



# Calculation of Participation Based Loss Factors using the Concept of Market Center in a Deregulated Power System

Arunachalam Sundaram<sup>1</sup>

<sup>1</sup> *Electrical and Electronic Engineering Technology, Jubail Industrial College, Jubail, Kingdom of Saudi Arabia*

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**Abstract:** The transmission loss factors play a vital role in determining the loss component of the Locational Marginal Price. If single slack bus power flow approach is used, then the component for loss in Locational Marginal Price is found to vary exorbitantly whenever the slack bus changes. This paper proposes a Participant Based Distributed Slack Power Flow model where the losses are distributed to each participant instead of each bus. Using the Jacobian Matrix of the Participant Based Distributed Slack Power Flow model participant based transmission loss factors are calculated which are slack bus independent and are obtained using a unique reference point. These loss factors overcome the problem of variations of loss component in Locational Marginal Price due to changes in slack bus. These loss factors are more suitable for competitive deregulated power market since it is calculated with respect to each participant and is different from existing techniques which calculates loss factors for each bus. This paper proposes the mathematical formulation of Participant based Distributed Slack Power Flow model and the calculation of Participant based loss factors. Case studies carried out using radial five bus system and IEEE 30 bus system indicate the effectiveness of the proposed model.

**Keywords:** Power Loss, Incremental Transmission Loss Factor, Loss Allocation, Deregulated Power System

## 1. INTRODUCTION

The vertically integrated power utility structures are being replaced by a deregulated competitive structure in many parts of the world. There are several reasons for this transition such as increase in social welfare due to competition and the latest development in the field of power generation and information technology. A deregulated structure will contain the separated entities of unbundled vertically integrated utility referred to as GENCO (GENerating COmpany), TRANSCO (TRANsmiSSion COmpany) and DISCO (DIStribution COmpany). Under this deregulated environment the consumers receive power from power pools or through privately negotiated bilateral contracts. The resources of the pool structure are pooled to increase reserve margins and to cut down the generation capacity while maintaining security levels.

Power losses in vertically integrated system were treated as an extra load. In the deregulated power system the cost of losses must be shared by each participant in a transparent and nondiscriminatory manner. Loss allocation is a procedure for subdividing the total transmission loss into fractions, the cost of which then becomes the responsibility of each participant of the deregulated power system. This process of loss allocation

is very challenging and contentious issue in deregulated power system since the power transmission losses are nonlinear functions of real power injection into the nodes. The main difficulty in allocating losses to each participant is that regardless of the approach the final loss allocation will always contain a degree of arbitrariness. As power transmission loss represent about ten percent of the total generation capacity, a quantity worth millions of dollars per year, their allocation to each participant has an impact on their benefits. For each GENCO participant this extra cost has to be subtracted from their revenue and for each DISCO participant the extra cost has to be added to their payments.

In literature many loss allocation schemes have been proposed to allocate transmission losses to each individual bus where generators and loads are connected. The energy that flows into the meshed network can be traced and so by using the principle of upstream power flow tracking and downstream power flow tracking loss allocation schemes to allocate the losses in deregulated power system is proposed in [1, 2]. One of the simplest techniques for loss allocation, Pro-Rata technique is used in Spain. The main drawback of this technique is it ignores the system configuration.



In [3] an iterative incremental load flow approach based on modified load flow calculation and marginal transmission loss approach based on Kron's loss formula has been proposed to allocate the transmission loss. In [4] it is clearly shown that depending upon the loss allocation technique involved the loss allocation to buses will change. Computationally efficient Z-bus loss allocation method exploits the sparseness of Y-bus matrix to allocate the transmission losses to each bus of the system [5]. The goal of this scheme is to take a solved power flow and systemically distribute the system transmission losses through current injected into the bus. Loss allocation based on ANN and support vector machine is proposed in [6, 7].

In [8] a method to calculate slack bus independent penalty factor for analysis of transmission loss in deregulated power system has been introduced. Loss allocation based on sensitivity relationship between the transmission losses and bus power injection, modification of bus admittance matrix, equivalent current injection and using graph theory has been proposed in [9], [10, 11], [12] and [13] respectively. In [14] discussion and comparison of the average loss pricing and competitive loss pricing in deregulated power system has been carried out.

The references reviewed so far have made significant contribution on loss allocation in vertically integrated power system and deregulated power system, but primary objective of any market operator is to recover the cost of loss from the participants of the deregulated power system.

The theory of spot price or nodal price was first developed [15] to price electricity in vertically integrated utility and was later extended to the deregulated power system. Since vertically integrated power system was monopolistic structure and deregulated power system is a market oriented competitive structure the extension of certain economic framework like Locational Marginal Price (LMP) calculation is perfect for vertically integrated power industry where LMP at a bus is calculated with respect to demand at a bus and the same price is used for settlement of both GENCO and DISCO participants connected to the same bus. But this has to be refined for deregulated power system since both GENCO and DISCO participants are competitive players.

While solving the problem of market clearing and settlement of double side auction market, using either ACOPF model or DCOPF model, if single slack bus power flow model is used, then the LMP component for loss is found to vary exorbitantly whenever the marginal bus (slack bus) changes [16]. This is because that loss is referred with respect to the slack bus. The conventional single slack bus power flow model has a drawback where the slack bus absorbs all the loss (mismatch). To

overcome this limitation various approaches have been proposed in the literature [16-18]. Even though the problem of large variation of loss costs was overcome in these approaches, an explicit location for loss balancing has not been indicated. Therefore the equitability and transparency of the loss factor cannot be established. The use of a suitable reference point called Market Center to calculate the loss contribution by both GENCO and DISCO participant is the main objective of this paper.

In a deregulated power system both GENCO and DISCO participants are competitive players the GENCOs loss contribution has to be measured from Load Center where they deliver power and DISCOs loss contribution has to be measured from Generation Center where they withdraw power. Even when both participants are present in the same bus their loss contribution would be different and so their loss factors will also be different. This issue is addressed in this paper.

This paper first proposes an improved model of distributed slack power flow called as Participant Based Distributed Slack Power Flow (PBDSPPF) where the loss contributed by each participant is calculated from a unique reference point similar to the one proposed in [17].

This paper is organized as follows. Section II provides a brief discussion on Market Center concept. Section III derives the PBDSPPF model. Using the Jacobian matrix obtained from section III, Section IV calculates the participant based incremental loss factors and section IV provide case studies and the comparison between existing and proposed method.

## 2. CONCEPT OF MARKET CENTER

This section discusses the concept of unique point for loss calculation in a deregulated power system called Market Center. In this paper the loss factor calculation of each GENCO/DISCO participant is with respect to this unique point.

### A. Load Center

In order to compare the relative contribution of different generating units to transmission loss, a reference point called "Load Center" was introduced in [19]. The "Load Center" is obtained by connecting each of the load buses through a zero impedance line to a common bus at which the total load current  $I_a$  is drawn. In Figure 1 each bus generator is connected to the load center through an equivalent radial line whose length is proportional to the respective incremental loss factor  $LFg_i'$ . While using the concept of load center the entire transmission loss is shared by the GENCOs only.

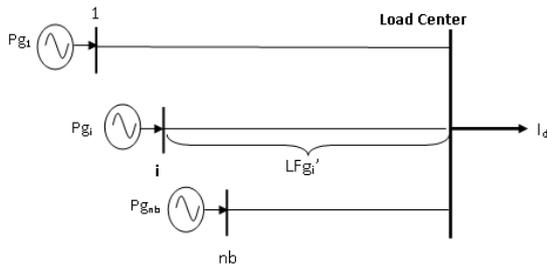


Figure 1. Loss Factor of Bus Generation referred to Load Center

**B. Generation Center**

In a deregulated power system DISCOs are also competing in the market and in order to compare the relative contribution of different bus demands to transmission loss, a reference point called “Generation Center” is introduced by us in [20, 21]. The “Generation Center” is obtained by connecting each of the generation buses through a zero impedance line to a common bus at which the total generation current  $I_g$  is supplied by all bus generation. In Figure 2 each bus demand is connected to the generation center through an equivalent radial line whose length is proportional to the respective incremental loss factor  $LFd_i'$ . While using the concept of generation center the entire transmission loss is shared by the DISCOs only.

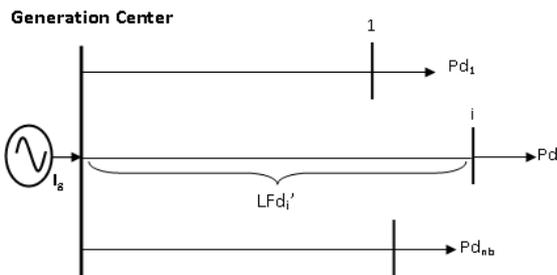


Figure 2. Loss Factor of Bus Demands referred to Generation Center

**C. Market Center**

In a market based restructured power system the total loss has to be shared equally between the suppliers and consumers since both parties use the transmission network for their own profit. In order to share half the losses equitably between the GENCO participants the calculated loss factors of generations  $LFg_i'$  shown in figure 1 are divided by two. Now the loss factor of generation at bus  $i$  is  $LFg_i$ . The next section explains in detail the procedure to calculate  $LFg_i$ .

If “Load Center” is the center of gravity of all loads then the Market Center is defined as a fictitious bus in the system which is located half the distance between each GENCO participant and the load center as shown in Figure 3.

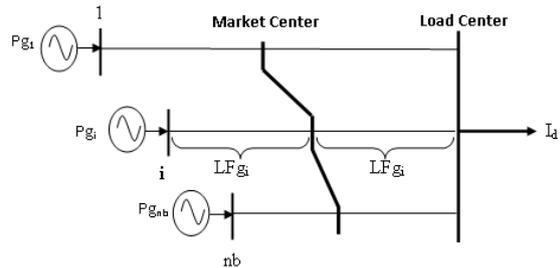


Figure 3. Load Center, Market Center and loss factors of Bus Generation

In order to share half the losses equitably between the DISCO participants the calculated loss factors of DISCOs  $LFd_i'$  shown in figure 2 are divided by two. Now the loss factor of demand at bus  $i$  is  $LFd_i$ . The next section explains in detail the procedure to calculate  $LFd_i$ . If the generation center is the center of gravity of all the bus generation, then the Market Center is defined as a fictitious bus in the system which is located half the distance between each bus DISCO participant and the generation center as shown in Figure 4.

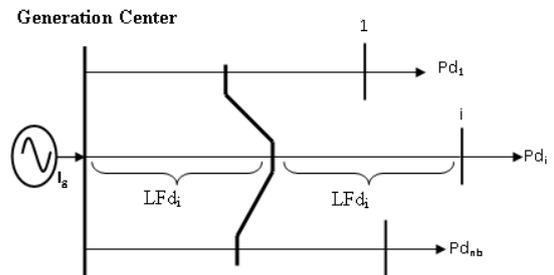


Figure 4. Generation Center, Market Center and loss factors of Bus Demands

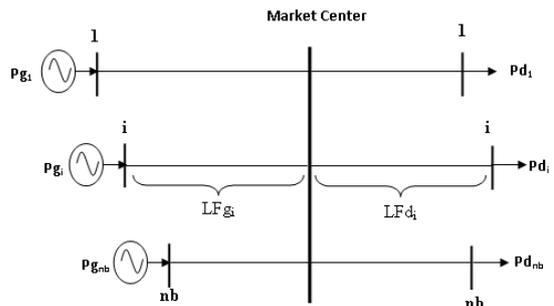


Figure 5. Unique Reference Point - Market Center



The Market center is located between each GENCO participant and load center and also between each DISCO participant and generation center.

The Market Center is defined in Figure 3 and Figure 4 are the same and hence both the figures can be merged to give Figure 5 [20, 21]. Each participant is connected to the market center using a radial line whose loss contribution characterizes the loss contributed by the participant in the power system.

### 3. PARTICIPANT BASED DISTRIBUTED SLACK POWER FLOW

The most widely used approach for modeling the steady state behavior of electric power transmission network is through the solution of the load flow. The power flow model used in vertically integrated power system and the distributed slack power flow model available in the literature considers a bus which has more generation than its load as a generation bus and a bus which has more load than its generation as a load bus to calculate the loss factors. This is one of the examples of well-established operating principles of vertically integrated power system carried forward to the deregulated power system. In the deregulated power market the access to the transmission system is open to all power market participants. In this environment it is necessary to calculate and account for the losses in an unambiguous, transparent and acceptable way for all market participants. Therefore even when both GENCOS and DISCOS are in the same bus the proposed formulation considers them as individual participants and not as equivalent injection/withdrawal as in conventional load flow analysis.

#### A. Existing Power Flow Models

For primary market, the power flow model is obtained by taking only the active power balance equation with the assumption that the bus voltage magnitudes are remaining constant. The power balance equation,  $F_i(\underline{\theta}) = 0; i = 1, 2 \dots nb$  used in active power flow model is given by (1)

$$F_i(\underline{\theta}) = f_i(\underline{\theta}) - (Pg_i - Pd_i) = 0; i = 1, 2 \dots nb \quad (1)$$

$Pg_i$  and  $Pd_i$  in (1) are the active power generation and demand at bus  $i$  respectively.  $f_i(\underline{\theta})$  in (1) denotes the active power injection into the grid at bus  $i$  through the lines connected at bus  $i$  expressed as a function of  $\underline{\theta}$ .

$$f_i(\underline{\theta}) = V_i \sum_{j=1}^{nb} V_j [G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}]; i = 1, 2 \dots nb \quad (2)$$

where  $G_{ij}$  is the  $ij^{th}$  element of bus conductance matrix and  $B_{ij}$  is the  $ij^{th}$  element of bus susceptance matrix.

Summing up all the  $nb$  equations in (1)

$$\sum_{i=1}^{nb} f_i(\underline{\theta}) - (\sum_{i=1}^{nb} Pg_i - \sum_{i=1}^{nb} Pd_i) = 0 \quad (3)$$

If a solution  $\underline{\theta}$  is obtained for the problem given in (1), then the first term in (3),  $\sum_{i=1}^{nb} f_i(\underline{\theta})$ , computed using this solution  $\underline{\theta}$  will give the transmission loss  $P_L$ . The second term in (3) will also be equal to transmission loss  $P_L$  which implies that the total generation is equal to total demand plus transmission loss.

However, there is a difficulty in specifying the injection data to the power flow problem in (1) since the bus demands are known and the transmission loss is not known a priori. To overcome this difficulty in the conventional Single Slack Bus Power Flow (SSBPF) model, the schedules of the generators at all the buses except one bus (slack bus) are specified. The slack bus takes the mismatch (the remaining load plus transmission loss). Instead of generation schedule, the phase angle of the slack bus is specified as zero. Assuming the slack bus as the last bus let us define the voltage phase angles vector  $\underline{\theta}'$  as  $\underline{\theta}' = [\theta_1, \theta_2 \dots \theta_{nb-1}]^T$ . Hence the equations to be solved in SSBPF model are given below

$$F_i(\underline{\theta}') = f_i(\underline{\theta}') - (Pg_i - Pd_i) = 0; i = 1, 2 \dots (nb - 1) \quad (4)$$

where the last bus  $nb$  is the slack bus,  $\theta_{nb} = 0$

In the Distributed Slack Power Flow (DSPF) model proposed in [17], the schedules of the generators at all the buses are specified assuming an estimated transmission loss. In this case the (3) will not be satisfied and there will be a mismatch,  $\delta$ , defined using (3) as

$$\delta \triangleq \sum_{i=1}^{nb} f_i(\underline{\theta}) - (\sum_{i=1}^{nb} Pg_i - \sum_{i=1}^{nb} Pd_i) \quad (5)$$

This mismatch  $\delta$  is distributed to all buses using participation factors  $\alpha_i; i = 1, 2 \dots nb$  such that  $\sum_{i=1}^{nb} \alpha_i = 1$ . In the DSPF model the equations at all the  $nb$  buses are taken for solution, the state vector  $\underline{x} = \begin{bmatrix} \underline{\theta}' \\ \delta \end{bmatrix}$ .

The DSPF model is given by

$$F_i(\underline{x}) = f_i(\underline{\theta}') - (Pg_i - Pd_i) - \alpha_i \delta = 0; i = 1, 2 \dots nb \quad (6)$$

#### B. PBDSPPF MODEL

In the proposed PBDSPPF formulation, for simplicity, let us assume that every bus has either GENCO participants or DISCO participants but not both. If a bus has both GENCO participants and DISCO participants the bus is split into two buses, one containing only the GENCO participants and other containing only the DISCO participants. There are  $ng$  GENCO participants distributed to  $ng$  generator buses which are numbered from  $1, 2 \dots ng$ . There are  $nd$  DISCO participants



distributed to  $nd$  demand buses which are numbered next as  $(ng + 1), (ng + 2), \dots (ng + nd)$ . The total number of buses in the network is  $nb$  which is equal to  $ng + nd$ . By taking the last bus as reference bus,  $\theta_{nb} = 0$  the voltage phase angle vector  $\underline{\theta}'$  is given as

$$\underline{\theta}' = [\theta_1, \theta_2 \dots \theta_{nb-1}]^T \quad (7)$$

In PBDSPP model for the generator buses, the active power injection balance equation,  $Fg_i(\underline{\theta})$  is written using (1) as

$$Fg_i(\underline{\theta}) = fg_i(\underline{\theta}) - Pg_i = 0; i = 1, 2 \dots ng \quad (8)$$

where  $fg_i(\underline{\theta}) = f_i(\underline{\theta})$ . For the load buses, the active power withdrawal balance equation,  $Fd_i(\underline{\theta})$  is written using (1) as

$$Fd_i(\underline{\theta}) = fd_i(\underline{\theta}) - Pd_i = 0; \quad (9)$$

$$i = (ng + 1), (ng + 2) \dots nb$$

where  $fd_i(\underline{\theta}) = -f_i(\underline{\theta}); i = (ng + 1), (ng + 2) \dots nb$

Summing up (8)

$$\sum_{i=1}^{ng} fg_i(\underline{\theta}) - \sum_{i=1}^{ng} Pg_i = 0 \quad (10)$$

Summing up (9)

$$\sum_{i=(ng+1)}^{nb} fd_i(\underline{\theta}) - \sum_{i=(ng+1)}^{nb} Pd_i = 0 \quad (11)$$

Subtracting (11) from (10)

$$\left[ \sum_{i=1}^{ng} fg_i(\underline{\theta}) - \sum_{i=(ng+1)}^{nb} fd_i(\underline{\theta}) \right] - \left[ \sum_{i=1}^{ng} Pg_i - \sum_{i=(ng+1)}^{nb} Pd_i \right] = 0 \quad (12)$$

In the PBDSPP model the schedules of all the generation and loads are specified assuming an estimated transmission loss or zero loss. When there is a mismatch or slack (or loss)  $\delta$  is defined as

$$\delta \triangleq \left[ \sum_{i=1}^{ng} fg_i(\underline{\theta}) - \sum_{i=(ng+1)}^{nb} fd_i(\underline{\theta}) \right] - \left[ \sum_{i=1}^{ng} Pg_i - \sum_{i=(ng+1)}^{nb} Pd_i \right] \quad (13)$$

The mismatch  $\delta$  is equally distributed between the two groups of participants, GENCO participants and DISCO participants. This half mismatch  $\delta/2$  is distributed to generator buses using participation factors  $\alpha_{gi}; i = 1, 2, \dots ng$ . The participation factor  $\alpha_{gi}$  is the ratio of generation at the bus  $i$  to the total generation and hence

$$\sum_{i=1}^{ng} \alpha_{gi} = 1 \quad (14)$$

The other half mismatch  $\delta/2$  is distributed to demand buses using participation factors  $\alpha_{di}; i = (ng + 1), (ng + 2), \dots (ng + nd)$ . The participation factor  $\alpha_{di}$  is the ratio of demand at bus  $i$  to the total demand and hence

$$\sum_{i=ng+1}^{ng+nd} \alpha_{di} = 1 \quad (15)$$

The PBDSPP model comprises two sets of equation namely (i) the generator bus active power injection balance equation given by (16) and (ii) the demand bus active power withdrawal balance equation as given by (17)

$$Fg_i(\underline{x}) = fg_i(\underline{\theta}') - (Pg_i + \alpha_{gi} \delta/2) = 0; \quad (16)$$

$$i = 1 \dots ng$$

$$Fd_i(\underline{x}) = fd_i(\underline{\theta}') - (Pd_i - \alpha_{di} \delta/2) = 0; \quad (17)$$

$$i = (ng + 1), (ng + 2), \dots nb$$

The proposed model is more suitable for the deregulated power market since its participant based model and considers each GENCO and DISCO as a participant and not as equivalent injection.

### C. Solution by Newton Raphson Algorithm

The above model is solved by Newton-Raphson (N-R) algorithm. The state correction equation which is obtained using the first order Taylor's series expansion of the GENCO participant's bus power injection equation (16) and DISCO participant's bus power withdrawal equation (17) is represented in matrix form as

$$\begin{bmatrix} \frac{\partial fg}{\partial \underline{\theta}'^{(h)}} & -\frac{\alpha_g}{2} \\ \frac{\partial fd}{\partial \underline{\theta}'^{(h)}} & \frac{\alpha_d}{2} \end{bmatrix}_{\underline{x}^{(h)}} \begin{bmatrix} \Delta \underline{\theta}'^{(h)} \\ \Delta \delta^{(h)} \end{bmatrix} = \begin{bmatrix} \Delta Fg(\underline{x}^{(h)}) \\ \Delta Fd(\underline{x}^{(h)}) \end{bmatrix} \quad (18)$$

where  $\underline{x} = [\underline{\theta}' \quad \delta]^T$  is the state vector and  $h$  is the iteration number. The mismatch vectors are calculated using (19) and (20)

$$\Delta Fg_i(\underline{x}^{(h)}) = Pg_i + \alpha_{gi}(\delta^{(h)}/2) - fg_i(\underline{\theta}'^{(h)}); \quad (19)$$

$$i = 1, 2, \dots ng$$

$$\Delta Fd_i(\underline{x}^{(h)}) = Pd_i - \alpha_{di}(\delta^{(h)}/2) - fd_i(\underline{\theta}'^{(h)}); \quad (20)$$

$$i = (ng + 1), (ng + 2), \dots nb$$

The load flow solution  $\underline{x}$  is obtained using Newton-Raphson algorithm. The final schedules  $\underline{Pg}^S, \underline{Pd}^S$  of the participants are calculated by distributing the mismatch  $\delta$  to each participant using the participation vector for bus generation  $\underline{\alpha}_g$  and the bus demand  $\underline{\alpha}_d$  as given by



$$Pg_i^S = Pg_i + \alpha_{gi}(\delta/2); i = 1, 2, \dots, ng \quad (21)$$

$$Pd_i^S = Pd_i - \alpha_{di}(\delta/2); i = (ng + 1), (ng + 2) \dots nb \quad (22)$$

#### D. Participant based incremental loss coefficients

Transmission loss is given by sum of the net power injection at all buses. This is obtained by subtracting the summed up equations of (16) from the summed up equations of (17) and is given by

$$\sum_{i=1}^{ng} f_{gi}(\underline{\theta}') - \sum_{i=(ng+1)}^{nb} f_{di}(\underline{\theta}') = \sum_{i=1}^{ng} Pg_i - \sum_{i=(ng+1)}^{nb} Pd_i + \delta \quad (23)$$

The sum of injections given by  $\sum_{i=1}^{ng} f_{gi}(\underline{\theta}') - \sum_{i=(ng+1)}^{nb} f_{di}(\underline{\theta}')$  is equal to transmission loss  $P_L$  and hence  $P_L$  can be written as

$$P_L = \sum_{i=1}^{ng} Pg_i - \sum_{i=(ng+1)}^{nb} Pd_i + \delta \quad (24)$$

The incremental transmission loss vector of the GENCO participants is obtained by differentiating (24) with respect to the real power generation vector  $\underline{Pg}$

$$\left[ \frac{\partial P_L}{\partial \underline{Pg}} \right]^T = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}_{ng} + \left[ \frac{\partial \delta}{\partial \underline{Pg}} \right]^T \quad (25)$$

The incremental transmission loss of the DISCO participants is obtained by differentiating (24) with respect to the real power demand vector  $\underline{Pd}$

$$\left[ \frac{\partial P_L}{\partial \underline{Pd}} \right]^T = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}_{nb} + \left[ \frac{\partial \delta}{\partial \underline{Pd}} \right]^T \quad (26)$$

$\partial \delta / \partial \underline{Pg}$  and  $\partial \delta / \partial \underline{Pd}$  can be obtained from the inverse of the Jacobian matrix given in (18). The inverse of the Jacobian matrix is represented as

$$\begin{bmatrix} \frac{\partial f_g}{\partial \theta'^{(h)}} & -\frac{\alpha_g}{2} \\ \frac{\partial f_d}{\partial \theta'^{(h)}} & \frac{\alpha_d}{2} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{\partial \theta'^{(h)}}{\partial \underline{Pg}} & \frac{\partial \theta'^{(h)}}{\partial \underline{Pd}} \\ \frac{\partial \delta^{(h)}}{\partial \underline{Pg}} & \frac{\partial \delta^{(h)}}{\partial \underline{Pd}} \end{bmatrix} \quad (27)$$

$\partial \delta^{(h)} / \partial \underline{Pg}$  and  $\partial \delta^{(h)} / \partial \underline{Pd}$  are calculated from the last row of the inverse of the Jacobian matrix given in (27).

The incremental transmission loss of GENCO participant connected to bus  $i$  is given by (25) is rewritten as

$$ITLg_i = \frac{\partial P_L}{\partial Pg_i} = 1 + \frac{\partial \delta}{\partial Pg_i}; i = 1, 2 \dots ng \quad (28)$$

and is defined as the change in loss due to 1 MW increase in injection of the GENCO participant at the bus  $i$  with respect to Market Center and the corresponding increase in the demand (1 MW-associated losses caused by the GENCO at the  $i^{th}$  bus) withdrawn at the Market Center by all the DISCO participants. Since GENCO participants and DISCO participants use the transmission network for their own profit, half the total loss ( $\frac{P_L}{2}$ ) has to be shared by the GENCO participants and another half ( $\frac{P_L}{2}$ ) has to be shared by DISCO participants. Among the loss contributed by the GENCO participants the incremental transmission loss factors characterize the loss contributed by each GENCO participant but these incremental transmission loss coefficients overestimate the loss. Since  $\sum_{i=1}^{ng} Pg_i ITLg_i > \frac{P_L}{2}$  these coefficients are normalized using the normalization factor given below

$$NFg = \frac{P_L/2}{\sum_{i=1}^{ng} Pg_i ITLg_i} \quad (29)$$

The loss factor of generation at bus  $i$   $LFg_i$  is defined as the normalized incremental transmission loss of generator at bus  $i$  given by

$$LFg_i = NFg \times ITLg_i; i = 1 \dots ng \quad (30)$$

Then

$$\frac{P_L}{2} = \sum_{i=1}^{ng} LFg_i Pg_i \quad (31)$$

The incremental transmission loss of DISCO participant connected to bus  $i$  is given by (26) is rewritten as

$$ITLd_i = \frac{\partial P_L}{\partial Pd_i} = -1 + \frac{\partial \delta}{\partial Pd_i}; i = (ng + 1), (ng + 2) \dots nb \quad (32)$$

and is defined as the change in loss due to 1 MW increase in withdrawal of the DISCO participant at the bus  $i$  with respect to the Market Center and the corresponding increase in the generation (1 MW + associated losses caused by the demand at the  $i^{th}$  bus) delivered at the Market Center by all the GENCO participants. Among the loss contributed by the DISCO participants the incremental transmission loss factors characterize the loss contributed by each DISCO participant but these incremental transmission loss coefficients overestimate the loss. Since  $\sum_{i=(ng+1)}^{nb} Pd_i ITLd_i > \frac{P_L}{2}$  these coefficients are normalized using the normalization factor given below



$$NFd = \frac{P_{L/2}}{\sum_{i=(ng+1)}^{nb} Pd_i ITLd_i} \quad (33)$$

The loss factor of demand at bus  $i$   $LFd_i$  is defined as the normalized incremental transmission loss of demand at bus  $i$  given by

$$LFd_i = NFd \times ITLd_i; i = (ng + 1), (ng + 2) \dots nb \quad (34)$$

Then

$$\frac{P_L}{2} = \sum_{i=(ng+1)}^{nb} LFd_i Pd_i \quad (35)$$

The loss factors  $LFg_i$  and  $LFd_i$  can be calculated using  $ITLg_i$  and  $ITLd_i$  respectively. The calculation of incremental loss factors requires  $\frac{\partial \delta}{\partial Pg}$  and  $\frac{\partial \delta}{\partial Pd}$  which can be obtained from the inverse of the Jacobian matrix.

#### E. Features of the Proposed Model

The important features of the proposed power flow model are

- The transmission losses are not balanced at the slack bus but are distributed to each participant.
- The transmission loss in the system depends only upon the network topology and present operating condition and not on the choice of the reference bus.
- Power flow results does not change even when slack bus changes.
- Enable us to obtain reference bus independent loss factors.

#### 4. CASE STUDY

The proposed PBDSPPF based N-R algorithm is compared with the well-known single slack bus based N-R algorithm. A case study is carried out on an IEEE 30 bus test system whose bus data is given in [21]. The system has 30 buses, 41 transmission lines, 6 generators, 23 loads, and the compensators are neglected. The generators are connected to bus 1, 2, 5, 8, 11 and 13. In the base case, the total system load is 283.4 MW and 126.2 MVAR. The comparison of the load flow solution obtained by single slack bus based N-R method and PBDSPPF based N-R method are shown in Table I. For single slack bus based load flow method bus 1 is taken as the slack bus whereas for PBDSPPF method bus 1 is taken as reference bus. The total system real power loss computed using Newton Raphson algorithm for single slack bus based load flow method is 6.59 MW. The total loss is added to the slack generator. The single slack bus based N-R method converges in four iteration for a tolerance of 0.01 MW.

TABLE I. COMPARISON OF THE LOAD FLOW SOLUTION FOR IEEE 30 BUS SYSTEM USING SSBPF METHOD AND PBDSPPF METHOD

Bus No.	SSBPF Method			PBDSPPF		
	Gen. MW	Load MW	Angle radians	Gen. MW	Load MW	Angle radians
1	139.99	0.00	0	134.88	0	0
2	40.00	21.70	-0.0475	40.44	0	-0.0454
3	-	2.40	-0.0773	-	2.37	-0.0747
4	-	7.60	-0.0925	-	7.52	-0.0893
5	60.00	94.20	-0.1187	60.67	0	-0.1140
6	-	0	-0.1101	0	0	-0.1065
7	-	22.80	-0.1230	0	22.55	-0.1187
8	10.00	30.00	-0.1184	10.11	0	-0.1145
9	-	0	-0.1408	-	0	-0.1363
10	-	5.80	-0.1676	-	5.74	-0.1628
11	10.00	0.00	-0.1223	10.11	0	-0.1176
12	-	11.20	-0.1430	-	11.08	-0.1382
13	30.00	0.00	-0.1058	30.33	0	-0.1006
14	-	6.20	-0.1606	-	6.13	-0.1555
15	-	8.20	-0.1631	-	8.11	-0.1580
16	-	3.50	-0.1579	-	3.46	-0.1530
17	-	9.00	-0.1683	-	8.90	-0.1634
18	-	3.20	-0.1771	-	3.16	-0.1719
19	-	9.50	-0.1820	-	9.39	-0.1768
20	-	2.20	-0.1794	-	2.18	-0.1743
21	-	17.50	-0.1754	-	17.31	-0.1704
22	-	0	-0.1750	-	0	-0.1701
23	-	3.20	-0.1731	-	3.16	-0.1680
24	-	8.70	-0.1806	-	8.60	-0.1755
25	-	0	-0.1840	-	0	-0.1791
26	-	3.50	-0.1915	-	3.46	-0.1864
27	-	0	-0.1813	-	0	-0.1766
28	-	0	-0.1196	-	0	-0.1157
29	-	2.40	-0.2032	-	2.37	-0.1981
30	-	10.60	-0.2188	-	10.48	-0.2135
31	-	-	-	-	21.46	-0.0454
32	-	-	-	-	93.16	-0.1140
33	-	-	-	-	29.67	-0.1145
Total	289.99	283.4		286.54	280.26	

In the proposed PBDSPPF method the buses 2, 5 and 8 which have both GENCO and DISCO participants are split into two using the technique given in [19] and are connected to buses 31, 32 and 33 respectively. The PBDSPPF method converges in three iteration for a tolerance of 0.01 MW. The line flows computed using SSBPF and PBDSPPF are given in appendix Table AI. The total real power loss computed using PBDSPPF method is 6.28 MW as against 6.59 MW for single slack bus based load flow method and is distributed to each participant proportional to its injection or withdrawal at the bus as shown in Table I. The voltage angle obtained by both the methods are also shown in Table I and the maximum difference in voltage angle solution between



the proposed PBDSPF method and single slack bus based load flow method is in bus 30 which is 0.0053 radians.

The loss computed by each method when slack bus changes are shown in Table II. The single slack bus based N-R method the loss computed varies widely whenever the slack bus changes. The percentage change in loss in single slack bus based load flow method is 7.18%. The main advantage of the PBDSPF based N-R method is that the loss computed is same irrespective of the selection of slack bus as shown in Table II.

TABLE II. COMPARISON OF TOTAL REAL POWER LOSS COMPUTED BY SSBPF METHOD AND PBDSPF METHOD WHEN THE MARGINAL BUS CHANGES

Marginal Bus	SSBPF (MW)	PBDSPF (MW)
1	6.595	6.285
2	6.407	6.285
5	6.182	6.285
8	6.153	6.285
11	6.162	6.285
13	6.289	6.285

## 5. CASE STUDY ON PARTICIPANT BASED INCREMENTAL LOSS FACTORS

The aim of the case studies is to show how the proposed participant based incremental loss factors allocate loss to each participant. Case study using a radial five bus test system is presented in this section.

### A. Loss Allocation of a radial five bus system

Consider a five-bus, four-line radial system shown in figure 6. All lines have the same resistance, reactance and half line charging capacitance of 0.005 p.u., 0.01 p.u., and 0.01 p.u. respectively. Base MVA is 100. The generator bus voltages are maintained at 1.02 p.u. The schedules obtained using Participant based Distributed Slack Power flow method with bus 1 as slack bus when reactive power consumption by loads taken as zero are given in Table III. The total transmission loss in the system is 8.671 MW.

The loss factors of each participant calculated using the Jacobian matrix of participant based load flow and loss allocation to the individual participants of the radial five-bus system are given in Table III. The generation center is located close to bus 1, the load center is located close to bus 5 and market center is located in between bus 2 and bus 3 close to bus 3.

The total loss in the system amounts to 8.671 MW and is equally shared between the suppliers and consumers. Among the loss factors for GENCOs the loss factor of GENCO connected to bus 3 is -0.00172 is less when compared to other GENCOs connected to bus 2

and bus 1 whose loss factors are 0.01159 and 0.02339 respectively.

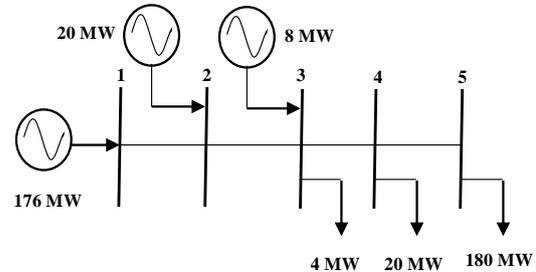


Figure 6. Radial Five Bus System

TABLE III. RESULTS OF THE LOSS ALLOCATION AND INCREMENTAL LOSS FACTORS COMPUTED WITH RESPECT TO MARKET CENTER FOR THE RADIAL FIVE BUS SYSTEM

Bus No.	Gen. MW $P_g^s$	Load MW $P_d^s$	Lf <sub>g</sub>	Loss allocated to Generator MW	LF <sub>d</sub>	Loss allocated to load MW
1	179.740	-	0.02339	3.740	-	-
2	20.425	-	0.01159	0.425	-	-
3	8.170	3.915	-0.00172	0.170	0.001719	0.085
4	-	19.575	-	-	0.010013	0.425
5	-	176.174	-	-	0.022934	3.826
Total	208.335	199.664		4.335		4.336

The loss factor of GENCO at bus 3 is negative since the market center is located between bus 2 and bus 3 nearer to bus 3 and the loss factor is less when compared to other GENCO participants since it is nearer to the market center. Similarly among the DISCOs the loss factors of the DISCO connected to bus 3 is 0.001719 which is less than the loss factor of other DISCOs connected to bus 4 and bus 5 whose loss factors are 0.010013 and 0.022934 respectively. The loss factor of DISCO at bus 3 is less since it is nearer to the market center. The GENCO participant connected to bus 1 and DISCO participant connected to bus 5 are far away from the market center and hence their loss factors are more when compared to the loss factors of other participants. Even though a GENCO and a DISCO are connected to bus 3, their loss factors are different as seen from the Table III. Table IV shows the loss allocation by Z-bus loss allocation method [5]. The loss factors calculated from the method overestimates the transmission loss and so they are normalized using (NLF) and shown in Table IV.



TABLE IV. RESULTS OF THE LOSS ALLOCATION AND INCREMENTAL LOSS FACTORS USING Z- BUS LOSS ALLOCATION

Bus No.	1	2	3	4	5
<b>Bus 1 as Slack</b>					
Gen. (MW)	<b>184.041</b>	20	8	-	-
Load (MW)	-	-	4	20	180
NLF	0.0497	0.0244	0.0039	-0.02	0.0113
Loss	3.9955	0.2090	0.2681	0.194	3.3790
<b>Bus 2 as Slack</b>					
Gen. (MW)	176	<b>27.872</b>	8	-	-
Load (MW)	-	-	4	20	180
NLF	0.0503	0.0253	0.0042	-0.02	0.0119
Loss	3.7489	0.2963	0.2656	0.193	3.3707

As seen from Table IV the losses are allocated to individual buses and losses change as current injection in that bus changes. When bus 1 is slack the loss allocated to bus 2 is 0.209 MW which changes to 0.2963 when bus 2 is changed as slack bus. Here the losses are allocated to the buses based on the magnitude of current injected at that bus where as in the proposed method losses are allocated to the participants based on their electrical distance from Market Center.

## 6. CONCLUSION

This paper proposes a Participant Based Distributed Slack Power Flow (PBDSPF) model for distributing the loss to all individual participants using participation vector. The proposed PBDSPF model is tested on IEEE 30 bus system and the voltage solution is compared with existing single slack bus based load flow method. The main advantage of the proposed PBDSPF method is that losses do not change with respect to the selection of the slack bus. The loss factors calculated by the proposed method are more suitable for the deregulated power system since they are calculated for each participant based on their distance from the Market Center and not bus wise as in existing techniques. Using the proposed model participant based non-linear and linear optimal power flow models can be developed for pricing of electricity in the deregulated power system.

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**Dr. Arunachalam Sundaram** He completed his Masters in Power system engineering with Gold Medal from Anna University in the year 2004. He received his Ph.D in Electrical Engineering from Anna University, India in the year 2014. He has over 13 years of academic experience and is currently working as Assistant Professor in the Department of Electrical,

Electronic Engineering and Technology, Jubail Industrial College, Al-Jubail, Kingdom of Saudi Arabia. He is a member of IEEE and his areas of interest are deregulated power system and optimization applied to power system engineering.

## APPENDIX A

Table AI. REAL POWER FLOWS COMPUTED USING SSBPF AND PBDSPF FOR IEEE 30 BUS SYSTEM

Line No.	From Bus – To Bus	SSBPF (MW)		PBDSPF (MW)		Line No.	From Bus – To Bus	SSBPF (MW)		PBDSPF (MW)	
		Real Power Flow	Real Power Loss	Real Power Flow	Real Power Loss			Real Power Flow	Real Power Loss	Real Power Flow	Real Power Loss
1	1-2	92.23	1.46	88.58 -	1.34	23	12-15	21.56	0.33	21.39	0.33
	2-1	-90.77		87.24			15-12	-21.23		-21.07	
2	1-3	47.77	0.96	46.30 -	0.91	24	12-16	11.03	0.13	10.99	0.13
	3-1	-46.81		45.40			16-12	-10.89		-10.85	
3	2-4	29.64	0.48	29.03	0.47	25	13-12	30.00	0.00	30.33	0.00
	4-2	-29.16		-28.56			12-13	-30.00		-30.33	
4	2-5	40.00	0.73	38.63	0.69	26	14-15	2.62	0.02	2.61	0.02
	5-2	-39.27		-37.95			15-14	-2.60		-2.59	
5	2-6	39.43	0.85	38.56	0.81	27	15-18	8.17	0.08	8.12	0.08
	6-2	-38.59		-37.75			18-15	-8.09		-8.05	
6	3-4	44.41	0.26	43.02	0.24	28	15-23	7.46	0.07	7.43	0.07
	4-3	-44.15		-42.78			23-15	-7.39		-7.36	
7	4-6	43.01	0.21	41.87	0.20	29	16-17	7.39	0.05	7.39	0.05
	6-4	-42.80		-41.66			17-16	-7.34		-7.33	
8	4-12	22.70	0.00	21.96	0.00	30	18-19	4.89	0.02	4.88	0.02
	12-4	-22.70		-21.96			19-18	-4.87		-4.86	
9	5-7	5.07	0.02	5.46	0.02	31	20-19	4.64	0.01	4.54	0.01
	7-5	-5.05		-5.44			19-20	-4.63		-4.53	
10	6-7	17.84	0.09	17.20	0.09	32	22-21	2.31	0.00	2.29	0.00
	7-6	-17.75		-17.11			21-22	-2.31		-2.29	
11	6-8	20.53	0.05	20.10	0.05	33	22-24	4.93	0.04	4.87	0.04
	8-6	-20.48		-20.05			24-22	-4.90		-4.84	



Line No.	From Bus – To Bus	SSBPF (MW)		PBDSPF (MW)		Line No.	From Bus – To Bus	SSBPF (MW)		PBDSPF (MW)	
		Real Power Flow	Real Power Loss	Real Power Flow	Real Power Loss			Real Power Flow	Real Power Loss	Real Power Flow	Real Power Loss
12	6-9	15.90	0.00	15.47	0.00	34	23-24	4.19	0.03	4.19	0.03
	9-6	-15.90		-15.47			24-23	-4.15		-4.16	
13	6-10	11.04	0.00	10.82	0.00	35	24-25	0.350	0.003	0.394	0.003
	10-6	-11.04		-10.82			25-24	-0.347		-0.390	
14	6-28	16.07	0.04	15.83	0.04	36	25-26	3.55	0.05	3.51	0.05
	28-6	-16.03		-15.79			26-25	-3.50		-3.46	
15	8-28	0.48	0.00	0.49	0.00	37	27-25	3.22	0.02	3.14	0.03
	28-8	-0.48		-0.49			25-27	-3.20		-3.12	
16	9-10	25.90	0.00	25.58	0.00	38	27-29	6.19	0.09	6.12	0.09
	10-9	-25.90		-25.58			29-27	-6.10		-6.04	
17	10-20	6.88	0.05	6.76	0.05	39	27-30	7.09	0.17	7.02	0.16
	20-10	-6.84		-6.72			30-27	-6.93		-6.85	
18	10-17	1.664	0.002	1.567	0.002	40	28-27	16.51	0.00	16.28	0.00
	17-10	-1.662		-1.564			27-28	-16.51		-16.28	
19	10-21	15.30	0.11	15.12	0.11	41	29-30	3.70	0.03	3.66	0.03
	21-10	-15.18		-15.02			30-29	-3.67		-3.63	
20	10-22	7.30	0.05	7.22	0.05	42	2-31	-	-	21.46	0.00
	22-10	-7.25		-7.17			31-2			-21.46	
21	11-9	10.00	0.00	10.11	0.00	43	5-32	-	-	93.16	0.00
	9-11	-10.00		-10.11			32-5			-93.16	
22	12-14	8.91	0.10	8.84	0.10	44	8-33	-	-	29.67	0.00
	14-12	-8.82		-8.74			33-8			-29.67	