A fuzzy evidential reasoning-based approach for risk assessment of deep foundation pit

Daojiang Wei[a] Dongsheng Xu[b*] Yong Zhang [c]

（[a] School of Civil Engineering and Architecture，Hubei University of art and science，Xiangyang Hubei 441053，China; [b] Salford Business School, the University of Salford, UK ; [c] School of Civil Engineering，Xi'an University of Architecture and Technology, Xi'an Shaanxi 710055，China）

Abstract: Traditional risk assessment methods, such as the probabilistic methods, are not effectively used in the construction works of a deep foundation pit (DFP) when data set collected are incomplete or vague input takes place. A new method based on fuzzy evidential reasoning approach is proposed in this paper to assess the overall risk level of a DFP construction project. Firstly, the method defines risks as the products of occurrence likelihood multiplying consequence severity, which is further depicted by trapezoidal fuzzy numbers. Thereafter, the fuzzy analytical hierarchy process is adopted to calculate the weighs of different hazardous events that may occur in a DFP construction project. The overall risk level of a DFP project therefore could be achieved through aggregating the risk level of all hazardous events based on evidential reasoning algorithm. However, due to the existence of intersections among more than two continuous fuzzy evaluation grades rather than between two adjacent grades, the prevailing aggregation method is not suitable any more. So, a new aggregated probability mass along with the reassigning method in relation to the degree of belief belonging to the fuzzy intersection of two grades is thus put forward in this paper, as a result to make the evidential reasoning possible. A case study on risk assessment of the DFP of underground traffic project of Zhengzhou comprehensive transportation hub in China is introduced to illustrate the application of the proposed method. The result indicates that the overall risk level of a DFP project could be assessed effectively under the scenario that more than two continuous fuzzy evaluation grades intersect rather than only two adjacent grades. Moreover, comparing with the traditional methods, the result obtained in the case study by using the proposed method seems to be more reasonable.

* Corresponding author.
E-mail addresses: Daojiang Wei: 295419709@qq.com; Dongsheng Xu*: d.xu1@edu.salford.ac.uk; Yong Zhang: 10929@hbuas.edu.cn

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1 Introduction

The risk management of DFP in the construction stage attracts widespread attention in the construction industry (Li et al., 2018; Xiong et al., 2018). According to the definition of risk management in the Subway and Underground Engineering Construction Risk Management Guidelines developed by Ministry of Construction, People's Republic of China (MoC, 2007), the risk management of DFP in the construction stage consists of two parts: risk assessment and risk control. Of which, the risk assessment is further divided into risk identification, risk analysis, and risk evaluation. Whether the construction risk of DFP can be assessed timely and objectively is not only related to the rationality of risk control, but also to the safety of DFP per se and the effectiveness of the protection measures on the surrounding environment of the DFP. Since the information available for risk assessment, including geotechnical parameters and hydrographical condition, are usually of uncertainty and incompleteness, a series of fuzzy methods were employed to assess the risk of DFP construction in previous researches. For example, a formalized procedure and a fuzzy-based risk assessment method developed by Choi et al. (2004); a fuzzy comprehensive evaluation model based on Bayesian network proposed by Zhou and Zhang (2011); and a hybrid framework integrating step-wise weight assessment ratio analysis with complex proportional assessment (Valipour et al., 2017). The common feature of the above approaches is that, the occurrence likelihood (L) and the consequence severity (S), the two parameters which measure the magnitude of risks that may happen, are usually estimated by risk assessors' human scoring. However, assessors are more likely to make qualitative assessments in the linguistic terms rather than precise scores. Therefore, the research of risk assessment for the construction of DFP under the linguistic environment keeps closer to the needs of construction practice. Moreover, since the risk assessment results are often exhibited in certain values in previous research, the extent to which the certain values are reliable is unrevealed.

Evidential reasoning (ER) is a method of evidence fusion proposed by Yang and Xu (2002) on the basis of evidence theory, which could be used to illustrate incomplete information directly and to deal with problems of assessment in the linguistic environment. The assessment result obtained using ER is a set of degree of beliefs associated with a predefined frame of discernment. In recent years, a number of scholars elaborated to combine ER with fuzzy set theory, which is usually called as fuzzy evidential reasoning (FER) approach, to carry out a systematic risk assessment. For example, a semi quantitative approach based on FER was proposed by Deng et al. (2011) and Liu et al. (2005) to perform risk analysis for complex systems due to lack of data and insufficient understanding of the failure mechanisms; a risk assessment model based on FER was adopted by Mokhtari et al. (2012), Yang & Wang (2015) and Zhang et al. (2016) owing to the objective data is sometimes incomplete in offshore engineering system; in the presence of multiple experts supplying different and uncertain judgments on risk parameters, Certa et al. (2017) conducted a failure mode and related effects analysis using FER; John et al. (2014) built a model based on FER to solve the problem of risk assessment of seaport operations in a fuzzy uncertain environment. The main shortcoming of the present researches is that only the intersection between two adjacent fuzzy evaluation grades is considered, however, in fact, there may exist the scenario that more than two continuous fuzzy evaluation grades intersect rather than only two adjacent grades.

In the field of DFP construction, Du et al. (2014) and Cheng et al. (2016) have ever tried to apply evidence theory to the field of risk assessment of DFP construction, but there exists some defects which need to be addressed. For example, in Du et al. (2014)'s application, the interrelationship among risk evaluation grades was not considered, which may lead to counter-intuitive results (Yang & Xu, 2013). After all, the evidence theory is based on a frame of discernment composed of a set of propositions that are mutually exclusive and collectively exhaustive (Shafer, 2016). With respect to Cheng et al. (2016)' application, it is unreasonable to use occurrence probability (likelihood) only rather than the product of occurrence likelihood and consequence severity as the basis for risk assessment.

A new method with respect to risk assessment for DFP construction based on FER is put forward in this paper. In the proposed method, risks are defined as the products of occurrence likelihood multiplying consequence severity; and, the scenario that more than two continuous fuzzy evaluation grades intersect rather than only two adjacent grades is depicted.

The remainder of this paper is organized as follows: Section 2 briefs the theoretical basis of the method; Section 3 describes how the risk data of DFP were obtained by assessors; using FER, the risk assessment model of DFP construction is established in Section 4; Section 5 conducts validating analysis about the applicability of the method through a case study; and, further discussion is delivered in in Section 6, conclusions of this paper are drawn in Section 7.

2 Theoretical Bases

2.1 Fuzzy Set Theory

Fuzzy set theory Zadeh (1965) is a generalization of classical set theory. Compared with classical set theory, fuzzy set theory could deal with the uncertain phenomenon relating to the rationale of 'both this and that' rather than the one of 'if not this, then that'. So far, it has been popularized in model identification (Certa et al., 2017; Jiang et al., 2017; Kim & Zuo, 2018; Liu et al., 2011; Liu et al., 2013; Liu et al., 2013), risk assessment (An et al., 2011; An et al., 2016), and uncertainty decision-making (Mokhtari et al., 2012). In different DFP construction stages, the information available for risk assessment is often incomplete and vague owing not only to the uncertain geological and hydrological conditions but also the complicated surrounding environment. Thereby, the linguistic terms, such as 'likely' and 'frequent' are usually employed to express the risk judgements from the risk assessors. Under such circumstance, the fuzzy set theory is a useful tool which through converting the assessors' subjective judgements into fuzzy numbers to quantify risk assessments. In general, there are two kinds of fuzzy numbers usually adopted, namely, triangular fuzzy number and trapezoidal fuzzy number. Since the former could be regarded as the special case of the latter, the trapezoidal fuzzy number is employed in this paper, as shown in Fig.1.

Where $\mu_A(x)$ represents the membership of x to A, and which is defined as:

$$\mu_A(x)=\begin{cases}(x-a)/(b-a), & x\in(a,b)\\ 1 & x\in[b,c]\\ (d-x)/(d-c), & x\in(c,d)\\ 0 & otherwise\end{cases} \tag{1}$$

In which, a denotes the pessimistic rating, b and c are two endpoints of the interval which denotes the most plausible rating, d denotes the optimistic rating (Li & Liao, 2007).

2.2 FER

FER is the extension to the original ER approach, which is proposed by Yang et al. (2006) to deal with the vagueness or fuzzy uncertainty in fuzzy assessment issues where the evaluation grades are no longer distinctive individual grades, but are dependent fuzzy grades. Suppose that the assessment object is evaluated at the $\tilde{L}$ attributes on the basis of N evaluation grades $H_n\ (n=1,2,\ldots,N)$, and the relative weights of the $\tilde{L}$ attributes are denoted by $\omega=(\omega_1,\omega_2,\ldots,\omega_{\tilde{L}})$, which are normalized to satisfy the condition: $0 \le \omega_j \le 1$ and $\sum_{j=1}^{\tilde{L}} \omega_j = 1$. The evaluation grades H_n are not independent from each other due to the expression using a linguistic form, such as 'critical' and 'very critical', then H_n can be labeled with fuzzy evaluation grades which are trapezoidal fuzzy sets in this research. In general, only the intersection between two adjacent fuzzy evaluation grades is considered, which can be depicted as Fig. 2.

There are often three steps to conduct fuzzy assessment using FER approach as follows:

- Converting the evaluated values, which are generated from risks assessors' judgement originally and expressed as trapezoidal fuzzy number uniformly then, into a belief structure denoted by $S(e_j)=\{(H_n,\beta_{n,j}),n=1,2,\cdots,N\}$. In which, $e_j\ (j=1,2,\cdots,\tilde{L})$ is attribute and $\beta_{n,j}$ is the degree of belief which refers to the evaluation object assessed to a grade H_n on an attribute e_j, and meets the conditions as follows: $\beta_{n,j} \ge 0$ and $\sum_{n=1}^{N} \beta_{n,j} \le 1$.
- Aggregating all the evidences in terms of belief structure using analytical (non-recursive) FER algorithm, so that the aggregated degree of belief β_n and $\beta_{n,(n+1)}$ could be calculated respectively.
- Redistributing $\beta_{n,(n+1)}$ into β_n and β_{n+1}, and finally the fuzzy assessment result which is denoted by $S(Object)=\{(H_n,\beta_n),n=1,2,\ldots,N\}$ could be arrived.

The detail of FER algorithm and the procedure of redistribution with respect to $\beta_{n,(n+1)}$ are omitted in this paper due to the consideration of brevity. Interested reader will get reference from Yang et al. (2006).

3 Preparation works for risk assessment

3.1 Allocation of expert indices to risk assessment

The complicated process of risk assessment on a DFP project enables few cases can be completed by a single assessor (expert). In practice, a number of experts with different backgrounds or domains in relation to DFP safety are usually involved in the risk assessment. Considering the different working experience and knowledge background of experts, the influence of individual expert on the overall decision-making results is different. Therefore, the concept of expert index (EI) is introduced to calculate the influence of expertise (An et al., 2011).

Definition 1: Expert index refers to the measurement of the influence of individual expert on the group decision-making results, which can be denoted by:

$$EI_{\tilde{i}} = \frac{RI_{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} RI_{\tilde{i}}} \qquad (2)$$

Where m is the number of experts involved in the risk assessment, $RI_{\tilde{i}}$ stands for the relevant importance of the $\tilde{i}th$ expert according to his experience, knowledge, and expertise, which takes a value in the universe of 1 to 9. RI is defined in a manner that '1' means less importance, whereas '9' means most importance (An et al., 2011).

3.2 Development of a risk framework

Many possible causes of risks may impact DFP safety. Developing a risk framework aims to decompose these risk contributors into adequate details in which different risks associated with a DFP construction could be efficiently assessed (An et al., 2011; An et al., 2016). A bottom-up approach is employed for the development of a risk framework. That is, through experts brainstorming, the hazardous events related to the construction of a DFP are numerated to the utmost, and then, the risks that may arise from the hazardous events are categorized on a layer-by-layer basis until the top layer of the risks framework is received. Typically, a risk framework of DFP breaks down into four layers: the hazardous event level, the hazard group level, the sub-object level, and the total object level (depicted in Fig. 3).

3.3 Acquisition of the risk level of the hazardous events

As MoC (2007) and Mokhtari et al. (2012) explained, there are parameters including occurrence likelihood (frequency) and consequence severity (impact) that may affect the risk level of every hazardous event. A common demonstration of risk level is simply to multiply the occurrence likelihood by consequence severity, which can be illustrated as:

$$R = L \times S \tag{3}$$

While R refers to the risk level of each hazardous event, L represents its occurrence likelihood and S represents the consequence severity. Since the information available for risk assessment is often incomplete and vague owing not only to the uncertain geological and hydrological conditions but to the complicated surrounding environment, it is more reasonable to ask experts for fuzzy instead precise risk assessment using qualitative linguistic variables. To measure the occurrence likelihood, for example, the qualitative scales such as being unlikely, infrequent, occasional, likely and frequent could be used. Likewise, the scales of being negligible, marginal, moderate, critical, and catastrophic could be adopted to assess the consequence severity.

Going further, the trapezoidal fuzzy number is selected to depict the aforementioned qualitative scales with the satisfaction of three properties: available domain knowledge, simplicity of the membership function, and possible parametric optimization of the fuzzy sets (Samantra et al., 2017; Yuen, 2014). Thus, according to MoC (2007), the classification criteria of each grade with respect to the occurrence likelihood and the consequence severity are shown respectively in Table 1 and Table2.

Table 1 The classification criteria of occurrence likelihood

Grade	Linguistic description	Numerical values	Membership function
1	Unlikely	0-0.01%	$\{0,0,5.0E-5,1.0E-4\}$
2	Infrequent	0.01%-0.1%	$\{5.0E-5,1.0E-4,1.0E-3,5.5E-3\}$
3	Occasional	0.1%-1%	$\{1.0E-3,5.5E-3,1.0E-2,5.5E-2\}$
4	Likely	1%-10%	$\{1.0E-2,5.5E-2,1.0E-1,5.5E-1\}$
5	Frequent	10%-1.0	$\{1.0E-1,5.5E-1,1.0,1.0\}$

Table 2 The classification criteria of consequence severity

Grade	Linguistic description	Numerical values*	Membership function
1	Negligible	0-500	$\{0,0,250,500\}$

2	Marginal	500-1000	$\{250, 500, 1000, 3000\}$
3	Moderate	1000-5000	$\{1000, 3000, 5000, 7500\}$
4	Critical	5000-10000	$\{5000, 7500, 10000, 55000\}$
5	Catastrophic	>10000	$\{10000, 55000, 100000, 100000\}$

Note*: according to MoC (2007), consequence severity can be represented by a variety of forms. But in this paper, only the form of direct economic losses is adopted (Unit: ten thousands RMB)

In addition, Table 3 represents the risk matrices (risk grade: I to V) derived from the combination of L and S, which are universally applied in China.

Table 3 The risk matrix derived from the combination of L and S

		Consequence severity				
		Negligible	Marginal	Moderate	Critical	Catastrophic
Occurrence Likelihood	Unlikely	I	I	II	III	IV
	Infrequent	I	II	III	III	IV
	Occasional	I	II	III	IV	V
	Likely	II	III	IV	IV	V
	Frequent	II	III	IV	V	V

Based on the aforementioned classification criterions, the assessment of occurrence likelihood and consequence severity of each hazard in Fig.3 could be conducted as follows:

Suppose that there are m experts involved in risk evaluation of a DFP construction. With respect to an individual hazard j, the occurrence likelihood assessed by $\tilde{i}th$ expert is denoted by $L_j^{\tilde{i}} = \left\{L_{j,a}^{\tilde{i}}, L_{j,b}^{\tilde{i}}, L_{j,c}^{\tilde{i}}, L_{j,d}^{\tilde{i}}\right\}$, where $\tilde{i} = 1,2,\ldots,m$ and $j = 1,2,\ldots,\tilde{L}$. Then the aggregated occurrence likelihood of hazard j could be determined by:

$$L_j = \left\{ \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times L_{j,a}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}}, \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times L_{j,b}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}}, \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times L_{j,c}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}}, \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times L_{j,d}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}} \right\} \quad (4)$$

Herein $EI_{\tilde{i}}$ stands for the $\tilde{i}th$ expert's EI .

Similarly, the consequence severity of hazard j could be obtained by:

$$S_j = \left\{ \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times S_{j,a}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}}, \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times S_{j,b}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}}, \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times S_{j,c}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}}, \frac{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}} \times S_{j,d}^{\tilde{i}}}{\sum_{\tilde{i}=1}^{m} EI_{\tilde{i}}} \right\} \quad (5)$$

Subsequently, the value-at-risk of hazard j could be calculated by Eq. (3). It is worth noting that the same level of risk corresponds to multiple value-at-risks in Table 3. Since these value-at-risks are all trapezoidal fuzzy numbers, the universe of discourse will be overlapped with each other inevitably. This paper proposes to integrate the overlapped state into a uniform frame, as a result, one single risk level corresponds to only one fuzzy number.

Suppose that the nth grade of the risk is represented by $H_n\,(n = 1, 2, \ldots, 5)$. H_n could be obtained from the combinations of l sets of occurrence likelihood and s sets of consequence severity, where $1 \le l \le 5$, $1 \le s \le 5$, and both are integers. Thereby the H_n is expressed as trapezoidal fuzzy number as:

$$H_n = \{R_a, R_b, R_c, R_d\}$$

$$= \left\{ \min\left(\left[L_{li,a} \otimes S_{si,a}\right]\right), \frac{\left[\sum_{li=1}^{l}\sum_{si=1}^{s} L_{li,b} \otimes S_{si,b}\right]}{[l \times s]}, \frac{\left[\sum_{li=1}^{l}\sum_{si=1}^{s} L_{li,c} \otimes S_{si,c}\right]}{[l \times s]}, \max\left(\left[L_{li,d} \otimes S_{si,d}\right]\right) \right\} \quad (6)$$

In Eq. (6), '[ ]'refers to the selection of the valid combinations only, because not all of the combinations should be classified as H_n. For example, when calculating the risk grade 'I' in Table 3, six results are generated from pairwise multiplications of three sets of occurrence likelihood i.e. L_1, L_2 and L_3 respectively, and two sets of consequence severity i.e. S_1 and S_2. However, four out of the six results which fall into the scope of risk grade 'I' are regarded as valid combinations: $L_1 \otimes S_1$, $L_1 \otimes S_2$, $L_2 \otimes S_1$, and $L_3 \otimes S_1$. The relationship between risk grade and the corresponding membership function which could be built up through Eq. (6) are shown in Table 4.

Table 4 The relationship between risk grades and the corresponding membership function

Grade*	Linguistic description	Control scheme	Membership function
H_1 (I)	Negligible	To conduct routine management and	$\{0, 0, 0.14, 2.75\}$

		monitoring	
H_2 (II)	Slight	To strengthen the routine management and examinations	$\{0,0.07,2.8,41.25\}$
H_3 (III)	Need-Consider	To rule with preventive and monitoring measures	$\{0,8.83,49.17,1650\}$
H_4 (IV)	Serious	To formulate precaution and warning measures	$\{0,199.38,641.67,30250\}$
H_5 (V)	Intolerable	To cease and initiate the contingency plan immediately	$\{100,9762.5,31250,100000\}$

Note*: For uniformity, the risk grades in Table 4 ae equivalent to what MoC (2007) regulates in Table 3

3.4 Determination on the weight of the hazardous events

Through the procedure of section 3.3, the risk level of an individual hazardous event could be received. Moreover, if we want to further obtain the overall risk level of the DFP, a procedure of multi-layer risk fusion should be conducted, which is from hazardous event level to hazard group level, and finally to the total object level (An et al., 2011). Given that the influence degree of each hazard to the overall risk level is different, the weighing factor is employed. The methods to determine the weighing factor are usually divided into three categories: subjective method, objective method, and hybrid method (Yang et al., 2017). As a kind of subjective method, the analytical hierarchy process (AHP) to obtain the influence of each factor is suitable for the scenarios of qualitative evaluation, and the result generated can reflect the subjective preference of the decision maker. Fuzzy-analytical hierarchy process (FAHP) is an important extension of the traditional AHP method (An et al., 2011; An et al., 2016), which uses a similar framework of AHP to conduct risk analysis but fuzzy ratios of relative importance replace crisp ratios to the existence of uncertainty in the risk assessment. An advantage of the FAHP is its flexibility of integrating with other techniques, for example, the integration with ER in risk analysis. There are six steps to calculate the weighing factors as described below (An et al., 2011).

Step1: To establish an estimation scheme

The same as the traditional AHP method, FAHP determines the weighing factors through pairwise comparison. The comparison is based on an estimation scheme, which lists the intensity of importance using qualitative descriptors. Each qualitative descriptor has a corresponding trapezoidal membership function that is employed to transfer expert judgments into a comparison matrix (Ahn, 2017; An et al., 2011; Bandeira et al., 2018; Ng, 2016; Ruiz-Padillo et al., 2016). Table 5 describes qualitative descriptors and their corresponding trapezoidal fuzzy numbers for risk analysis in DFP.

Table 5 Fuzzy-AHP estimation scheme

Qualitative descriptors	Description	Trapezoidal fuzzy numbers
Equal importance	Two risk contributors contribute equally	$\{1,1,1,2\}$
Week importance	Experience and judgment slightly favor one risk contributor over another	$\{1,2,2,3\}$
Between week and strong importance	When compromise is needed	$\{2,3,4,5\}$
Strong importance	Experience and judgment strongly favor one risk contributor over another	$\{4,5,5,6\}$
Between strong and very strong importance	When compromise is needed	$\{5,6,7,8\}$
Very strong importance	A risk contributor is favored very strongly over the other	$\{7,8,8,9\}$
Absolute importance	The evidence favoring one risk contributor over another is of the highest possible order of affirmation	$\{8,9,9,9\}$

Step 2: To compare risk contributors

Suppose that there are two risk contributors denoted by h_1 and h_2. If h_1 is of stronger importance than h_2, a fuzzy number of (4, 5, 5, 6) is then assigned to h_1 based on the estimation scheme as shown in Table 5. Correspondingly, risk contributor h_2 has a fuzzy number of (1/6, 1/5, 1/5, 1/4). If there are m risk contributors in the index system, a total of $\left(m\left(m-1\right)/2\right)$ pairs needs to be compared.

Step 3: To aggregate the comparative results

Generally, multiple experts are involved in the risk assessment and their judgment may be different. Therefore, the comparative result from each individual expert should be aggregated into a synthetic result for each risk contributor. The process is the same as what has been described in Eq. (4) or Eq. (5).

Step 4: To construct comparison matrix

Based on the synthetic results obtained in Step 3, a comparison matrix could be constructed.

Suppose that $h_1, h_2, \ldots, h_m$ are risk factors in a hazard group, $A_{x,y}$ is the synthetic result representing the quantified judgment on h_x comparing with h_y. The pairwise comparison between h_x and h_y in the hazard group thus yields a $m \times m$ matrix shown as:

$$M = \left[A_{x,y}\right] = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,m} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m,1} & A_{m,2} & \cdots & A_{m,m} \end{bmatrix} \tag{7}$$

$$x, y = 1, 2, \cdots, m, \quad A_{x,y} = \left\{a_{x,y}, b_{x,y}, c_{x,y}, d_{x,y}\right\}, \quad A_{y,x} = \left\{1/d_{x,y}, 1/c_{x,y}, 1/b_{x,y}, 1/a_{x,y}\right\}$$

Where $a_{x,y}$, $b_{x,y}$, $c_{x,y}$, and $d_{x,y}$ are the numbers of $A_{x,y}$.

Step 5: To calculate weighing factors

The weighing factors can be calculated by using geometric mean method (An et al., 2011; Kim & Zuo, 2018; Liu et al., 2013). The geometric mean $\bar{A}_x$ of the xth row in the comparison matrix is defined as:

$$\bar{A}_x = \left\{\bar{a}_x, \bar{b}_x, \bar{c}_x, \bar{d}_x\right\} = \left\{\sqrt[m]{\prod_{y=1}^{m} a_{x,y}}, \sqrt[m]{\prod_{y=1}^{m} b_{x,y}}, \sqrt[m]{\prod_{y=1}^{m} c_{x,y}}, \sqrt[m]{\prod_{y=1}^{m} d_{x,y}}\right\} \tag{8}$$

Then the weighing factor FW_x of risk factor h_x can be received by:

$$FW_x = \left\{a_x, b_x, c_x, d_x\right\} = \left\{\frac{\bar{a}_x}{\sum_{x=1}^{m} \bar{d}_x}, \frac{\bar{b}_x}{\sum_{x=1}^{m} \bar{c}_x}, \frac{\bar{c}_x}{\sum_{x=1}^{m} \bar{b}_x}, \frac{\bar{d}_x}{\sum_{x=1}^{m} \bar{a}_x}\right\} \tag{9}$$

3.4.6 Step 6: Defuzzification and normalization

Since the outputs generated in Step 5 are fuzzy numbers, defuzzification and normalization are conducted to convert fuzzy numbers into normalized crisp values as:

$$W'_x = \frac{a_x + b_x + c_x + d_x}{4} \tag{10}$$

$$W_x = \frac{W'_x}{\sum_{x=1}^{m} W'_x} \tag{11}$$

4 Assessment on overall risk level of DFP

4.1 Conversion of the risk level of hazardous event into belief structure

The risk level of each hazardous event provides evidence supporting that the overall risk of DFP reaches to a certain level. Thereby, the FER is employed subsequently to aggregate these pieces of evidence contributed by the risk level of all hazardous events depicted in Fig. 3 to reflect the overall risk of DFP construction. In order to implement the FER, a belief structure to represent the risk level of hazardous events should be realized firstly.

Suppose that the risk level of hazardous event j ($j=1,2,\cdots,\tilde{L}$) is denoted by R_j, and the frame of discernment associated with the risk grade shown in Table 4 is defined as $H=\{H_n, n=1,2,\ldots,5\}$. There are four steps to convert R_j in the form of trapezoidal fuzzy number into the belief structure.

Step 1: To plot out the curve of membership function according to Table 4.

Fig.4 below portraits the trapezoidal curves of membership function with respect to different risk grades of DFP ($H_n, n=1,2,\ldots,5$).

Step 2: To Plot out the curve of R_j in Fig.4.

Set $R_j = L_1 \otimes S_3$ as an example, it could be obtained from Table 1 and Table 2 that $L_1=\{0,0,5.0E-5,1.0E-4\}$ and $S_3=\{1000,3000,5000,7500\}$. Then the R_j could be arrived through Eq. (3) that $R_j=\{0,0,0.25,0.75\}$. The curve of R_j is plotted out in Fig. 4 with the bold line as shown in Fig.5.

Step 3: To calculate the extent with respect to R_j contributing to H_n $(n=1,2,\cdots,5)$.

Suppose the intersection set formed by R_j and H_n is denoted by S_n^j, meanwhile, the area surrounded by both H_n and the coordinate axis is represented by S_n, then calculate the extent with respect to R_j contributing to H_n, which is defined as ratio of S_n^j to S_n. According to Fig.5, it can be worked out that $S_n^j=\{0.50,0.47,0.03,1.40E-3,0\}$ and $S_n=\{1.45,21.99,845.17,15346.15,60693.75\}$, then the results of S_n^j to S_n are obtained as $3.44E-1$, $2.11E-2$, $3.55E-5$, $9.12E-8$ and 0 respectively.

Step 4: To work out the belief structure.

Normalize the results obtained in Step 3 to receive the degree of belief $\beta_{n,j}$

$(n=1,2,\cdots,5; j=1,2,\cdots,\tilde{L})$, and then the belief structure of R_j which expressed by $S(R_j)=\{(H_n,\beta_{n,j}), n=1,2,\ldots,5\}$ could be obtained. Still think R_j in Fig.5 as an example, the belief structure of R_j is shown as follow:

$$S(R_j)=\{(H_1,0.94),(H_2,0.06),(H_3,9.72E-5),(H_4,2.50E-7),(H_5,0)\}$$

4.2 Fusion of risk based on the FER algorithm

The principle of DFP risk assessment based on FER is that the risk level of each hazardous event provides evidence supporting that the overall risk of DFP reaches to a certain level. Based on this, the overall risk level of DFP could be obtained through evidential fusion. In general, given that the weighing factors and belief structures of the hazardous events are available, the basic probability mass $m_j\{H_n\}$ and the unassigned degree of belief $m_j\{H\}$ on hazardous event e_j could be drawn out by the follow equations:

$$m_j\{H_n\}=\omega_j\beta_{n,j} \qquad (12)$$

$$m_j\{H\}=1-\sum_{n=1}^{5}m_j\{H_n\} \qquad (13)$$

Furthermore, $m_j\{H\}$ could be divided into two parts, i.e., $\bar{m}_j\{H\}$ and $\tilde{m}_j\{H\}$. Where $\bar{m}_j\{H\}$ is caused by the relative importance of the attribute e_j and $\tilde{m}_j\{H\}$ by the incompleteness of the assessment on e_j, denoted by the following equations:

$$\bar{m}_j\{H\}=1-\omega_j \qquad (14)$$

$$\tilde{m}_j\{H\}=\omega_j\left(1-\sum_{n=1}^{5}\beta_{n,j}\right) \qquad (15)$$

$$m_j\{H\}=\bar{m}_j\{H\}+\tilde{m}_j\{H\} \qquad (16)$$

After that, the FER algorithm is adopted to acquire the aggregated value of the probability mass $m_{1-\tilde{L}}\{H_n\}$, $m_{1-\tilde{L}}\{\bar{H}_{n,(n+1)}\}$, and $\bar{m}_{1-\tilde{L}}\{H\}$ as follows:

$$m_{1-\tilde{L}}\{H_n\}=k\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\{H_n\}+m_j\{H\}\right]-\prod_{j=1}^{\tilde{L}}m_j\{H\}\right\},\quad n=1,2,\ldots,5 \qquad (17)$$

$$m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+1)}\right\} = k\mu_{H_{n,(n+1)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+1}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$

$$\left.-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+1}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}, \quad n=1,2,\ldots,4 \qquad (18)$$

$$\bar{m}_{1-\tilde{L}}\left\{H\right\} = k\left\{\prod_{j=1}^{\tilde{L}}\bar{m}_j\left\{H\right\}\right\} \qquad (19)$$

$$k=\left\{\sum_{n=1}^{4}\left(1-\mu_{H_{n,(n+1)}}^{\max}\right)\left(\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right)\right.$$

$$+\sum_{n=1}^{4}\mu_{H_{n,(n+1)}}^{\max}\left(\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+1}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+1}\right\}+m_j\left\{H\right\}\right]\right)$$

$$\left.+\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_5\right\}+m_j\left\{H\right\}\right]\right\}^{-1} \qquad (20)$$

While $H_{n,(n+1)}$ is the intersection of two adjacent assessment grades H_n and H_{n+1}, $\bar{H}_{n,(n+1)}$ is a normalized fuzzy subset for the fuzzy intersection subset $H_{n,(n+1)}$ whose maximum degree of membership is represented by $\mu_{H_{n,(n+1)}}^{\max}$ (Yang et al., 2006).

However, as Fig. 4 illustrates, the total number of intersections is more than the scenario in traditional fuzzy set, due to the fact that intersections exist not only between two adjacent fuzzy evaluation grades, but also the non-adjacent two fuzzy evaluation grades. For example, there exists intersection between H_1 and H_3. Therefore, the expression of $m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+1)}\right\}$ should be changed to $m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+t)}\right\}$, where $1 \le t \le 4$ and $n+t \le 5$. Accordingly, the varied equation is described as follows:

$$m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+t)}\right\} = k\mu_{H_{n,(n+t)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$

$$\left.-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\} \qquad (21)$$

$$k=\left\{\sum_{n=1}^{5}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}\right.$$

$$+\sum_{t=1}^{4}\sum_{n=1}^{5-t}\mu_{H_{n,(n+t)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$

$$\left.\left.-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}^{-1} \qquad (22)$$

The proofs of Eq. (21) and Eq. (22) are described in the Appendix.

After the risk levels (evidences) of $\tilde{L}$ hazardous events have been assembled, the aggregated degree of belief β_n and $\beta_{n,(n+t)}$ could be calculated respectively by:

$$\beta_n = \frac{m_{1-\tilde{L}}\{H_n\}}{1-\bar{m}_{1-\tilde{L}}\{H\}} \tag{23}$$

$$\beta_{n,(n+t)} = \frac{m_{1-\tilde{L}}\{\bar{H}_{n,(n+t)}\}}{1-\bar{m}_{1-\tilde{L}}\{H\}} \tag{24}$$

Where $\beta_{n,(n+t)}$ denotes the degree of belief to which the overall risk level of DFP lies when the intersection of grade H_n and H_{n+t} exists. However, no corresponding evaluation grade as $H_{n,(n+t)}$ appears in the fuzzy evaluation grades which is also referred to the frame of discernment in Yang et al. (2006), therefore, $\beta_{n,(n+t)}$ has to be redistributed into β_n and β_{n+t}. According to the different circumstance of the intersections of grade H_n and H_{n+t}, there are two situations to redistribute $\beta_{n,(n+t)}$:

First scenario: The maximum membership degree of the intersection of two fuzzy evaluation grades is lower than one， which is shown as Fig. 6a.

Suppose that $\bar{H}_{n,(n+t)}$ is the normalized result of $H_{n,(n+t)}$, and $\bar{H}_{n,(n+t)}$ intersects H_n with an area of $\left(S_n + S_{n,(n+t)}\right)$ and H_{n+t} with an area of $\left(S_{n+t} + S_{n,(n+t)}\right)$, where $S_{n,(n+t)}$ is the common area among $\bar{H}_{n,(n+t)}$, H_n, and H_{n+t}. The minimum distance between the peaks of $\bar{H}_{n,(n+t)}$ and H_n is denoted as d_n and that between the peaks of $\bar{H}_{n,(n+t)}$ and H_{n+t} as d_{n+t}, then $\beta_{n,(n+t)}$ can be redistributed by Eq. (25) and Eq. (26) (Yang et al., 2006):

$$\beta_n^{n,(n+t)} = \frac{S_n + \mathrm{AF}_n^{n,(n+t)} \cdot \mathrm{S}_{n,(n+t)}}{S_n + \mathrm{S}_{n,(n+t)} + S_{n+t}} \cdot \beta_{n,(n+t)} \tag{25}$$

$$\beta_{n+t}^{n,(n+t)} = \frac{\mathrm{AF}_{n+t}^{n,(n+t)} \cdot \mathrm{S}_{n,(n+t)} + S_{n+t}}{S_n + \mathrm{S}_{n,(n+t)} + S_{n+t}} \cdot \beta_{n,(n+t)} \tag{26}$$

Where $\beta_n^{n,(n+t)}$ and $\beta_{n+t}^{n,(n+t)}$ denote the magnitude of redistribution to β_n and β_{n+t} respectively, $\mathrm{AF}_n^{n,(n+t)}$ and $\mathrm{AF}_{n+t}^{n,(n+t)}$ refer to allocation factors meeting the conditions as:

$$\mathrm{AF}_n^{n,(n+t)} = \frac{1}{2}\left[\left(1-\frac{d_n}{d_n+d_{n+t}}\right)+\frac{S_n}{S_n+S_{n+t}}\right] \tag{27}$$

$$\mathrm{AF}_{n+t}^{n,(n+t)} = \frac{1}{2}\left[\left(1-\frac{d_{n+t}}{d_n+d_{n+t}}\right)+\frac{S_{n+t}}{S_n+S_{n+t}}\right] \tag{28}$$

Second scenario: The maximum membership degree of the intersection of two fuzzy evaluation grades is equal to one， which is shown as Fig. 6b.

Compared with Fig. 6a, S_n and S_{n+t} are degraded to 0 in Fig. 6b. Therefore, the Eq. (25) and Eq. (26) will be changed as follows:

$$\beta_n^{n,(n+t)} = AF_n^{n,(n+t)} \cdot \beta_{n,(n+t)} \tag{29}$$

$$\beta_{n+t}^{n,(n+t)} = AF_{n+t}^{n,(n+t)} \cdot \beta_{n,(n+t)} \tag{30}$$

Suppose S'_n refers to the remaining area of H_n deducted by $S_{n,(n+t)}$ and S'_{n+t} refers to the remaining area of H_{n+t} deducted by $S_{n,(n+t)}$. A small S'_n together with a large $S_{n,(n+t)}$ should imply a high degree of belief to which $\bar{H}_{n,(n+t)}$ belongs to H_n, and vice versa. So the allocation factors $\mathrm{AF}_n^{n,(n+t)}$ and $\mathrm{AF}_{n+t}^{n,(n+t)}$ can be defined by:

$$\mathrm{AF}_n^{n,(n+t)} = 1-\frac{S'_n+S_{n,(n+t)}}{S'_n+2S_{n,(n+t)}+S'_{n+t}} \tag{31}$$

$$\mathrm{AF}_{n+t}^{n,(n+t)} = 1-\frac{S_{n,(n+t)}+S'_{n+t}}{S'_n+2S_{n,(n+t)}+S'_{n+t}} \tag{32}$$

After all values of $\beta_{n,(n+t)}$ ($n=1,2,\dots,4$, $t=1,2,\dots,4$, and $n+t\le 5$) have be redistributed, the overall risk level of the DFP in belief structure could be depicted as: $S(R_{overall}) = \{(H_n,\beta_n), n=1,2,\dots,5\}$. It is worth noting that the result of β_n is the combination of two parts: one comes from aggregated result generated by Eq. (23), and the other one from the redistribution of $\beta_{n,(n+t)}$.

5 Case Study

5.1 Project Overview

The underground traffic project of Zhengzhou Comprehensive Transportation Hub is located on the east side of Zhengzhou East Station, China. The construction area in total is 113,367.8m^2, which is an underground three-story reinforced concrete structure. The DFP excavation depth is 20m down to the underground and the DFP construction is divided by the tunnel of Metro Line One into two areas: the south half and the north half, which are linked by three connecting passages. The two parts of the DFP is surrounded by underground continuous wall. Concrete cast-in-place pile is used in three areas, i.e. around the area used Bottom-Up Method, the area used Top-Down Method nearby both sides of the tunnel and the area on both sides of the three connecting passages. The minimum net distance between the bottom of the connecting passage and the top of the tunnel structure is only four meters. Therefore, the protection of the interval tunnel for normal operation of the metro line is the key part of this DFP construction project. The construction area of the foundation pit is shown in Fig.7.

5.2 Data collection

Five experts (assessors), A, B, C, D, and E, were invited to assess the DFP construction risks. Expert index $EI_{\tilde{i}}$ calculated by Eq. (2) are shown in Table 6. In terms of Definition 1 in the section 3.1 of this paper, the relevant importance ($RI_{\tilde{i}}$) of expert D valued as 9 due to the fact that he has got the richest working experience. Conversely, expert E received the minimum value $RI_{\tilde{i}}$ as 1 because of his weakest experience comparatively. The relevant importance ($RI_{\tilde{i}}$) of the remaining three experts are obtained by the interpolation method (An et al., 2011).

Table 6 Expert index $EI_{\tilde{i}}$ of five experts

Experts	Years of experience	$RI_{\tilde{i}}$	$EI_{\tilde{i}}$
A	15	3.29	0.14
B	20	5.19	0.24
C	18	4.43	0.19
D	30	9	0.39
E	9	1	0.04

Brainstorming session was introduced among the five experts to enumerate all the hazardous events related to the DFP construction. Risks in relation to nineteen hazardous events, nine hazard groups, and three sub-objects were identified and defined as follows:

(1) The first sub-object risk identified is the technical one which includes four hazard groups

i.e. earth excavation, dewatering, excavation bracing, and structural works. Pit landslide and upheaval in the bottom are thought to be two critical hazards in earth excavation. Both rush of confined water and leakage of foundation pit are two common hazards in the process of dewatering. Destabilization of support and excessive deformation of enclosure are regarded as two key causes of excavation bracing failure. There are three representative hazards, according to the experts, need to be cautioned in structural works i.e. concrete cracking, template failure, and upward displacement of structure.

(2) The second sub-object risk is the management one which consists of three hazard groups: safety awareness, safety regulations, and safety facilities management. Each of hazard groups covers two hazardous events. The safety awareness includes insufficient preparation and illegal operation; the hazard group of safety regulation involves two aspects including defected safety regulation and unimplemented regulation; the hazards from the facet of facilities management come from the low facilities quality and scarcity in quantity.

(3) Environmental risk is regarded as another sub-object one which encompasses two hazard groups: differential settlement and damage to the third-party. Differential settlement covers excessive deformation of the metro tunnel and nearby road damage in the process of excavation. Damage to the third-party happens with falls from height and mechanical injuries due to human misconducts, severe weather condition, or other unforeseen factors.

A risk framework of this DFP construction project has been developed and shown in Fig. 8.

The pairwise comparisons were conducted among the risk contributors at the same level and within the same parent node in Fig.8; the results of comparison were quantified according to Table 5; After that, Eq. (7) ~ Eq. (11) were employed to calculate the weighing factors as shown in Table 7.

Table 7 Weighing factors of risk contributors

Risk contributors at sub-object level (local weights)	Risk contributors at hazard group level (local weights)	Risk contributors at hazardous event level (local weights)	Global weights
Technical risk（0.2）	Earth excavation（0.18）	e_1（0.54）	0.019
		e_2（0.46）	0.017

	Dewatering（0.24）	e_3（0.68）	0.033
		e_4（0.32）	0.015
	Excavation bracing（0.37）	e_5（0.52）	0.038
		e_6（0.48）	0.035
	Structural works（0.21）	e_7（0.22）	0.009
		e_8（0.17）	0.007
		e_9（0.61）	0.026
Management risk（0.07）	Safety awareness（0.44）	e_{10}（0.35）	0.012
		e_{11}（0.65）	0.02
	Safety regulations（0.35）	e_{12}（0.7）	0.017
		e_{13}（0.3）	0.007
	Safety facilities（0.21）	e_{14}（0.53）	0.008
		e_{15}（0.47）	0.007
Environmental risk（0.73）	Differential settlement（0.72）	e_{16}（0.92）	0.484
		e_{17}（0.08）	0.042
	Damage of third-party（0.28）	e_{18}（0.66）	0.135
		e_{19}（0.34）	0.069

The occurrence likelihood and consequence severity of all hazardous events were assessed by five experts, and then the combined results were received by Eq. (4) and Eq. (5). On the basis of the combined results, Eq. (3) was used to calculate the risk level of each hazardous event (shown in Table 8).

Table 8 The occurrence likelihood, consequence severity and risk level of all hazardous events

Hazardous events	Occurrence likelihood（L）	Consequence severity（S）	Risk level（R）
e_1	{4.42E-2,2.43E-1, 4.42E-1,7.21E-1}	{0,0,250,500}	{0,0,110.5,360.5}
e_2	{1.0E-3,5.5E-3,1.0E-2,5.5E-2}	{0,0,250,500}	{0,0,2.5,27.5}
e_3	{1.0E-2,5.5E-2,1.0E-1,5.5E-1}	{970,2900,4840,7320}	{9.7,159.5,484,4026}

e_4	{1.0E-1,5.5E-1,1.0,1.0}	{0,0,250,500}	{0,0,250,500}
e_5	{1.0E-3,5.5E-3,1.0E-2,5.5E-2}	{250,500,1000,3000}	{0.25,2.75,10,165}
e_6	{1.0E-2,5.5E-2,1.0E-1,5.5E-1}	{240,480,970,2900}	{2.4,26.4,97,1595}
e_7	{8.29E-2,4.56E-1, 8.29E-1,9.15E-1}	{0,0,250,500}	{0,0,207.25,457.25}
e_8	{1.0E-3,5.5E-3,1.0E-2,5.5E-2}	{0,0,250,500}	{0,0,2.5,27.5}
e_9	{5E-5,1.0E-4,1.0E-3,5.5E-3}	{1000,3000,5000,7500}	{0.05,0.3,5,41.25}
e_{10}	{9.64E-2,5.3E-1, 9.64E-1,9.82E-1}	{0,0,250,500}	{0,0,241,491}
e_{11}	{6.13E-2,3.37E-1, 6.13E-1,8.07E-1}	{0,0,250,500}	{0,0,153.25,403.25}
e_{12}	{5.0E-5,1.0E-4,1.0E-3,5.5E-3}	{0,0,250,500}	{0,0,0.25,2.75}
e_{13}	{6.13E-3,3.37E-2, 6.13E-2,3.37E-1}	{0,0,250,500}	{0,0,15.33,168.58}
e_{14}	{7.84E-3,4.31E-2, 7.84E-2,4.31E-1}	{0,0,250,500}	{0,0,19.6,215.6}
e_{15}	{8.29E-3,4.56E-2, 8.29E-2,4.56E-1}	{0,0,250,500}	{0,0,20.73,227.98}
e_{16}	{1.99E-3,1.09E-2, 2.01E-2,1.1E-1}	{6200,18900,31600,65800}	{12.36,206.35,634.53,7266.95}
e_{17}	{1.0E-1,5.5E-1,1.0,1.0}	{0,0,250,500}	{0,0,250,500}
e_{18}	{4.21E-4,2.21E-3, 4.51E-3,2.48E-2}	{82.5,165,497.5,1325}	{0.03,0.36,2.24,32.87}
e_{19}	{1.0E-3,5.5E-3,1.0E-2,5.5E-2}	{0,0,250,500}	{0,0,2.5,27.5}

5.3 Risk assessment

Firstly, all of the risk levels in Table 8 were converted into belief structures using the method described in Section 4.1, the results obtained were depicted in Table 9.

Table 9 The belief structures of all hazardous events

Hazardous events	Belief structure				
	$\{H_1\}$	$\{H_2\}$	$\{H_3\}$	$\{H_4\}$	$\{H_5\}$
e_1	0.466	0.466	0.064	0.004	0.000
e_2	0.590	0.402	0.008	0.000	0.000
e_3	0.000	0.022	0.840	0.128	0.009
e_4	0.429	0.429	0.135	0.008	0.000
e_5	0.290	0.641	0.067	0.002	0.000
e_6	0.001	0.349	0.617	0.031	0.001
e_7	0.418	0.418	0.158	0.006	0.000
e_8	0.590	0.402	0.008	0.000	0.000
e_9	0.465	0.523	0.012	0.000	0.000
e_{10}	0.412	0.412	0.169	0.007	0.000
e_{11}	0.429	0.429	0.137	0.005	0.000
e_{12}	0.937	0.062	0.000	0.000	0.000
e_{13}	0.475	0.475	0.049	0.001	0.000
e_{14}	0.468	0.468	0.063	0.002	0.000
e_{15}	0.466	0.466	0.066	0.002	0.000
e_{16}	0.000	0.066	0.712	0.202	0.021
e_{17}	0.429	0.429	0.135	0.008	0.000
e_{18}	0.519	0.472	0.010	0.000	0.000
e_{19}	0.590	0.402	0.008	0.000	0.000

Subsequently, the basic probability mass $m_j\{H_n\}$ and the remaining degree of belief $m_j\{H\}$ could be calculated by Eq. (12) and Eq. (13). The risk level of each hazardous event provided evidence supporting that the overall risk of DFP reaches to a certain level; then, the aggregated probability masses $m_{1-\tilde{L}}\{H_n\}$ and $m_{1-\tilde{L}}\{\bar{H}_{n,(n+t)}\}$ ($\tilde{L}=19,\ \ n=1,2,\ldots,5,\ \ t=1,2,\ldots,4$, and $n+t\leq 5$) could be obtained by Eq. (14)~ Eq. (22), which were shown as Table 10.

Table 10 Aggregated probability masses of risk assessment

n	$m\{H_n\}$	$m\{\bar{H}_{1,n}\}$	$m\{\bar{H}_{2,n}\}$	$m\{\bar{H}_{3,n}\}$	$m\{\bar{H}_{4,n}\}$
1	0.091				
2	0.112	0.026			
3	0.270	0.017	0.061		
4	0.067	0.000	0.003	0.006	
5	0.007	0	0	0	0
$k=1.123$; $m\{H\}=0.340$; $\bar{m}\{H\}=0.340$					

Eq. (23) and Eq. (24) were used to generate the aggregated degree of belief β_n and $\beta_{n,(n+t)}$; and then $\beta_{n,(n+t)}$ was redistributed by Eq. (25) ~ Eq. (32). Ultimately, the belief structure of overall risk of the foundation pit was received as Table 11.

Table 11 The belief structure of overall risk of the foundation pit

	$\{H_1\}$	$\{H_2\}$	$\{H_3\}$	$\{H_4\}$	$\{H_5\}$
β_n	0.197	0.193	0.489	0.110	0.010

As demonstrated in Table 11, the most probability of the overall DFP risk was valued as 0.489, which falls into the risk grade Ⅲ. Since risks under grade Ⅲ are illustrated as Need-Consider in Table 4, the control strategy adopted in present was to rule with preventive and monitoring measures.

6 Discussion

6.1 Sensitivity analysis

Through the above processes, the assessment of overall risk level of the DFP construction could be arrived, but the degree of each potential hazard that contributes to the overall risk level has not been revealed. In other words, which hazardous event should be paid more attention in risk control is not explicit yet. Therefore, the sensitivity analysis is employed to examine the sensitivity of individual hazards.

Suppose that the numerical utility values of β_n ($n=1,2,\ldots,5$) are linearly assigned as follows:

$U(\beta_1)=0$, $U(\beta_2)=0.3$, $U(\beta_3)=0.5$, $U(\beta_4)=0.7$, $U(\beta_5)=1$

Then, the overall risk score of the DFP could be calculated by:

$$Score(R)=\sum_{n=1}^{5}\beta_n \times U(\beta_n)$$

$$=0.39 \tag{33}$$

Subsequently, the belief structure of each hazard is varied in turn to the same extent to observe the impact on the overall risk score of the DFP. Intuitively, the greater the impact generates, the more sensitive the hazard is. For example, with respect to each hazard, the degree of belief that belongs to 'H_1'rises by 0.05, correspondingly, the degree of belief that belongs to 'H_5'decreases by 0.05. If the degree of belief attached to 'H_5'is less than 0.05, then the scant degree of belief [0.05-β_5] can be deducted from 'H_4', this process continues until the 0.05 of degree of belief is consumed. The impact of the above operation on the overall risk score of the DFP is shown in Fig.9.

It is demonstrated in Fig. 9 that a minor decline or increment in the input data, i.e. degree of belief for any hazard, may lead to a decrease or an increase of the overall risk score correspondingly.

Let VD_j ($j=1,2,\ldots,19$) be defined as the extent of variation in overall risk score resulting from the variation of belief structure in relation to hazardous event e_j, which is denoted by:

$$VD_j = \left| Score^{j}_{varied}(R) - Score_{original}(R) \right| \tag{34}$$

While $Score^{j}_{varied}(R)$ refers to the overall risk score generated after the belief structure of hazardous event e_j has varied, $Score_{original}(R)$ represents the original risk score, and '$|\ |$'denotes the operation of take absolute value.

Thus, the results of VD_j obtained, as the belief structure of hazardous event varied in turn, are displayed in Table 12.

Table 12 The results of VD_j obtained as the belief structure of hazardous event varied in turn

Hazardous events	VD_j generated as 'H_1' is added by 0.05 in e_j	VD_j generated as 'H_1' is deducted by 0.05 in e_j	Average of VD_j	Ranking

e_1	4.955*	7.134	6.044	10
e_2	2.680	6.373	4.526	12
e_3	11.563	5.852	8.708	7
e_4	3.979	5.618	4.798	11
e_5	9.792	14.436	12.114	5
e_6	10.986	8.561	9.773	6
e_7	2.367	3.359	2.863	15
e_8	1.096	2.610	1.853	19
e_9	4.340	9.798	7.069	8
e_{10}	3.181	4.487	3.834	14
e_{11}	5.243	7.515	6.379	9
e_{12}	2.381	6.370	4.375	13
e_{13}	1.782	2.610	2.196	18
e_{14}	2.038	2.984	2.511	16
e_{15}	1.786	2.610	2.198	17
e_{16}	**254.089**	**206.672**	**230.381**	**1**
e_{17}	11.288	15.992	13.640	4
e_{18}	23.170	54.384	38.777	2
e_{19}	11.171	26.654	18.912	3

Note*: For brevity, all of the VD_j have been amplified ten thousand times, and keep three decimal fraction.

It can be drawn out from Table 12 that, the biggest variation in overall risk score results from the variation of belief structure in relation to e_{16}, which also occupies biggest global weight in Table 7. When comparing Table 12 with Table 7, it could be found that the bigger the global weight of the hazard is, the greater the impact on the overall risk score happens, which matches well with people's intuition. That is, the hazardous event with bigger global weight denotes more sensitive factor contributing to the overall risk level of the DFP construction, which deserves to paid more attention in risk control operations.

6.2 Comparison with previous studies

Comparison with the previous methods, for example, the FER method by Mokhtari et al. (2012) and the fuzzy reasoning approach by An et al. (2011), is presented to validate the effectiveness of the method in this paper. It is worth mentioning that the membership function of each risk grade using in fuzzy reasoning approach is not grounded on the product of occurrence likelihood multiplying consequence severity, instead, it depends on the domain knowledge of risk experts involved. So, by learning lessons from An et al. (2011) and Mokhtari et al. (2012), a new arrangement of the membership function in relation to five-grade risk scale is drawn (as shown in Table 13).

Table 13 The five-grade risk levels and the corresponding membership functions

Risk grade	Linguistic description	Membership function
H_1	Negligible	$\{0,0,1,2\}$
H_2	Slight	$\{1,2,3,4\}$
H_3	Need-Consider	$\{3,4,5,6\}$
H_4	Serious	$\{5,6,7,8\}$
H_5	Intolerable	$\{7,8,9,9\}$

Three methods are introduced in turn to the case study in section 5. The results obtained are listed in Table 14.

Table 14 The comparison of results obtained by using three methods respectively

Adopted method	The obtained results				
	$\{H_1\}$	$\{H_2\}$	$\{H_3\}$	$\{H_4\}$	$\{H_5\}$
FER used in this paper	0.197	0.193	0.489	0.110	0.010
FER used in Mokhtari et al. (2012)	0.103	0.139	0.351	0.310	0.097

Fuzzy reasoning approach used in An et al. (2011)	0.000	0.000	1.000	0.000	0.000

Table 14 demonstrates that the risk level of the DFP is evaluated as H_3, i.e. Need-Consider using three methods, but with different degree of belief. The reason accounting for the different degree of belief could be elaborated as two-fold: (1). The fuzzy reasoning approach is grounded on Mamdani method (An et al., 2011; Bandeira et al., 2018; Markowski & Mannan, 2008) which determines the rule to be involved in reasoning through using series of Minimal and Maximal Operators. In other words, the values between the minimal and maximal ones are abandoned in the process of reasoning, which enables the evaluated result to be either a singular number, i.e., the degree of belief equals to one just as the status shown in Table 14, or even numbers with the degree of beliefs less than one respectively but equal to one in total when the aggregated risk score locates in the intersection of two adjacent fuzzy risk grades. (2). Two distinct ways in developing belief structure may lead to the difference between the methods proposed in this paper and Mokhtari et al. (2012), although FER is employed in both methods. In this paper, the area of intersection between input fuzzy set and fuzzy risk grade is adopted to determine the degree of membership which is further converted into belief structure. However, it is the maximum ordinate value of the intersecting point, but not the area of intersection, that is used as the basis for developing belief structure in Mokhtari et al. (2012).

It seems more reasonable to use the intersecting area rather than the ordinate value for building belief structure. Just as Fig. 10 indicates that the area of intersection between input fuzzy set R_j and fuzzy risk grade H_n increases gradually as the location of input fuzzy set R_j changing from Fig. 10(a) to Fig. 10(b). Instead, the maximum of ordinate value of the intersecting point keeps as a constant, i.e. equals to one, which denotes that the degree of membership remains the same as the location of fuzzy set R_j changing.

7 Conclusion

Risk assessment is an essential element in ensuring the effective construction management of DFP. Based on the FER approach, a new method is proposed in this paper to assess the overall risk level of DFP construction. In this method, the occurrence likelihood, consequence severity, and risk grade are firstly classified by trapezoidal fuzzy numbers according to the regulations defined in MoC (2007). Then, an approach of data acquisition taking into account the impacting factor from the risk assessors' expertise is adopted to obtain more reliable results of risk assessment on the potentially happened hazards. Applying FER algorithms, the risk level of each possible hazardous event is aggregated into the overall risk of DFP construction. A case study on DFP risk assessment of underground traffic project of Zhengzhou comprehensive transportation hub, China, is introduced in this paper to verify the application of the proposed method.

Comparing with the previous methods, the advantages of the proposed method can be summarized as: (1). the dilemma that different combinations of likelihood and consequence assigned with identical risks in traditional risk matrices has been overcome through depicting the specific risk grade with a sole trapezoidal fuzzy number; (2). the method which engage the impacting factors of expertise in the process of data acquisition, enables the results of risk assessment on the potentially happened hazards more objective; (3). the proposed method makes an attempt on implementing FER under the scenario that more than two continuous fuzzy evaluation grades intersect rather than only two adjacent grades; (4). the result of risk assessment obtained by the new method may be more reasonable, owing to its coincidence with the fact that the bigger the global weight of the hazard is, the greater the impact on the overall risk score is generated.

The successful application of the proposed method in this study indicates its practicality, whereas there are two limitations which deserve further consideration. First, the amount of computation is tremendous in this method, which may be a major obstacle for its universal application in practice. It is advised here to develop a computerized application to reduce the workload of manual computation and thus, to make the method more applicable. Second, the FAHP method with critiques on its subjective nature was adopted in this research to determine the weighing factors of the hazardous events that may happen. It is necessary for further studies to develop objective methods. The results comparison on risk assessment when using different methods would provide implications to the weights determination of the possible hazards, not only for Deep Foundation Pit, but for other construction projects as well.

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Appendix: The derivation of $m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+t)}\right\}$

Suppose that there is an intersection between each two sets among three consecutive fuzzy evaluation grades as shown in Fig. 11.

It is easy to prove that the equations of aggregated probability masses i.e. $m_{1-\tilde{L}}\left\{H_n\right\}$, $m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+1)}\right\}$ and $\bar{m}_{1-\tilde{L}}\left\{H\right\}$ are the same with Eq. (17) ~ Eq. (19) respectively. Readers can refer to (Yang et al., 2006) to acquire relevant contents about the proof. The expression of $m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+t)}\right\}$ is displayed as follows:

$$m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+t)}\right\} = k\mu_{H_{n,(n+t)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$
$$\left.-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\} \qquad (35)$$

$n=1,2,\cdots,N-1$, $\quad t=1,2$, $\quad n+t\le N$

Proof:

When $t=1$, $m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+t)}\right\}=m_{1-\tilde{L}}\left\{\bar{H}_{n,(n+1)}\right\}$, the expression of which is the same with Eq. (18).

When $t=2$, the combined probability mass generated by aggregating the two attributes is given as follow:

$$m_{1-2}\left\{H_{n,(n+2)}\right\}=m_1\left\{H_n\right\}m_2\left\{H_{n+2}\right\}+m_2\left\{H_n\right\}m_1\left\{H_{n+2}\right\}$$

$$=\left[m_1\left\{H_n\right\}+m_1\left\{H_{n+2}\right\}\right]\left[m_2\left\{H_n\right\}+m_2\left\{H_{n+2}\right\}\right]-m_1\left\{H_n\right\}m_2\left\{H_n\right\}-m_1\left\{H_{n+2}\right\}m_2\left\{H_{n+2}\right\}$$

$$=\left[m_1\left\{H_n\right\}+m_1\left\{H_{n+2}\right\}+m_1\left\{H\right\}\right]\left[m_2\left\{H_n\right\}+m_2\left\{H_{n+2}\right\}+m_2\left\{H\right\}\right]$$

$$-\left[m_1\left\{H_n\right\}+m_1\left\{H\right\}\right]\left[m_2\left\{H_n\right\}+m_2\left\{H\right\}\right]$$

$$-\left[m_1\left\{H_{n+2}\right\}+m_1\left\{H\right\}\right]\left[m_2\left\{H_{n+2}\right\}+m_2\left\{H\right\}\right]+m_1\left\{H\right\}m_2\left\{H\right\}$$

$$=\prod_{j=1}^{2}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{2}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{2}\left[m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{2}m_j\left\{H\right\}$$

Suppose the following equation is true for combing the first $\left(\tilde{L}-1\right)$ attributes:

$$m_{1-(\tilde{L}-1)}\left\{H_{n,(n+2)}\right\}=\prod_{j=1}^{\tilde{L}-1}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}-1}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]$$

$$-\prod_{j=1}^{\tilde{L}\text{-}1}\left[m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}\text{-}1}m_j\left\{H\right\}$$

The above combined probability mass is further aggregated with the $\tilde{L}th$ attributes. The combined probability mass is then given below:

$$m_{1-\tilde{L}}\left\{H_{n,(n+2)}\right\}=m_{1-(\tilde{L}-1)}\left\{H_n\right\}m_{\tilde{L}}\left\{H_{n+2}\right\}+m_{\tilde{L}}\left\{H_n\right\}m_{1-(\tilde{L}-1)}\left\{H_{n+2}\right\}$$

$$=\left[m_{1-(\tilde{L}-1)}\left\{H_n\right\}+m_{1-(\tilde{L}-1)}\left\{H_{n+2}\right\}\right]\left[m_{\tilde{L}}\left\{H_n\right\}+m_{\tilde{L}}\left\{H_{n+2}\right\}\right]$$
$$-m_{1-(\tilde{L}-1)}\left\{H_n\right\}m_{\tilde{L}}\left\{H_n\right\}-m_{1-(\tilde{L}-1)}\left\{H_{n+2}\right\}m_{\tilde{L}}\left\{H_{n+2}\right\}$$

$$=\left[m_{1-(\tilde{L}-1)}\left\{H_n\right\}+m_{1-(\tilde{L}-1)}\left\{H_{n+2}\right\}+m_{1-(\tilde{L}-1)}\left\{H\right\}\right]\left[m_{\tilde{L}}\left\{H_n\right\}+m_{\tilde{L}}\left\{H_{n+2}\right\}+m_{\tilde{L}}\left\{H\right\}\right]$$

$$-\left[m_{1-(\tilde{L}-1)}\left\{H_n\right\}+m_{1-(\tilde{L}-1)}\left\{H\right\}\right]\left[m_{\tilde{L}}\left\{H_n\right\}+m_{\tilde{L}}\left\{H\right\}\right]$$
$$-\left[m_{1-(\tilde{L}-1)}\left\{H_{n+2}\right\}+m_{1-(\tilde{L}-1)}\left\{H\right\}\right]\left[m_{\tilde{L}}\left\{H_{n+2}\right\}+m_{\tilde{L}}\left\{H\right\}\right]+m_{1-(\tilde{L}-1)}\left\{H\right\}m_{\tilde{L}}\left\{H\right\}$$

$$=\prod_{j=1}^{\tilde{L}-1}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]\left[m_{\tilde{L}}\left\{H_n\right\}+m_{\tilde{L}}\left\{H_{n+2}\right\}+m_{\tilde{L}}\left\{H\right\}\right]$$
$$-\prod_{j=1}^{\tilde{L}-1}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\left[m_{\tilde{L}}\left\{H_n\right\}+m_{\tilde{L}}\left\{H\right\}\right]$$

$$-\prod_{j=1}^{\tilde{L}-1}\left[m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]\left[m_{\tilde{L}}\left\{H_{n+2}\right\}+m_{\tilde{L}}\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}-1}m_j\left\{H\right\}m_{\tilde{L}}\left\{H\right\}$$

$$=\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]$$

$$-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}$$

Since the fuzzy subset $H_{n,(n+2)}$ is the intersection of the two fuzzy evaluation grades H_n and H_{n+2}, its maximum degree of membership is normally not equal to 1. In order to capture the exact probability mass assigned to $H_{n,(n+2)}$, its membership function needs to be normalized(Yang et al., 2006):

$$m_{1-\tilde{L}}\left\{\overline{H}_{n,(n+2)}\right\}=k\mu_{H_{n,(n+2)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$

$$-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+2}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\Bigg\}$$

Where k can be determined using the following normalization constraint condition:

$$\sum_{n=1}^{N}m\left\{H_n\right\}+\sum_{t=1}^{2}\sum_{n=1}^{N-t}m\left\{\bar{H}_{n,(n+t)}\right\}+m\left\{H\right\}=1$$

From the above equation, it can be received:

$$k=\left\{\sum_{n=1}^{N}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}\right.$$

$$+\sum_{t=1}^{2}\sum_{n=1}^{N-t}\mu_{H_{n,(n+t)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$

$$\left.\left.-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}^{-1}$$

Suppose there is an intersection between each two sets among N consecutive fuzzy evaluation grades as shown in Fig. 12.

In this case, the expression of $m_{1-\tilde{L}}\left\{H_{n,(n+t)}\right\}$ is the same with Eq. (35), but the value of t is defined as: t=1,2,$\cdots$,N-1， and $n+t\le \mathrm{N}$, the process of proof ibids.

Also, k can be determined using the following normalization constraint condition:

$$\sum_{n=1}^{N}m\left\{H_n\right\}+\sum_{t=1}^{N-1}\sum_{n=1}^{N-t}m\left\{\bar{H}_{n,(n+t)}\right\}+m\left\{H\right\}=1$$

From which it can be obtained:

$$k=\left\{\sum_{n=1}^{N}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}\right.$$

$$+\sum_{t=1}^{N-1}\sum_{n=1}^{N-t}\mu_{H_{n,(n+t)}}^{\max}\left\{\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_n\right\}+m_j\left\{H\right\}\right]\right.$$

$$\left.\left.-\prod_{j=1}^{\tilde{L}}\left[m_j\left\{H_{n+t}\right\}+m_j\left\{H\right\}\right]+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}+\prod_{j=1}^{\tilde{L}}m_j\left\{H\right\}\right\}^{-1}$$

Fig.1.

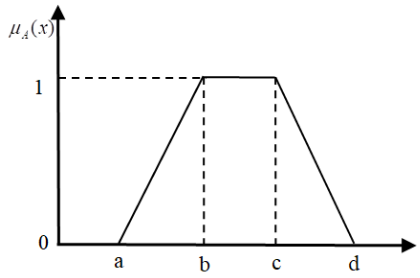


Fig.1 A trapezoidal fuzzy number

Fig. 2.

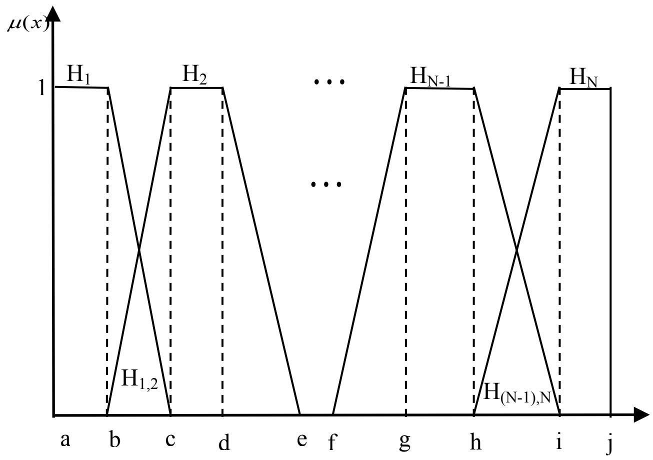


Fig.2 Intersection exists between two adjacent fuzzy assessment grades

Fig. 3

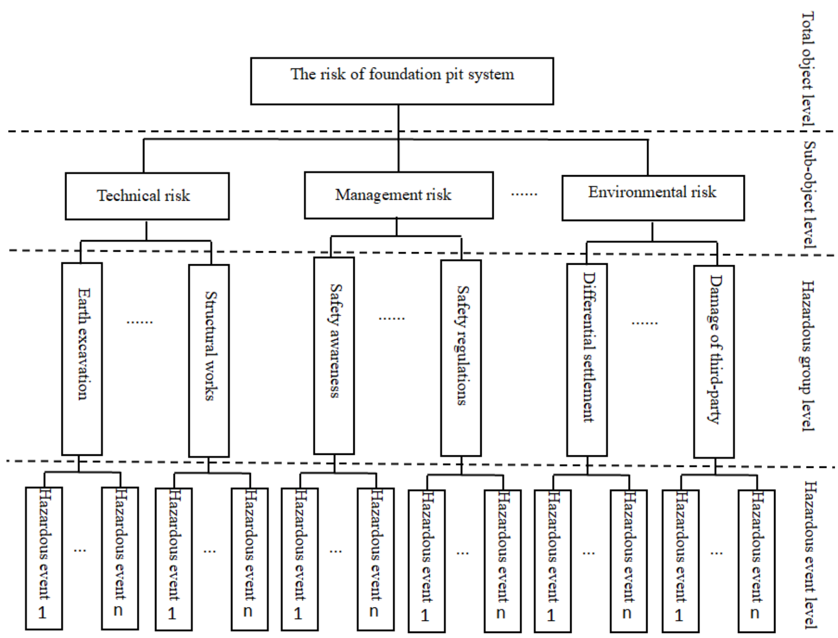


Fig.3 A typical risk framework of DFP

Fig. 4.

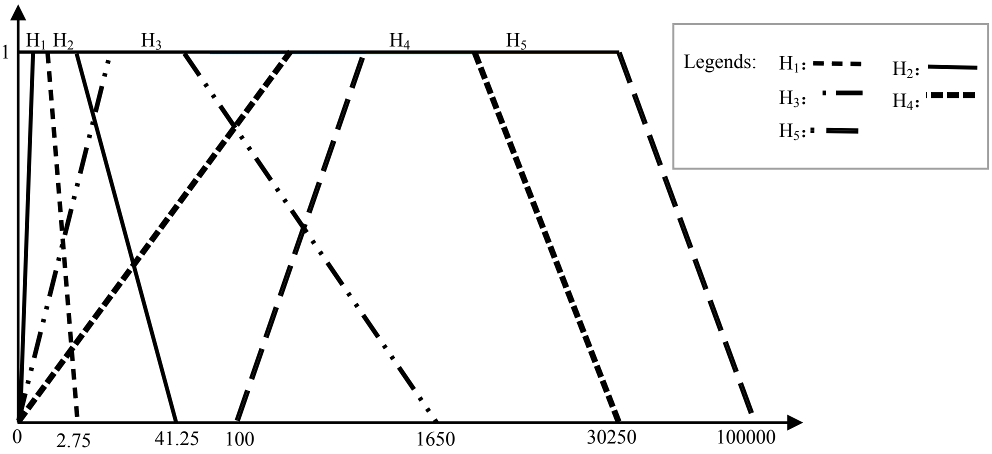


Fig.4 Curves of membership function with respect to risk grades

Fig.5.

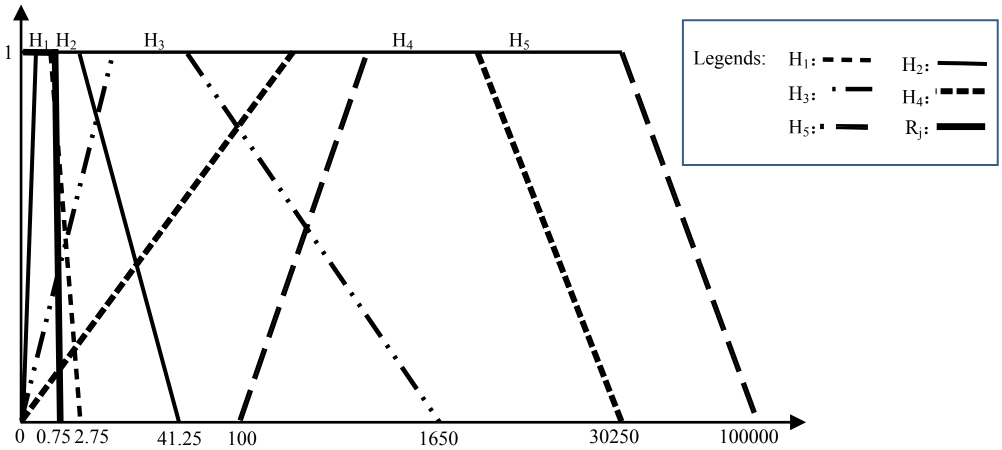


Fig. 5 Location of R_j in curves of membership function

Fig. 6a.

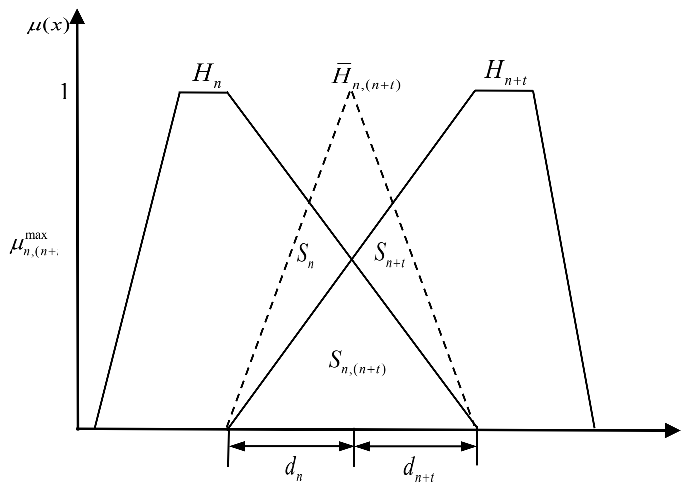


Fig.6a Redistribution of $\beta_{n,(n+t)}$ as maximum membership is less than one

Fig. 6b.

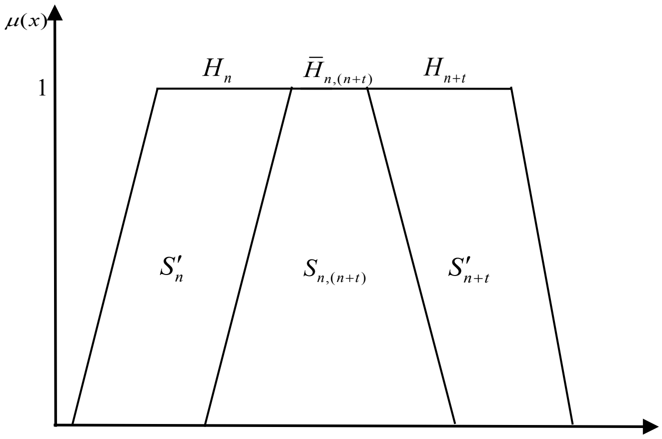
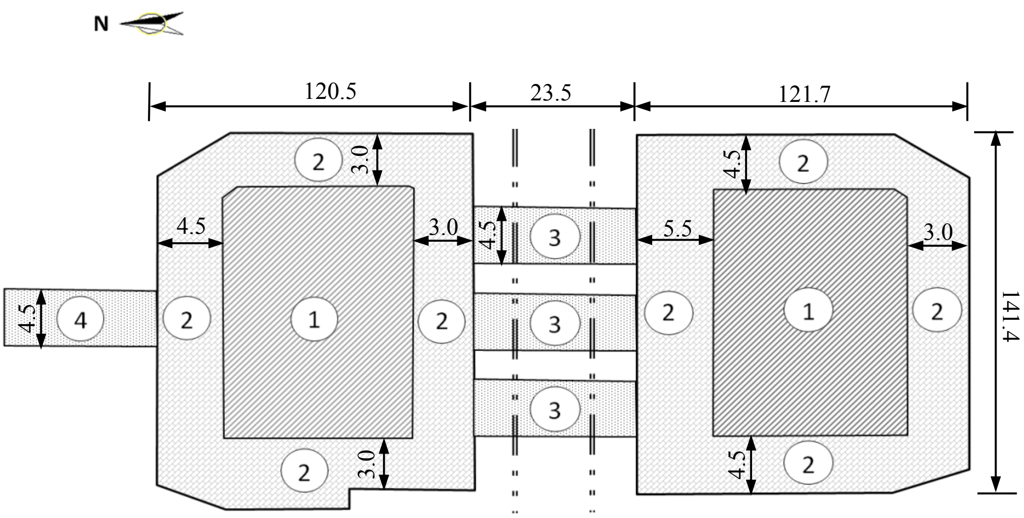


Fig.6b Redistribution of $\beta_{n,(n+t)}$ as maximum membership equals to one

Fig. 7.



Notes: ①Area used Bottom-Up Method; ②Area used Top-Down Method; ③ Connecting passage; ④Connecting passage of north entrance; Unit: meter

Fig.7 Construction division of the DFP

Fig. 8

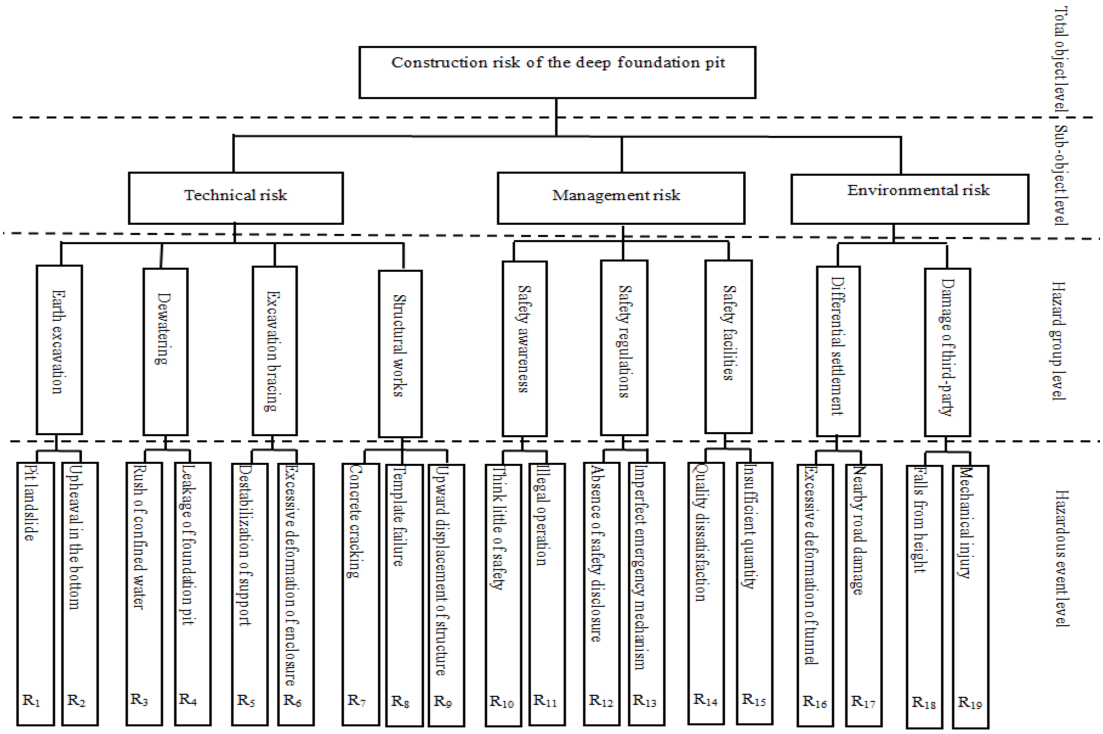


Fig.8 Risk framework developed under this DFP construction project

Fig.9.

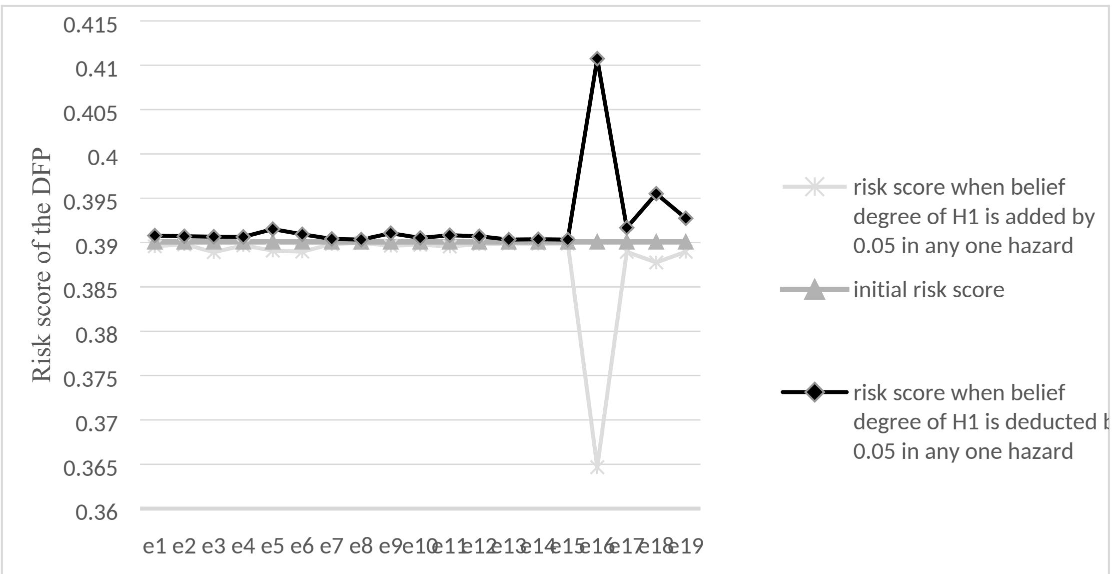


Fig.9 The variation of overall risk score as the belief structure of each hazard varied in turn

Fig. 10.

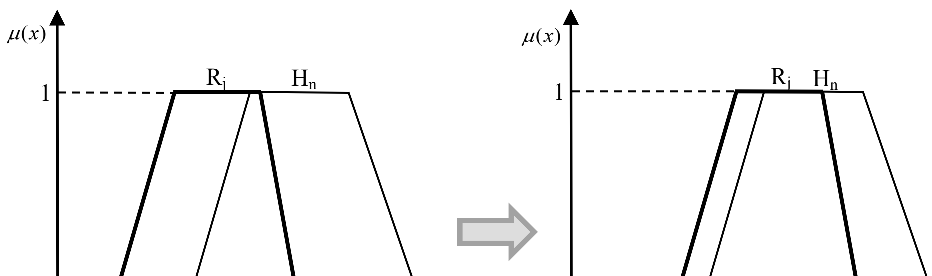


Fig. 9 Comparison of the location between hazard R_j and risk grade H_n

Fig. 11.

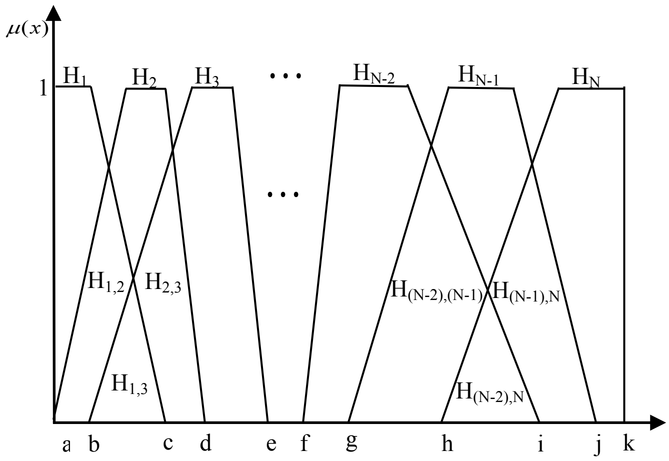


Fig.11 Intersection exists between each two sets among three consecutive fuzzy assessment grades

Fig. 12.

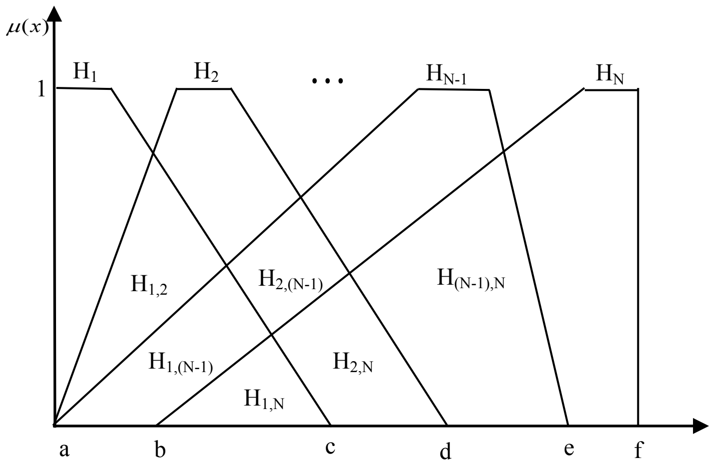


Fig.12 Intersection exists between each two sets among N fuzzy assessment grades