d(T) = \int_0^T \lambda du = \lambda T$ since the number of defects arriving in the interval for example $(u, u + du)$ is λdu.

4.3.4 Numerical Examples

- **Two components system case**

To simplify the problem, we firstly consider a case where PM is perfect and the delay time is exponentially distributed, namely for component j

$$F_j(h) = 1 - e^{-\alpha_j h}, \tag{4.12}$$

where $\frac{1}{\alpha_j}$ is the mean delay time of component *j*. First, we assume that a system has 2 series components and each component has different defect arrival rate and delay time distribution. The mean downtimes for each component per failure are assumed known.

We assume that the data is as shown below. Further, in inspection activities, the system components are repaired in parallel, which means inspection and repair are undertaken by duplicate repair teams consisting of two parallel capabilities. Now we calculate the expected downtime for the component model and system model.

For component 1

Defect arrival rate (λ_1) : 0.4 per unit time.

Delay time parameter (α_1) : 0.05 per hour.

Mean downtime per failure (d_{f1}) : 0.9 hours.

Mean duration of PM activity (d_{p1}) : 1.0 hour.

Inspection cost (C_{p1}) : 10 units.

Failure repair cost (C_{f1}) : 30 units.

Inspection repair cost (C_{d1}) : 2 units.

Since we assume that delay time distribution is exponential, the expected number of failures for component 1 over time (0, *T*) is given by $\int_0^T 0.4(1-e^{-0.05h})dh$, see equation (4.1). The expected downtime per unit time over PM interval (0, *T*) for component 1 can be obtained from equation (4.2).

For component 2

Defect arrival rate (λ_2) : 0.5 per unit time.

Delay time parameter (α_2) : 0.02 per hour.

Mean downtime per failure (d_{f2}) : 0.5 hours.

Mean duration of PM activity (d_{p2}) : 1.0 hour.

Inspection cost (C_{p2}) : 10 units.

Failure repair cost (C_{f2}) : 30 units.

Inspection repair cost (C_{d2}) : 2 units.

Similarly, the expected number of failures and the expected downtime for component 2 can be obtained from equations (4.1) and (4.2). The total expected downtime for two components in system, $ED_c(T)$, is the sum of the downtime of component 1 and component 2. That is,

$$ED_c(T) = ED_1(T) + ED_2(T). \tag{4.13}$$

This assumes there is no saving in inspecting both components at the same time.

For total system

In this system which consists of two components, from assumption (6) we have the defect arrival rate of system, $\lambda =$ 0.9 per unit time since we assume $\lambda = \lambda_1 + \lambda_2$ from assumption (6). From equation (4.10), we can obtain the *cdf.* of the combined delay time distribution, namely, $F(h) = 1 - 0.444e^{-0.05h} - 0.556e^{-0.02h}$. Also, costs are taken as inspection cost, C_p = 15 units, failure and inspection repair cost C_f = 30 units and C_d = 2 unit, respectively. Here as we mention before, the inspection downtime and cost will be reduced by a reduction factor reflecting the advantage of grouping. Here we assume that the inspection downtime of the grouping reduction factor, a_{dp} is 0.5 which d_p = 1 and the inspection cost of the grouping reduction factor, a_{dc} is 0.75. Then the expected total downtime for system per unit time can be also obtained from the equation (4.8), namely

$$ED_s(T) = \frac{d_p + d_f \int_0^T \lambda F(h)dh}{T + d_p}, \tag{4.14}$$

where $d_p = \sum_{j=1}^{m} d_{pj} \times a_{dp}$ and m is the total number of components.

Now, Figure 4.3 shows the expected total downtime per hour as a function of PM cycle length for each component and system model. For each component model, the optimal inspection interval of component 1 is 12 hours with an expected total downtime of 0.159 per hour, and the optimal inspection interval for component 2 is 22 hours with an expected total downtime of 0.089 per hour.

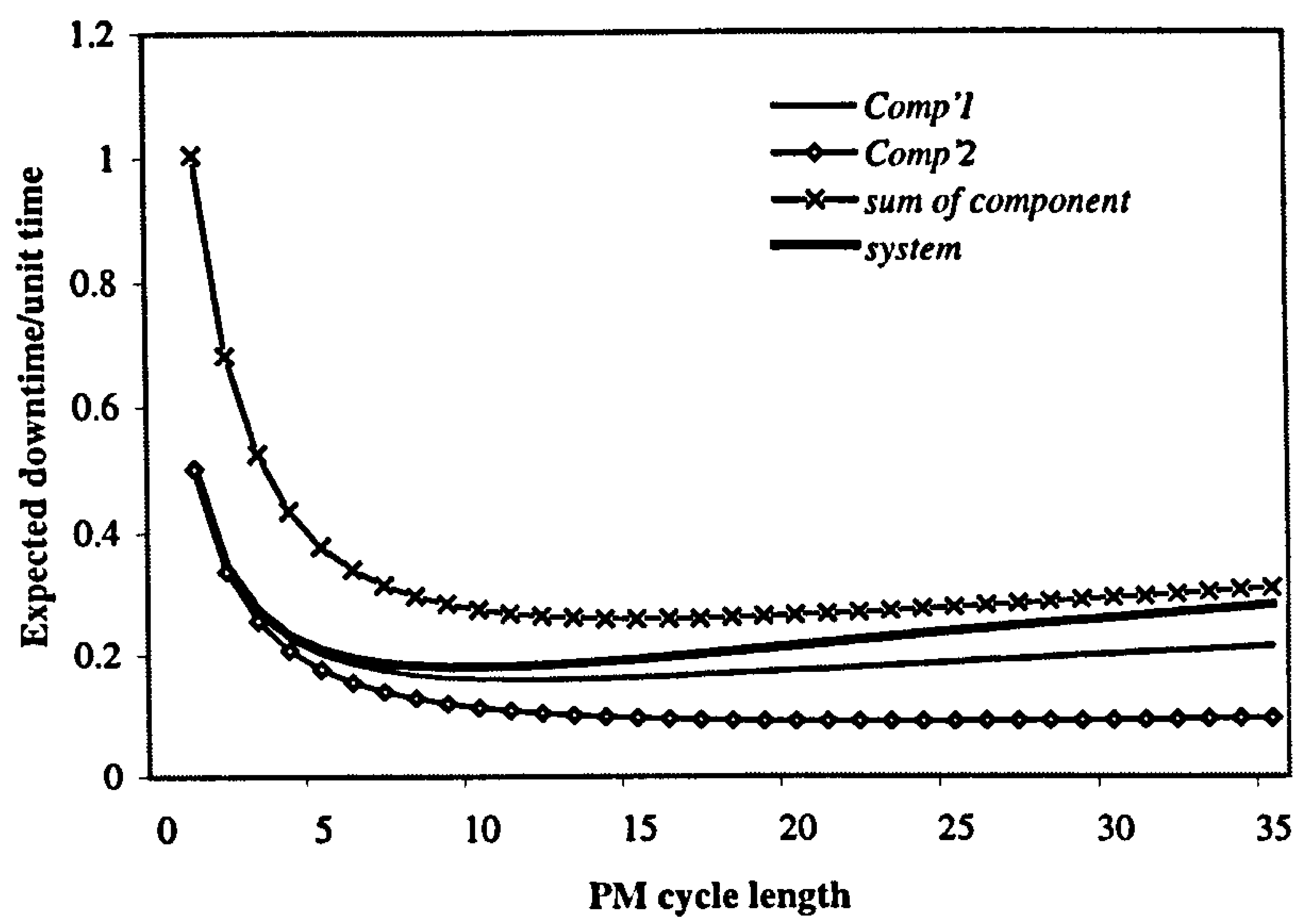


Figure 4.3. Expected total downtime per hour according to PM cycle length

The total expected downtime per unit time, using equation (4.3) for the two components is also shown in Figure 4.3. The expected total downtime for the sum of two components on a common inspection period is 0.257 per hour. For the system model, equation (4.8), is also shown in Figure 4.3, and the optimal inspection interval is 10 hours with an expected total downtime of 0.18 per hour. Since the system considered has an assumed partial parallel repair facility modelled via the reduction factor, a_{dp}, it is obvious that system PM model is more beneficial than each unit PM model. If we assume that the inspection reduction factor a_{dp} = 1, the optimal inspection interval of the system PM model is 14hours with an expected total downtime of 0.239 per hour. When the inspection reduction factor a_{dp} = 1, the system PM model is less downtime of that of sum of component PM model. Suppose for now that PM is carried out individually on each component, and the system considered here is a series system consisting of 2 components, then at the time of checking the first component, the system has to be shutdown. This implies that component 2 is in a non-production state while component 1 is checked, which evidently increases the total downtime.

Now we consider the cost model of this system. Curves of objective functions (4.4) and (4.11) are presented in Figure 4.4 for the above data. This figure illustrates expected total cost for the two components model and the system model. For component 1, the optimal PM interval is 5 hours with an expected total cost of 3.409 per hour, and for unit 2, the optimal PM interval is 7 hours with an expected total cost of 2.944 per hour. For the system model, equation (4.11), the optimal inspection interval is 5 hours with an expected total cost of 5.6395 per hour. It is noted that the PM cost of system PM model may less than that of sum of component PM model with the same value of the grouping reduction factor $a_{dc} = 0.75$.

It is useful to determine the sensitivity of the optimal decision policy to the input data for various factors such as maintenance downtime, or costs.

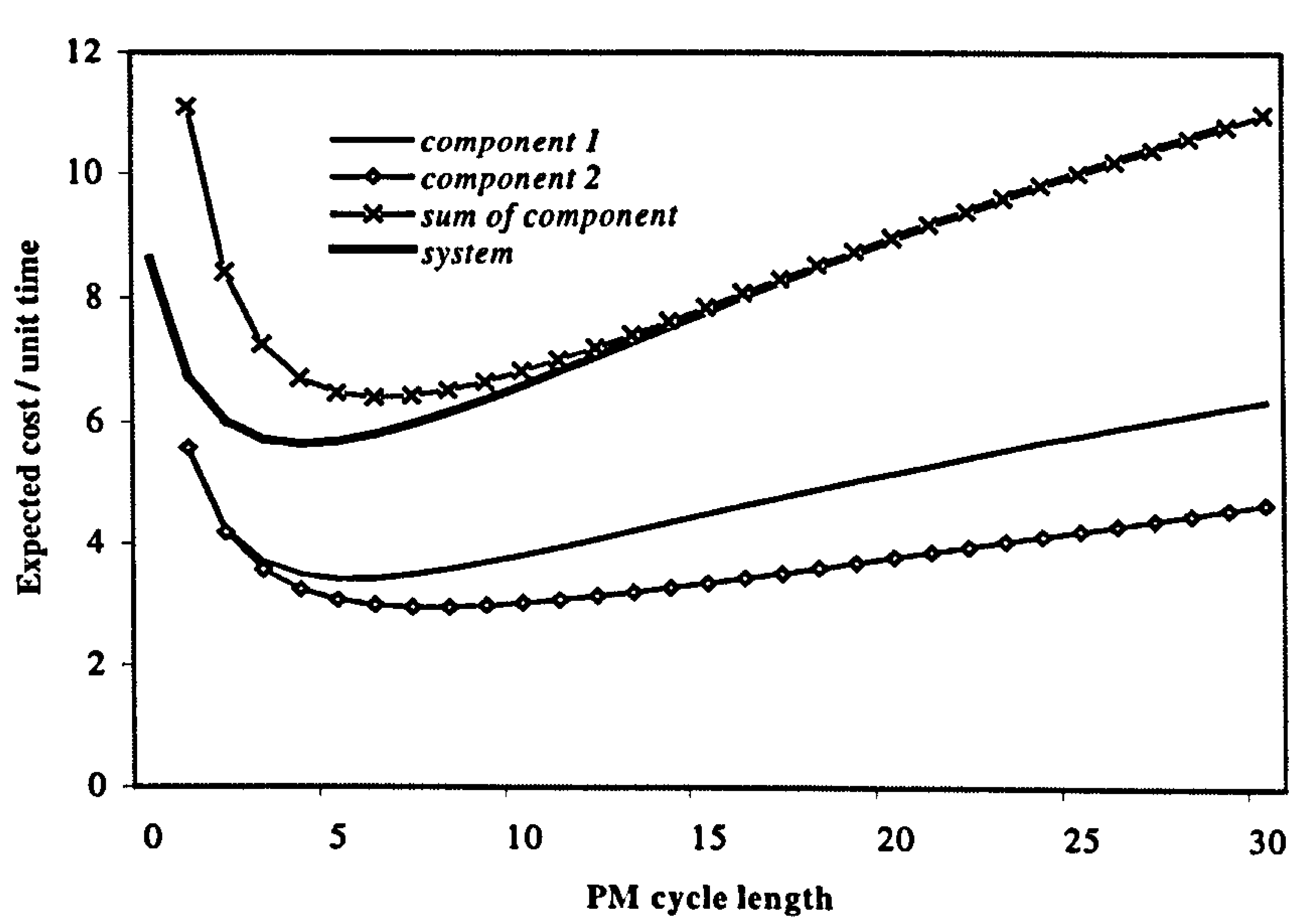


Figure 4.4. Expected total cost per unit time

Here, we investigate the sensitivity of inspection downtime for the two models, the sum of components PM model and system model for inspection time.

In a system model, since all components in the system are checked in parallel, the downtime due to PM is less than or equal to that for a component based PM model. To know how much additional PM time is available by grouping and paralleling PM activities without changing the total downtime, let the inspection downtime, $d_p^* = (\sum_j d_{pj} \times a_{dp}) + \theta$, where $a_{dp} = 0.5$,

and then

$$f(\theta) = ED_s(T;\theta) - ED_c(T) = 0, \tag{4.15}$$

where $ED_s(T;\theta) = \dfrac{(d_p + \theta) + EN_f(T)d_f}{T + (d_p + \theta)}$,

and $ED_c(T) = \dfrac{d_{p1} + EN_{f1}(T)d_{f1}}{T + d_{p1}} + \dfrac{d_{p2} + EN_{f2}(T)d_{f2}}{T + d_{p2}}$.

The θ obtained from equation (4.15) is the additional time gained by doing PM in parallel.

We also use the same data above for two components with equations (4.13) and (4.14). From equations (4.13) and (4.14), when the inspection interval is 10 time units the system PM model is optimum with an expected total downtime per unit time of 0.18 per hour and the sum of component PM model has an expected total downtime per unit time is 0.273 per hour at the same inspection interval as 10 hours. Therefore, the system PM model already has more benefit as the difference with 0.077 per hour than the component PM model for the expected total downtime per unit time. Figure 4.5 shows the sensitivity analysis for optimal downtime of both models. In this case, an additional time (θ) gained by doing PM in parallel is 1.4 hours. Therefore we may know that how much time can save from PM activities by grouping or paralleling.

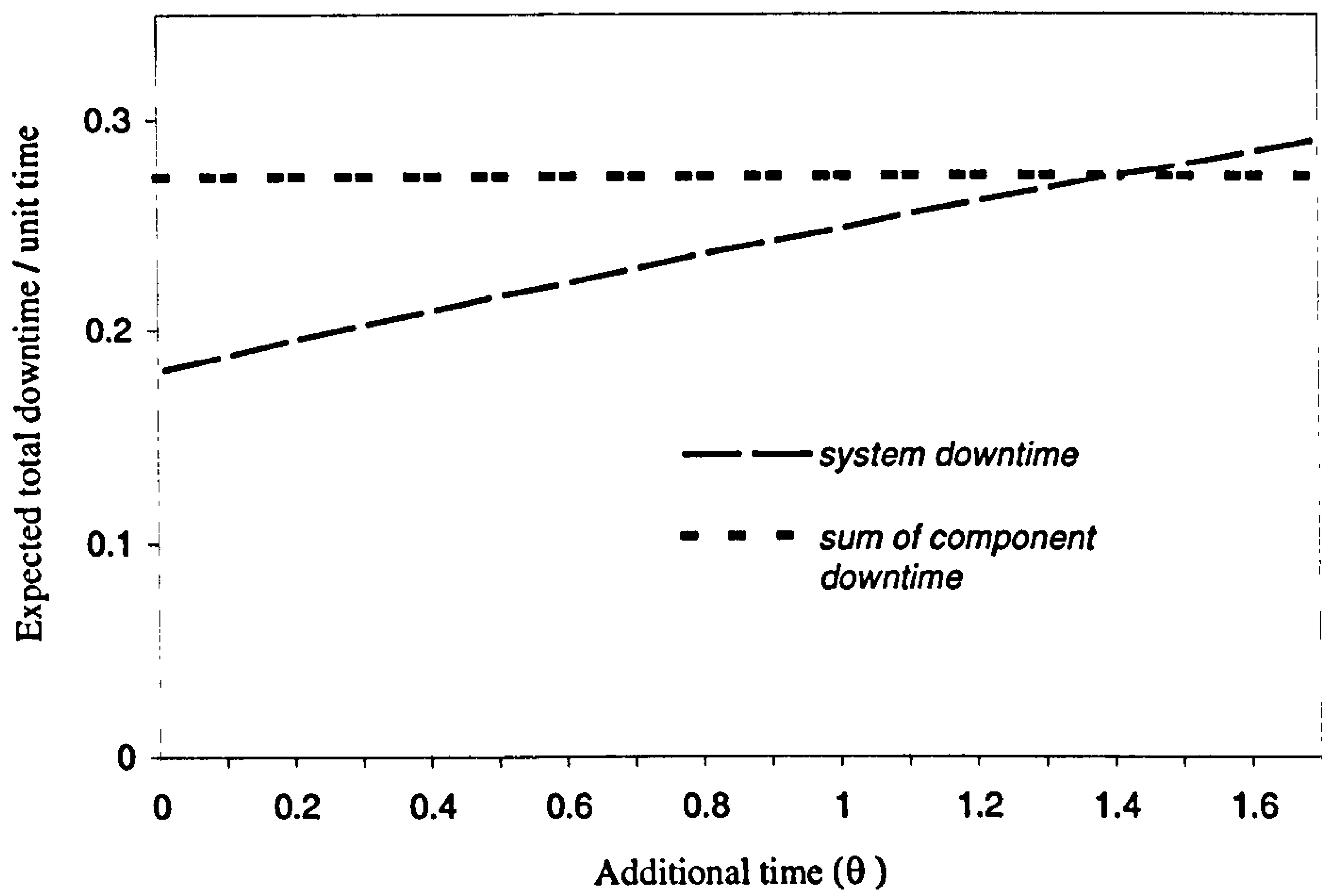


Figure 4.5. Sensitivity curve to System model when d_{p1} =1, d_{p2} = 1 and d_p is considered as a 1 with the reduction factor a_{dp} = 0.5

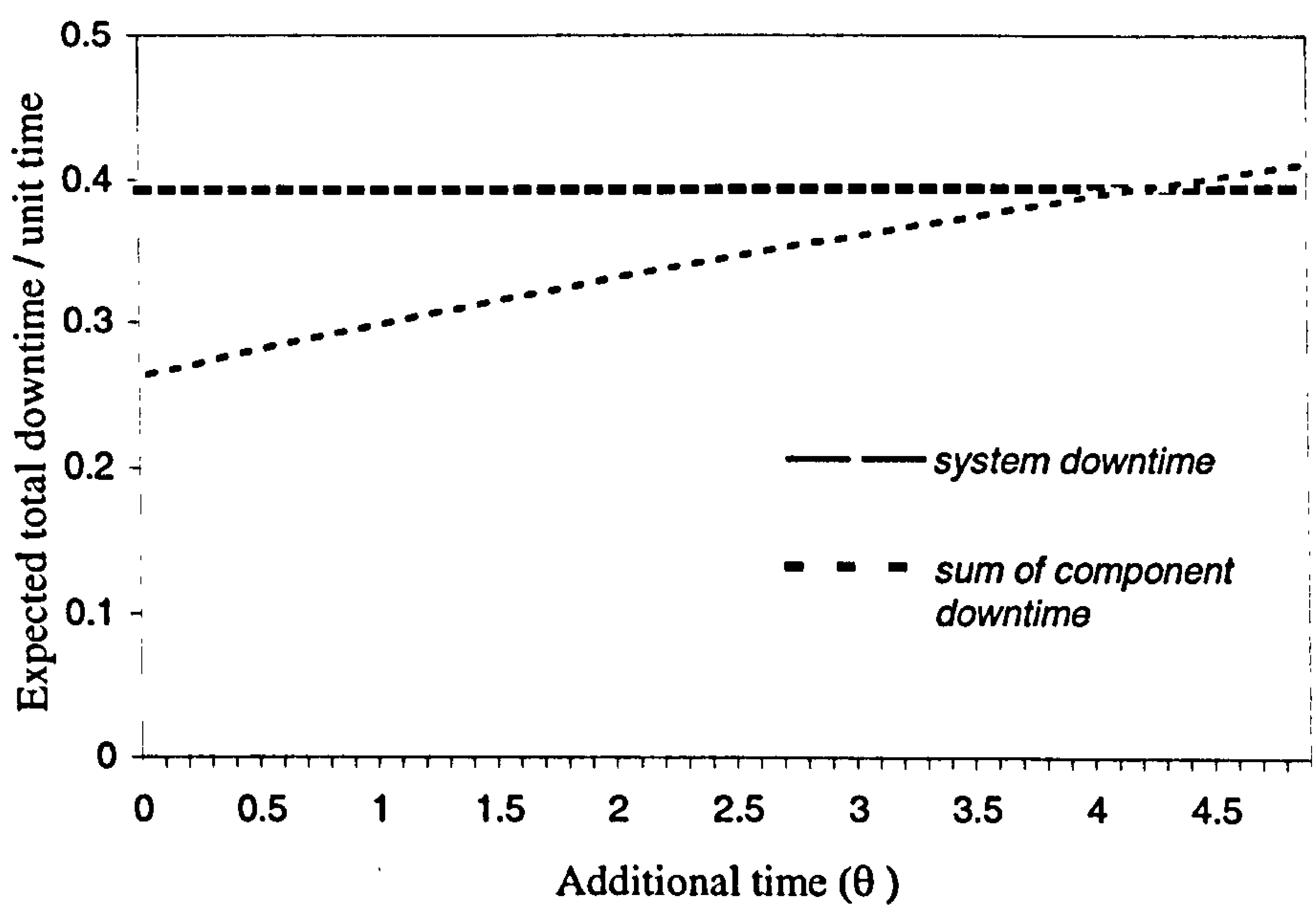


Figure 4.6. Sensitivity curve to System model when d_{p1} = 2, d_{p2} = 3 and d_p is considered as 2.5 with the reduction factor a_{dp} = 0.5

Figure 4.6 also shows the result when each component of a system has a different inspection time as the component 1 has 2 hours and the component 2 has 3 hours, and the inspection time for a system PM model is considered as 2.5 hours with the reduction factor, $a_{dp} = 0.5$ from equations (4.13) and (4.14).

When the optimal inspection interval for a system PM model is 17 hours, the expected total downtime is 0.262 per hour, and the expected total downtime of component PM model is 0.393 per hour at the same inspection interval with the system PM model. Therefore, we obtain the value of θ as 4.2 from equation (4.15). That is, a system PM model can share the workforce as much as the value of θ.

- **More complex components system case**

In maintaining a more complex component system, there are usually several policy options for decision alternatives available to management for consideration. There is, for example, the possibility of reducing downtime or costs for maintenance and repairs. However, it will not always be possible or economically sensible, to reduce to any extent the level of maintenance demand. Here, we consider the various PM policies for a system which consisting of 3 components. Using the above components, we add one more component, namely, component 3, to the system. Component 3 information and data is assumed as follows:

For component 3

Defect arrival rate (λ_3) : 0.7 per unit time.

Delay time parameter (α_3) : 0.01 per hour.

Mean downtime per failure (d_{f3}) : 0.35 hours.

Mean duration of PM activity (d_{p3}) : 1.0 hour.

Inspection cost (C_{p3}) : 10 units.

Failure repair cost (C_{f3}) : 30 units.

Inspection repair cost (C_{d3}) : 2 units.

First we consider 4 different PM policies of this system with assuming the perfect inspection. Here we assume that for inspection reduction factor in combining of components will use the longest inspection time of components. And the inspection cost of the grouping reduction factor, a_{dc} is assumed as 0.75 and the failure and the inspection repair cost of the system are also assumed as $a_{dc} = 1/m$, where m is the number of components. The average downtime of all combined components is applied as same equation as (4.6).

- Case model I : Separate component PM policy model (Com.1, Com.2 and Com.3).

  Each component in the system has PM performed independently. Here we assume that each component has a different inspection time, d_{pj}, but identical inspection cost C_{pj}.

- Case model II : Mixed PM policy model (Com.1 + Com.2 combined, and Com.3 separate).

Component1 and Component2 are considered as a system PM model and Component3 is considered an independent component PM model. Therefor for component 1 and 2 the inspection reduction factor is applied, which is the longest inspection time of components, the inspection cost of the grouping reduction factor, a_{dc} is applied as 0.75, the failure and the inspection repair cost of the system are applied as 0.5.

- Case model III : System PM policy model (Com.1+Com.2+Com.3).

  All the components are considered as a system PM model. Therefore, this case model is assumed that the downtime of inspection is longest time of components, that is the system PM policy has a parallel maintenance facility.

- Case model IV : Combined PM policy and failure based model (Com.1 + Com.2 combined, and No PM for Com.3).

  This case model assumes that Component1 and Component2 are considered as a system PM model and Component3 has no PM plan, but is subject to breakdown maintenance only.

Now from the downtime models and cost models for components, namely equations (4.2), (4.4), (4.8) and (4.11), we can obtain the downtime model and cost model for each of the above cases.

Here, for demonstration we let the mean inspection downtimes for each component be d_{p1} = 1 hour, d_{p2} =1.2 hours, d_{p3} = 0.5 hours. We also assume that the inspection downtime when grouping PMs is the maximum of the inspected component's individual inspection downtime. For the reduction factor of grouping cost for inspection we take, $a_{dc} = 0.75$.

For Case model I (separate component PM policy model), the expected total downtime model and cost mode are given by

$$ED_I(T) = \sum_{j=1}^{3} ED_j(T), \tag{4.16}$$

where $ED_j(T) = \dfrac{d_{pj} + d_{fj} EN_{fj}(T)}{T + d_{pj}}$, where $EN_{fj}(T)=\int_0^T \lambda_j F_j(h)dh$, and $F_j(h)=1-e^{-\alpha_j h}$

and

$$EC_I(T)=\sum_{j=1}^{3} EC_j(T), \tag{4.17}$$

where $EC_j(T)=\dfrac{EN_{fj}(T)C_{fj}+(EN_{dj}(T)-EN_{fj}(T)C_{fj}+C_{pj}}{T+d_{pj}}$ and $EN_{dj}(T)=\lambda_j T$.

For Case model II (Mixed PM policy model), the expected total downtime model and cost mode also given by (Since $d_{p1}<d_{p2}$)

$$ED_{II}(T)=ED_s(T)+ED_3(T)=\frac{d_{p2}+d_{f1,2}EN_{f1,2}(T)}{T+d_{p2}}+\frac{d_{p3}+d_{f3}EN_{f3}(T)}{T+d_{p3}}, \tag{4.18}$$

where $EN_{f1,2}(T)=(\lambda_1+\lambda_2)\int_0^T F_{1,2}(h)dh$, $EN_{f3}(T)=\lambda_3\int_0^T F_3(h)dh$,

$F_{1,2}(h)=\left\{\dfrac{(\lambda_1 F_1(h)+\lambda_2 F_2(h))}{(\lambda_1+\lambda_2)}\right\}$, $F_3(h)=1-e^{-\alpha_3 h}$ and

$$d_{f1,2}=\frac{d_{f1}\times No.of\ failures\ of\ component\ 1+d_{f2}\times No.of\ failures\ of\ component\ 2}{Total\ No.of\ failures\ of\ component\ 1,2},$$

and

$$EC_{II}(T)=EC_s(T)+EC_3(T), \tag{4.19}$$

where $EC_s(T)=\dfrac{EN_{f1,2}(T)C_{f1,2}+(EN_{d1,2}(T)-EN_{f1,2}(T))C_{d1,2}+C_{p1,2}}{T+d_{p2}}$,

$EN_{d1,2}(T) = (\lambda_1 + \lambda_2)T$, where $C_{f1,2} = (C_{f1} + C_{f2})/2$, $C_{d1,2} = (C_{d1} + C_{d2})/2$, and $C_{p1,2} = (C_{p1} + C_{p2}) \times a_{dc}$

and $EC_3(T) = \dfrac{EN_{f3}(T)C_{f3} + (EN_{d3}(T) - EN_{f3}(T))C_{d3} + C_{p3}}{T + d_{p3}}$,

where $EN_{d3}(T) = \lambda_3 T$, $EN_{f3}(T) = \lambda_3 \int_0^T F_3(h)dh$.

For Case model III (System PM policy model), the expected total downtime model and cost model is given by (Since assumed that $d_{p3} < d_{p1} < d_{p2}$)

$$ED_{III}(T) = ED_s(T) = \frac{d_{p2} + d_f EN_f(T)}{T + d_{p2}} \tag{4.20}$$

and

$$EC_{III}(T) = \frac{EN_f(T)C_f + (EN_d(T) - EN_f(T))C_d + C_p}{T + d_{p2}}, \tag{4.21}$$

where $EN_d(T) = \sum_{j=1}^{3} \lambda_j T$, $EN_f(T) = \sum_{j=1}^{3} \lambda_j \int_0^T F(h)dh$,

$F(h) = \left\{ \dfrac{(\lambda_1 F_1(h) + \lambda_2 F_2(h) + \lambda_3 F_3(h))}{(\lambda_1 + \lambda_2 + \lambda_2)} \right\}$ and $C_f = (C_{f1} + C_{f2} + C_{f3})/3$, $C_d = (C_{d1} + C_{d2} + C_{d3})/3$, and $C_p = \sum_{j=1}^{3} C_{pj} \times a_{dc}$.

For Case model IV (Combined PM policy and failure based model), the expected total downtime model and cost mode also given by (Since $d_{p1} < d_{p2}$ and $d_{p3} = 0$)

$$ED_{IV}(T) = \frac{d_{p2} + d_{f1,2}EN_{f1,2}(T)}{T + d_{p2}} + \frac{d_{f3}EN_{f3}(T)}{T} \quad (4.22)$$

where

$$d_{f1,2} = \frac{d_{f1} \times No.of\ failures\ of\ component\ 1 + d_{f2} \times No.of\ failures\ of\ component\ 2}{Total\ No.of\ failures\ of\ component\ 1,2},$$

$EN_{f3}(T) = \lambda_3 \int_0^T F_3(h)dh$ and

$$EC_{IV}(T) = \frac{EN_{f1,2}(T)C_{f1,2} + (EN_{d1,2}(T) - EN_{f1,2}(T))C_{d1,2} + C_{p1,2}}{T + d_{p2}} + \frac{EN_f(T)C_{f3}}{T} \quad (4.23),$$

where $C_{f1,2} = (C_{f1} + C_{f2})/2$, $C_{d1,2} = (C_{d1} + C_{d2})/2$, and $C_{p1,2} = (C_{p1} + C_{p2}) \times a_{dc}$.

Figures 4.7 to 4.14 illustrate the behavior in expected downtimes and costs per unit time for various PM policy of the system.

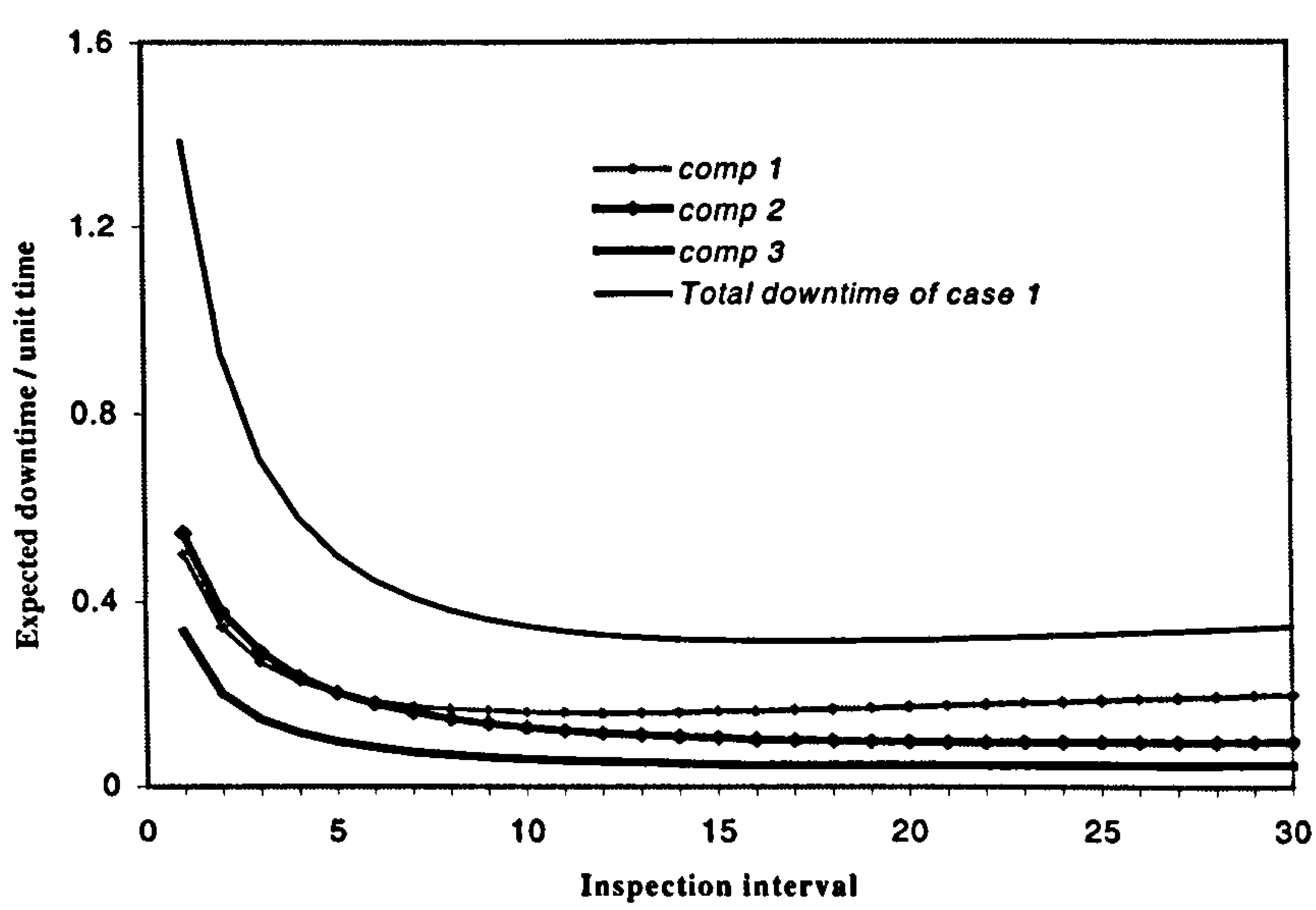


Figure 4.7. The expected total downtime for case Model I

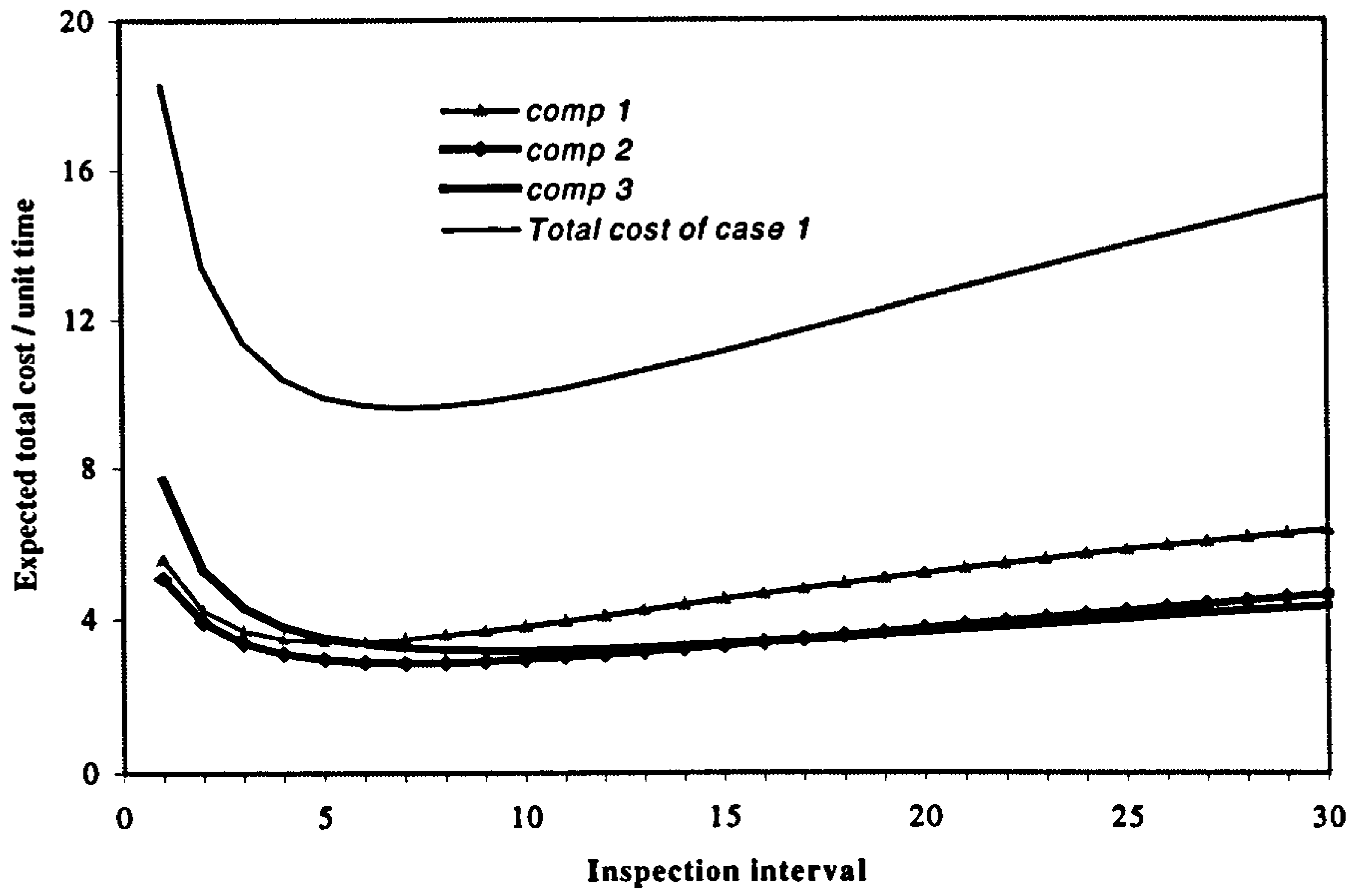


Figure 4.8. The expected total cost for case Model I

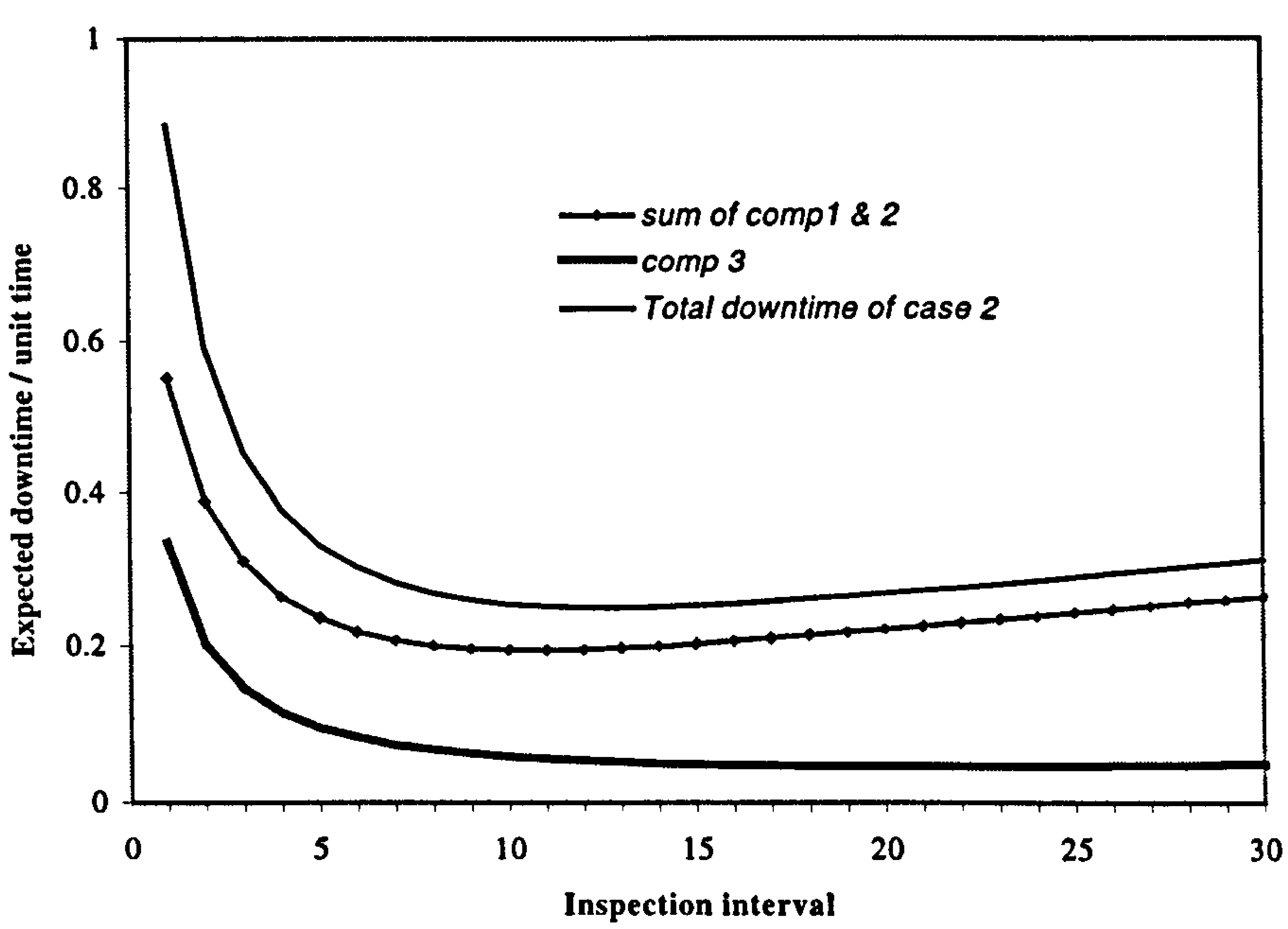


Figure 4.9. The expected total downtime for case Model II

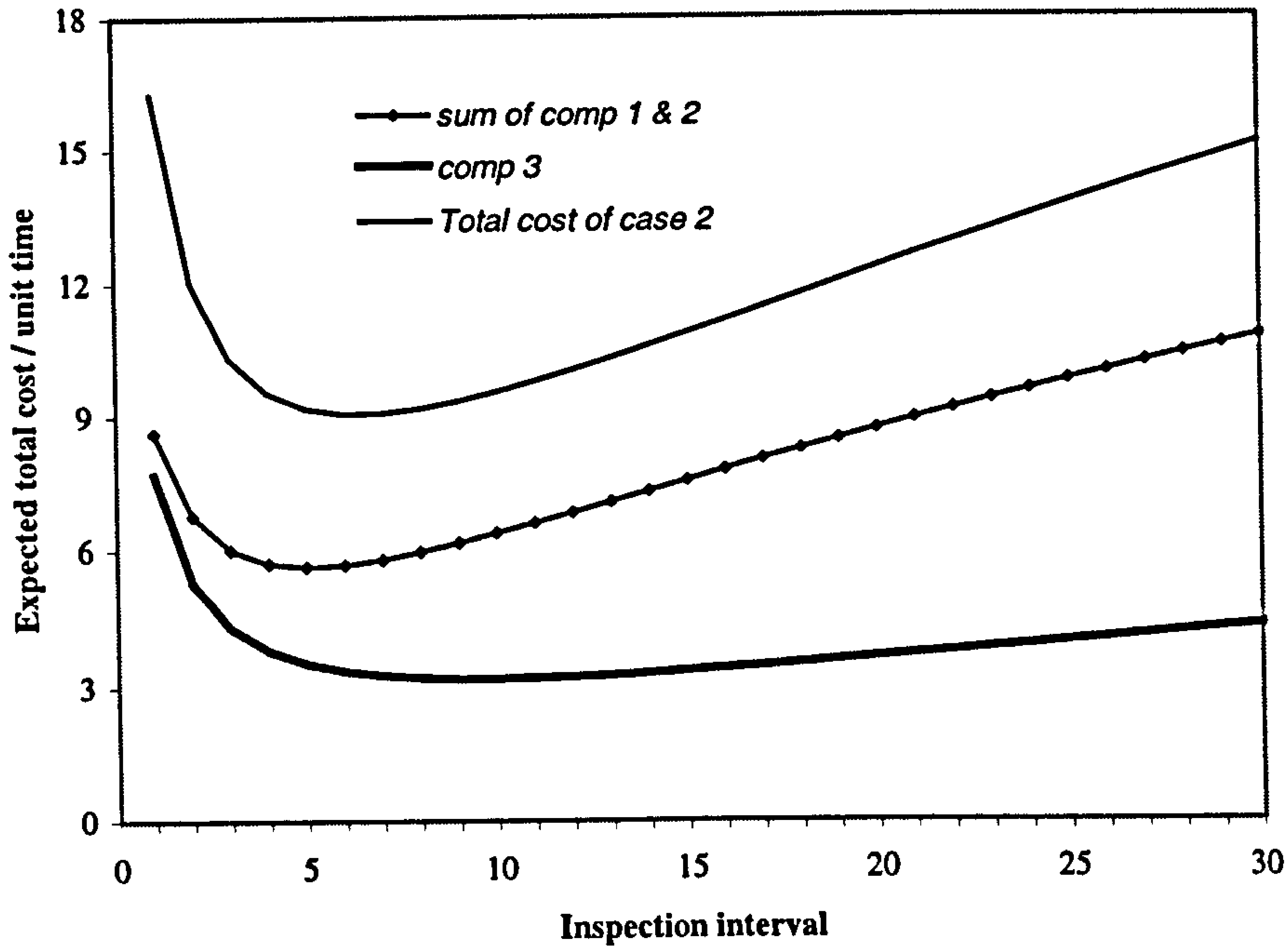


Figure 4.10. The expected total cost for case Model II

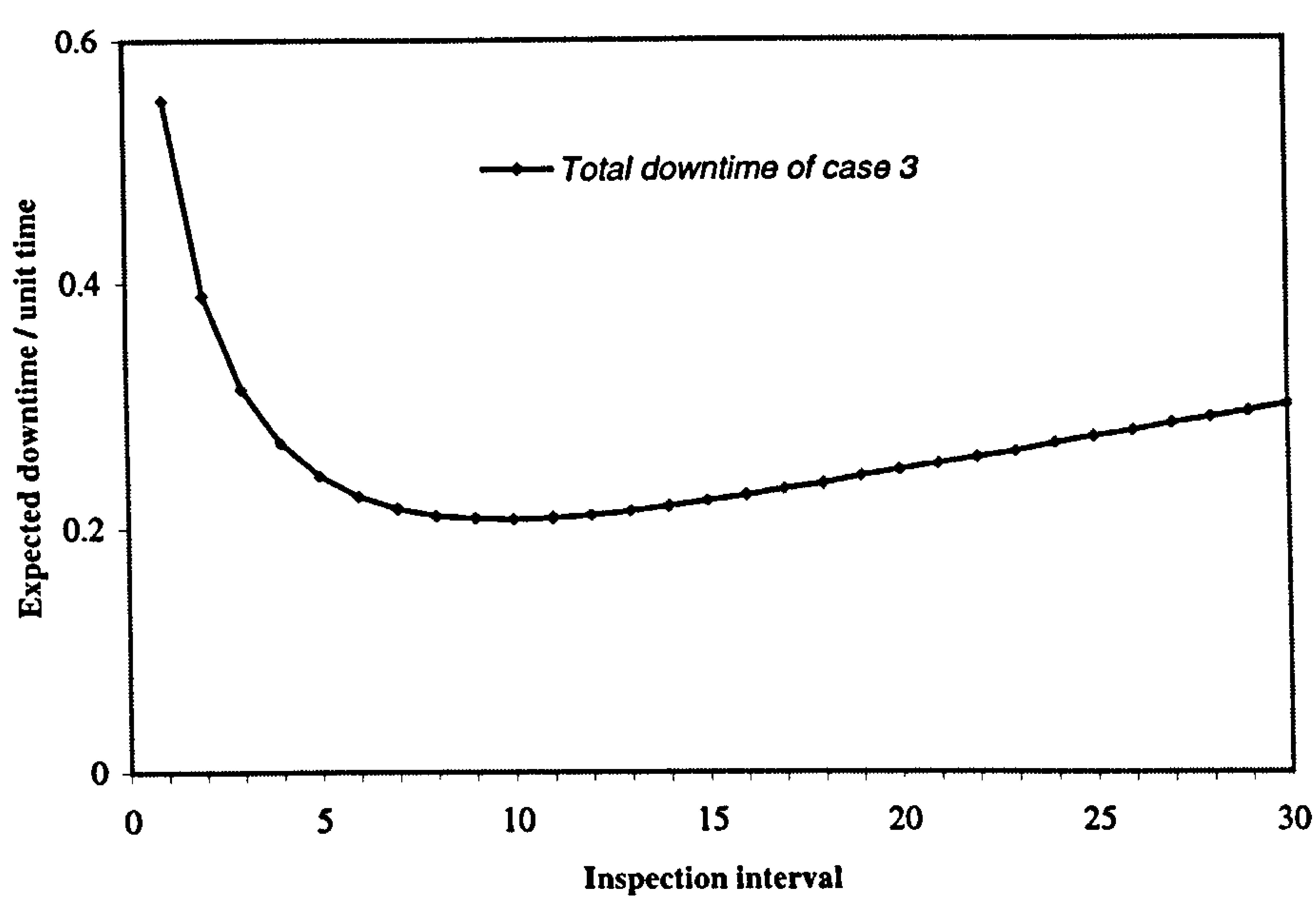


Figure 4.11. The expected total downtime for case Model III

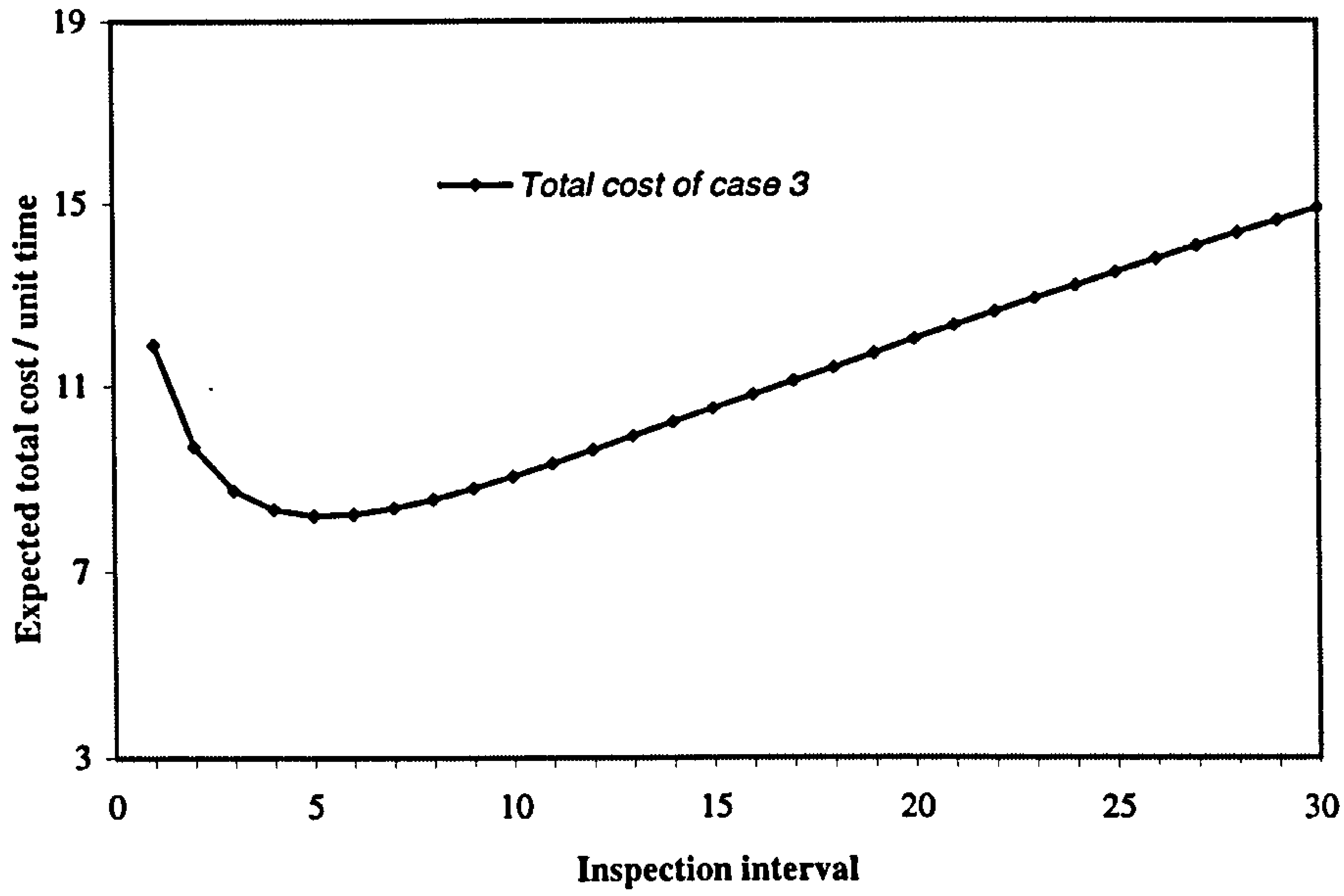


Figure 4.12. The expected total cost for case Model III

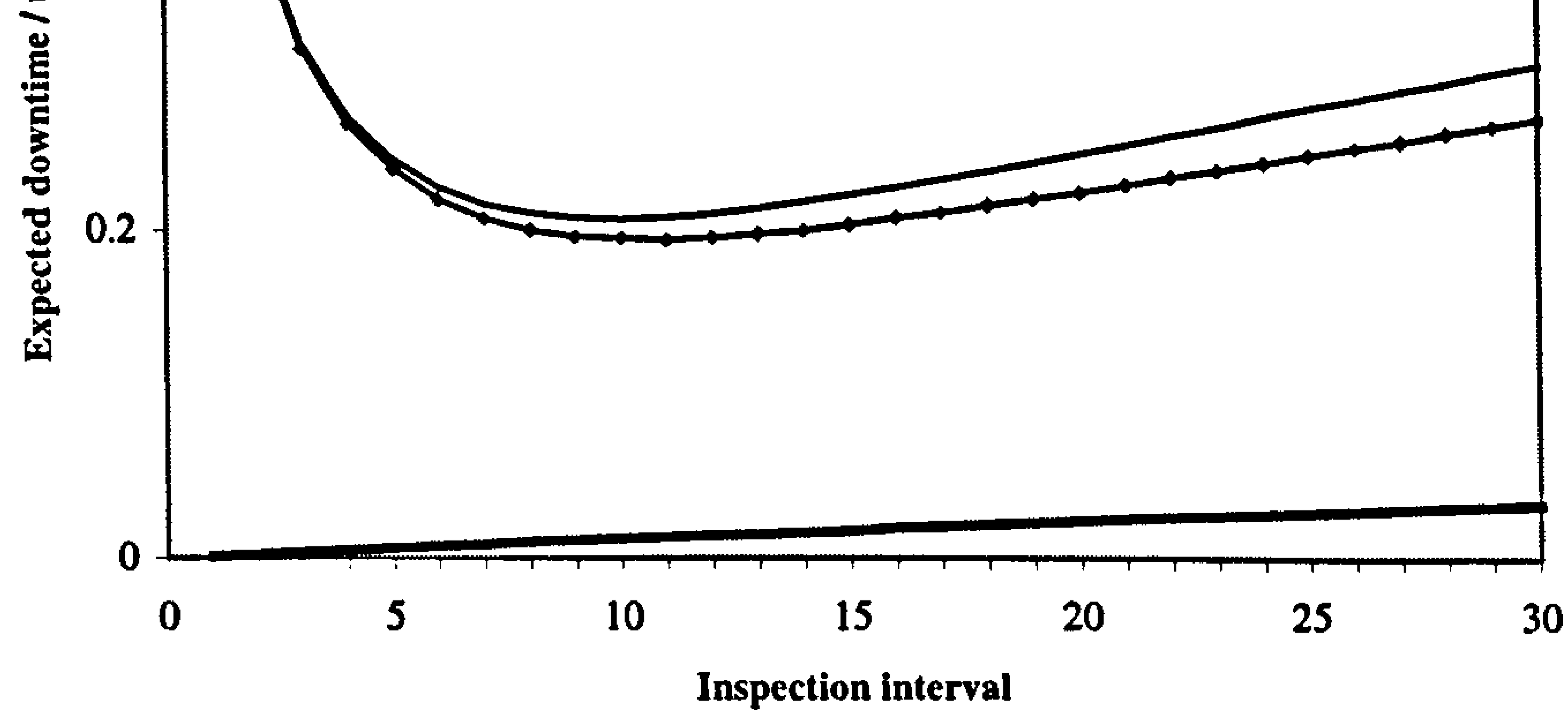


Figure 4.13. The expected total downtime for case Model IV

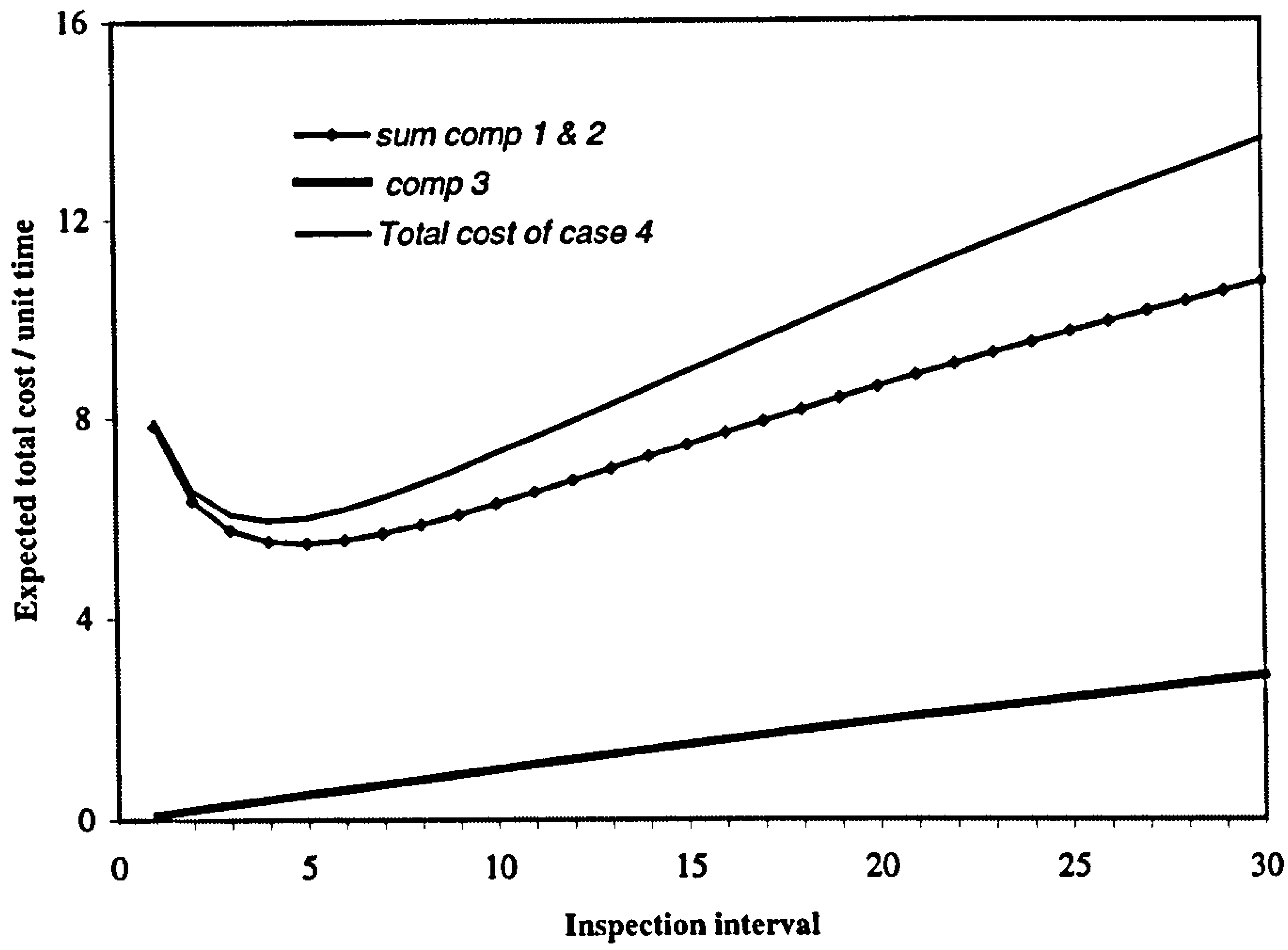


Figure 4.14. The expected total cost for case Model IV

From the various PM policies model, we can evaluate the case models in terms of downtimes and cost. In Figure 4.15 we can notice that the system PM model III is the best for reducing downtime. This is because we have assume that all components in the system are checked by parallel workforces, that is the inspection time, d_p, of the system PM model, namely Case model III, can be reduced by parallel inspections and repairs. It is also evident that Case model IV has very little difference in terms of the values of downtime with the Case model IV. The reason is that the total downtime over the PM interval for the Case model IV is only slightly influenced by the total value of downtime, since we considered that inspection time, d_p, is the maximum individual inspection time of the combined components, which is component 2, namely $d_{p2} = 1.2$. Also, in figure 4.13 we can seen that the expected total downtime of component 3, which is failure based maintenance, is relatively low. Therefore, the expected total downtime over the PM period of the Case model IV is not much relevant to the Case model III.

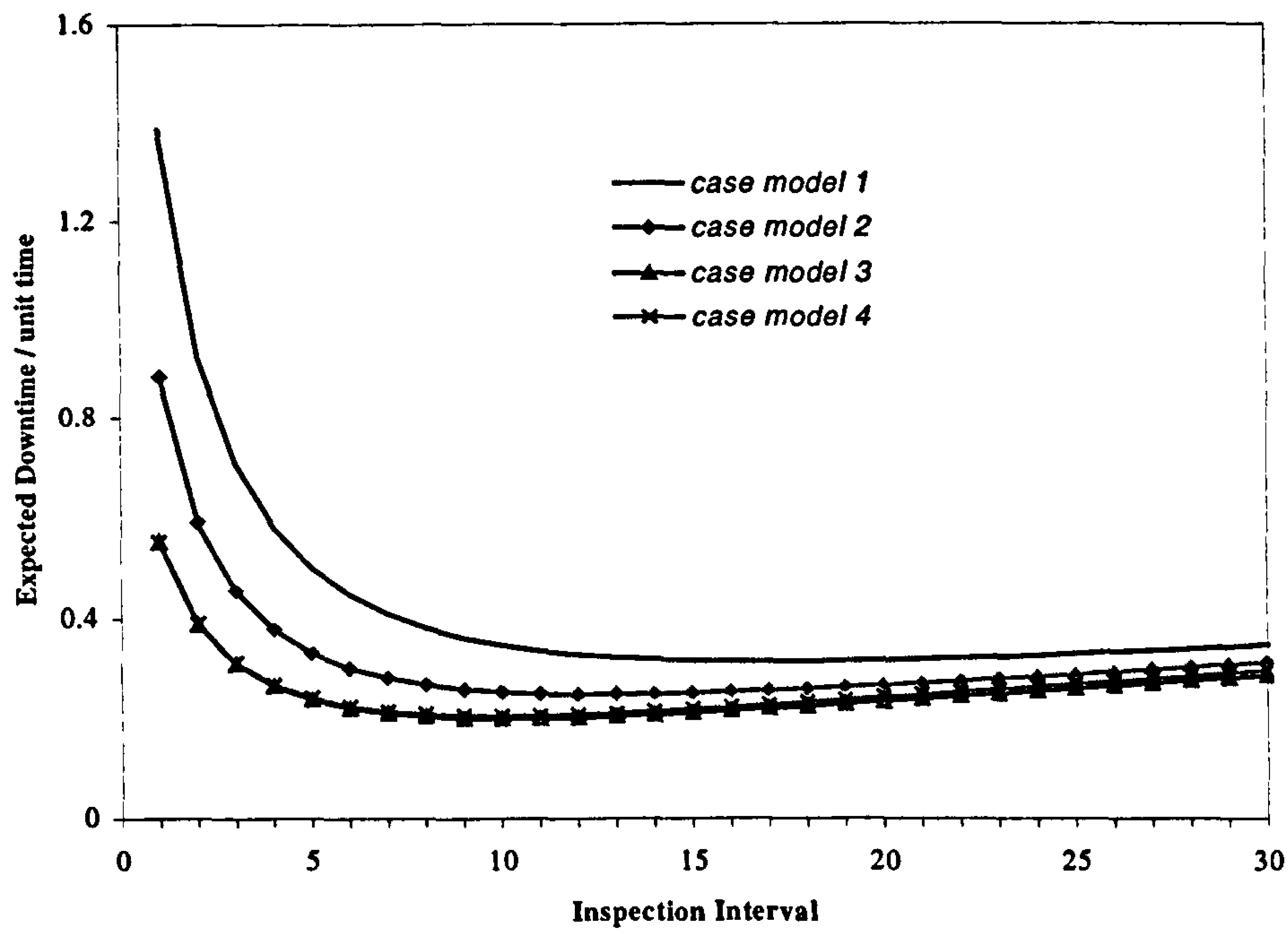


Figure 4.15. Expected total downtime per unit time for all models

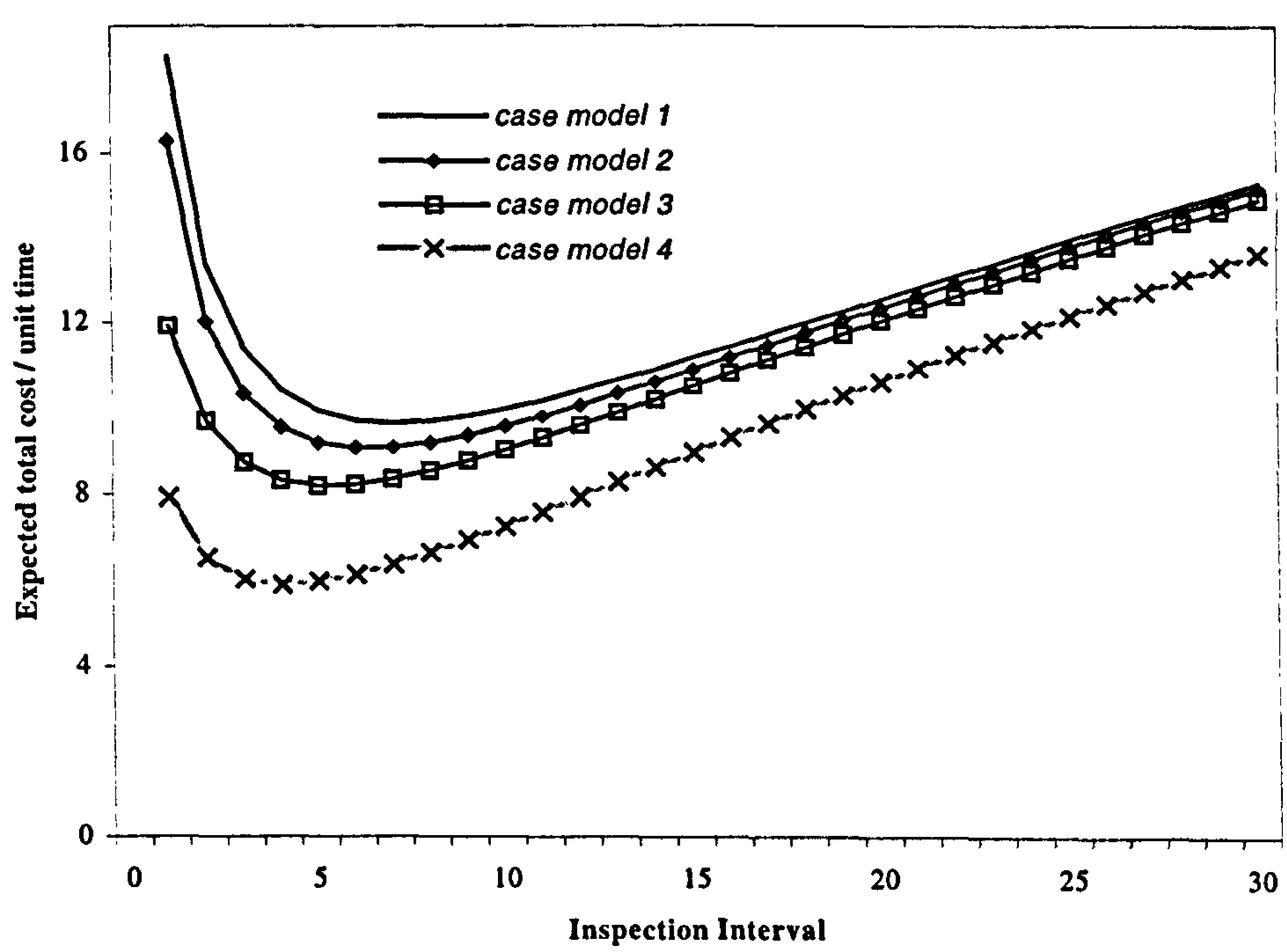


Figure 4.16. Expected total cost per unit time for all models

By contrast, in Figure 4.16, Case model IV is the best model, which has the lowest cost per unit time. In the Case model IV, we only considered the inspection cost of the component 1 and 2 since component 3 has breakdown maintenance only, with the rate of failure for component 3 the lowest. Therefore the expected total cost per unit time for the Case model IV is much smaller than that of the other Case models.

Again from figure 4.15, we can see that curves of the Case model III and Case model IV are very close to the values of the expected total downtime. The Case model IV may be considered as Case III with component 3 removed and maintenance as failure based. Since component 3 has no PM plan but is subject to breakdown maintenance only and the rate of failure is relatively low, the total expected downtime is not much difference for the downtime models for cases III and IV, that is equations (4.21) and (4.23).

Table 4.1 presents optimal results of figures of downtime and costs of case models based on various PM policies. Under the given maintenance information, we have obtained the expected unit downtimes and expected total unit cost of the various cases. In this system, the Case model III dominates the other models in terms of downtime.

Models	Model I	Model II	Model III	Model IV
Expected unit downtimes	0.316	0.249	0.20	0.206
Optimum inspection period	17	13	10	10
Expected total unit costs	9.619	9.047	8.185	5.91
Optimum inspection period	7	6	5	4

Table 4.1. Optimal results for downtimes and costs for all models

However, although the Case model III has the lowest downtime per unit time, the Case model IV has lower expected total cost per unit time. Therefore, if cost data is available, it may be more appropriate to compare not only downtimes but also maintenance cost.

4.4 Extending the Downtime Modelling

4.4.1 General assumptions

So far we have discussed in section 4.3 downtime and cost models which depend upon various PM policies for a system. In this section we present an extension to the downtime model, which deal with exploring the actual operating time and downtime when calculating the expected number of failures.

Previously, the analysis centers around the application of a periodic inspection of a system where independent defects may arise which have a delay time h. Here we also consider the general case of an inspection policy, which may be characterized by the following assumptions. (1) PM is undertaken every T time units, requires d_p time units and all defects are repaired. We suppose for now that (2) this inspection is perfect in that, if a defect is present at the time of inspection, it will be identified. (3) Defects are assumed to arise within the system at a constant rate λ over any PM period, failures during operating time are repaired immediately with downtime d_f, and failure repair time is independent of the defect's delay time. (4) Accumulated failure over a PM period T as assumed to impose a small amount of downtime, and defects are assumed to only deteriorate and lead to failures whilst the system is operating. We assume here that (5) the delay time h of a random defect is independent of its time origin and has *pdf* $f(\bullet)$ and *cdf* $F(\bullet)$. In complex plant, the notion (λ is constant or not) of failure rate has no meaning, as such, only rate of occurrence of failure.

In explaining our development of delay time modelling, we first briefly present the inspection models based on above basic assumptions. The model presented here is a development of the delay time models put forward by Christer and Waller (1984a) and Christer and Redmond (1990). Then we discuss the non-negligible downtime model extension in section 4.4.3.

4.4.2 Models of Inspection

- *Basic Model*

The basic model is first considered as the simplest possible case of an inspection policy which may be characterized by based upon above general assumptions of section 4.4.1 (see Christer and Waller,. 1984a). From the above assumption, the expected number of defects arising in the PM period T is λT. This ignores the downtime due to breakdowns, during which no defects would arise since the machinery is idle. However, if this downtime is small compared with T, then the error will be small. We will discuss this point in the section 4.4.3. Here we suppose that a defect arising within the period $(0, T)$ has a delay time in the interval $(h, h + dh)$, the probability of this event being $f(h)dh$. This defect will be repaired as a failure repair if the defect arises in period $(0, T - h)$, otherwise as an inspection repair. The probability of the defect arising before $(T - h)$, given that a defect will arise, is $(T\text{-}h)/T$. We have, therefore, that the probability that a defect is repaired as a failure and has delay time in $(h, h + dh)$ is $(T-h)/Tf(h)dh$.

Summing over all possible h, we have that the probability of a defect resulting in a failure, $b(T)$, (see equation (3.1)), is

$$b(T) = \int_0^T (\frac{T-h}{T}) f(h)dh. \tag{4.24}$$

Therefore, if the average downtime for a failure repair is d_f, the expected downtime per unit time to be incurred operating an inspection policy of period T is given by, (see equation (3.4)),

$$ED(T) = \frac{\lambda T b(T) d_f + d_p}{T + d_p}, \tag{4.25}$$

where λ is the arrival rate of defects per unit time and d_p denotes the mean duration of the PM activity.

- *Imperfect inspection case model*

In some cases it may be necessary to introduce a probability that a specific defect will be identified at an inspection, and a corresponding probability $(1-r)$ that it will not. If inspections are imperfect, there is a probability $r \le 1$ that a defect present at an inspection will be identified. Here the probability of detecting a defect at successive inspections is assumed to be independent and constant. Here equation (4.24) for $b(T)$ will need to be modified as below. We have from Christer and Waller (1984a) and at section 3.5.2 in Chapter 3 that the probability that a defect arises as a failure as opposed to being identified at an inspection, $b(T)$, (see equation (3.15)), is

$$b(T) = 1 - \int_0^T \sum_{n=1}^{\infty} \frac{r}{T}(1-r)^{n-1} R(nT-y)dy, \tag{4.26}$$

where $R(\bullet) = 1 - F(\bullet)$.

- *Non-homogeneous defect arrival rate case*

In the real situation, the system's reliability after a repair or replacement may not be the same as before. And if machine is subject to wear or ageing, the instantaneous rate of failure occurrence could vary. If most repairs involve the replacement of a very large fraction of a system's constituent parts, the system's reliability after a repair is not the same as it was immediately before failure occurred. This situation may lead to the non-homogeneous failure arrival rate. That is, the rate of occurrence of failure varies with time rather than being a constant. The NHPP model for the arrival process of failures of a repairable system, endorsed by Ascher and Feingold (1984), incorporates the concept of delay time by allowing the ROCOF be a convolution of the defect arrival rate and the delay time *pdf* under the assumption of independence. The inspection point for the case of imperfect inspection with NHPP defect arrivals is not a system renewal implying that the defect arrival rate $\lambda(t)$ cannot be considered identical in each inspection interval. Ageing can be modelled by assuming non-identical defect rates, $\lambda(t)$, over each inspection interval. It is also possible to allow the delay time of a defect to be dependent on u and the inspection interval in which the defect occurred, see Christer and Wang (1992). The process of failures then would not necessarily be an NHPP.

Assume now that the instantaneous rate of defect occurrence at time u after PM is not constant but is given by $\lambda(u)$. Thus, the expected number of defect arising in the small interval $(u, u + du)$ is $\lambda(u)du$. Therefore, the expected number of defects arising in the interval $(0,T)$, (see equation (3.6)), is

$$EN_d(T) = \int_0^T \lambda(u)du. \tag{4.27}$$

A defect arising within the period $(0, T)$ has a delay time in the interval $(h, h + dh)$, with probability $f(h)dh$. Therefore the expected number of failures resulting from defect arising in $(u, u + du)$ is

$$EN_f(u,u+du) = \lambda(u)du\int_0^{T-u} f(h)dh = \lambda(u)F(T-u)du, \tag{4.28}$$

where $F(x) = \int_0^x f(h)dh$. Accordingly, the expected number of failures during the time period $(0, T)$, (see equation (3.8)), is

$$EN_f(T) = \int_0^T \lambda(u)F(T-u)du, \tag{4.29}$$

and if we assumed that downtime, failure time and defect delay time are independent. Then the expected total downtime over operating cycle, T, is

$$ED(T) = \frac{EN_f(T)d_f + d_p}{T + d_p}. \tag{4.30}$$

4.4.3 Non-Negligible Downtime Model

In the downtime modelling of the delay time, it has been generally assumed that the downtime of a failure, d_f is very small compared with PM cycle length, T, that is, the downtime due to failure has not been deducted from the operating time T when estimating the number of failures which will arise in that period. However in some case, it is possible that the downtime of a failure repair is not very small (Chilcott and Christer, 1991). Figure 4.17 shows the actual failure process with some downtime of a failure repair over PM time period $(0, T)$. In this case, we need more assumptions for formulating a PM model. Therefore we considered here that (a) defects and failures can not arive during downtimes, and (b), any outstanding delay time of defects will be frozen over the downtime period. This implies deterioration only occurs in the plant when in use.

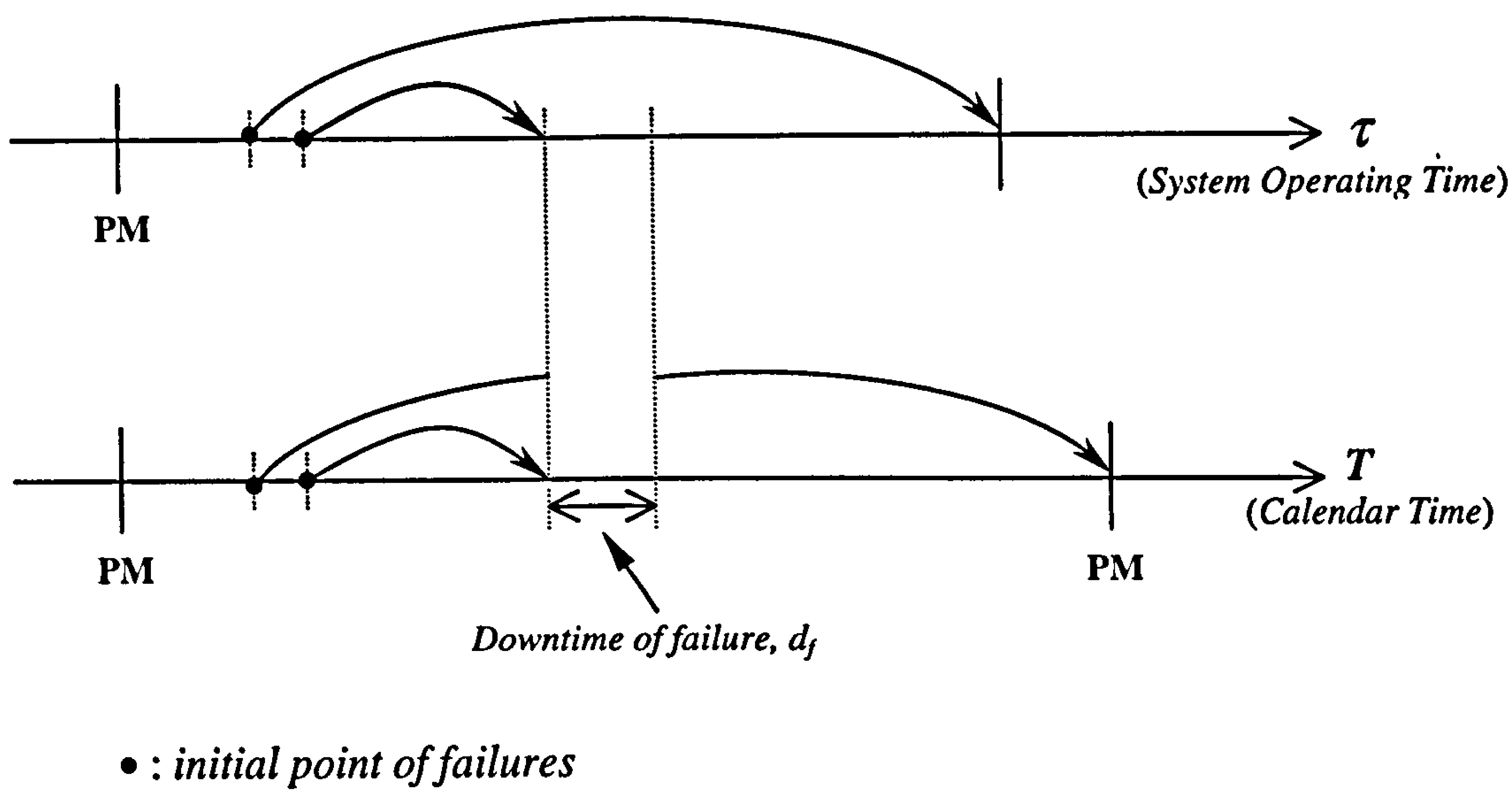


Figure 4.17. Actual failure process based on failure downtime in system

For example, in the prototype model of the delay time, if we do not consider the downtime for failures, the number of defects arising and the delay time of defects may be under or over-estimated respectively since defects would not normally arise during machine downtime. Therefore, we consider a modified downtime model allowing for downtime caused by failures from the above sources, over the PM time period, T. We define

T : calendar time of PM cycle for the system.

τ : actual system operating time over the time (0, T) of the system, *i.e.*, $\tau(\bullet)$.

$EN_f(T\,;d_f)$: the expected number of failure over the time period (0, T).

$EN_f(\tau)$: the expected number of failure over the operating time period (0, τ).

$EN_d(\tau)$: the expected number of defect arising over the operating time period (0, τ).

Other parameters are as before. If there are N_f failures, over the time (0, T), the actual system operating time is $(T - N_f d_f)$. Therefore, the effective actual system operating time τ over the PM time period (0, T) is

$$\tau = (T - N_f d_f). \tag{4.31}$$

Thus when the downtime of failure repairs is $d_f \neq 0$, the expected number of failures as a function of the actual system operating time, τ, is given by

$$EN_f(\tau) = E(N_f(\tau\,; d_f = 0)). \tag{4.32}$$

That is, if the downtime of failure, $d_f = 0$, the calendar time of PM cycle for the system, namely, T, and actual system operating time(τ) will be equal. However, if the downtime of failures, $d_f \neq 0$, the expected number of failures will not be equal. We have

$$\begin{aligned} E[N_f(T;d_f)] &= E[N_f(\tau, d_f = 0)] \\ &= E[N_f(\tau)], \end{aligned} \tag{4.33}$$

where $\tau = (T - N_f(\tau) d_f)$.

The basic downtime model of equation (4.25) may now be updated to the non-negligible downtime case. If the downtime of failure, d_f, is not zero, the expected number of defects arising and the expected number of failures depend upon the expected system operating time τ. The expected number of defects arising in the interval $(0, \tau)$ is

$$EN_d(\tau) = \lambda\tau. \tag{4.34}$$

Also the probability that one of these defects leads to a failure during the operating time period $(0, \tau)$ is

$$b(\tau) = \int_0^\tau \left(\frac{\tau - h}{\tau}\right) f(h)dh, \tag{4.35}$$

and the expected numbers of failures arise over $(0, \tau)$ is

$$N_f(\tau) = (\lambda\tau)b(\tau), \tag{4.36}$$

where τ is the actual system operating time over the PM period, T.

Therefore, from equation (4.25), the expected total downtime per unit time for the revised downtime model allowing for non-zero downtime is given by the equation pair

$$ED_\tau(T) = \frac{\lambda\tau b(\tau)d_f + d_p}{T + d_p}, \tag{4.37}$$

and

$$\tau = T - N_f(\tau)d_f, \tag{4.38}$$

where d_f is the mean downtime of failure repair, d_p is the mean duration of PM activity, and $b(\tau)$ is given by equation (4.35). It is noted that when the downtime due to failures may be ignored, *i.e.*, $\tau = T$, we have that equation (4. 38) reduces to equation (4.30) as expected. For given T, λ, d_f, d_p and $f(h)$, equations (4.37) and (4.38) may be evaluated for $ED_\tau(T)$.

Since $N_f(\tau) = (\frac{T - \tau}{d_f})$ from equation (4.38), we can readily calculate the value of τ which satisfies this equation for given T. If we know λ and α, for given τ we know $N_f(\tau)$ from equation (4.36), and may plot this function and $(T - \tau / d_f)$ on a common graph, seen in Figure 4. 18. The solution value of τ is $\tau^*(T)$. If d_f increase, actual operating time (τ) will be decrease to minimum of T and also as λ decreases , the actual operating time (τ) increases to a maximum of T.

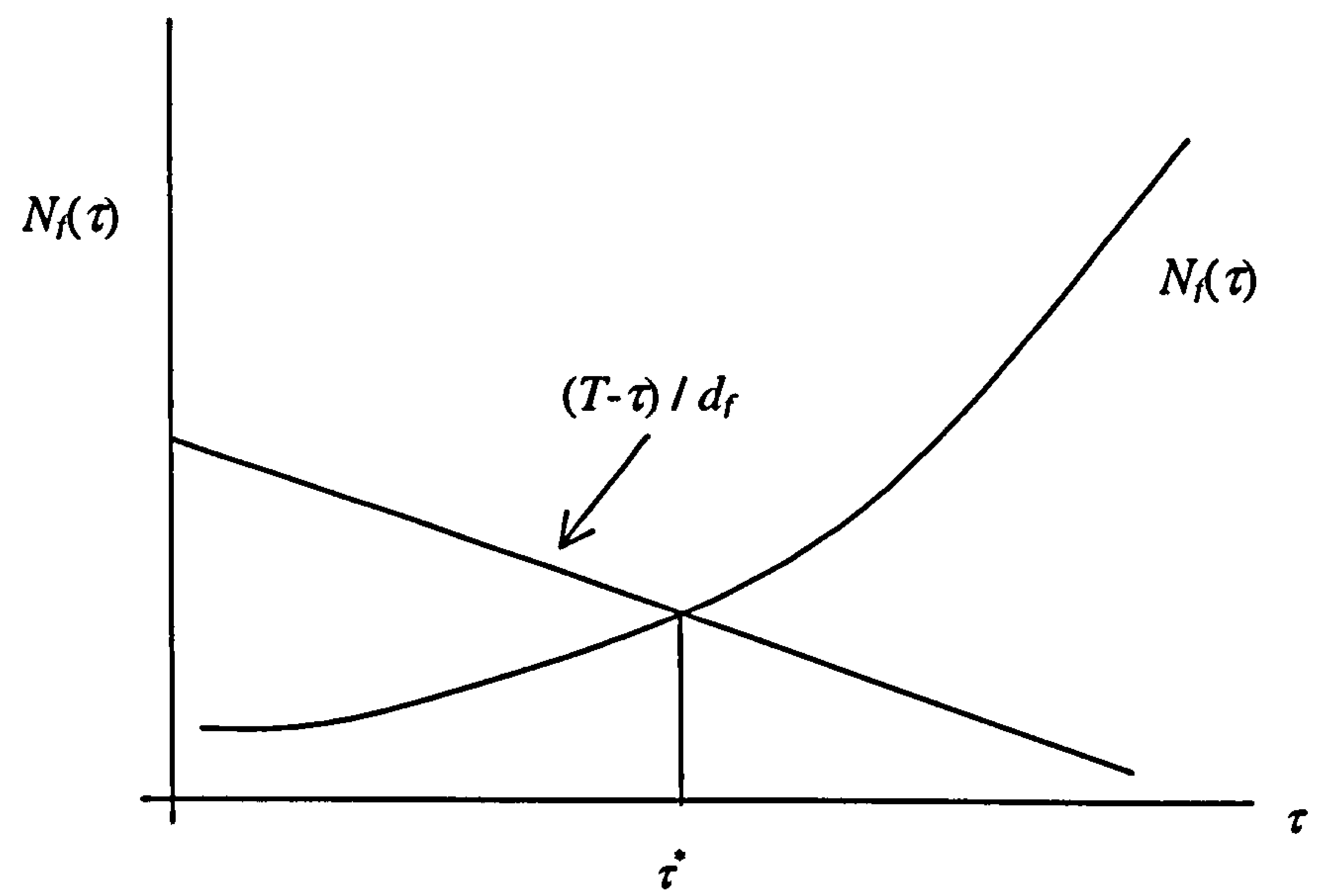


Figure 4.18. Relations of the number of failure and actual operating time (τ).

4.4.4 Numerical Examples

Numerical examples of the downtime model variant outlined above have been evaluated for demonstration purposes. The data used has been chosen arbitrarily as being broadly representative of some kinds of situation which may be encountered in reality.

For prototype PM models, we assume that the *pdf* of delay time is a negative exponential distribution, $f(h) = \alpha e^{-\alpha h}$, with $\alpha = 0.05$ giving an average delay time of 20 hours and the average defect frequency has been taken as 0.2 defects per hour. The mean downtime for a failure $d_f = 0.85$ hours and the mean downtime for PM, $d_p = 0.35$ hours.

The objective function for the downtime has the form given below,

$$ED(T) = \frac{Expected\ Number\ of\ failures\ in\ a\ cycles(0,T) \times d_f + d_p}{T + d_p}. \tag{4.39}$$

- ***For basic model***

Neglecting the influence of downtime, when the *pdf* of delay time is $f(h) = \alpha e^{-\alpha h}$, the probability $b(T)$ that a defect arises as a failure from equation (4.24), is

$$b(T) = \int_0^T \frac{T-h}{T} \alpha e^{-\alpha h} dh = 1 + \frac{1}{\alpha T}(e^{-\alpha T} - 1). \tag{4.40}$$

Therefore, the expected total downtime per unit time, $ED(T)$ is

$$ED(T) = \frac{kT(1 + \frac{1}{\alpha T}(e^{-\alpha T} - 1))d_f + d_p}{T + d_p}. \tag{4.41}$$

- ***Imperfect inspection case***

It is shown equation (4.26), that in the case r ≤ 1,

$$
\begin{aligned}
b(T) &= 1-\int_0^T \sum_{n=1}^{\infty} \frac{r}{T}(1-r)^{n-1}\{1-(1-e^{-\alpha(nT-y)})\}dy \\
&= 1-\frac{r}{\alpha T}\sum_{n=1}^{\infty}\{(1-r)^{n-1}e^{-\alpha nT}(e^{-\alpha T}-1)\} \\
&= 1-\frac{r}{\alpha T}(-e^{-\alpha T}-1)\sum_{n=1}^{\infty}\{(1-r)^{n-1}e^{-\alpha nT}\} \\
&= 1-\frac{r(1-e^{-\alpha T})}{\alpha T(1-(1-r)e^{-\alpha T})}.
\end{aligned}
\tag{4.42}
$$

Here, the probability of a defect present during an inspection being detected, r, is taken as $r = 0.3$. The expected total downtime per unit time for imperfect inspection case model, $ED(T)$ is given by

$$
ED(T) = \frac{kTb(T)d_f + d_p}{T + d_p}. \tag{4.43}
$$

- ***Non-homogeneous defect arrival rate case***

Since most materials, structures, and devices wear out with time, the class of failure distributions for which the failure rate function, $r(t)$, is increasing is one of special interest (Barlow and Proschan, 1965). Suppose that an equipment is subjected to a constantly increasing stress. Then the instantaneous defect arrival rate of this equipment may be increase. It should be noted that when the defect arrival rate increases, such as for the normal distribution, this indicates an ageing or wear-out effect (Jardine, 1973). Since $\lambda(t)$ represents the expected defect arrival for a component over time, we are considering the case when defect arrival frequency at time u after perfect inspection is non-homogeneous. The following defect frequency, $\lambda(u)$, has been taken for demonstration purposes from Christer and Waller, 1984 ;

$$\lambda(u) = 0.2 - 0.06e^{-0.2u}. \tag{4.44}$$

Then, the expected number of failures arising in (0, T) from equation (4.29), ENf(T) is

$$EN_f(T) = \int_0^T (0.2 - 0.06e^{-0.2y})\{1 - (1 - e^{-0.05(T-y)})\}dy, \tag{4.45}$$

and the expected total downtime per unit time, $ED(T)$, in the non-homogeneous defect arrival rate case is

$$ED(T) = \frac{EN_f(T)d_f + d_p}{T + d_p}. \tag{4.46}$$

Other cases exist but we are not considering here.

- ***Revised PM model for non-negligible downtime***

This model also assumes that the expected number of defects arising in the PM period T is λT, as in the basic model. We have for the PM interval of length T, the number of failures over an actual operating time $(0, \tau)$ from equation (4.35). Therefore, assuming perfect inspection, we have $b(\tau)$

$$b(\tau) = \int_0^\tau \frac{\tau - h}{\tau} \alpha e^{-\alpha h} dh = 1 + \frac{1}{\alpha\tau}(e^{-\alpha\tau} - 1), \tag{4.47}$$

where $\tau = T - k\tau b(\tau)d_f$. It follows from equation (4.41) that the expected total downtime per unit time for the revised PM model is given by

$$ED_{\tau}(T)=\frac{k\tau(1+\frac{1}{\alpha\tau}(e^{-\alpha\tau}-1))d_f+d_p}{T+d_p}, \tag{4.48}$$

where $\tau=T-k\tau b(\tau)d_f$. (4.49)

In a similar way, $b(\tau)$ for imperfect PM model case from equation (4.42) is

$$\begin{aligned} b(\tau)&=1-\int_0^{\tau}\sum_{n=1}^{\infty}\frac{r}{\tau}(1-r)^{n-1}\{1-(1-e^{-\alpha(n\tau-y)})\}dy \\ &=1-\frac{r(1-e^{-\alpha\tau})}{\alpha\tau(1-(1-r)e^{-\alpha\tau})}, \end{aligned} \tag{4.50}$$

It also follows from equation (4.43) that the expected total downtime per unit time for the revised PM model is given by

$$ED(T)=\frac{k\tau b(\tau)d_f+d_p}{T+d_p}. \tag{4.51}$$

For the non-homogeneous defect arrival rate case of revising PM model, we also assume that the instantaneous rate of defect occurrence at time u after PM is not constant but is given by $\lambda(u)$ and a defect arising within the period (0, τ) has a delay time in the interval (h, $h+dh$), with probability $f(h)dh$. Therefore, the expected number of failures arising over actual operating time $(0,\tau)$ of non-homogeneous defect arrival rate case from equation (4.29), $EN_f(\tau)$ is given by

$$EN_f(\tau)=\int_0^{\tau}(0.2-0.06e^{-0.2y})\{1-(1-e^{-0.05(\tau-y)})\}dy. \tag{4.52}$$

Therefore, the expected total downtime for revising downtime model from equation (4.30) is

$$ED(T) = \frac{EN_f(\tau)d_f + d_p}{T + d_p}. \tag{4.53}$$

Now we shall consider consequence to the downtime for the various models in terms of calendar time, T, and actual operating time, τ. The results for the models outlined above are shown in Figures 4.19, 20 and 21.

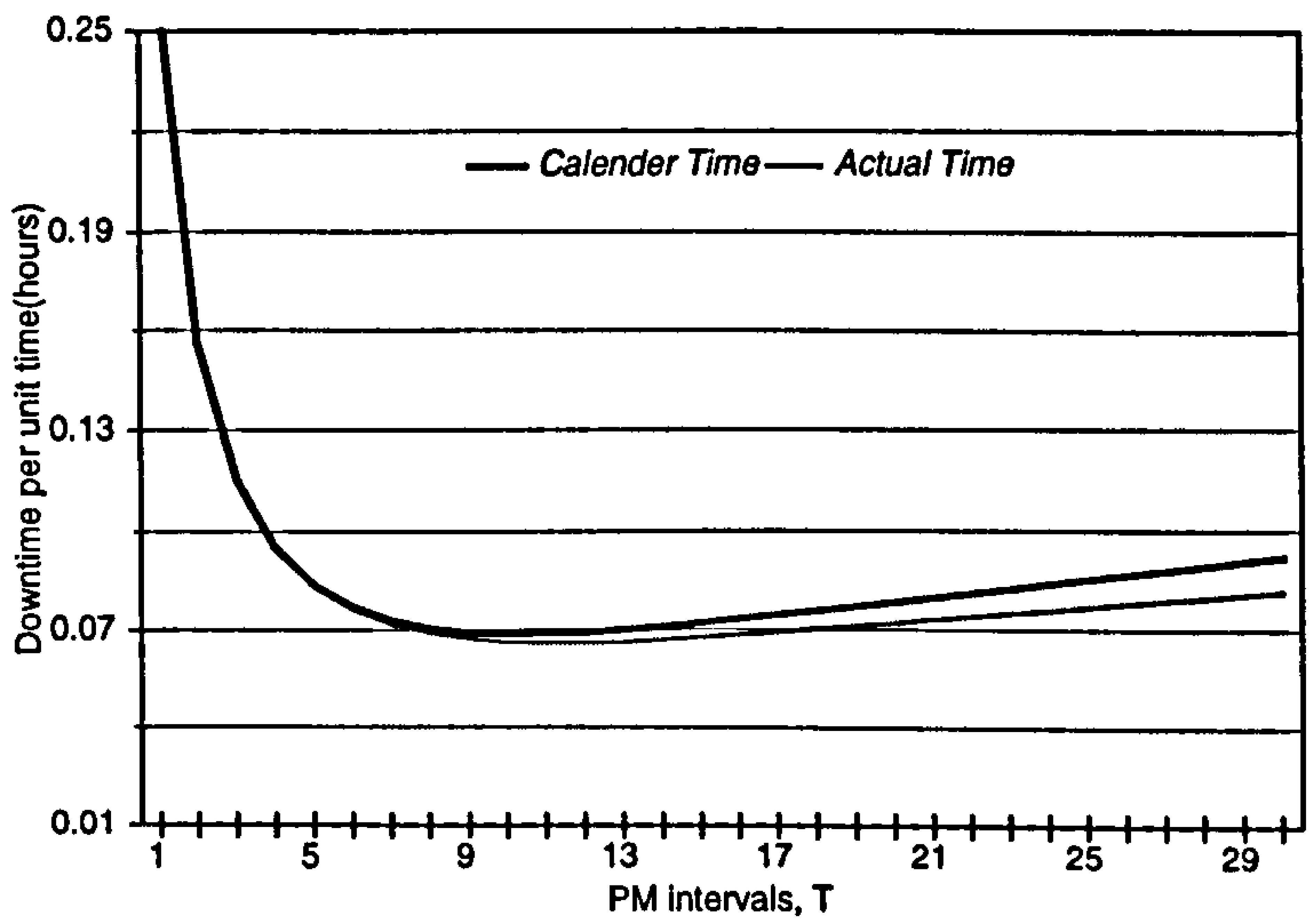


Figure 4.19. Expected downtime for basic model and revised model

First, we shall look at the basic model. It can be seen from Figure 4.19 that the optimal point based on the calendar time, T, is 10 hours and whilst the optimal point for the update model which is based on the actual time, τ, is 12 hours. That is, the expected total downtime for the basic model based on the calendar time, T, is slightly high, as would be expected since the model overestimates the number of failures. Secondly, we shall look at the imperfect inspection case.

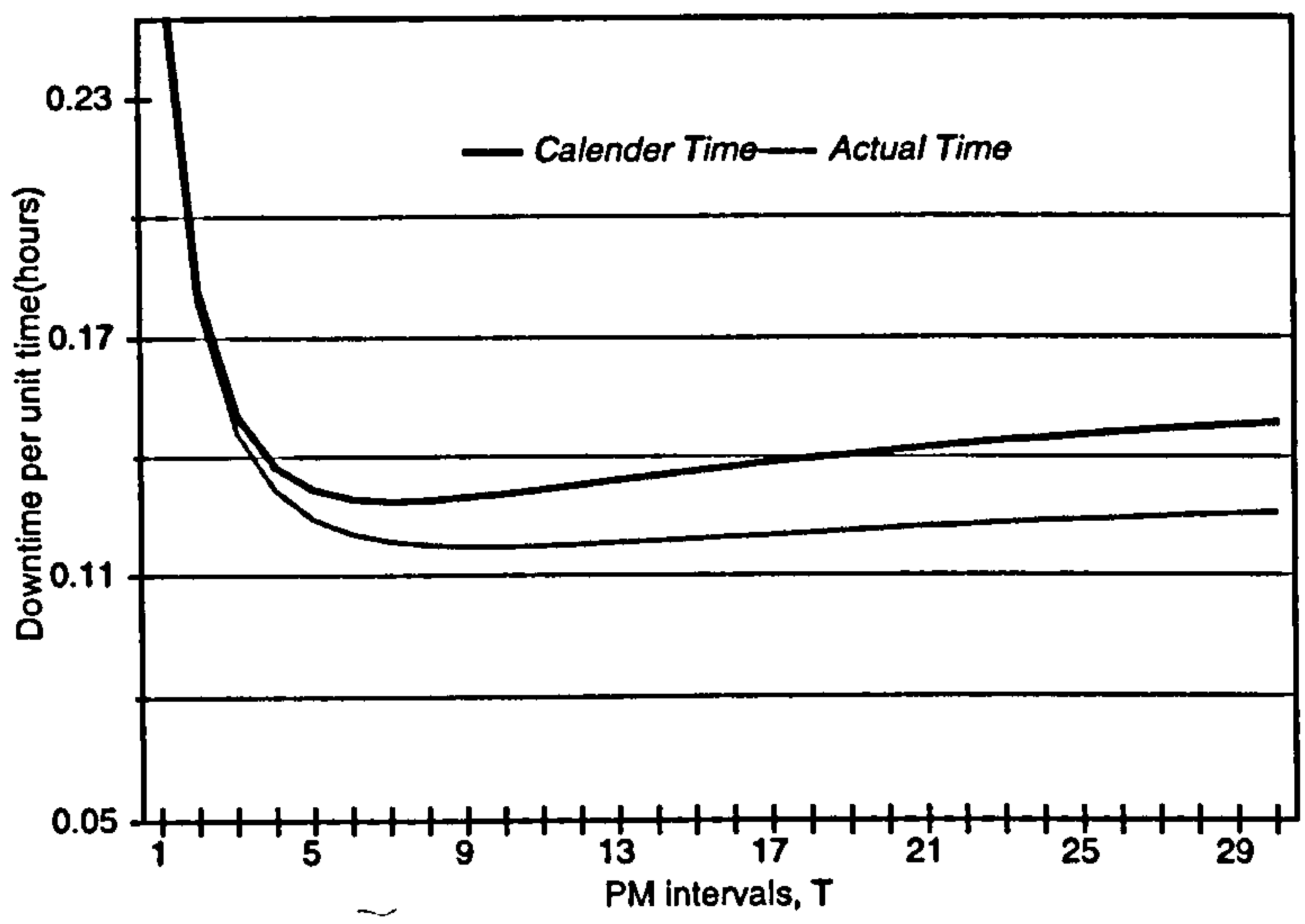


Figure 4.20. Expected downtime for imperfect PM case model and revising model

In Figure 4.20 the graphs for $ED(T)$ for the imperfect PM case based on the calendar time, T and actual operating time, τ. It also can be seen that the optimum values of $ED(T)$ for the different times between calendar time and actual operating time has resulted from updating $b(T)$. The optimal interval of the prototype model, which is based on calendar time T, in the case of imperfect inspection is 7 hours and the expected total downtime is 0.129 hours.

Otherwise, the optimal interval for the revised model, in the case of the imperfect inspection model based upon actual operating time, τ, is 10 hours and the expected total downtime is 0.117 hours. This result shows that there is a greater difference between the basic and downtime revised model in the non-perfect inspection case compared to the perfect inspection case. This is due to the fact that the model for imperfect inspections has a higher frequency of failures than the basic model.

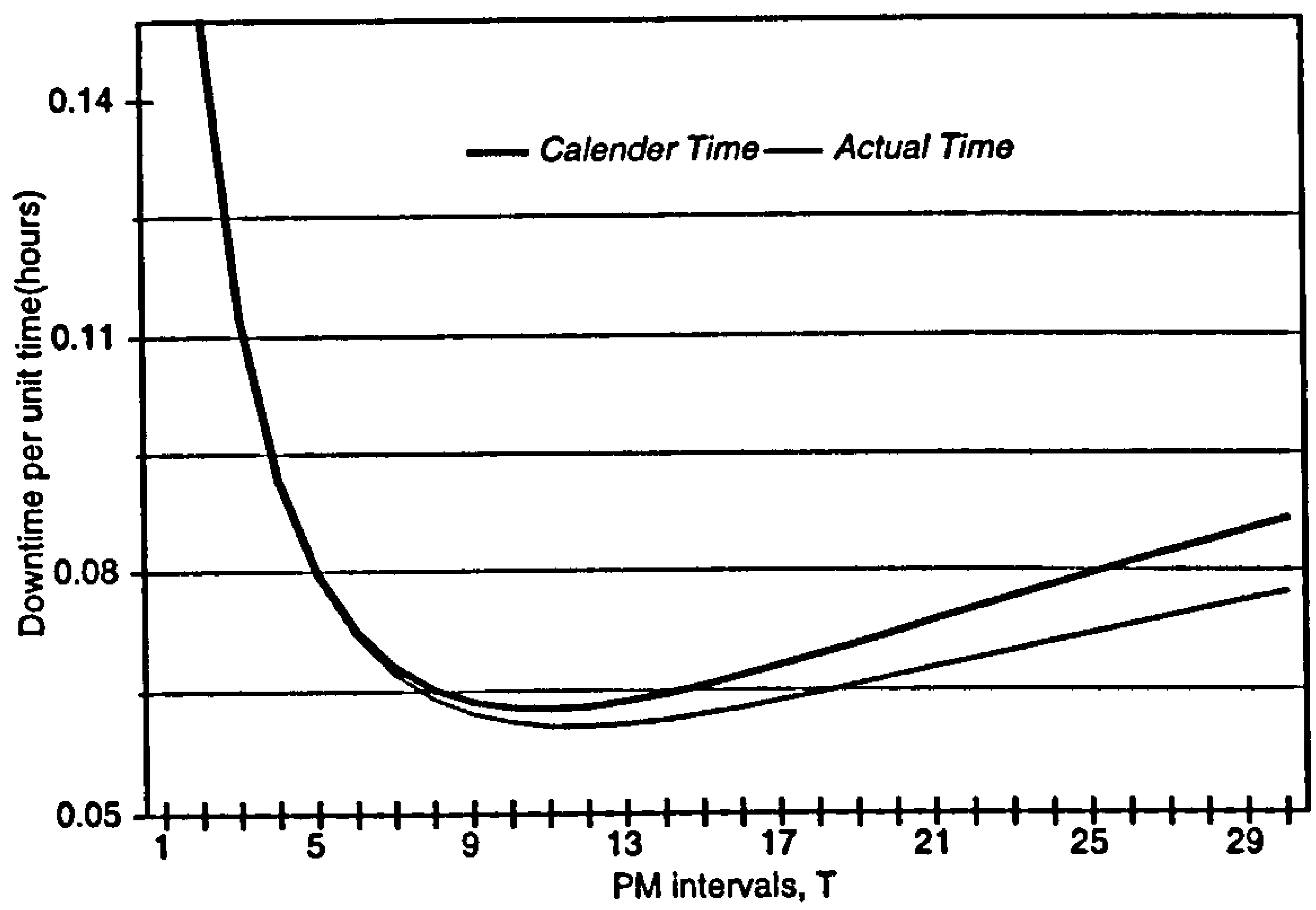


Figure 4.21. Expected downtime for non-Homogeneous case model and revised model

Again, Figure 4.21 shows the expected total downtime of the model for a non-homogeneous defect arrival rate over the calendar time and actual operating time. This case also shows the optimal interval and the expected total downtime for the calendar time based model, namely, 11 hours and 0.063 hours respectively. The optimal interval and the expected total downtime for revising model are 12 hours and 0.061 hours. Also, the model for non- homogeneous defect arrival rate shows that the expected downtime is slightly less in the revised downtime formulation case, as would be expected. If this model has a high frequency of failures, a grater difference would be expected between the two models based upon calendar time and actual operating time for a non- homogeneous defect arrival rate.

Although the percentage savings in total downtime in the cases considered above is small, the financial consequences, which depend on cost of such saving and their value, may be very attractive. Therefore, revised PM model can be provided the accuracy for good decision-making of maintenance activities.

4.5 Conclusions

Delay time analysis has already proved useful in the rudimentary applications made so far. Its scope for development has still to be really explored (Christer and Redmond, 1990). In the delay time modelling of inspection practice, we used to assume that all components are inspected in one inspection schedule. However in some cases, there are components that need not be inspected as often than others. For instance, one set of components may be inspected as regularly as every day, whilst the remaining can be inspected during the optional planned week. First in this Chapter, we have presented an approach for formulating a system PM model which allows various PM policies, where the system consists of many components with possible economic dependence.

Here, a technique is suggested in which decisions about complex systems are aided by inspection options relating how to group the components into sets and when to perform the inspection for each set. In modelling the options, component PM models and system PM model have been presented. The system model is constructed under the assumption that defects have a common delay time distribution. The component PM model track on defects and failures individually. In contrast, a system PM model is built by combining the all components of a system. So both modelling concepts apply to a system which consists of many components with different PM policies. Therefore, as a consequence of an optimal inspection policy, a proportion of maintenance work is identified and clustered at specific points in time, so giving the maintenance organization the opportunity to allocate its resources appropriately and rectify the defects in a more efficient manner than would otherwise be the case.

As expected, the result from modelling spilt and integrated inspection schedules may support the expectation of management that by splitting or integrating the inspection task, a better quality of inspections could be achieved. This would be mainly in terms of a reduced expected number of failures or costs due to failures. From these modelling options, best inspection policy also can be identified.

In general, if there is some reduction factor in downtime for inspection activities, the system PM model may reduces the expected total downtime since the PM is carried out at the same time for the whole system, but PMs of components are carried out individually. In the situation of PM being individually carried out, the system has to be stopped while one of its components is being checked. By combining PM, production loss of a system may reduce. On the other hand, to decrease operating costs, combining of components is applicable for a system if there exist the economical benefit such as the cost reduction for grouping of PM.

In the section 4.4, we have discussed a revising the PM model to allow for the downtime incurred at failures and its impact upon the expected number of failures. A revision was undertaken, which updated the downtime model. This model is used to predict the effectiveness of maintenance activity using the resultant downtime of the system as the relevant measure. Using the revising PM model it is evident that the more accurate economic PM interval may obtain. If the downtime of failures is not small compared with the PM cycle length, then the expected downtime of failures over the PM intervals may be overestimated without allowing for downtime in modelling the expected number of failures. This implies that revised downtime model would be a more sensitive model with which to determine the actual downtime or cost.

Chapter 5

PARAMETER ESTIMATION OPTIONS WITH AND WITHOUT PM INFORMATION

5.1 Introduction

In this chapter, we present a simulation study undertaken to further investigate and verify parameter estimation methods. In reality, some plant managers may keep maintenance records which may be used, for example, to calculate production efficiency. However, it is rare to find perfect or even good maintenance data in practice. In our experience, data is usually incompletely recorded or lost. Restrictions upon the availability of data required for modelling is common place.

Simulation is one of the most widely used techniques in operations research and management science, and by all indications its popularity is on the increase (Law and Kelton, 1982). We test parameter estimation methods using simulated data to check the consequences of different volumes of data upon the accuracy of parameter estimates for maintenance models. Simulation programs, written in FORTRAN, have been used to simulate data generation assuming the delay time process for specific and realistic sets of input parameters and assumptions. Sometimes a simulation language, *e.g.*, GPSS, SIMSCRIPT, SLAM, SIMULA, DYNAMO, may have merits above that of a general-purpose language like FORTRAN for programming simulation models. However, since complicated numerical calculation is not easy in certain simulation languages, we use FORTRAN here.

Once parameters have been estimated from objective data, they need to be tested for amongst other things, the goodness of fit of the consequential PM model relative to the true maintenance model.

This Chapter demonstrates the estimation of DTM model parameters given maintenance record data, which includes the failure times, or number of failures per day, and the number of defects identified at PM.

5.2 Maintenance Data Generation using Simulation

A simulation must involve sampling, or generating random variables from one or more distributions. These distributions often are specified as a result of fitting some appropriate distributional form, e.g., exponential, or Weibull, to observed data. Thus, once a distribution has already been specified including the values of the parameters, we can generate random variables with this distribution in order to run the simulation model.

Before we begin the process of developing a model, we need to understand the structural building blocks from which models are constructed. In our delay time modelling, data we needed is inspection and failure information. This data includes the number of failures per working day or failure times and the number of defects at PM. Therefore we have to define the output variable or variables that are of inspections and failures information. Generally, in a basic delay time model, there are two random variables, that is the initial point u and the delay time h. In order to carry out a simulation of a system having the initial point u and delay time h, we have to specify the probability distributions of these random variables. Then, given that these random variables follow particular distributions, the simulation proceeds by generating values of these random variables from the appropriate distribution. In this pilot study, we select the exponential and the Weibull distributions as the distributions for these random variables for a system model.

Consider that an initial point, namely u, has defect arrival rate of λ defects per unit time and the delay time h has the exponential distribution with parameter $1/\alpha$, $\alpha > 0$, that is $f(h) = \alpha e^{-\alpha h}$. Since the inter-arrival time of defects X has an exponential distribution with the mean $1/\lambda$, the *pdf* of the inter-arrival time X is given by $g(x) = \lambda e^{-\lambda x}$.

There are many techniques for generating random variables, and the particular algorithm used must, of course, depend on the distribution from which we wish to generate. The basic ingredient needed for methods of generating random variables from any distribution or random process is a source of independent identically distributed (IID) U(0, 1) random variables. For this reason, it is very important that a statistically reliable U(0, 1) random-number generator be available. Most computer installations and simulation languages have a convenient random-number generator. We also here select the particular algorithm, called the *inverse-transform method*, which is a popular one, for generating random variables.

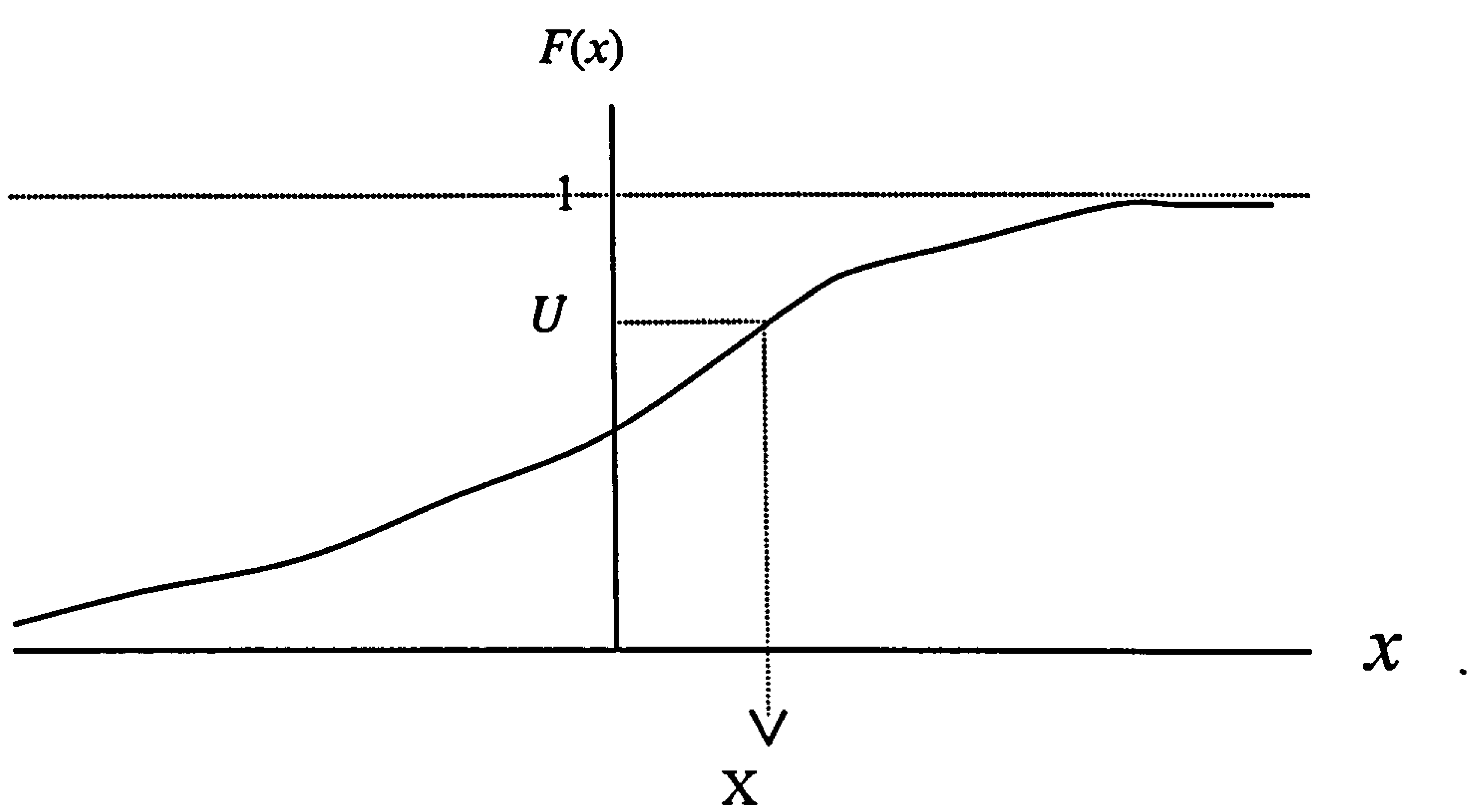


Figure 5.1. Inverse-transform method for continuous random variables
(Law and Kelton, 1982)

Using the inverse transform method, the algorithm for generating a random variable X having distribution function $F(\bullet)$ is as follows (Law and Kelton, 1982):

(1) Generate U, distributed as U(0, 1).

(2) Set $X = F^{-1}(U)$ and return.

It is note that $F^{-1}(U)$ will always be defined, since $0 \leq U \leq 1$ and the range of F is [0, 1]. Figure 5.1 illustrates the algorithm graphically.

For the exponential distribution with parameter $1/\lambda$, the distribution function is

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{5.1}$$

and to find F^{-1}, we set $u = F(x)$ and solve for x to obtain

$$F^{-1}(u) = -\frac{1}{\lambda}\ln(1-u). \tag{5.2}$$

Thus, to generate the desired random variable we first generate a $U \sim$ U(0, 1), and then form $X = -\frac{1}{\lambda}\ln U$. It is possible in this case to use U instead of $1 - U$, since $1 - U$ and U have the same U(0, 1) distribution. This saves a subtraction.

If we also consider that a random variable X has the Weibull distribution function with the scale parameter $\alpha(>0)$ and shape parameter $\beta(>0)$, that is $f(x) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}$, the sample data generation algorithm using the *inverse-transform method* is ;

(1) Generate $U \sim$ U(0, 1).

(2) Set $X = \alpha(-\ln U)^{1/\beta}$ and return.

Once the random variables of initial point u and the delay time h are generated, the simulation model can be built to generate synthetic maintenance data corresponding to the system description of the basic DTM. For example, suppose the following assumptions will initially be assumed to apply for the system to be simulated:

(1) Defects are independent of each other and arise as a homogeneous Poisson process (HPP), with rate of occurrence of defects λ.

(2) The delay time h of a defect is independent of the time of origin, and all defects share a common delay time *pdf* $f(h)$ and *cdf* $F(h)$.

(3) The condition of the system can be observed by inspections only, and a failure will be observed immediately at its occurrence.

(4) Inspections are perfect in that any defect present within the system will be identified at inspection, and no new defect generated because of inspection.

(5) An inspection is undertaken every T time units and requires d_p time units.

(6) All identified defects at an inspection will be repaired within allocated inspection time, d_p.

(7) A failure will be observed immediately at its occurrence. The component is repaired immediately upon failures and mean time for a failure repair is d_f time units.

(8) The component is as good as new after repair.

By the assumption (1), defects are independent of each other and arise as a HPP with the rate λ. If X_n, $n \geq 1$, denotes the time between the initial points of $(n\text{-}1)$st and nth defects (see Figure 5.2), the sequence $\{X_n, n \geq 1\}$ is called the sequence of inter-arrival times of defects and X_n are IID exponential random variables having mean $1/\lambda$. If the assumption (4) is relaxed to allow an imperfect inspection, such inspection practice may be characterized by the following changed assumption (4'):

(4') Inspections are assumed to be imperfect in that a defect present will be identified with probability r, $0 \leq r \leq 1$. The probabilities of detecting a defect at successive inspections are assumed to be independent and constant.

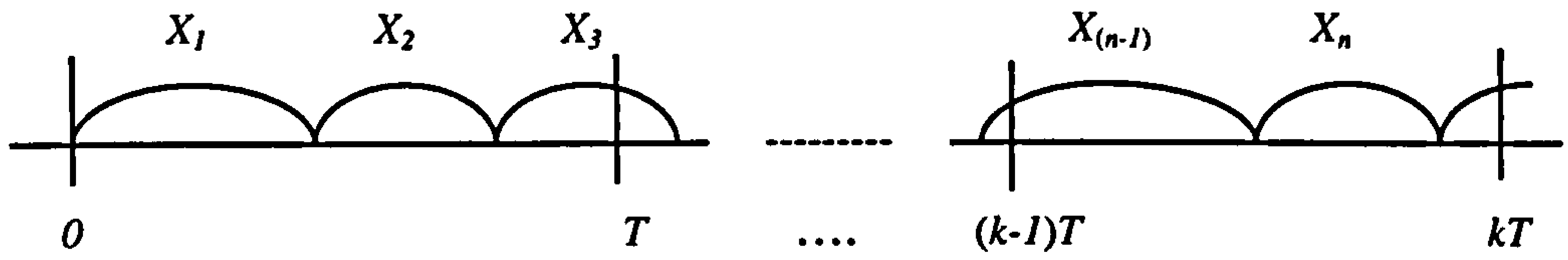


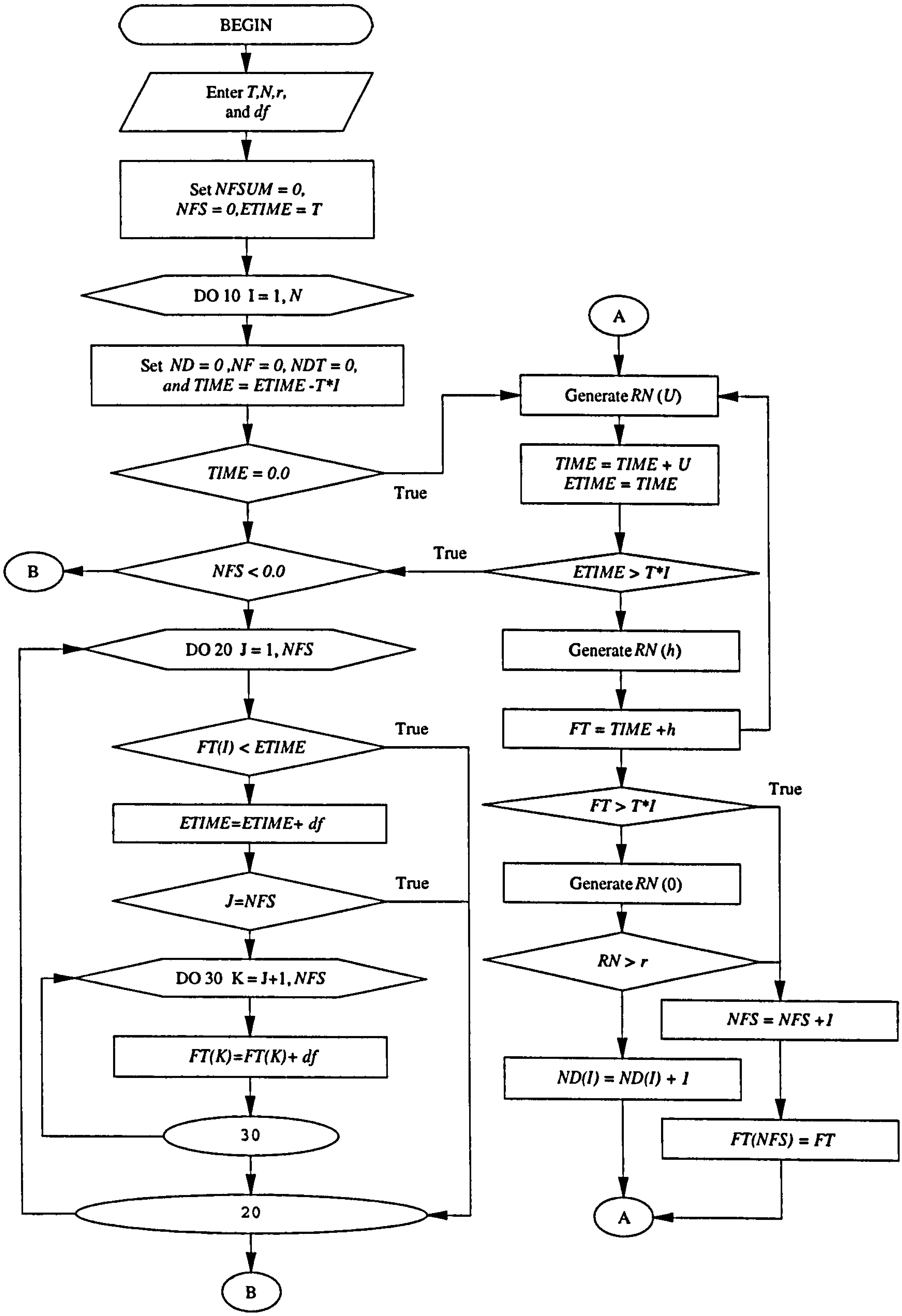
Figure 5.2. The sequence of inter-arrival times

By setting a value for the probability of identifying a defect an inspection, r, we can generate both a set of data for a perfect inspection (r = 1) policy, and for an imperfect inspection policy, $r \neq 1$.

In the first case, we consider that there are no records of times of failures during the day, but there are records of the number of failures on each day. Therefore, in the data generating simulation program, only the number of failures on each day and the number of defects identified at PM are recorded. We also consider a second and richer case where data are available recording the times of failures and the number of defects identified at PM. Figure 5.3 illustrates the simulation and presents the flow chart in which the observed events are the number of failure per working day or the times of failures and the number of defects identified at PM.

Case Model	**Case A**	**Case B**
Data Available	1. Number of failures per working day. 2. Number of defect identified at PM	1. Times of failures. 2. Number of defect identified at PM

Table 5.1. The generated data set by the simulation program


BEGIN
Enter T,N,r, and df
Set NFSUM = 0, NFS = 0, ETIME = T
DO 10 I = 1, N
Set ND = 0, NF = 0, NDT = 0, and TIME = ETIME - T*I
TIME = 0.0
True
NFS < 0.0
B
DO 20 J = 1, NFS
FT(I) < ETIME
True
ETIME=ETIME+ df
J=NFS
True
DO 30 K = J+1, NFS
FT(K)=FT(K)+ df
30
20
B
A
Generate RN (U)
TIME = TIME + U
ETIME = TIME
True
ETIME > T*I
Generate RN (h)
FT = TIME +h
FT > T*I
True
Generate RN (0)
RN > r
NFS = NFS +1
ND(I) = ND(I) + 1
FT(NFS) = FT
A

Continued

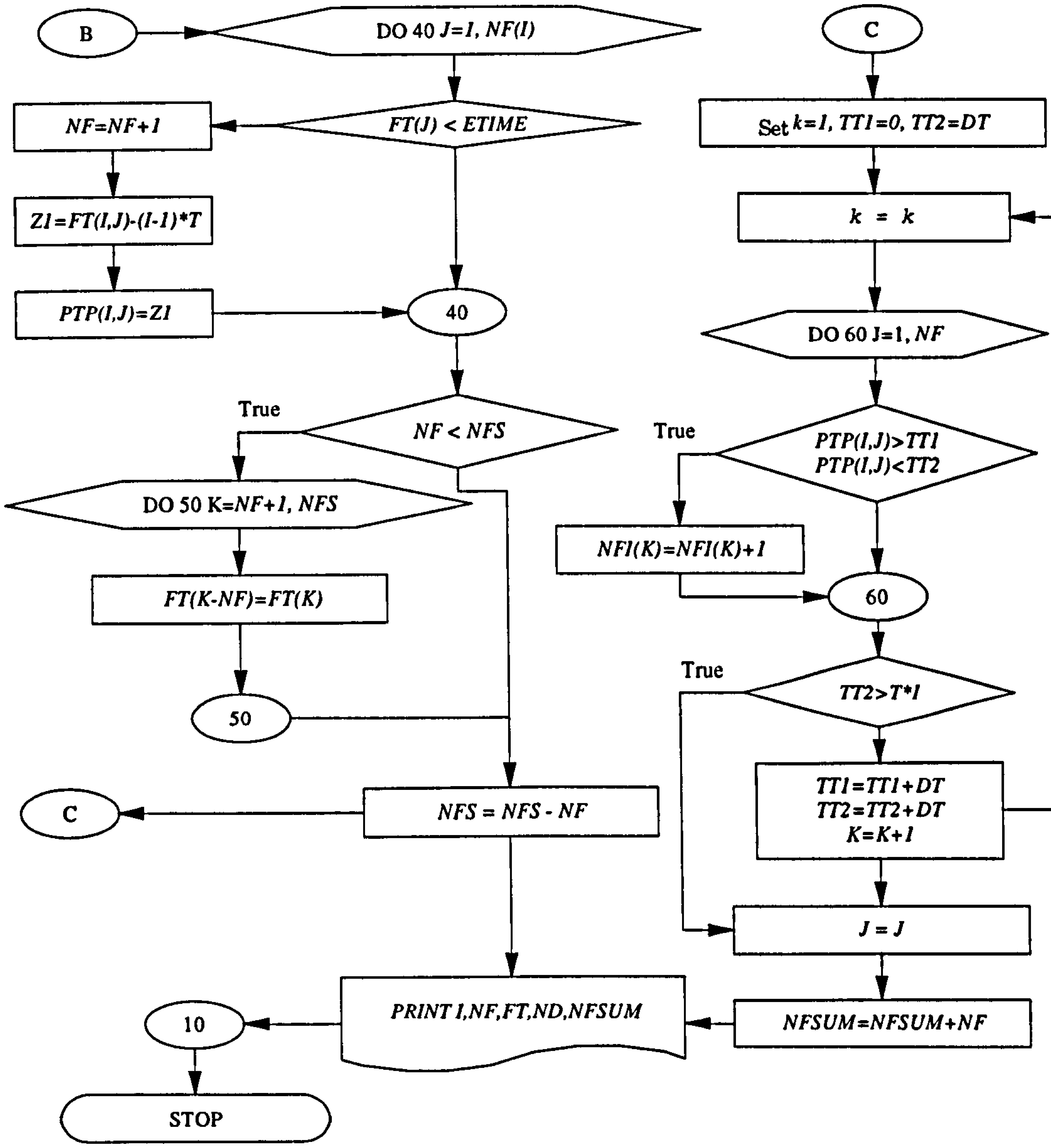


Figure 5.3. Flow chart for generating the data recording the number of failures per working day, or failure time data and the number of defect identified at PM

In the flowchart of Figure 5.3 the following mnemonic symbols are not yet defined. T is a present inspection time period and N is the number of inspection for computer running and r is a parameter for inspection quality. d_f is a downtime for a failure. *ND* and *NF* are the number of defects and failures arising within a present inspection period, respectively. U is an inter-arrival time of defect and h is a delay time. *FT*(*J*) is the *Jth* failure time point and *Z1* is the failure time point within a present inspection period. *NFSUM* represents for total number of failure for whole inspection period. Chosen values of system parameters are consisted with observations from a data set for the IMA machine.

Validation is the process of bringing to an acceptable level the user's confidence that any inference about a system derived from the simulation is correct. The strongest verification of any simulation model occurs when we demonstrate that the model can successfully predict events which have not yet transpired. Even though we have succeeded in developing a model that appears to be reasonable and adequately reproduces the past performance of the system, question still remains. When validating a model we read in the historical data, have these data processed by the simulation program, obtain simulation output, compare this simulation output with the historical output, and decide whether the model is realistic or not. Once we have validated the model we shall use it to predict the response for certain system variants.

Now we compare the simulation output with historical output. From historical data, parameters were estimated by using maximum likelihood estimation. The rate of occurrence of defects (ROCOD) is 9.518 and the distribution of delay time is a negative exponential, $F(h) = 1 - e^{-1.816}$. The probability of a defect being identified also was estimated as $r = 0.071$, we know that PM is imperfect (see Table 6.4). We input these values in the simulation model, and generate a set of data by simulating the process. We compare only the number of failures, since available field information of historical record is the number of failures. The simulation output is run 10 times, with an average number of failures of 2127 and standard deviation, 33. Table 5.2 shows comparison of the historical data and simulation output. The mean of simulation outputs is nearly close to the historical data.

	Historical data	Simulation output (Average)
Number of failures	2133	2127

Table 5.2 Comparison of historical data and simulation output

Therefore we use the mean of 10 simulation results in further study. The similar studies have been explored by Christer, *et. al.*(1995) and Christer, Wang and Choi (1998).

Having generated synthetic data, it is adopted as the data set for parameter estimation. The objective here is to re-capture the initial parameter set used to simulate the data. To estimate the given parameter from the synthetic failure data, use is made of the NAG library of numerical routines available for the personal computer. The routines are intended to be called by programs written in FORTRAN, and are of considerable use in computing the log-likelihood equations. The NAG function minimizer E04JAF was used to minimize minus the log-likelihood. E04JAF is an easy-to -use quasi-Newton algorithm for finding a minimum of a function $F(x_1, x_2, \ldots, x_n)$, subject to fixed upper and lower bounds on the independent variables $x_1, x_2, \ldots, x_n$, using function values only. From the starting point supplied by the user there is generated, on the basis of estimates of the gradient and the curvature of $F(x)$, a sequence of feasible points which is intended to converge to a local minimum of the constrained function. Attempt is made to verify that the final point is a minimum. D01AJF is used here for calculating an approximation to the integral of a function $F(x)$ over a finite interval (A, B):

$$\int_A^B F(x)dx \tag{5.3}$$

where $F(x)$ is defined by the user, either at a set of point $(x_i, F(x_i))$, for $i = 1, 2, \ldots, n$ where $A = x_1 < x_2 < \ldots < x_n = B$, or in the form of a function. To estimate the value of an integral, this quadrature rule uses an approximation in the form of a weighted sum of integrand values, *i.e.*

$$\int_A^B F(x)dx \cong \sum_{i=1}^{n} w_i F(x) \tag{5.4}$$

The points x_i, within the interval [A, B] are known as the abscissae, and the w_i are known as the weights.

For each simulation, we simulated 5 different sample sets of 10, 50, 100, 150, and 200 PMs with depending of parameters. For each number of PMs, the number of or times of failures and the number of defects at PM data are generated by simulation model. Here inspection interval used is 30days. In perfect inspection case, we set the probability of defect identified at PM, $r = 1$ and imperfect inspection case with $r < 1$ are simulated in Figure 5.3. After simulating the data with known parameters, we estimate these parameters by using the maximum likelihood method. Then we check how well recapture the given parameters in likelihood formulation

5.3 Parameter Estimation Methods With and Without PM Information

5.3.1 Method A ; When the Number of failures in each working day and the number of defects identified at PM times are available and not available.

First, it is assumed for the moment that observations of number and downtime of failures, and the number of defects identified at PM are available. We define the notation (see Christer *et al.*, 1995) for modeling the likelihood of this data set.

Let

λ : the constant rate of occurrence of defects within the system.

h : the delay time of a defect with *pdf* $f(\bullet)$ and *cdf* $F(\bullet)$.

r : the probability of detecting a defect at PM, if it is present.

T_i : time of the *i*th PM from last inspection, $i = 1, 2, ..., n, ...$

t : failure time from last inspection.

Δt : a time period to be defined.

$EN_f(t, t + \Delta t)$: the mean number of failures over $(t, t + \Delta t)$.

$EN_p\,(T_n)$: the mean number of defects identified and removed at T_n .

$P(t,\ t + \Delta t \mid u)$: the probability of a failure in $(t,\ t + \Delta t)$ resulting from a defect arising at time u.

Accepting the above notation, we now assume t is such that assume $T_{n-1} < t < T_n$ for some n.

Consider the probability of a failure over $(t,\ t + \Delta t)$ for $T_{n-1} \leq t \leq T_n$ resulting from a defect arising at time u , $u \leq t + \Delta t$, that is, $P(t,\ t + \Delta t \mid u)$. The defect could have arisen since the last inspection, T_n, or during one of several earlier inspection periods, but have not been detected, see Figure 5.4.

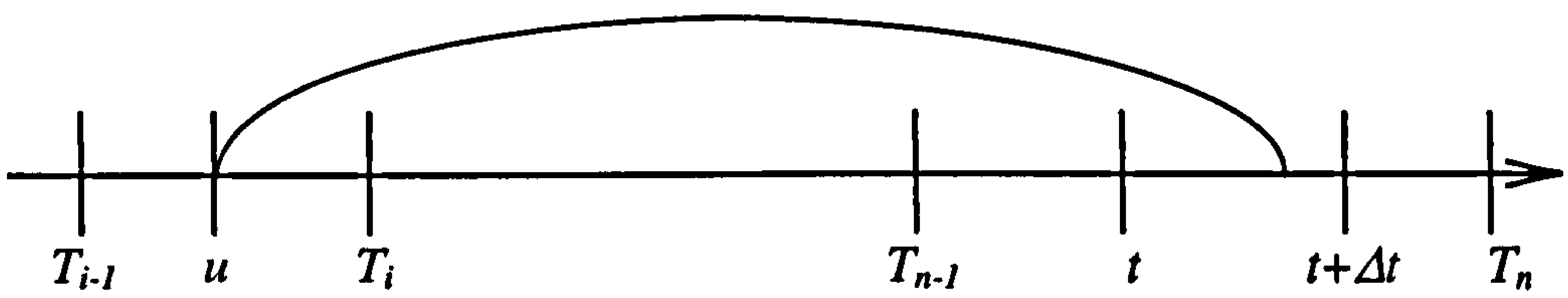


Figure 5.4. The failure process of a defect arising in (T_{i-1}, T_i)

Now we have that

$$P(t,t+\Delta t \mid u)\begin{cases}(1-r)^{n-i}[F(t+\Delta t-u)-F(t-u)] & T_{i-1}<u<T_i, i\in\{1,\ldots n-1\}\\ F(t+\Delta t-u)-F(t-u) & T_{n-1}<u<t\\ F(t+\Delta t-u) & t<u<t+\Delta t\\ 0 & u>t+\Delta t.\end{cases} \tag{5.5}$$

The expected number of failures over $(t,\ t+ \Delta t)$, for $T_{n-1} < t < T_n$, is , therefore, given by

$$EN_f(t,t+\Delta t) = \sum_{i=1}^{n-1}(1-r)^{n-i}\lambda\int_{T_{i-1}}^{T_i}[F(t+\Delta t-u)-F(t-u)]\,du$$

$$+\lambda\int_{T_{n-1}}^{t}[F(t+\Delta t-u)-F(t-u)]\,du$$

$$+\lambda\int_{t}^{t+\Delta t}F(t+\Delta t-u)du. \quad (5.6)$$

Changing the variable of integration, let $x = t + \Delta t - u$, then $-du = dx$ and when $u = T_i$ and $u = T_{i-1}$, $x = t + \Delta t - T_i$ and $t + \Delta t - T_{i-1}$ respectively. Rearranging the sequence, after some manipulation we have for the expected number of failures over period $(t, t + \Delta t)$ due to defects arising within $(0, t + \Delta t)$ may be written as,

$$EN_f(t,t+\Delta t)=\lambda\int_t^{t+\Delta t}\sum_{i=1}^{n-1}(1-r)^{n-i}[F(x-T_{i-1})-F(x-T_i)]dx+\lambda\int_t^{t+\Delta t}F(x-T_{n-1})dx. \quad (5.7)$$

For the probability of identifying the defect at PM time T_n resulting from a defect arising at time u, the probability is given by

$$P(T_n \mid u)=\begin{cases}(1-r)^{n-i}r(1-F(T_n-u)) & T_{i-1}<u\le T_i \;\; i=1,2,\ldots,n-1.\\ r(1-F(T_n-u)) & T_{n-1}<u<T_n\\ 0 & otherwise.\end{cases} \quad (5.8)$$

Therefore, the expected number of defects founded at PM time T_n resulting from defects arising at any time period to T_n, $EN_p(T_n)$, is

$$EN_p(T_n)=\lambda\int_0^\infty P(T_n \mid u)du$$
$$=\sum_{i=1}^{n-1}(1-r)^{n-i}r\lambda\int_{T_{i-1}}^{T_i}[1-F(T_n-u)]du+r\lambda\int_{T_{n-1}}^{T_n}[1-F(T_n-u)]du. \quad (5.9)$$

Since defects are assumed to arise according to a Poisson process, as a generalization of Proposition 2.3.2 in Ross (1983), namely, if an event arrival process follows a Poison process with λ, the number of events that occur by the time t, $N(t)$, is an independent Poisson random variable having mean given by $\lambda\int_0^t P(s)ds$, where $P(s)$ is the probability that the event occurs independently all else at time s, the number of failures in $(t, t + \Delta t)$ follows a Poisson process. Therefore, the probability of m failures over $(t, t + \Delta t)$, where $T_{n-1} < t < T_n$, is given by

$$P(\, m\, failures\ in\ (t,\, t + \Delta t)) = \frac{[EN_f(t, t+\Delta t)]^m e^{-EN_f(t,t+\Delta t)}}{m!}, \tag{5.10}$$

where $EN_f(t, t + \Delta t)$ is the mean number of failure over $(t, t + \Delta t)$.

Christer *et al* (1995) presented the following Lemma 5.1 as a generalisation of the above proposition in Ross (1983).

Lemma 5.1

If the defect arrival process follows a Poisson process with rate λ, the number of defects identified at time t if there is an inspection at time t is Poisson distributed with a mean given by equation (5.9).

Also therefore the number of defects identified at PM follows a Poisson distribution with means defined by equations (5.9) if the defect arrival process is assumed to arise according to a Poisson process. We have that the probability of n defects being identified at T_n is

$$P\,(\, n\ defects\ identified\ at\ T_n\,) = \frac{[EN_p(T_n)]^n e^{-EN_p(T_n)}}{n!} \tag{5.11}$$

where $EN_p(T_n)$ is the mean number of defects identified at T_n (see for proof, Christer and Wang, 1995).

In this case, since the observed events are the number of failures in each working day and the number of defects identified at PM times, the likelihood function of observed events may be formulated in the following way. Suppose data for l PM's has been recorded and that n_i defects have been observed at the ith PM time ($n = 1,2, \ldots, l$), and the PM interval (T_{n-1}, T_n) is now divided into k non-overlapping subintervals I_j^n of equal length Δt, where $\Delta t = 1$ day, that is,

$$I_j^n = [T_{n-1} + (j-1)\Delta t,\ T_{n-1} + j\Delta t],\ (j = 1, \ldots, k), \tag{5.12}$$

where $T_{n-1} + k\Delta t = T_n$.

It follows from equation (5.7) that the mean number of failures occurring in I_j^n over (T_{n-1}, T_n) is, see Figure 5.5,

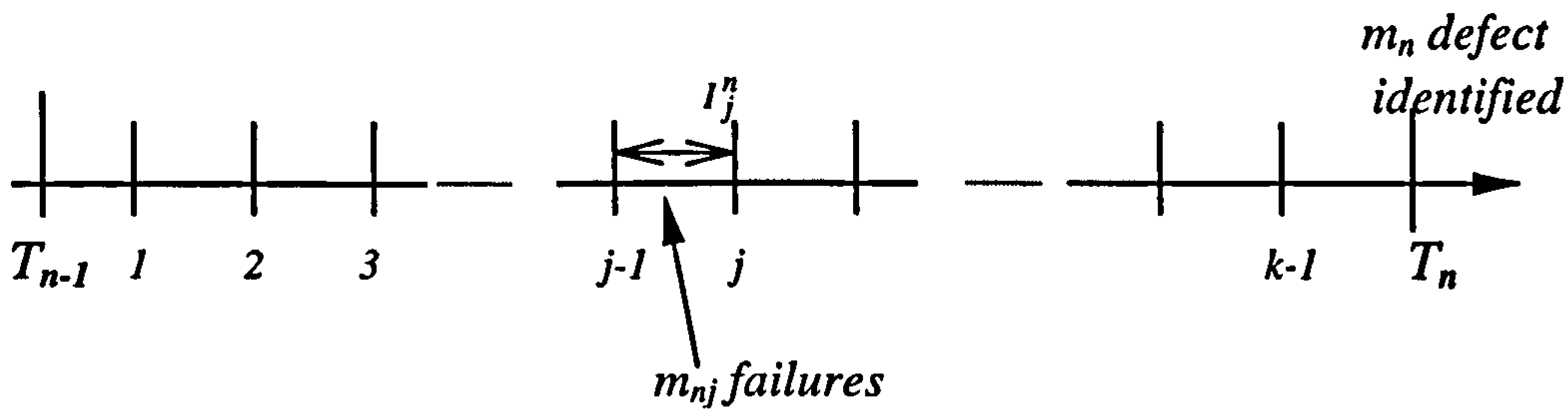


Figure 5.5. The observed number of failures and defects over time (T_{n-1}, T_n)

$$EN_f(I_j^n) = \lambda \int_{T_{n-1}+(j-1)\Delta t}^{T_{n-1}+j\Delta t} \sum_{i=1}^{n-1} (1-r)^{n-i} [F(x - T_{i-1}) - F(x - T_i)] dx$$
$$+ \int_{T_{n-1}+(j-1)\Delta t}^{T_{n-1}+j\Delta t} \lambda F(x - T_{n-1}) dx. \tag{5.13}$$

Now, if m_{nj} denotes the number of failures occurring in I_j^n over (T_{n-1}, T_n) and m_n denotes the defects identified at PM, T_n, we have from equations (5.10) and (5.11) that

$$P(m_{nj}\ failures\ in\ I_j^n) = \frac{[EN_f(I_j^n)]^{m_{nj}} e^{-EN_f(I_j^n)}}{m_{nj}!} \tag{5.14}$$

and

$$P(m_n \textit{ defects identified at } T_n) = \frac{[EN_p(T_n)]^{m_n} e^{-EN_p(T_n)}}{m_n!}. \quad (5.15)$$

Then, the likelihood function for the observation set $\{m_{nj}, m_n\}$, $j = 1,\dots, k$ and $n = 1, \dots, l$ is, given by,

$$L = \prod_{n=1}^{l} \{P(m_n \textit{ defects identified at } T_n) \prod_{j=1}^{k} P(m_{nj} \textit{ failures in } I_j^n)\}. \quad (5.16)$$

Substituting from equation (5.14) and (5.15) into equation (5.16), and taking the logarithm, we have that

$$Log\ L = \sum_{n=1}^{l} [m_n \log EN_p(T_n) - EN_p(T_n) - \log m_n!]$$

$$+ \sum_{n=1}^{l} \sum_{j=1}^{k} [m_{nj} \log EN_f(I_j^n) - EN_f(I_j^n) - \log m_{nj}!], \quad (5.17)$$

where as before, l is the number of PMs. In the case where inspection data are not available, information is lost and the log likelihood function (5.17) reduces to

$$Log\ L = \sum_{n=1}^{l} \sum_{j=1}^{k} [m_{nj} \log EN_f(I_j^n) - EN_f(I_j^n) - \log m_{nj}!]. \quad (5.18)$$

See also Christer *et al.* (1995). Using the above maximum likelihood equations , we can estimate the parameters of the process. In the current case of synthetic simulation data, using expressions (5.17) and (5.18) we hope to recapture the delay time parameters used to generate the data.

5.3.2 Method B ; When time of each failure and the number of defects identified at PM times are available and not available.

Here the available data consists of the number of defects identified at PM times and times of each failure. In this case the likelihood formulation for the observed data to estimate parameters has been established (see Christer, Wang and Choi, 1998). First we need to define our notation. Let T_n denotes the time of the nth PM from new, $n = 1, 2, \ldots$, let $t_{(i-1)j}$ denotes the time of the jth failure occurring in (T_{i-1}, T_i), $j = 1, 2, \ldots, k_{i-1}$, let $t_{(i-1)k_{i-1}}$ denotes the time of the last failure in (T_{i-1}, T_i), and let Δt denotes a time interval sufficiently small that only one event at most can arise within it, see Figure 5.6.

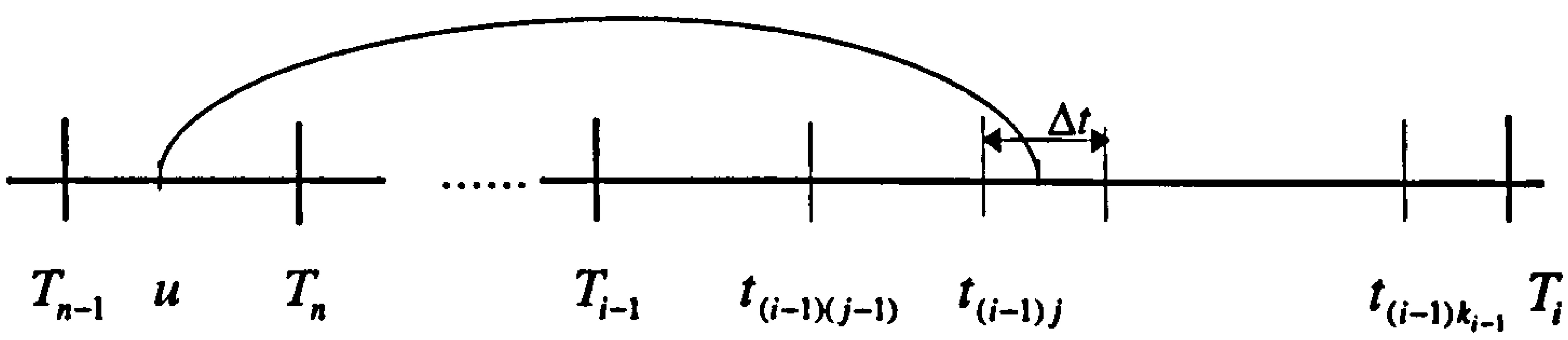


Figure 5.6. The failure process of a fault arising in (T_{n-1}, T_n)

Consider all observation in (T_{i-1}, T_i), namely the PM results at T_i and failure times in (T_{i-1}, T_i). The likelihood function is the product of the probabilities of these observations arising. For the PM results, we can consider the probability of detecting and removing n_i defects from the system if they are present, $P(n_i \; defects \; identified \; at \; T_i)$. For failure times in (T_{i-1}, T_i), we consider the probabilities of a failure arising at times $t_{(i-1)j}$, $j = 1, 2, \ldots, k_{i-1}$, and of having no further failure between failures. Therefore, the likelihood function L is given by

$$L = \prod_{i=1}^{m} \{ P(n_i \; defects \; identified \; at \; T_i)$$

$$\cdot \prod_{j=1}^{k_{i-1}} P(a\, failure\, at\, time\, t_{(i-1)j}) \cdot P(no\, further\, failure\, between\, failures)\} \quad (5.19)$$

and the log likelihood function is given by

$$Log\ L = \sum_{i=1}^{m} \{\log P(n_i\ defects\ identified\ at\ T_i)$$

$$+ \sum_{j=1}^{k_{i-1}} \log P(a\, failure\, at\, time\ t_{(i-1)j}) + \log P(no\, failure\, between\, failures)\}, \quad (5.20)$$

where m is the number of PM

To compute the above log likelihood function, firstly, we consider the probability of a failure in $(t, t+\Delta t)$, $T_{i-1} < t \le T_i$, resulting from a defect arising at time u in (T_{n-1}, T_n), namely $P(t, t+\Delta t|u)$, (see Figure 5.7).

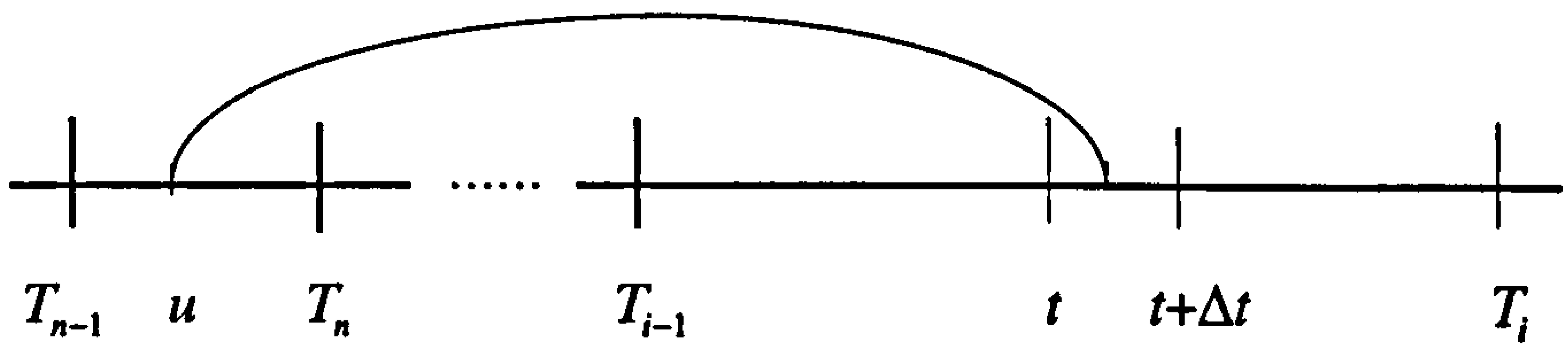


Figure 5.7. The failure process of a defect arising in (T_{n-1}, T_n)

We have, therefore, the probability of a failure in $(t, t+\Delta t)$ caused by a defect arising at time u in (T_{n-1}, T_n) is given by

$$P(t, t+\Delta t|u) = \begin{cases} (1-r)^{i-n}[F(t+\Delta t-u) - F(t-u)] & T_{n-1} < u < T_n,\ n = 1, 2, \cdots, i-1 \\ F(t+\Delta t-u) - F(t-u) & T_{i-1} < u < t \\ F(t+\Delta t-u) & t < u < t+\Delta t \\ 0 & u > t+\Delta t. \end{cases} \quad (5.21)$$

From equations (5.21), in a similar ways to that used to establish $EN_p(T_n)$ in equation (5.8) we can obtain the expected number of defects identified at the ith PM, namely $EN_p(T_i)$ as

$$\mathrm{E}N_p(T_i) = \lambda \sum_{n=1}^{i-1} (1-r)^{i-n} r \int_{T_{n-1}}^{T_n} (1 - F(T_i - u))du + \lambda r \int_{T_{i-1}}^{T_i} (1 - F(T_i - u))du \,. \qquad (5.22)$$

Because the number of defects identified at PM follows a Poisson distribution with means defined by equation (5.22), the probability of n_i defects identified at T_i is

$$P(n_i \; defects\; identified\; at\; T_i) = \frac{(EN_p(T_i))^{n_i} e^{-EN_p(T_i)}}{n_i!}. \qquad (5.23)$$

Using equation (5.21), Christer Wang and Choi (1998) presented the following Lemma 5.2.

Lemma 5.2

If the defect arrival process follows a HPP with the rate of λ, we have that the failure arrival process follows a NHPP with the rate function given by

$$v(t) = \lambda \{ \sum_{n=1}^{i-1} (1-r)^{i-n} [(F(t - T_{n-1}) - F(t - T_n)] + F(t - T_{i-1}) \}, \; T_{i-1} < t \le T_i. \quad (5.24)$$

Given a defect arising at time u, the probability density function of time t to failure is given by

$$p(u;t) = \lim_{\Delta t \to 0} \frac{P(t, t + \Delta t \mid u)}{\Delta t}, \qquad (5.25)$$

where, as before, $P(t, t + \Delta t \mid u)$ is the probability of having a failure over $(t, t + \Delta t)$ resulting from a defect arising at time u. Therefore we have that the failure rate function is given by

$$v(t) = \int_0^t \lambda \lim_{\Delta t \to 0} \frac{P(t, t + \Delta t | u)}{\Delta t} du. \tag{5.26}$$

Also, for $T_{n-1} < u < T_n$, $n = 1, 2, \ldots, i\text{-}1$,

$$\lim_{\Delta t \to 0} \frac{P(t, t + \Delta t \mid u)}{\Delta t}$$
$$= (1-r)^{i-n} \lim_{\Delta t \to 0} \frac{F(t + \Delta t - u) - F(t-u)}{\Delta t} = (1-r)^{i-n} f(t-u), \tag{5.27}$$

and for $T_{i-1} < u < t$,

$$\lim_{\Delta t \to 0} \frac{P(t, t + \Delta t \mid u)}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{F(t + \Delta t - u) - F(t-u)}{\Delta t} = f(t-u). \tag{5.28}$$

Therefore, equation (5.26) becomes

$$v(t) = \int_0^t \lambda \lim_{\Delta t \to 0} \frac{P(t, t + \Delta t | u)}{\Delta t} du$$
$$= \lambda \{ \sum_{n=1}^{i-1} (1-r)^{i-n} \int_{T_{n-1}}^{T_n} f(t-u)du + \int_{T_{i-1}}^{t} f(t-u)du)$$
$$= \lambda \{ \sum_{n=1}^{i-1} (1-r)^{i-n} [(F(t-T_{n-1}) - F(t-T_n)] + F(t-T_{i-1})\} \tag{5.29}$$

We now define expression for the probability of specific failure events in (T_{i-1}, T_i). Using Lemma 5.2, we can obtain the probability of a failure arising in time interval $(t_{(i-1)j}, t_{(i-1)j} + \Delta t)$. For sufficiently small Δt, we have

$$P(a\,failure\ in\ time\ (t_{(i-1)j}, t_{(i-1)j} + \Delta t)) = v(t_{(i-1)j})\Delta t + 0(\Delta t). \tag{5.30}$$

Since the probability of having no failure in $(t_{(i-1)(j-1)}, t_{(i-1)j})$ is given by the Poisson property as

$$P(no\,failure\ in\ (t_{(i-1)(j-1)}, t_{(i-1)j})) = e^{-\int_{t_{(i-1)(j-1)}}^{t_{(i-1)j}} v(t)dt}, \tag{5.31}$$

the total summation of the log Probability(of having no further failure between failures within (T_{i-1}, T_i)) is given by

$$\sum \log P(no\,further\,failure\ between\ failures) = \sum_{j=1}^{k_{i-1}} (-\int_{t_{(i-1)(j-1)}}^{t_{(i-1)j}} v(t)dt) - \int_{t_{(i-1)k_{i-1}}}^{T_i} v(t)dt. \tag{5.32}$$

If we define $t_{(i-1)0} = T_{i-1}$, without loss of generality, the equation (5.32) becomes

$$\sum \log P(no\,further\,failure\ between\ failures) = -\int_{T_{i-1}}^{T_i} v(t)dt. \tag{5.33}$$

In equation (5.33), log *P(no further failure between failures)* is necessary because of the complex component nature of the plant, and would not apply if it is single component item. If no further failures occurred in (T_{i-1}, T_i), from equation (5.31), the probability of having no further failure in (T_{i-1}, T_i) is given by

$$P(no\,failure\ in\ (T_{i-1}, T_i)) = e^{-\int_{T_{i-1}}^{T_i} v(t)dt}. \tag{5.34}$$

Dividing the equation (5.30) by Δt and taking the logs of equations (5.23) and (5.33), the log likelihood function becomes

$$Log\,L = \sum_{i=1}^{m} \log P(n_i \; defects\; identified\; at\; T_i)$$

$$+\sum_{i=1}^{m}\{\sum_{j=1}^{k_{i-1}} \log \frac{P(a\, failure\, at\, time\, t_{(i-1)j})}{\Delta t} + \sum \log P(no\, further\;\; failure\, between\, failures)\}$$

$$= \sum_{i=1}^{m}(n_i \log EN_p(T_i) - EN_p(T_i) - \log n_i!) + \sum_{i=1}^{m}\{\sum_{j=1}^{k_{i-1}} \log v(t_{(i-1)j}) - \int_{T_{i-1}}^{T_i} v(t)dt\}. \quad (5.35)$$

In particular, for case where inspection data are not available, the contracted log likelihood function is given by

$$Log\,L = \sum_{i=1}^{m}\{\sum_{j=1}^{k_{i-1}} \log v(t_{(i-1)j}) - \int_{T_{i-1}}^{T} v(t)dt\}. \quad (5.36)$$

Using the above likelihood equations, we can estimate the parameters of the process from actual simulated data.

5.4 Evaluation of Estimated Parameter

To evaluate the estimated parameters, the downtime model of delay time, developed by Christer *et al.*, has been used here. Given an acceptable model for the failure and PM process of the system, a downtime model of maintenance may be established. If we assume that the major concern of maintenance activities is to reduce the downtime caused by failures and PM activities the conventional downtime measure can be the expected downtime per unit time over a long future period.

Based upon assumptions of the section 5.2, the long term measure of the expected downtime per unit time, $D(T)$, is

$$D(T) = \frac{d_f EN_f(T) + d_p}{T}. \tag{5.37}$$

where d_f is the mean downtime per failure when the PM cycle length is T, $EN_f(T)$ is the expected number of failures over T and d_p is the mean duration of the PM activity.

Figure 5.8 shows the envisaged relationships between expected downtime and inspection interval. Using the downtime model, we can calculate the mathematically optimum inspection period from true parameter values, that is the parameter values used to generate the simulated data.

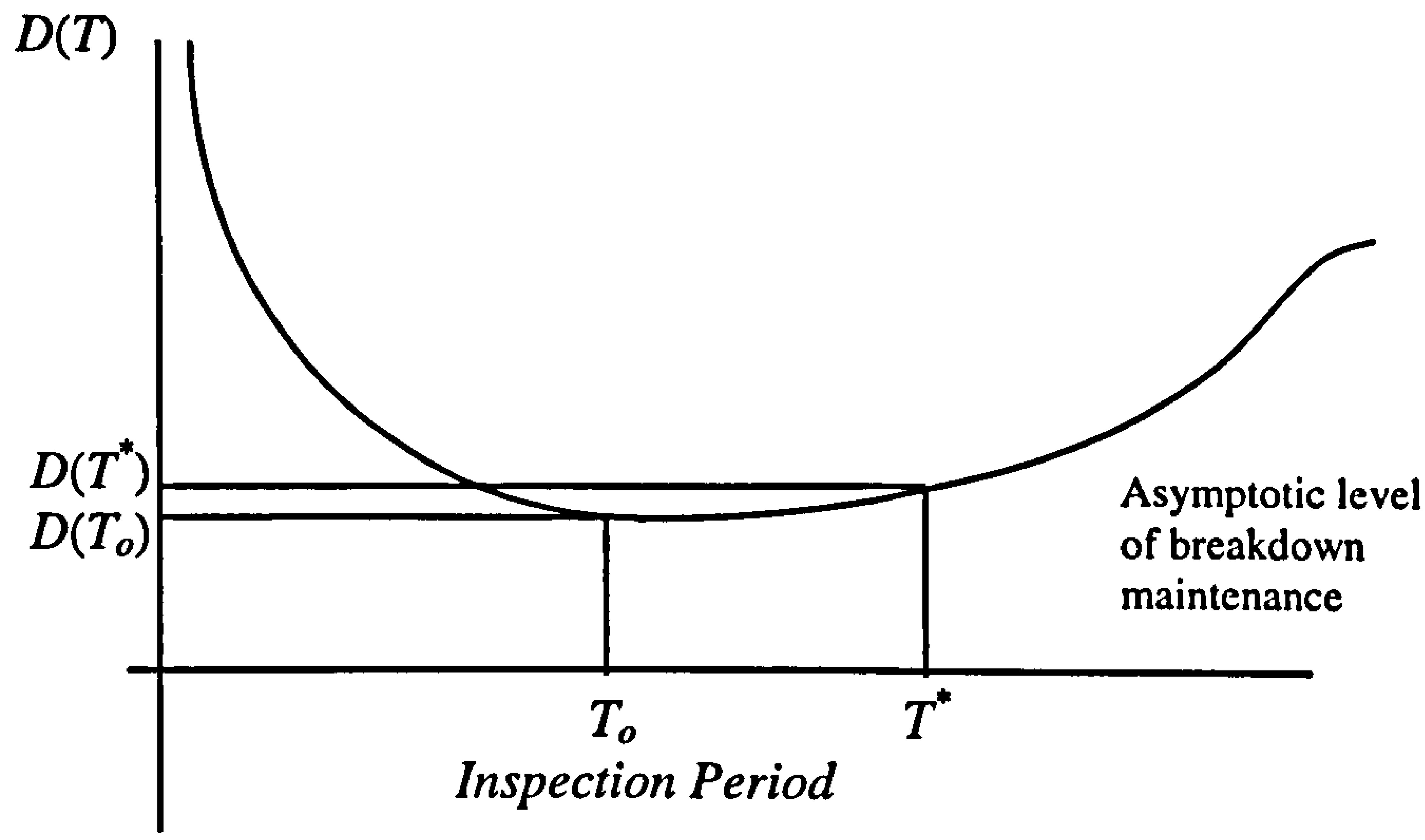


Figure 5.8. Expected downtime against Inspection interval

Thus, if we have estimated parameter values, we can obtain a measure of the decision consequences, namely the differences between the 'true' values and the model based estimated values. This we call the inspection period error (called *I*err) with respect to the downtime model. That is, *Ierr* is can be written as below

$$Ierr = \frac{T^* - T_o}{T_o} \times 100, \tag{5.38}$$

where T_o is the optimum inspection period obtained from true parameter values and T^* is the optimum inspection period obtained estimated parameter values.

Similarly since we have obtained the optimal inspection interval from the true given parameter values, it is possible to compare the difference between the two parameter sets of the expected downtime. This measure is called the downtime error, (called *Derr*), which is also given by

$$Derr = \frac{D(T^*) - D(T_o)}{D(T_o)} \times 100, \tag{5.39}$$

where $D(T^*)$ is the downtime for estimated parameter values leading to T^* and $D(T_o)$ is the true expected downtime corresponding to the given parameter values leading to T_o.

One way to evaluate parameters estimated here is to use statistical decision theory. Sometimes it may be difficult to obtain the optimum value for the downtime model, depending upon given conditions. In such a case we can use decision theory.

Consider a situation where, given the data, it is necessary to make an inspection/PM decision. Further, assume that the consequences of decisions are known and that they can be evaluated numerically. Given the necessary background, the problem is to decide on optimum decision rules with reference to some performance measure. Some general features will be found in any formulation of a decision problem as an 'inference' about a parameter θ. The parameter θ represents perfect information, because when θ is known any further information is irrelevant. In statistical decision theory, it is usual to formulate a *loss function* $L(d, \theta)$, which expresses how bad it would be to make decision d if the parameter value was θ. The optimal decision is to minimize expected loss. For predictive inference, θ can be replaced by some future observation, and $f(\theta)$ represents the appropriate predictive distribution.

First consider the problem of point estimation, where the decision consists of asserting a single parameter as an estimate of θ. We therefore identify the decision set $\wp$ as the set of possible values of θ. The ideal estimate is $d = \theta$, and it is natural to define $L(\theta, \theta) = 0$ for all θ. Then it should also be the case that if θ is further from d_1 than from d_2, according to some appropriate measure of distance, $L(d_1, \theta) \geq L(d_2, \theta)$. Now we suppose that θ is scalar, and therefore so is d. The *quadratic loss* function $L(d, \theta) = (d - \theta)^2$, is also known as the squared-error loss (see, O'hagan, 1994, pp50-52)

$$E(L(d,\theta)) = d^2 - 2dE(\theta) + E(\theta^2) = (d - E(\theta))^2 + \text{var}(\theta). \tag{5.40}$$

This is minimized at $d = E(\theta)$, which is therefore the optimal estimate.

For example, in this study, we have four true parameter values in the model. If we also have four estimated parameter values, then we can obtain $L(d, \theta)$ as the minimize expected loss.

$$E(L(d,\theta)) = \sum_{i=1}^{4} (d_i - \theta_i)^2 , \tag{5.41}$$

where d_i is the set of true parameter values and θ_i is the corresponding set of estimated parameters. Therefore, if we have values of estimated parameter from simulated PM data set, we can evaluate the expected loss for the case methods A and B from equation (5.40).

Now we use the evaluation model, which is equations (5.38) and (5.39) for under perfect inspection policy for comparing method A and B. And in case of difficulty of obtaining the optimum value for the downtime model, the statistical decision model, which is the equation (5.40), will be use later for under imperfect inspection policy.

5.5 Parameter Estimation Results based upon Simulation Tests

5.5.1 Under Perfect Inspection

First we consider assume both inspection and repair are perfect, and choose the delay time as exponential, namely $F(h) = 1 - e^{-\alpha h}$, where $1/\alpha$ is the mean delay time. The rate of occurrence of defects, λ, is assumed constant. To generate a set of PM data using simulation, we assume for demonstration purposes that the data have been given in the real-world situation or simulation, the estimates of λ and α are $\lambda = 0.9$ and $\alpha = 0.05$. Based upon the simulated PM data, the results of the parameter estimation process based upon the equations (5.17) and (5.35) for case with inspection data, and equations (5.18) and (5.36) for the case without inspection data, are shown in Tables 5.2 and 5.3 respectively. Once parameters are estimated, the inspection period error and the downtime error are obtained from equations (5.38) and (5.39).

Here we arbitrarily choose the mean duration of PM activity, d_p, to be d_p = 1hour, and the average downtime per failure, d_f, to be $d_f = 0.28$ hours. When the given true values are $\lambda = 0.9$ and $\alpha = 0.05$, the optimum inspection interval and the expected downtime per day can obtain as 16days and is 0.1410 hours per day. These values are obtained using the equation (5.37).

As an example of demonstration of the evaluation models, if we assume that estimated parameter values are $\hat{\lambda} = 0.5$ and $\hat{\alpha} = 0.06$, the optimum inspection interval and the expected downtime per day can obtain as 26days and is 0.1494hours using the equation (5.37). Therefore, from equations (5.38) and (5.39), the inspection period error(*I*err) and the downtime error(*D*err) are 62.5 % and 5.96 %.

The estimate inspection period is overestimated by 62.5 % for true optimum inspection period. And the downtime error by an estimated inspection period is 5.96 % from true asymptotic level of breakdown maintenance. Figure 5.9 illustrates the expected downtime against inspection interval when true parameter values are $\lambda = 0.9$ and $\alpha = 0.05$, estimated parameter values are $\hat{\lambda} = 0.5$ and $\hat{\alpha} = 0.06$.

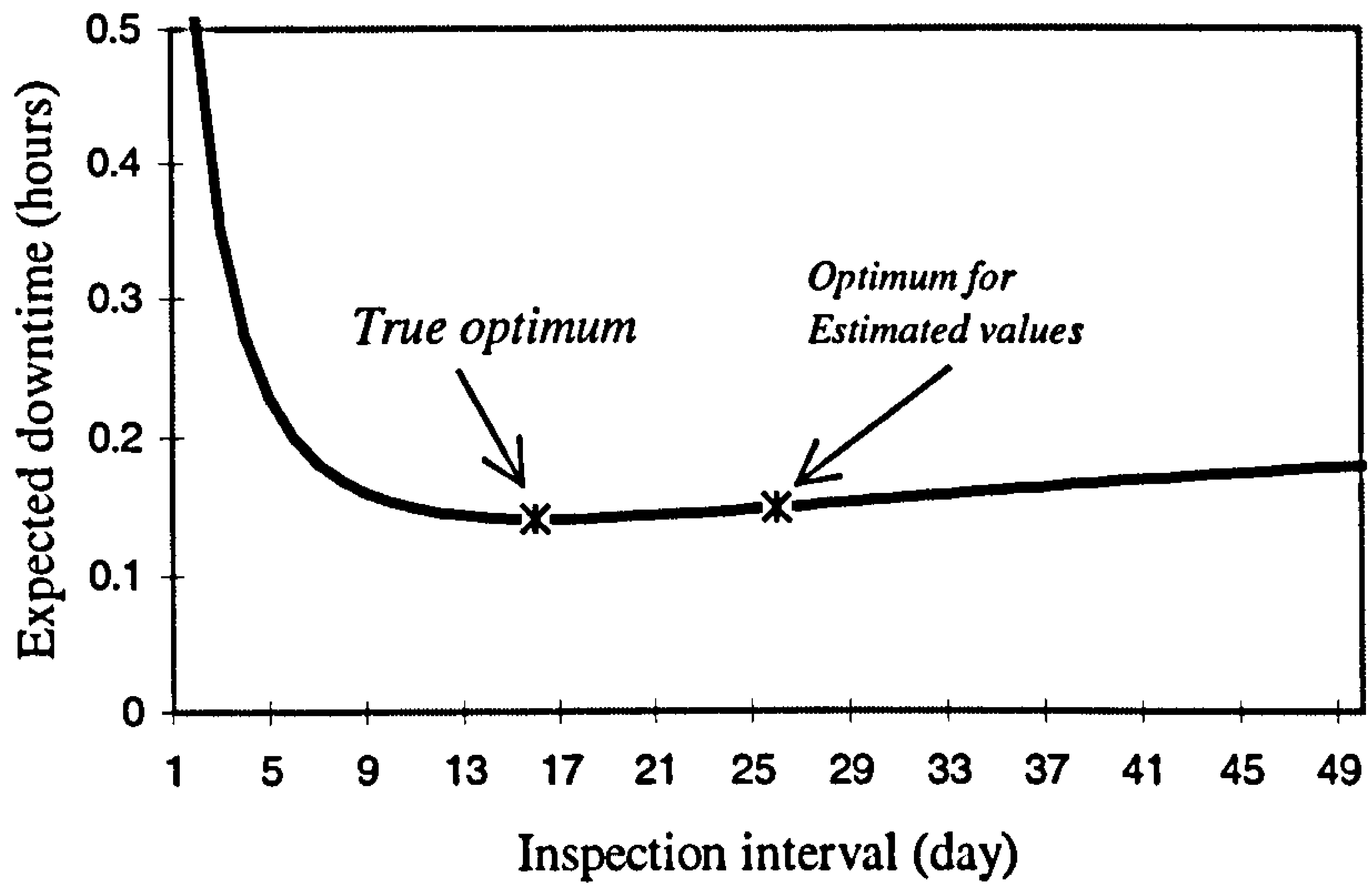


Figure 5.9. Expected downtime against inspection interval
(when true parameter values are $\lambda = 0.9$ and $\alpha = 0.05$ and estimated parameter values are $\hat{\lambda} = 0.5$ and $\hat{\alpha} = 0.06$.)

For given true values, λ = 0.9 and α = 0.05, Table 5.2 shows the parameter estimation from simulated data with PM information. This table shows that if sufficient PM information is available, the maximum likelihood estimates recover the underlying process defect origination, parameter λ, and that even when there is a small number of PMs in the two methods based on the equations (5.17) and (5.35) the delay time parameter α is determined in this case remarkably accurately. In terms of the downtime error (*Derr*) and inspection period error (*Ierr*), the two methods are, to within the accuracy of the calculations, close to the true parameter values. Table 5.3 also shows that the estimation result is a symmetric because the number of defect identified is same and only difference of methods is between the number of failures and times of failures within PM from equations (5.17) and (5.35). Therefore, it means that the variance of the estimation of samples is very small in case of PM data available. In the Table, '0' figure of under *D*err and *I*err represents 0 % of errors.

Table 5.3. Parameter estimation from simulated data with PM information

Sample size		Method A: Estimation given number of failure				Method B: Estimation given times of each failure			
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	*Derr(%)*	*Ierr(%)*	$\hat{\lambda}$	$\hat{\alpha}$	*Derr(%)*	*Ierr(%)*
10	144	0.993	0.050	0.243	-6.25	0.993	0.050	0.243	-6.25
50	659	0.927	0.049	0	0	0.927	0.049	0	0
100	1358	0.936	0.050	0.021	6.25	0.936	0.050	0.021	6.25
150	2001	0.922	0.050	0	0	0.922	0.050	0	0
200	2600	0.904	0.050	0	0	0.904	0.050	0	0
True values		0.9	0.05			0.9	0.05		

*λ is the rate of occurrence of defects, α is the delay time parameter, *D*err is downtime error and *I*err is the inspection period error.

Table 5.4. Parameter estimation from simulated data without PM information

Sample size		Method A: Estimation given number of failure				Method B: Estimation given times of each failure			
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	*Derr(%)*	*Ierr(%)*	$\hat{\lambda}$	$\hat{\alpha}$	*Derr(%)*	*Ierr(%)*
10	144	0.959	0.053	0.243	-6.25	0.981	0.051	0.243	-6.25
50	659	0.750	0.070	0.021	6.25	0.753	0.070	0.021	6.25
100	1358	0.856	0.059	0	0	0.860	0.058	0	0
150	2001	0.902	0.052	0	0	0.901	0.052	0	0
200	2600	0.870	0.053	0	0	0.869	0.053	0	0
True values		0.9	0.05			0.9	0.05		

*λ is the rate of occurrence of defects, α is the delay time parameter, *D*err is downtime error and *I*err is the inspection period error.

Table 5.4 also shows that estimates in the case when PM information is not available. This case also recovers the underlying process of failures and defects in the given example, but only when the data set is sufficiently large. The two case methods with time of failures and day of failures are not very difference to each other for estimated parameter values and also downtime error and inspection period error.

Now we compare the two methods when the delay time has the Weibull distribution $F(h) = 1 - e^{-\alpha h^{\beta}}$, where α is the scale parameter and β is the shape parameter. As before, we generate a data set using simulation and adopt the simulated data as observed data. The rate of occurrence of defects (ROCOD) is assumed constant, λ = 0.5, and the distribution of delay time is Weibull, $F(h) = 1 - e^{-\alpha h^{\beta}}$, with scale parameter, $\alpha = 0.15$ and shape parameter β taking values 0.6, 0.9, 1.1, 1.5 and 2.0. Thus mean values of the delay time distributions are approximately 35.53, 8.661, 5.414, 3.198 and 2.288 respectively, corresponding to ranged 0.6 to 2.0 of shape parameter, β.

For comparison, if shape parameter, β =1, and the scale parameter, $\alpha = 0.15$, the mean values of this Weibull delay time distribution is 6.667. Then using the equations (5.17) and (5.35), and (5.18) and (5.36) we obtain the estimated parameter values in the cases when the PM information is available, and the PM information is unavailable. This results in methods A and B from the simulation data set.

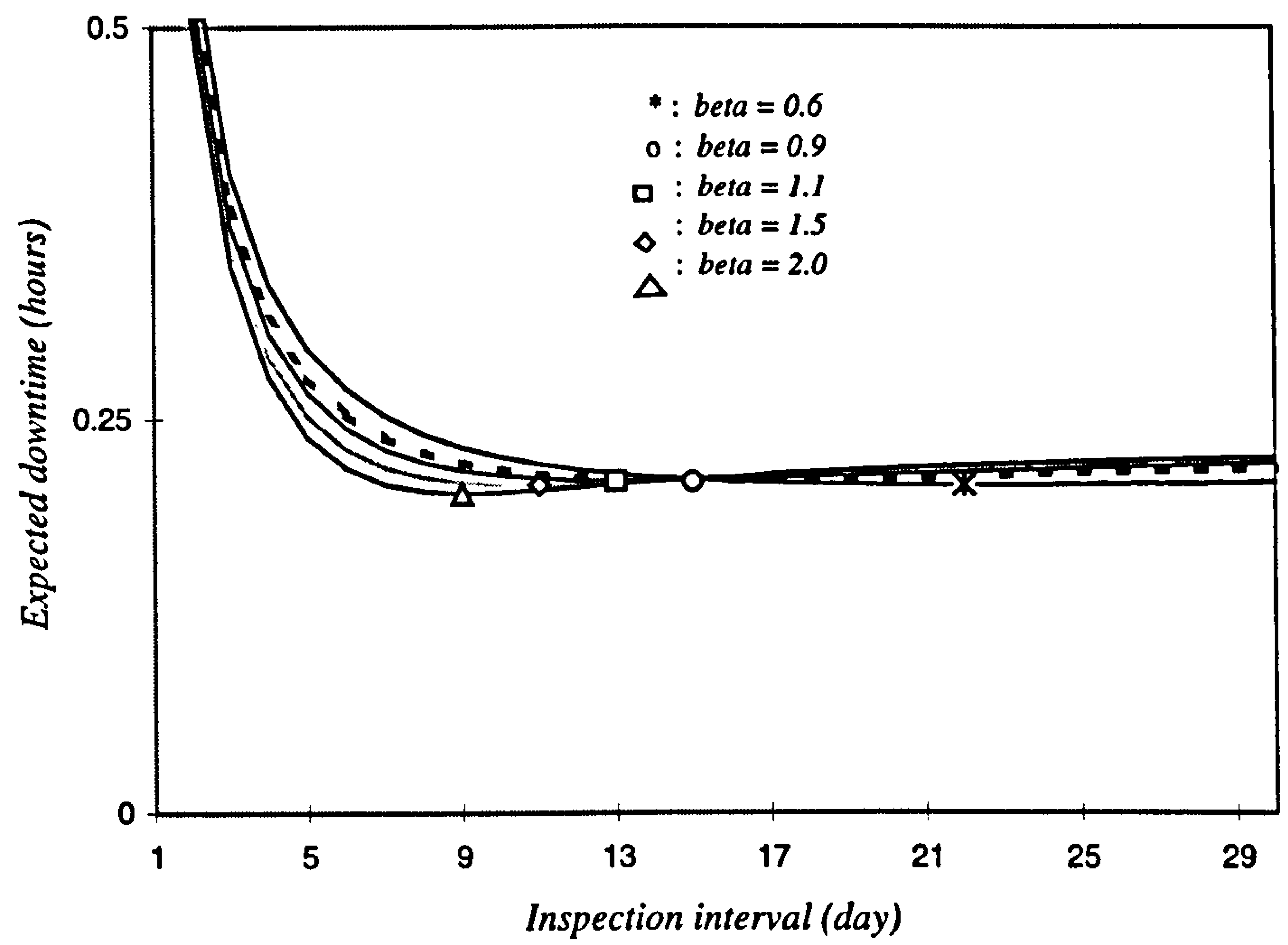


Figure 5.10. Expected downtime against inspection interval when parameter values are $\lambda = 0.5$, $\alpha = 0.15$ and β ranges over 0.6.0.9, 1.1 and 2.0 (Perfect inspection case, r = 1)

Tables 5.5. and 5.6 show the estimation result. Figure 5.10 also shows the optimum inspection interval for given true values as β varies between 0.6, 0.9, 1.1, 1.5

and 2.0. For parameter values are λ = 0.5 α = 0.15 and β = 0.6 the optimum inspection interval is 22 days and downtime per day is 0.2086 hours. If β is 0.9, 1.1, 1.5 and 2.0, optimum inspection intervals are 15 ,13, 11 and 9 days respectively. Further the correspondingly optimum expected downtimes are 0.2112, 0.2107, 0.2077 and 0.2037.

Table 5.5. Parameter estimation from simulated data with Weibull distribution when α = 0.15 and β ranges over 0.6 ~ 2.0.(with PM information in perfect inspection case)

Sample size		Method A: *Estimation given number of failure*					Method B: *Estimation given times of each failure*				
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	*Derr (%)*	*Ierr (%)*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	*Derr (%)*	*Ierr (%)*
10	131	0.553	0.199	0.592	0.011	-4.5	0.553	0.189	0.639	0.181	-13.6
50	580	0.522	0.151	0.603	0.011	-4.5	0.522	0.149	0.622	0.068	-9.1
100	1153	0.519	0.142	0.693	0.068	-9.1	0.519	0.140	0.707	0.365	-18.2
150	1732	0.522	0.140	0.695	0.365	-18.2	0.522	0.139	0.706	0.365	-18.2
200	2269	0.516	0.140	0.668	0.181	-13.6	0.516	0.139	0.674	0.181	-13.6
True values		0.5	0.15	0.6			0.5	0.15	0.6		
10	139	0.553	0.206	0.848	0.013	6.7	0.553	0.199	0.910	0	0
50	610	0.522	0.155	0.861	0	0	0.522	0.154	0.875	0	0
100	1212	0.519	0.147	0.984	0.113	-6.7	0.519	0.146	0.998	0.113	-6.7
150	1825	0.522	0.145	1.011	0.399	-13.3	0.522	0.145	1.020	0.926	-20.0
200	2391	0.515	0.145	0.975	0.113	-6.7	0.516	0.145	0.978	0.113	-6.7
True values		0.5	0.15	0.9			0.5	0.15	0.9		
10	140	0.553	0.190	1.117	0.134	-7.7	0.553	0.193	1.065	0	0
50	622	0.522	0.155	1.025	0	0	0.522	0.155	1.031	0	0
100	1236	0.519	0.148	1.181	0.134	-7.7	0.519	0.148	1.180	0.134	-7.7
150	1854	0.522	0.146	1.211	0.134	-7.7	0.522	0.146	1.218	0.134	-7.7
200	2435	0.515	0.146	1.169	0.134	-7.7	0.515	0.146	1.173	0.134	-7.7
True values		0.5	0.15	1.1			0.5	0.15	1.1		
10	142	0.553	0.192	1.302	0	0	0.553	0.189	1.392	0.051	-9.1
50	631	0.522	0.154	1.317	0	0	0.522	0.154	1.324	0	0
100	1260	0.519	0.150	1.520	0.051	-9.1	0.519	0.150	1.521	0.051	-9.1
150	1884	0.522	0.147	1.611	0.051	-9.1	0.522	0.147	1.620	0.051	-9.1
200	2474	0.516	0.147	1.565	0.051	-9.1	0.516	0.147	1.560	0.051	-9.1
True values		0.5	0.15	1.5			0.5	0.15	1.5		
10	142	0.553	0.180	1.864	0.608	-11.1	0.553	0.181	1.754	0	0
50	634	0.522	0.152	1.714	0	0	0.522	0.153	1.720	0	0
100	1262	0.519	0.148	1.902	0	0	0.519	0.148	1.921	0	0
150	1880	0.522	0.145	2.090	0	0	0.522	0.145	2.092	0	0
200	2464	0.516	0.144	1.980	0	0	0.516	0.144	2.005	0	0
True values		0.5	0.15	2.0			0.5	0.15	2.0		

λ is the rate of occurrence of defects, α and β are the scale and the shape parameters of delay time distribution, *D*err is downtime error and *I*err is the inspection period error.

Table 5.6. Parameter estimation from simulated data with Weibull distribution when $\alpha = 0.15$ and β ranges over = 0.6 ~ 2.0.(without PM information in perfect inspection case)

Sample size		Method A					Method B				
		Estimation given number of failure					*Estimation given times of each failure*				
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	*Derr (%)*	*Ierr (%)*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	*Derr (%)*	*Ierr (%)*
10	131	5.432	9.9e-5	0.372	0.644	31.8	6.120	9.9e-5	0.391	1.027	-27.3
50	580	4.617	9.9e-5	0.365	0.365	-18.2	2.351	0.001	0.390	0.365	-18.2
100	1153	0.684	0.059	0.555	0.638	-22.7	0.627	0.078	0.598	0.638	-22.7
150	1732	0.558	0.113	0.646	0.365	-18.2	0.541	0.125	0.678	0.365	-18.2
200	2269	0.583	0.094	0.592	0.365	-18.2	0.562	0.106	0.617	0.365	-18.2
True values		0.5	0.15	0.6			0.5	0.15	0.6		
10	139	10.91	9.9e-5	0.473	0.113	-6.7	12.26	9.9e-5	0.491	0.113	-6.7
50	610	0.772	0.052	0.629	0	0	0.698	0.068	0.674	0	0
100	1212	0.594	0.102	0.836	0.113	-6.7	0.584	0.106	0.862	0.113	-6.7
150	1825	0.559	0.121	0.917	0.399	-13.3	0.553	0.124	0.936	0.399	-13.3
200	2391	0.558	0.117	0.874	0.113	-6.7	0.554	0.119	0.884	0.113	-6.7
True values		0.5	0.15	0.9			0.5	0.15	0.9		
10	140	0.800	0.069	0.753	0.134	-7.7	0.884	0.054	0.699	0	0
50	622	0.627	0.095	0.828	0	0	0.611	0.101	0.851	0	0
100	1236	0.570	0.116	1.033	0.134	-7.7	0.569	0.116	1.035	0.134	-7.7
150	1854	0.542	0.132	1.134	0.134	-7.7	0.541	0.133	1.145	0.134	-7.7
200	2435	0.540	0.129	1.082	0.134	-7.7	0.539	0.129	1.089	0.134	-7.7
True values		0.5	0.15	1.1			0.5	0.15	1.1		
10	142	0.651	0.118	1.023	2.249	-27.3	0.631	0.127	1.121	0	0
50	631	0.551	0.133	1.194	0.051	-9.1	0.552	0.131	1.196	0	0
100	1260	0.554	0.126	1.360	0.677	-18.2	0.555	0.125	1.362	0	0
150	1884	0.535	0.138	1.531	0.067	-18.2	0.535	0.138	1.541	0.051	-9.1
200	2474	0.528	0.137	1.491	0.067	-18.2	0.528	0.137	1.485	0.051	-9.1
True values		0.5	0.15	1.5			0.5	0.15	1.5		
10	142	0.590	0.150	1.771	0.608	-11.1	0.592	0.149	1.663	0	0
50	634	0.535	0.142	1.620	0.291	11.1	0.535	0.142	1.632	0.291	11.1
100	1262	0.544	0.131	1.749	0	0	0.543	0.132	1.771	0	0
150	1880	0.528	0.141	2.043	0	0	0.528	0.141	2.047	0	0
200	2464	0.520	0.141	1.951	0	0	0.519	0.142	1.981	0	0
True values		0.5	0.15	2.0			0.5	0.15	2.0		

λ is the rate of occurrence of defects, α and β are the scale and the shape parameters of delay time distribution, *D*err is downtime error and *I*err is the inspection period error.

Table 5.5 shows that the maximum likelihood estimates recovers the parameters well in both methods A and B, the underlying process of failures and the defects origination process for the particular case. From the downtime model, the optimal inspection interval is 22 days and the expected downtime is 0.2086 hours for the given true values, $\lambda = 0.5$ $\alpha = 0.15$ and $\beta = 0.6$. Table 5.5 also shows that downtime error (*Derr*) and inspection period error (*Ierr*) are less than about 0.4 % and 20 %. In case of $\beta = 0.9$, the maximum likelihood estimates also recover well the underlying process of failures and defects origination in both methods A and B. The optimum inspection interval is 15 days and expected downtime per day is 0.2112 hours. Downtime error and inspection period error are less than 0.4 % and 20 %. There is no evident difference between the two methods.

For $\beta = 1.1 \sim 2.0$, the maximum likelihood estimates also more recovers well the underlying process of failure and defect arrival as the set of inspection data and the number of inspections increase, and downtime error and inspection period error are less than 0.13 % and 9.1 %. In the case of $\beta = 1.1$ and 1.5, downtime error (*Derr*) and inspection error (*Ierr*) are the same error even the value of estimates is not identical. The reason is that optimal value is not very much sensitive to the model parameter and there is not a lot of difference of between estimates model parameters Table 5.6 depicts the case of PM information is not available shows that the estimation results of maximum likelihood for variable values of shape parameter $\beta = 0.6 \sim 2.0$.

From Table 5.6, we can see irrespective of the form of failure record that as the number of inspection increase the estimation of parameters based upon equations (5.17) and (5.35) is more accurate only in case of $\beta = 2.0$. And downtime error and inspection interval error also are not much different for methods A and B in case of $\beta < 1.1$. However, in case of $\beta = 1.5$ and 2.0, we can see that Method B is slightly more accurate to Method A. Figure 5.11 shows the regenerated number of failures for the method A and B against given true number of failures when number of PMs are 100, $\lambda = 0.5$, $\alpha = 0.15$ and the value of β is varied as 0.6 to 2.0 of a Weibull. It also shows that there are no relevant differences for method A and B.

In conclusion here, under the perfect inspection policy, for two case methods A and B, that is, method A assumes PM data and the number of failures per working day, and method B assumes PM data and times of failures available, the maximum likelihood estimates recover the underlying process failure and defect origination for both cases. Even without PM information, with a Weibull distribution of delay time, for the shape parameter $\beta = 2.0$, as the number of inspections increase the estimation process can recover the underlying process. There is also no relevant different between the estimation results from methods A and B for exponential and Weibull distribution of delay time. Therefore, comparing method A and B, under perfect inspection policy we appear safe in assuming that the likelihood formulation doesn't introduce a serious bias in parameter estimation accuracy for different formats of failure data, and if PM information is available or unavailable.

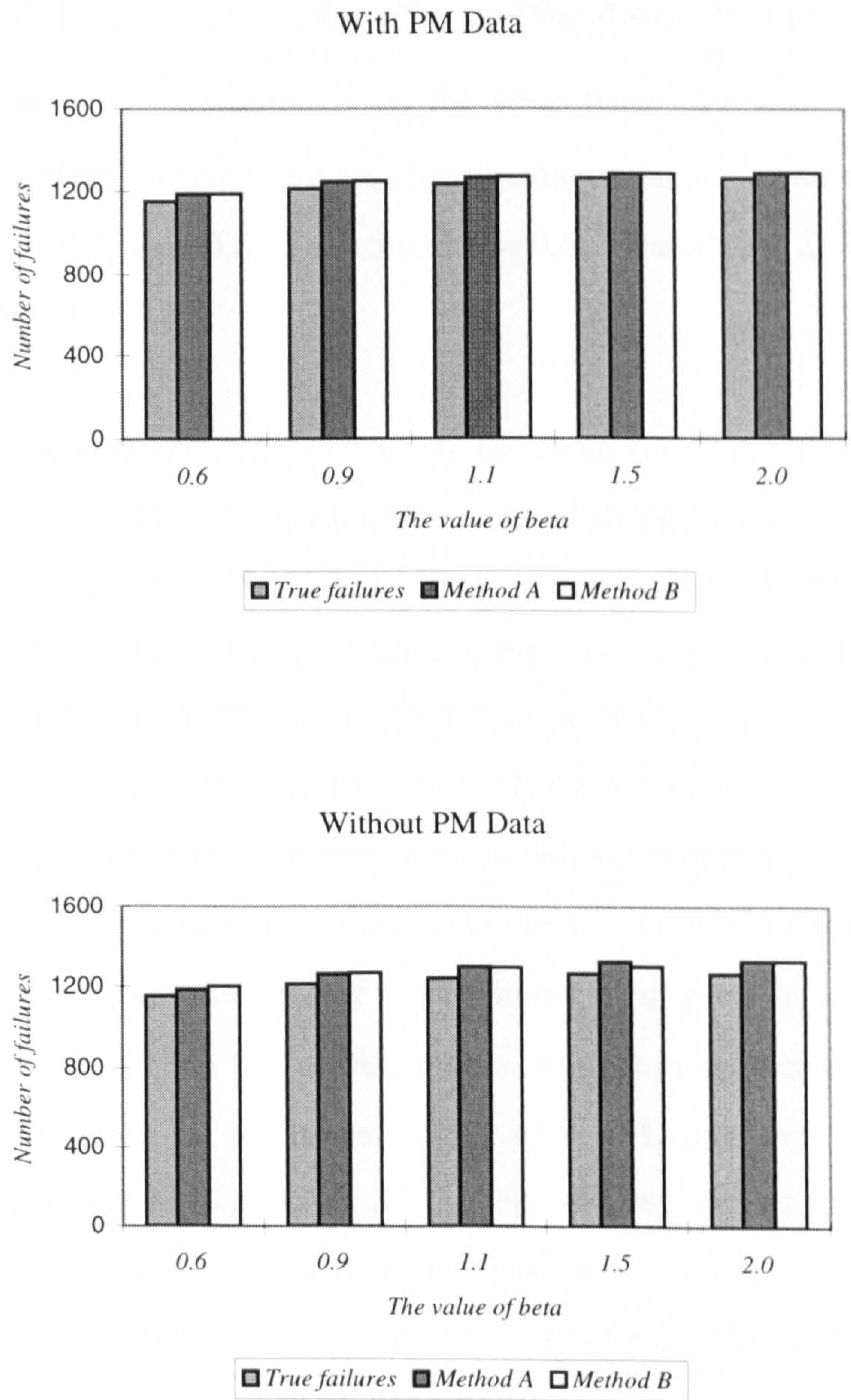


Figure 5.11. Comparison of Method A and Method B against the true number of failures when number of PMs are 100 and delay time is a Weibull distribution. (Perfect inspection case, $r = 1$)

So far, we assumed that the rate of occurrence of defects (ROCOD) is a constant, λ. Much of the recent work on modelling and analysis of repairable system is based on the assumption of a special type of NHPP known as a Weibull process (Bain and Engelhardt, 1991). The name Weibull process derives primarily from the resemblance of the intensity function of the process to the hazard function of a Weibull distribution, $\lambda = \lambda_1 \lambda_2 u^{\lambda_2 - 1}$.

In the following case is assumed that the rate of occurrence of defects (ROCOD) at time u is $\lambda(u)$, namely $\lambda(u) = \lambda_1 \lambda_2 u^{\lambda_2 - 1}$, and the distribution of delay time has a Weibull, $F(h) = 1 - e^{-\alpha h^{\beta}}$, where α is the scale parameter and β is the shape parameter. As before, to generate a set of maintenance data using simulation, we first assume that $\lambda_1 = 0.5$, $\lambda_2 = 0.6$, $\alpha = 0.5$ and $\beta = 0.6$. Values of λ_2 are β are then increased to 0.9, 1.1 and 2.0.

For given Weibull delay time distribution, the mean values are 4.777, 2.273, 1.812, 1.433, and 1.253 when the scale parameter is 0.5 and shape parameter are 0.6, 0.9, 1.1, 1.5 and 2.0. For equations (5.17) and (5.35) we can replace λ into $\lambda(u)$ with no loss of generality. Then the parameter estimation results based upon the new equations of (5.17) and (5.35), for different values of the λ_2 and β and various number of inspections are given by Table 5.7. Table 5.7, in cases of $\lambda_2 = 0.6$ and 2.0 and $\beta = 0.6$ and 2.0, show that the maximum likelihood estimate does not in all cases recover the underlying process of failure and defects origination based upon the new equations (5.17) and (5.35) in spite of large number of inspections. In case of $\lambda_2 =$ 0.9 and 1.1 and $\beta = 0.9$ and 1.1, as the number of inspections increase the maximum likelihood estimates recover the underlying process of failure and defect origination based upon equations (5.17) and (5.35). For all these cases, since the rate of occurrence of defects (ROCOD) at time u, $\lambda(u)$, is changing, it is more difficult to estimate the parameter than the constant rate of occurrence of defects. As the delay time parameter is Weibull, becomes more difficult to estimate parameters.

However, even when the maximum likelihood estimate does recover the underlying process, we see that the downtime error and inspection interval error can not be obtain from the downtime model in some cases since it has no optimal inspection interval from the estimated parameters. When we have a true given values, $\lambda_1 = 0.5$, $\lambda_2 = 0.6$, $\alpha = 0.5$ and $\beta = 0.6$, and $\lambda_1 = 0.5$, $\lambda_2 = 0.9$, $\alpha = 0.5$ and $\beta = 0.9$ the optimal inspection interval for $d_f = 1.0$ and $d_p = 0.5$ is not effectively infinite, that is do not inspect. In the case of $\lambda_2 = 1.1$ and 2.0, $\beta = 1.1$ and 2.0, the best inspection intervals are 11 and 2 days, respectively.

Table 5.7. Parameter estimation from simulated data with Weibull distribution when λ_1 and $\alpha = 0.5$, λ_2 and β ranges over 0.6 ~ 2.0 of case with PM information

Sample size		Method A: *Estimation given number of failure per day*						Method B: *Estimation given times of each failure*					
PMs	*Failures*	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\alpha}$	$\hat{\beta}$	*Derr (%)*	*Ierr (%)*	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\alpha}$	$\hat{\beta}$	*Derr (%)*	*Ierr (%)*
10	96	0.126	1.292	0.466	0.809	-	-	0.781	0.824	0.238	1.496	-	-
50	493	0.129	1.274	1.581	0.179	-	-	0.481	0.955	0.467	0.637	-	-
100	968	0.203	1.160	0.994	0.412	-	-	0.441	0.969	0.431	0.683	-	-
150	1451	0.272	1.082	0.685	0.537	-	-	0.424	0.975	0.451	0.698	-	-
200	1988	0.281	1.075	0.853	0.473	-	-	0.355	1.002	0.553	0.612	-	-
True values		0.5	0.6	0.5	0.6			0.5	0.6	0.5	0.6		
10	142	0.253	1.197	0.613	0.832	-	-	0.632	0.953	0.411	2.014	-	-
30	429	0.367	1.088	0.424	1.568	-	-	0.605	0.971	0.423	2.224	-	-
50	684	0.351	1.090	0.388	1.534	-	-	0.657	0.955	0.380	1.595	-	-
100	1383	0.426	1.038	0.435	1.437	-	-	0.574	0.979	0.451	1.173	-	-
150	2099	0.449	1.028	0.462	1.262	-	-	0.523	0.994	0.473	1.114	-	-
200	2722	0.463	1.018	0.485	1.090	-	-	0.549	0.987	0.483	1.049	-	-
True values		0.5	0.9	0.5	0.9			0.5	0.9	0.5	0.9		
10	162	0.440	1.074	0.840	0.720	3.402	145.5	0.528	0.016	0.725	0.735	-	-
50	762	0.469	1.042	0.727	0.689	7.041	209.9	0.634	0.975	0.597	0.782	-	-
100	1510	0.488	1.028	0.618	0.850	11.19	509.1	0.606	0.982	0.547	0.938	-	-
150	2268	0.531	1.004	0.530	0.971	-	-	0.575	0.991	0.519	0.995	-	-
200	3001	0.543	0.995	0.537	1.007	-	-	0.602	0.984	0.545	0.989	-	-
True values		0.5	1.1	0.5	1.1			0.5	1.1	0.5	1.1		
10	171	0.399	1.114	0.633	1.118	130.5	466.7	0.563	1.013	0.495	1.054	502.8	3367
50	805	0.402	1.095	0.513	1.448	145.1	633.3	0.658	0.977	0.466	1.841	-	-
100	1594	0.477	1.043	0.452	1.569	246.3	1100	0.626	0.986	0.466	1.700	-	-
150	2414	0.472	1.049	0.485	1.622	235.9	1033	0.592	0.995	0.494	1.707	-	-
200	3183	0.499	1.030	0.471	1.512	324.9	1667	0.618	0.988	0.492	1.603	-	-
True values		0.5	1.5	0.5	1.5			0.5	1.5	0.5	1.5		
10	172	0.609	1.121	0.621	1.502	330.0	166.7	0.562	1.014	0.547	1.915	7749	5900
50	818	0.495	1.039	0.512	1.738	2000	2000	0.637	0.984	0.507	2.421	-	-
100	1626	0.499	1.036	0.511	1.628	2839	2200	0.616	0.990	0.515	1.864	-	-
150	2451	0.501	1.036	0.521	1.580	2905	2250	0.590	0.997	0.528	1.551	-	-
200	3229	0.489	1.039	0.508	1.581	2706	2100	0.623	0.989	0.515	1.614	-	-
True values		0.5	2.0	0.5	2.0			0.5	2.0	0.5	2.0		

λ_1 and λ_2 are parameters of rate of occurrence of defects, α and β are scale and shape parameters of a Weibull distribution of delay time. *Derr* is downtime error and *Ierr* is the inspection period error. " - " means there is no optimal value.

However, in both cases, Table5.7 also shows that downtime error and inspection interval error are very large for method A. In this case, as the rate of occurrence of defects is changing, not surprisingly perhaps, it is more difficult to estimate parameters than in the constant parameter case.

Figure 5.12 shows the regenerated number of failures for methods A and B against the true number of failures when λ_1 and α are 0.5, λ_2 and β are varied as 0.6 to 2.0 and the number of PMs are 100.

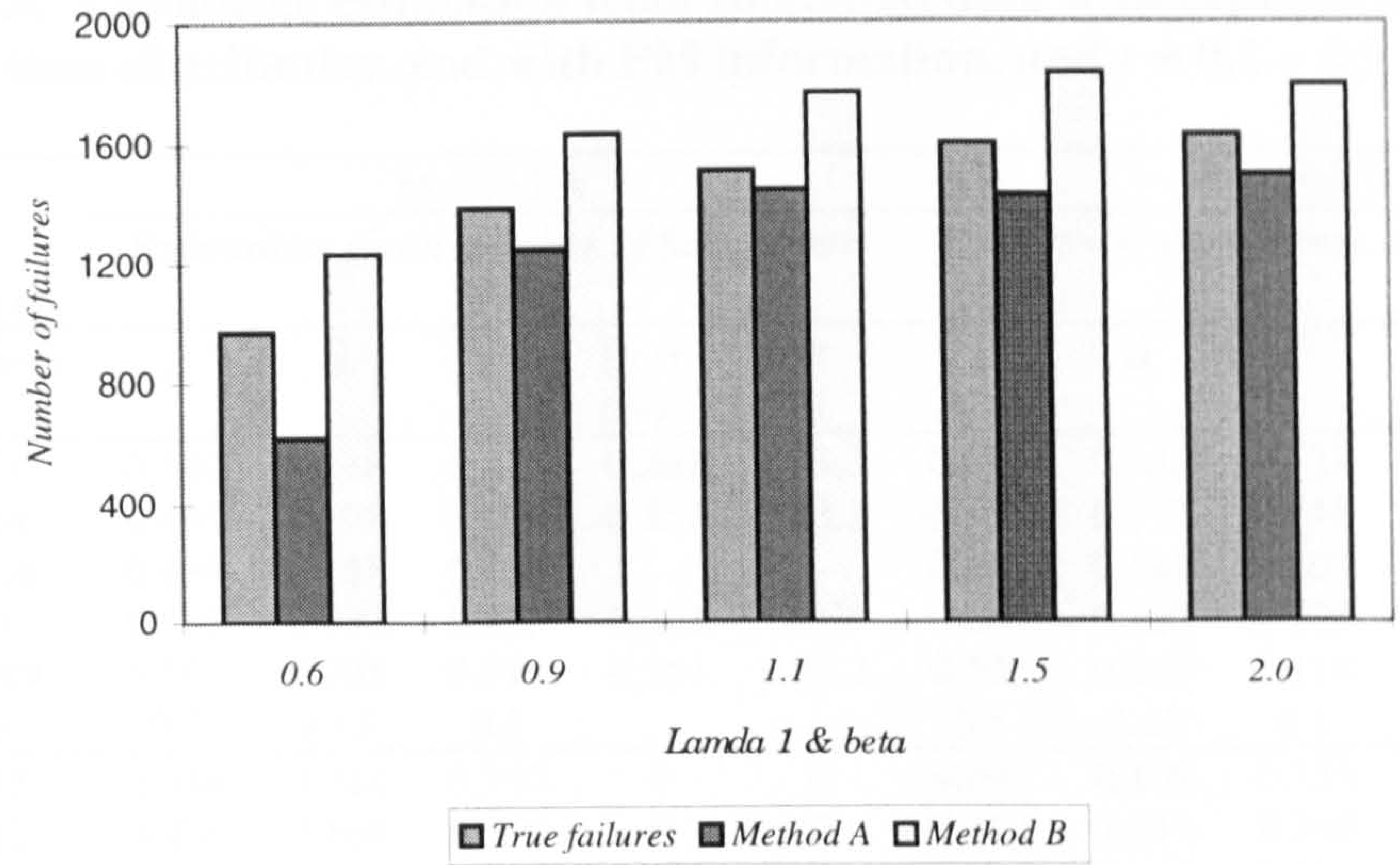


Figure 5.12. Comparison of methods A and B against the true number of failures when number of PMs are 100 and delay time is a Weibull distribution. (Perfect inspection case, $r = 1$)

5.5.2 Under Imperfect Inspection

Now, assuming that inspection is not perfect. If maintenance data were available, the objective method would be used to estimate the probability of identifying a defect upon inspection, r. For methods A and B, we now compare these assuming $r \neq 1$. Lets assume again that the rate of occurrence of defects (ROCOD) is a constant, λ = 0.5, and that the distribution of delay time is exponential, $F(h) = 1 - e^{-\alpha h}$, where α = 0.15. Inspection quality, that is the probability of a defect being properly identified, r, is considered as 0.1, 0.3, 0.6 and 0.9. Again we generate a set of the maintenance data using simulation. Then from the simulated data set, the delay time parameters are estimated via the process based upon the equations (5.17), (5.18), (5.35) and (5.36), and the results are given in Tables 5.8 and 5.9.

Table 5.8. Parameter estimation from simulated data with exponential delay time distribution and with PM information, and r = 0.1 ~ 0.9

Sample size		Method A: Estimation given number of failure per day					Method B: Estimation given times of each failure				
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{r}$	*Derr* (%)	*Ierr* (%)	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{r}$	*Derr* (%)	*Ierr* (%)
10	161	0.550	0.238	0.127	0.262	133.3	0.550	0.232	0.122	0.351	183.3
50	704	0.480	0.198	0.120	0.351	183.3	0.480	0.192	0.115	0.525	300.0
100	1434	0.488	0.188	0.110	-	-	0.488	0.181	0.105	-	-
150	2211	0.503	0.189	0.123	0.001	16.7	0.503	0.186	0.120	0.001	16.7
200	2949	0.501	0.205	0.119	0.351	183.3	0.501	0.203	0.118	0.351	183.3
True values		0.5	0.15	0.1			0.5	0.15	0.1		
10	155	0.548	0.324	0.742	0	0	0.691	0.109	0.725	0	0
50	682	0.480	0.166	0.256	0	0	0.493	0.151	0.205	0.908	50.0
100	1392	0.490	0.158	0.253	0	0	0.496	0.149	0.151	0	0
150	2166	0.507	0.144	0.221	0	0	0.510	0.147	0.160	0	0
200	2865	0.504	0.171	0.272	0	0	0.504	0.172	0.273	0	0
True values		0.5	0.15	0.3			0.5	0.15	0.3		
10	150	0.551	0.254	0.873	1.323	-33.3	0.551	0.244	0.833	1.323	-33.3
50	638	0.480	0.148	0.515	0	0	0.480	0.146	0.508	0	0
100	1304	0.491	0.145	0.505	0	0	0.491	0.143	0.498	0	0
150	2001	0.501	0.161	0.554	0	0	0.501	0.159	0.548	0	0
200	2639	0.497	0.172	0.609	0	0	0.497	0.169	0.598	0	0
True values		0.5	0.15	0.6			0.5	0.15	0.6		
10	143	cannot find optimal			-	-	0.550	0.204	1.000	0	0
50	601	0.481	0.148	0.768	0	0	0.481	0.151	0.781	0	0
100	1213	0.491	0.150	0.817	0	0	0.491	0.149	0.811	0	0
150	1864	0.501	0.155	0.835	0	0	0.501	0.153	0.826	0	0
200	2459	0.498	0.160	0.875	0	0	0.498	0.157	0.861	0	0
True values		0.5	0.15	0.9			0.5	0.15	0.9		

λ is the rate of occurrence of defects, α is the scale parameter of delay time distribution and r is the probability of a defect being recognized at inspection. *D*err is downtime error and *I*err is the inspection period error. " - " means there is no optimal value.

Table 5.8 shows that for methods A and B with PM information, the maximum likelihood estimates recover quite well the underlying process of failure and defect origination based upon equations (5.17) and (5.35). When $r = 0.1$, the downtime error and inspection interval error have maximum errors for method A of about 0.35 % and 183 % and about 0.5 % and 300 % for method B. However, as the probability of a defect being identified, r, increase downtime error and inspection interval error reduce and the model converges to the true optimum value. This is to be expected since information on r is contained within inspection data, and if r is small, very little information on r will be available from failure data. This will change as r increases.

Table 5.9. Parameter estimation from simulated data with exponential delay time distribution in case of no PM information when $r = 0.1 \sim 0.9$

Sample size		Method A *Estimation given number of failure per day*					Method B *Estimation given times of each failure*				
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{r}$	*Derr* (%)	*Ierr* (%)	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{r}$	*Derr* (%)	*Ierr* (%)
10	161	0.688	0.114	0.746	1.034	-50	0.691	0.109	0.725	1.034	-50
50	704	0.495	0.152	0.219	3.533	-66.7	0.493	0.151	0.205	1.034	-50
100	1434	0.496	0.151	0.160	1.034	-50	0.496	0.149	0.151	1.034	-50
150	2211	0.511	0.148	0.165	1.034	-50	0.510	0.147	0.160	1.034	-50
200	2949	0.514	0.138	0.179	3.553	-66.7	0.514	0.138	0.176	3.553	-66.7
True values		0.5	0.15	0.1			0.5	0.15	0.1		
10	155	0.697	0.111	0.875	0.908	50.0	0.698	0.110	0.869	0.908	50.0
50	682	0.498	0.128	0.326	0	0	0.498	0.126	0.316	0	0
100	1392	0.503	0.125	0.287	0	0	0.503	0.123	0.283	0	0
150	2166	0.520	0.114	0.252	0	0	0.520	0.115	0.253	0	0
200	2865	0.528	0.110	0.314	0	0	0.528	0.111	0.315	0	0
True values		0.5	0.15	0.3			0.5	0.15	0.3		
10	150	0.717	0.096	0.913	0	0	0.721	0.093	0.899	0	0
50	638	0.513	0.107	0.555	0	0	0.515	0.105	0.550	0	0
100	1304	0.521	0.103	0.524	0	0	0.523	0.101	0.520	0	0
150	2001	0.532	0.113	0.562	0	0	0.533	0.110	0.559	0	0
200	2639	0.528	0.120	0.609	0	0	0.530	0.117	0.604	0	0
True values		0.5	0.15	0.6			0.5	0.15	0.6		
10	143	0.777	0.078	1.000	0	0	0.771	0.079	1.000	0	0
50	601	0.531	0.102	0.773	0	0	0.528	0.105	0.781	0	0
100	1213	0.539	0.104	0.806	0	0	0.540	0.103	0.802	0	0
150	1864	0.545	0.110	0.819	0	0	0.547	0.108	0.815	0	0
200	2459	0.540	0.116	0.819	0	0	0.542	0.113	0.809	0	0
True values		0.5	0.15	0.9			0.5	0.15	0.9		

λ is the rate of occurrence of defects, α is the scale parameter of delay time distribution and r is the probability of a defect being recognized at inspection. *D*err is downtime error and *I*err is the inspection period error.

Table 5.9 also shows estimated parameter for methods A and B without PM information, that is failure information only. The maximum likelihood process used to recover estimates of the underlying process of failure and defect origination was based upon equations (5.18) and (5.36).

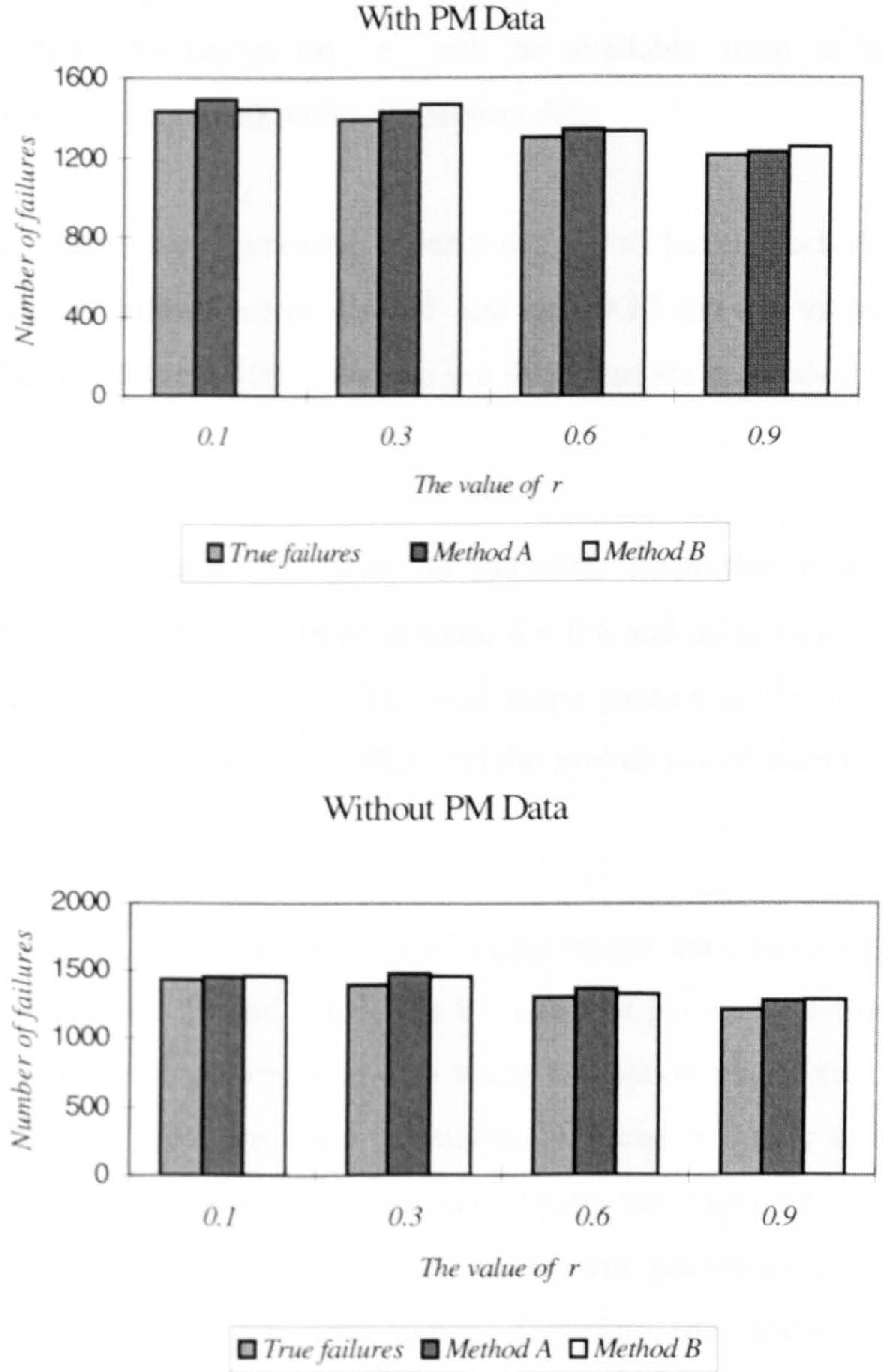


Figure 5.13. Comparison of methods A and B against the true number of failures when number of PMs are 100 and delay time is exponential distribution

It is noted that as the probability of a defect being identified, r, increase, the estimation of parameters is more accurate. If r is over 0.3, parameter estimated is accurate in spite of a low number of inspections for both methods A and B. For downtime error and inspection interval error, when $r = 0.1$ maximum errors of method A and B both are about 3.55 % and -66.7 %. However as the probability of a defect identified, r, increases the downtime error and inspection interval error reduce and parameter converge to the true optimum value. This is also expected that if r is small, very little information on r will be available from failure data since information on r is contained within inspection data.

Figure 5.13 shows the regenerated number of failures for methods A and B against the true number of failures when $\lambda = 0.5$ and $\alpha = 0.15$ and r is varied as 0.1 to 0.9 and the number of PMs are 100. We can see that there are no evident differences for methods A and B.

Next, it is also assume that under an imperfect inspection policy, the rate of occurrence of defects (ROCOD) is a constant, $\lambda = 0.6$ and delay time distribution is a Weibull, with scale parameter $\alpha = 0.03$ and shape parameter $\beta = 1.5$, where mean value and variance of h is 9.35 and 40.3, and the probability of defect identified, r, is between 0.1 and 0.9.

Using simulation, we simulate a set of maintenance data based upon the above assumptions. Tables 5.10 and 5.11 show the result of parameter estimation process based on the simulated maintenance data using estimation equations (5.17), (5.18), (5.35) and (5.36). Here we used the statistical decision theory to minimize the expected loss due to estimated parameters, where the expected loss, $L(d,\theta)$, is obtained from equation (5.40), where d is the true parameter value and θ is corresponding of estimated parameter value. Therefore, the ideal estimate is $d = \theta$, and it is natural to define $L(\theta, \theta) = 0$ for all θ. The function provides a measure of the extent to which estimated parameter values different from their given true value. Results are also given in Tables 5.10 and 5.11. For case methods A and B, in case of with PM information, Table 5.10 shows that the maximum likelihood estimate can recover quite well the underlying process of failure and defect origination based upon equations (5.17) and (5.35) regardless of the probability of a defect being identified at PM and the existence of PM data.

Table 5.10. Parameter estimation from simulated data with Weibull delay time distribution in case of with PM information when $r = 0.1 \sim 0.9$

		Method A					Method B				
Sample size		*Estimation given number of failure per day*					*Estimation given times of each failure*				
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{r}$	$L(d,\theta)$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{r}$	$L(d,\theta)$
10	160	0.660	0.029	1.442	0.105	0.007	0.660	0.029	1.431	0.104	0.008
50	818	0.629	0.028	1.416	0.113	0.008	0.630	0.028	1.405	0.112	0.01
100	1665	0.625	0.033	1.623	0.120	0.016	0.625	0.033	1.597	0.119	0.01
150	2435	0.608	0.031	1.560	0.111	0.004	0.608	0.031	1.536	0.111	0.001
200	3194	0.599	0.031	1.584	0.115	0.007	0.599	0.031	1.564	0.114	0.004
True values		0.6	0.03	1.5	0.1		0.6	0.03	1.5	0.1	
10	122	0.600	0.032	1.463	0.301	0.001	0.600	0.031	1.418	0.297	0.007
50	659	0.626	0.030	1.538	0.312	0.002	0.626	0.029	1.504	0.309	0.0008
100	1350	0.629	0.024	1.137	0.245	0.136	0.629	0.023	1.126	0.243	0.144
150	1992	0.612	0.025	1.148	0.250	0.127	0.612	0.026	1.456	0.440	0.022
200	2570	0.599	0.025	1.150	0.264	0.124	0.599	0.025	1.143	0.263	0.129
True values		0.6	0.03	1.5	0.3		0.6	0.03	1.5	0.3	
10	102	0.600	0.032	1.578	0.466	0.007	0.600	0.032	1.560	0.467	0.005
50	529	0.627	0.025	1.450	0.426	0.009	0.627	0.025	1.454	0.427	0.008
100	1089	0.633	0.027	1.400	0.460	0.013	0.633	0.027	1.409	0.459	0.011
150	1602	0.619	0.026	1.425	0.441	0.009	0.619	0.026	1.456	0.440	0.006
200	2059	0.604	0.029	1.883	0.479	0.147	0.604	0.029	1.883	0.478	0.147
True values		0.6	0.03	1.5	0.5		0.6	0.03	1.5	0.5	
10	71	0.625	0.022	1.202	0.634	0.094	0.625	0.021	1.188	0.630	0.103
50	391	0.620	0.030	1.949	0.684	0.202	0.620	0.030	1.892	0.684	0.154
100	787	0.607	0.031	1.520	0.713	0.0006	0.607	0.031	1.527	0.712	0.0009
150	1178	0.597	0.029	1.433	0.690	0.005	0.597	0.029	1.440	0.689	0.004
200	1551	0.600	0.030	1.578	0.698	0.006	0.600	0.030	1.576	0.697	0.006
True values		0.6	0.03	1.5	0.7		0.6	0.03	1.5	0.7	
10	70	0.634	0.035	1.849	0.845	0.126	0.634	0.035	1.823	0.844	0.109
50	313	0.632	0.031	1.806	0.847	0.097	0.632	0.031	1.804	0.845	0.096
100	624	0.636	0.030	1.496	0.891	0.001	0.636	0.030	1.516	0.888	0.002
150	910	0.620	0.030	1.483	0.890	0.0008	0.620	0.030	1.493	0.888	0.0006
200	1186	0.617	0.030	1.601	0.886	0.011	0.617	0.030	1.602	0.886	0.011
True values		0.6	0.03	1.5	0.9		0.6	0.03	1.5	0.9	

λ is the rate of occurrence of defects, α is the scale parameter and β is the shape parameter of delay time distribution and r is the probability of a defect being identified. $L(d,\theta)$ is the expected loss.

Table 5.11. Parameter estimation from simulated data with Weibull delay time distribution in case of without PM information when $r = 0.1 \sim 0.9$

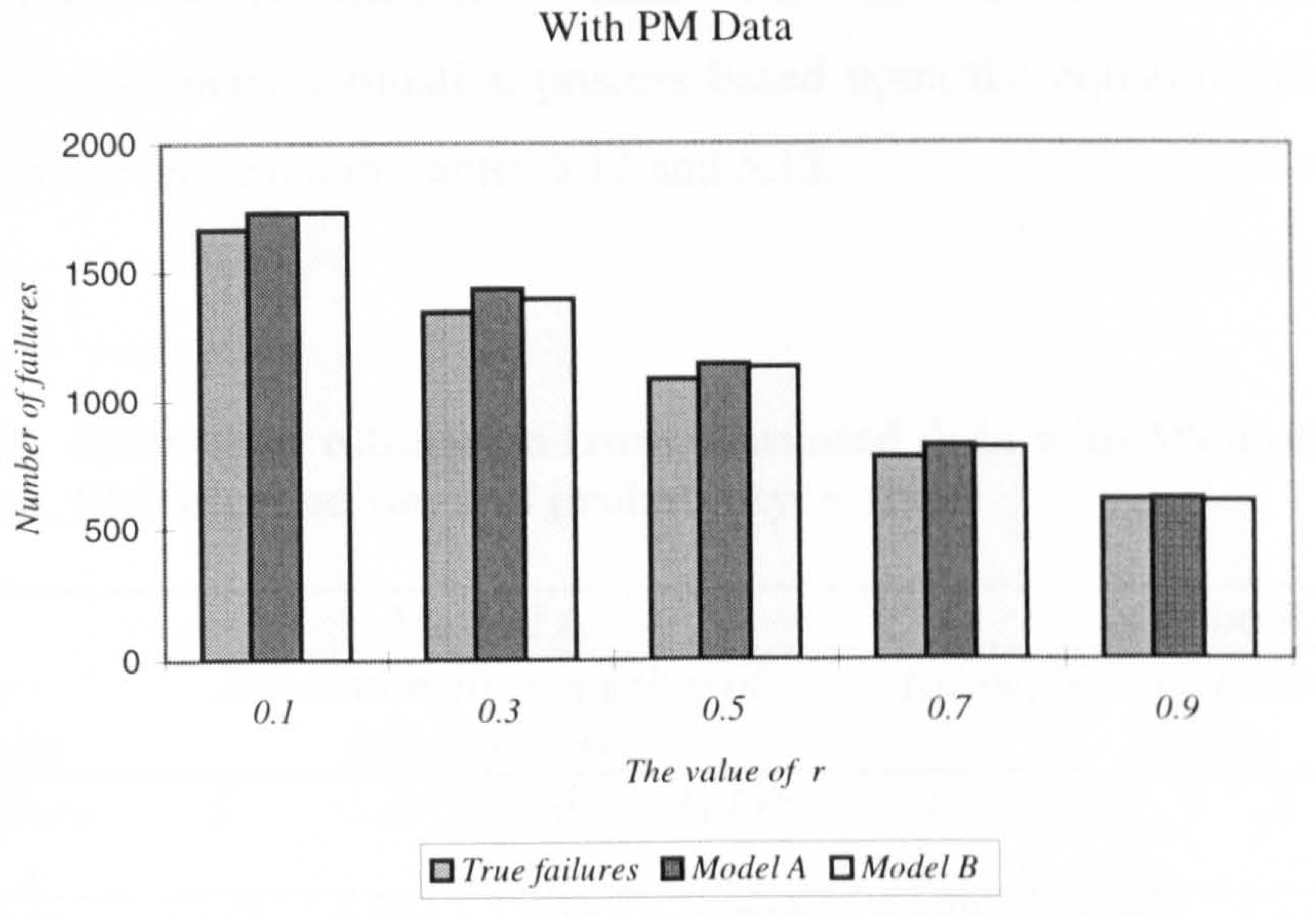
Sample size		Method A *Estimation given number of failure per day*					Method B *Estimation given times of each failure*				
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{r}$	$L(d,\theta)$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{r}$	$L(d,\theta)$
10	160	0.893	0.021	1.328	0.268	0.144	0.889	0.021	1.313	0.264	0.145
50	818	0.579	0.031	1.343	0.041	0.029	0.578	0.031	1.331	0.039	0.033
100	1665	0.670	0.028	1.446	0.160	0.01	0.669	0.028	1.443	0.158	0.011
150	2435	0.605	0.031	1.475	0.105	0.007	0.604	0.031	1.448	0.103	0.003
200	3194	0.590	0.032	1.538	0.106	0.002	0.591	0.032	1.514	0.104	0.0003
True values		0.6	0.03	1.5	0.1		0.6	0.03	1.5	0.1	
10	122	0.686	0.008	0.590	0.025	0.912	0.691	0.008	0.586	0.025	0.920
50	659	1.091	0.007	0.836	0.295	0.683	1.119	0.007	0.821	0.295	0.731
100	1350	0.863	0.009	0.781	0.233	0.591	0.856	0.009	0.776	0.231	0.595
150	1992	0.829	0.010	0.785	0.246	0.567	0.825	0.010	0.782	0.245	0.57
200	2570	0.726	0.014	0.853	0.259	0.436	0.717	0.014	0.860	0.257	0.425
True values		0.6	0.03	1.5	0.3		0.6	0.03	1.5	0.3	
10	102	2.781	0.001	0.729	0.252	5.014	54.94	9.9e-5	0.693	0.261	300.52
50	529	1.893	0.004	0.918	0.406	2.025	1.901	0.004	0.924	0.408	2.034
100	1089	13.57	0.001	0.914	0.472	168.61	12.96	9.9e-5	0.925	0.471	153.02
150	1602	5.685	0.001	0.890	0.455	26.23	8.886	0.001	0.900	0.454	69.02
200	2059	4.175	0.002	1.012	0.479	13.02	3.341	0.003	1.038	0.494	7.727
True values		0.6	0.03	1.5	0.5		0.6	0.03	1.5	0.5	
10	71	3.905	0.001	0.812	0.537	11.424	16.32	9.9e-5	0.781	0.524	247.54
50	391	0.439	0.046	2.444	0.637	0.921	0.419	0.049	2.485	0.629	1.008
100	787	12.26	0.001	1.157	0.729	273.45	17.68	0.001	1.164	0.727	292.63
150	1178	12.26	0.001	1.069	0.708	130.09	6.106	0.002	1.086	0.705	30.43
200	1551	6.385	0.002	1.182	0.716	33.57	2.188	0.007	1.238	0.711	2.694
True values		0.6	0.03	1.5	0.7		0.6	0.03	1.5	0.7	
10	70	0.409	0.056	2.415	0.792	0.886	0.400	0.057	2.468	0.785	0.991
50	313	0.344	0.058	3.485	0.762	4.026	0.343	0.058	3.542	0.758	4.257
100	624	0.443	0.041	1.796	0.872	0.113	0.429	0.048	1.858	0.867	0.159
150	910	0.564	0.033	1.537	0.886	0.003	0.497	0.039	1.641	0.878	0.035
200	1186	0.543	0.031	1.677	0.882	0.035	0.503	0.038	1.734	0.878	0.065
True values		0.6	0.03	1.5	0.9		0.6	0.03	1.5	0.9	

λ is the rate of occurrence of defects, α is the scale parameter and β is the shape parameter of delay time distribution and r is the probability of a defect being identified. $L(d,\theta)$ is the expected loss.

Table 5.11, in case of $r = 0.1$ and $r = 0.9$, shows that as the number of PMs increase, the maximum likelihood estimates recover the underlying process of failure and defect origination based upon failure data only using equations (5.18) and (5.36), for case methods A and B. In the case of $r = 0.3$, 0.5 and $r = 0.7$, Table 5.11 shows that the maximum likelihood estimates do not recover the underlying process of failure and defect origination based upon equations (5.18) and (5.36) in spite of the number of PMs, for case methods A and B in case of without PM information.

From Table 5.11, we can see that when the probability of a defect being identified is, $r = 0.1$ and 0.9, the estimation of parameters is more accurate than when $r = 0.3$, 0.5 and 0.7. In this case, if we have very low or high probability of defect identification at PM, like $r = 0.1$ or 0.9, without PM information we can estimate the parameters well if the number of PM cycles is over 50. In this case of no PM information, when the probability of a defect being identified at PM is 0.1, the estimation of parameters relies mainly on failure information even if PM data were available. This is because relatively few defect will be identified at PM and only a minor modification to the likelihood function arise between the with and without PM cases. However, if the probability of defect being identified at PM is sufficiently high, such as 0.9, the PM point can be regarded as renewal point so that in the case of relatively long PM intervals, failure data is again capable providing most of the information needed in the parameter estimation process. In the middle ground case of $r = 0.3 \sim 0.7$, it is more difficult to estimate the parameters without PM information.

As a result of Table 5.11, we see that if the quality of inspection is relatively very low or high, we can estimate the parameters accurately, but it is more difficult to estimate in the range of $r = 0.3 \sim 0.7$. From Table 5.11 we can also see that there is no consistent pattern in the range of $r = 0.3 \sim 0.7$, comparing method A and B. Figure 5.14 shows the regenerated number of failures of methods A and B against the true number of failures based on given true values when $\lambda = 0.6$, $\alpha = 0.03$, $\beta = 1.5$, the probability of a defect being identified, r, is varied between 0.1 to 0.9, and the number of PMs is 100. In contrast to the exponential distribution of delay time for estimating parameters, see Tables 5.8 and 5.9, maximum likelihood estimates recover the parameters well in the with PM data case, but the estimates are more unstable for no PM data.



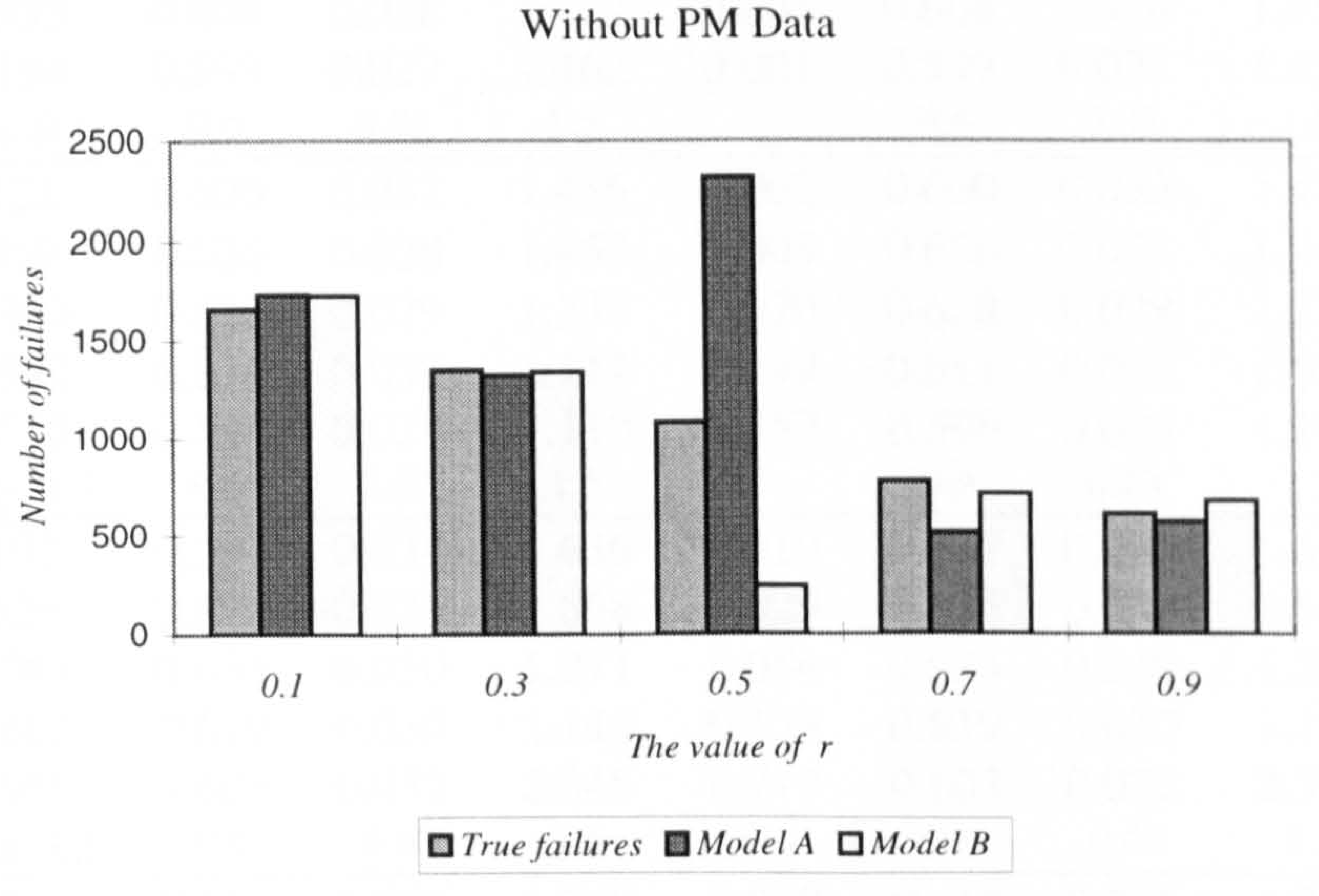


Figure 5.14. Comparison of methods A and B against the true number of failures when number of PMs are 100 and delay time distribution is a Weibull.

Next, we consider that the probability a defect is identified at inspection is given by the subjective method, and that the distribution of delay time is a Weibull, $F(h) = 1 - e^{-\alpha h^{\beta}}$, where α is the scale parameter and β is the shape parameter. Also to generate a set of data using a simulation, we assume that $\lambda = 0.6$, $\alpha = 0.03$ and $\beta = 1.5$. The probability a defect is identified, r, is fixed as 0.1, 0.3, 0.5, 0.7

and 0.9. From the simulated set of data, with r fixed at its estimated values, the results of the parameter estimation process based upon the equations (5.17), (5.18), (5.35) and (5.36) are given in Tables 5.12 and 5.13.

Table 5.12. Parameter estimation from simulated data with Weibull delay time distribution, PM information, and probability r fixed

Sample size		Method A *Estimation given number of failure per day*				Method B *Estimation given times of each failure*			
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$L(d,\theta)$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$L(d,\theta)$
10	160	0.662	0.028	1.421	0.010	0.662	0.028	1.412	0.012
50	818	0.631	0.025	1.342	0.026	0.631	0.025	1.335	0.028
100	1665	0.626	0.028	1.468	0.002	0.626	0.028	1.457	0.003
150	2435	0.608	0.028	1.472	0.001	0.608	0.028	1.459	0.002
200	3194	0.599	0.027	1.462	0.001	0.599	0.027	1.452	0.002
r is fixed as 0.1		0.6	0.03	1.5		0.6	0.03	1.5	
10	122	0.600	0.032	1.456	0.002	0.600	0.032	1.429	0.005
50	659	0.626	0.028	1.457	0.003	0.626	0.028	1.443	0.004
100	1350	0.628	0.029	1.237	0.070	0.628	0.029	1.223	0.078
150	1992	0.611	0.030	1.114	0.149	0.611	0.030	1.092	0.167
200	2570	0.598	0.029	1.110	0.152	0.598	0.029	1.104	0.157
r is fixed as 0.3		0.6	0.03	1.5		0.6	0.03	1.5	
10	102	0.597	0.034	1.636	0.019	0.597	0.034	1.613	0.013
50	529	0.625	0.030	1.668	0.029	0.625	0.030	1.668	0.029
100	1089	0.633	0.030	1.271	0.054	0.633	0.030	1.291	0.045
150	1602	0.619	0.030	1.143	0.128	0.619	0.030	1.199	0.091
200	2059	0.603	0.032	2.348	0.719	0.603	0.032	2.368	0.753
r is fixed as 0.5		0.6	0.03	1.5		0.6	0.03	1.5	
10	71	0.619	0.025	1.242	0.067	0.619	0.025	1.238	0.069
50	391	0.620	0.031	1.961	0.213	0.620	0.031	1.899	0.16
100	787	0.607	0.030	1.572	0.005	0.607	0.030	1.571	0.005
150	1178	0.597	0.030	1.387	0.013	0.597	0.030	1.392	0.012
200	1551	0.600	0.030	1.570	0.005	0.600	0.030	1.563	0.004
r is fixed as 0.7		0.6	0.03	1.5		0.6	0.03	1.5	
10	70	0.631	0.036	1.593	0.010	0.631	0.036	1.541	0.003
50	313	0.631	0.031	1.468	0.002	0.631	0.031	1.459	0.003
100	624	0.636	0.030	1.454	0.003	0.636	0.030	1.458	0.003
150	910	0.620	0.030	1.433	0.005	0.620	0.030	1.431	0.005
200	1186	0.617	0.030	1.523	0.0008	0.617	0.030	1.523	0.0008
r is fixed as 0.9		0.6	0.03	1.5		0.6	0.03	1.5	

λ is the rate of occurrence of defects, α is the scale parameter and β is the shape parameter of Weibull delay time distribution. $L(d,\theta)$ is the expected loss.

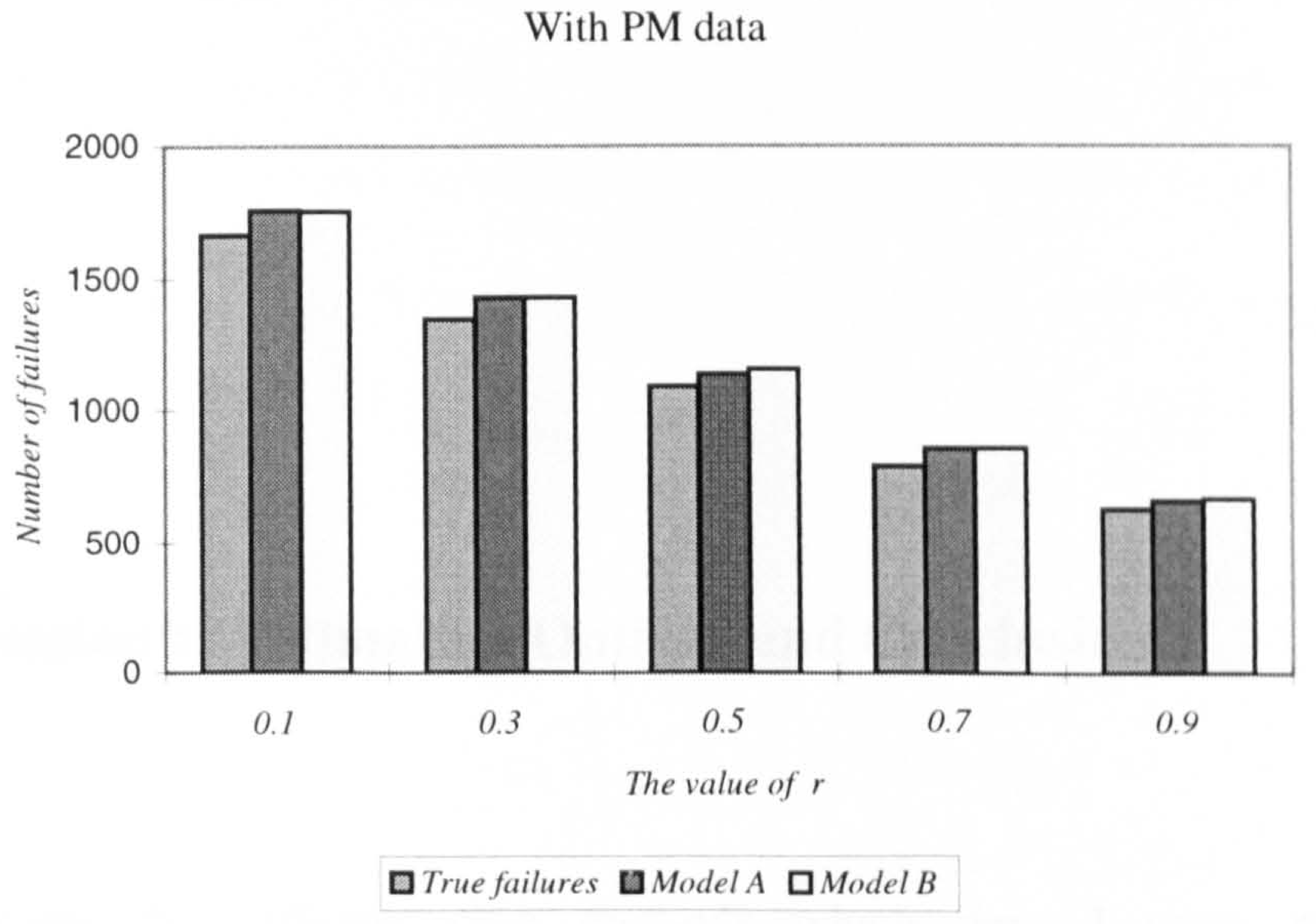
Table 5.12 shows that for case methods A and B, maximum likelihood estimates recover quite well the underlying process of failure and defect origination based upon equations (5.17) and (5.35) regardless of the probability of defect identified and a set of PM data. From Table 5.12, there is no evident difference between estimate parameters for methods A and B.

Table 5.13. Parameter estimation from simulated data with Weibull delay time distribution, no PM information, and probability r fixed

Sample size		Method A: Estimation given number of failure per day				Method B: Estimation given times of each failure			
PMs	*Failures*	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$L(d,\theta)$	$\hat{\lambda}$	$\hat{\alpha}$	$\hat{\beta}$	$L(d,\theta)$
10	160	0.680	0.025	1.256	0.066	0.681	0.025	1.246	0.071
50	818	0.618	0.029	1.362	0.019	0.618	0.029	1.350	0.023
100	1665	0.623	0.029	1.427	0.006	0.623	0.029	1.410	0.009
150	2435	0.601	0.031	1.471	0.0008	0.602	0.031	1.466	0.001
200	3194	0.588	0.032	1.533	0.001	0.589	0.032	1.511	0.0002
r is fixed as 0.1		0.6	0.03	1.5		0.6	0.03	1.5	
10	122	15.96	9.9e-5	0.699	228.3	70.683	9.9e-6	0.687	4912
50	659	1.134	0.007	0.832	0.732	1.175	0.006	0.816	0.799
100	1350	5.618	0.0002	0.607	25.98	5.041	0.004	0.606	23.85
150	1992	1.000	0.008	0.732	0.75	2.223	0.001	0.606	3.434
200	2570	1.174	0.005	0.669	1.021	1.084	0.006	0.692	0.888
r is fixed as 0.3		0.6	0.03	1.5		0.6	0.03	1.5	
10	102	91.29	9.9e-5	1.008	8225	90.400	9.9e-5	1.006	8064
50	529	33.76	1.8e-4	0.932	1000	24.620	0.003	0.944	577.27
100	1089	20.255	2.0e-4	0.909	386.7	19.417	0.003	0.922	354.41
150	1602	11.423	0.012	0.950	117.44	14.893	0.003	0.865	204.69
200	2059	6.021	0.001	0.955	29.64	6.661	0.001	1.004	36.98
r is fixed as 0.5		0.6	0.03	1.5		0.6	0.03	1.5	
10	71	45.66	9.9e-5	0.877	2031	0.329	0.077	2.498	1.072
50	391	224.65	0.0002	1.290	50200	30.911	0.001	1.288	918.80
100	787	7.861	0.002	1.222	52.8	9.842	0.002	1.218	85.50
150	1178	9.784	0.001	1.094	84.51	13.04	0.001	1.092	155.02
200	1551	0.902	0.002	1.322	0.124	0.838	0.023	1.450	0.022
r is fixed as 0.7		0.6	0.03	1.5		0.6	0.03	1.5	
10	70	72.80	4.7e-4	1.213	5231	91.70	0.0003	1.154	8299
50	313	8.990	0.002	1.173	70.5	18.12	0.001	1.158	306.99
100	624	0.597	0.033	1.470	0.001	0.590	0.033	1.480	0.0005
150	910	0.760	0.024	1.367	0.043	0.708	0.026	1.384	0.025
200	1186	0.836	0.022	1.432	0.06	0.756	0.024	1.456	0.026
r is fixed as 0.9		0.6	0.03	1.5		0.6	0.03	1.5	

λ is the rate of occurrence of defects, α and β are scale and shape parameters of Weibull delay time distribution. $L(d,\theta)$ is the expected loss.

Table 5.13 shows that in case of $r = 0.1$ and 0.9, maximum likelihood estimates recover the underlying process of failure and defect origination based upon equations (5.18) and (5.36) as the number of PMs increase for case methods A and B. However, in case of $r = 0.3$, 0.5 and 0.7, estimations are not recovered well, unlike the case with PM data, see Table 5.12.



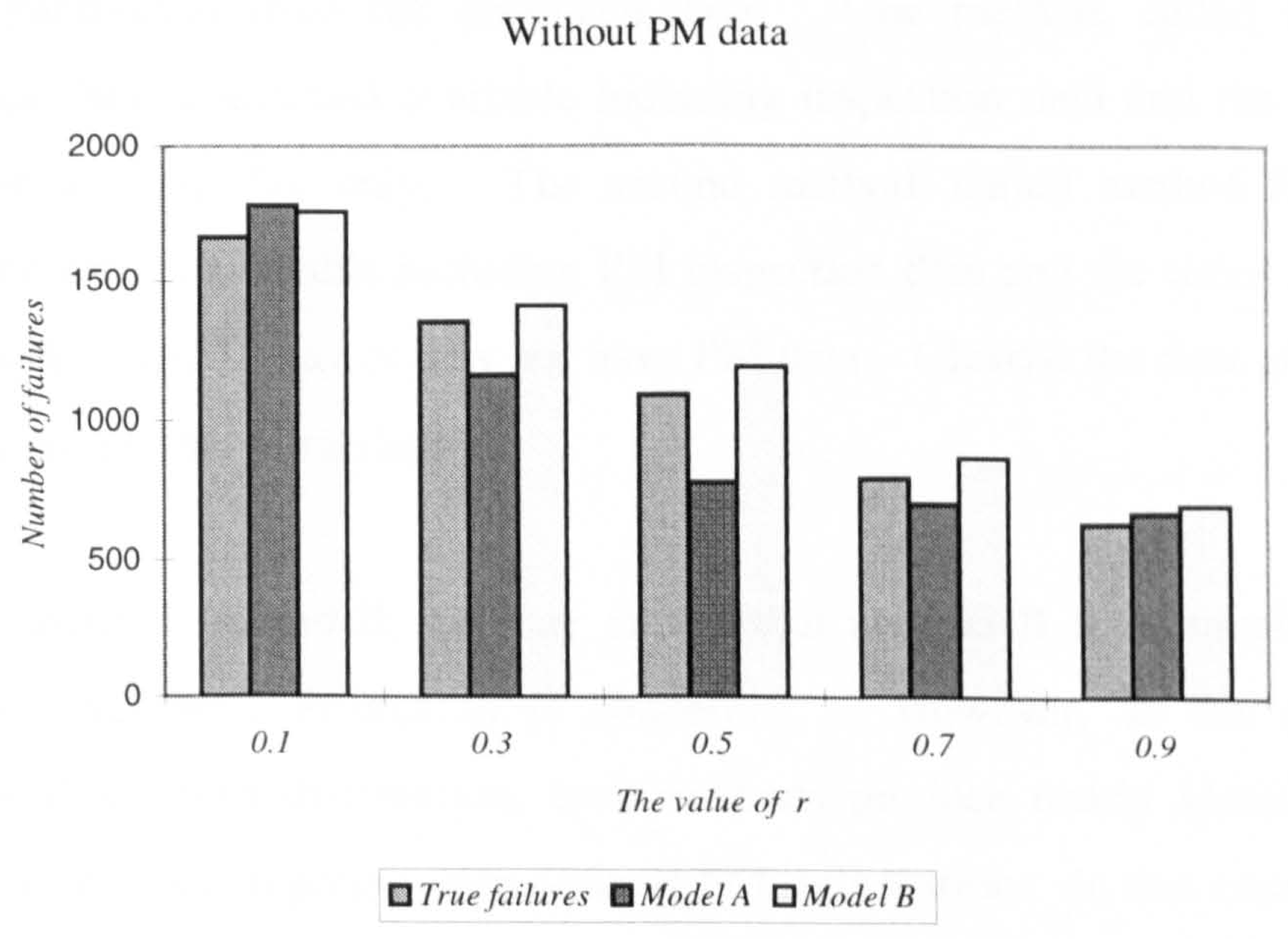


Figure 5.15. Comparison of methods A and B against the true number of failures when number of PMs are 100 and delay time distribution is a Weibull.

It also mean that if the quality of inspection is very low or high, the maximum likelihood estimates recovers the underlying process in the case when the rate of occurrence of defect is high and the delay time is short. Figure 5.15 shows the regenerated number of failures based upon the analysis of methods A and B against the true number of failures based on given values when $\lambda = 0.6$, $\alpha = 0.03$, $\beta = 1.5$, the probability of defect identified, r varies between 0.1 and 0.9, and the number of PMs is 100.

5.6 Discussion of Estimation Options and Conclusions

To compare the different case methods which depend upon maintenance information, we generated a set of PM data using the simulation method and then estimated parameters from the simulated data. One method, called method A, maintenance data is assumed available including inspection data and the number of failures per working day only. The second method, called method B, assumes maintenance data is available including PM inspection data and the times of failures. Both methods A and B may or may not have PM data. Clearly, the data of method A is included within that of method B.

For case methods A and B, we may expect that method B is dominant as far as accuracy of parameter estimation is concerned. However, in the case of an exponential delay time distribution, both methods produce nearly identical results, under perfect inspection policy even without PM information. In this case, since two methods have a constant rate of occurrence of defect and delay time is exponentially distributed, there is no useful difference in the comparison of downtime error and inspection period error for methods A and B. If the defect arrival rate is a constant and the distribution of delay time is exponential, we can use both methods with no fear of bias in parameter estimation due to different levels of information available.

When we assumed that delay time distribution is a Weibull and a perfect inspection policy, maximum likelihood estimates also recover well the underlying process of failure and defect origination of both methods A and B. Even though in the case of no PM information being available, just failure data, maximum likelihood estimates can recover parameters as the number of PMs increase. When we assumed that the rate of occurrence of defects at time u is $\lambda(u)$, namely $\lambda(u) = \lambda_1\lambda_2 u^{\lambda_2 - 1}$, and the distribution of delay time is a Weibull, $F(h) = 1 - e^{-\alpha h^\beta}$, maximum likelihood estimates do not recover well the underlying process of failures and defects origination for methods A or B, even under perfect inspection. It is evidently more difficult to find optimal value as the rate of occurrence of defects change and the delay time distribution is a Weibull. From Table 5.6, we can see that if λ_2 and the shape parameter of the Weibull distribution is close to 1, as in the case of an exponential distribution, maximum likelihood estimates recover the underlying process of methods A and B. However, in this case it is also difficult to find the downtime and inspection period error from the parameters estimated because the variation of parameters estimated is too large. In the downtime model, the optimum is more sensitive to the parameters estimated.

In this PM modelling, PM data is important for most practical cases. When $r <$ 0.9, the estimation process is complicated because of a correlation between estimates of r and λ. If PM data is not available, it is best to obtain a subjective estimation of either r or λ and than proceed to use the maximum likelihood estimation (MLE) to obtain the remaining estimates. A similar case study for lack of PM data was carried out by Christer, Wang and Choi (1998) to model preventive maintenance (PM) practices of a complex machine used in commercial vehicle break lining manufacturing. And in this point of view, a case study of modelling option of practical maintenance plant for Tea Bag production machine will be present in Chapter 6 (see also Christer, Lee and Wang 1997).

For computing time using a Pentium-PC, as the number of parameters to estimate and the number of inspections increase, it requires more time to estimate parameters using the NAG library. Comparing with method A and B of computing time for estimating the parameters, method A required more time to calculate the equations (5.17) or (5.18) than method B (equations 5.35 or 5.36) since method A has more integration routines. For example, in the case that the number of parameters to estimate is 3, the computing process for method B using the NAG library took approximately 1hour using a Pentium-PC, but method A took more than twice this. Therefore, when we have better data giving a more exact time of failure, we may use method B, and save time.

Under imperfect inspection policy, two methods give very similar results for with PM data. In contrast to method B, method A requires more computer running time because method A has a more integrated routine within the maximum likelihood function. However in the case that the only data available is the number of failures per working day, and PM data, we choose and may used method A to estimate the parameter from the observed data. Also if information is available giving exact times of failures and PM data, we would choose method B with no evident loss due to bias.

Chapter 6

A Case Study of Modelling Plant Maintenance for a Tea Bag Production Machine

6.1 Introduction

This Chapter describes a modelling study of preventive maintenance (PM) of production plant in a local company with a view to improving current practice. At the time of the study, a planned maintenance (PM) system was operated consisting of regular inspections with corrective actions as required. Of interest to management was whether or not the existing PM practice contributed to reducing downtime, and whether or not a more appropriate PM schedule was possible. To establish the relationship between PM and operating measures such as downtime, the delay time concept has been used.

In delay time modelling, one of the important tasks is the estimation of the delay time parameters which are usually the rate of occurrence of defects, the distribution of the underlying delay time h of a defect, and the probability of identifying and removing a defect at PM. Two basic methods have been developed, namely, objective data methods where there is a sufficient valid data, and the subjective assessment method where objective data is scarce or even non-existent. Applied studies using the delay time concept within industrial situations have been reported in which the model parameters are estimated from the synthesis of subjective opinions of maintenance engineers, Christer & Waller (1984a,b), Chilcott & Christer (1991), and Desa (1995). A recent development in delay time modelling has established that these parameters can also be adequately estimated using objective data obtained from maintenance records of failures and records of defects found at PM, Baker & Wang (1992, 1993). For complex machinery with many components, some modelling has been carried out for actual plant using the delay time concept with objective estimation of parameters, Christer *et al.* (1995), Christer and Wang (1995). They developed a model utilising both failure and PM data for parameter estimation.

Here we report on the two studies of the same problem of modelling preventive maintenance. We have two objectives. One objective is to compare the model formats and parameter values resulting from the two parameter estimation methods, and the another objective is to consider the degree of consistency between the subsequent decision consequence of the two methods. Therefore we present first the study of an objective data based modelling of the tea-bag plant. This study attempts to use the objective estimation technique, but with failure data only as opposed to failure and PM data as in previously published cases. The model developed is based upon the delay time concept where because of an absence of PM data, the process parameters and the delay time distribution are estimated from failure data only using the method of maximum likelihood (see Chrster, Lee and Wang, 1997). Particular attention is paid to the problem arising during the parameter estimating process because of the inadequate recording of PM data and the implied correlation between model parameters. The case of data deficiency explored in the study is important because it is a relatively general situation in practice. An inspection model is finally proposed to identify the best inspection policy based upon the estimated model parameters and the delay time distribution. Next, the subjective data method is applied to the same problem, where the initial subjective estimates are obtained via a questionnaire survey. On the basis of the data analysis and delay time modelling, PM policy and procedures were proposed to increase the effectiveness and efficiency of PM. The two studies of the same problem provide a unique opportunity to compare the models formats and parameter values resulting from the two approaches, and to consider the degree of consistency between the subsequent decision consequence of the two methods.

6.2 The Production Plant and Maintenance Practice

The Tea pack production plant consists of four main production lines, called P1, N1, N2 and N3. The P1 line consists of the following three primary machines, the IMA machine, the Marden Edwards and the Europack. N1, N2 and N3 production lines also have similar types of machines. The first stage of the production process is the IMA machine which forms a continuous tea bag strip. Here tea bag paper runs on a continuous belt with tea dropping onto the paper, and the strip cut and sealed into separate tea bags by the Marden Edwards machine. These bags are shrink packed into boxes and wrapped by Europack machine. Finally, products are moved by a conveyor system to the storage depot. A flow chart of the production process of the P1 line is shown in Figure 6.1.

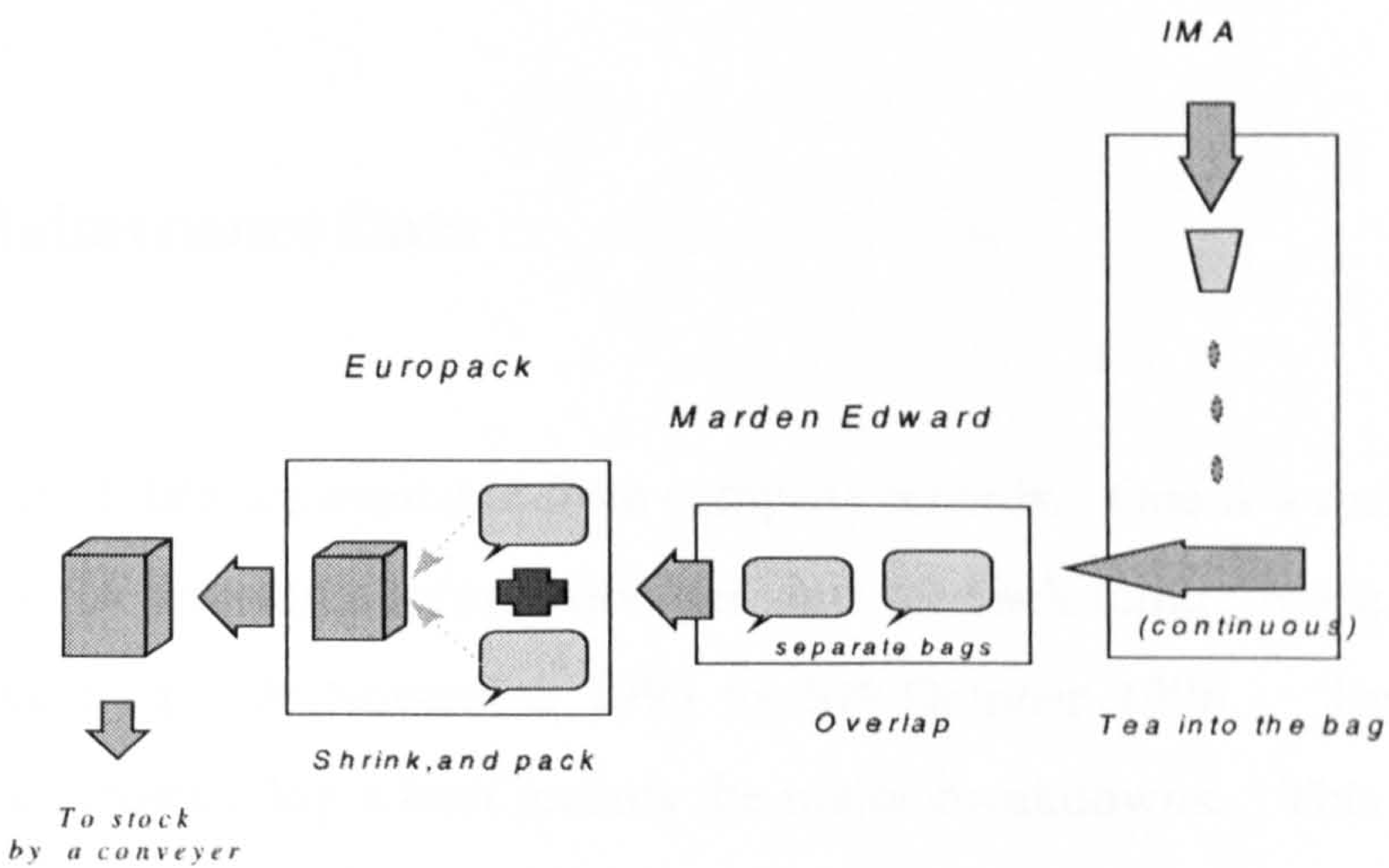


Figure 6.1. A flowchart of the production process of a tea production line

The company has two different planned preventive maintenance schedules, the first operates every 5/6 week and lasts 8 hours, and the second, called the extended maintenance plan, is for the IMA machine only and runs every 3 month with 16 hours downtime. This additional attention is because the IMA machine is a more complicated high-speed machine. The PM plan of the production line is shown in Table 6.1 and includes routine maintenance and inspection. However, no record is kept of PM activity other than the date; information such as the number of defects identified and repaired is currently not recorded. This is not an uncommon observation, but imposes a problem in model parameter estimation, which will be discussed later.

Lines	Machine	PM period	PM downtime
P1	IMA	5/6 weekly	8 hours
		3 monthly	16 hours
	ME	5/6 weekly	8 hours
	Europack	5/6 weekly	8 hours

Table 6.1. The PM plans of the P1 production line

6.3 The Maintenance Data

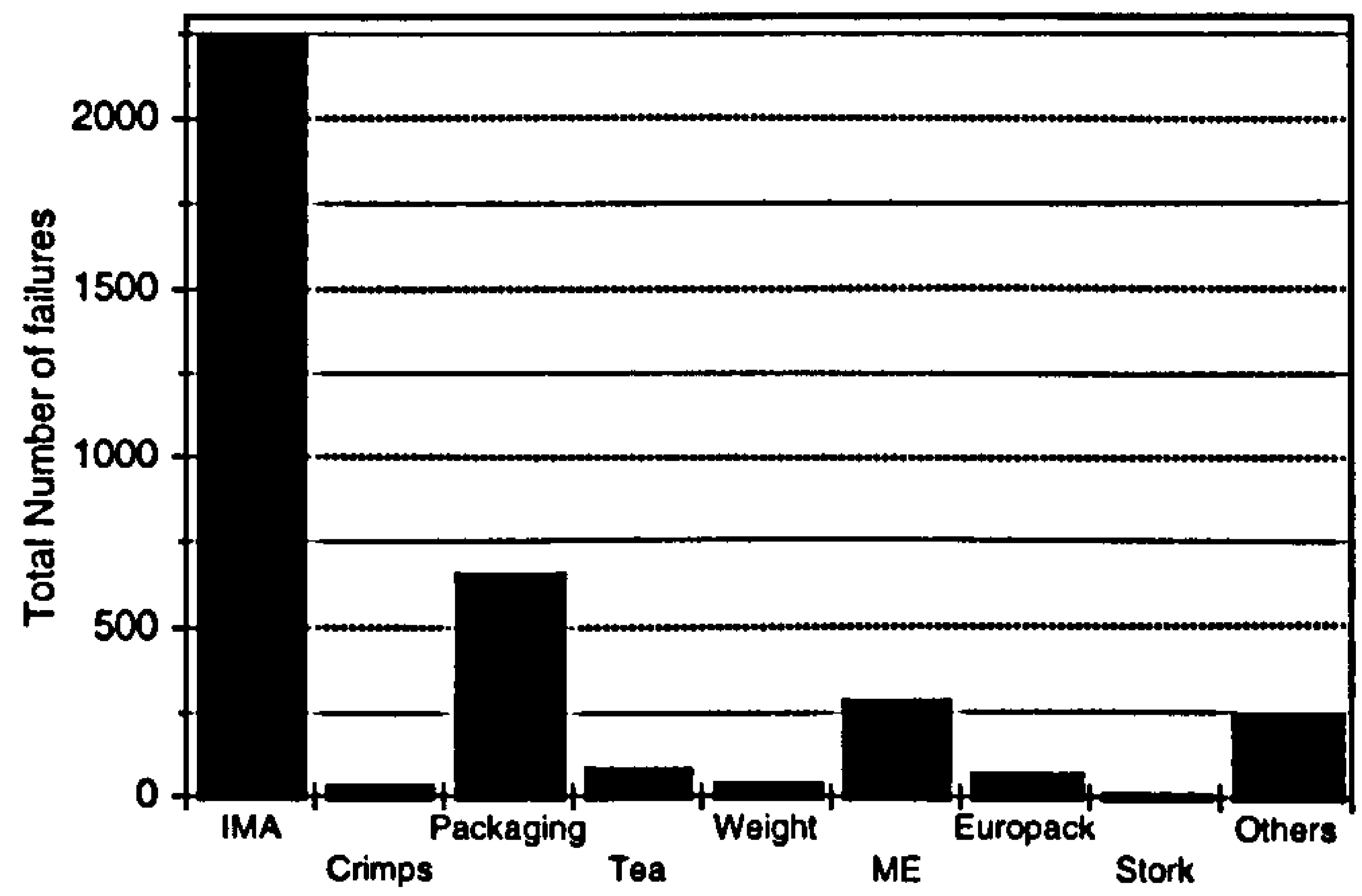
Two sources of data are available from company records. One is a spreadsheet file which includes the downtime information per shift for each failure mode. This data set is available from 20th November 1995 to 30th October 1996. The other is a breakdown maintenance log which records the major breakdowns. This information is, however, included in the spread sheet data set.

All stoppages per shift are recorded by the production line operator for the spread sheet file. This data includes the planned downtime, the routine production stoppages, and unexpected stoppages. This data is used to calculate the company's production efficiency. The plant operates 24 hours a day (three shifts), 5 days per week, with sometimes an extra 12 hours of production on Saturday or Sunday.

In this study, we focus on the key machines in the P1 line as a pilot study, since the other lines have the similar structure.

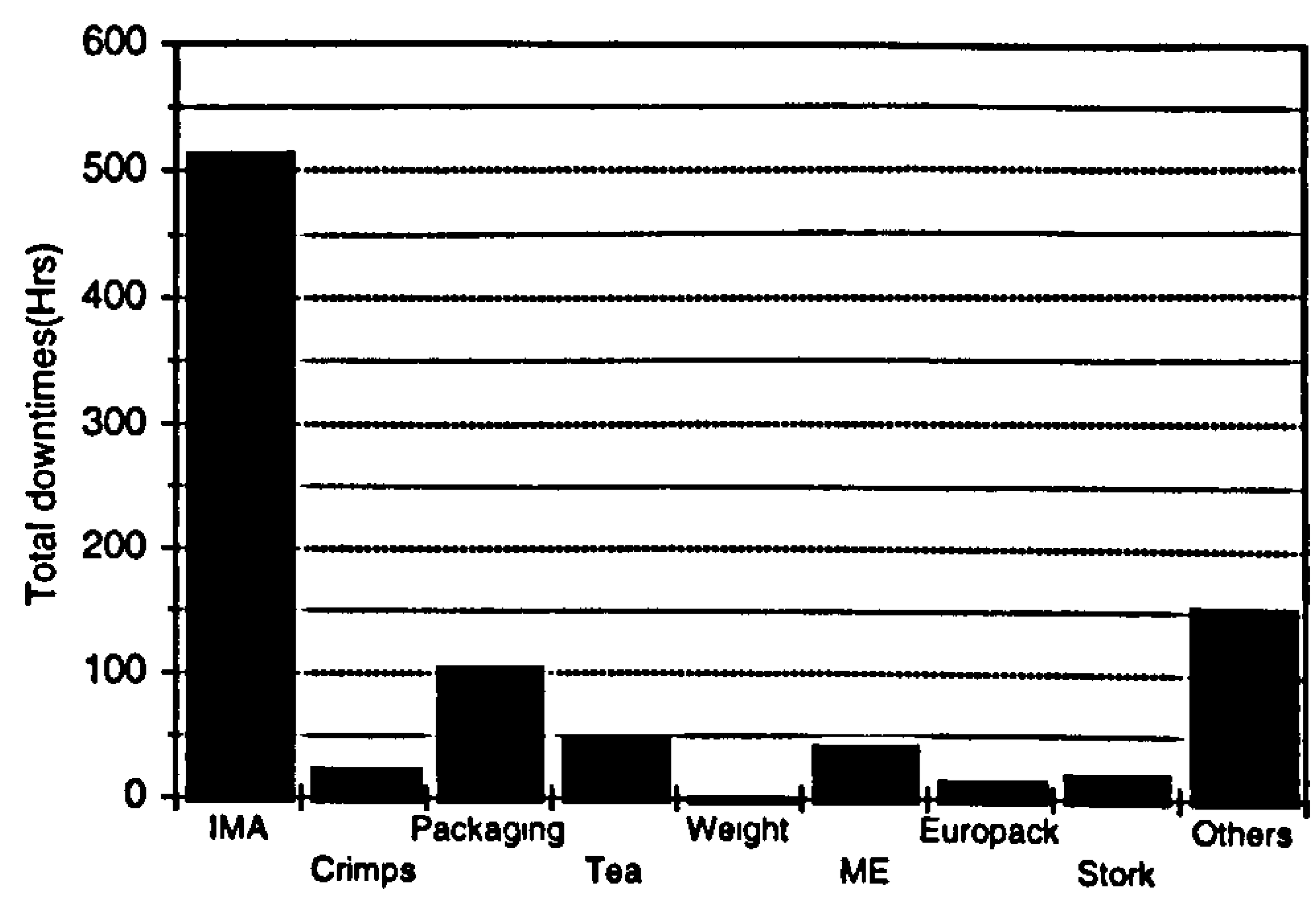
6.4 Data Analysis

The failure data from the spreadsheet is available over an 11 month period. It consists of the failure mode, downtime periods during each hour of each shift, the date and the plant operator's name. Figures 6.2 and 6.3 below show the major failure areas of P1 line. The detailed data can be found in Table 6.2.



(Data : 20/11/95 -30/10/96)

Figure 6.2. Failure frequency of P1 line



(Data : 20/11/95 -30/10/96)

Figure 6.3. Downtimes of P1 line

Component	Failure Modes	Number of Failure	Downtimes (Min)	Average (Min)
IMA *(2251)*	Bad stacking	*505*	13513	26.76
	Cartons jam/former/over-load	*411*	3410	8.30
	Running on one paper reel	*372*	936	2.52
	Alignment problem	*220*	2368	10.76
	Starting up	*219*	1562	7.13
	Paper extraction	*208*	2033	9.77
	Pad/cutter cleaning	*82*	1678	20.46
	Filling gum	*79*	290	3.67
	Vacuum	*51*	1089	21.35
	T-plate broken	*18*	600	33.33
	Stacking light	*18*	176	9.78
	Centre knife	*17*	671	39.47
	Tea in seams	*17*	160	9.41
	Carton light	*11*	135	12.27
	Cutter change	*7*	1835	262.14
	Tension roller	*6*	72	12.00
	Pusher plate broken	*5*	120	24.00
	Loli-pop stick bent	*4*	18	4.50
	Break on reel too tight	*1*	130	130.00
Crimps(31)	Heater/light problem	*31*	1372	44.26
Packaging *(654)*	Lids not inter-locking	*227*	1335	5.88
	Paper reel snapping/not splicing	*220*	2535	11.52
	Paper jamming	*98*	973	9.93
	Poor cartons	*96*	966	10.06
	Waiting for materials	*13*	480	36.92
Tea *(83)*	Tea feed/blockage/no tea	*60*	2741	45.68
	Move tea	*14*	168	12.00
	Check magnets	*9*	59	6.56
Weights(39)	Re-calibrate check-weigher	*39*	424	10.87
ME *(283)*	Boxes sticking together	*224*	1339	5.98
	Cellophane alignment	*23*	442	19.22
	Lift/elevator jams	*21*	192	9.14
	Missing wraps	*9*	136	15.11
	ME stopping mid-cycle	*6*	110	18.33
Europack *(69)*	Jamming	*49*	760	15.51
	Not sealing/bad wraps	*20*	1124	56.20
Stork(9)	Conveyor	*9*	133	14.78
Other *(243)*	Waiting for engineer	*35*	2453	70.09
	No operator	*19*	1220	64.21
	Side plates	*13*	350	26.92
	No agency	*4*	32	8.00
	Others	*168*	5160	30.71
	SUM	**3662**	**55300**	**15.10**

(Data : 20/11/95 -30/10/96)

Table 6.2. The number and downtime of failures of P1 line

The available data within the plant record include dates and numbers of each downtime occurrence due to both PM and failures. Each failure also incurs production downtimes. The plant operates 24 hours a day (three shifts), 5 days per week, with sometimes an extra 12 hours of production on Saturday or Sunday.

The PMs are performed on three primary machines of a tea production line, namely IMA machine, the Marden Edwards and the Europack. PM work for these machines carried out every 5/6week and lasts 8 hours. For the IMA machine, additional PM is performed every 3 months with 16 hours downtime. The PM includes routine maintenance which involves adjustments or repairs if the defects found can be rectified within the PM downtime, and inspection of machinery. However, there is no record of PM activity undertaken available.

The maintenance data of company includes the downtime information per shift for each failure mode. The failure mode of a component is defined as the effect by which a failure is observed. The failure modes are the effects of failure causes on the component function. Table 6.2 shows the number of failures, downtimes and average downtime for each failure mode. The high numbers of failures in the IMA machine due to Bad stacking, Cartons jam/former/over-load and Running on one paper reel are associated with this machine being a complicated high-speed machine.

It can be seen from Figures 6.2 and 6.3 that the IMA plant causes the main problem. The total number of failures for the IMA over the data collection period is 2251, with mean downtime per failure of 0.228 hours. Thus we observe the IMA machine accounts for 61.5 % of total breakdowns and 55.6 % of the total downtime for the production line. In the remainder of this paper, we concern ourselves with only the IMA machine of the P1 line since most failures occurred on this machine. An alternative study based upon a study of failure mechanism is also possible. This would be part of engineering process to design out defects, also known as reliability growth. It is related directly to the notion and purpose of snap-shot modelling (Christer and Whitelaw, 1983). It is also discussed in section 6.7.

Figures 6.4 and 6.5 show the average number of failures and downtime since the beginning of the data collection period of the IMA machine, see also Table 6.3 for details. The number of completed PM cycles is 10 over the data collection period. Table 6.3 is presented for the IMA data and Figures 6.6 and 6.7 are based from Table 6.3.

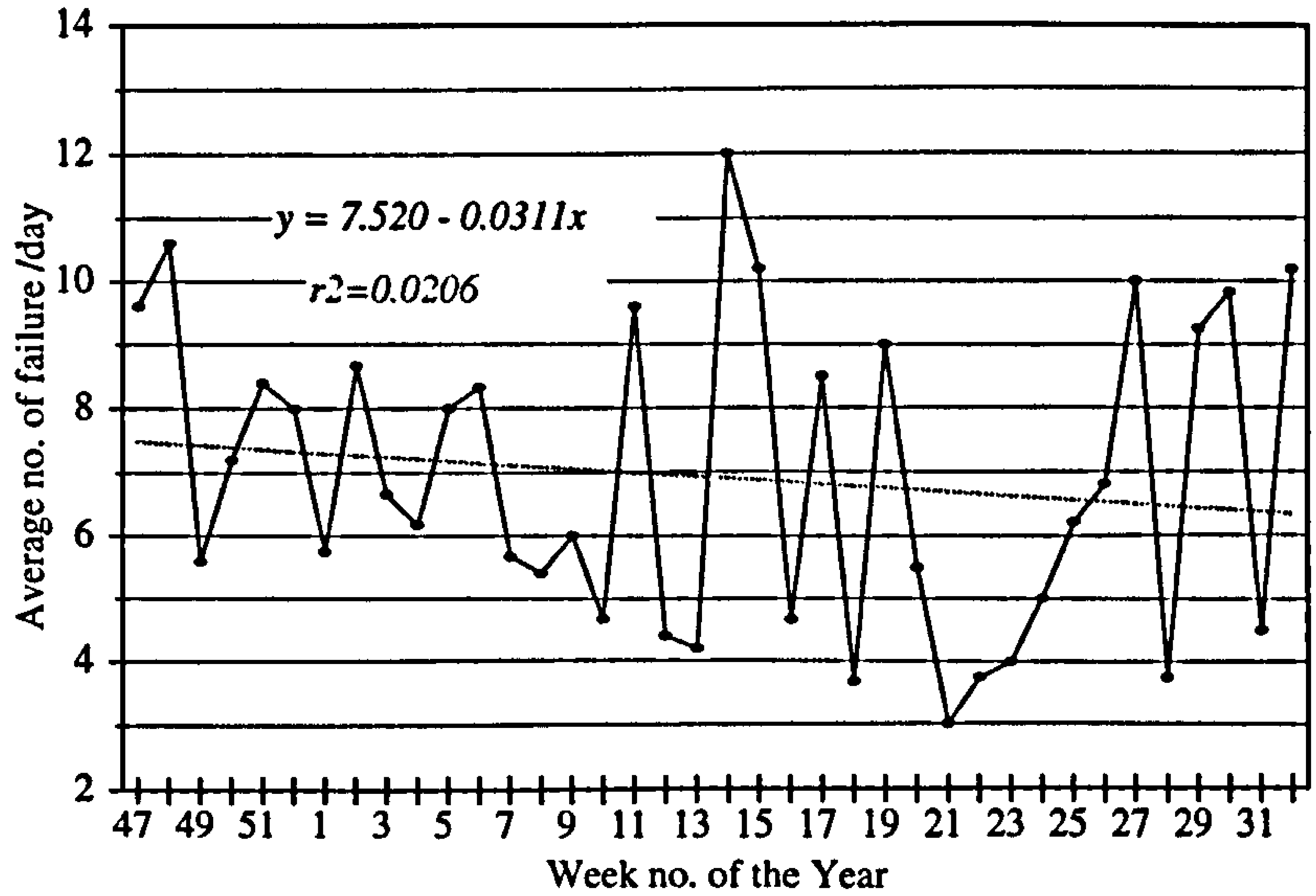


Figure 6.4. Trend analysis for IMA failure data since the start of PM

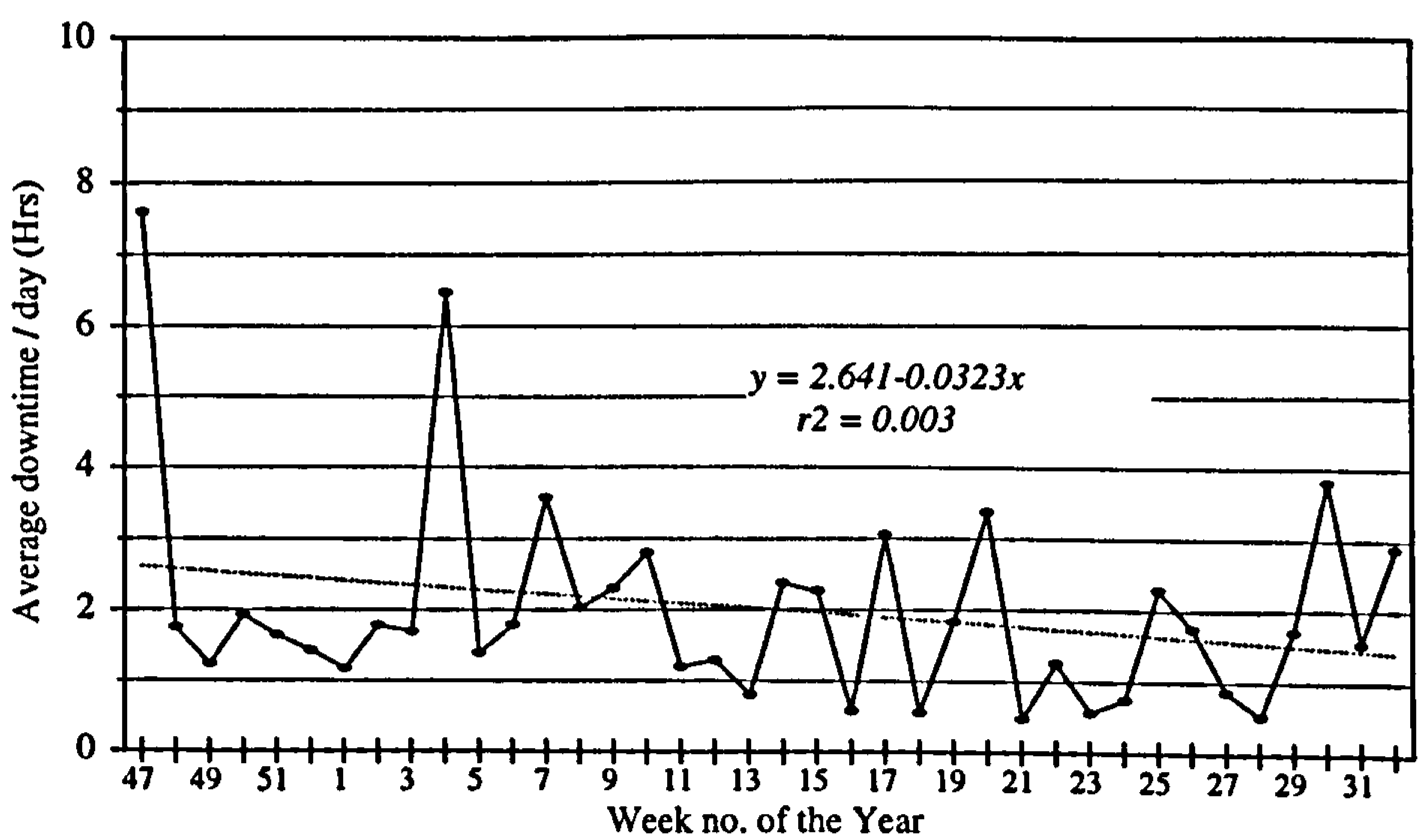


Figure 6.5. Trend analysis for IMA downtime data since the start of PM

A *t*- test accepts the hypothesis that the failure occurrence rate per week is constant from the data of Figure 6.4. The same is true for the data of Figure 6.5. These findings are important since a constant defect arrival rate can be assumed in the subsequent modelling process.

Days since the last PM	Total number of failures	Sample size	Average number of failure	Average downtimes (Hours)	Days since the last PM	Total no. of failures	Sample size	Average no. of failure	Average downtimes (Hours)
1	84	10	8.40	3.02	19	73	7	10.43	0.97
2	95	10	9.50	2.83	20	63	6	10.50	1.79
3	112	10	11.20	2.45	21	92	6	15.33	2.06
4	101	10	10.10	2.37	22	64	5	12.80	2.06
5	72	10	7.2	1.90	23	14	3	4.67	1.09
6	130	10	13.0	2.29	24	28	3	9.33	3.86
7	80	10	8.00	1.46	25	7	2	3.50	1.07
8	115	10	11.50	1.65	26	17	2	8.50	1.00
9	115	10	11.50	1.75	27	13	2	6.50	1.63
10	104	10	10.40	1.59	28	13	2	6.50	1.75
11	74	10	7.40	2.31	29	4	2	2.00	2.50
12	119	10	11.90	1.97	30	8	2	4.00	6.25
13	80	10	8.00	2.00	31	14	2	7.00	7.83
14	79	10	7.90	3.23	32	11	2	5.50	0.63
15	87	10	8.70	2.53	33	2	1	2.00	0.42
16	111	9	12.33	1.84	34	7	1	7.00	6.42
17	69	8	8.63	1.84	35	11	1	11.00	2.67
18	60	8	7.50	1.85	36	5	1	5.00	0.83

Table 6.3. The average number and downtimes of failures since the last PM

We wish to investigate whether or not the 5/6 weekly PM has any effects in reducing failures. Given effective PM, it is expected that the number of failures per day would have an increasing trend following the last PM. An indication of the effectiveness of PM for the IMA machine is shown in Figures 6.6 and 6.7. It can be seen that the number of failures of the IMA machine increases slightly, but the average downtimes decrease since the last PM. It would, however, not be correct to imply from these figures that the PM performed for the IMA might be effective in reducing the number of failures, but not in reducing the downtime due to failures. An evident trend line is exaggerated by scale, and the trend in Figures 6.6 and 6.7 are actually very weak, which suggests that PM might be imperfect in that some defects present are not recognised and cause subsequent failures. A *t*-test also accepts the hypothesis that the average number of failures per day and average downtimes per day since the last PM of the IMA machine are both constant and, as in the case of Figures 6.4 and 6.5, the trend lines of Figures 6.6 and 6.7 may be assumed to have zero slope.

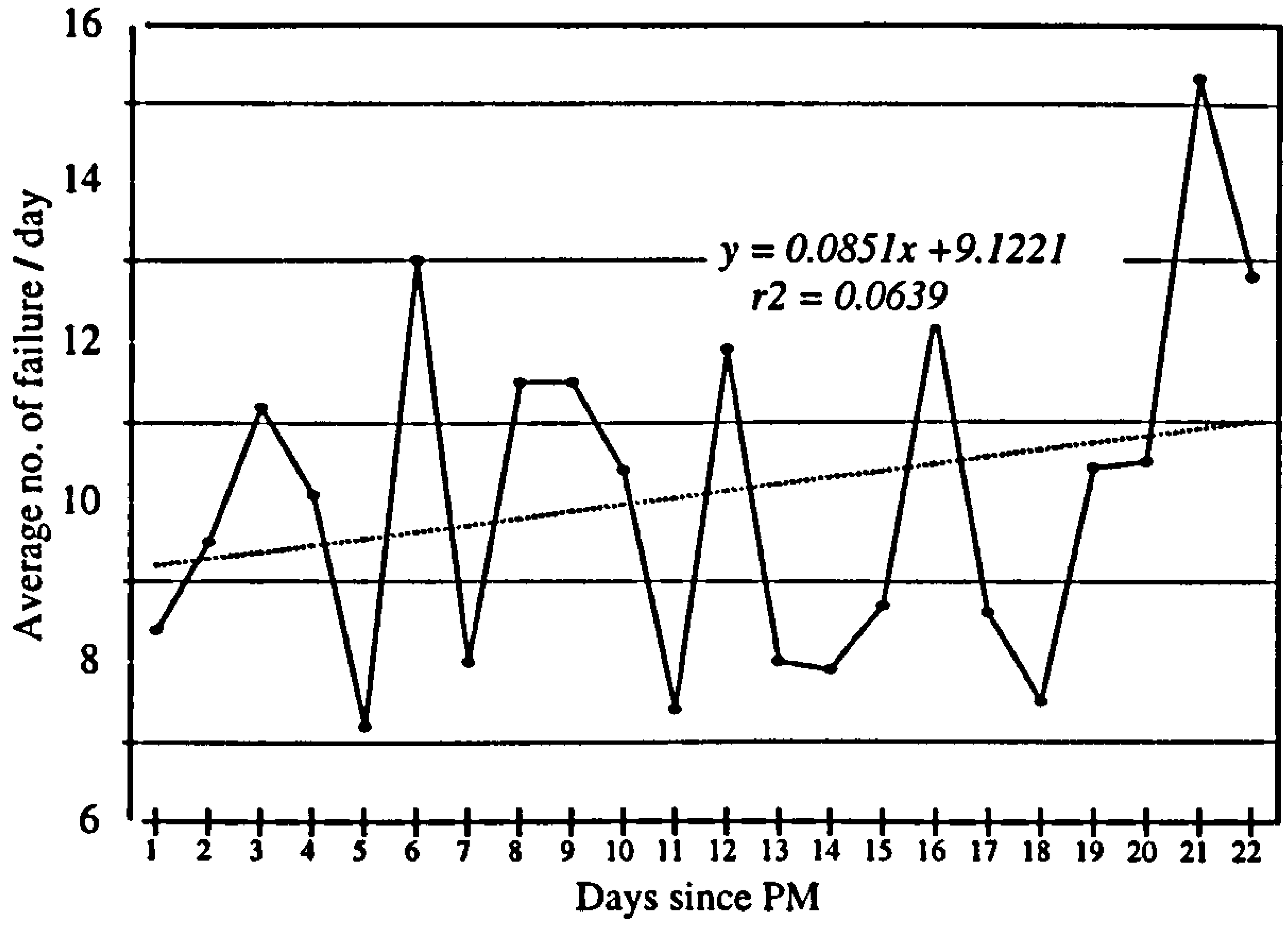


Figure 6.6. The effectiveness of PM for the IMA (Number of failure)

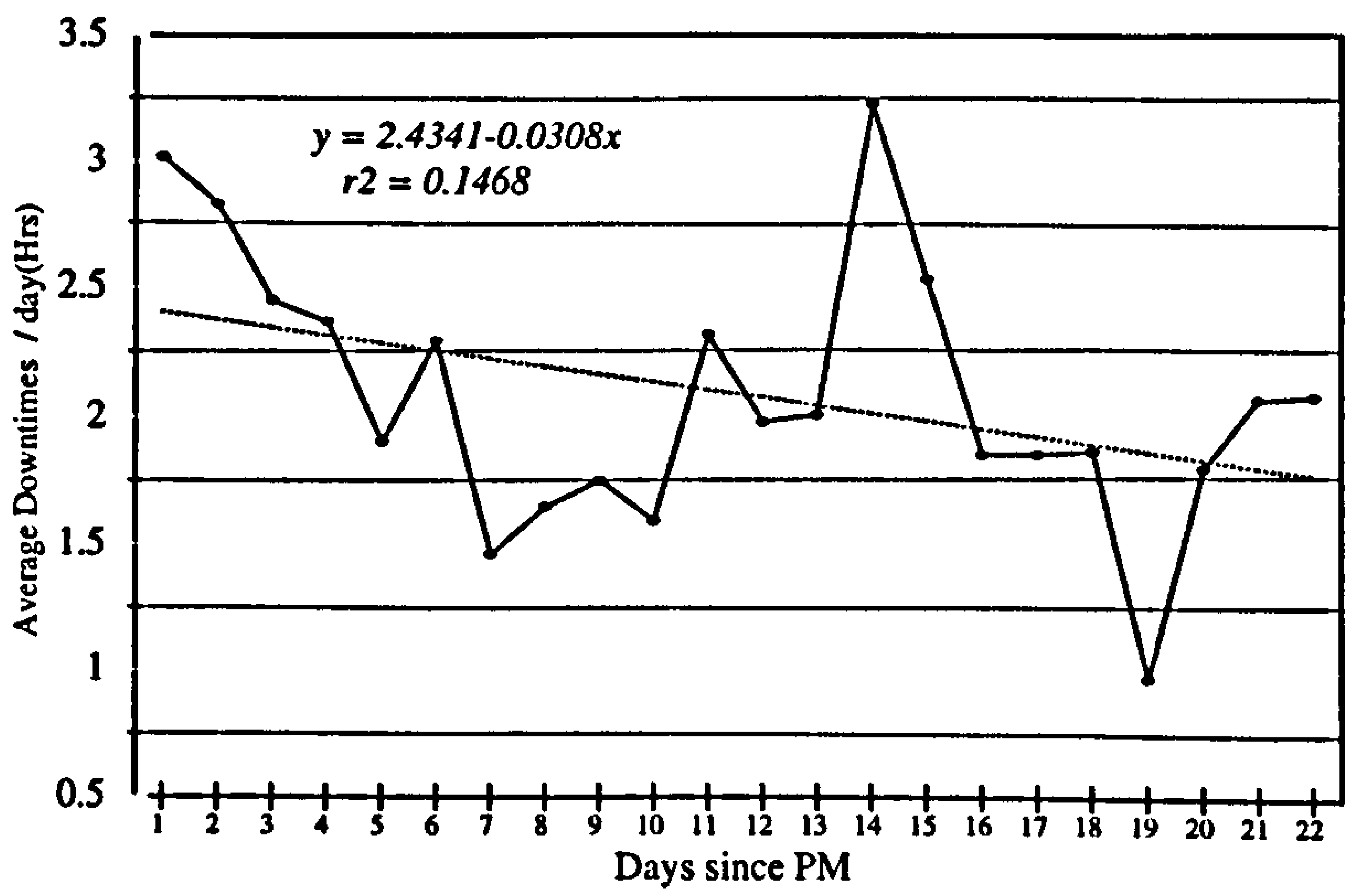


Figure 6.7. The effectiveness of PM for the IMA (Downtimes)

6.5 The Objective Parameter Estimation

6.5.1 General model assumptions

The first objective of this study is to build a statistical model describing the operating process under which the data was recorded, which is used to estimate the parameters of the delay time model, e.g. the fault arrival process and the delay time distribution. Having a delay time model for the process by which defects and failures arise, a maintenance model for decision-making may be constructed. Based upon the data analysis and previous experience, the following assumptions are proposed for the estimation model.

(a) Defects arise as a homogeneous Poisson process (HPP), and the instantaneous rate of occurrence of defects (ROCOD), at time u, is λ. In general, as a complex machine ages, a nonhomogeneous Poisson process (NHPP) is believed to be a good approximation to the fault-arrival process for a complex machine. However, in a simpler context, Barlow & Proschan (1965) proved that, for a complex machine with negligible repair times, the failure process does indeed in the limit follow a homogeneous Poisson process (HPP). This is supported by Figures 6.4 and 6.5. The concept of ROCOD is assumed in chapter 4.

(b) Defects are assumed to arise independently of each other.

(c) The delay time of a failure is independent of its origin and has *pdf.* $f(h)$ and *cdf.* $F(h)$. Assumption (b) and (c) are common in delay time modelling since they greatly simplify the modelling work and have been validated by real-world observations.

(d) Inspections carried out at PM are assumed to be imperfect in that a defect present can only be identified with a probability r. Also, the probability of detection of a defect is independent of the number of times it may have been previously inspected and not detected. Assumption (d) recognises that inspection work during PM might be imperfect. The simpler assumption of perfect inspection is likely inconsistent with the observation of a more or less constant failure rate after PM.

(e) All identified faults are rectified by repairs or replacements during the time of the PM.

(f) Failures are identified immediately, and repairs or replacements are made as soon as possible. Assumptions (e) and (f) embody the maintenance practice currently adopted.

6.5.2 Likelihood formulation

It is assumed for the moment that observations of the number and downtime of failures, and of the number of defects identified and removed at PM are available. We define the notation for modelling the likelihood of this data.

Let

λ : the rate of occurrence of defects within the system.

h : the delay time of a fault with *pdf* $f(\bullet)$ and *cdf* $F(\bullet)$.

r : the probability of detecting a defect at PM, if it is present.

t_i : time of the ith PM from new, $i = 1,2, ..., n, ...$

t : failure time from new.

Δt : A time period to be defined.

$EN_f(t, t+\Delta t)$: the mean number of failures over $(t, t+\Delta t)$.

$EN_p(t_n)$: the mean number of defects identified and removed at t_n.

$P(t, t+\Delta t \mid u)$: the probability of a failure in $(t, t+\Delta t)$ resulting from a defect arising at time u.

In the above notion, we assume $t_{n-1} < t \leq t_n$.

Consider the probability of a failure over $(t, t+\Delta t)$ resulting from a defect arising at time u, $u \leq t + \Delta t$, that is, $P(t, t+\Delta t \mid u)$. The defect could have arisen since the last inspection, t_n, or during several inspection periods before, but not have been detected, see Figure 6.8.

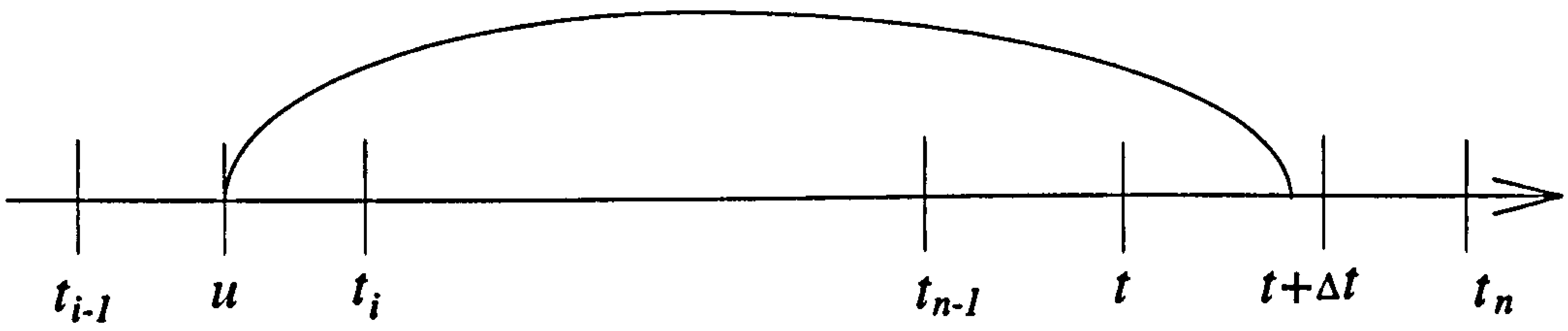


Figure 6.8. The failure process of a defect arising in (t_{i-1}, t_i)

We have, Christer and Wang (1995), that

$$P(t,t+\Delta t\mid u)\begin{cases}(1-r)^{n-i}[F(t+\Delta t-u)-F(t-u)] & t_{i-1}<u<t_i; i\in\{1,...n-1\}\\ F(t+\Delta t-u)-F(t-u) & t_{n-1}<u<t\\ F(t+\Delta t-u) & t<u<t+\Delta t\\ 0 & u>t+\Delta t.\end{cases} \quad (6.1)$$

The mean number of failures over (t, t+Δt), for $t_{n-1} < t \leq t_n$, is, therefore, given by

$$EN_f(t,t+\Delta t)=\sum_{i=1}^{n}(1-r)^{n-i}\lambda\int_{t_{i-1}}^{t_i}[F(t+\Delta t-u)-F(t-u)]du$$
$$+\lambda\int_{t_{n-1}}^{t}[F(t+\Delta t-u)-F(t-u)]du$$
$$+\lambda\int_{t}^{t+\Delta t}F(t+\Delta t-u)du. \quad (6.2)$$

Changing the variable of integration and rearranging the sequence, after some manipulation we have for the expected number of failures over period (t, t + Δt), (see equation (3.38)),

$$EN_f(t,t+\Delta t)=\lambda\int_t^{t+\Delta t}\sum_{i=1}^{n-1}(1-r)^{n-i}[F(x-t_{i-1})-F(x-t_i)]dx+\lambda\int_t^{t+\Delta x}F(x-t_{n-1})dx. \tag{6.3}$$

In a similar way, it is seen that the expected number of failures found at PM time t_n, $EN_p(t_n)$, (see equation (3.39)), is

$$EN_p(t_n)=\sum_{i=1}^{n-1}(1-r)^{n-i}r\lambda\int_{t_{i-1}}^{t_i}[1-F(t_n-u)]du \quad +r\lambda\int_{t_{n-1}}^{t_n}[1-F(t_n-u)]du, \tag{6.4}$$

see also Christer and Wang (1995). Since defects are assumed to arise according to a Poisson process, as a generalization of Proposition 3.3.2 in Ross (1983), the number of failures in $(t, t+\Delta t)$ also follows a Poisson process, Christer and Wang (1995). Therefore, the probability of m failures over $(t, t+\Delta t)$, where $t_{n-1} < t \le t_n$ is given by, (see equation (3.40)),

$$P(\,m\ \textit{failures in}\ (t,\,t+\Delta t))=\frac{[EN_f(t,t+\Delta t)]^m e^{-EN_f(t,t+\Delta t)}}{m!}. \tag{6.5}$$

It can be shown, Christer and Wang (1985), that the number of defects identified and removed at PM also follows a Poisson distribution, and we have, see also equation (3.41),

$$P(\,n\ \textit{defects identified and removed at}\ t_n\,)=\frac{[EN_p(t_n)]^n e^{-EN_p(t_n)}}{n!}. \tag{6.6}$$

If the observed events are the number of failures in each working day and the number of defects identified at PM times, the likelihood function of the observed events may be formulated in the following way. Suppose that n_i defects have been observed at the ith PM time ($n = 1, 2, \ldots, l$), and the PM interval (t_{n-1}, t_n) is now divided into k non-overlapping subintervals of equal length Δt, as 1 day, namely, (see equation (3.42)),

$$I_j^n = [t_{n-1} + (j-1)\Delta t, t_{n-1} + j\Delta t], (j = 1, \ldots, k), \tag{6.7}$$

where $t_{n-1} + k\Delta t = t_n$.

It follows from equation (6.3) that the mean number of failures occurring in I_j^n over (t_{n-1}, t_n), is, (see also equation (5.13)),

$$EN_f(I_j^n) = \lambda \int_{t_{n-1}+(j-1)\Delta t}^{t_{n-1}+j\Delta t} \sum_{i=1}^{n-1} (1-r)^{n-i} [F(x - t_{i-1}) - F(x - t_i)] dx$$

$$+ \int_{t_{n-1}+(j-1)\Delta t}^{t_{n-1}+j\Delta t} \lambda F(x - t_{n-1}) dx. \tag{6.8}$$

Now, let m_{nj} denote the number of failures occurring in I_j^n over (t_{n-1}, t_n) and m_n denote the defects identified and removed at t_n, we have from equations (6.5) and (6.6) that, (see equations (5.14) and (5.15)),

$$P(\,m_{nj} \mathit{failures\ in}\ I_j^n\,) = \frac{[EN_f(I_j^n)]^{m_{nj}} e^{-EN_f(I_j^n)}}{m_{nj}!}, \tag{6.9}$$

and

$$P(\,m_n\ \mathit{defects\ identified\ and\ removed\ at}\ t_n\,) = \frac{[EN_p(t_n)]^{m_n} e^{-EN_p(t_n)}}{m_n!}. \tag{6.10}$$

The likelihood function for these observations is, therefore,

$$L = \prod_{n=1}^{l} (P(m_n\ \mathit{defects\ identified\ and\ removed\ at}\ t_n) \prod_{j=1}^{k} P(m_{nj}\ \mathit{failures\ in}\ I_j^n)). \tag{6.11}$$

Substituting from equations (6.9) and (6.10) into equation (6.11), and taking the logarithm, we have that, (see equation (5.17)),

$$\log L = \sum_{n=1}^{l} [m_n \log EN_p(t_n) - EN_p(t_n) - \log m_n!]$$

$$+ \sum_{n=1}^{l} \sum_{j=1}^{k} [m_{nj} \log EN_f(I_j^n) - EN_f(I_j^n) - \log m_{nj}!], \quad (6.12)$$

where l is the number of PMs. In the case where PM data are not exists, the log likelihood function (6.12) reduces to, (see equation (5.18)),

$$\text{Log } L = \sum_{n=1}^{l} \sum_{j=1}^{k} [m_{nj} \log EN_f(I_j^n) - EN_f(I_j^n) - \log m_{nj}!]. \quad (6.13)$$

6.5.3 Results of the Model Fit

In this study, we seek to fit to a model to the data from the IMA plant. Possible models for $f(\bullet)$ could be exponential or mixed delta-exponential, or Weibull or a mixed delta-Weibull distribution. In this study, we consider 4 models for the delay time distribution, that is (1) exponential, (2) mixed delta-exponential, (3) Weibull, (4) mixed delta-Weibull. The reason to choose a mixed delta distribution is the possibility that some defects may have a zero delay time, whilst other defects have a stochastic delay time. This situation may be modelled by the mixed delay time distribution with a *pdf* given by $(1\text{-}P)f(h) + P\delta(h)$, where $f(h)$ is the *pdf* of the non identically zero delay time h, $\delta(h)$ is the Dirac delta function, and P is the proportion of defects that have zero delay time.

However, we have a problem when fitting the likelihood formulation to the data. Since PM data is not available, the likelihood function does not converge for 3 out of 4 of the targeted distributions, namely, the exponential, the mixed delta-exponential and the Weibull. The reason for this is the strong correlation between model parameters when PM data is not available, particularly between r and the rest of parameters. The problem may be solved by fixing r at an appropriate level and maximising the likelihood function in terms of the other parameters. This shifts the problem to deciding the value of r, which is usually done using a subjective survey technique. Here, we adopt the one value of r that was obtainable from the likelihood function in the one case that it converged, namely, $r = 0.071$. This seemed very low to the authors, but we accept it for now and return to it later.

In maximum log likelihood estimation, the goodness of fit of values of parameters of a specific model may be measured by the expected log likelihood, namely, the larger the expected log likelihood, the better is the fit of model of parameters. The log likelihood is usually regarded as an estimator of expected log likelihood. The parameters set that maximizes the likelihood is not necessarily the best fit to the data, since the more parameters one has, the lager the likelihood function can become. Because of this, the choice of the distribution from a family of plausible distributions for h is made using the criterion of minimum Akaike Information Criterion (*AIC*), Baker and Wang (1992). *AIC* is derived under the assumption that the true distribution can be described by the given model when its parameters are suitably adjusted. A model which minimises the *AIC* is considered to be the most appropriate model.

$$AIC = -2\times(\textit{maximum log likelihood of the model}) \\ + 2\times(\textit{number of free parameters of the model})$$

Although the value of the *AIC* may be the lowest of the options considered, if the *chi-squared* test for a goodness of fit is not acceptable at some significance level, the chosen model may be invalid. For this reason we also need to consider the *chi-squared* goodness of fit test, which is given by

$$\chi^2 = \sum_{i=1}^{k} \frac{(n_i - \hat{n}_i)^2}{\hat{n}_i}, \tag{6.14}$$

where the range of data is divided into k suitable classes, n_i is the number of the events in the ith class, and $\hat{n}_i$ is the expected number of events in the ith class calculated from the fitted model. If $N_f(I_j)$ denotes the observed number of failures in the jth class, using equation (6.14), the *chi-squared* test statistic in our case is simply

$$\chi^2 = \sum_{j=1}^{k} \frac{(N_f(I_j) - EN_f(I_j))^2}{EN_f(I_j)}, \tag{6.15}$$

and the number of degrees of freedom is k - v, where v is the number of model parameters.

Given $r = 0.071$, Table 6.4 shows the fitted values of parameters, their AIC values and the results of *chi-squared* test statistic. From Table 6.4, exponential distribution is selected as having the lowest AIC and χ^2 statistic. From Table 6.4, we see for the chosen exponential distribution that the mean delay time is 0.55 day and the arrival rate of defects is 9.5 per day.

Table 6.4. Models and fitted values of parameters based upon IMA machine data

Models	Exponential Distribution	Mixed delta-Exponential	Weibull Distribution	Mixed delta-Weibull
ROCOD ($\hat{\lambda}$)	9.518	9.745	9.535	9.853
Scale ($\hat{\alpha}$)	1.816	1.816	6.981	1.223
Shape ($\hat{\beta}$)			1.482	15.326
$\hat{P}$		0.023		0.032
r	0.071	0.071	0.071	0.071
Maximum Log likelihood	-1118.138	-1118.138	-1119.995	-1116.282*
χ^2	2.135*	3.790	2.226	5.098
AIC	2240.276*	2242.276	2245.990	2242.564

Notes : P is the proportion of identically zero delay times, ROCOD is the rate of occurrence of defects, r is the probability a defect is identified and rectified at PM, '*' denote the optimal choice for the criterion.

Using equation (6.15) with the model parameters from the exponential distribution of Table 6.4, we compare the observation data of the number of failures of the IMA with the model predictions, see Figure 6.9. To check the validity of this fit, we compare the *chi-squared* test statistic with the critical value of the *chi-squared* distribution table. Since the critical χ^2 value, ($\chi^2_{(0.05,3)}$) is 7.815, we can conclude with χ^2 = 2.135 that the model fit is acceptable.

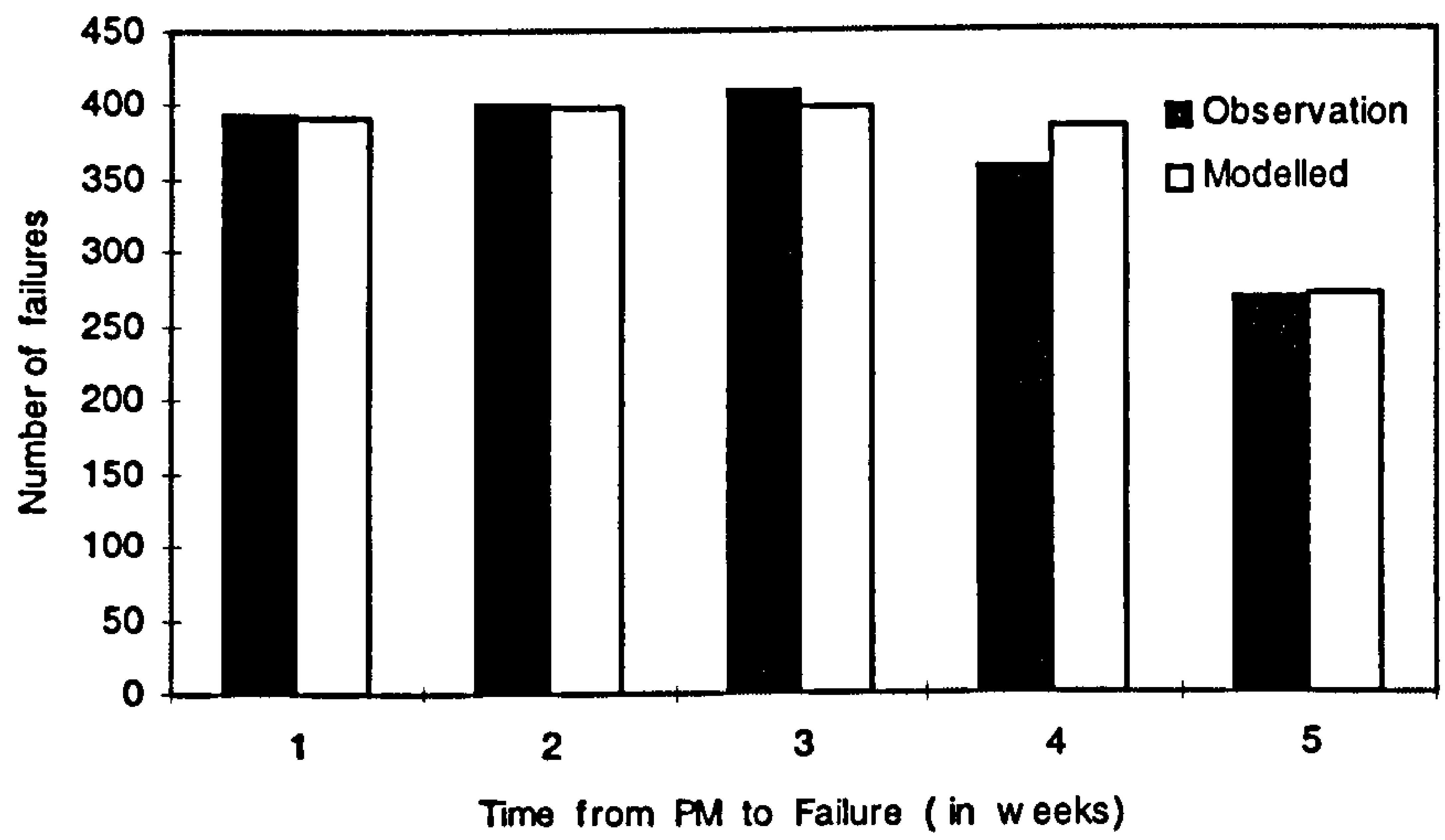


Figure 6.9. Histogram of failures for IMA

6.5.4 The PM model and Results

Based on an acceptable model for the failure and PM process of the IMA, a PM model of the maintenance practice of the machine can be established. We model the downtime since the major concern of the company is to reduce the downtime caused by failures and PM activities. The conventional downtime measure is the expected downtime per unit time over a long period. The key issue in the model is the expected number of failures over different PM cycles, which is given in equation (6.1)

If d_f denotes the mean downtime per failure and d_p denotes the mean duration of PM activity, it follows that the long term measure of the expected downtime per unit time, *ED* (*T*), is

$$ED(T) = \frac{d_f EN_f(T) + d_p}{T}. \tag{6.16}$$

Using the fitted model parameters of Table 6.4, we obtain the result of the model output shown in Figure 6.10.

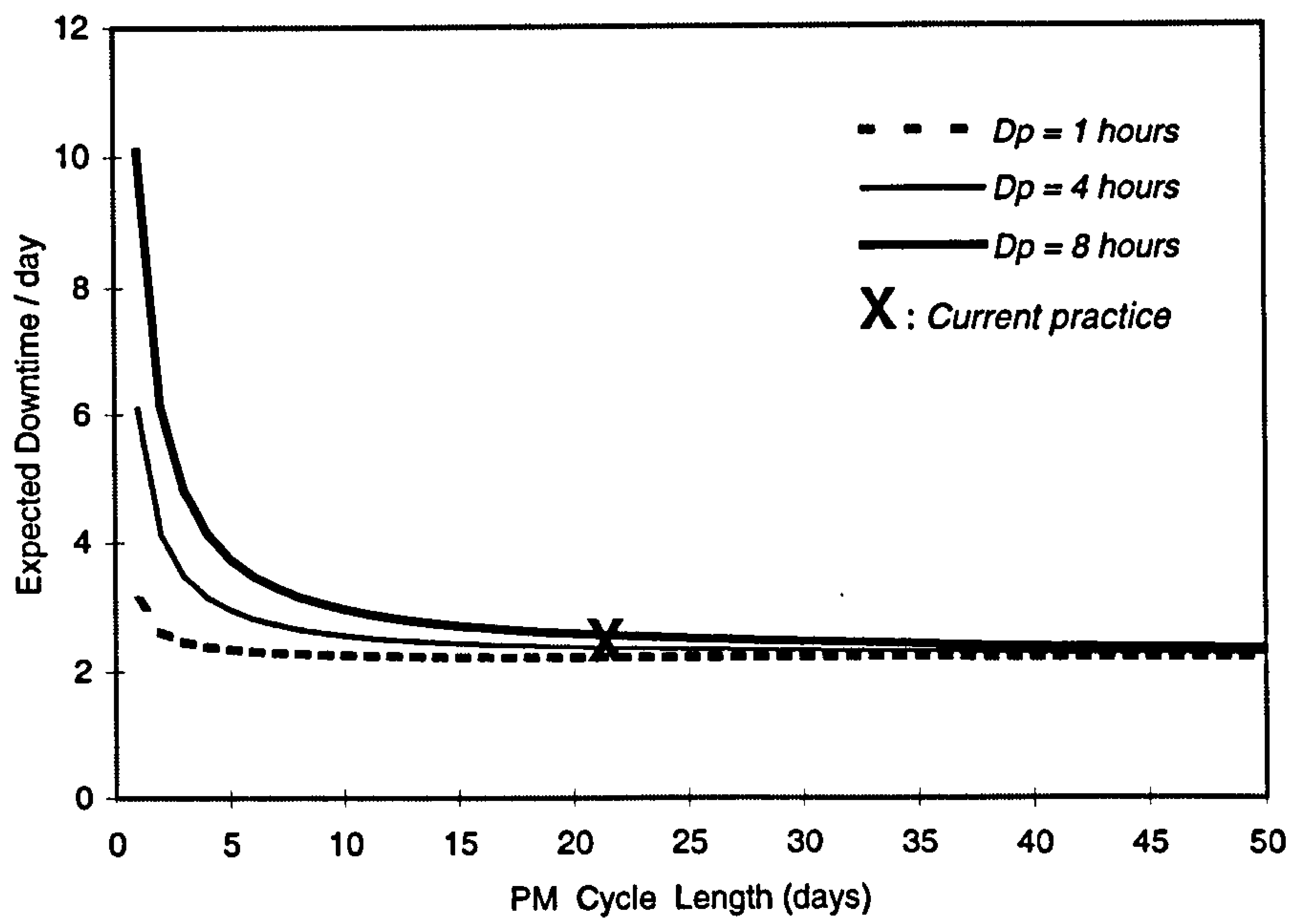


Figure 6.10. Expected downtime against PM cycle length

It should be noted that not all 8 hours PM time is dedicated to the inspection of the IMA plant, which may only occupy a small portion of the total PM time. For this reason, we set d_p = 1, 4 and 8 hours in Figure 6.10, which also indicates the observed downtime level of current practice. This is very close for the various cases. It can be seen from Figure 6.10 that there is no indication of any inspection element of periodic PM being beneficial to the IMA concerned if the PM inspection downtimes are 1, 4 or 8 hours. This suggests strongly that the current PM policy for IMA may not be appropriate and an alternatively way needs to be found to reduce downtime due to failures.

Discussion with engineers revealed that one of the reasons that PM is not as good as expected is the lack of proper inspection at PM, which is reflected by the low value of r. If the inspection element of PM is enhanced, or the efficiency of inspection is increased to $r = 0.85$, say, an optimal PM cycle length may exists when d_p = 1hour, see Figure 6.11, but with very modest returns over a breakdown system. Figure 6.11 shows that if $r = 0.85$, the optimal PM cycle length is 3.3 days with the expected total downtime of 2.166 hours per day and for $r = 0.9$, the optimal PM interval is 2.3 days with the expected total downtime of 2.144 hours per day.

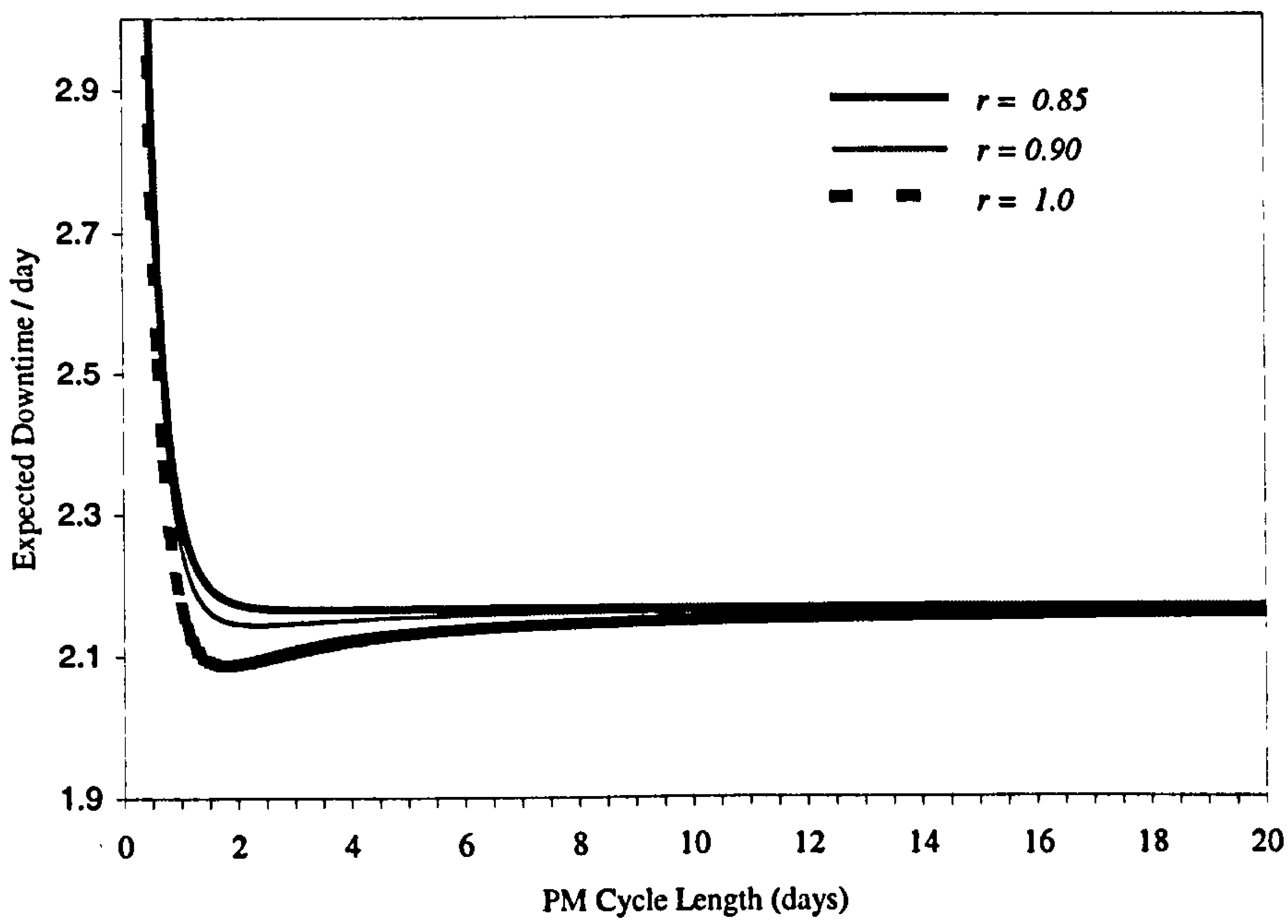


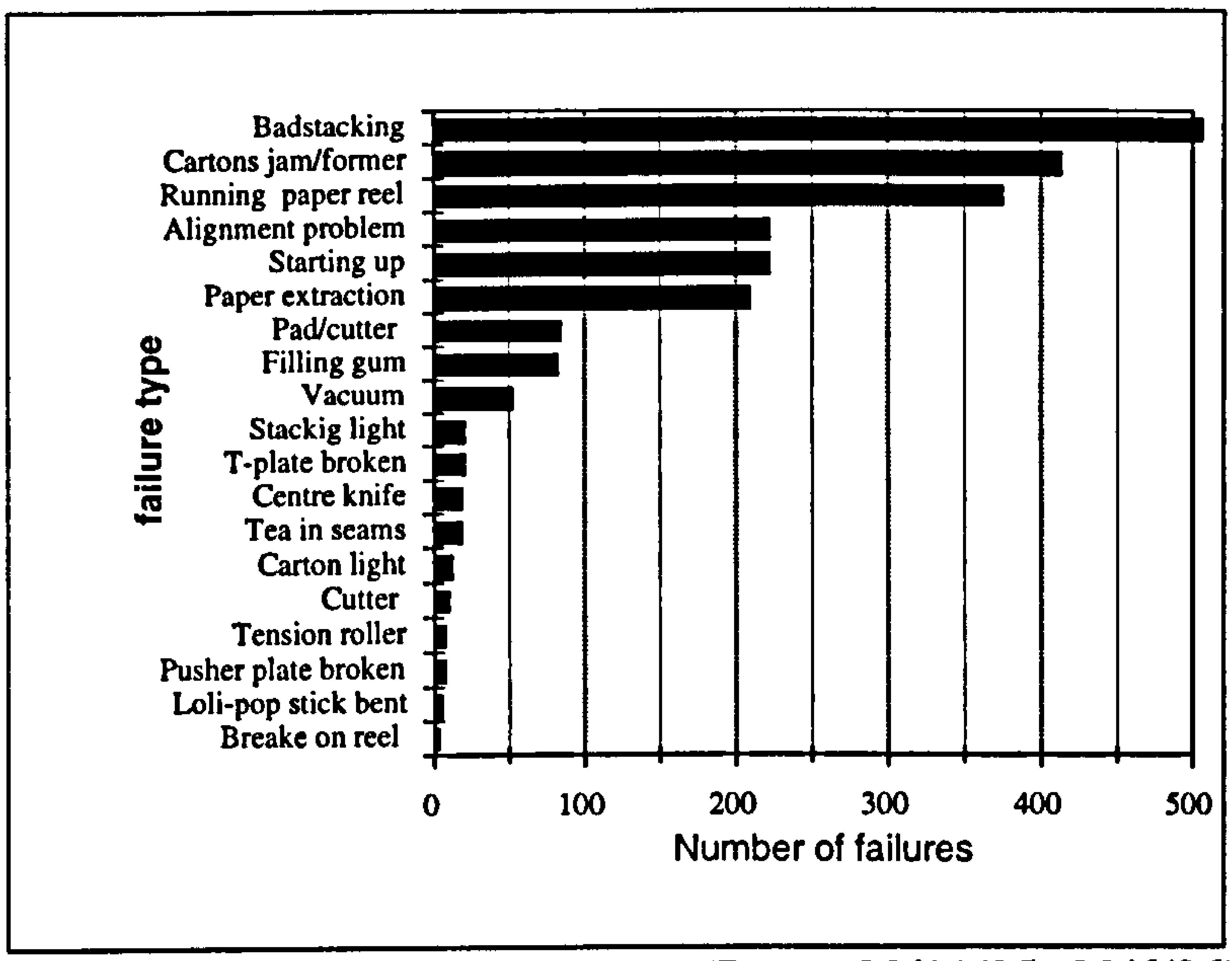
Figure 6.11. Expected downtime when r is 0.85, 0.9 and 1.0

If we assume PM inspection is perfect, namely $r = 1$, the optimal PM cycle length is 1.8 days and the expected total downtime per day is 2.087 hours. It is noted that as the quality of PM inspection level is increase, the expected total downtime per day may decrease. However, the point here is that since the delay time has a small mean value, 0.5 days, the inspection quality measure, r, is not a sensitive parameter in the downtime model. To reduce downtime further may require a snap-shot type of surveys, Christer and Whitelaw (1983), to identify the nature of causes of fault, and whether or not engineer solutions such as re-design, or maintenance solutions such as an improved and deeper inspection, can reduce downtime. A snapshot approach can assist with and inform all of these developments. This type of survey is discussed in the subsequent section 6.7.

6.6 The Subjective Parameter Estimation

6.6.1 The parameter estimation of delay time from subjective data

So far, the objective method for parameter estimation has been used. In the current case, although there have been plant inspections, there is no record of maintenance activities and defects found at PM. Therefore, the subjective method in delay time estimation might be appropriate. The subjective method using subjective opinions of experts in the estimation of delay time parameters has been developed by Christer and Waller (1984a, b). The approach used is based upon the analysis of historic data of failures and the delay time concept where the distribution of the delay time was estimated from the subjective data obtained from the expert. Wang (1997) proposed a revised method for obtaining the subjective delay time estimate. Wang attempts to measure directly the distribution of delay time, as opposed to using a process of synthesis of numerous individual estimates. Here we use the survey methodology which used in Wang (1997). The maintenance engineer performs regularly the preventive maintenance for the three major components, namely IMA, ME and Europack. At the time of study, the IMA component only was selected for detailed study in this case study.



(Data : 20/11/95 -30/6/96)

Figure 6.12. Failure statistic of IMA at P1 Line

One of maintenance engineers responsible for the plant was chosen as our expert to provide subjective estimates. With the help of the chosen expert to augment existing data, delay time estimates and estimates of failure types were obtained, see Figure 6.12 and Figure 6.13.

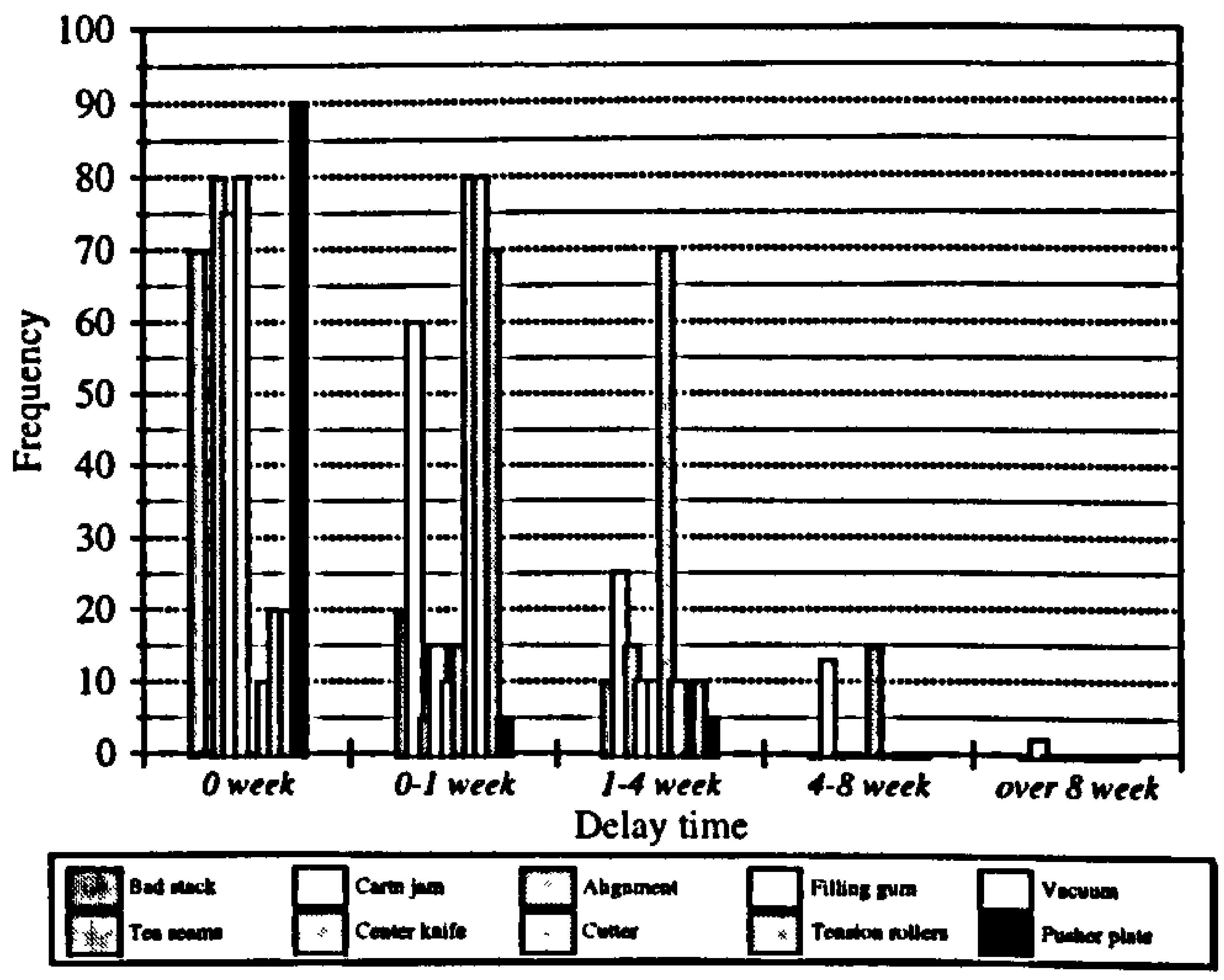


Figure 6.13. Histogram of the delay time distributions of failure types

It is noted that 9 of the failure types shown in Figure 6.12 are in the category of the zero delay time and, as suggested by the expert can not be identified at any inspection. They are, therefore, are not shown in Figure 6.13.

However, as the delay time estimates were obtained on the basis of component units, it is necessary to combine these to obtain the delay time distribution for all failures of machine. For a combined delay time distribution,

Let

N denote the number of dominant failure types.

d_{ij} denote the subjectively estimated number of delay times in ith interval specified for failure type j, $i = 1, \dots, m$ and $j = 1, \dots, N$ (see Table 6.5).

w_j denote the weighted values of failure type j.

If the estimate of the number of the delay times over each interval in the required histogram of the delay time distribution is available, it follows that the combined estimate of the number of the delay times in the ith interval for all failure types within the system is given by

$$n_{di} = \sum_{j=1}^{N} w_j d_{ij} , \tag{6.17}$$

where $w_i = \dfrac{\textit{Number of failures for failure type j}}{\textit{Total number of failure}}$.

Therefore, a combined delay time distribution of all failure types is obtained by using the equation (6.17), see Figure 6.13.

Table 6.5. The subjectively estimated number of delay times

Failure types	Number of failures	Weighted values (w_i)	0 week	0 – 1 week	1 – 4 week	4 – 8 week	Over 8 week
Bad Stacking	505	0.383	70	20	10	0	0
Carton jam/former	411	0.312	0	60	25	13	2
Alignment problem	220	0.167	80	5	15	0	0
Filling gum	79	0.060	75	15	10	0	0
Vacuum	51	0.039	80	10	10	0	0
Tea in seams	17	0.013	0	15	70	15	0
Center knife	17	0.013	10	80	10	0	0
Cutter	7	0.005	20	80	0	0	0
Tension rollers	6	0.005	20	70	10	0	0
Pusher plate broken	5	0.004	90	5	5	0	0
Total	1318	1.000					

For a fitted distribution of the delay time of failures given the data in Figure 6.12, a *chi-squared* test method has been used to achieve the best fit. The *chi-squared* test statistic used here, (see also equation (6.14)), is

$$\chi^2 = \sum_{j=1}^{N} \frac{(n_j - \hat{n}_j)^2}{\hat{n}_j}, \tag{6.18}$$

where N is the total number of classes in the histogram in Figure 6.14, n_j is the estimated number of the delay times in the jth classes of the histogram in Figure 6.13 or 6.14, and $\hat{n}_j$ is the calculated number of the delay times in the jth classes from the fitted delay time distribution.

Minimising equation (6.18) in terms of unknown parameters in fitted delay time distribution will give the estimated values of these parameters. From the Figure 6.14 it is obvious that the chosen delay time distribution should be a mixed distribution with parameter P representing the probability of the delay time concentrating at zero and $(1-P)G(h)$ denoting the remaining part of the continuous delay time distribution where h denotes the random variables of the delay time.

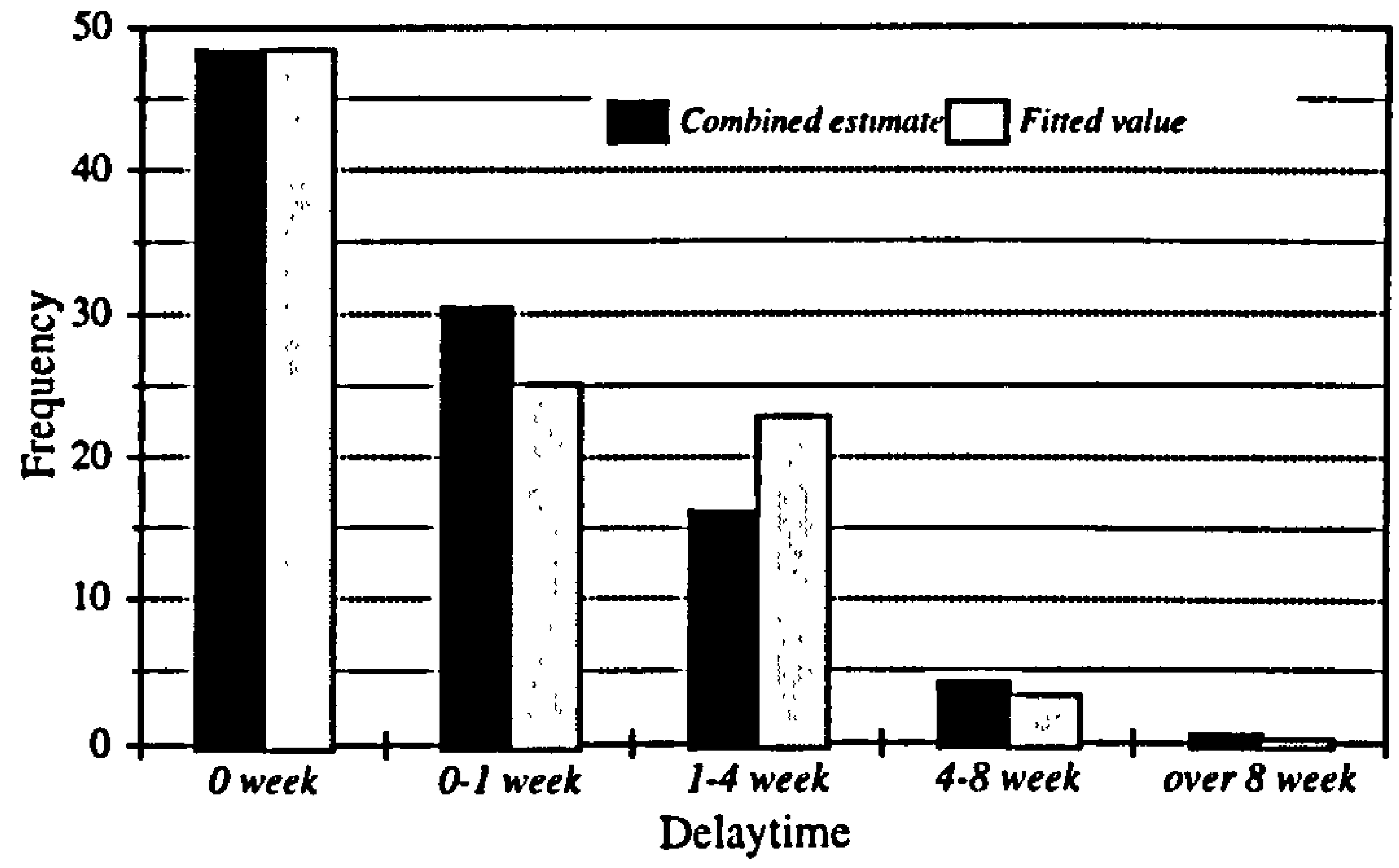


Figure 6.14. Histogram of the delay time distributions of all failure types

The parameter values of the delay time distribution and associated values of equation (6.18) are shown in Table 6.6. From the shape of the histogram in Figure 6.14, $G(h)$ is chosen to be exponential since exponential distribution has a better *chi-squared* result. Thus the complete distribution for the delay time is as follows:

$$G(h) = 1 - (1 - P)e^{-\alpha h}, \qquad (6.19)$$

where α denotes the scale parameter of the exponential distribution and P is the proportion of failures with the zero delay time. In the case of failure data only, the estimated parameter of $G(h)$ is obtained by minimising equation (6.18) namely 0.0954. Figure 6.14 also indicates the number of the delay times in each class of the histogram calculated from the fitted delay time distribution. Both theoretically and visually the fit is acceptable with $\chi^2 = 3.9209$.

In order to complete parameter estimates of the delay time model, it is necessary to determine the rate of occurrence of defects (ROCOD), and establish the probability r of detecting a defect at a PM. If the process is in a steady state, the ROCOD can be estimated by $\hat{\lambda} = N/t$, where N is the total number of failures and defects collected by a survey methodology which used in Wang (1997) and t is the time length of the survey. In this case $\hat{\lambda} = 2251/239 = 9.418$ per day. The probability of detecting a defect at a PM can also be obtained by asking the maintenance engineers to estimate a mean figure. In discussion with the engineer in the company, $\hat{r} = 0.02$ was considered an appropriate figure. Table 6.6 shows the estimated parameter values, which completes the determination of parameters of the delay time model from subjective data.

$\hat{\lambda}$	$\hat{\alpha}$	$\hat{r}$	$\hat{P}$
9.418	*0.0954*	*0.02*	*0.484*

Table 6.6. Values of estimated parameters

6.6.2 The PM model and results

We also use the equation (6.16) for PM modelling in the subjective assessment case. To establish the relationship between the PM interval and total downtime, we assume that defects arise from a homogeneous Poisson process, with an imperfect inspection at PM. Again, d_f denotes the mean downtime per failure and d_p denotes the mean duration of PM activity, it follows that the long-term measure of the expected total downtime per unit time, $ED(T)$, (see equation (6.20), is

$$ED(T) = \frac{d_f EN_f(T) + d_p}{T}, \tag{6.20}$$

where $EN_f(T)$ denotes the expected number of failures over T. It is known that if the initial defect origination process is a Poisson process, the failure process caused by these defects also follows a Poisson process (Christer *et al.* (1995) and Ross (1983)). Therefore now, let t_i denote the time of the ith PM from now, we have $T = t_i - t_{i-1}$ for $i = 1, 2, \dots, n$. We assume that the system is in a steady state since it has gone through n PMs where $n >> 1$. Supposing a defect arise in (t_{i-1}, t_i) and cause a failure in (t_{n-1}, t_n) for $n > i$, we must have that it has a delay time $t_{n-1} - t_i < h < t_n - t_{i-1}$ and that it is not identified at the jth PM for $j = i, i+1, \ldots, n-1$. Then, the expected number of failures in (t_{n-1}, t_n) from a defect in (t_{i-1}, t_i) is given by

$$EN_f(t_{n-1}, t_n) = \hat{\lambda}\int_{t_{i-1}}^{t_i} \sum_{i=1}^{n-1} (1-\hat{r})^{n-i} \{F(t_n - u) - F(t_{n-1} - u)\} du + \hat{\lambda}\int_{t_{n-1}}^{t_n} F(t_n - u) du. \quad (6.21)$$

Also, since the delay time follows a mixed delta-exponential distribution, equation (6.19), the expected number of failures over a PM cycle can be obtained as follow. Letting $n \to \infty$ in equation (6.21), we may sum the resultant geometric series of equation (6.21) over n. In this case, the expected number of failures over (t_{n-1}, t_n) is given by,

$$EN_f(T) = \frac{\hat{\lambda}\hat{r}(e^{\hat{\alpha}T} - 1)(\hat{P} - 1)}{\hat{\alpha}(e^{\hat{\alpha}T} - 1 + \hat{r})} + \hat{\lambda}T. \quad (6.22)$$

With the fitted delay time distribution obtained in the subsection section 6.6.1 and the average downtime per failure, d_f = 0.228 hours from Table 6.2, here we now assume that the average downtime per PM, d_p, is (based on the Table 6.1), 8 hours.

Figure 6.15 shows the result from the downtime per unit time equation (6.20). From Figure 6.15, we note the result is consistent with the objective parameter estimation method, namely that the subjective parameter estimation method also shows that there is no indication of any period for the current PM system which would be beneficial to the plants concerned if the PM requires 8 hours. This argument is important since it shows that inspection is not invariably appropriate to all machines. In fact, we would expect this finding in our case since almost half of the total failure types have no delay time, and the remainder has a relatively short delay time, see Figure 6.14. This would indicate that the current inspection programme is unlikely to be useful to the IMA.

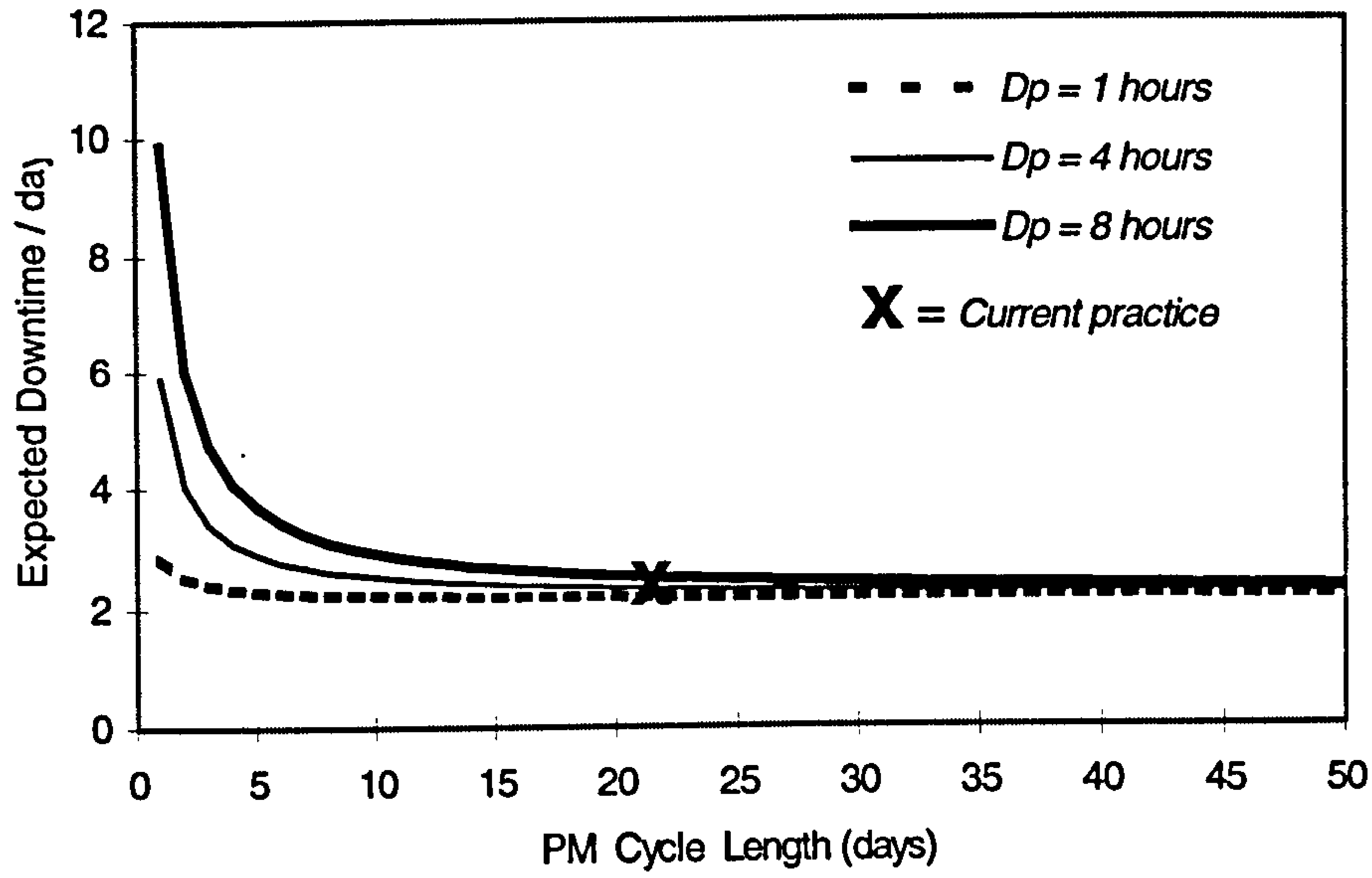


Figure 6.15. Expected downtime against PM cycle length

However, the true conclusion depends very much upon the downtimes associated with failures and PM, the parameter values of λ and the delay time distribution. Figure 6.15 also shows that the expected downtime per unit time when the downtime due to PM is d_p = 1, 4 and 8 hours, and indicates the observed downtime level of current practice. In respect to the current PM practice, this result which is Figure 6.15 giving the expected downtime against PM cycle length is very close to the results obtained from objective method, Figure 6.10, and the same conclusions follow. As before, in this case if the inspection element of PM is enhanced, or the efficiency of inspection is increased to r = 0.85, say, an optimal PM cycle length may exists even when d_p = 8hours. Figure 6.16 shows that when d_p = 1hour and if r = 0.85, the optimal PM cycle length is 5 days with the expected total downtime of 1.5214 hours per day and for r = 0.9, the optimal PM interval is also 5 days with the expected total downtime of 1.5017 hours per day. Again, if we assume PM inspection is perfect, namely r = 1, the optimal PM cycle length is 5 days and the expected total downtime per day is 1.4661 hours. It is noted that for increasing quality of inspection, the expected total downtime per day can decrease. As already mentioned in the subsection 6.5.4, we need to find an alternative way of inspection which occupies little production downtime, but which is powerful enough to detect more defect if they exists. Although the current PM inspection is ineffective, an alternative inspection practice might be effective, and one-off design modification might significantly influence the parameter λ. This possibility needs to be considered.

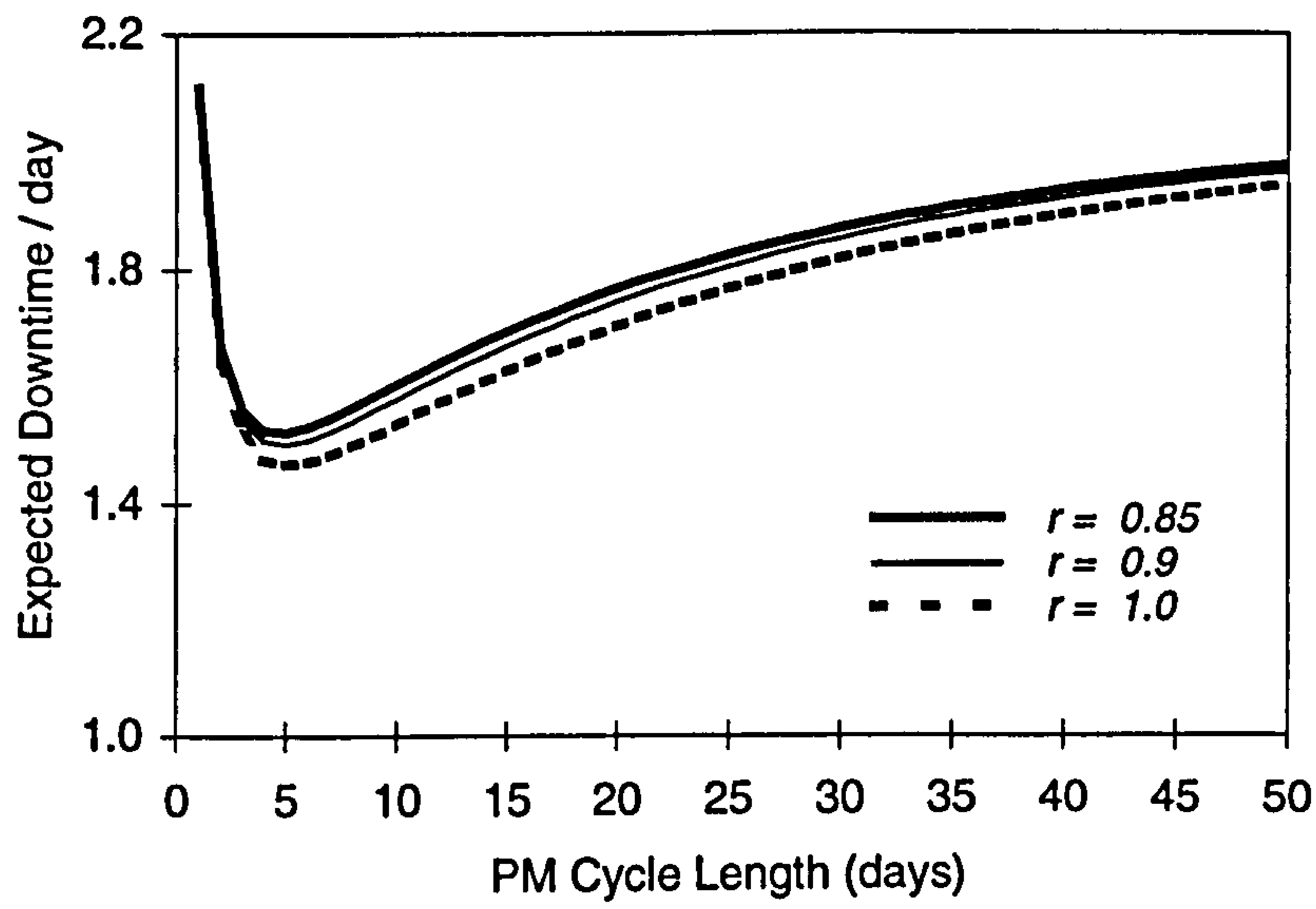


Figure 6.16. Expected downtime against PM cycle when r = 0.85, 0.9 and 1.0

6.7 The Snap-shot Survey and Analysis

In previous sections, using the objective and subjective parameter estimation methods, values of parameters have been estimated, and the results of PM models have been shown. As a consequence of the PM model, it was seen that the current PM policy is not an effective choice. Therefore we need to find an alternative way for reducing production downtime. In this study, we use the snap-shop survey technique to diagnosis the problem.

In general, to determine the causes of the problem, the analysis of the problem's causes can be at structural or functional level (Wagner, 1993). What caused a machine to jam in operation, for instance, may be explained at structural level as 'the operator was not alert with the warning light on the panel instrument'. However, at functional level where a more detail understanding of the system's behaviour is required, there may be a causal explanation, namely not enough oil in the lubrication system due to an oil leak. Consequently, depending on the level of causal analysis, different solution strategies may be generated.

In delay time modelling, a snap-shot approach has first used by Christer and Whitelaw (1983), to identify the nature of causes of fault, and whether or not engineer solutions such as re-design, or maintenance solutions such as an improved and deeper inspection, can reduce downtime. Desa (1995) also used a snap-shot modelling technique to assist the process of recognizing the nature of the actual maintenance problems of a bus company.

A similar approach, which is called 'critical analysis technique', is described by Corder (1976). Applying this technique, analyses are carried out based on plant history records to determine the plants or areas within the plants, which are critical in terms of fault or failure rate, maintenance labour, maintenance costs and or downtime. The results from such analysis define the actual problem and lead to engineering actions such as to review the inspection schedule, the quality of work, or perhaps to design-out defect through plant design changes.

Brombacher *et. al.* (1996) discuss the Maturity Index on Reliability (MIR) which reflects the capability of an organization on managing reliability. The MIR level of an organization is analyzed by analyzing the relevant reliability information flows in an orgatization. The MIR concept uses four MIR levels, which are quantification, identification, cause and improvement, to describe the (increasing) learning capability to not only detect reliability problems but also to resolve these problems for current and future products. In this paper, they present that existing, component based, analysis techniques show a very low correlation with the actual reliability as observed during the lifecycle of this product. They also note that MIR technique can analyse the deployment of reliability information in, for the lifecycle of the product relevant, business processes.

Christer and Whitelaw (1983) suggested the data or information need to be collected over a certain period of time via a specifically designed survey form. That is, (a) causes of fault: This could be attributed to operator error, poor maintenance, wear and ageing, etc. Data of this type could be used to establish the nature of the source of the problem within the plant; (b) consequences of fault: Data of this type may included the time lost or the downtime due to waiting for repair crews or collecting spares, and the repair itself, and also the cost incurred. This data could be used in identifying the factors that constitute the downtime and the cost; and (c) means of prevention: It may be possible to identify the viable means or procedures for preventing or delaying the fault or failure from recurring. Such procedures could be some form of preventive maintenance or replacement, redesigning or operator training.

From the results of the previous sections 6.5 and 6.6 we know that there is a potential scope for improvement of the regular PMs. Since the current PM practice is not appropriate for the IMA machine, it is necessary to find an alternative way for reducing production downtime. As a supplement to the previous modelling analysis, we seek the opinions of the engineers who run the plant as to what the causes of failures are and their influence upon the downtime model through its parameter values. It is of interest to know the consequences to the downtime model if some of the model parameters are changed.

On the basis of the failure mode analysis conducted in section 6.5, we ask a maintenance engineer his opinion on engineering and maintenance features of specific types of failure within IMA machines. We have four basic questions on each of the failure types in Figure 6.12. The questions are (a) failure category, for example, whether they are mechanical or electrical failure, (b) causes of failures, (c) nature of rectification of failures, and (d) means of prevention of failures.

In order to investigate the failures and downtime influence upon the production loss of IMA machine, the results from the survey are presented in graphical forms as Figures 6.17 to 6.20 for IMA machine.

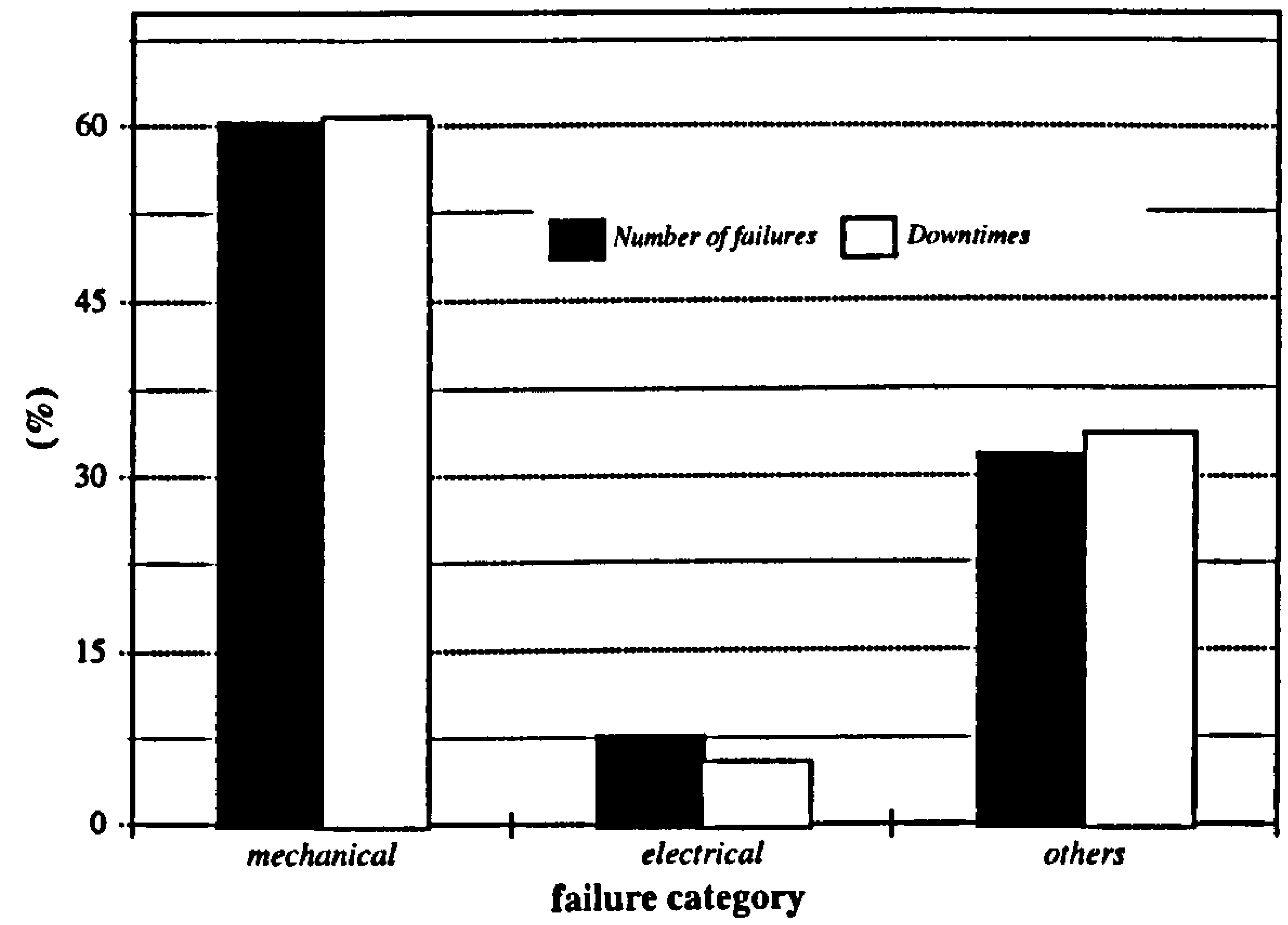


Figure 6.17. Failure category for IMA machine

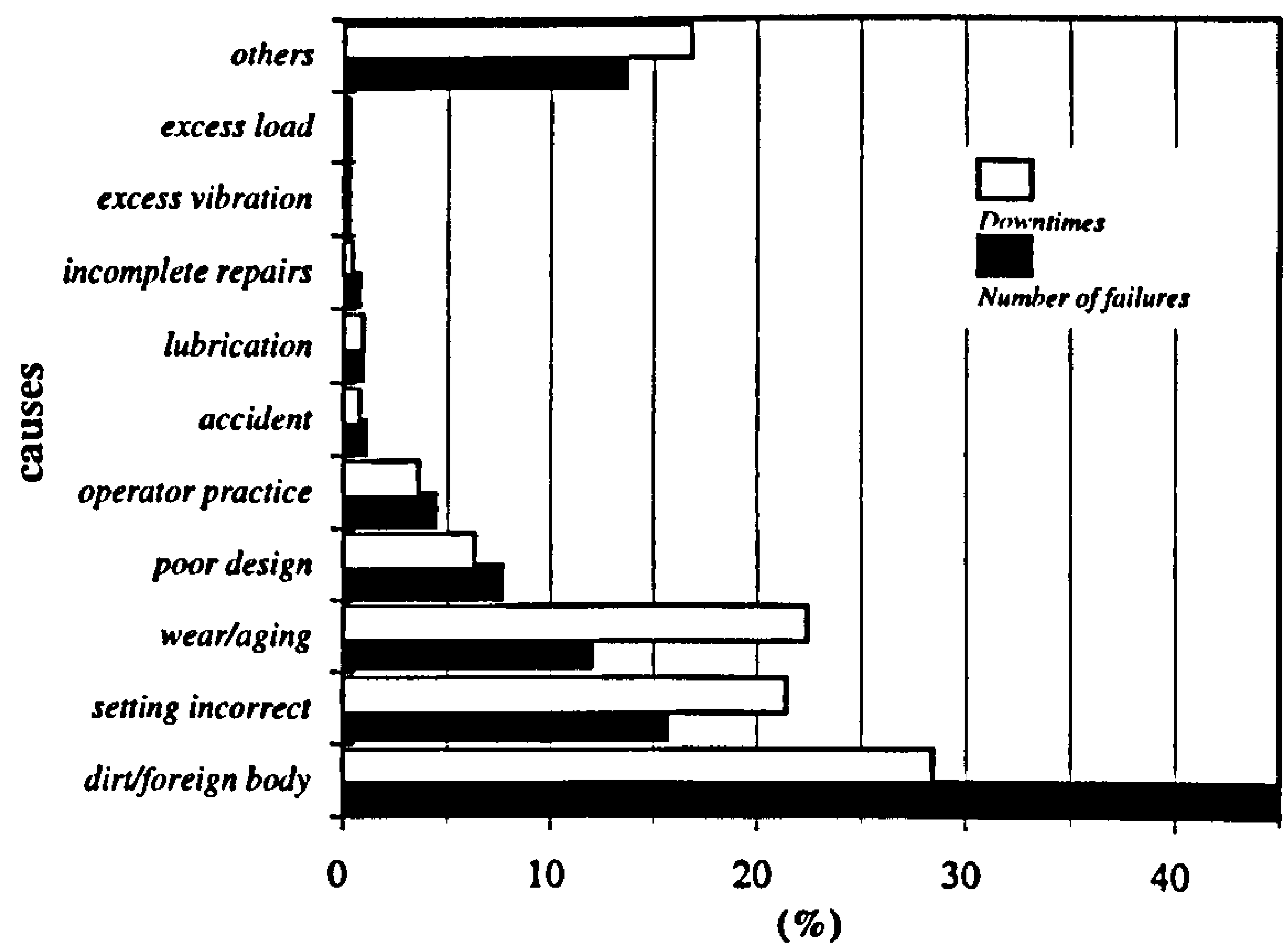


Figure 6.18. Causes of failures and downtimes for IMA machine

Based on the subjective assessments, Figure 6.17 shows an analysis of failures classified as mechanical or electrical in nature. The dark areas correspond to the analysis of number of failures and the light areas represent to the analysis of downtime for IMA machine. From Figure 6.17, we can see that about 60 % of failures or downtimes are mechanical, while electrical problems only account for less than 10%.

Also from Figure 6.18, it can be seen from the subjective assessments that dirt/foreign bodies the top cause of failures and occupy a lot of downtimes. If the objective is to reduce the number of failures, then the indications are that some form of mechanical preventive maintenance cleaning for dirty area and setting components should be considered a first measure. Although, in this case study we pay particular attention to reducing failure downtimes, it is virtually the same as reducing the number of failures.

Of the causes of failure listed in Figure 6.18, some causes, for example dirty/foreign body and setting incorrect, could have been observed prior to failure. Wearing/ageing for this machine, periodic preventive replacement of component may provide the potential for removing about 20 % of the total downtime for IMA. These major causes which are wear/ageing, setting incorrect and dirty/foreign body may consider as a non-homogeneous Poisson process for the defect arrival. And the defect arrival mechanism for each of the three/four most frequently occurring failure is unlikely to be HPP. However, the superposition of all these failure processes within a repairable complex plant may well be modelled be a HPP arrival rate.

Figure 6.19 shows that 33 % of total failure present are not fully fixed, that is 67 % of total failures are fully fixed, which implies imperfect repair exist. Figure 6.20 illustrates the potential means for the prevention of failures. It shows that 'some form of PM' and 'material' have the greatest potential to prevent downtime, namely, over 50 %. If some form of PM and material quality can be improved, the rate of fault arrival will be reduced significantly.

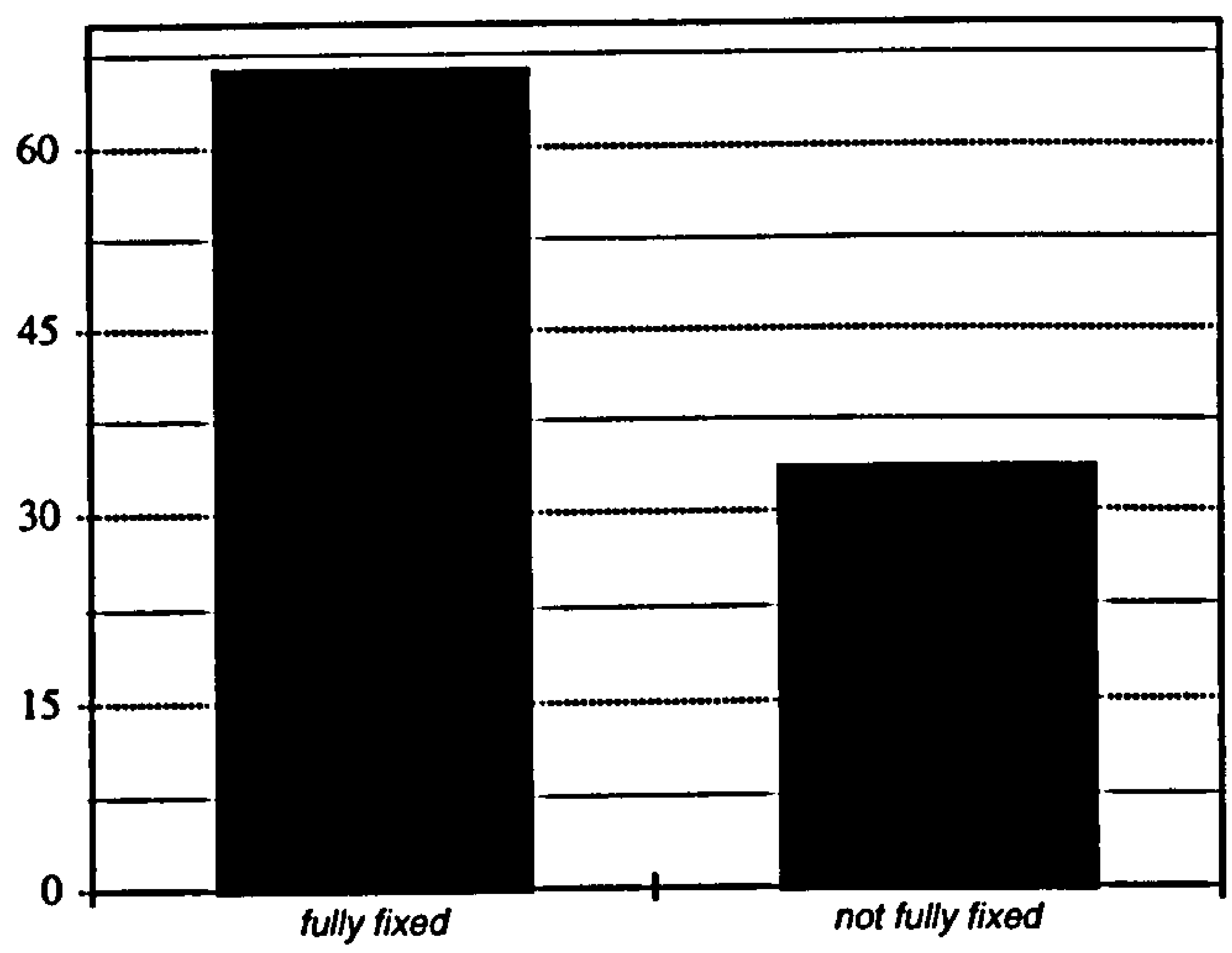


Figure 6.19. Nature of the rectification of failures for IMA machine

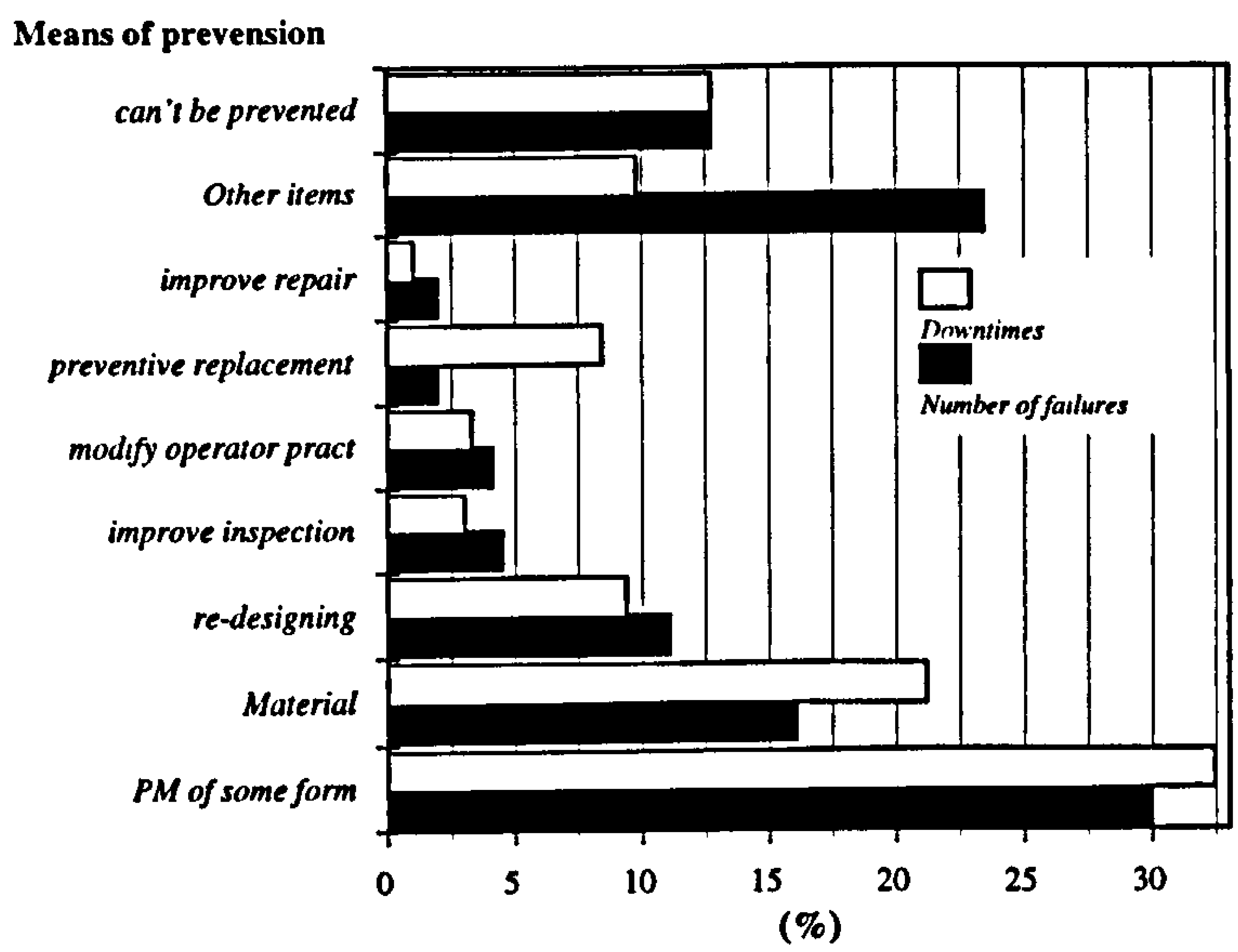


Figure 6.20. The means of prevention of failures and downtimes

Here we consider the resulting downtime per day if, where possible, failures are removed by some form of PM and by improving material quality, which would result in the order of a 53% reduction in fault arrival rate as indicated in Figure 6.20.

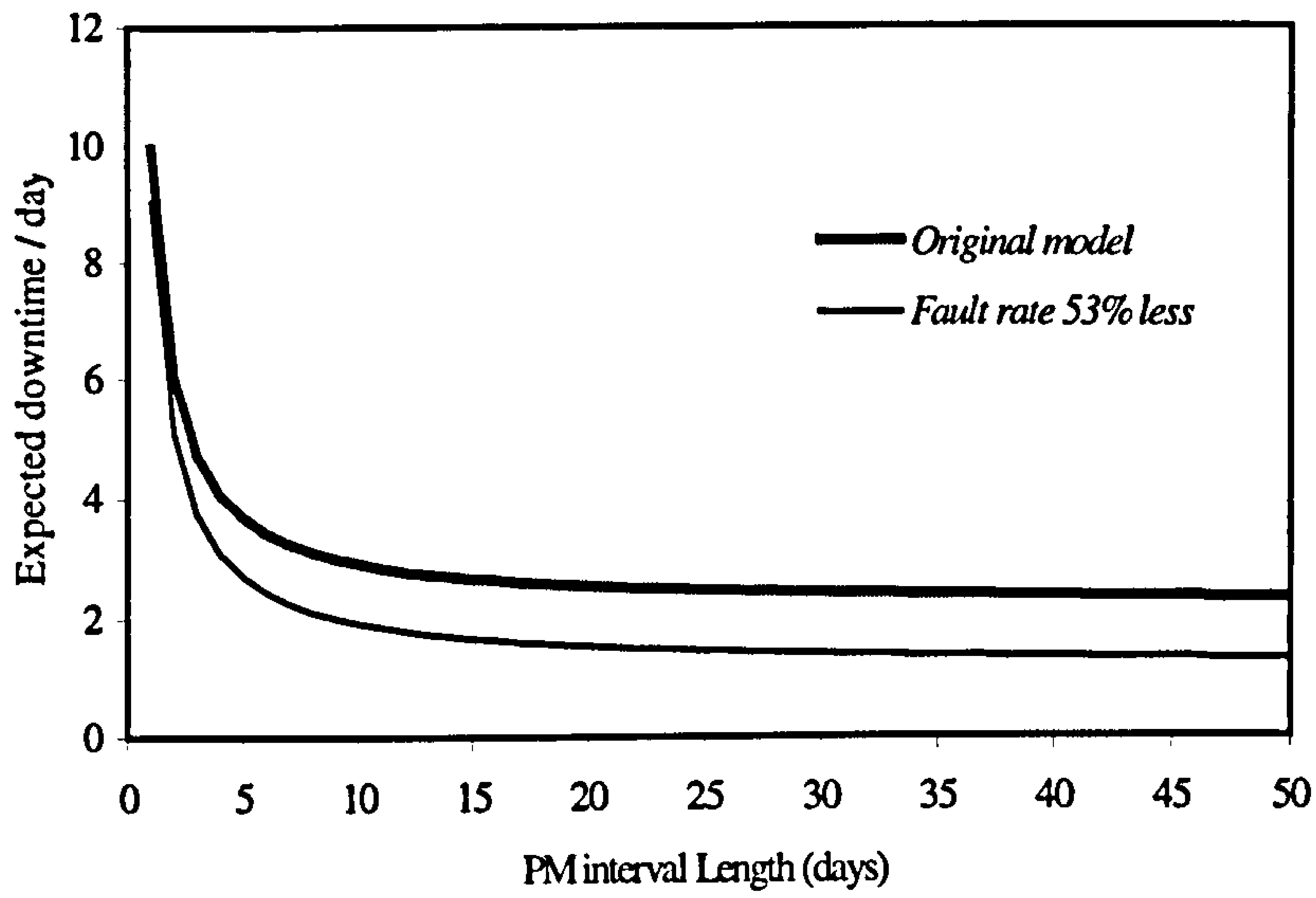


Figure 6.21. Comparison of expected downtime when the fault rate is reduced

By adjusting model parameters appropriately, the consequence of such some form of PM and improving material quality is readily calculated and is shown in Figure 6.21. Here the original model represents as before shown as Figure 6.15 when d_p = 8hour. The gain from these actions is obvious, but how to achieve it may not be straightforward. And it is noted that these exploratory investigations and resulting quantitative estimates become possible because of the existence of a maintenance model for IMA machine. Such a model is essential if improvements and developments in plant management and performance are themselves to be managed on a rational cost benefit basis.

6.8 Discussion and Conclusions

The study reported here is a pilot study for the plant which needs further refinement and extension to include all the components. However, it shows the importance of OR modelling in aid of maintenance decision making. The problem addressed here has generality since incomplete data, particularly lack of PM data, is a common problem in industry.

In this study the objective method and the subjective method of parameter estimation are used. First of all, based upon the estimated delay time distribution from objective data, a PM model has been established to give the recommended PM interval if it is applicable. It is reassuring to note how good the model fit is to current practice. This is a standard check to be expected of any modelling exercise. However, the model shows that the inspection element of PM does not contribute to the plant concerned, since the total downtime cause by failures and PM is higher than that to be expected from a breakdown only maintenance policy. Two of the reasons for this may be attributable to the fact that the mean delay time is small (about 1/2 day), and the fact that the manual PM occupies a relatively long production downtime. However, the model also shows that if the PM can identify the defects with little downtime, a finite optimal PM policy may exist. In particular, the model shows that the current probability value of a defect being identified at PM is 7 %, which is rather low, and clearly suggests that the current inspection may not be useful for the IMA machine.

Secondly, based upon the estimated delay time distribution from subjective data, a PM model has also been established to give the recommended PM interval if it is applicable. In this case, the results of the PM model are consistent with these of the objective estimation method, see Figures 6.10 and 6.14. Consequently, current PM practice for IMA machine in this plant is not effective. Therefore, it could be that the wrong types of elements are being inspected, or perhaps the wrong type of inspection technique is being applied. Since the mean delay time is short and most failures are recurrent ones, designing out weak points in troublesome components units might also be an appropriate option. Eitherway, engineering attention is required and the snap-shot model suggested by Christer and Whitelaw (1983) can be considered to highlight the true nature of problems to be addressed, and to inform and aid engineering choice. Suffice it to say, with the current maintenance practice, the indications are that the plant is being over maintained.

Thus, in this respect, we have the snap-shot survey from the company. From the snap-shot survey, it is estimated that more than 60 % of the downtime is preventable by attention to 'some form of PM', 'material' and 'redesign'. Returning to the plant after the survey to discuss progress revealed that the company had replaced the material, which is tea pack paper, to high quality paper. An engineer mentions that available production time was much increased. The other options proposed here would need to be costed to complete a cost-benefit analysis. This quality change will reduce the value of λ in the downtime model and lead to a reduced downtime measure. It will not change the conclusion that maintenance inspection is ineffective. PM inspection will only become justified for this plant if

(a) A revised inspection technique gives a larger delay time for a sufficient sub-group of defects, or

(b) The time required for PM decreases substantially or

(c) The quality of inspection increases substantially, along with elements of (a) and (b).

In an attempt to assess the practicability of the snap-shot approach for analyzing maintenance problems within the plant, it was evident that the technique can be a useful tool for problems recognition and basic discussion. The immediate impact from the problem recognition activity was that, by identifying the location, nature, and the causes of faults, some pragmatic solutions were immediately and naturally generated by the company staff themselves and implemented.

Chapter 7

Conclusion

This Chapter presents a summary of the works and results accomplished. The aim of this thesis is investigating, understanding and extending the role and consequence of different modelling options and parameter estimation options for modelling a complex plant. Maintenance optimization models provide a quantitative balancing of costs and benefits of alternative maintenance policies. The first maintenance optimization models appeared in the sixties. Since that time several papers have been published in that field.

Chapter 2 has reviewed the literature on a single-component and multi-component system maintenance models. Numerous models in the literature deal with the problem of finding optimal inspection policies for systems. Most of these models consider systems with a single component. Not as many models only have considered a multi-component system consisting of independent or dependent components. Valdez-Flores and Feldman (1989) surveyed preventive maintenance models where an optimal policy for a single-component system, or a system that can be modeled as a single entity, is being determined. They note that although a system may consist of several components, it is sometimes practical to consider the system as a single unit that behaves in such a way that individual components do not directly affect the reliability of the system. Therefore, in multi-component system, if all components in the system are economically and stochastically independent of one another, a maintenance policy for the single component models may be applied to the multi-component maintenance problem. On the other hand , if any component in the system are economically or stochastically dependent upon each other, then an optimal decision on the repair or replacement of one component is not necessarily optimal for the whole system (see Cho and Parlar, 1986). In this case a decision must be made to improve the whole system, rather than any subsystem.

To model the maintenance of multi-component system, it is necessary to develop or extend modelling. In practice many dependencies, both economic and stochastic, may exist between components of a system. The improved modelling techniques are required for looking at such system in a systematic way, using specific models for modelling the maintenance of individual components, and combining the output of such modelling in order to schedule the maintenance of the system itself.

With the models discussed above, much work has been carried out in relation to inspection maintenance using the delay time concept, and the models have found application. The challenge is how to make simple OR models available and accessible to practitioners (Scarf, 1997). The delay time theory has been reviewed in Chapter 3. This Chapter has presented a basic account of the method, and its historical evolution in which a modified basic preventive maintenance model has been successfully used in many case studies since 1982. Work to date since the genesis of the delay time model is of two kinds. The first is model developments to include factors that seem likely to be important in practice, such as imperfection of inspection, irregular timing of inspections and stochastically timed inspections. The other kind is that of fitting DTMs to data in case studies, with emphasis on parameter estimation, model validation, and post-modelling verification (Baker, 1996).

The delay time concept defines a two stage stochastic process where the first stage is the initiating phase of a defect, and the second is the stage where the defect leads to a failure. The time lapse from when a defect can be first identified at an inspection to the time that the defect causes a failure is called the delay time. Clearly from the definition of the delay time, it is a random variable which, in most cases, would not be directly measurable. Therefore, the successful use of the delay time concept in maintenance modelling depends upon how well the underlying delay time distribution can be estimated from available information sources. We have seen in Chapter 3 the previous work on the DTM, and its present status, summarised. It should be noted that the delay time concept in maintenance modelling has provided a powerful tool in modelling and validating the relationship between maintenance actions, such as preventive maintenance or inspections, and the consequence of these actions.

In Chapter 4, first we have been concerned with investigating and understanding the role and consequence of different modelling options which are based upon delay time concept for modelling a complex plant. The key options are regular PM/inspections for the system modelled as a whole, and as a set of separate component models. In the latter case, the option of a mix of different component PM/inspection schedule has been considered. By using two models, that is the system model and a collection of sub-system models, the effectiveness of maintenance scheduling for a system has been analysed. So both model concepts may apply to a system which consists of many components and has various PM policies. Therefore, as a consequence of an optimal inspection policy, a proportion of maintenance work is identified and clustered at specific points in time, so giving the maintenance organization the opportunity to allocate its resources appropriately and rectify the defects in a more efficient manner than would otherwise be the case. This is reflected in efficiency adjustments to parameters.

As expected, the result from modelling split and integrated inspection schedules may support the expectation of management that by splitting or integrating the inspection task, a better quality of inspections could be achieved. This would be mainly in terms of a reduced expected number of failures or costs due to failures. From these modelling, the best inspection policy for the system can also be identified.

Second, we have revised the downtime model to embrace the case when the downtime due to failures of system is not very small. In previous models, the tendency has been to ignore the downtime due to failures when calculating the expected number of failures over a period. Having a non-negligible downtime can change the failure process over the PM period $(0, T)$. The actual operating time over the calandar time $(0, T)$ of system is obtained, and assuming the plant can only deteriorate and fail when in use, the downtime models extended and based upon the actual operating time.

This model is used to predict the effectiveness of maintenance activity using the resultant downtime of the system as the relevant measure. Using the revising PM model it is evident that the more accurate economic PM interval may obtain for assuming that defect would not arise when the machine is idle. This implies that revised downtime model would be appropriate for the practical use in realistic models for determining downtime (or cost)-based maintenance policies.

Parameter estimation and model validation are often neglected in theoretical developments. The parameter estimation options with and without PM information has been presented in Chapter 5. As seen in Chapter 3, there are two methods for estimating the parameters of the delay time modelling, namely the subjective method and the objective method. If there are objective data available, the objective estimation method is both theoretically and practically possible to estimate the delay time distribution from objective data, that is, data from maintenance records of failures and defects found at inspections or PM (Baker and Wang, 1992,1993).

The likelihood maximization method is useful because it produces efficient estimators of the model parameters, and the method can cope with missing data (Baker and Christer, 1994). Sometimes, to formulate a model which could be adequate, the likelihood function can be used as a means of deriving parameters for a DTM for complex plant. Given a likelihood function for a DTM, one can obtain maximum likelihood estimates, $\hat{\theta}$ of model parameters, θ.

Two case methods considered have been presented which reflect experience in data availability. The main difference is data recording the number of failures per day, method A, and data recording the time of failures, method B. Parameter estimation experiments have been undertaken to test estimation methods in terms of their ability to recapture known parameters. Simulated data has been used to check the consequences of different volumes and types of data upon the accuracy of parameter estimates for maintenance models. The given maintenance record data includes the failures times, or number of failures per day, and the number of defects identified at PM. The important practical cases of with PM data and no PM data recorded, have also been studied within the investigation.

In the case of an exponential delay time distribution, both methods produce nearly identical results under perfect inspection policy even without PM information. When we assumed that delay time distribution is a Weibull under perfect inspection, the maximum likelihood estimates also recover well the underlying process of failure and defect origination of both methods. Even though in the case of no PM information being available, just failure data, maximum likelihood estimates can recover parameters as the number of PMs increase. However, when we assumed that the rate of occurrence of defects at time u is $\lambda(u)$, namely $\lambda(u) = \lambda_1 \lambda_2 u^{\lambda_2 - 1}$, and the distribution of delay time is a Weibull, $F(h) = 1 - e^{-\alpha h^{\beta}}$, maximum likelihood estimates do not recover well the underlying process of failures and defects origination of methods even under perfect inspection. It is evidently more difficult to find optimal value as the rate of occurrence of defects change and delay time distribution is a Weibull. Under imperfect inspection policy, two methods give a very similar results for with PM data. In contrast to method B, method A is require more computer running time because method A has a more integrated routine of the maximum likelihood function. However in the case that the only data available is the number of failures per working day, and PM data, we may use method A to estimate the parameter from the observed data. Also, if information is available giving exact times of failures and PM data, we choose method B.

The existence of PM data is important for most practical cases. When $r < 0.9$, the estimation process is complicated because of a correlation between estimates of r and λ. If PM data is not available, it is best to obtain a subjective estimation of either r or λ and than proceed to use the maximum likelihood estimation (MLE) to obtain the remaining estimates.

In Chapter 6 we have presented two modelling studies of preventive maintenance (PM) policy of production plant in a local company with a view to improving current practice. An objective data based model is developed based upon the delay time concept where because of an absence of PM data, the process parameters and the delay time distribution were estimated from failure data only using maximum likelihood. Confidence to do this is gained from the numerical investigation of Chapter 5.

Particular attention is paid to the problems arising during the parameter estimating process because of the in adequate recording of PM data and the implied correlation between model parameters. The case of data deficiency explored in the study is important because it is a relatively generation situation in practice. A subjective data based method is also carried out at the same company, which parallels the objective data based study. The two studies of the same problem provide a rare opportunity to compare the model formats and parameter values resulting from the two approaches and to consider the degree of consistency between the subsequent decision consequences of the two methods. They both indicate current over-maintenance. In addition, to reduce downtime further in this case study, a snap-shot type of survey technique has also been presented. From the two modelling studies of the plant, it has been realized that there is a potential scope for improvement of the regular PMs. As we expected, there are possible means to reduce downtime, and it has been shown that some form of PM and changing the material used can potentially reduce downtime by over 50%. Therefore, it has been noted that if some form of PM and the material can be improved, the rate of fault arrival may be reduced. Finally, this study demonstrates the practicability of this approach for analyzing maintenance problems within plant, and can be very useful tool for problems recognition, basic discussion, and modelling.

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