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Hybrid Algorithm for the Classification of Fractal Designs and Images

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Abstract

The fractal patterns are recursive patterns and are self similar in nature. The fractal geometry provides better understanding of natural patterns as compared to the euclidean geometry. The fractal designs have been used extensively in the fields of applied sciences due to the systematic methods used for their generation. These methods provide benchmarks to analyze the roughness, narrow/ broad vision of the objects. In the fields of architecture and design, computational methods for fractal generation can prove to be more reliable tools. The fractal patterns can be simulated and the architecture can be modeled, with several options, before implementing it practically. During this research, this strategy is opted to design novel fractal tile designs. Several designs are selected from a series of simulations, based on the final visionary evaluation, according to the requirement of the walls of different zones in modern buildings. Four fractal patterns are simulated with several orientations and final designs are documented with corresponding geometrical evaluation.

Keywords: fractals; Tile design; Geometry processing; computational framework; Simulations.

1. Introduction

The mathematical graphic designs and the resulting art has improved the decorating patterns in the fields of architecture and building designs. fractal patterns are used extensively to design the tiles and walls. Art and mathematics have a close connection. Artists can use mathematical models to create art. The resulting aesthetic factors play an important role in the study of mathematics; thus, in both ways, mathematics is regarded as an art [1, 2, 3].

The popularity of mathematical art graphics has helped to enhance the vitality of the tile patterns, in order to decorate the floors, roofs and walls. As a result, the novel designs for the tiles printing have attracted attention of the researchers and the improved variants can help to boost the impression of the walls and thus the buildings. Architecture designs are themselves symbols of the success of mathematics, such as the Fibonacci number and the golden ratios were used by the Greeks in architecture and were prevalent in Renaissance architecture. The applied mathematical programming is thus helpful in almost all domains of science and technology [4, 5, 6, 7, 8]. With the aid of mathematical models, programming and simulations, the designs can be improved in novel ways.

The advancement of modern computer technology helped the tile pattern designers to provide new resources for the tile patterns, decorative tile-work, and tile art.

There are numerous types of tile patterns that are made up of a large number of tiny irregularly positioned, neutral components. Such components are usually made up of glass but sometimes of ceramic or stone [9]. These patterns contribute to an architecture's finishing, worth, beauty and elegance. In history, fractal patterns have been used to design buildings [10, 11]. With the passage of time, this field of research evolved by the virtue of modern programming tools [12, 13]. fractal geometries in the history of art have served as a language that has proven its worth through its applications. For this reason, during this research a series of fractal patterns is simulated using novel programming tools.

The history of fractal generation and its mathematical programming dates back since the pioneering work of Mandelbrot, the famous American French mathematician. During his research work on fractal geometry, he identified it as a subject that uses regular or irregular geometry as the research object, in which nature is viewed as a fine structure with infinite nesting levels and some self-similar properties at different scales [14]. Therefore, the naturally occurring objects, such as trees, coastlines, and clouds have fractal properties, and much of the prevailing interest in the subject stems from attempts to recreate such natural phenomena using visual effects. The availability of modern computer graphics techniques have provided new insight into the design of such patterns. Users can generate fractals mathematically that can be reproduced at any magnification or reduction, and the representation of each part looks exactly like the original, or at least has similar patterns [15].

fractal tiling with self-similarity can generate more dynamic patterns for the buildings and have a strong visual appearance. However, the fractal tilings that have been discovered so far are very limited in number because creating such fractal tilings requires knowledge of resulting percentages of colors used, image magnifications, geometrical aspects, special talent and skill. This paper presents a general method for generating tile designs based on the idea of hierarchically subdividing adjacent tiles [16].

Design patterns and tile designs are both the outcome of a systematic repetition of a unique geometrical form in modern times. It is expected from the designers to build efficient and dynamic production facilities in order to get the best potential results in terms of profitability, throughput, and lead time due to international competition. It is imperative to study the performance measurements in the field of tile production designs in order to improve the efficiency and to optimize the processes. Simulating the performance of a real system without making any changes to the real system is one approach for studying its performance. It is extremely likely that the behavior of the real system can be observed and understood via simulation, since the simulations are cost effective and can help to forecast the resulting patterns. The recurring geometrical form in a design pattern has no limits because the result is a series of independent geometrical forms that are more or less close to each other. When it comes to tile designs, the repetitive form necessitates a specific shape in order to minimize gaps or overlapping. Any figure, picture, or drawing utilized as a unit motif in a pattern design is referred to as a "geometrical form" [17].

During this research, the numerical model of the tiling design is simulated using MATLAB^{TM} software, the main goal of simulation in tiling pattern or tiling design is to facilitate decision-making so that desired goals can be met quickly. To enhance the system, simulation tools can produce better results that indicate a significant gain in productivity, cycle time, and throughput time. In the literature, evidence is available, where the numerical simulations have been built to analyze the behavior of complex patterns and to simplify the nonlinear mathematical models. The numerical simulations help the reader to better interpret and utilize the research findings [18, 19, 20, 21].

In this manuscript, we have provided the step by step description of the method with numerical algorithms and innovative tile structures. The statistical analysis of the resulting images and patterns is conducted with the aid of machine learning regression and classification tools. In the next section, the methods are illustrated with the flow chart. Next useful results and innovative conclusions are drawn.

2. Materials and Methods

2.1. Materials

In 1975, the mathematician Benoit Mandelbrot coined the term "fractal" from the Latin word fractus, which means irregular or fragmented, to describe objects that were too irregular to fit into a traditional geometrical setting. fractal is a new type of subject that has emerged in the last 40 years. The mathematical concept of fractal is difficult to classify formally, even for mathematicians, but key features can be understood with a little mathematical background. These erratic and fragmented shapes can be found almost in every architecture, designed with innovative approaches. fractals, at their most basic, are a graphic illustration of a repeating pattern or formula that begins simple and becomes increasingly complex. fractals are broadly classified into two types: random fractals (or non-deterministic) and deterministic fractals.

The random fractals basically lack the self-similarity, these are not similar to their systematic counterparts, their non-uniform appearance is often more akin to natural phenomena and is the preferred method when modeling coastlines, topographical surfaces, or cloud boundaries, is the subject. However, random fractal implies that the rules for generating fractal graphics remain unchanged. So, while the graphics produced by two operations at different times may have the same fractal dimension, the final generated graphics will differ due to the influence of random factors, inferring that random fractals are not repeatable.

Among the fractal definitions available in the literature, the approach that is used is the Falconer's, which, rather than a computationally efficient definition, provides a concise indication in the form of a list of properties. A fractal, G is taken to be a collection of objects that exhibit many of the following characteristics:

- The structure must be fine and recursive.
- The traditional geometric language is not sufficient to define G.
- There is self-similarity (statistical) in G.
- Dimension of G is greater than its topological dimension. recursively.

2.2. Methods

2.2.1. *fractal Patterns Algorithms*

The fractal patterns can be generated using analytic models as well as by virtue of numerical simulations, using simple and parallel pool algorithms [22]. Different softwares provide different tools to generate fractals.

Wang and Zuo [23] generated grid map from the sampling data, converted map from space to Fourier domain, with the aid of fast Fourier transformation and obtained the power spectrum. Their algorithm then followed the process of segmentation and filtration. Finally inverse Fourier approach was utilized to generate the final results.

Similarly, Lu et'al. [24] discussed in detail the algorithms available for fractal generation for the architecture design with the aid of smart programming tools. Several other studies were reported during the past decade to support the importance of algorithms and their smart design in generating fractals [1].

One of the leading softwares, Matlab *TM* provides the users very swift algorithms for fractal generation. These include fractal-Toolkit, Hausdorff-fractal-Dimension tool, Minkowski's loop fractal toolbox, Koch-curve-fractal tool box, 2D-Lyapunov fractal, Icosahedral-fractal, Icosahedral-fractal and many more. An important thing to keep in view, before utilizing these fractal tool boxes is that most of these algorithms require good processors and parallel processing.

In the next section we will outline the properties of fractal patterns and the algorithmic approach, used during this research, to generate tile patterns. The algorithm designed during this research is comparatively cost effective than the other algorithms outlined above and is thus user friendly.

2.2.2. *fractal Patterns Application*

Inspired from the available algorithms for fractal generation [25], during this research, a numerical algorithm is used that is looped to generate four different patterns, in 16 different orientations. These patterns are defined as the Julia-fractal, Mandelbrot-fractal, Newton-fractals and Nova-fractal respectively.

The four approaches that we have used while designing the algorithm, can be better interpreted with the aid of the following schematic 1. The fractal graphic design program starts with the master node, where the master node transmits work to the next node as shown in the frame diagram. The key node will then receive the row number and start working on the classification and derivation of fractal art graphics, it will then start further processing on the measurement scale range of fractal art graphics.

To proceed, the algorithm will require the input for the expected designs based on the requirement of the architect, and the algorithm will select the preferable technique accordingly. To begin the process, we choose one of the four fractal designs from our four options. The node passes the task to tile engineering design, to process pattern coloring and display the fractal art when the measurement scale of fractal art graphics is finally simulated. If the display design does not meet the expected outcomes, the process will mutate and restart at the master node. However, if the outcome is satisfactory, the algorithm will proceed to interpret and transmit the image. At that point, the algorithm will stop and exit the program.

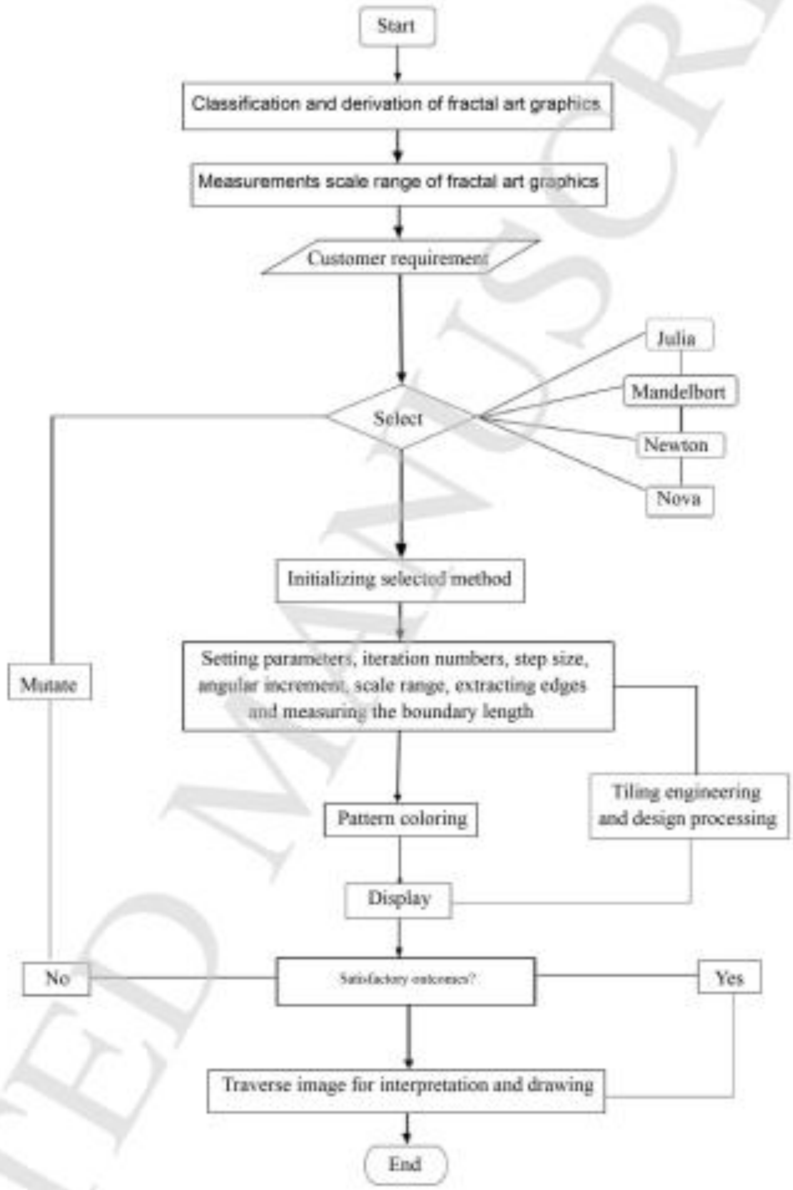


Figure 1: The classification framework of innovative fractal art graphics of tiles printing, based on the needs of the consumer.

3. Results and Discussion

In this section, we will discuss the numerical results generated with the aid of four approaches.

3.1. Julia fractals Graphic

The Julia set and the Fatou set are two complementary sets (Julia "laces" and Fatou "dusts") defined from a function in the context of complex dynamics.

The Julia set of a function f is commonly denoted **J(f)**, and the Fatou set is denoted by **F(f)**. Julia sets are a great example of how a simple process can result in wonderfully intricate sets. Julia sets are born from the iteration of functions in the complex plane and are named after the French mathematician Gaston Julia. They are based in the field of complex analysis [26].

theoremet f : C → C be a polynomial of degree $n \geq 2$ with complex coefficients, $f(z) = b_0 + b_1 z + b_2 z_2 + \ldots + b_n z_n$ where $b_i \in C$, $b_n \neq 0$. To start looking at Julia sets, we'll need the following:

L . • Let $f^k(w) = f(f(\ldots(w)\ldots))$ be the k-fold composition of f with itself.

- If $f(w) = w$ for some $w \in C$, we call w a fixed point of f.
- If $f^p(w) = w$ for some integer $p \geq 1$, $w \in C$, we call w a periodic point of f.
- If w is a periodic point of f, the least such $p \in N_{\geq 1}$ such that $f^p(w) = w$ is called the period of w.
- If w is a periodic point with period p, then $f(w), \ldots, f^p(w)$ is the period p orbit of w.
- If w is a periodic point with period p, and $(f^p)'\ w = \lambda$, where prime denotes the complex differentiation.

super attractive periodic point of t if $|\lambda| = 0$

attractive periodic point of t if $0 \leq |\lambda| < 1$

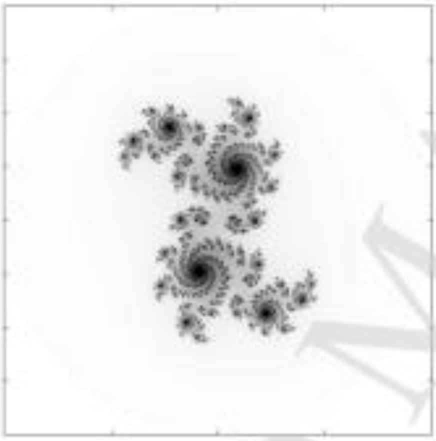
indifferent periodic point of f if $|\lambda| = 1$

repelling periodic point of t if $|\lambda| > 1$.

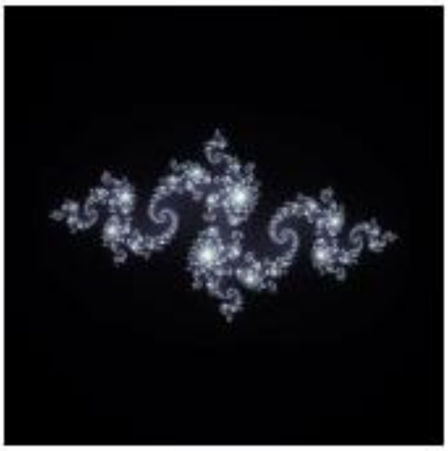
- The Julia set J(f) of f may be defined as the closure of the set of repelling periodic points of f.
- The Fatou, or stable, set $F(f) = C/J(f)$ of f is the complement of the Julia set of f.

Based on the algorithm (figure 1), the following patterns (figure 2) are generated with the aid of Matlab^{TM} framework.

Julia fractals caught a lot of attention because the graphics are spectacular. This Julia fractal tiling is suitable for building decoration and interior design. The fractal pattern design presented in Julia fractal images (figure 3) has some advantages such as it has the ability to effectively reflect the characteristics of real scenery and texture. The contrast of white and black (light and darkness, day and night) has a long history of metaphorical use. We present a revolutionary pattern design technique combining fractal geometry and visual texture generation. These Julia fractal structures apply to manipulate texture generation with light colors, where the shade of the pattern could be easily changed. Experiments revealed that patterns with various styles can provide multiple forms to decorate the environment effectively while using the new tile designs.



((a)) $f(z) = z^2 + c$, c=-0.28+0.008i



((b)) $f(z) = z^2 + c$, c=-0.79+0.15i

Figure 2: Different orientations of Julila fractals.

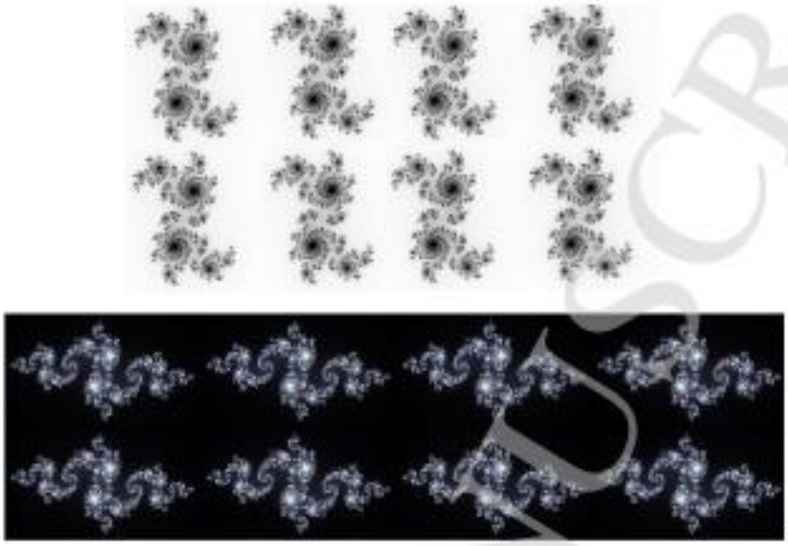


Figure 3: A wall visualizations with fractal tile pattern using Julia.

3.2. Mandelbrot fractals Graphic

The Mandelbrot set is made up of iterations in the complex plane of specific quadratic functions. It arises from the study of Julia sets of functions of the form $f_c(z) = z^2 + c$; those which we have briefly studied in the previous section.

theoreme define the Mandelbrot set to be the set of parameters c for which the Julia set of $J(f_c)$ is connected.

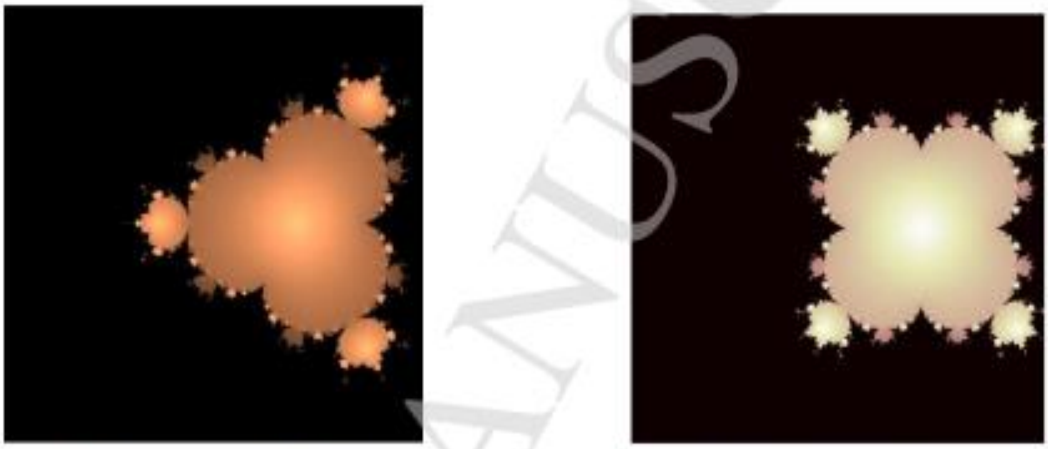
$$\mathbf{W}\ .\ M = c \in C : J(f_c) \text{ is connected.}$$

The Julia set and the Mandelbrot set have a significant difference in terms of iteration. We must iterate z always starting from 0 and varying the value of c in the Mandelbrot set. The Julia set iterates for a constant value of c and a variable value of z. In other words, the Mandelbrot set is in the parameter space, or c-plane, whereas the Julia set is in the dynamical space, or z-plane.

Following the same procedure explained in section 2.2, figure 4 provides interesting patterns.

We can see that these gloomy tiles are darker in color and are typically used in more professional environments, such as offices. Dark diagonal-patterned tiles in small kitchens deceive the eye into believing the area is larger than it is. Dark flooring can produce a rich contrast or an unexpected impression in a well-appointed lavatory, as shown in these

Mandelbrot images (see figure 5), respectively). Because they are easy to clean and do not retain dirt and dust for longer duration, they can be used in restrooms, desk areas, offices, and restaurants. These images can be used for a variety of dark tile surfaces, including glazed finish, glossy finish, matte finish, and textured finish.



((a)) $f(z) = z^4 + c$, c=-0.6+0i ((b)) $f(z) = z^5 + c$, c=-0.6+0i

Figure 4: Two different presentations of Mandelbrot graphs.

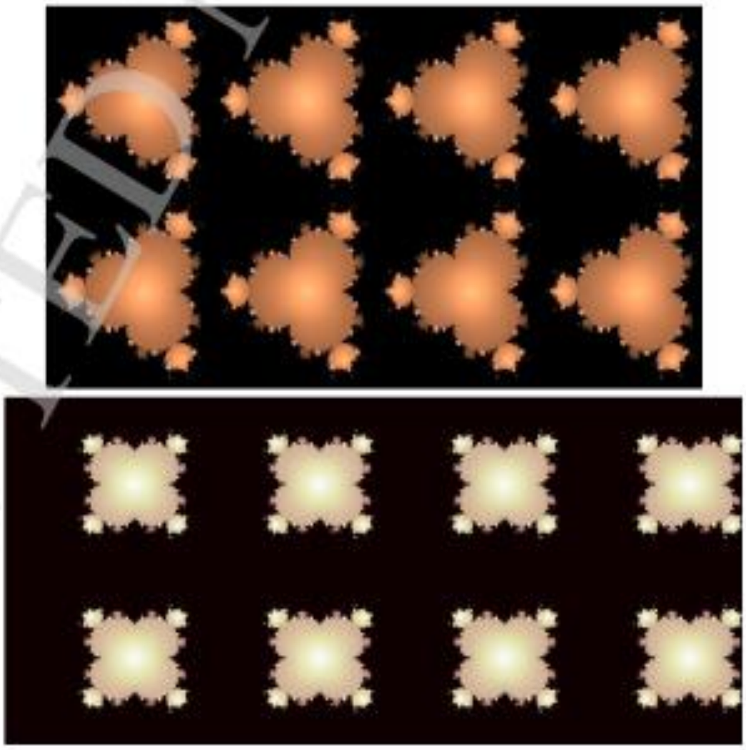
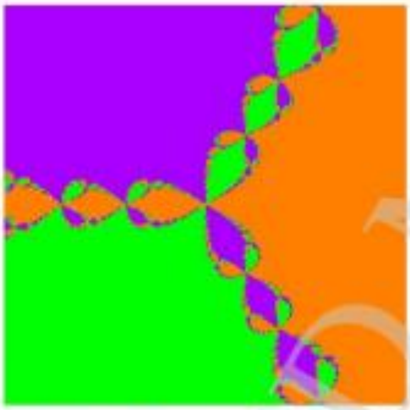


Figure 5: A wall visualization with fractal tile pattern using Mandelbrot.

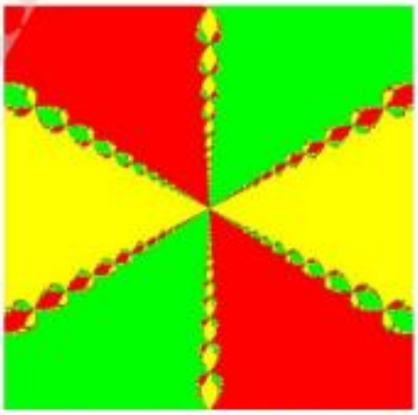
3.3. Newton fractals Graphic

"Isaac Newton's method" was used to create Newton fractals. They have a completely unique appearance compared to most fractal sets. The existence of Newton fractals is proof that Isaac Newton was a brilliant mathematician in his time.

These fractals are ideally suited for bathrooms, kitchens, and dining rooms. Because bright colors create the illusion of light. For a strong, saturated mix of shades. The fractal patterns in figures 6 and 7 gives a different impression. Using Newton fractal art with vibrant colors may create a rich contrast and draw attention to generate tiles for a stunning floor with a constantly repeated design. These patterns have high design parameters, a big occupation area and a sophisticated design process.



((a)) The basins of attraction, $f(z)= z^3+1$, c=-0.5-0.8660254i



((b)) Magnification of the middle, $f(z)=z^3+1$, c=-0.5-0.8660254i

Figure 6: Different magnifications for Newton's fractals.

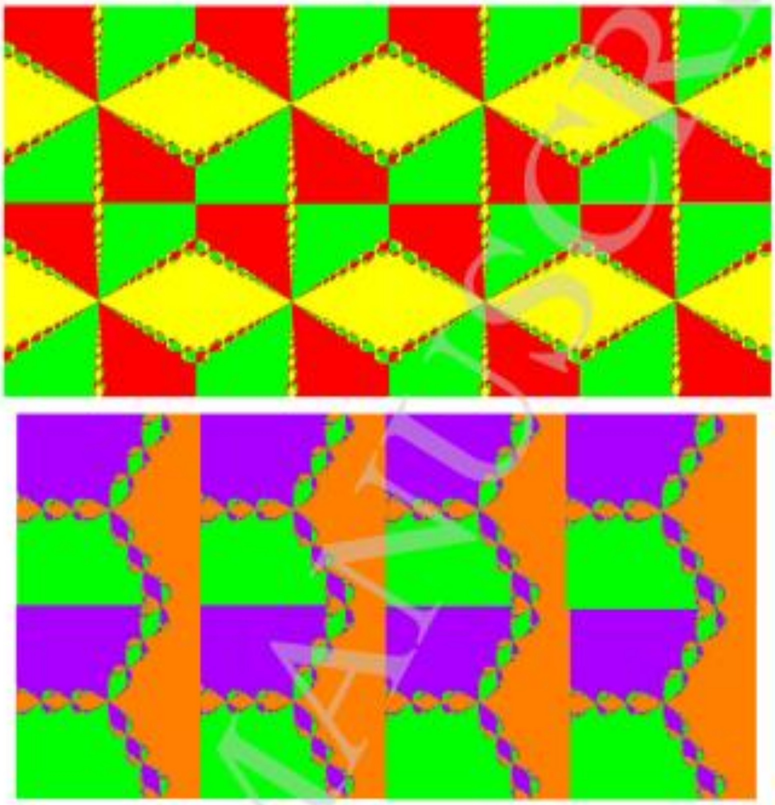


Figure 7: A wall visualization with fractal tile pattern using Newton.

3.4. Nova fractals Graphic

Nova is really a large class of fractals since any function f(z) can be used. Varying '*R*' changes the shape of the fractal. The classic Nova fractal uses $f(z) = z_p - 1$, and therefore produces the following equation that is iterated:

$$z_{n+1} = z_n - R\,\frac{f(z_n^p)}{f'(z_n^{p-1})} + c$$

Mostly images use p=3, which represents the search for the cubic roots of unity, and yields:

$$z_{n+1} = z_n - R\,\frac{(z_n^3)}{3(z_n^2)} + c$$

The new designs in tile construction can be driven by the creative usage of Nova fractal patterns in various shapes, textures, and patterns. The numerical results generated by using Nova present the fractal patterns in a more artistic way, matte colors in these tile textures, offer a softer, warmer impression as we can see from figures 8 and 9 respectively. These encaustic tiles and imitations provide a Victorian style. They can be used in many types of projects, from kitchen back splashes, floors, outdoor spaces and decorative wall features.

((a)) $p(z) = z^3 - 1$, c ranges from -1 to 1

((b)) $p(z) = z^3 - 1$, $c = \frac{1}{2}e^{i\theta}$, θ ranges from 0 to 2π

Figure 8: Comparison of two designs with different ranges.

Figure 9: Wall visualization with fractal tile pattern using Nova.

3.4.1. *SVM-Classification*

The fractal patterns, designs and resulting images generated during this research were processed using the image segmentation and filtration. The resulting statistical data was classified using the state of the art classification tools [27], with least loss of important features of the patterns. The judgment/ selection criteria was defined according to the user's choice, features (broad/ narrow, recursive, size, shape), architecture type and colour contrast respectively. The classification results from the support vector machine learning classifier are presented in figure 10. These results were generated following the step (1) the fractal patterns with different orientations are used to produce different images. These images were segmented and the statistical data, including the entropy, contrast and homogeneity was processed and classified. The two response variable was taken to be the type of illusion that the images made, broad or narrow. The results obtained showed 84.3% accuracy.

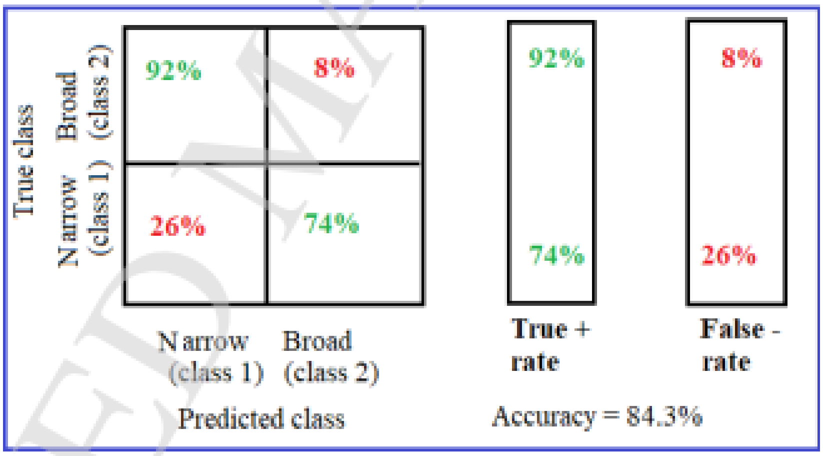


Figure 10: Classification of fractal patterns, according to wall broad or narrow illusion.

A summary of all the algorithms is provided in table 1.

Method	Iterative Description	Limitation	Characterization
Julia	c non-variable	infinite	dynamical Plane
Mandelbrot	variable c	infinite	parametric Space
Newton	class of fractal	infinite	polymetric space
Nova	variable c	infinite	polymetric space

Table 1: Comparison of the algorithms.

4. Conclusions

Integrated fractals in a tiling design with geometric repetition provide creative patterns with continuity. During this research, a unique iterative technique is used to create several classical fractal patterns that combine multiple characteristics and different orientations. The creation of these methods by using MATLAB^{TM} with the repetition principle of fractals provides the production of various forms with different complexities. Innovative fractal patterns are created and tested for accuracy and perfection, when used in architecture. In order to carry out the essential research, fractal art on tiles was combined with different pattern styles, keeping in view the applications of fractal art with smart tools. The discovery of fractal Geometry has made it possible to automatically find thousands of attractive tile themes, and designers can now run large numbers of simulations without limitations using a digital system. fractals can be more frequently and swiftly employed in the future design sector, and in the field of product design to generate a higher artistic level.

Compliance with Ethical Standards:

Acknowledgments:

Not applicable.

Declaration

The authors declare that there is no conflict of interest.

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