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OPERATIONAL MEASUREMENT OF FREQUENCY RESPONSE FUNCTIONS

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ABSTRACT

A common problem that arises during modal testing is the inability to directly excite degrees-of-freedom to obtain frequency response functions (FRFs). It has been shown previously that reliable reproduction of FRFs via indirect excitation is difficult. This issue led to further investigation into the reconstruction of FRFs, with the most successful method arising called the 'Round-trip' method. In one arrangement, the round-trip identity can be used to determine the point receptance between two components, by replacing local excitations with remote ones. This original formulation requires three receptance terms to be measured. Depending on the combination two of these terms represent either a velocity or force transmissibility, which may be determined operationally. This alternative formulation called the 'Operational Round-trip' method, leaves just one receptance term to be measured via modal testing. Presented in the paper is an experimental example demonstrating the application of the ORT method. Furthermore, an investigation into the sensitivity of the approach when either the velocity or force transmissibility is analysed.

Keywords: *Round-trip method, frequency response functions, transmissibility, operational*

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1. INTRODUCTION

Obtaining a full FRF matrix for a system is not always easy in practical test situations. This can be due to: DoFs not being easily accessible for direct excitation via modal testing, the test structure only allowing for in-situ testing (such as essential infrastructure that cannot be shut down), and there being a large number of DoFs that exciting every one becomes too time consuming. These issues has further led research into reconstructing a full M × M FRF matrix without needing to apply excitations at every DoF (I.e. using only one column to synthesise the full matrix). There are methods that have proven to be successful as demonstrated by [1], [2], and [3], where they require only a single excitation at one DoF and repeated tests with varying mass loading. However this approach is error prone if the FRFs have a very small change due to the mass loading. [4] demonstrated theoretically that reconstruction was possible without repeated tests, but this was proven to be impractical for systems with multiple modes as the method implies that due to a certain frequency range, some residual modes have to be added to account for the effect of the modes outside the frequency range. [5] builds upon Ewin's work by presenting a similar method that cancels the effects of all transducer masses. Using a time-reversal technique, [6] set forth the Round-trip method, based on the cavity equation [7]. FRF matrix reconstruction is determined by applying excitations to a 'remote' subset of DoFs either side of the 'target' subset of DoFs. This was the first method to determine an FRF matrix without directly exciting the target DoFs. The Round-trip identity (which is shown in the next section) consists of mobility terms which are to be determined via experimental modal testing. However this paper will present a novel version of the method called the Operational Round-trip (ORT) method, which seeks to make the method applicable to output-only test cases. This is particularly useful in civil engineering applications where ambient excitation is preferred over artificial. The theory behind ORT will be outlined in the next section, but in essence the method uses the concept of generalised transmissibility to replace two of the three mobility terms in the original round-trip formula. Transmissibilities are particularly useful in this application for two main reasons: they can be determined operationally, and when defined implicitly using the blocked force/in-situ method [8] [9] they can be quantified for a certain component or sub-structure which is essential to replacing the mobility terms in the round-trip formula. These transmissibilities are an 'invariant' property of a component as shown by [10], meaning that a transmissibility calculated for a given component is solely the property of that sub-structure. The theory behind the round-trip method will be outlined in Section 2, along with some definitions of transmissibilities. In the latter part of the section the two definitions will be tied together in which the novel ORT method will be introduced. In Section 3 an experiment investigating this method is outlined and the results of the ORT method are displayed. The method is validated by comparing the results against a direct measurement, as well as the original round-trip identity. Following this Section 4 discusses the results of the experiment, as well as any points of the study that need further investigation. Lastly, Section 5 presents the concluding remarks of the study.

2. BACKGROUND THEORY

The round-trip method proposed by [11] constructs an $\mathbf{M} \times \mathbf{M}$ FRF matrix at a coupling interface without needing to excite the DoFs along it directly. This technique replicates point FRFs at the interface by utilising subsets of remote DoFs either side of the coupling interface that are easier to excite. In the past there have been successful attempts at reconstructing this matrix by measuring the responses at all $\mathbf{M}$ positions due to an excitation applied at a single DoF. [6] briefly outlines how those previous methods are subject to error and presents a new method based off the "cavity equation" by [7]. Namely the method is a generalised version of the equation where the time reversed response at a 'receiver' sub-structure due to an excitation at a 'source' sub-structure, is expressed via convolution of impulse responses at both points across a virtual interface.

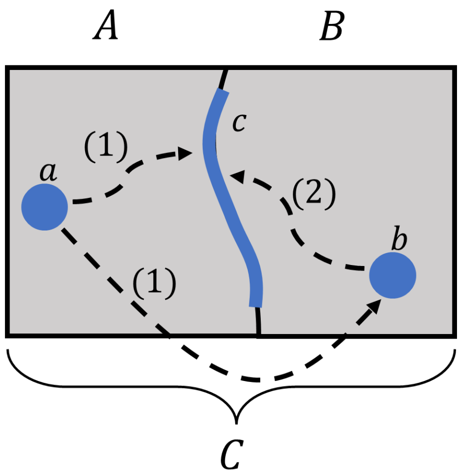

Figure 1. Diagram specifying the order and path of the measured FRFs using the round-trip method.

Considering an arbitrary structure such as Fig.1, we have two sub-domains A and B with multiple DoFs in a given area of each, named as a and b. The interface c is also a multi-point measurement location, dividing the coupled structure into two sub-domains. Suppose A and B are 'receiver' and 'source' sub-structures respectively. Let's take the excitations applied at the subset b, called $\mathbf{f}_b$. Our definitions for the velocity experienced at subset a and c are defined as,

$$\mathbf{v}_a = \mathbf{Y}_{C_{ab}} \mathbf{f}_b \tag{1}$$

$$\mathbf{v}_c = \mathbf{Y}_{C_{cb}} \mathbf{f}_b \tag{2}$$

As blocked forces are utilised at c, defining the velocity at the interface requires taking the reaction force measured at c which is due to excitations applied at b. I.e. the blocked force at the interface $-\bar{\mathbf{f}}_c$, is equal to the applied force at at b, denoted $\mathbf{f}_b$. This assumption implies the two forces form the same velocity field in the receiver A. Thus the velocity at the interface is defined as,

$$\mathbf{v}_c = -\mathbf{Y}_{C_{cc}} \bar{\mathbf{f}}_c \tag{3}$$

A definition must be made for the velocity in the receiver sub-structure A with our blocked force definition,

$$\mathbf{v}_a = -\mathbf{Y}_{C_{ac}} \bar{\mathbf{f}}_c \tag{4}$$

If we take Eq. 1 and 2, rearrange them for $\mathbf{f}_b$ so the applied force terms cancel we arrive at,

$$\mathbf{v}_c = \mathbf{Y}_{C_{cb}} \mathbf{Y}_{C_{ab}}^{-1} \mathbf{v}_a \tag{5}$$

A requirement of the round-trip is that the number of DoFs in the subset of the receiver sub-structure should be more than or equal to the number of DoFs in the source subset. In this case where n represents the number of DoFs, $n_a \geq n_b$. Taking our definitions due to a blocked force made in Eq. 3 and 4 and substituting them into Eq. 5, we have the following definition,

$$\mathbf{Y}_{C_{cc}} \bar{\mathbf{f}}_c = \mathbf{Y}_{C_{cb}} \mathbf{Y}_{C_{ab}}^{-1} \mathbf{Y}_{C_{ac}} \bar{\mathbf{f}}_c \tag{6}$$

The $\bar{\mathbf{f}}_c$ term cancels on both sides of Eq.6, and so the point mobility round-trip identity is obtained,

$$\mathbf{Y}_{C_{cc}} = \mathbf{Y}_{C_{cb}} \mathbf{Y}_{C_{ab}}^{-1} \mathbf{Y}_{C_{ac}} \tag{7}$$

Equally by reciprocity,

$$\mathbf{Y}_{C_{cc}} = \mathbf{Y}_{C_{cc}}^T = \mathbf{Y}_{C_{ca}} \mathbf{Y}_{C_{ba}}^{-1} \mathbf{Y}_{C_{cb}}^T \tag{8}$$

The reconstructed interface is denoted $\mathbf{Y}_{C_{cc}}$, providing the full mobility matrix of all DoFs at the interface c. This calculation is due to three measured mobilities via modal testing, denoted in Eq.7 as $\mathbf{Y}_{C_{cb}}$, $\mathbf{Y}_{C_{ab}}^{-1}$, and $\mathbf{Y}_{C_{ac}}$. These mobilities are represented by the arrows in Fig.1. It is clear from this figure why the method is termed 'round-trip'. Note that the mobility $\mathbf{Y}_{C_{cc}}$ is to be determined for linear systems (as well as time invariant ones). Eq.8 represents the same equation that satisfies Maxwell-Betti reciprocal theorem. With the first part of the foundational theory behind the ORT method covered, the second part is presented by introducing the concept of invariant generalised transmissibilities [10].

A transmissibility is defined as the ratio between two like quantities. This is usually in terms of force, displacement, or velocity.

$$\begin{pmatrix} -\bar{f}_1 \\ \vdots \\ -\bar{f}_N \end{pmatrix} = \begin{bmatrix} T_{11}^f & \dots & T_{1M}^f \\ \vdots & \ddots & \vdots \\ T_{N1}^f & \dots & T_{NM}^f \end{bmatrix} \begin{pmatrix} f_1 \\ \vdots \\ f_M \end{pmatrix}, T_{ij}^f = \left. \frac{-\bar{f}_i}{f_j} \right|_{f_{k \neq j}=0} \tag{9}$$

The force transmissibility T_{ij}^f is defined as the relationship between an applied force f_j at the DoF j, and the blocking force $-\bar{f}_i$ at the DoF i, whilst all other DoFs that are excited are subject to a zero force constraint $f_{k \neq j} = 0$. For Eq.9 the excitation and blocking force DoFs are part of different sets ($j \in M$, $i \in N$). If more than one blocking DoF is used then a constraint of $v_{i \in N} = 0$ is applied to those DoFs. Transmissibility in terms of velocity (also sometimes referred to as the displacement transmissibility), is defined as T_{ij}^v. It is introduced in Eq.10 that the velocity v_j found for DoF j, and velocity v_i for DoF i are shown to be related due to an applied force f_k. All the other DoFs ($j \in N$) are rigidly constrained.

$$\begin{pmatrix} v_1 \\ \vdots \\ v_M \end{pmatrix} = \begin{bmatrix} T_{11}^v & \dots & T_{1N}^v \\ \vdots & \ddots & \vdots \\ T_{M1}^v & \dots & T_{MN}^v \end{bmatrix} \begin{pmatrix} v_1 \\ \vdots \\ v_N \end{pmatrix}, T_{ij}^v = \left. \frac{v_i}{v_j} \right|_{v_{k \neq j}=0}. \tag{10}$$

Now that the velocity and force transmissibilities are defined one can see there are similarities between them. Both require rigidly constraining all blocking DoFs to zero when measuring at or between distinct DoFs. This is an important requirement, as the author explains why in detail in another paper [12]. If the blocking (interface) DoFs that separate a system into sub-structures have the constraints applied, the sub-structure of interest will be sufficiently characterised as an independent entity. Essentially the target sub-structure has blocked the dynamic behaviour of any coupled sub-structures and are unable to influence it. Thus the transmissibility measured in the target sub-structure is an invariant property of it. The similarities between the two transmissibilities does not stop there. In [13] it is shown that through a series of matrix operations the two are related. This relationship is expressed in Eq.16. The following part of this section will show the derivation of said equation.

Although force and velocity transmissibilities can be expressed in terms of dynamic stiffness (impedance) due to the equations of motion, the following derivations will take form of mobilities and are in reference to Fig.2. The reason for using mobilities is because direct measurement of impedance is rather difficult and impractical in comparison, requiring a constraint on all DoFs other than the one with the applied excitation. Firstly let's define the force transmissibility in terms of mobilities. By equivalent field theorem [8] [9], due to an applied force $\mathbf{f}_a$ an identical velocity response field along c can be made by applying the negative blocked force at the interface,

$$\mathbf{v}_c = \mathbf{Y}_{C_{ca}} \mathbf{f}_a = -\mathbf{Y}_{C_{cc}} \bar{\mathbf{f}}_{A_c} \tag{11}$$

$\mathbf{Y}_{C_{ca}}$ is defined as the coupled transfer mobility between the interface and remote subset of DoFs a, while $\mathbf{Y}_{C_{cc}}$ is the point interface mobility. By taking the inverse of point interface mobility $\mathbf{Y}_{C_{cc}}^{-1}$ and pre-multiplying both sides of Eq.11, the following definition is made,

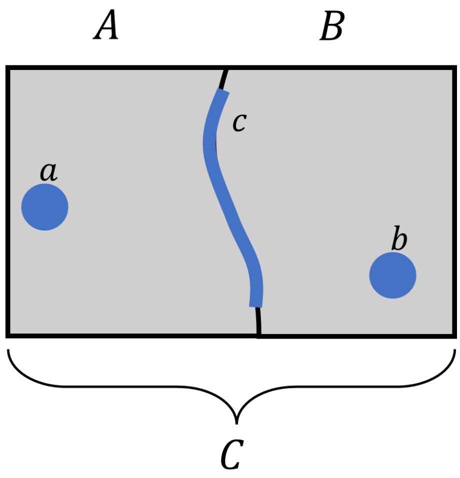

Figure 2. Diagram representing 2 sub-structures A and B which contain subsets of DoFs a and b respectively. These sub-structures are coupled at the interface c where another subset of DoFs lie.

$$\bar{\mathbf{f}}_{Ac} = \mathbf{Y}_{C_{cc}}^{-1}\mathbf{Y}_{C_{ca}}\mathbf{f}_a \tag{12}$$

The definition in Eq.12 relates the blocking force $\bar{\mathbf{f}}_{Ac}$ to an external force $\mathbf{f}_a$. One should notice the similarity to Eq.9, where it is obvious the transmissibility term represents the mobility terms $\mathbf{Y}_{C_{cc}}^{-1}$ and $\mathbf{Y}_{C_{ca}}$.

$$\mathbf{T}_{A_{ca}}^{f} = \mathbf{Y}_{C_{cc}}^{-1}\mathbf{Y}_{C_{ca}} \tag{13}$$

The derivation behind the velocity transmissibility in terms of mobilities starts with Eq.3, and Eq.4. Equating these gives the following equation,

$$\mathbf{v}_c = \mathbf{Y}_{C_{cc}}\mathbf{Y}_{C_{ac}}^{-1}\mathbf{v}_a \tag{14}$$

Similarly to Eq.12, Eq.14 relates the velocity at the interface $\mathbf{v}_c$, to the velocity at the subset of DoFs in A, $\mathbf{v_a}$. Comparison to Eq.10 shows that the mobility terms $\mathbf{Y}_{C_{cc}}$, and $\mathbf{Y}_{C_{ac}}^{-1}$ represent the velocity transmissibility.

$$\mathbf{T}_{A_{ca}}^{v} = \mathbf{Y}_{C_{cc}}\mathbf{Y}_{C_{ac}}^{-1} \tag{15}$$

Comparison between Eq.13 and Eq.15 shows there is a relationship between the two transmissibilities. By applying matrix inversions, [13] showed that the force and velocity transmissibilities are related. This is expressed by Eq.16, where the force transmissibility is equal to the inverse transpose of the velocity transmissibility and vice versa.

$$\mathbf{T}_{A_{ca}}^{f} = \left(\mathbf{Y}_{C_{cc}}\mathbf{Y}_{C_{ac}}^{-1}\right)^{-T} = \left(\mathbf{T}_{A_{ca}}^{v}\right)^{-T} \tag{16}$$

Collecting FRF data often requires human intervention via experimental modal test, which can be rather time consuming. This is especially the case where in some instances where regions of a system are inaccessible to excite by modal hammer. The measurement of the mobility terms in Eq.13 and Eq.15 may require some planning beforehand. Instead of defining these transmissibilities in terms of mobility, one may do so via measured responses. The remainder of this section will derive $\mathbf{T}_{A_{ca}}^{v}$ and $\mathbf{T}_{A_{ca}}^{f}$ in an output-only approach. This is not the same for FRFs where the force and response inputs are known. All quantities are in reference to Fig.2. For this method to work the vibro-acoustic source should not be the sub-structure selected. I.e. the transmissibility should be calculated for a 'receiver' subdomain. Firstly, N amount of linearly independent operational states must be measured. An operational state can be interpreted as an external force $\mathbf{F}_c$. These states represent a time window during an operational measurement. By ordering the external force vectors for the columns of a matrix the following is given:

$$\mathbf{F}_c = \left[\mathbf{f}_c^{(1)} \ldots \mathbf{f}_c^{(N)}\right] \tag{17}$$

The Eq.17 definition results in a corresponding velocity matrix $\mathbf{V_c}$ at those DoFs. Redefining Eq.3 and Eq.4 results in the following equations,

$$\mathbf{V}_a = \mathbf{Y}_{C_{ac}}\mathbf{F}_c \tag{18}$$

and,

$$\mathbf{V}_c = \mathbf{Y}_{C_{cc}}\mathbf{F}_c \tag{19}$$

Applying the same rearrangement and substitution as shown previously for in Eq.14 to the above equations 18 and 19 obtains the following,

$$\mathbf{V}_c = \mathbf{Y}_{C_{cc}}\mathbf{Y}_{C_{ac}}^{-1}\mathbf{V}_a \tag{20}$$

From this it can be clearly deduced that the mobility terms in Eq.20 represent the velocity transmissibility matrix in terms of operational response,

$$\mathbf{T}_{A_{ca}}^{v} = \mathbf{Y}_{C_{cc}}\mathbf{Y}_{C_{ac}}^{-1} = \mathbf{V}_c\mathbf{V}_a^{-1} \tag{21}$$

Implementing the inverse transpose relationship as shown in Eq.16 obtains the force transmissibility in terms of operational responses,

$$\mathbf{T}^{f}_{A_{ca}} = \left(\mathbf{V}_c\mathbf{V}_a^{-1}\right)^{-T} = \mathbf{V}_c^{-T}\mathbf{V}_a^{T} \tag{22}$$

A means of applying operationally determined transmissibilities to the round-trip equations Eq.7 and Eq.8 will now be detailed. Firstly taking Eq.7 and applying a post-multiplication matrix operation of $\mathbf{Y}_{C_{ac}}^{-1}$ to both sides results in Eq.23.

$$\mathbf{Y}_{C_{cc}}\mathbf{Y}_{C_{ac}}^{-1} = \mathbf{Y}_{C_{cb}}\mathbf{Y}_{C_{ab}}^{-1} \tag{23}$$

As seen before the left hand side of this equation is defined as velocity transmissibility. A small side note should be made that this equation indicates another mobility definition of the transmissibility $\mathbf{T}^{v}_{A_{ca}}$. It can be said that this definition also equals the output-only definition, which will be integrated into the round-trip equation.

$$\mathbf{Y}_{C_{cb}}\mathbf{Y}_{C_{ab}}^{-1} = \mathbf{V}_c\mathbf{V}_a^{-1} \tag{24}$$

Inserting the definition made in Eq.24 into the round-trip Eq.7, or by rearranging Eq.15, sets forth the transmissibility approach of the method.

$$\mathbf{Y}_{C_{cc}} = \mathbf{V}_c\mathbf{V}_a^{-1}\mathbf{Y}_{C_{ac}} = \mathbf{T}^{v}_{A_{ca}}\mathbf{Y}_{C_{ac}} \tag{25}$$

As well as there being a velocity transmissibility application, there is also a means of applying the force transmissibility to the round-trip method. Therefore taking Eq.24 and applying an inverse transpose matrix operation, we have another mobility definition for the force transmissibility,

$$\mathbf{T}^{f}_{A_{ca}} = \mathbf{Y}_{C_{cc}}^{-1}\mathbf{Y}_{C_{ca}} = \mathbf{Y}_{C_{bc}}^{-1}\mathbf{Y}_{C_{ba}} = \mathbf{V}_c^{-T}\mathbf{V}_a^{T} \tag{26}$$

In order for the force transmissibility to be applied to the round-trip equation, an inverse operation needs to be made.

$$(\mathbf{T}^{f}_{A_{ca}})^{-1} = \mathbf{V}_a^{-T}\mathbf{V}_c^{T} \tag{27}$$

The inverse force transmissibility definition in Eq.27 is seen in the transposed round-trip Eq.8. Thus the force transmissibility application to the round-trip method is defined as,

$$\mathbf{Y}_{C_{cc}} = \mathbf{Y}_{C_{ca}}\mathbf{V}_a^{-T}\mathbf{V}_c^{T} = \mathbf{Y}_{C_{ca}}(\mathbf{T}^{f}_{A_{ca}})^{-1} \tag{28}$$

The definitions in Eq.25 and Eq.28 are a useful means of applying the round-trip when the mobility terms cannot be measured. Eq.25 only requires one FRF measurement to obtain the mobility $\mathbf{Y}_{C_{ac}}$ and an operational measurement of the velocity transmissibility. The $\mathbf{Y}_{C_{ca}}$ term requires excitation at the interface, but applying the transpose of this will allow one to apply the excitation at the remote DoF a instead. Modal hammer excitations that are applied to a remote set of DoFs are often more desirable as these DoFs will be positioned for accessibility.

3. RESULTS

A similar system to Fig.2 was constructed, where the component A is a steel beam resiliently coupled to B which is a large hard acrylic plate.

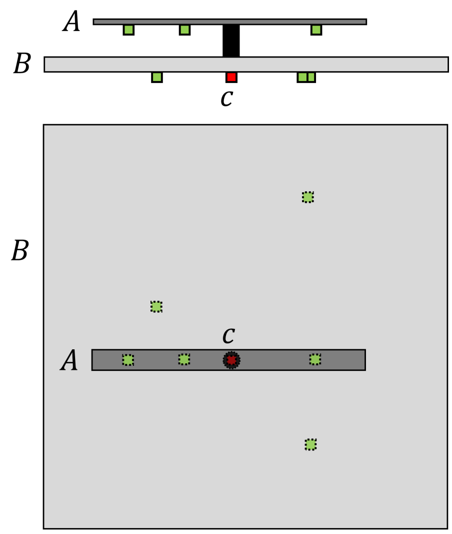

Figure 3. 2D diagram of beam-isolator-plate test. Above illustration is a side-on view, while the one below is a top-down view.

In this experiment two rounds of tests were conducted where the 'source' beam was interchanged, in order to show that the transmissibility term in the ORT method is indeed invariant and thus the point mobility determined is representative of that particular system when compared to a direct measurement. Dimensions of Beam 1 and 2 are 56x4x1cm and 44x4x1cm respectively. Instead of a shaker, an impact hammer was used to apply excitations within the source component A. The hammer was constantly exciting a particular point for 20 seconds, where each time window is taken at every 1 second. The points on the source were excited at the exact same locations during the FRF measurements. Firstly, we compare the original round-trip formulation of point mobility against the directly measured point mobility at the interface, presented in Fig.4. For this system, Fig.4 shows the original round-trip formulation accurately representing the point mobility at the interface up till 3 kHz, where a significant increase in artefacts and noise can be observed.

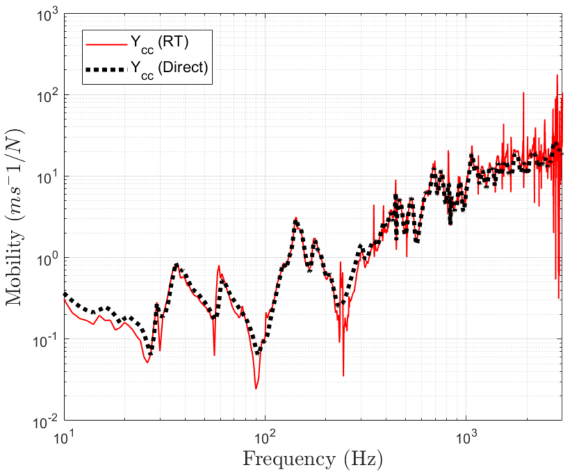

Figure 4. $\mathbf{Y}_{C_{cc}}$ due to beam 1: Round-trip vs. direct measurement

Fig.5 compares the ORT method using the velocity transmissibility, to the direct measurement. Similarly to Fig.4, this version of the ORT method accurately follows the direct measurement, albeit with more noise and artefacts. It should be noted that no regularisation techniques were applied to the ORT result. A study on the use of regularisation to the ORT method will be presented in a future paper. In Fig.6 the ORT formula that include the force transmissibility also shows an accurate representation when compared to the direct measurement. This version shows less noise than the velocity transmissibility application, all the while still showing artefacts. To prove that the transmissibility term in the ORT formula is an invariant property of the receiver sub-structure B, Fig.7 displays the velocity transmissibility version of the ORT method for when

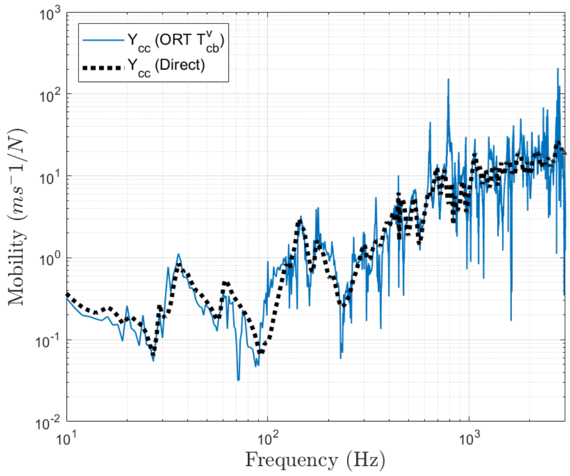

Figure 5. $\mathbf{Y}_{C_{cc}}$ due to beam 1: $\mathbf{T}^{v}_{B_{cb}}$ ORT vs. direct measurement

source 'Beam 2' is coupled to the system. It is clear from Fig.7 that the ORT formulation accounts for the new source component, thus the point mobility determined is representative of the new system. The increased noise and artefacts are visible similarly to the source 'Beam 1' scenario in Fig.5.

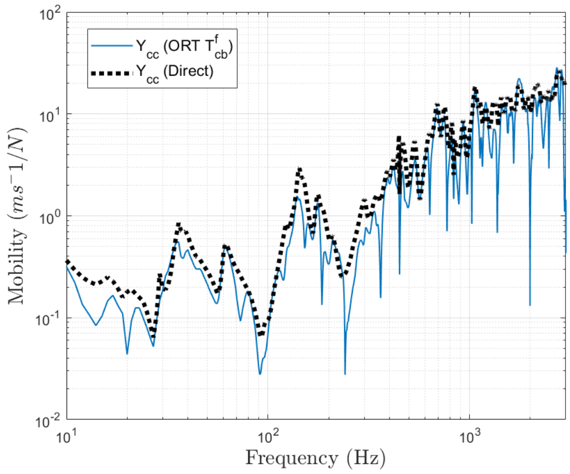

Figure 6. $\mathbf{Y}_{C_{cc}}$ due to beam 1: $\mathbf{T}^{f}_{B_{cb}}$ ORT vs. direct measurement

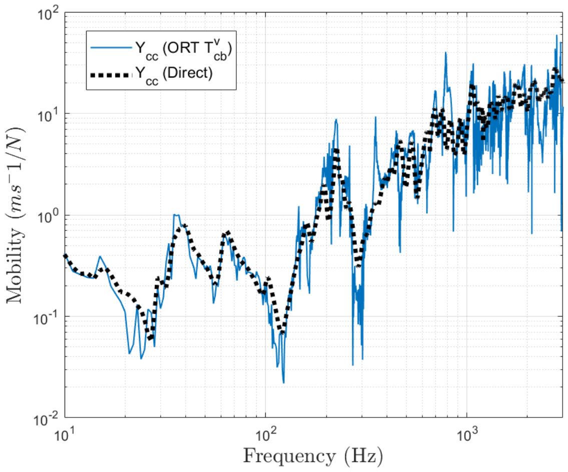

Figure 7. $\mathbf{Y}_{C_{cc}}$ due to beam 2: $\mathbf{T}^{v}_{B_{cb}}$ ORT vs. direct measurement

4. DISCUSSION

Overall this study has shown there is good agreement between the operational round-trip method and their directly measured point mobility counterparts. While artefacts were present in both transmissibility cases (and noise in the velocity transmissibility ORT), they both followed the trend of the direct measurement with good accuracy. The reasoning for the force transmissibility ORT not including as much noise as the velocity transmissibility ORT is not fully understood yet. This could be due to the transmissibilities being applied differently to the ORT formula (I.e. force transmissibility is inverted when the velocity transmissibility is not). This is a point of investigation for a future study. While the ORT method has proven to be accurate in this study, the original round-trip method is slightly more precise. One must take into account that small reduction in precision of the ORT method brings a large benefit of it being almost completely operational, leaving only one impact hammer measurement instead of three. As part of a future study the ORT method will be compared to the original round-trip when regularisation is applied. The study will seek to show what regularisation techniques will aid the ORT in reducing noise and artefacts, so that it resembles the 'cleaner' nature of the original round-trip method.

5. CONCLUSIONS

- Increase in noise and artefacts for the ORT formula Eq.25.
- Noise not as prominent for the ORT formula Eq.28, but still an increase in artefacts is observed.
- Both versions of the ORT formula show an accurate depiction of the point mobility at the interface, albeit with slightly less precision compared to the original round-trip method. Nevertheless, the slight reduction in precision comes with a large added benefit of having a nearly completely operational measurement.
- Further investigation is needed on the application of regularisation techniques to ORT method.

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