Julia sets in relaxed Schröder and Newton-Raphson maps: periodic points, escape points, symmetry-breaking

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TALK ABSTRACT

The Schröder algorithm is a generalization of the well-known Newton-Raphson (NR) iterative scheme for finding approximate roots of functions [A. S. Househölder, *Principles of Numerical Analysis* (Dover, New York, 1974)]. In this talk, both methods are deployed on the complex plane to study a simple problem: computing the fourth roots of –1. The root-finding aspect is not of principal concern. Rather, our analysis is from the standpoint of discrete dynamical systems where maps, their higher-order iterates, and their Julia sets (fractals) take centre stage. Ultimately, our interest lies with uncovering the detailed nature of the building blocks that make up the Julia sets for our Schröder and NR maps—periodic orbits. For greater generality, we have introduced a relaxation parameter whose impact on the structure of the periodic orbits can be nontrivial.

We will present some recent results showing how Julia sets for the Schröder and NR maps can be decomposed. Particular emphasis is placed upon the derivation and numerical solution of polynomial equations whose roots are period-M points, where $M = 2, 3, 4,\ldots$ For the NR map, the polynomial degree increases as $\exp[\ln(4)M]$ while, for the Schröder map, the divergence is an even more severe $\exp[\ln(8)M]$. In all cases, the associated periodic orbits are unstable and show extreme sensitivity to arbitrarily-small perturbations. They also break-up into distinct families, each with its own signature and which (through linearization) can be classified according to its instability growth rate. We will conclude by estimating the dimension of the computed Julia sets.

Keywords: Discrete dynamics, Julia set, Schröder, Newton-Raphson, symmetry-breaking.