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Effect of viscous dissipation and internal heat source on mono-diffusive thermoconvective stability in a horizontal porous medium layer

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ABSTRACT

A mathematical model is developed for studying the onset of mono-diffusive convective fluid flow in a horizontal porous layer with temperature gradient, internal heat generation and viscous dissipation effects. Darcy's model is used for the porous medium which is considered to be isotropic and homogenous. A linear instability analysis is conducted and transverse or longitudinal roll disturbances are examined. The dimensionless emerging eigenvalue problem is solved numerically with Runge-Kutta and shooting methods for both cases of disturbances i.e. longitudinal and transverse rolls. Critical wave number and critical vertical thermal Rayleigh number R_z are identified. For higher value of Gebhart number Ge , a significant destabilizing effect of Hadley-Prats flow as computed. Internal heat generation also strongly modifies critical vertical Rayleigh number. Extensive interpretation of the solutions related to the onset of convection is provided. The study is relevant to geophysical flows and materials processing systems.

KEYWORDS *Thermo-convective instability; viscous dissipation, internal heat source, horizontal porous layer; eigenvalues; critical wave number; Gebhart number; materials processing.*

1. INTRODUCTION

The analysis of the thermo-convective instability in a horizontal infinite porous layer saturated with the fluid has many applications such as insulation of buildings [1,27], underground energy transport [2], groundwater transport [3], chemical engineering micro/nano-devices [4] and oil recovery [5] and chemical reactor engineering [27]. The occurrence of mono-diffusive convection (in the appearance of scalars diffusing at various rates) in a saturated porous medium is of high practical significance in various branches of engineering and science, such as chemical engineering, oceanography, transport of contaminants in saturated soil and geophysics. The mathematical analysis of these flows provides useful insights and a strong compliment to experimental investigations which is beneficial in optimizing thermofluid characteristics in for example materials processing systems. In these applications the porous medium is frequently simulated as a Darcian regime in which the flow is dominated by viscous forces. Many complex characteristics arise in such flows including linear and nonlinear hydrodynamic stability, convection rolls, oscillatory behaviour etc. Several interesting studies have been conducted for such flows. Thermal convection between two parallel infinite horizontal porous layers caused by temperature variations in the middle of the boundaries was originally examined by Horton and Rogers [6] and Lapwood [7]. More recently mono-diffusive convection between two horizontal parallel porous layers with internal heat generation has attracted some attention. A comprehensive linear instability analysis was presented by Nield [8]. This has been subsequently extended by various researchers with considering the additional effect of viscous dissipation on transport characteristics. However releatively few researchers have examined the thermal convection instability with the combined effects of viscous dissipation and internal heat generation. Barletta *et al.* [9] studied the case of inclined infinite horizontal porous layers with internal heat generation in which parallel boundaries were considered as isothermal. A linear instability analysis provides the sufficient condition for the perturbation of the basic steady state solution to be destabilized. A lucid description of porous media thermal instability analysis has been documented for a range of practical applications in the monograph of Nield and Bejan [10]. Hill [11] examined thermosolutal instability in porous horizontal layers in the presence of internal heat generation which is a function of concentration. Weber [12] considered thermal instability with both vertical and horizontal temperature gradients, although this analysis was confined to the case of small horizontal gradient.

Viscous dissipation which is associated with frictional heating arises in many industrial processes including both internal and external convection flows such as thermal mixing devices, boundary layer coating systems etc. This effect is significant in real viscous flows and its inclusion therefore furnishes more accurate appraisal of the heat transfer behaviour in engineering systems. Important studies of viscous dissipation in thermal convection flows have been presented by Barletta and Nield [13-14] and an early seminal boundary layer analysis has been presented by Gebhart [15]. Turcotte *et al.* [16] computed the effects of viscous dissipation on Bénard convection with an adiabatic temperature gradient. They evaluated in detail the finite amplitude convection response and observed that both viscous heating and temperature gradient substantially decelerate the flow, eventually inducing stability. Investigations of Hadley-Prats flows in porous media include Barletta and Storesletten [17], Nield *et al*. [18], Barletta and Nield [19] and Deepika and Narayana [20].These studies have generally shown that instabilities induced by viscous dissipation can originate even when there is no temperature increase or decrease in the vertical direction of the porous medium.

Convection with *internal heat generation* has also been assessed by a number of researchers [21], [22]. Internal heat generation arises for example in geophysical flows where the earth's mantle is heated internally. Internal heat generation strongly effect on the vertical motion [24-25, 28]. Brinkman number measures the internal heat generation by the act of viscous dissipation to the temperature difference [20]. Parthiban and Patil [23] deployed a Galerkin method to simulate the influence of horizontal temperature gradients due to non-uniform heating of the boundaries on the onset of convection in saturated porous medium with uniformly distributed internal heat sources. They showed that the onset of convection is encouraged with internal heat source, in particular with stronger horizontal temperature gradients and that there is a boost in the critical Rayleigh number with higher horizontal gradients. Further studies have been presented by Matta *et al.* [24] (who included mass flux effects) and also Matta [25] (who considered gravity variation).

The motivation for the present study is to generalize previous investigations by considering the collective influence of viscous dissipation and internal heat source. It also included the additional efftes of temperature gradient on mono-diffusive Haldey-Prats thermoconvective stability in a horizontal saturated porous medium layer. Darcy's model is implemented for the porous medium. The transport equations are non-dimensionalized and transformed into a robust eigenvalue problem. Numerical solutions are evaluated with a Runge-Kutta (RK) shooting method [24, 25, 27, 28, 29]. In particular the impact of viscous heating and internal heat generation on the onset of convection in Hadley-Prats flow for both cases of disturbances i.e. longitudinal and transverse rolls, is addressed. Critical wave number and critical vertical thermal Rayleigh number R_z are identified. For higher value of Gebhart number Ge, a significant destabilizing effect of Hadley-Prats flow is computed. This research paper is organized as follows. Section 2 provides the details of the physical system where the fluid flow takes place with the derivation of associated governing equations. Section 3 describes the basic steady state solutions. Section 4 presents the equations for perturbations. Section 5 deals with linear stability analysis of the perturbation equations. Section 6 documents the numerical and graphical results with interpretation. Section 7 presents the conclusions.

2. MATHEMATICAL FORMULATION

The basic model consists of a homogeneous infinite horizontal fluid saturated porous layer with height d. The upward vertical axis is z' and the horizontal axis is x' . The imposed horizontal temperature gradient is β_θ with internal heat generation Q'. The physical model is depicted in Fig.

1.

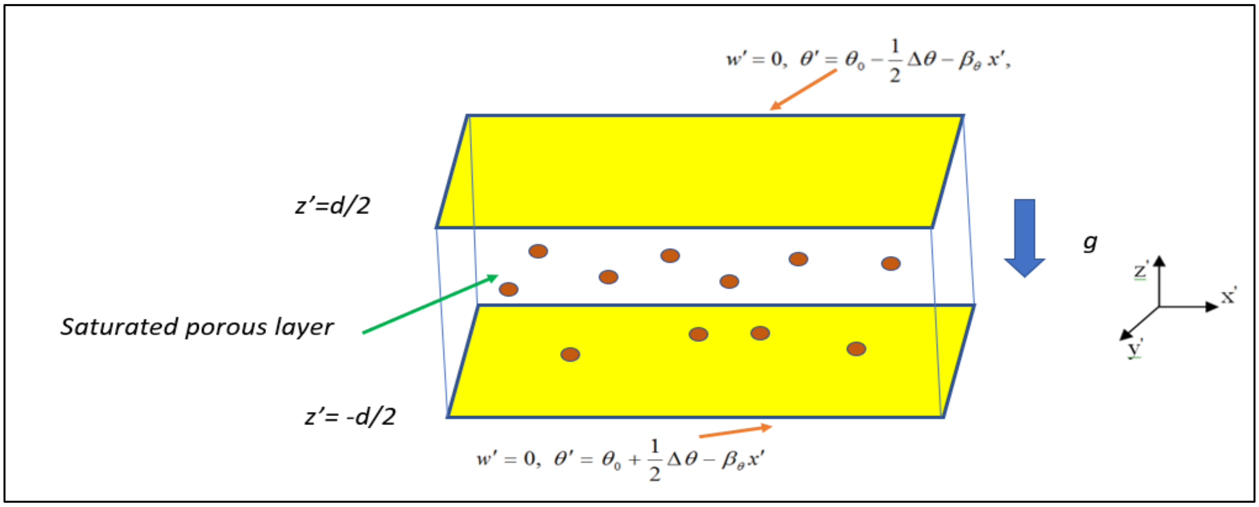

Figure 1: Schematic representation of the physical system.

$\Delta\theta$ is the perpendicular thermal variation through the horizontal boundaries. The linear Oberbeck-Boussinesq approximation (density deviations such that are supposed to be sufficiently small to be overlook every term exclusion of the body force term) [8, 24, 25, 27, 28, 29] is applied and Darcy's law is deployed for the porous medium. Under these assumptions, the governing equations for the thermal convection in the horizontal porous layer with associated boundary conditions are as follows [8,20,24-25]:

$$\nabla'.q' = 0, \tag{2.1}$$

$$\frac{\mu}{K}q' + \nabla'P' - \rho_f' g k = 0, \tag{2.2}$$

$$(\rho c)_m \frac{\partial\theta'}{\partial t'} + (\rho c_P)_f q'.\nabla'\theta' = k_m \nabla'^2\theta' + (\rho c_P)_f \frac{\vartheta}{Kc} q'.q' + Q', \tag{2.3}$$

$$\rho_f' = \rho_{0[1-(\theta'-\theta_0)\gamma_\theta]}. \tag{2.4}$$

The conditions on the horizontal boundaries are defined by:

$$w' = 0, \quad \theta' = \theta_0 - \frac{1}{2}(\mp\Delta\theta) - \beta_\theta x', \quad \text{at} \quad z' = \mp\frac{d}{2}. \tag{2.5}$$

In the above Eqns., the seepage (Darcy) flow velocity is $q' = (u', v', w')$, P' denotes the pressure and θ' is the temperature. The subscripts f and m designate respectively the fluid and the porous medium. Additionally, μ, c, k_m, K, γ_θ, ϑ and ρ_0 represents dynamic viscosity, specific heat, thermal conductivity, permeability of the porous medium, a fluid property that is indicative of the extent to which a fluid expands upon heating (thermal expansion coefficient) in the infinite porous layer, kinematic viscosity, and the density of the fluid medium, respectively. The following dimensionless variables and parameters are invoked [8, 12, 20, 24, 25, 27, 28, 29]:

$$(x, y, z) = \frac{1}{d}(x', y', z') \ , \quad t = \frac{\alpha_m t'}{a d^2} , \quad (u, v, w) = q = \frac{d q'}{\alpha_m}, \quad P = \frac{K(P' + \rho_0 g z')}{\mu \alpha_m}$$

$$\theta = \frac{R_z(\theta' - \theta_0)}{\Delta\theta} , \qquad Q = \frac{d^2 Q'}{k_m \Delta\theta} , \qquad \alpha_m = \frac{k_m}{(\rho c_p)_f} , \qquad a = \frac{(\rho c)_m}{(\rho c_p)_f} . \quad (2.6)$$

Here a is the ratio of the heat capacity of the porous and fluid media and α_m is the thermal diffusivity of the fluid medium. Q is the internal heat generation parameter.

Under these non dimensional transformations, the governing Eqns. (2.1) to (2.5) emerge as follows:

$$\nabla . q = 0 \, , \quad (2.7)$$

$$q + \nabla P - \theta k = 0 \, , \quad (2.8)$$

$$\frac{\partial \theta}{\partial t} + q . \nabla \theta = \nabla^2 \theta + Ge \, q . q + Q R_z \, , \quad (2.9)$$

The prescribed boundary conditions at the horizontal walls assume the form:

$$w = 0, \qquad \theta = -\frac{1}{2}(\mp R_z) - R_x x, \qquad \text{at} \qquad z = \mp \frac{1}{2}. \quad (2.10)$$

In the above Eqns. (2.7)-(2.10), the vertical and horizontal thermal Rayleigh numbers are denoted as R_z, and R_x, Ge represents the Gebhart number (viscous heating parameter). These dimensionless numbers are defined as follows [8, 12, 20, 24, 25, 27, 28, 29]:

$$R_x = \frac{\rho_0 g \gamma_\theta K d^2 \beta_\theta}{\mu \alpha_m} \ , \qquad R_z = \frac{\rho_0 g \gamma_\theta K d \Delta\theta}{\mu \alpha_m} \ , \qquad Ge = \frac{\gamma_\theta g d}{c} . \quad (2.11)$$

Eqn. (2.10) indicates that linear conditions for the temperatures on the horizontal boundaries are utilized.

3. STEADY STATE SOLUTIONS

Eqns. (2.7) to (2.9) have basic steady state solutions, subject to the conditions at the horizontal boundaries as follows:

$$\theta_s = \widetilde{\theta}\,(z) - R_x x\,, \tag{3.1}$$

$$(u_s\,, v_s\,, w_s\,) = (u(z), 0, 0), \qquad P_s = P(x, y, z) \tag{3.2}$$

This is a solution provided that:

$$u_s = -\frac{\partial P}{\partial x}\,, \qquad 0 = -\frac{\partial P}{\partial z} + \tilde{\theta}(z) - R_x x\,,$$

$$D^2\tilde{\theta}(z) = -QR_z - u_s R_x - Geu_s^2. \tag{3.3}$$

Here $D = \frac{d}{dz}$, the net flow along the direction of x-axis is $\int_{-\frac{1}{2}}^{\frac{1}{2}} u(z)\,dz = 0$. The basic steady state solutions therefore now become:

$$u_s = R_x z \tag{3.4}$$

$$\tilde{\theta} = -R_z z + \frac{R_x^2}{24}\left(z - 4z^3 - 2Ge(z^4 - 4)\right) + \frac{QR_z}{8}(1 - 4z^2) \tag{3.5}$$

4 DISTURBANCE EQUATIONS

In this section, the basic steady state solutions are perturbed as $q = q_s + \bar{q}, \theta = \theta_s + \bar{\theta},$ and $P = P_s + \bar{P}$. Replacing these perturbations in the non dimensional equations (2.7) to (2.9), leads to the following perturbation equations:

$$\nabla.\bar{q} = 0, \tag{4.1}$$

$$\bar{q} = \bar{\theta}k - \nabla\bar{P}, \tag{4.2}$$

$$\frac{\partial\bar{\theta}}{\partial t} + q_s.\nabla\bar{\theta} + \bar{q}.\nabla\theta_s + \bar{q}.\nabla\bar{\theta} = \nabla^2\bar{\theta} + Ge(2q_s.\bar{q} + \bar{q}.\bar{q}) \tag{4.3}$$

Here $\nabla\theta_s = -\left(R_x, 0, R_z - \tilde{A}\right)$, $\tilde{A} = \frac{R_x^2}{24}(1 - 12z^2 - 8Gez^3) - QR_z z.$

Now the conditions at the horizontal boundaries are defined as follows:

$$\bar{w} = 0, \qquad \bar{\theta} = 0 \quad \text{at} \quad z = \mp\frac{1}{2}. \tag{4.4}$$

These boundary conditions indicate that there is no orthogonal flow velocity and no porous medium temperature disturbances arising at the boundaries.

5 LINEAR INSTABILITY ANALYSIS

In this section, thermal instability is addressed via a linear stability analysis. To execute a linear stability analysis, the nonlinear terms appearing in Eqn. (4.3) are neglected. The linearized disturbance equations therefore emerge as:

$$\nabla.\bar{q} = 0, \tag{5.1}$$

$$\bar{q} = \bar{\theta}k - \nabla\bar{P}, \tag{5.2}$$

$$\frac{\partial\bar{\theta}}{\partial t} + q_s.\nabla\bar{\theta} + \bar{q}.\nabla\theta_s = \nabla^2\bar{\theta} + 2Ge(q_s.\bar{q}) \tag{5.3}$$

Here $\nabla\theta_s = -\left(\boldsymbol{R_x}, \boldsymbol{0}, \boldsymbol{R_z} - \frac{R_x^2}{24}(1 - 12z^2 - 8Gez^3) + QR_z z\right)$,

The linearized conditions at the boundaries take the form:

$$\bar{w} = 0, \qquad \bar{\theta} = 0 \quad \text{at} \quad z = \mp\frac{1}{2}. \tag{5.4}$$

Now *linearized perturbation equations* become:

$$\nabla.\bar{q} = 0, \tag{5.5}$$

$$\bar{q} = \bar{\theta}k - \nabla\bar{P}, \tag{5.6}$$

$$\frac{\partial\bar{\theta}}{\partial t} + u_s\frac{\partial\bar{\theta}}{\partial x} - R_x\bar{u} + \bar{w}\left(D\tilde{\theta}\right) = \nabla^2\bar{\theta} + 2Ge(u_s\bar{u}) \tag{5.7}$$

Adopting a Fourier mode solution to Eqns. (5.6) to (5.7) along with the boundary conditions (5.4) we have:

$$[\bar{q}, \bar{\theta},\ \bar{P}\ ] = [q(z), \theta(z), P(z)]\ exp\{i(kx + ly - \sigma t)\} \tag{5.8}$$

Eliminating the pressure term P from the equation (5.6) yields [8, 24, 25, 27]:

$$(D^2 - \alpha^2)\ w + \alpha^2\theta = 0, \tag{5.9}$$

$$\left(D^2 - \alpha^2 + i(\sigma - ku_s)\right)\theta + \frac{ik}{\alpha^2}(R_x + 2\ Ge\ u_s)\ Dw - \left(D\tilde{\theta}\right)w = 0, \tag{5.10}$$

The above equations (5.9) and (5.10), conditional on $w = \theta = 0$ at both the boundaries $z = \frac{1}{2}$ and $z = \frac{-1}{2}$, satisfy:

$$D\tilde{\theta} = -R_z + \frac{R_x^2}{24}(1 - 12z^2 - 8\ Ge\ z^3) - QR_z z\ , \tag{5.11}$$

The eigenvalue problem is therefore defined for the vertical thermal Rayleigh number R_z with σ, R_x, Q, Ge, k and l as variables. Furthermore $\alpha = \sqrt{k^2 + l^2}$ represents the overall wave number.

6 RESULTS AND DISCUSSION

In the previous sections, a linear stability analysis has been conducted to examine the consequence of a viscous dissipation and internal heat generation on mono-diffusive celebrated Hadley-Prats flow in an infinite horizontal porous layer. Following the numerical strategy of Barletta and Nield [13, 14] the eigenvalue problem i.e. Eqns. (5.9) to (5.10) is derived. A linear stability analysis, is conducted in which, vertical thermal Rayleigh number (R_z) is considered as the eigenvalue of the stability problem. Ideal Fourier mode analysis is implemented to investigate the thermal instability of Hadley-Prats flow. The value of critical R_z is determined as the lowest magnitude of all R_z values as α is changed. The matching wave number is defined as a row vector such as $\alpha = (k; l; 0)$. Let us consider the stationary convection which is accomplished by substituting $\sigma = 0$ as described in the earlier work of Nield [8]. The *type* of the longitudinal perturbations is registered by setting $k = 0$ in the eigenvalue problem encountered from the stability analysis. Ge is a non-dimensional number represents the viscous dissipation and this non-dimensional number represents the rate of viscous heating and its significance will not change with several numbers and its importance is the work done by the shear forces will converts in to heat energy and similarly Q is an internal heat source parameter. The MATLAB software is used for numerical calculations and Origin software is used for the plotting the graphs.

	R_x	0	10	20	30	40
$Ge = 0$	R_z	39.4784	42.0076	49.5486	61.9566	78.9671
	α	3.1399	3.1399	3.1499	3.1599	3.1999
$Ge = 5$	R_z	39.4784	41.9821	49.1156	59.5244	70.3359
	α	3.1399	3.1399	3.1699	3.1999	3.1999
$Ge = 10$	R_z	39.4784	41.9055	47.7973	52.3438	46.2155
	α	3.1399	3.1499	3.1999	3.1999	3.1999

Table 1 : Critical R_z at $Q = 0$

Table 1 summarizes the computations for $Q = 0$ and $Ge = 0$. It is clear that values of critical vertical Rayleigh number, R_z is reduced as Ge is elevated from 0 to 10, Thus, the destabilization of Hadley-Prats flow is observed at elevated values of the Gebhart number. The linear results are constituted by the curves in **Figures 2-7**.

Visualization of the vertical Rayleigh number (R_z) variation with horizontal Rayleigh number (R_x) for $Q = 0.5$ (fixed internal heating) and various Gebhart number values (*Ge*) is given in **Fig. 2**. Significant deviation is induced with increasing Gebhart number. Vertical Rayleigh number grows strongly with horizontal Rayleigh number for *Ge = 0 and Ge = 5*; however with *Ge* =10, it initially grows and then there is a downsurge (means decreasing trend or fall down) in vertical Rayleigh number at a critical value of horizontal Rayleigh number. In **Fig. 3** (with zero horizontal Rayliegh number prescribed i.e. $R_x = 0$) there is a slight reduction in vertical Rayleigh number (*Rz*) with increase in *Ge* up to a critical value of *Q*; beyond this *Q* value there is a strong enhancement in vertical Rayleigh number with a *Ge= 0,* although the decrease in vertical Rayleigh number with *Ge >0* is sustained. Also the vertical Rayleigh number initially decreases with increasing Q up to the critical point, and thereafter it ascends strongly with subsequent increasing Q values, for only the case of *Ge* = 0 (absence of viscous heating).

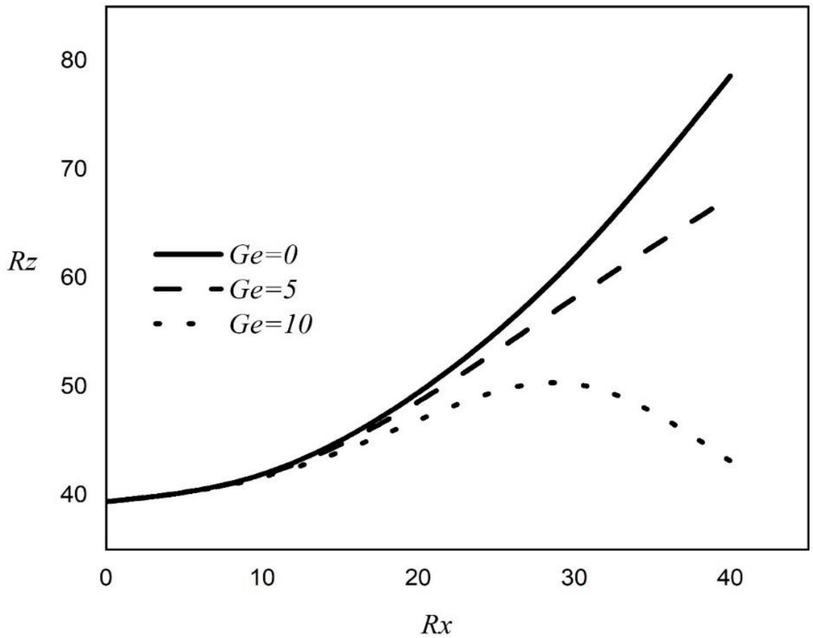

Figure 2: Deviations of R_z with R_x at $Q = 0.5$

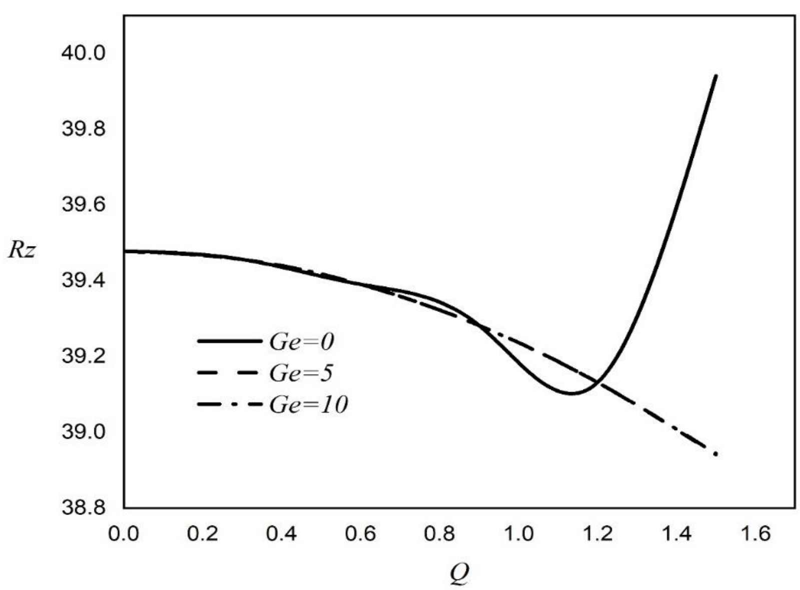

Figure 3: Deviations of R_z with Q at $R_x = 0$

The deviations of R_z are recognized as a function of Q in the appearance and disappearance of R_x and the Gebhart number Ge in **Figures 3 and 6**, respectively. It is also identified that, when $Ge = 0$ and $Ge = 5$, the value of R_z is reduced with enlargement of the value of internal heat generation Q, indicating the flow is unstable and instability is connected to the appearance and disappearance of the Gebhart number Ge. An enlargement of the heat generation boosts the overall temperature of the horizontal porous layer. Additionally, it is established that, the value of R_z is reduced with enlargement in Gebhart number Ge. Overall vertical Rayleigh number, R_z is reduced by an increase in viscous dissipation.

In **Fig. 4** the distribution of vertical Rayleigh number (R_z) with horizontal Rayleigh number (R_x) with $Ge = 3$ for various Q values is depicted. A monotonic increase in vertical Rayleigh number accompanies an increase in horizontal Rayleigh number. There is also a weak refunction (means it decreases) in vertical Rayleigh number with increasing Q values.

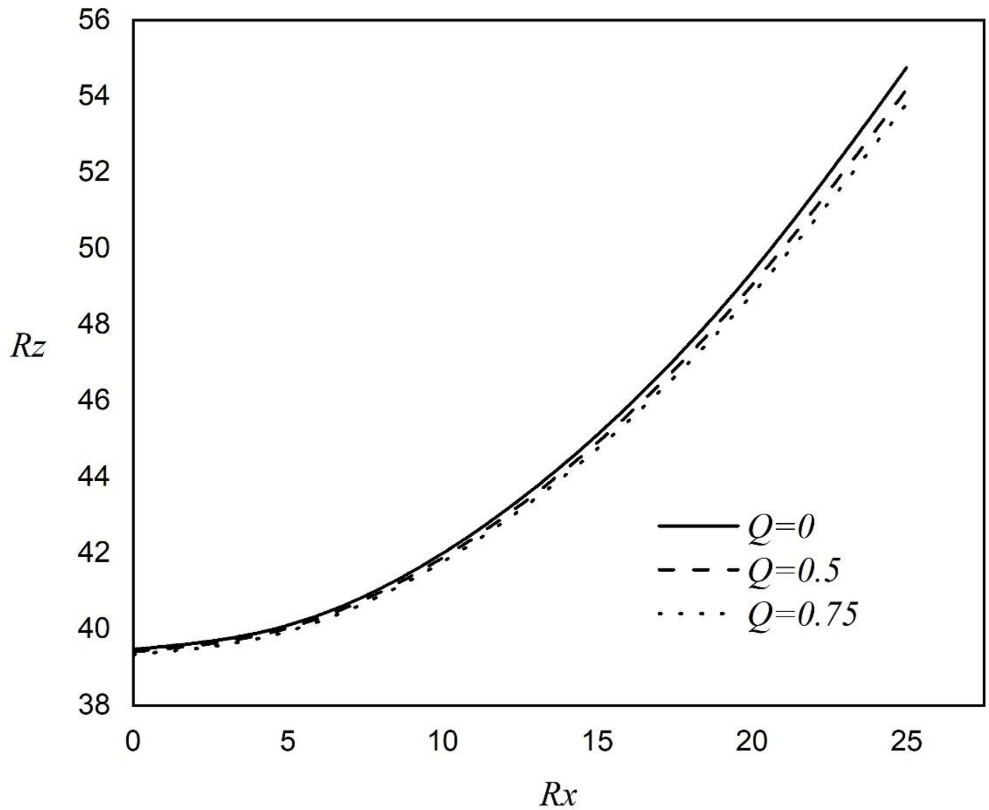

Figure 4: Deviations of R_z with R_x at $Ge = 3$

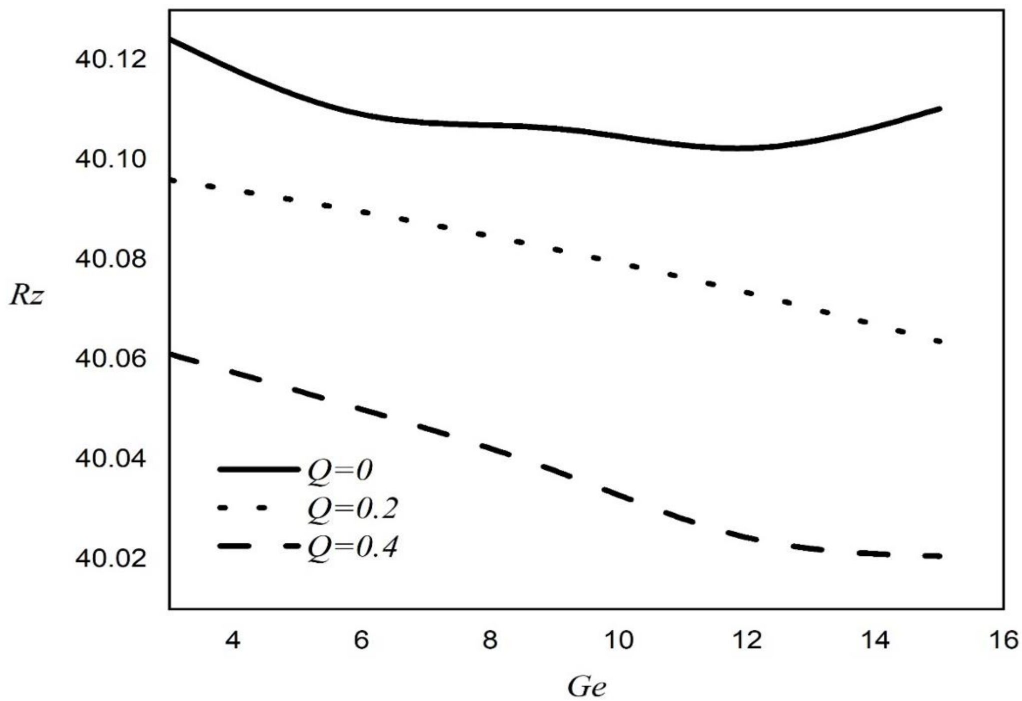

Figure 5: Deviations of R_z with Ge at $R_x = 5$

The critical R_z versus Gebhart number Ge is shown in Figures 5 and 7. Figure 5 represents deviations of R_z as a function of the Gebhart number Ge, for variation in internal heat generation Q at $R_x = 0$. An enlargement in the value of the heat generation diminishes the vertical thermal critical Rayleigh number R_z. Additionally, it is evident that greater viscous dissipation effect also boosts the critical value of R_z. The flow is destabilized at high values of viscous dissipation and internal heat generation in the porous medium. Fig. 6 shows that a strong decrement is produced in vertical Rayleigh number with increasing heat generation values (Q). However there is a sustained increment in critical vertical Rayleigh number (R_z) with increasing horizontal Rayleigh number (R_x).

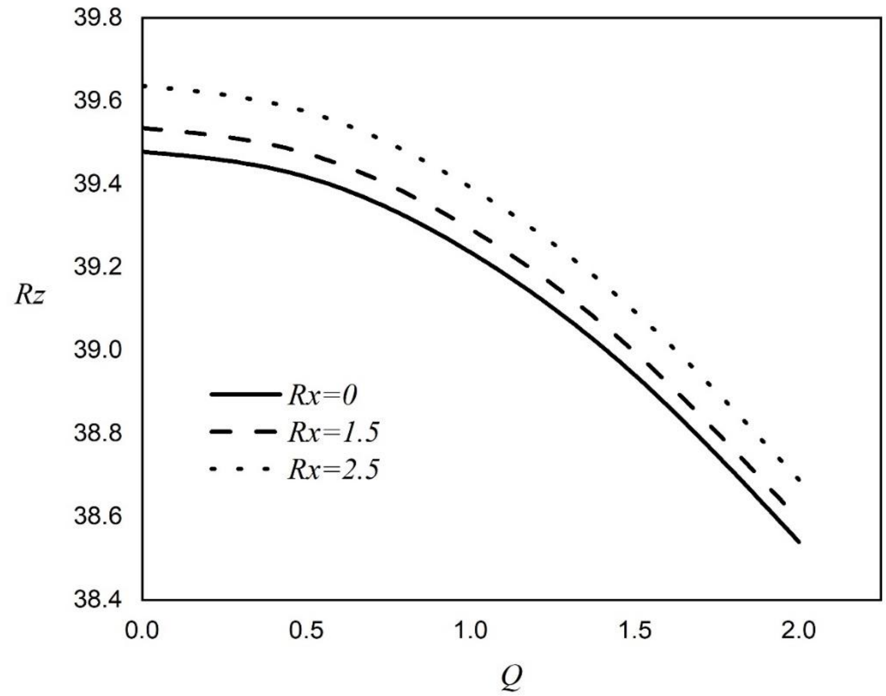

Figure 6: Deviations of R_z with Q at $Ge = 0.2$

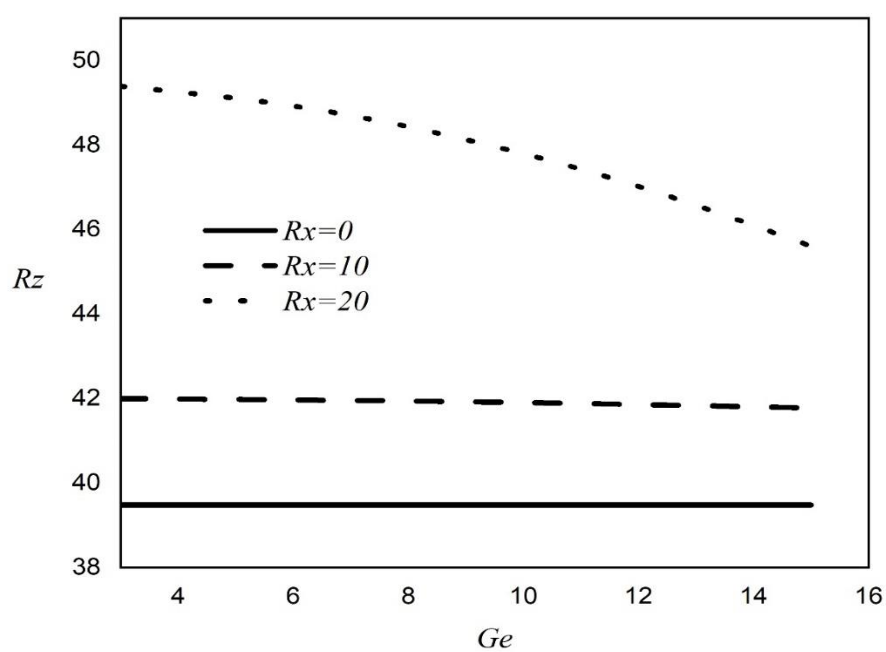

Figure 7: Deviations of R_z with Ge at $Q = 0$

Ultimately, larger values of Ge and R_x, encourage the flow to be destabilized in the porous medium. Internal heat source exert a strong influence on the value of critical R_z.

7 CONCLUSIONS

A mathematical model has been derived to study the thermoconvective instability in a horixontal porous layer with heat generation, viscous dissipation and horizontal temperature gradients. A linear stability analysis of the thermal convection in a Hadley-Prats flow has therefore been presented to evaluate the impact of internal heat generation and viscous dissipation on convection. The critical values of the vertical thermal Rayleigh number R_z is established for different values of the emerging parameters. The principal findings of the study can be summarized as follows.

- An enlargement in the porous medium internal heat generation parameter induces flow destabilization since it strongly modified the critical vertical Rayleigh number, and increases the overall temperatures of the system.
- The results show some qualitative changes in stability of the fluid flow with increasing Gebhart number (viscous heating) for dissimilar combinations of internal heat generation Q and horizontal thermal Rayleigh numbers R_x.
- Increased Gebhart number and thermal horizontal gradients have a dual impact on the critical values of vertical Rayleigh number R_z and modify the stability of the incipient thermal convection; strong destabilization of the Hadley-Prats flow is computed for elevated values of the Gebhart number Ge.
- Combined effects of internal heat source and viscous dissipation may cause instability in the fluid system.

The present model for mono-diffusive Hadley-Prats flow can be extended to consider other cases including nanofluids with species diffusion i.e. double-diffusive convection [26] and efforts in this direction are currently underway. Further, the present work on linear stability analysis can be extended to examine others types of fluids and boundary effects including variable viscosity flows [30], Maxwell viscoelastic fluids [31], particle-fluid suspensions [32], Rabinowitsch non-Newtonian fluids [33], electro-magnetic fluids [34], slip hydrodynamics at the boundaries [35], hybrid nanofluids [36] and nanoparticle-doping [37], [38]. All these areas are relevant to refining the current study with applications in geophysical transport and materials processing systems and will be explored soon.

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