

Ternary and quadriphase sequence diffusers

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A room acoustic diffuser breaks up reflected wavefronts, and this can be achieved by presenting a spatially varying surface impedance. In hybrid surfaces, varying impedance is achieved by patches of absorption and reflection, giving reflection coefficients nominally of 0 and 1. These surfaces are hybrids, absorbing some of the incident sound while diffusing any reflected energy. A problem with planar hybrid surfaces is that specular energy is only removed by absorption. By exploiting interference, by reflecting waves out-of-phase with the specular energy, it is possible to diminish the specular energy further. This can be achieved by using a diffuser based on a ternary sequence that nominally has reflection coefficients of 0, -1 , and $+1$. Ternary sequences are therefore a way of forming hybrid absorber-diffusers that achieve better scattering performance without additional absorption. This paper discusses methods for making ternary sequence diffusers, including giving sequence generation methods. It presents prediction results based on Fourier and boundary element method models to examine the performance. While ternary diffusers have better performance than unipolar binary diffusers at most frequencies, there are frequencies at which the performances are the same. This can be overcome by forming diffusers from four-level, quadriphase sequences. © 2006 Acoustical Society of America. [DOI: 10.1121/1.2139632]

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I. INTRODUCTION

Diffusers can be used to improve the acoustics of enclosed spaces to make music more beautiful and speech more intelligible.¹ Early research in diffusers began by considering nonabsorbing surfaces, such as Schroeder diffusers.² Recent developments have concerned the development of “diffsorb-ers” of hybrid absorber-diffusers; these are surfaces where partial absorption is inherent in the design, and any reflected sound is dispersed. In hybrid surfaces, wavefront dispersal is achieved via a spatial distribution of impedances, which is achieved by patches of absorbing and reflecting material. These surfaces are hybrids somewhere between pure absorbers and nonabsorbing diffusers, usually providing sound diffusion at high frequencies, and crossing over to absorption below some cutoff frequency.

The use of absorptive patches to generate dispersion is not particularly new. In studio spaces, people have been arranging absorption in patches rather than solid blocks for many years. In recent times, however, a new breed of surface has been produced, where the absorbent patches are much smaller, and the arrangement of these patches is determined by a pseudorandom sequence to maximize the dispersion generated. For instance, the binary amplitude diffuser (BAD) panel^{3–5} is a flat hybrid surface with the location of the absorbent patches determined by a maximum length sequence (MLS). Figure 1 shows a typical construction for a device designed to scatter in a single plane.

A problem with planar hybrid absorber-diffusers is that energy can only be removed from the specular reflection by absorption. While there is diffraction caused by the impedance discontinuities between the hard and soft patches, this is not a dominant mechanism except at low frequencies. At high frequencies, when the patch becomes smaller than half the wavelength, the specular reflection is only attenuated by about 6 dB for a surface with 50% absorptive area. To improve performance, it is necessary to exploit interference and reflect waves out-of-phase with the specular energy. This can be achieved by using a diffuser based on a ternary sequence.

In the next section, a brief outline of a simple theory for sound scattering is given; this is needed to understand the diffuser design. Section III then outlines the principles behind the construction of a ternary sequence diffuser. Section IV then details various construction methods that can be used to make the diffusers and also includes some results for the scattering performance based on a simple prediction model. Repeating diffuser units concentrates energy in diffraction lobes, which can degrade scattering performance, consequently Sec. V examines how repetition can be avoided in ternary diffusers using modulation techniques. Up to this point, rather narrow and short sequences have been used to outline the general principles. In Sec. VI, longer sequences that are more useful for application are considered and more accurate prediction models are used. For many frequencies, ternary sequence diffusers have improved performance over diffusers based on unipolar binary sequences. However, at even multiples of the design frequency, the performances of the two diffuser types are the same. Consequently, Sec. VII examines how this is solved using quadriphase (four-level) sequences. Up until this point, the paper will only have dealt

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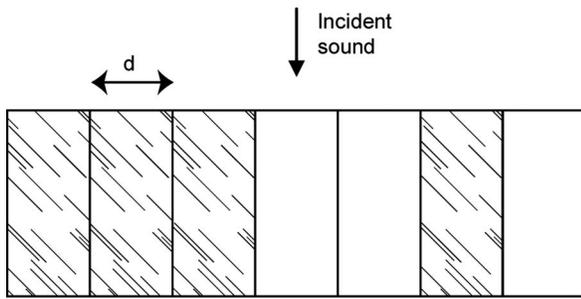


FIG. 1. A binary amplitude diffuser where the white patches are made of hard material and are reflecting, and the shaded patches are made of absorbent material and so are absorbing. Based on an $N=7$ MLS $\{1\ 1\ 1\ 0\ 0\ 1\ 0\}$.

with diffusers that scatter in a single plane, and so in Sec. VIII appropriate methods for constructing ternary sequence diffusers that scatter hemispherically are given.

II. THEORY

Consider a flat diffuser surface with a distribution of reflection coefficients R_n . The diffuser is illuminated with a plane wave normal to the surface. When the observer is in the far field, this results in a far-field pressure, $p(\theta)$ of

$$p(\theta) = \sum_{n=0}^{N-1} R_n e^{-jn\Omega}, \quad (1)$$

where $\Omega = kd \sin(\theta)$, d is the spacing of the patches (see Fig. 1), k is the wave number, and θ is the angle of reflection with respect to the normal to the surface of the diffuser. This equation is a discrete Fourier transform, which means that the far-field polar pattern is related to the reflection coefficients by a Fourier transform relationship. What is needed is to find structures with reflection coefficients that have uniform magnitude Fourier transforms, such as Schroeder diffusers.² The Wiener-Khinchin theorem states that the squared Fourier transform magnitude of a sequence is equal to the Fourier transform of its autocovariance (or autocorrelation function). Thus a sequence of reflection coefficients whose autocovariance is a Kronecker delta function will form a good diffuser, because the autospectrum will be uniform.

III. A SIMPLE TERNARY DIFFUSER

Figure 1 shows a typical hybrid absorber-diffuser. The hard and soft patches produce reflection coefficients that are nominally either 1 (hard) or 0 (absorbing). By changing the relative proportion of hard and soft patches on the surface, it is possible to control the absorption coefficient. By changing the ordering of the patches, it is possible to control how the reflected sound is distributed. If a periodic arrangement of patches is used, then the autocovariance will contain a series of peaks, and so the autospectrum will be uneven. From Eq. (1), this means that at each frequency the reflected sound will be concentrated in particular directions due to spatial aliasing; these are grating lobes. If a good pseudorandom sequence is used to choose the patch order, one with a delta-function like autocovariance—say a Barker sequence⁶—then the scattering will be more even.

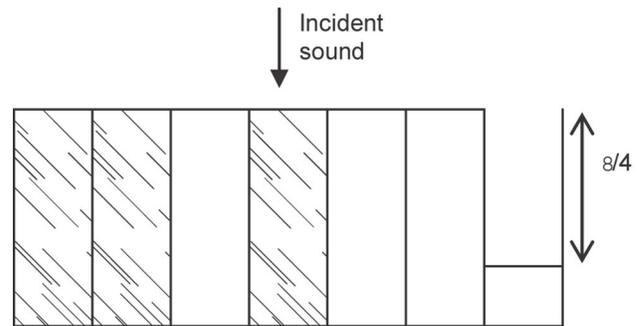


FIG. 2. A ternary diffuser based on the sequence $\{1\ 1\ 0\ 1\ 0\ 0\ -1\}$. The last well is a quarter of a wavelength deep at the design frequency to provide a reflection coefficient of -1 .

However, whatever the arrangement of the patches, at high frequency, the best that can be achieved is an attenuation of 7 dB of the specular reflection lobe, because $3/7$ of the surface forms a flat plane surface that reflects mostly unaltered by the presence of the absorbing material. For the specular reflection direction, $\theta=0$, the scattered pressure is simply a sum of the reflection coefficients R_n . So unless some of these coefficients are negative, the suppression of the specular lobe is limited. Figure 1 is an example of a “unipolar” binary diffuser because the sequence of reflection coefficients is only in one direction with respect to zero; i.e., it is composed of 0’s and 1’s. (Schroeder’s original maximum length sequence diffuser² was bipolar, because the binary sequence was composed of -1 ’s and $+1$ ’s).

Ternary diffusers offer a chance to introduce some negative reflection coefficients. An example of a ternary diffuser is shown in Fig. 2. The final well has a depth of a quarter of a wavelength at the design frequency, f_0 , and so at odd multiples of this frequency the well has a reflection coefficient, $R=-1$. Therefore, the surface reflection coefficient distribution is a sequence of -1 ’s, 0’s, and $+1$ ’s. The well produces waves out-of-phase with the sections of the diffuser producing the specular energy (the patches with $R=+1$), thus enabling better reduction of the specular energy. In this case, the suppression of the specular lobe will be up to $\approx 20 \log_{10}(7/2) \approx 11$ dB.

IV. SEQUENCES

To compare the performance of unipolar binary and ternary sequences, it is necessary to construct some diffusers, and for this sequences with the best patch order are needed. For diffusers with a small number of patches, it is possible to find the sequences with the best autocovariance by an exhaustive search of all possible combinations using a computer. The computer judges the quality of each sequence’s autocovariance using a merit factor.⁶ For the unipolar case, there can be no cancellation within the autocovariance calculation because $R=0$ or 1; in this case, the merit factor used for optical sequences is appropriate; this is the maximum value of the out-of-phase autocovariance function. For the ternary sequence, there can be cancellation when calculating the out-of-phase autocovariance values, and so the merit factor is the total out-of-phase energy.

There are many combinations of patches that are not useful because they would form diffusers that are too absorbing or too reflecting, and so these sequences are excluded from the search. (It is assumed that the $R=-1$ wells are non-absorbing, however, as shall be seen later, they can generate absorption by putting significant energy into the reactive field in conjunction with the $R=1$ patches.) In the results presented below, there were four reflecting and three absorbing elements. The sequences shown in Figs. 1 and 2 are the result of this search; however, there are many more length 7 sequences of equally good merit.

With a larger number of patches, it is not possible to construct the ternary diffuser by searching all combinations. Consequently, methods from number theory must be drawn upon. However, many of the ternary sequences that have been generated for other applications are inappropriate because they do not have the right balance of -1 , 0 , and $+1$ elements. Many sequences have very few zero elements in them and consequently the diffusers made from these sequences would not be very absorbing. This arises because most applications of number theory try to maximize the efficiency of the sequence—efficiency in this context meaning the power carried by a signal based on the sequence. In the case of hybrid diffusers, more zero terms are required in a sequence; fortunately, there is a method that can achieve this.

A. Correlation identity derived ternary sequences

Correlation identity derived ternary sequences⁷ have a nominal absorption coefficient of ≈ 0.5 provided that the design parameters are chosen correctly. They are formed from two MLSs of length $N=2^m-1$, with the constraint that the order of the sequences $m \neq 0 \pmod 4$.

First it is necessary to find a pair of MLSs with suitable cross-covariance properties. The process is to form an MLS, and then sample this sequence at a different rate to form a complementary sequence. For example, if the sample rate Δn is 2, then every second value from the original signal is taken. The sample rate is chosen using either $\Delta n=2^k+1$ or $\Delta n=2^{2k}-2^k-1$. A parameter e is defined as $e=\text{gcd}(m,k)$ where $\text{gcd}()$ is the greatest common divisor. This must be chosen so that m/e is odd as this gives the correct distribution of cross-covariance values.

Under these conditions, the two MLSs have a cross-covariance $S_{ab}(\tau)$, which has three values,

$$S_{ab}(\tau) = \begin{cases} -1 + 2^{(m+e)/2} & \text{occurs } 2^{m-e-1} + 2^{(m-e-2)/2} \text{ times} \\ -1 & \text{occurs } 2^m - 2^{(m-e)} - 1 \text{ times} \\ -1 - 2^{(m+e)/2} & \text{occurs } 2^{m-e-1} - 2^{(m-e-2)/2} \text{ times.} \end{cases} \quad (2)$$

The total number of 1's and -1 's in the sequence will be given by $\approx N(1-2^{-e})$. This is therefore the amount of reflecting surface on the diffuser, and so at high frequency, when the wavelength $\lambda < d$, it would be anticipated that the absorption coefficient of the diffuser, α , would be $\approx 1-2^{-e}$. If the aim is to achieve a diffuser with $\alpha \approx 0.5$, this means choosing $e=1$, which means the order of the MLS, m , must be odd.

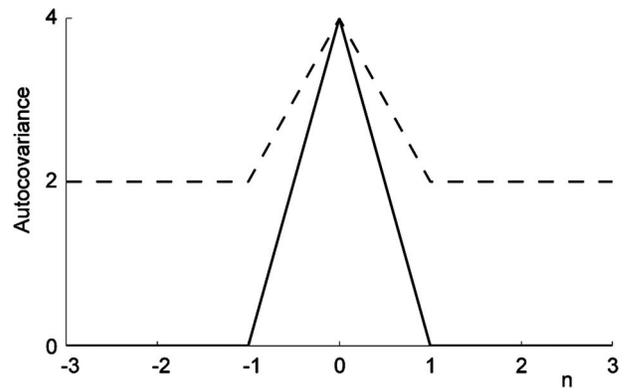


FIG. 3. Autocovariance for two sequences: - - -, unipolar binary and —, ternary.

Consider an example of $N=31=2^5-1$. e is required to be a divisor of m so that m/e is odd and this can be achieved with $k=1$ as this makes $e=\text{gcd}(k,m)=1$. A possible sample rate is $\Delta n=3$.

The first part of the first MLS used was $\{1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ \dots\}$. Taking every third value then gives the second MLS: $\{1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 1\ \dots\}$. This then gives a cross-covariance where $S_{ab}=7, -1$, or -9 (7 occurs 10 times, -1 occurs 15 times, and -9 occurs 6 times).

The ternary sequence c_n is formed from this cross-covariance, a rather surprising and remarkable construction method—the sequence is $2^{-(m+e)/2}(S_{ab}(\tau)+1)$. This sequence has an ideal autocovariance with a peak value of 2^{m-e} and out-of-phase values that are zero. Applying this to the above pair of MLSs yields the ternary sequence $\{0\ 0\ 1\ 1\ -1\ 1\ -1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ -1\ -1\ 0\ 1\ 0\ \dots\}$.

The autocovariance indicates the advantages that might be expected from ternary sequence diffusers in comparison to unipolar binary sequence diffusers. The autocovariance functions for the ternary and unipolar binary sequences are shown in Fig. 3. The binary sequence has constant out-of-phase values, but they are not zero. This leads to diffusers with a significant specular component in their polar pattern. Perfection can be achieved using a ternary sequence as the out-of-phase values are all zero.

In terms of scattering, the ternary sequence has the better reflection coefficient autospectrum because it is constant; this is shown in Fig. 4. It would be anticipated that the scattering from the ternary sequence would be more even with reflection angle if one repeat of the device was tested. For a periodic structure, that is, one in which many repeats of the diffuser are placed side by side but not an infinite number, this will result in all the grating lobes having the same energy for the ternary sequence. For the binary sequence, on the other hand, the specular lobe will have a higher level when compared to the other lobes.¹ (Note: The autospectra for MLS usually seen in the literature have a reduced $N_f=0$ value, however this is for bipolar sequences.)

Figure 5 shows the scattering from the ternary and unipolar binary diffusers alongside the scattering from a plane surface. A simple Fourier prediction model is used.¹ $d=10$ cm. Figure 5(a) is at the design frequency. As expected,

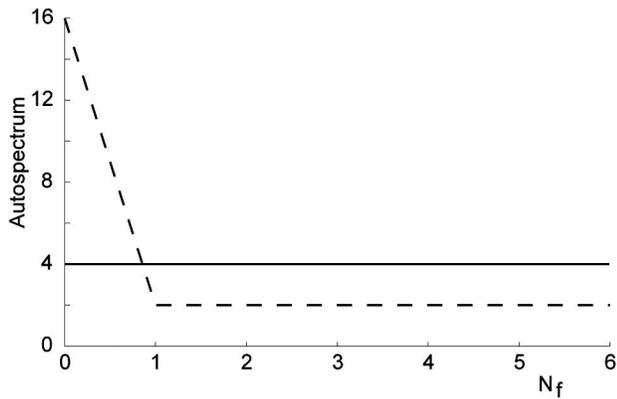


FIG. 4. Autospectrum for two sequences:---, unipolar binary and —, ternary.

the ternary diffuser has three lobes all of the same energy, whereas the specular lobe is not so well suppressed for the unipolar binary diffuser. Figure 5(b) shows the case one octave higher. At this frequency, the last well in the ternary sequence no longer has a reflection coefficient of -1 . Now the well is half a wavelength deep, and the reflection coefficient is $+1$. In fact, the sequence of reflection coefficients is now the same as for the unipolar binary sequence, and hence the two diffusers have the same polar responses. Hence, the ternary diffuser provides better scattering than the unipolar binary diffuser at odd multiples of the design frequency and the same scattering at even multiples of the design frequency. This trend continues at higher frequencies as illustrated by the plot of diffusion coefficient versus frequency in Fig. 6.

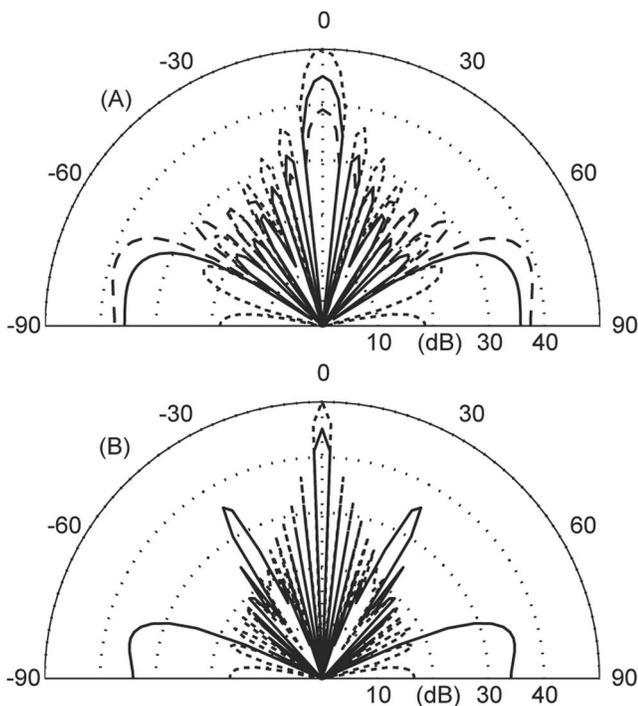


FIG. 5. The predicted scattered polar response from three surfaces: —, unipolar binary diffuser; ---, ternary diffuser; and plane surface. (a) At the design frequency and (b) twice the design frequency. In (b), the two diffuser responses are identical and overlay each other.

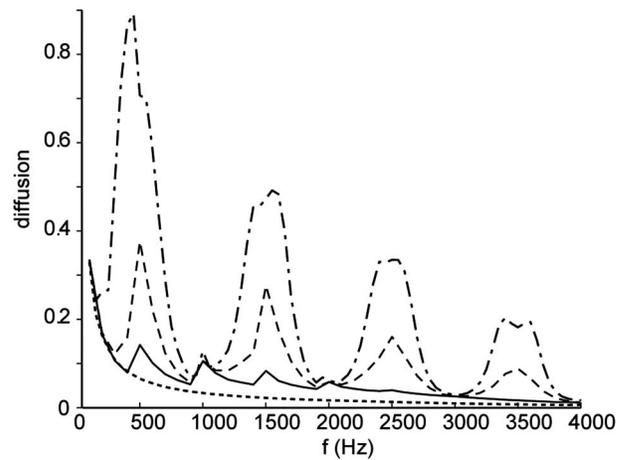


FIG. 6. Predicted diffusion coefficient as a function of frequency for four surfaces: —, periodic unipolar binary diffuser; ---, periodic ternary diffuser; -·-, modulated ternary diffuser; and plane surface.

The diffusion coefficient is evaluated using AES-4id-2001,⁸ and a higher value indicates better dispersion.

So far the performance has only been discussed at harmonics of the design frequency. Between these frequencies, the phase of the reflection coefficient offered by the well of fixed depth is neither 180° nor 0° . The waves reflected from this well will be partly out-of-phase with the waves from the parts of the diffuser with $R=+1$. Consequently, the performance is improved over the unipolar binary diffuser for these in-between frequencies, a finding confirmed by Fig. 6.

V. MODULATION AND PERIODICITY

The overall performance could be improved at many frequencies by removing the repetition of the diffusers as this would remove the defined grating lobes. This could be achieved either by using much longer sequences or by modulating two sequences.^{9,10} Using one long sequence is normally avoided because of manufacturing cost, and so the use of modulation is considered here.

For Schroeder diffusers, one method is to modulate a diffuser with its inverse.¹ Two sequences are chosen that produce the same magnitude of scattering, but with opposite phase. So if the first ternary sequence is $\{1\ 1\ 0\ 1\ 0\ 0\ -1\}$, then the complementary sequence used in modulation is the inverse of this $\{-1\ -1\ 0\ -1\ 0\ 0\ 1\}$. Given these two base diffusers, a pseudorandom sequence is used to determine the order these diffusers are used; this reduces repetition.

Figure 7 shows the scattering at the design frequency for a periodic and modulated arrangement of the ternary sequences illustrating the removal of the three lobes via modulation. Figure 6 shows the diffusion coefficient versus frequency. This shows that great improvement can be achieved but only over certain bandwidths. At even multiples of the design frequency, the two base shape reflection coefficients are identical, and so at these frequencies the structure is periodic and grating lobes reduce performance. Consequently, while inverting a sequence is good for modulating Schroeder diffusers, they are not optimal here.

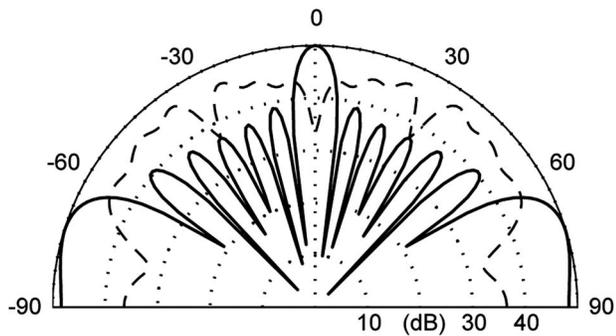


FIG. 7. The predicted scattered polar response from two ternary diffuser arrangements at their design frequency: —, periodic and ---, modulated.

Single base shape asymmetric modulation¹¹ is where a single sequence is used, but the order of the sequence is reversed between different diffusers. For example, if the first ternary sequence is {1 1 0 1 0 0-1}, then the second sequence used is {-1 0 0 1 0 1 1}. An added advantage of this method is that only one base shape needs to be made. At even multiples of the design frequency, the reflection coefficients all revert to 0 and 1, but the structure will not be completely periodic. However, it is found that periodicity is only partly removed, and that the grating lobes are still present because the two sets of reflection coefficient are very similar. Consequently, when choosing a sequence for asymmetrical modulation, it is necessary to find ones that are as asymmetric as possible at even multiples of the design frequency.

VI. BOUNDARY ELEMENT MODELING

Having established the general principles, more exacting predictions will be presented using boundary element methods (BEMs). BEMs have been shown to give accurate results for hybrid surfaces before when compared with measurements¹² and also to give accurate results for Schroeder diffusers.¹³ Consequently, it would be anticipated that the BEM will be accurate for ternary diffusers. The model used here is a 2D BEM based on the standard Helmholtz-Kirchhoff integral equation. The open well in the ternary diffuser is modeled assuming plane-wave propagation in the well, and using an element at the well entrance with the appropriate surface impedance. Previous experience¹³ indicates this is a reasonable model, but becomes less accurate for oblique incidence and reflection. For the absorptive patches, the impedance was modeled using an empirical formulation¹⁴ for mineral wool with a flow resistivity of 50000 N m^{-4} and a porosity of 0.98. The scattering was predicted in the far field at discrete frequencies. The results were converted into 1/3 octave bands by integrating the scattered energy from nine discrete frequencies within each 1/3 octave band using Simpson's rule. The source was normal to the surface.

Two diffusers were used. The first was an $N=31$ unipolar binary diffuser based on a MLS. A little over ten periods of the device were used, the patch width was 2 cm, and the total width was 6.3 m. The second diffuser was an $N=31$

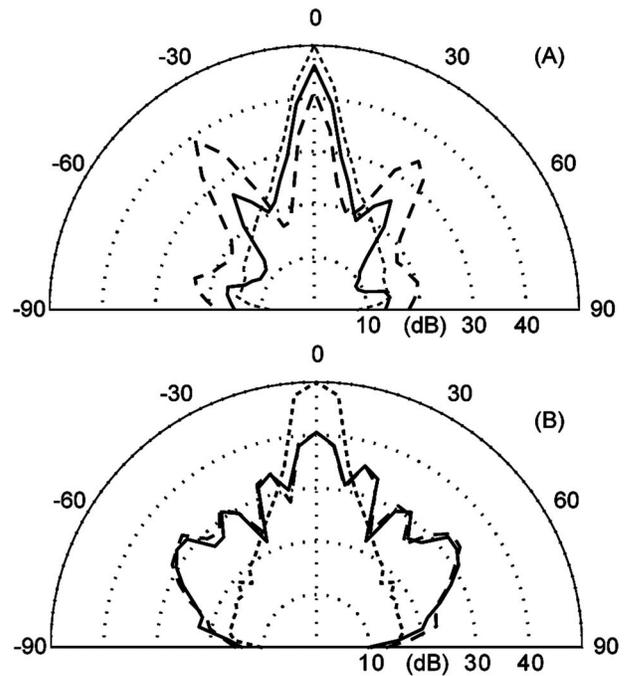


FIG. 8. Scattering from three diffusers predicted using a BEM model. —, binary diffuser; ---, ternary diffuser; and , plane surface, for the 1/3 octave whose center frequency is (a) the design frequency and (b) twice the design frequency.

ternary diffuser, with the same overall dimensions and patch size. The wells were set to be 8.5 cm deep and so the design frequency was 1 kHz.

A. Results

Figure 8(a) shows the scattering for the 1/3 octave band centered on the design frequency, Fig. 8(b) shows the scattering at an octave higher. The results confirm the simple analysis provided earlier. At even multiples of the design frequency, the scattering from the unipolar binary and ternary diffusers is similar. At odd multiples of the design frequency, the ternary diffuser offers more even scattering and a reduced specular lobe. It is also found that at frequencies that are not multiples of the design frequency, the ternary diffuser is better than the unipolar binary diffuser.

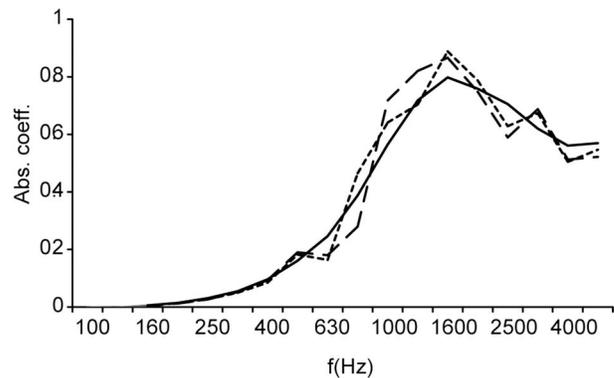


FIG. 9. Predicted absorption coefficient for three surfaces: —, binary diffuser; ---, ternary diffuser; and , quadriphase diffuser estimating using a BEM model.

Using the BEM results, it is possible to estimate the normal incidence absorption coefficients. The results in Fig. 9 are typical for hybrid absorber-diffusers. The low-frequency response is dominated by the onset of the absorption provided by the mineral wool. At high frequency the absorption coefficient is determined by the open area and is about 0.5. As the system is essentially a perforated resonant absorber, there is a peak of absorption at midfrequency. The absorption coefficient response is less smooth for the ternary diffuser. It is assumed that this is due to reflections from wells providing out-of-phase reflections when compared to other parts of the diffuser, and therefore the waves can combine to put energy into the reactive field. Overall, however, the absorption is similar for all diffuser types.

VII. QUADDIFFUSERS

A. Design

It is impossible to greatly improve the performance of the ternary diffusers at even multiples of the design frequency when the diffuser only has reflection coefficients of 0 and 1. To overcome this, more well depths need to be used. For only a few absorbent wells and many different depth wells, it would be possible to use index sequences.¹⁵ However, this would complicate the construction of the surface, and the absorption coefficient would be relatively small. Another solution would be to use active elements. It has been shown¹⁶ that with active impedance technologies it is possible to create $R=-1$ across a 3–4 octave bandwidth. However, this can only be achieved at low- to midfrequencies due to limitations of the active technologies, and, furthermore, active diffusers are very expensive. Another solution would be to bend and shape the diffuser so the front face was no longer flat, and therefore use corrugation to break up the specular reflection; this works well for binary amplitude diffusers.¹⁷

Another simple approach is to use one more well depth. Consequently, diffusers with four different reflection coefficients will be considered. At the design frequency, these coefficients should be $R=-1, 0, +1,$ and ξ . It is assumed that the last coefficient, ξ , is generated by a rigid walled well of a certain depth, and consequently $|\xi|=1$, and the well purely causes a change in the phase of the reflection. In choosing an appropriate value for ξ , it is necessary to consider not just the design frequency f_0 , but also the effects at multiples of the design frequency, because the idea behind introducing this additional wave depth is to improve performance at even multiples of the design frequency. For instance, if $\angle \xi = \pi/2$ at f_0 , then at $2f_0$, the reflection coefficient would be -1 . However, at $4f_0$, the reflection coefficient would be $+1$ and so a poor performance at this frequency would be expected. Using a depth related by relatively prime fractions, e.g., $1/2, 1/3, 1/5, 1/7,$ etc. to the $\lambda/4$ well depth, or maybe prime rationals, e.g., $1/2, 3/5, 7/11,$ etc., ensures that there are no frequencies in the audible frequency range for which all the nonabsorbing parts of the diffuser reflect in phase. Consequently, at the design frequency the $R=-1$ wells are set to a

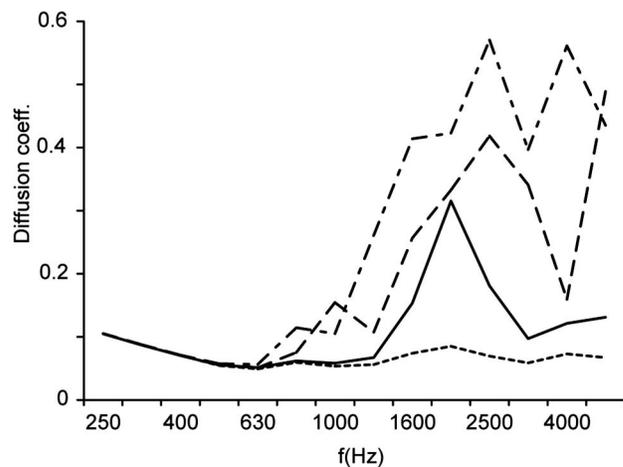


FIG. 10. Predicted diffusion coefficient for three diffusers and a plane surface: —, binary diffuser; ---, ternary diffuser; - · -, quadriphase diffuser; and ·····, plane surface.

depth of $\lambda/4$, and the $R=\xi$ wells are set to $\lambda/6$. This puts the frequency at which these two well types radiate in phase at $24f_0$.

Choosing an appropriate number sequence for this design is no longer simple. While there are quadriphase sequences in number theory, these do not normally have zero terms in them. For a 31-element diffuser, there are too many combinations to exhaustively search all combinations. Consequently, the approach used is to adapt the current ternary sequence. It is assumed that the same open area is required, and consequently the zeros in the sequence will be maintained in their current locations. Then all that remains is to determine which -1 's and 1 's in the sequence need to be changed to $\lambda/6$ wells. In the original ternary sequence, there are 16 -1 's and 1 's, and consequently it is possible to search all possible combinations to find the best arrangement. The search is for the best average merit factor for the first five harmonics of the design frequency, as these are in the frequency range (1–5 kHz) of interest here.

B. Results

Figure 10 shows the diffusion coefficient versus frequency response predicted using the BEM. The use of multiple well depths in the quadriphase diffuser produces better scattering than the other diffusers except at 1 kHz, where the ternary diffuser performs better. However, this diffusion response needs to be reviewed alongside the absorption coefficients shown in Fig. 9. Only above ≈ 2 kHz is the diffusion performance of these devices important, because in the frequency range 1–2 kHz the devices are essentially just absorbers, and below 1 kHz the surface has a decreasing effect on the sound wave because of insufficient absorption. At frequencies, such as 4 kHz, where the unipolar and ternary diffusers produce identical scattering, the quadriphase diffuser is performing better. The scattering at 4 kHz is shown in Fig. 11; the design is working as expected. The absorption coefficient (Fig. 9) is similar to that for ternary diffusers.

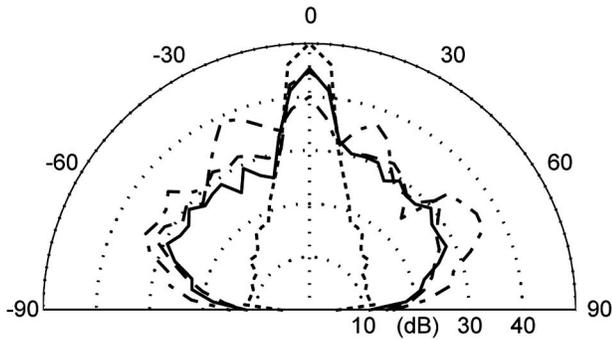


FIG. 11. Predicted scattering for three diffusers and a plane surface at four times the design frequency: —, unipolar binary diffuser; ---, ternary diffuser; -·-, quadriphase diffuser; and ·····, plane surface.

VIII. HIGHER DIMENSION ARRAYS

So far this paper has been concerned with diffusers that scatter in one plane. However, there are many applications where diffusers with hemispherical reflection patterns are required. To form hemispherical diffusers, two-dimensional ternary sequences (in number theory these would be called sequence arrays) are needed. There are a variety of methods for making multidimensional binary¹⁸ and ternary arrays.¹⁹

Consider constructing a ternary diffuser of dimensions (in terms of number of patches) of $N \times M$. There should be $\approx NM/2$ zeros in the sequence for a nominal 50% absorption. Whether a sequence can be constructed depends on the values of N and M . There are three standard construction methods: folding, modulation (Kronecker product in number theory), and periodic multiplication. Even so, there will be many array sizes that cannot be made with ideal two-dimensional autocovariance properties.

A. Folding

Schroeder²⁰ showed that a folding technique called the Chinese Remainder Theorem could be applied to phase grating diffusers based on polyphase sequences. D'Antonio⁴ used the same technique for a binary hybrid diffuser. References 1 and 5 give comprehensive descriptions of the process. The folding process wraps a 1D sequence into a 2D array while preserving the autocovariance properties. This can also be applied to ternary sequences. To use this method, N and M must be co-prime. The requirement for 50% absorptive patches means a correlation identity derived ternary sequence must be used with length $NM = 2^m - 1$, with m being odd.

The folding technique can be viewed as an indexing process. The 1D sequence, a_k , will be indexed using $k = 1, 2, 3, 4, \dots, NM$. The elements of the 2D array, $b_{p,q}$, are given by $b_{p,q} = a_k$ with $p = k \bmod N$ and $q = k \bmod M$, where (p, q) are the coordinates of the elements in the folded array.

Consider a real ternary sequence with $N=9$ and $M=7$: $a_k = \{0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, -1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 1, -1, 0, 0, 1, 0, -1, 0, 0, 0, 0, 1, -1, 0, 0, -1, 0, 0, -1, 0, 0\}$.

The folded 2D array is then

$$\begin{Bmatrix} 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix}$$

The autocovariance of this array is ideal; it is a Kronecker delta-function.

The number of sequences that can be constructed using this method with 50% absorptive patches is rather limited (only 7×73 , 23×89 , $7 \times 31 \times 151$, 217×151 , 31×1057 , or 7×4681 for $NM \leq 2^{16}$), and consequently other construction methods are needed. However, the folding process will be useful again later because it will allow the resizing of other arrays.

B. Modulation

Modulation was a process that was used to allowed diffusers to be arranged in a nonperiodic fashion by modulating one or more base shapes with a binary sequence. Another way of viewing the outcome of this process is that it forms a single longer length sequence. A very similar process can be used to form arrays using ternary and binary sequences and arrays.

1. Ternary and binary modulation

By modulating a ternary sequence with a perfect aperiodic binary array, a ternary array with ideal autocovariance properties can be obtained. (Note, it is important to modulate the array by the sequence and not vice versa.) Consider a length 7 correlation identity derived ternary sequence $a = \{1, 1, 0, 1, 0, 0, -1\}$. This is used to modulate the perfect aperiodic binary array, $b: b = \begin{Bmatrix} -1 & -1 \\ -1 & 1 \end{Bmatrix}$ to form a 2×14 length array, c , given by

$$c = \begin{Bmatrix} -1 & -1 & -1 & -1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 1 & 1 \\ -1 & 1 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \end{Bmatrix}$$

This array has ideal periodic autocovariance properties. As the binary array has no zeros, the modulated array has the same proportion of absorptive patches as the original ternary sequence, 40% in this case. For long sequences, the proportion tends toward 50%.

An issue that is not often discussed in the number theory literature is the imbalance between the distribution of -1 and 1 's in a sequence. This is important to the diffusers because the proportion of -1 and 1 's changes the amount of attenuation of the specular reflection. In this case, the modulation has produced an array with a more even balance of -1 and 1 's than the original ternary sequence, and consequently it would be expected to perform better at attenuating the specular reflection.

There is only one known perfect aperiodic binary sequence, the one shown above. Consequently, 2×14 , 2×62 , 2×254 , 14×146 , 46×178 , and 2×16382 are the array sizes that can be constructed by this method with $\approx 50\%$ efficiency for $NM \leq 2^{16}$; again the allowable array sizes are rather few. Furthermore, as the resulting array sizes have N and M , which are not co-prime, it is not possible to re-fold these arrays to get other sizes.

2. Ternary and ternary modulation

The efficiency (proportion of zeros) of the derived array by modulation is a product of the efficiency of the original array and sequence. Consequently, it is possible to modulate a ternary array by a ternary sequence, provided the product of their efficiencies is about 50%. Two aperiodic perfect ternary arrays with 67% zeros are

$$d_1 = \begin{Bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \end{Bmatrix}, \quad d_2 = \begin{Bmatrix} 1 & 1 \\ 0 & 0 \\ 1 & -1 \end{Bmatrix}. \quad (3)$$

Consequently, if either of these is combined with a perfect periodic ternary sequence with 75% zeros, the overall design goal of an array with 50% zeros is achieved.

In this case, the correlation identity derived ternary sequences are not useful because they have too low an efficiency. On the other hand, some Ipatov ternary sequences and those based on the Singer difference sets are appropriate.^{21,22} If the efficiency goal is set to be between 45% and 55%, then there are four Ipatov ternary sequences that can be used of length 13, 121, 31, and 781. While these achieve an efficiency of about 50%, respectively, there is an imbalance between the number of +1 and -1 in the sequence, leading to somewhat less than optimal specular reflection absorption, so these are not considered further.

By combining two binary sequences based on Singer difference sets, it is possible to form a ternary sequence with the desired efficiency and a better balance of -1's and 1's. Difference sets are used to form sequences with the best possible autocovariance. The Singer difference set is a particular class of difference sets, and has the following parameters:

$$(N, k, \gamma) = \left(\frac{q^{2r+1} - 1}{q - 1}, \frac{q^{2r} - 1}{q - 1}, \frac{q^{2r-1} - 1}{q - 1} \right), \quad (4)$$

where N is the length of the sequence, k the number of 1's in the two binary sequences formed, and γ the maximum out-of-phase autocovariance of the two binary sequences. q and r are constants and are specified below. The efficiency of the ternary sequence formed by combining the binary sequences is approximately given by $(q-1)/q$. Since the requirement is to find a sequence with 75% efficiency, $q=4$ is used. This meets a necessary requirement that $q=2^s$, where s is an integer.

The possible sequences are $N=21, 341, 5461, \dots$, which are the cases for $r=1, 2$, and 3. Consider the case of $N=21$, for example. The two Singer difference sets for this case are $D1=\{3, 6, 7, 12, 14\}$ and $D2=\{7, 9, 14, 15, 18\}$, where the numbers in the brackets indicate the locations of the 1's in

the sequence. The unipolar binary sequence for $D1$ is $a = \{0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0\}$ and for $D2$ is $b = \{0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0\}$.

To form the ternary sequence, the cross-correlation between these two sequences is found: $s_{ab} = \{2, 0, 0, 1, 0, 2, 1, 1, 0, 2, 2, 0, 1, 2, 1, 2, 0, 2, 2, 2, 2\}$. The final sequence, c , is then given by

$$c = q^{1-r} \left(s_{ab} - \frac{q^{2r-1} - 1}{q - 1} \right), \quad (5)$$

which in this case yields $c = \{1, -1, -1, 0, -1, 1, 0, 0, -1, 1, 1, -1, 0, 1, 0, 1, -1, 1, 1, 1, 1\}$. This sequence has ideal autocovariance properties.

Having obtained the necessary ternary sequence, it is now possible to form the array. The sequence c is modulated with the first perfect aperiodic ternary array d_1 shown in Eq. (3) to form an array that has size 63×2 and has ideal autocovariance properties with a maximum value of 64. Hence, the absorption coefficient at high frequency in this case is nominally 0.51. The array has 28 values at -1 and 36 values at +1, and so there is good attenuation of the specular reflection at odd multiples of the design frequency.

Array sizes that are square will be more useful, because if the 63×2 diffuser is used periodically, the small repeat distance in one direction will reduce performance. By applying the Chinese Remainder Theorem in reverse, it is possible to unfold this array into a 126×1 sequence, and then re-fold it into two other array sizes 18×7 and 14×9 , which are squarer.

3. Periodic multiplication

The final design process is to use periodic multiplication. Two arrays can be multiplied together to form a larger array. Consider array 1 to be $b_{p,q}$ of size $N_b \times M_b$, which has an efficiency of E_b , and array 2 to be $c_{p,q}$ of size $N_c \times M_c$, which has an efficiency of E_c . Then the new array is a product of the periodically arranged arrays, $b_{p,q} \cdot c_{p,q}$ of size $N_b N_c \times M_b M_c$, and the efficiency will be $E_b E_c$. A necessary condition is that N_b and N_c are co-prime, and so are M_b and M_c , otherwise the repeat distance for the final arrays will be the least common multiples of N_b and N_c in one direction and M_b and M_c in the other.

For example, the ternary sequence derived previously from Singer difference sets, c , can be folded into an array that is 7×3 ,

$$\begin{matrix} \{1, & 0, & 1 \\ 0, & 1, & 0 \\ -1, & 1, & -1 \\ -1, & 1, & -1 \\ 1, & 0, & 1 \\ -1, & 1, & -1 \\ 1, & 0, & 1 \} \end{matrix}$$

which has an efficiency of 76%. This can then be multiplied by the ternary array d_2 , which has an efficiency of 67% to form a 21×6 array with ideal autocovariance properties and an efficiency of 51%. There is a slight imbalance between

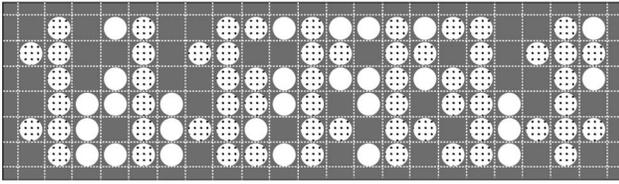


FIG. 12. A visualization of a 21×6 ternary array mask. The white holes open up to nonabsorbing wells ($R=-1$), the shaded holes open up to mineral wool ($R=0$), and the rest of the mask is reflecting ($R=1$).

the number of -1 and 1 's with 28 and 36, respectively, of each. Figure 12 shows a visualization of a mask for this sequence.

This multiplication process can involve a binary array multiplied by a ternary array, or two ternary arrays multiplied together. Except for the perfect 2×2 binary array, perfect binary arrays will have an imbalance between the number of $+1$ and -1 terms, which could lead to an imbalance in the final array design and insufficient attenuation of the specular reflection lobe. In general, perfect binary arrays of size $N \times M$ have $NM \bmod 4=0$ and $NM=(2k)^2$, where k is an integer, and they have an imbalance of \sqrt{NM} .

C. Array discussions

Once the array is formed, any periodic section can be chosen and many other manipulations can be done and still preserve the good autocovariance. Procedures that can be done on their own or in combination include the following.

- (i) Using a cyclic shift to move the pattern around. $c_{p,q} = b_{p+u, q+v}$, where u and v are integers and the indexes $p+u$ and $q+v$ are taken modulo N and M , respectively.
- (ii) Mirror image the array $c_{p,q} = b_{\pm p, \pm q}$.
- (iii) Invert the sequence $c_{p,q} = -b_{p,q}$.
- (iv) Rotation by 90° $c_{p,q} = b_{q,p}$.
- (v) Under sample the array, $c_{p,q} = b_{up, vq}$, provided both u, N and v, M are co-prime.

These will not change the acoustic performance, but may change the visual aesthetic. It can also help to make the array more asymmetric, which can be useful in modulation.

The main problem in forming these arrays is that there is only a limited set of arrays, which provide ideal autocovariance properties, the required efficiency to give the desired absorption coefficient, and a reasonable balance between the number of -1 's and 1 's in the sequence leading to good suppression of the specular lobe. In work on binary sequences,²³ it has been shown that relaxing the requirement for ideal autocovariance enables more different length sequences to be formed. This should also be possible for the ternary sequence case. For example, where there are a large number of elements in a sequence, it may be possible to truncate the sequence, losing one or two elements, and still gain good (but not ideal) autocovariance properties. This type of truncation might then give the right sequence length for folding into an array with the desired size.

IX. CONCLUSIONS

The performance of a new sort of room acoustic diffuser that is based on ternary sequences has been discussed. The

rationale and performance of these have been analyzed using simple prediction models and concepts, as well as more accurate boundary element models. By adding wells to hybrid diffuser surfaces, thus forming ternary diffusers, a useful improvement in performance is achieved for a modest depth penalty. While this produces an improved performance for odd multiples of the design frequency, at even multiples the performance of the ternary structures is the same as for the hybrid diffuser surfaces made from unipolar binary sequences. Using two different well depths reduces this problem and allows more broadband improvements in scattering. A method for obtaining the design sequence for these four-level (quadrphase) diffusers is discussed. Design methods for transforming the sequences into arrays and thus producing hemispherical diffusers have been outlined. While there are a number of methods available for construction, the number of different arrays available with ideal autocovariance properties is rather small. Future work will undertake measurements to confirm the performance of the surfaces, and examining methods of construction so the surfaces can be easily and cheaply made.

- ¹T. J. Cox and P. D'Antonio, *Acoustic Absorbers and Diffusers: Theory, Design and Application* (Spon Press, 2004).
- ²M. R. Schroeder, "Diffuse sound reflection by maximum-length sequences," *J. Acoust. Soc. Am.* **57**, 149–150 (1975).
- ³J. A. S. Angus, "Sound diffusers using reactive absorption grating," Proceedings of the 98th Convention of the Audio Engineering Society, paper no. 3953 (1995).
- ⁴P. D'Antonio, "Planar binary amplitude diffusor," U.S. Pat. No. 5,817,992 (1998).
- ⁵J. A. S. Angus and P. D'Antonio, "Two dimensional binary amplitude diffusers," Proceedings of the Audio Engineering Society, paper no. 5061 (D-5) (1999).
- ⁶P. Fan and M. Darnell, *Sequence Design for Communication Applications* (John Wiley and Sons, Inc., 1996).
- ⁷P. Fan and M. Darnell, *Sequence Design for Communication Applications* (John Wiley and Sons, Inc., 1996), pp. 180–183.
- ⁸AES-4id-2001, "AES Information document for room acoustics and sound reinforcement systems—Characterisation and measurement of surface scattering uniformity," *J. Audio Eng. Soc.* **49**, 149–165 (2001).
- ⁹J. A. S. Angus, "Using grating modulation to achieve wideband large area diffusers," *Appl. Acoust.* **60**, 143–165 (2000).
- ¹⁰J. A. S. Angus and C. I. McManmon, "Orthogonal sequence modulated phase reflection gratings for wide-band diffusion," *J. Audio Eng. Soc.* **46**, 1109–1118 (1998).
- ¹¹P. D'Antonio and T. J. Cox, "Aperiodic tiling of diffusers using a single asymmetric base shape," Proceedings of the 18th ICA, paper Mo2.B2.3 (2004).
- ¹²L. Xiao, T. J. Cox, and M. R. Avis, "Active diffusers: Some prototypes and 2D measurements," *J. Sound Vib.* **285**, 321–339 (2005).
- ¹³T. J. Cox and Y. W. Lam, "Prediction and evaluation of the scattering from quadratic residue diffusers," *J. Acoust. Soc. Am.* **95**, 297–305 (1994).
- ¹⁴M. E. Delaney and E. N. Bazley, "Acoustical properties of fibrous absorbent materials," *Appl. Acoust.* **3**, 105–116 (1970).
- ¹⁵M. R. Schroeder, "Phase gratings with suppressed specular reflections," *Acustica* **81**, 364–369 (1995).
- ¹⁶L. Xiao, T. J. Cox, and M. R. Avis, "Maximum length sequence and Bessel diffusers using active technologies," *J. Sound Vib.* **289**, 807–829 (2006).
- ¹⁷T. J. Cox and P. D'Antonio, "Optimized planar and curved diffusers," Proceedings of the 107th Convention of the Audio Engineering Society, paper no. 5062, New York (1999).
- ¹⁸T. J. Cox and P. D'Antonio, "Optimized planar and curved diffusers," Proceedings of the 107th Convention of the Audio Engineering Society, paper no. 5062, New York (1999), pp. 297–310.
- ¹⁹M. F. M. Antweiler, L. Bomer, and H. D. Luke, "Perfect ternary arrays," *IEEE Trans. Inf. Theory* **36**, 696–705 (1990).
- ²⁰M. R. Schroeder, *Number theory in science and communication: With*

applications in cryptography, physics, digital information, computing, and self-similarity (Springer, 1997).

²¹M. R. Schroeder, *Number theory in science and communication: With applications in cryptography, physics, digital information, computing, and self-similarity* (Springer, 1997), pp. 178–183.

²²V. P. Ipatov, “Ternary sequences with ideal autocorrelation properties,” *Radio Eng. Electron. Phys.* **24**, 75–79 (1979).

²³H. D. Luke, H. D. Schotten, and H. Hadinejad-Mahram, “Binary and quadriphase sequences with optimal autocorrelation properties: A survey,” *IEEE Trans. Inf. Theory* **49**, 3271–3282 (2003).