Discrete dynamics on $\mathbb{C}$: periodic orbits, fractal boundaries, and basin entropy

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Iterative processes defined on the complex plane $\mathbb{C}$ provide a convenient arena for studying nonlinearity and possibly even chaos in two dimensions. Here, our attention will focus primarily on maps of two quite general classes in the presence of quadratic feedback—*analytic* (satisfying the Cauchy-Riemann conditions everywhere) and *nonanalyic*. Despite their mathematical simplicity, under repeated iteration both maps can lead to remarkably complicated behaviours and fractal structures in their respective dynamics.

This talk will present an overview of some recent work that decomposes fractal basin boundaries on $\mathbb{C}$ into their fundamental constituents, namely unstable periodic points and their pre-images. The analysis involves deriving, then solving, hierarchies of polynomial equations whose roots are period-N points (where $N = 2, 3, 4, ...$) and whose degrees increase exponentially with N. These orbits may be ordered according to their linear-instability growth rate but, for symmetry reasons, they tend not to combine into distinct solution families. The maps considered here tend to have just two attractors, and we have estimated numerically the uncertainty dimension of the fractal boundary between the two basins of attraction. Sets of results will also be given for numerical calculations of the entropy associated with these basins and their boundary.