A review on maintenance optimization

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Abstract

To this day, continuous developments of technical systems and increasing reliance on equipment have resulted in a growing importance of effective maintenance activities. During the last couple of decades, a substantial amount of research has been carried out on this topic. In this study we review more than two hundred papers on maintenance modeling and optimization that have appeared in the period 2001 to 2018. We begin by describing terms commonly used in the modeling process. Then, in our classification, we first distinguish single-unit and multi-unit systems. Further sub-classification follows, based on the state space of the deterioration process modeled. Other features that we discuss in this review are discrete and continuous condition monitoring, inspection, replacement, repair, and the various types of dependencies that may exist between units within systems. We end with the main developments during the review period and with potential future research directions.

Keywords: Maintenance, Replacement, Repair

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1. Introduction

Industrial systems are in general subject to degradation because of usage and exposure to environmental factors. This degradation ultimately leads to system failure, in turn resulting in safety issues, equipment damage, quality issues, and unexpected machine unavailability. A few decades ago, maintenance was mostly seen as something that had to be done after such a failure, but it was also something that was difficult to manage. Nowadays, maintenance is widely recognized as an essential business function and a critical element of asset management. Organizations increasingly realize that they can improve their efficiency and reliability by planning maintenance interventions more effectively. This results in more preventive maintenance actions that are also better aligned with other business functions, such as production scheduling and spare parts control. For instance, companies in process and chemical industries can significantly increase profits by avoiding unplanned stoppages and bad quality production (Alsyouf, 2007).

The recognition of the importance of effective maintenance planning is fueled by an ongoing automation of production processes and an increasingly competitive marketplace. Due to the

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former, a substantial part of the total costs of industrial assets is nowadays related to maintenance (Zio & Compare, 2013). For instance, the operations and maintenance costs for offshore wind farms contribute a quarter of the life-cycle costs, making it one of the largest cost components for these assets (Snyder & Kaiser, 2009; Irawan, Ouelhadj, Jones, Stalhane & Sperstad, 2017). Furthermore, in both the process industry and the chemical industry over a quarter of the total workforce deal with maintenance operations (Waeyenberg & Pintelon, 2002). The fierce competition among enterprises has led many of them to revise their maintenance policies for production systems (Cherkaoui, Huynh & Grall, 2018).

Maintenance optimization consists of the development and analysis of mathematical models aimed at improving or optimizing maintenance policies. It has been the subject of many studies, and in this paper we review these. We focus on papers that have appeared in the period 2001 to 2018. Furthermore, due to the very large number of papers published on maintenance optimization in this period, we consider studies in the following journals (~~sorted~~ by the number of papers in this review): Reliability Engineering and System Safety (112 papers), European Journal of Operational Research (35), IEEE Transactions on Reliability (25), International Journal of Production Research (12), IISE Transactions (formerly known as IIE Transactions; 11), International Journal of Production Economics (8), Computers & Industrial Engineering (6), Journal of the Operational Research Society (5), Production and Operations Management (3), Journal of Applied Probability (2), Operations Research (2), Mathematics of Operations Research (1), and OR Spectrum (1).

The main criterion for including a paper in this review is that decision making about maintenance interventions should be part of the analysis. This means for instance that we do include studies that simultaneously optimize maintenance and spare parts ordering decisions, but that we do not include studies that consider spare parts ordering for a component that will only be replaced after failure. The latter does not consider maintenance decisions. Next to the properties of the models that are considered and the characteristics that are taken into account, we also indicate the methodologies that are used to analyze the models and to determine optimal policies.

We note that the scientific literature on maintenance optimization mainly focuses on the development and extension of maintenance models, and on analyzing those. The progress in this respect is also the focus of this review. The exact features that studies on maintenance optimization consider are generally inspired by practical situations, but only a small number of studies in this review apply their model to a real life case. We have indicated these studies throughout the review. Dekker (1996) points out that the application of maintenance optimization models can be challenging because models must be interpreted, decision support systems need to be developed, and data are required. This author also provides a review on the applications of maintenance optimization models that existed at the time of the review. A follow-up of this study would be interesting to understand the extent to which developments in maintenance optimization are currently applied in practice. As the scientific literature mainly focuses on new models and because practitioners have little tendency to publish their results, such a study could largely be based on discussions with practitioners.

Returning to our review, it is our purpose that it is useful for both researchers and practitioners. Researchers can use our review to identify studies that are related to their problem, to position their study in the literature, and to get inspiration for future research. Practitioners may use our review to provide guidance on the identification of models that most closely match their maintenance problem.

The current review can be seen as a continuation of the general review on maintenance policies by Wang (2002). In the concluding section of this paper we will indicate how the field has developed since then. An earlier general review is provided by McCall (1965). There also exist more specific reviews that provide a more detailed overview of certain subareas. Multi-unit systems have been reviewed by Cho & Parlar (1991), Dekker, Wildeman & Van Der Duyn Schouten (1997) and Nicolai & Dekker (2008). Other more specific reviews are provided by Haque & Armstrong (2007) on the machine interference problem, by Budai, Dekker & Nicolai (2008) on the joint consideration of maintenance and production, by Van Noortwijk (2009) on the application of gamma processes in maintenance, by Wang (2012) on the delay-time model, by Shafiee & Chukova (2013) on maintenance models in warranty, by Van Horenbeek, Buré, Cattrysse, Pintelon & Vansteenwegen (2013a) on the joint optimization of maintenance and spare parts inventory, by Alaswad & Xiang (2017) on condition-based maintenance, and by Olde Keizer, Flapper & Teunter (2017a) on condition-based maintenance for multi-unit systems. By not just considering one certain subarea, we provide a thorough review of the research on maintenance optimization since 2001, which will result in more general conclusions and suggestions for future research. By both considering single-unit and multi-unit systems, for instance, we are able to identify research avenues for multi-unit systems based on features that have only been considered for single-unit systems so far.

The remainder of this paper is organized as follows. In Section 2 we describe terms commonly used in the modeling process. In Section 3 we review studies that consider single-unit systems, and in Section 4 we review studies that consider multi-unit systems. These two sections are further subdivided into subsections that respectively consider deterioration processes with two states, three states, a finite state space, and a continuous state space. Dependent on the number of studies per subsection, a further distinction into subsections is made to improve the readability of the paper. In Section 5 we end with conclusions and avenues for future research.

2. Terminology

In this section, we describe common terms used by agents in the modeling process to refer to assets (e.g. system, unit, component, part), to their deterioration state spaces (e.g. two states, three states, discrete state space, continuous state space), and to the maintenance actions carried out on these assets (e.g. preventive and corrective maintenance, replacement, repair, inspection and condition monitoring). We also discuss the dependencies that can exist between units of a system. This discussion frames the structure of the classification of the papers that we review. We note that maintenance management is part of *asset management*, which is defined as coordinated activity of an organization to realize value from assets (ISO 55000, 2014). Asset management concerns the whole life cycle of assets, whereas maintenance management is mainly relevant during the operational phase of assets. By improving maintenance decisions during this phase, maintenance management can increase the value realized by assets.

Firstly then, an *agent* is a stakeholder in the modeling process. The agent may be the decision maker, modeler, operator, or system owner. An agent may have multiple roles (e.g. operator and decision-maker).

Next, on assets, a *system* is an asset that performs an operational function. It is subject to deterioration and therefore also to maintenance interventions. A system has a hierarchical structure: a system may be viewed as comprising sub-systems, with each sub-system comprising sub-systems, and so on, depending on the level of resolution. A *unit* or *component* is part of a system that is subject to maintenance interventions and that cannot be further subdivided into sub-units that are individually subject to any maintenance intervention. *Single-unit* or *single-component* systems consist of a single unit or component. Otherwise, a system is a *multi-unit* or *multi-component* system. Whether a system is modeled as a single-unit system or as a multi-unit system depends on the collective viewpoint of the agents. Thus, at an advanced military supply depot, an engine may be regarded as a single-unit system to be replaced on failure, whereas in the rearward repair workshop it is a multi-unit system to be repaired. Finally, there exist systems that are not easily classified as single-unit or multi-unit, for example, a continuously welded rail or a road pavement. Discussion of such systems is interlinked with the notion of repair, which we discuss shortly.

On the *deterioration state space*, we distinguish deterioration processes with two states, three states, a discrete state space, and a continuous state space, and devote subsections to each of these categories. Although two and three states are special cases of a discrete state space, we devote separate subsections to these. A deterioration process with two states means that there only exist a functioning and a failed state, implying that there is no condition information available. A deterioration process with three states adds one state between the functioning and the failed state and is often referred to as the delay-time model. This model considers failure processes as two-stage processes (Wang, 2012). After the first stage a defect ~~can be identified.~~ The time between the first moment that a defect can be identified and failure is called the delay-time ~~of the failure~~. In the delay-time model, the states are often called good, defective and failed. This is the simplest model that includes condition information of functioning units. We consider this model separately because it is used a lot by studies that model deterioration. Models with a discrete state space have a countable, generally finite number of states, whereas in models with a continuous state space the deterioration level can take any value within a particular interval. The number of deterioration states in a model is closely related to what can be measured and what will be measured in practice. It may not be possible to obtain any condition information, or the quality of the information may not justify the use of detailed deterioration models. Furthermore, the number of states is sometimes reduced for analytical tractability.

On maintenance actions, we distinguish *preventive* and *corrective* maintenance actions. A preventive maintenance action is performed before failure of a unit, a corrective maintenance action after failure. Preventive maintenance actions should be planned or triggered, for instance, based on time, age, usage or condition information. The terms prognostic, predictive and condition-based are merely jargon associated with preventive maintenance that is planned based on condition information; we refer to these preventive maintenance types as condition-based maintenance (CBM). Condition monitoring is a prerequisite for using condition-based maintenance, and is an intervention at which the state of a unit becomes to an extent known. The state may refer to the functional state (e.g. functioning or failed; off or on), or the condition state (e.g. good, defective or failed; at wear level X). Two types of condition monitoring can be distinguished: condition that can be monitored continuously; and condition monitored by performing inspections. The latter can be further subdivided into periodic and aperiodic inspections (either scheduled in advance or dynamically). If inspections are needed for monitoring condition, a further distinction can be made concerning the identification of failures. Failures are either self-announcing (i.e. no inspection is needed to identify them), or not self-announcing (also called silent or hidden failures).

We now discuss the notions of *replacement* and *repair* in mathematical models of maintenance. A unit is either *repairable* or *non-repairable*. Maintenance of a non-repairable unit is always a replacement; maintenance of a repairable unit can be a replacement or a repair. A repair may be perfect, whereby the repair returns the unit to the as-good-as-new state, or imperfect, whereby the state of the unit is not returned to as-good-as-new. A corrective repair may be a *minimal repair*, whereby the unit is restored to its state immediately prior to failure (i.e. it is restored to the functioning state but without modifying the stochastic properties of the failure process of the unit). Immediately following such a minimal repair, the unit is termed as-bad-as-old. A corrective repair may even be worse than minimal so that the unit is functioning but worse-than-old. A replacement is as-good-as-new if a homogeneous set of spare components is considered. In this case replacement and perfect repair are the same action mathematically. A replacement can be imperfect or better-than-new if a heterogeneous set of spare components is considered.

The notions of repair and single- and multi-unit systems are interlinked. In modeling terms, a multi-unit system can always be repaired by replacing or repairing some of its units. Thus, i) a multi-unit system is a repairable system, and ii) a non-repairable system is a single-unit system. In each case, the converse is not true because there exist repairable single-unit systems (e.g. a tyre). There exist systems for which it is debatable whether they should be modeled as single-unit or multi-unit systems, e.g. a continuously welded rail or road pavement. We would suggest that such systems be classified as multi-unit systems because they can be divided, albeit arbitrarily, into sub-units that are individually subject to maintenance actions, e.g. division of a continuous road pavement into sections of certain length, with repair (re-laying of asphalt) carried out section-wise.

Then, with the set of repair-effects, the set of systems, and the set of mathematical models of repair defined, an agent should choose one member from each when specifying a mathematical model of maintenance for optimization. In so doing, it is important that the agent is explicit about their choices in a particular application context, so that their systems and maintenance actions of interest are defined unambiguously.

On *dependencies*, in reality, units never operate in isolation, and the operation and maintenance of units depend to a greater or lesser extent on the operation and maintenance of other units. When planning maintenance activities, we can choose to ignore these dependencies, or we can take these dependencies into account. Thus, the replacement of a brake pad or a single light bulb could be modeled as a single-component system, or as a multi-component system including dependencies between brake pads and calipers or between light bulbs in a traffic light system.

Traditionally, three types of dependencies between units are distinguished in the maintenance literature, namely *economic dependence*, *structural dependence*, and *stochastic dependence* (e.g. Thomas, 1986; Laggoune, Chateauneuf & Aissani, 2010). Economic dependence exists when the cost of maintaining or inspecting multiple units simultaneously is different from the cost of maintaining or inspecting these units separately, for instance due to a fixed setup cost. Stochastic dependence exists when deterioration processes or failure times of various units are, too some extent, stochastically dependent. This is for example the case if deterioration of multiple units is influenced by common external factors. Olde Keizer et al. (2017a) make a distinction between structural dependence from a technical point of view and structural dependence from a performance point of view. Structural dependence from a technical point of view exists for instance if maintenance of a unit requires other units or sub-systems to be dismantled as well. This may induce deterioration or failure of these other units. Structural dependence from a performance point of view exists when the performance of a system depends on the configuration of its units, and when it is not just the total of the performances of the individual units.

Multi-unit systems with structural dependence from a performance point of view can have arbitrary structures. The simplest are systems of identical components in parallel (e.g. two identical pumps connected in parallel supplying potable water to a soft drinks manufacturing line, Wang, Scarf & Smith, 2000), systems of identical components in series (e.g. the step-train of an escalator), systems of non-identical components connected in series (e.g. a brake disk, pads, caliper, hose, fluid, and lever in a motorcycle brake), and k-out-of-N systems of identical components (e.g. 16 traction motors in an electric multiple unit such that 8 motors are sufficient, but not ideal for providing motive power, Scarf, Dwight & Al-Musrati, 2005).

Besides economic, stochastic and structural dependence, Olde Keizer et al. (2017a) identify *resource dependence* as a fourth type of dependence that can exist. This type of dependence could also be seen as *logistical dependence* and exists for example if limited repairmen are responsible for the maintenance activities of various units or systems, if a single, finite stock of spare parts is used for the replacement of multiple units, or if time windows during which maintenance can be carried out have limited lengths.

Finally, we note that various *optimality criteria* are considered when specifying maintenance policies. Examples are minimization of the long-run cost rate, minimization of the total (discounted) costs during a finite time horizon, maximization of the long-run availability, and maximization of the reliability during subsequent missions.

3. Single-unit systems

In this section, we review studies that consider single-unit systems. Thus, maintenance interventions apply to the system as a whole, and there are no sub-units that are maintained individually. As a consequence, these studies can be classified based on the number of condition states of the units. We distinguish two states (functioning or failed), three states (good, defective or failed), multiple discrete states, and a continuous state space. We note that systems can have multiple competing failure modes each with a separate deterioration process. In this case the study is classified based on the most detailed deterioration process.

3.1. Two condition states

We start with studies that assume that all maintenance actions make the unit as-good-as-new, i.e., maintenance interventions are perfect and can be seen as replacements. Thereafter, we consider studies that consider imperfect repairs.

3.1.1. Replacements

We start the discussion of studies that consider perfect maintenance with those that consider time-based maintenance with known model parameters, followed by studies that do so for an uncertain lifetime distribution. Thereafter, we discuss studies that assume that failures are not self-announcing, but that an inspection is required to observe failure. We finish with studies that consider a production setting and with a single study that includes spare parts ordering. Studies in this section generally use renewal theory to provide a mathematical expression for the performance indicator of interest (typically cost), and use numerical analysis to obtain optimal policies.

Various studies consider time-based maintenance with known model parameters. Jiang (2009) presents an approach to determine an approximately optimal age-based maintenance policy for a finite time horizon. Sidibé, Khatab, Diallo & Kassambara (2017) consider the optimal replacement age for a unit with a random and unknown initial age. Dekker & Plasmeijer (2001) consider opportunities for maintenance that arrive according to a Poisson process. Maintenance is carried out at the first opportunity after a certain threshold age, or when a certain fixed maintenance age is reached, whichever occurs first. Li, Wang & Peng (2016b) consider a similar setting and extend this with non-negligible maintenance durations. Finkelstein, Shafiee & Kotchap (2016) extend the age replacement policy to the case in which the output of the system is decreasing as a function of its age. Aramon Bajestani & Banjevic (2016) introduce the age-based replacement policy in which preventive replacements can only be carried out at specific calendar times. These opportunities are used if the age exceeds a certain threshold. Their model is applied to an example with real data on wooden poles in the distribution system of a Canadian electricity company. He, Maillart & Prokopyev (2017) consider the age replacement policy and investigate the impact of unpunctual preventive maintenance. The actual maintenance times differ from the scheduled ones in a probabilistic manner. Cha, Finkelstein & Levitin (2017) consider age-based maintenance for a unit with a failure rate that increases over time. Furthermore, external shocks lead to a sudden increase of the failure rate with a probability that depends on the age. Cha, Finkelstein & Levitin (2018) assume that all shocks result in an increase of the failure rate. Preventive maintenance is carried out when a certain age is reached or after a certain number of shocks. Wang & Su (2016) propose a two-dimensional warranty under which an item is preventively maintained when it reaches a certain age or a certain amount of usage, whichever occurs first. Ke & Yao (2016) study the block replacement policy and consider three optimality criteria, namely the chance value, the expected value, and the critical value. De Jonge & Jakobsons (2018) optimize the block-based maintenance policy under random usage of a unit. Rebaiaia, Ait-Kadi & Jamshidi (2017) compare the age replacement and the block replacement policy using the expected cost function as the principle criterion.

There are also studies that consider an unknown, i.e. uncertain, lifetime distribution in a setting with time-based maintenance. De Jonge, Klingenberg, Teunter & Tinga (2015b) consider the optimal age-based maintenance policy under uncertainty in the parameters of the lifetime distribution. Fouladirad, Paroissin & Grall (2018) analyze three time-based maintenance policies under uncertainty in the parameters of the lifetime distribution under the assumption that a large data set is available. Zitrou, Bedford & Daneshkhah (2013) use the concept of expected value of perfect information to explore the effect of parameter uncertainty on maintenance optimization problems. The age replacement and block replacement policies are considered as examples. Coolen-Schrijner & Coolen (2004) use a nonparametric approach and study how the optimal maintenance age adapts when more data become available. They use the long-run cost rate as the optimality criterion, and combine numerical analysis and simulation to study the performance of their approach. On the other hand, Coolen-Schrijner & Coolen (2007) also use simulation, but use the cost rate over a single cycle as the optimality criterion. Bunea & Bedford (2002) consider model uncertainty in settings with multiple competing risks, and show that the error that is made when optimizing costs using the wrong model can be substantial. Sidibé, Khatab, Diallo & Adjallah (2016) consider different operating environments and assume that the lifetime distribution depends on the environment. The lifetime distributions are unknown and estimated using kernel estimators, and an age-based maintenance policy is adapted.

Another branch of research considers failures that can only be revealed by inspections, and the main question is then when to carry out these inspections. Berrade, Cavalcante & Scarf (2013a) study a model without preventive maintenance and with imperfect inspections (both false positives and false negatives). Two models are considered, one in which a positive inspection is followed by a further investigation, and one in which it is followed by a replacement. Berrade, Scarf & Cavalcante (2015) consider periodic inspections that can also be imperfect. Furthermore, they study the effect of the quality of replacements by assuming that new units come from a heterogeneous population. He, Maillart & Prokopyev (2015) consider periodic inspections that only detect failure with a particular probability; preventive maintenance occurs after a fixed number of inspections. Leung (2001) derive optimal inspection policies under an unknown lifetime distribution. In special cases of the model, the author includes inspections that are imperfect and not instantaneous. Yeung, Cassady & Schneider (2007) consider a continuously operating manufacturing system. Samples of finished products are used to identify the out-of-control state. A joint maintenance and statistical process control policy is determined based on a partially observable Markov decision process.

Chelbi & Ait-Kadi (2001) include the ordering of spare components in a block-based maintenance setting. They simultaneously optimize the preventive maintenance interval, the length of the replenishment cycles, and the inventory level at which a new order for spare components is placed.

The following studies jointly consider maintenance and production. Cassady & Kutanoglu (2005) consider a fixed set of jobs that need to be processed and aim to minimize the total expected weighted completion time. A minimal repair is carried out upon failure after which the current job can be resumed. Preventive maintenance is scheduled in between jobs. Najid, Alaoui-Selsouli & Mohafid (2011) use an integer linear programming formulation to consider the multi-item capacitated lot-sizing problem combined with periodic preventive maintenance and minimal repairs after failure. Borrero & Akhavan-Tabatabaei (2013) use a Markov decision process formulation to consider a single-product workstation, and include maintenance costs and either a cost for holding inventory or a cost for not satisfying demand when the machine is not available for production. Assid, Gharbi & Hajji (2015) develop a simulation model to study the joint optimization of production, setup and maintenance activities of an unreliable manufacturing system that is used to produce two types of products. Finally, Zhao, Mizutani & Nakagawa (2015) consider a unit that is processing successive jobs with random processing times. The age replacement policy that results in the interruption of jobs is compared to replacement after the completion of a fixed number of jobs, and to replacement after the job during which the maintenance age is reached.

We conclude that many extensions of the standard time-based maintenance models have been developed, and that a significant number of these take uncertainty in the lifetime distribution into account. The latter is mostly considered in the most basic models, whereas the effect of model uncertainty would also be interesting in studies on e.g. hidden failures, spare parts ordering, and production planning, especially because model uncertainty is likely to exist in practice in these cases.

3.1.2. Repairs

We begin this section on imperfect repairs for single-unit systems by reviewing studies that use virtual age modeling. Thereafter, we discuss studies that consider eventual perfect replacements in conjunction with imperfect repairs, cost analysis over a finite time horizon, two types of failures or failure modes, and a production setting. Again, unless indicated otherwise, numerical calculations based on renewal theory are used for the analysis in these studies.

Studies that consider imperfect repairs in a time-based maintenance setting generally use virtual (or effective) age modeling. Repairing a unit does not bring its age back to zero, and the failure rate (or hazard rate) is higher than that of a new unit. Finkelstein (2015) considers a system that is only repaired at failure. The author models the cost of a repair as a function of the level of repair and considers the optimization of the repair level of the system. Jack, Iskandar & Murthy (2009) consider a repairable product under a two-dimensional warranty (time and usage). Upon failure under warranty the product is either repaired or replaced. Wang, Liu &

Liu (2015a) consider a two-dimensional warranty, consisting of a basic warranty and an extended warranty. Failures are rectified by minimal repairs and imperfect preventive repairs are carried out periodically. Su & Wang (2016) also consider a two-dimensional warranty, and assume that the extended warranty is optional for interested customers. This additional warranty can be bought either at the start or at the end of the basic warranty. Park, Jung & Park (2018) consider the optimal periodic preventive maintenance policy after the expiration of a two-dimensional warranty. Lim, Qu & Zuo (2016) consider age-based maintenance with a replacement at the maintenance age. They assume that either a minimal repair or a perfect repair is carried out upon failure. Cha & Finkelstein (2016) consider the optimal long-run periodic maintenance and age-based maintenance policy in the case that maintenance actions are imperfect. Badia, Berrade, Cha & Lee (2018) distinguish catastrophic failures that are rectified by replacements, and minor failures that are rectified by worse-than-old repairs. Preventive maintenance is initiated based on the age and on the number of minor failures.

Zhou, Xi & Lee (2007) consider a system with imperfect preventive and corrective repairs that is replaced after a fixed number of repairs. The optimal maintenance interval is decreasing because the repairs are imperfect. Lin, Huang & Fang (2015) consider a system that is replaced after a fixed number of preventive repairs and that is minimally repaired at failure. They consider an adjusted preventive maintenance interval. Coria, Maximov, Rivas-Davalos, Melchor & Guardado (2015) assume a similar model and consider periodic preventive maintenance. Lee & Cha (2016) propose failures that occur according to a generalized version of the non-homogeneous Poisson process. Component failure and subsequent corrective maintenance lead to system degradation and an increase in the failure rate function. Repairs are therefore 'worse-than-minimal'. Periodic imperfect preventive maintenance is carried out, and the system is replaced after a fixed number of preventive maintenance actions. Truong Ba, Cholette, Borghesani, Zhou & Ma (2017) consider a system that is minimally repaired upon failure, and preventively replaced at a certain age. Furthermore, opportunities that arrive according to a non-homogeneous Poisson process can also be used for maintenance. Zhao, Qian & Nakagawa (2017) assume minimal repair after failure and replacements that are carried out periodically and after a certain number of repairs.

A finite time horizon is explicitly considered by a number of studies. Lugtigheid, Jiang & Jardine (2008b) use stochastic dynamic programming to consider the repair and replacement decision for a component that can only be repaired a certain number of times. The failure intensity is not age-related, but it increases at each repair. Yeh & Lo (2001) study the optimal imperfect preventive maintenance scheme during a warranty period of fixed length. Failures are minimally repaired. Zhou, Li, Xi & Lee (2015b) consider preventive maintenance scheduling for leased equipment. Preventive maintenance is imperfect, reduces the age by a certain factor, and failures are minimally repaired. The lease period is divided into multiple phases with periodic maintenance within each phase. Belyi, Popova, Morton & Damien (2017) consider the optimal preventive maintenance schedule when the failure rate is increasing and when it is bathtub-shaped. Again, failures are minimally repaired.

Various studies distinguish two types of failures or failure modes. Wang & Zhang (2013) distinguish repairable and non-repairable failures. The mean time until failure is decreasing in the number of repairs, and the system is replaced after a fixed number of repairable failures, or at a non-repairable failure. Sheu, Tsai, Wang & Zhang (2015d) distinguish minor failures and catastrophic failures. The system is repaired after a minor failure and is replaced after a certain number of minor failures, at a catastrophic failure, or when a certain working age is reached, whichever occurs first. Failures can only be revealed by inspections and the length of the inspection interval depends on the number of minor failures. Fan, Hu, Chen & Zhou (2011) consider a system that is subject to two failure modes that affect each other. The hazard rate of one failure mode depends on the accumulated number of failures caused by the other failure mode. Preventive maintenance actions are imperfect, corrective maintenance actions are minimal, and the system is replaced after a fixed number of preventive maintenance actions. Sheu, Liu, Zhang & Tsai (2018) consider a machine that is used for working projects with random lengths. One type of failure can be removed by minimal repair, the other must be rectified by replacement. Preventive replacement is carried out when a certain age is reached or after a certain number of working projects. Chang (2018) also considers minor failures followed by minimal repairs and catastrophic failures followed by corrective replacement. Preventive replacement is carried out at a certain age or at the completion of a working time. Furthermore, a spare part is needed that is ordered at time 0 and that has a random lead time. Wang & Pham (2011) consider shocks that are either fatal, or that result in an increase of the failure rate. They use a genetic algorithm to determine the imperfect preventive maintenance interval, and the number of preventive repairs after which replacement is carried out. The aim is to simultaneously minimize unavailability and cost.

We continue with studies that consider repair decisions in a production setting. Chang (2014) considers a system that processes jobs at random times. Failures are either repairable and rectified by a minimal repair, or non-repairable and followed by a corrective replacement. Various preventive maintenance policies are evaluated and compared. Khojandi, Maillart & Prokopyev (2014) consider a system with a fixed initial lifetime that generates reward at a decreasing rate as the virtual age increases. Repairs can be carried out to reduce the virtual age of the system, but they also shorten the remaining lifetime. They use stochastic dynamic programming to determine maintenance policies that maximize the expected reward during the lifetime. Nourelfath, Nahas & Ben-Daya (2016) consider a production system that is either in-control or out-of-control. The latter implies that a fraction of the produced items are nonconforming. Random samples are drawn periodically and imperfect preventive maintenance is carried out that reduces the age of the machine proportionally to the level of maintenance. Jbili, Chelbi, Radhoui & Kessentini (2018) consider a transportation vehicle for which both the optimal delivery sequence and the customers at which preventive maintenance is carried out should be determined. Failures are followed by minimal repairs. The analysis is based on the formulation of an integer program.

Various authors address the topic of uncertainty in the parameters of the lifetime distribution in the context of repair. De Jonge, Dijkstra & Romeijnders (2015a) consider time-based repairs and use simulation to investigate the benefits of initially postponing preventive maintenance actions to reduce this uncertainty. Juang & Anderson (2004) consider periodic repairs and a failure rate function that depends on the number of repairs. Either a major or a minimal repair is carried out upon failure, depending on the random repair cost at failure. A Bayesian approach is used to update the parameters of the lifetime distribution. Sheu, Yeh, Lin & Juang (2001) also uses Bayesian updating in a model with age-based preventive repairs, corrective or minimal repair at failure depending on a random repair cost, and replacement after a certain number of repairs. Each repair results in an increase of the failure rate.

It turns out that many studies on repairs consider a setting with warranties. On the other hand, only limited studies include uncertainty in the lifetime distribution. Especially in the more complex models with e.g. multiple failure modes, the amount of uncertainty is likely to be significant in practice. Also the effect of imperfect repairs themselves may be uncertain. This could be studied by assuming that repairs have a random effect, and that the distribution of this random effect is unknown. Finally, only a single study on repairs takes the ordering of spare components into account. In practice, a viable policy may be to carry out repairs as long as no spare is available, and to use replacement when a spare is on stock.

3.2. Three condition states

The deterioration model with three states is often called the delay-time model, and contains a functioning or good state, a defective state and a failed state. Sometimes this model is referred to as a two-phase system, with a fault free phase, a worn phase, and ultimately failure. Inspections are typically required to observe the defective state. We begin this section by reviewing various extensions of the standard delay-time model, followed by models that include external shocks. Then, we consider studies that either include imperfect or failure-inducing inspections, and we finish with production systems for which three condition states are distinguished. Similar to studies of models with two deterioration states, most studies on single-unit systems with three deterioration states use numerical analysis based on renewal theory. We indicate studies that use a different methodology.

Aven & Castro (2009) consider the delay-time model with periodic inspections and two types of safety constraints: (i) on the probability of at least one failure and (ii) on the fraction of time the system is in the defective state. Van Oosterom, Elwany, Çelebi & Van Houtum (2014) also consider periodic inspections and allow for postponed replacement after an inspection. Berrade, Scarf & Cavalcante (2017) explore in more detail conditions for cost-effective postponement. Both perfect and imperfect periodic inspections are considered. Wang, Wang & Peng (2017) assume a delayed first inspection, periodic inspections thereafter, and replacement at an age-limit. Furthermore, if an inspection reveals the defective state and the remaining time until the replacement age-limit is below a certain threshold, maintenance is delayed until this age-limit. Maillart & Pollock (2002) consider aperiodic inspections and introduce an operating-characteristic representation to explore the trade-off between monitoring and maintenance costs. The analysis is extended to the case in which scarce monitoring resources are allocated among multiple systems. Keleş, Tekin & Bakir (2017) consider a system that is periodically inspected and that deteriorates according to a continuous-time Markov chain with three states. Failures are not self-announcing, and both repairs and replacements are considered. Cavalcante, Lopes & Scarf (2018) assume that components arise from a mixed population of weak and strong components. Initially, components are inspected periodically. After a fixed number of inspections a component is replaced when a certain age is reached or when an opportunity arrives.

The following studies combine the delay-time model with the arrival of external shocks. Yang, Ma, Peng, Zhai & Zhao (2017) assume shocks that arrive according to a non-homogeneous Poisson process and that lead to immediate failure. The intensity of shocks depends on the system state. Periodic inspections are combined with age-based maintenance. Zhao, Guo & Wang (2018b) consider a system that is subject to shocks that are either damaging or self-healing. After a certain number of damaging shocks, the system loses its self-healing ability. Various maintenance policies are considered, both with continuous monitoring and with inspections. Zhao, Cai, Wang & Song (2018a) distinguish damaging and non-damaging shocks. After reaching the defective state the system fails when the number of damaging shocks exceeds a threshold, or when it suffers a run of damaging shocks with a certain length. Three replacement policies are considered.

Various studies acknowledge that condition monitoring can be imperfect. These studies include both the case that the defective state is not identified (false negatives) and the case that an inspection wrongly indicates the defective state (false positive) are considered. Berrade, Scarf, Cavalcante & Dwight (2013b) consider imperfect periodic inspections. MacPherson & Glazebrook (2014) combine perfect inspections with an error-prone sensor that continuously monitors the system. The system can be repaired or replaced, and heuristic policies are studied. Driessen, Peng & Van Houtum (2017) assume that the probability of an inspection error changes over the operating time of the system.

Two studies consider failure-inducing inspections. Flage (2014) combines this with the possibility of both false positives and false negatives. Scarf & Cavalcante (2012) consider a component that is replaced when it is found in the defective state, or when it reaches an age-limit. Two suppliers of the component are considered, the supplier that charges a higher cost provides fewer components that are likely to fail early. Both studies consider periodic inspections.

Several studies consider a production system that can be in-control, out-of-control, or failed. Suliman & Jawad (2012) assume that periodic preventive maintenance is carried out, and that a shift to the out-of-control state is detected instantaneously and followed by a restoration action after a preparation period. The total cost rate for maintenance, inventory, non-conforming items and shortage is minimized. Yin, Zhang, Zhu, Deng & He (2015) use a genetic algorithm to optimize a policy in which the production quality is used to infer the process state. Preventive maintenance is carried out when the out-of-control state is detected or at an age-limit. Beheshti Fakher, Nourelfath & Gendreau (2018) consider a system that produces a set of different product types during a finite planning horizon that is subdivided into periods of equal length, and that is subject to two types of failures. A type I failure means a shift to the out-of-control state where a fraction of nonconforming items are produced. An inspection is required to detect a type I failure, after which a repair is carried out. Type II failures are self-announcing and followed by a minimal repair. Panagiotidou & Tagaras (2010) assume that a number of imperfect measurements, taken at aperiodic inspections, are used to infer the process state. The inspection times and the number of measurements per inspection are decision variables. Panagiotidou & Tagaras (2012) assume that production samples of a particular size are taken at predetermined inspection instances. If a resulting sample statistic falls outside a control limit an investigation is carried out that reveals the exact state of the system. Imperfect maintenance, carried out if the system is out-of-control, returns the system to the in-control state without changing its age. Perfect preventive maintenance is carried out if an age-limit is reached and corrective maintenance is carried out upon failure. Liu, Wang & Peng (2015) consider a system that produces multiple product types alternately. The aim is to determine the set-up points for switching to another product type at which preventive maintenance should be carried out. The model is applied to production data from a centrifugal system. Yang, Ma, Zhai & Zhao (2016) consider a system that executes successive missions with random durations. The system is inspected both periodically and after the completion of each mission. Immediate replacement is carried out upon failure or at a positive (defect found) periodic inspection. At a positive post-mission inspection, replacement occurs if the time to the subsequent periodic inspection is less than a pre-determined limit. Taghipour & Azimpoor (2018) use a dynamic program to consider the joint optimization of the job sequence and the inspection policy for a system that has to process a given set of jobs with different processing times. An interrupted job due to failure should be restarted after corrective maintenance.

In short, single-unit systems that deteriorate according to a delay-time model are well-studied. Various extensions have been considered, but it may be difficult to determine the corresponding model parameters in practice. For instance, both false positive and false negative inspections are considered and the corresponding probabilities are assumed to be known. In practice, the decision maker first needs to learn these probabilities. This even becomes more complicated for phenomena that are not directly observable, such as external shocks that have a random effect.

3.3. Discrete state space

We begin this section on deterioration processes with a discrete state space by reviewing studies that use a discrete-time Markov chain, followed by those that use continuous-time Markov chains, semi-Markov chains, and proportional hazards models with a failure rate that depends on the condition. We end with studies on production systems.

Various studies consider deterioration according to a Markov chain in a discrete-time setting; these studies use a Markov decision process formulation. Maillart (2006) assumes that inspections are needed to reveal the deterioration state. At any time, it is possible to carry out an inspection, so that the inspection schedule is aperiodic. Preventive maintenance can be carried out immediately after an inspection, or delayed. Cases with perfect and with imperfect inspections are considered. Icten, Shechter, Maillart & Nagarajan (2013) assume a finite supply of spare components, and that the system expires if it fails. The objective is to schedule replacements to maximize system-life. Kurt & Kharoufeh (2010) assume that each repair influences the transition probability matrix of the Markov chain. After a particular number of repairs, a replacement should be carried out. The objective is to minimize the expected discounted cost over an infinite horizon. Van Oosterom, Peng & Van Houtum (2017) assume a heterogeneous set of spare components that deteriorate according to different transition probabilities. The type of the current component must be inferred from the observed deterioration information. Kim (2016) assumes imperfect condition signals and uses Bayesian learning. The study focuses on situations in which the decision maker distrusts the model specification.

Other studies use a continuous-time Markov chain to model deterioration. Chen & Trivedi (2002) use numerical analysis and consider exponential durations between inspections and optimize the mean duration. A minor repair is performed if the deterioration level at an inspection exceeds a first threshold, and a perfect repair if it exceeds a second threshold and upon failure. Makis & Jiang (2003) consider imperfect periodic inspections. The observed system state is stochastically related to the system state through an observation probability matrix. An optimal stopping problem is formulated to minimize the long-run cost rate. Le & Tan (2013) use stochastic dynamic programming and propose a strategy that combines inspections and continuous monitoring. Continuous monitoring is imperfect, whereas inspections are perfect. An inspection is carried out when the continuous monitoring indicates a transition to another state.

Wang, Zhao & Peng (2014) extend the delay-time model and distinguish minor and severe defects. State-durations are non-exponential, which results in a semi-Markov deterioration process. Imperfect inspections identify minor defects with a particular probability and major inspections identify both defect types perfectly. The inspection interval is shortened if a minor defect is identified, and maintenance is carried out either immediately or at the next maintenance window when a severe defect is identified. A numerical analysis is carried out. Chen & Trivedi (2005) also use a semi-Markov deterioration process and distinguish minimal and major preventive maintenance. Semi-Markov decision processes are used to determine optimal polices. Srinivasan & Parlikad (2014) assume that inspections are imperfect and that the observed state is stochastically related to the true system state through an information matrix. A partially observable semi-Markov decision process is formulated to optimize inspection and repair decisions. Crespo Marquez & Sánchez Heguedas (2002) use a semi-Markov decision process and distinguish various types of preventive and corrective maintenance actions that require different amounts of time. They discuss an application in the mining industry.

Another stream of papers considers the proportional hazards model, in which the failure rate is a function of both the age and the condition of equipment. In these studies, that are based on numerical calculations, the condition is modeled by a Markov chain. Vlok, Coetzee, Banjevic, Jardine & Makis (2002) consider perfect periodic inspections and present a case study based on real data, whereas Ghasemi, Yacout & Ouali (2007) consider imperfect periodic inspections. Golmakani (2012) considers aperiodic inspections. Lam & Banjevic (2015) consider multiple covariates and a myopic policy to schedule aperiodic inspections. They provide a case study relating to transmission systems of mining transportation systems.

Various studies that consider a deterioration process with a discrete state space combine maintenance and production decisions. Batun & Maillart (2012) reconsider the study of Sloan & Shanthikumar (2000), in which they assume that the system condition affects the yield of production. In each period, the system may either be maintained, or used to produce a single unit of any of a set of different product types. Linear programming is used to determine optimal policies. Sheu, Chang, Chen & Zhang (2015a) do not use the state of the system for maintenance decisions, but assume that it influences the production rate. Customer demand sets the production rate. Policies with major, minimal and imperfect repairs are considered. The remaining studies that consider production decisions are based on semi-Markov decision processes. Iravani & Duenyas (2002) consider a make-to-stock system that produces, undergoes maintenance, or is idle. Costs are incurred for inventory and for lost demand. A continuous-time model is used and all durations are exponential. Jafari & Makis (2015) model failure with the proportional hazards model, and jointly optimize the economic manufacturing quantity and preventive maintenance. The system is inspected after each production run, and preventive maintenance is performed if either the age or the failure rate exceeds a threshold. Kazaz & Sloan (2013) maximize the long-run reward rate for a production system that deteriorates when producing and improves when maintenance is carried out. Multiple products can be produced and different maintenance actions are available.

A discrete state space allows application of the framework of Markov decision processes, either in a discrete-time or in a continuous-time setting. In this setting, standard maintenance models have been studied, as well as models that take production decisions into account. A note to make is that the transition probabilities are in general assumed to be known with certainty.

3.4. Continuous state space

In this section, we consider deterioration processes with continuous states. We begin with studies that consider units that are monitored continuously. Then we consider studies that assume that inspections are required to obtain deterioration information. We finish with miscellaneous studies that, for instance, consider multiple competing failure modes.

3.4.1. Continuous monitoring

Our discussion of studies that assume continuous monitoring begins with studies that consider perfect maintenance, followed by studies that incorporate imperfect maintenance.

De Jonge, Teunter & Tinga (2017) use simulation to study the benefits of condition-based maintenance compared to time-based maintenance. Preventive maintenance is initiated when a deterioration threshold is reached. The influence of planning time, imperfect condition information and randomness in the failure deterioration level are considered. The other studies in this section use numerical calculations based on renewal theory. Grall, Dieulle, Bérenguer & Roussignol (2006) assume preventive maintenance that is scheduled when an alarm threshold is reached and that it is carried out after a fixed planning time. They show how to compute the asymptotic failure rate of the system. Van Der Weide, Pandey & Van Noortwijk (2010) assume that shocks occur at random moments in time and that each shock causes a random amount of damage. Preventive maintenance is carried out when the deterioration level exceeds a threshold. Saassouh, Dieulle & Grall (2007) assume that the deterioration process of a system switches to a deterioration mode with a higher deterioration rate after a random time. Preventive maintenance requires a planning time and is scheduled based on the current mode and the deterioration level. Chien, Sheu & Zhang (2012) consider a discrete time setting. Preventive maintenance is carried out when a deterioration threshold level is exceeded or after a fixed number of periods, whichever occurs first. Peng & Van Houtum (2016) consider the joint optimization of maintenance and production lot-sizing. When a deterioration threshold is reached, preventive maintenance is carried out after the production of the current lot. Corrective maintenance interrupts a production lot and is more expensive and requires more time.

The following two studies combine continuous monitoring with imperfect maintenance. Liao, Elsayed & Chan (2006) assume that the effect of imperfect maintenance decreases in the number of maintenance actions, and after a particular number of maintenance actions the system is replaced. Mercier & Castro (2013) assume that maintenance actions require a planning time. Preventive maintenance is imperfect and the system is replaced when deterioration is above a threshold.

The number of studies on single-unit systems with continuous monitoring of a continuous deterioration level is limited. All studies assume that the stochastic process that models deterioration is known with certainty. This may be unrealistic in certain practical situations. Furthermore, only a single study includes imperfect condition monitoring.

3.4.2. Inspections

We now review models that use continuous deterioration states and that require inspections to obtain condition information. We make a distinction between periodic inspections and aperiodic inspections. Again, the majority of studies use renewal theory, and we will indicate when another methodology is used.

The majority of models assume a periodic inspection schedule. Guo, Wang, Guo & Si (2013) consider a system that performs missions of equal length. After each mission, the system is inspected, and the revealed deterioration level is used to determine whether the system is imperfectly maintained. Zhang, Gaudoin & Xie (2015) consider imperfect preventive repairs and perfect preventive and corrective replacements. Maintenance actions affect the deterioration speed of the system. Fouladirad, Grall & Dieulle (2008) assume degradation processes with accelerated deterioration due to sudden changes in the environment. The real environment is not observed and must be inferred from inspection data. Wang & Wang (2015) consider a continuous deterioration process with two stages, corresponding to the good and defective states of the delay-time model. Imperfect inspections are carried out. If, according to such an inspection, a first threshold is exceeded, a perfect inspection is carried out, followed by preventive maintenance if the unit is defective. If a second threshold is exceeded, immediate preventive maintenance is performed. Yang, Zhao & Ma (2018a) also consider an increased deterioration speed if the defective state is reached. They combine this with failures due to environmental shock loads that exceed a certain threshold. The inspection interval is shortened when the defective state has been observed. Zhang, Huang, Fang, Zhou, Zhang & Li (2016) consider a system that is in steady state, worn-out, or failed. The deterioration process accelerates when the system is worn-out. Preventive maintenance is carried out if the system is worn-out, if the deterioration level reaches a threshold, or if the system reaches an age-limit, whichever occurs first. Wu, Niknam

& Kobza (2015) assume a given preventive maintenance threshold and optimize the inspection interval and the degree of each imperfect preventive repair. The cost of a preventive repair depends on its degree. Elwany, Gebraeel & Maillart (2011) assume that degradation evolves according to a parametric stochastic model. The parameters of this model are uncertain, and the uncertainty is updated in a Bayesian manner. Optimal policies are determined based on a Markov decision process. The optimal preventive maintenance threshold turns out to increase over time. Liu, Liang, Parlikad, Xie & Kuo (2017a) consider systems that are subject to aging and cumulative damage and use the proportional hazards model to characterize the joint effect of these. The system is preventively maintained if the degradation level or the age exceeds a certain threshold at an inspection. They apply their model to maintenance decisions for bridge joints. Xiang, Cassady & Pohl (2012) use simulation and consider a system that operates in an environment that is modeled by a continuous time Markov chain with three states. The instantaneous rate of deterioration depends on the environment. Preventive maintenance is performed when the deterioration level at an inspection exceeds a threshold. They also consider the effect of inspection errors. Liu, Wu, Xie & Kuo (2017b) use a Markov decision process and consider an operating cost that increases in the system age and the degradation level. After an inspection, the age and degradation level determine whether the system is replaced. The model is extended with imperfect repairs.

Now we consider aperiodic inspections. Grall, Bérenguer & Dieulle (2002a) propose a discrete-time setting with aperiodic inspections that are sequentially scheduled based on observed deterioration levels. The preventive replacement threshold is optimized. Grall, Dieulle, Bérenguer & Roussignol (2002b) and Dieulle, Bérenguer, Grall & Roussignol (2003) use an inspection scheduling function that prescribes the time until the next inspection as a function of the present system condition. Flage, Coit, Luxhoj & Aven (2012) also use inspection scheduling functions, but assume uncertainty in the parameters of the deterioration process. These uncertainties are updated after each inspection in a Bayesian manner. Do, Voisin, Levrat & Iung (2015b) schedule sequential inspections such that the probability of failure before the next inspection does not exceed a certain limit. They consider imperfect preventive maintenance when a threshold is exceeded, and perfect preventive maintenance after a number of imperfect actions.

Similar to studies on continuous monitoring, the studies that consider inspections almost all assume that all model parameters are known with certainty. This is doubtful in practice, especially when a more complex model is considered, for instance with deterioration behavior that can change over time. Studies that adopt an aperiodic inspection interval schedule inspections based on heuristics; future research could consider optimal dynamic scheduling of inspections.

3.4.3. Miscellaneous

We begin this section with a number of studies that distinguish multiple failure modes. Thereafter, we discuss models that include either a heterogeneous set of spare components, the ordering of spare parts, or production decisions. Unless stated otherwise, the studies in this section use numerical calculations based on renewal theory.

A number of studies consider multiple competing failure modes. Mercier & Pham (2012) consider a continuously monitored system with a bivariate deterioration process. The occurrence of failure depends on both components of this process. Deloux, Castanier & Bérenguer (2009) consider deterioration and external shocks. Deterioration is modeled by a Gaussian process and external shocks result in failure with a probability that depends on the amount of deterioration. Huynh, Barros, Bérenguer & Castro (2011) also consider deterioration and shocks and compare the optimal condition-based maintenance policy to block-based maintenance. Caballé, Castro, Pérez & Lanza-Gutiérrez (2015) assume that each external shock initiates a degradation process, and preventive maintenance is performed when degradation reaches a threshold. Cha, Sangüesa & Castro (2016) consider external shocks that arrive according to a non-homogeneous Poisson process. Shocks either are catastrophic, or increase the deterioration level of the unit. A control-limit preventive maintenance policy with periodic inspections is proposed. Rafiee, Feng & Coit (2015) assume that shocks arrive according to a homogeneous Poisson process, and that each of them is fatal with a particular probability. Deterioration accrues continuously, and a non-fatal shock causes a sudden additional amount of deterioration. The system is inspected periodically and two thresholds are used, respectively for carrying out imperfect repair and for preventive replacement. Yang, Zhao, Peng & Ma (2018b) consider a gradual deterioration process and the arrival of external shocks. These shocks cause an abrupt deterioration increment and can also lead to immediate failure. A combination of age-based replacement and periodic inspections with varying control limits is implemented.

Zhang, Ye & Xie (2014) consider a heterogeneous population of components. They develop a maintenance policy under which a single early inspection is carried out to identify defective products. The component is either replaced immediately at this inspection, or a preventive replacement age is chosen based on the observed deterioration level.

Wang, Hu, Wang, Kong & Zhang (2015b) consider a system that deteriorates according to a Wiener process and for which a spare part is needed to carry out maintenance. A stochastic lead time for ordering the spare part is considered, and a spare part ordering and system replacement policy based on condition information is proposed. Wang, Huang, Li & Yang (2016) consider a continuously monitored system and a deterministic lead time for spare parts. The model includes a scheduling threshold that initiates the order of a new spare part and a maintenance threshold. Elwany & Gebraeel (2008) update the remaining lifetime distribution each time that a deterioration level is observed, and use this to determine the optimal replacement time and optimal inventory ordering time.

Cheng, Zhou & Li (2018) use a simulation-based optimization approach and consider a make-to-stock production system with constant production and demand rates. The proportion of defective items increases in the deterioration level. The production lot size and the inventory threshold that triggers a new production run are decision variables. The system is inspected at the end of each production run, and is imperfectly maintained if the degradation level exceeds a threshold.

When continuous deterioration states are assumed, quite some models include multiple competing failure modes, but fitting these models to practical situations will be difficult because large amounts of data are required. Components that differ in quality, the ordering of spare components, and production systems have received very little attention, and future research could further investigate these extensions.

4. Multi-unit systems

In this section, we review studies that consider multi-unit systems. A system is considered as a multi-unit system if at least some of the considered maintenance interventions apply to individual units of the system. Analogous to single-unit systems, we classify the studies based on the number of condition states of the units, again by distinguishing two states (functioning or failed), three states (good, defective or failed), multiple discrete states, and a continuous state space. We note that the number of deterioration states may not be the same for each unit. In this case, the study is classified in the section of the most detailed deterioration processes.

4.1. Two condition states

We start the discussion of multi-unit systems without condition information with studies that consider economic dependence, then we consider studies that consider another type of dependence. We finish with studies that take either spare parts ordering or production decisions into account.

4.1.1. Economic dependence

Two types of economic dependence have been considered. We start with studies that consider a fixed setup cost for maintenance, independent of the number of units that are maintained. Thereafter, we consider systems that require inspections to identify failed units, and that assume that the inspection cost does not depend on the number of units that are inspected.

A fixed setup cost for maintenance that is shared if multiple units are maintained simultaneously is adopted by various studies. Lugtigheid, Banjevic & Jardine (2008a) use a weighted sum of the ages of the components as the state of the system, and plan the next maintenance activity based on this state. A myopic policy that is analyzed numerically prescribes which units should be maintained at a planned maintenance activity or upon failure of a unit. Canh Vu, Do, Barros & Bérenguer (2014) develop a grouping approach for maintenance activities based on a genetic algorithm. They acknowledge that grouping can both have a positive economic impact because of shared setup costs, as well as a negative impact because it can lead to a shutdown of the system. Moghaddam & Usher (2011) determine which components should be maintained at the end of each time period. Maintenance actions are imperfect and reduce the effective age of components. A nonlinear mixed-integer program is formulated to optimize the maintenance policy. Laggoune et al. (2010) consider a setting in which each unit has its own preventive maintenance interval. When a unit fails the policy prescribes whether other units should be maintained as well. Uncertainty in the parameters of the lifetime distributions is considered, and the Bootstrap technique is applied to model these uncertainties. Heuristic decision rules are evaluated based on numerical calculations, and the analysis is applied to the real case of a centrifugal compressor. Zhou, Lu & Xi (2012) consider a dynamic job shop and develop a heuristic to dynamically determine which units to maintain at the end of each job.

Other studies consider economic dependence in the sense that a fixed inspection cost is only incurred once regardless of the number of units inspected. Liu, Yeh, Xie & Kuo (2017c) determine heuristic policies for a system consisting of non-identical components. Each component is inspected periodically to detect failures. The next studies only consider specific policies and are therefore able to base the analysis on renewal theory. Zhang & Yang (2015) assume that periodic inspections are carried out to identify failed components. Two policies are considered. The first assumes that the entire system is perfectly replaced when a failure is detected. The second assumes that only the failed component is imperfectly repaired. The inspection interval is optimized. Alebrant Mendes, Coit & Ribeiro (2014) compare various policies for redundant systems that are inspected periodically and for which only one working component is required. The components have exponential lifetimes. Taghipour, Banjevic & Jardine (2010) consider a system with two groups of components. Failures of components in the first group are fatal and self-announcing. Failures of components in the second group are silent and are revealed only by periodic inspection. These silent failures are penalized by a downtime cost. Failures are minimally repaired during a finite time horizon.

Various heuristic policies have been studied for multi-unit systems with a fixed setup cost for maintenance. Future research could aim to improve these heuristics or, although challenging, focus on determining optimal policies.

4.1.2. Other dependencies

We start this section with studies that consider structural dependence. Thereafter, we discuss various types of resource dependence, and we finish with a single study that includes stochastic dependence.

Various studies include a structural dependence. Zhou, Huang, Xi & Lee (2015a) use simulation-based optimization to consider structural dependence from a technical point of view. If one of the units fails or reaches its reliability threshold, preventive maintenance opportunities for the other units arise. The disassembly sequence of a maintenance action influences the maintenance duration and cost. Wu, Chen, Wu & Wang (2016) consider structural dependence from a performance point of view. Systems with an arbitrary structure are considered, and an importance measure is introduced to determine which units should be preventively maintained when a unit fails. Heuristic decision rules are optimized based on simulation. Canh Vu, Do & Barros (2016) consider systems with an arbitrary complex structure and propose preventive maintenance decision-making rules that take the mean remaining lifetimes and the Birnbaum's importance measures of units into account. Zhu, Fouladirad & Bérenguer (2016) also consider structural dependence from a performance point of view and combine this with economic dependence. They consider a system of systems, each consisting of a number of critical and a number of non-critical components. Periodic routine, reactive, and opportunistic maintenance are considered, and the analysis is based on renewal theory. Hajipour & Taghipour (2016) use a genetic algorithm and consider systems with hard-type and soft-type components and k-out-of-N systems. Failures of components are unrevealed as long as they do not lead to system failure and aperiodic inspections are optimized. There is no preventive maintenance, and failed components are either minimally repaired or replaced. Sanchez, Carlos, Martorell & Villanueva (2009) consider periodic imperfect maintenance and minimal repairs upon failure. They assume that the effect of imperfect repairs is uncertain and use a genetic algorithm to minimize both unavailability and cost. A case study relating to standby safety systems of nuclear power plants is presented.

Now we consider resource dependence. Armstrong (2002) considers a set of machines that are maintained by a limited number of technicians. Two maintenance policies are analyzed by using simulation. The single age repair policy sends units for preventive repair if an age-limit is reached. The double age repair policy uses two age-limits. If the lower limit is reached a unit is only sent for repair if a technician is available. De Smidt-Destombes, Van Der Heijden & Van Harten (2004) use exact and approximate calculations to study the availability of a k-out-of-N system for which maintenance is initiated when the number of failed components exceeds a threshold. Failed components are repaired by a repair shop with limited capacity. De Smidt-Destombes, Van Der Heijden & Van Harten (2007) consider a similar setting but assume a block replacement policy. They use approximations that are validated using simulation. De Smidt-Destombes, Van Der Heijden & Van Harten (2009) develop a heuristic algorithm to jointly optimize the fixed number of spare parts in the system, the maintenance frequency and the repair capacity for a k-out-of-N system. Zhong & Jin (2014) use renewal theory to analyze a two-component system with a working component and a cold standby spare component. The working component undergoes periodic preventive maintenance, provided that the spare component is available. The system fails if the working component fails while the other component is being maintained. Phan & Zhu (2015) develop a heuristic multi-stage algorithm that can be applied to large-scale infrastructure systems. The units are first divided into groups and the inspection interval per group is determined. Thereafter, the inspection schedule for the whole system is determined, accounting for workforce and budget constraints. The framework is applied to the inspection of fire hydrants. Rasmekomen & Parlikad (2013) consider a system consisting of a number of parallel machines that each process raw material into a single critical machine. A single maintenance crew can only maintain one machine at a time.

The following studies consider resource dependence in the sense that only a limited amount of time is available for maintenance in between successive missions. Pandey, Zuo, Moghaddam & Tiwari (2013) consider a series-parallel system. For each component the degree of maintenance should be determined to maximize the reliability of the next mission. The problem is formulated as a non-linear program and evolutionary algorithms are used to solve it. Diallo, Venkatadri, Khatab & Liu (2018) develop a two-stage heuristic algorithm to select components for maintenance, and to determine the degrees of repair. Do, Canh Vu, Barros & Bérenguer (2015a) consider a series system for which a limited number of repairmen is available to carry out preventive maintenance between missions. Failures are rectified by immediate minimal repairs. Khatab & Aghezzaf (2016) assume that various maintenance levels are available. The quality of maintenance actions is stochastic and there is a reliability threshold for the next mission.

A model is provided, but the optimization is left for future research. Duan, Deng, Gharaei, Wu & Wang (2018) actually analyze a similar model by using a simulated annealing algorithm. Khatab, Aghezzaf, Diallo & Djelloul (2017) consider a similar setting for a system consisting of subsystems in series, each subsystem consisting of components in parallel. The durations of maintenance actions, missions and breaks are stochastic. A non-linear stochastic program is provided and some small problem instances are analyzed.

Another type of resource dependence arises when a set of geographically distributed assets have to be maintained by a limited number of technicians. Camci (2014) defines the traveling maintainer problem in which a set of such assets needs to be maintained by a single technician. A static model is formulated taking into account that the time at which an asset is reached influences its failure probability. A genetic algorithm is used to obtain solutions. Camci (2015) also considers geographically distributed assets. Based on a genetic algorithm, the assets that require maintenance during a finite time horizon are first distributed over a number of days. Thereafter, the order of maintenance visits per day is determined. If a failure occurs prior to scheduled maintenance, a new schedule is determined for the remaining days. López-Santana, Akhavan-Tabatabaei, Dieulle, Labadie & Medaglia (2016) use a two-stage heuristic and assume that multiple technicians are available to visit assets. First, a time window is determined for maintenance of each asset. Thereafter, a routing model schedules the tasks for each technician.

Zhang, Fouladirad & Barros (2017) consider a two-component load-sharing system with stochastic dependence. The failure rates of the components are time and load dependent. When a component is failed or being maintained, the failure rate of the other component increases. Three maintenance policies with imperfect repairs are analyzed based on renewal theory.

Similar to settings with economic dependence, multi-unit systems with another type of dependence are also mainly analyzed by approximate methods, leaving room for future research. Other points are that structural dependence from a technical point of view and stochastic dependence have received relatively little attention.

4.1.3. Spare parts ordering and production

In this section we first review studies that include the ordering of spare components, after which we discuss studies that adopt a production setting.

The ordering of spare components is included by a number of studies. Brezavššček & Hudoklin (2003) use numerical analysis for the joint optimization of block-based maintenance and spare parts ordering. Failed components are replaced immediately upon failure if spares are available, and all components are replaced at the end of each maintenance interval. Reorder points are chosen such that new spares arrive at the end of the maintenance intervals. The other studies that include spare parts decisions use simulation models. Bjarnason, Taghipour & Banjevic (2014) consider a k-out-of-N system. A failed component is either minimally repaired or replaced with a spare from inventory, depending on its age. Orders for new spares can be placed periodically and arrive after a lead time. Van Horenbeek, Scarf, Cavalcante & Pintelon (2013b) jointly optimize maintenance and inventory decisions. The age-based maintenance policy is used for each component. A replacement is carried out immediately when a spare component is available, and is delayed otherwise. New spares are ordered periodically up to a fixed level. The stockpile of spares is heterogeneous and consists of weak and strong components. Poppe, Basten, Boute & Lambrecht (2017) consider a set of identical components and continuous review for spare components. Simulation is used to compare purely corrective maintenance to periodic maintenance. The authors also extend this to a condition-based maintenance setting with linear degradation.

There exist a number of studies that combine maintenance scheduling with production planning. Nourelfath, Fitouhi & Machani (2010) develop an integrated production and preventive maintenance model for a system that should produce given quantities of a number of product types during a planning horizon. Block-based maintenance with minimal repair upon failure is used for all units, and a genetic algorithm is used to determine policies. Nourelfath & Châtelet (2012) formulate a mixed integer program to study a production system with a similar production requirement and with a set of parallel components. Multiple components can fail simultaneously due to a common cause. The components are simultaneously renewed periodically. Component failure lowers the production rate of the system. Xiao, Song, Chen & Coit (2016) assume that a given set of jobs should be processed on a series system. They use a genetic algorithm to jointly determine the production schedule and the preventive maintenance interval. Failures are minimally repaired. Lu & Zhou (2017) consider series-parallel multistage manufacturing systems. Products go through different process routes and deterioration of a machine not only affects the product at the current stage, but also propagates to down-stream stages. An opportunistic preventive maintenance policy is combined with minimal repairs at failure. Specific policies are studied using numerical calculations. Bouslah, Gharbi & Pellerin (2018) consider a production line with a constant demand rate consisting of two machines in series. The machines are subject to aging which leads to decreased product quality. The second machine is also subject to quality-dependent failures caused by defective parts produced by the first machine. A simulation-based optimization approach is used.

Because the inclusion of spare parts ordering or production planning results in more complicated models, simulation is used to a large extent to analyze these models. Future research could aim to use other methodologies.

4.2. Three condition states

Deterioration processes with three states are mostly used by studies on single-unit systems. Only a limited number of studies on multi-unit systems uses this deterioration model. De Jonge, Klingenberg, Teunter & Tinga (2016) consider a system with units that first give an alert signal, and later an alarm signal. After an alarm signal preventive maintenance is always performed in time to avoid failure. Since maintenance costs consist of a fixed setup cost, two clustering policies are compared with a policy that individually maintains all units. The analysis is based on renewal theory and on simulation. Arts & Basten (2018) use maintenance policies for single-unit systems to develop a periodic maintenance policy for multi-unit systems with minimal repairs upon failure. Part of the units of the system have two states, others deteriorate according to the delay-time model. The model can be analyzed based on renewal theory. De Smidt-Destombes, Van Der Heijden & Van Harten (2006) consider a k-out-of-N system in which each component is in the as-good-as-new state, the degraded state, or the failed state. The number of spares and the repair capacity are limited. Maintenance is initiated when the number of failed components exceeds a threshold. After a fixed lead time, all failed and possibly also all degraded components are replaced. The specific policies that are considered can be analyzed numerically. Panagiotidou (2014) distinguishes minor and major failures. Minor failures can be repaired, major failures require a replacement. Failures are not self-announcing and the system is inspected periodically. The policy determines when and how many spare parts are ordered while taking into account order, holding and shortage costs. A periodic review and a continuous review policy are studied based on numerical calculations. Seyedhosseini, Moakedi & Shahanaghi (2018) use simulation-based optimization to study a two-unit system consisting of a unit with two states and hidden failures and a unit with three states and self-announcing failures. Imperfect periodic inspections are carried out, and failure of the second unit increases the failure rate of the first unit. Berrade, Scarf & Cavalcante (2018) consider a similar two-unit system, but assume that failure of the first unit induces a defect in the second unit with a certain probability. Their analysis is based on renewal theory. Zahedi-Hosseini, Scarf & Syntetos (2017) consider a system consisting of identical units that is inspected periodically. They take the ordering of new spare components into account. Simulation is used to compare various periodic and continuous review policies.

The delay-time model for multi-unit systems has been studied in much less detail than for single-unit systems. Extensions that have been considered for single-unit systems could also be included in studies on multi-unit systems, e.g., delayed maintenance interventions, delayed first inspections, aperiodic inspections, external shocks, or a heterogeneous set of spare components.

4.3. Discrete state space

Most of the studies on multi-unit systems with a discrete state space consider structural dependence from a performance point of view. We start this section with these, after which we discuss a few studies that consider different types of dependencies.

The following studies assume structural dependence from a performance point of view. Marseguerra, Zio & Podofillini (2002) consider a series-parallel system and simultaneously optimize profit and availability. Each unit has its own preventive maintenance threshold and a genetic algorithm is used to determine policies. Zhou, Zhang, Ran Lin & Ma (2013) consider a series-parallel system. The components deteriorate according to a discrete time Markov chain. The policy uses a preventive maintenance threshold and a lower opportunistic maintenance threshold for each component. Deterioration levels are observed only at inspection, and the time until the next inspection depends on the current deterioration level. An optimization method based on the theory of stochastic ordering is developed, and simulation is used to evaluate the performance of this method. Zhou, Lin, Sun, Bian & Ma (2015c) consider a similar system but assume that the components deteriorate according to a continuous-time Markov chain and are monitored continuously. The policy uses a preventive maintenance threshold and multiple opportunistic maintenance thresholds for each component. Maintenance actions are imperfect. A method based on linear programming is developed to reduce the decision space.

Zhou, Ran Lin, Sun & Ma (2016) consider a parallel-series system. Components deteriorate according to a discrete-time Markov chain, and the model includes stochastic dependence, economic dependence and resource dependence. Olde Keizer, Teunter & Veldman (2016) consider a k-out-of-N system with a penalty cost for system failure and a setup cost for maintenance. They use a Markov decision process to determine the optimal policy and compare the performance of this policy with various basic policies. Alebrant Mendes, Ribeiro & Coit (2017) numerically analyze a two-component cold standby system that is inspected periodically. Both components have an excellent, good, poor, and failed state. When the active component fails, the cold standby component is activated. A component in the poor or failed state at inspection is repaired. Olde Keizer, Teunter, Veldman & Zied Babai (2018) use a Markov decision process formulation to determine optimal replacement decisions for a parallel system with load sharing due to redundant components and with economic dependence. Dao & Zuo (2017b) consider structural dependence both from a performance and from a technical point of view. The units are connected in series, and a directed graph is used to represent the order of disassembling. A genetic algorithm is used to decide on the maintenance activities during breaks between missions.

Olde Keizer, Teunter & Veldman (2017b) use a Markov decision process to jointly optimize condition-based maintenance and spare parts ordering. Components share a pool of spare parts and ordered spare parts arrive after a fixed lead time. Maintenance actions and orders for spare parts depend on the deterioration levels of the units, the number of spares on hand, and the outstanding orders. Zhou, Guo, Lin & Ma (2018) consider a system consisting of multiple production units in series with intermediate buffers. To avoid the curse of dimensionality, a multi-agent factored Markov decision process is used to determine heuristic policies.

Gürler & Kaya (2002) subdivide the discrete states into a number of good states, a number of doubtful states, a preventive-maintenance-due state and a down state. A unit is immediately maintained preventively or correctively when it respectively reaches the preventive-maintenance-due state or the down state. Furthermore, the entire system is replaced when the number of components in doubtful states exceeds a threshold at this time. A genetic algorithm is used to determine policies. Tian & Liao (2011) consider a system with identical units that are either functioning or failed. A single covariate with a discrete state space influences the failure rates of the units, implying a stochastic dependence. The system is inspected periodically and a unit is maintained preventively if its failure rate exceeds a threshold. A fixed maintenance cost is incurred if at least one unit is replaced. A second threshold is used for opportunistically maintaining units. The thresholds are optimized numerically, the inspection interval is fixed.

Dao & Zuo (2017a) use simulation to analyze a series system that has to carry out missions under variable, unknown loading conditions. After each mission the set of components and associated maintenance actions have to be determined that maximize the expected system reliability in the next mission. Liu, Chen & Jiang (2018) consider a system that has to execute a sequence of missions with breaks between them. Because of limited resources a selective maintenance model is developed to maximize the probability of successfully completing the next mission. Stochastic durations of breaks and maintenance actions are taken into account, and ant colony optimization is used to determine policies.

Studies on multi-unit systems with a discrete state space mainly assume structural dependence from a performance point of view. The other types of dependencies have received little or no attention, although those are also present in practice. The main methodology for single-unit systems with a discrete state space is Markov decision processes. Because of the curse of dimensionality, we observe a shift to heuristic policies and approximate methodologies for multi-unit systems. Markov decision processes are still used, but are only applied to systems with a small number of units.

4.4. Continuous state space

We begin this section on a continuous deterioration state space with studies that consider economic dependence, followed by studies that consider stochastic dependence. We will end with studies that include the ordering of spare parts.

4.4.1. Economic dependence

The following studies combine continuous deterioration levels with economic dependence. Barata, Guedes Soares, Marseguerra & Zio (2002) assume that imperfect preventive maintenance is carried out if the deterioration level of a component reaches its threshold, and that a component is replaced upon failure. Simulation is used to compare values of the decision variables. Bouvard, Artus, Bérenguer & Cocquempot (2011) develop an algorithm that uses heuristic decision rules to dynamically schedule maintenance actions at each periodic inspection. Maintenance actions are grouped to reduce maintenance costs. Zhu, Peng & Van Houtum (2015) numerically analyze a system consisting of non-identical units that is inspected periodically. Failed units are maintained correctively at inspection, and each unit has its own preventive maintenance threshold. A high setup cost has to be paid for each inspection, also if no replacement is carried out. Castanier, Grall & Bérenguer (2005) consider two-unit systems with aperiodic inspections that are dynamically scheduled based on the current, observed deterioration levels. A setup cost is incurred only once if both units are inspected or replaced. They numerically evaluate the performance of heuristic decision rules. Van Horenbeek & Pintelon (2013) consider a random failure level and imperfect maintenance actions that reduce the degradation level by a fixed factor. Maintenance is first scheduled on the unit level. Thereafter, the first maintenance action for each unit is shifted in order to group maintenance actions and share setup costs. Simulation is used to assess the performance of heuristic decision rules. Shafiee, Finkelstein & Bérenguer (2015) consider a system consisting of identical components. Defects arrive according to a non-homogeneous Poisson process and propagate according to gamma processes. A component is maintained if a defect-size exceeds a certain threshold, and all components are replaced at an operational age-limit. A policy with two decision variables is analyzed numerically. Nguyen, Do & Grall (2015) use simulation-based optimization to study a system with both economic dependence and structural dependence from a technical point of view. Inspections are carried out periodically, and a set of decision rules are used to select the group of components that will be maintained. Poppe, Boute & Lambrecht (2018) combine condition-based maintenance for one monitored component with periodic maintenance for the other components of the system.

Two degradation thresholds are used to determine when to maintain the monitored component. These are optimized numerically based on renewal theory.

Due to their complexity, models with continuous deterioration levels and economic dependence are mainly studied using heuristic decision rules and simulation. Future research could aim to improve the resulting policies.

4.4.2. Stochastic dependence

Various studies consider stochastic dependence between units. Sheu, Liu, Zhang & Ke (2015c) consider a two-unit system. Unit A has a binary state, unit B a continuous state. Two types of shocks occur with age-dependent probabilities of occurrence. Type I shocks cause a minor failure of unit A, followed by a minimal repair, and a random amount of degradation for unit B. Type II shocks cause a system replacement. A two-dimensional maintenance policy is optimized numerically: Preventive maintenance is carried out at a certain age or when the degradation level of unit B exceeds a certain threshold. Sheu, Liu, Zhang & Chien (2015b) numerically analyze a similar setting but assume that the system is also replaced after a certain number of type I shocks. Tsai, Sheu & Zhang (2017) consider a two-unit system in which unit 1 is subject to both minor and catastrophic failures that both can only be detected by inspections. Minor failures are minimally repaired and result in a random amount of deterioration to unit 2. The system is replaced when the number of minor failures of unit 1 reaches a threshold, or when the deterioration level of unit 2 reaches a threshold. A policy with three decision variables is optimized numerically. Hong, Zhou, Zhang & Ye (2014) use gamma processes to model the degradation of the individual components, and use a copula to model the dependency. Periodic inspections are carried out and a preventive maintenance threshold is used. Simulation-based optimization is used and the variation in costs is also addressed. Li, Deloux & Dieulle (2016a) also use copulas to model stochastic dependence between components. A two-unit system is considered in which each unit has its own periodic inspection interval and preventive maintenance threshold. Cha & Castro (2015) assume that external shocks arrive according to a non-homogeneous Poisson process. Each shock either leads to immediate failure or a random amount of deterioration of all components. The system is inspected periodically and is replaced if the deterioration level of any component exceeds a threshold. The numerical analysis to determine the optimal policy is based on renewal theory. Rasmekomen & Parlikad (2016) consider a multi-unit system for which the deterioration states of certain units could affect the deterioration rates of other units. They use simulated annealing to determine a heuristic policy. Zhang, Fouladirad & Barros (2018) consider two versions of a two-component system with failure interactions. Failure of the first component either causes a random amount of damage to the other component, or it results in failure of the other component with a certain probability. Three preventive maintenance policies are analyzed numerically.

Studies that include stochastic dependence in a setting with continuous deterioration levels mainly focus on two-unit systems, leaving room for future research on general numbers of units. A lot of data are needed to fit models with stochastic dependence to practical situations. This makes the practical applicability of these models challenging.

4.4.3. Spare parts ordering

A number of studies include the ordering of spare components. Nguyen, Do & Grall (2017) use simulation to jointly decide on maintenance and spare parts ordering for systems consisting of non-identical components. Deterioration levels and structural importance measures are used to determine thresholds for ordering spare parts and for carrying out maintenance. Opportunistic maintenance decision rules are used to take advantage of economic dependence. The next two studies use a genetic algorithm. Wang, Chu & Mao (2009) assume a random deterioration level at which failure occurs. Each unit is inspected periodically and preventive maintenance is carried out according to the control-limit policy. New spares are ordered up to the maximum stock level when the reorder level is reached and arrive after a lead time. If no spares are available when the preventive maintenance threshold is reached, the unit continues to operate, without inspection, until new spares arrive. Zhang & Zeng (2017) also consider a system that is inspected periodically. A preventive maintenance threshold is used to initiate maintenance actions, and an opportunistic maintenance threshold is used to cluster maintenance actions. Spares are ordered when the number of spares is below a certain safety stock level.

The number of studies on spare parts ordering for multi-unit systems with continuous deterioration levels is limited, and these studies consider heuristic decision rules. Further studies could consider this area in more detail.

5. Conclusion and future research avenues

In this final section on conclusions and future research avenues we first elaborate on the general progress in the area of maintenance optimization. Then, we discuss applications of the models, the data that are needed, and model uncertainty. Thereafter, we devote a section to methodologies, and we finish with various model extensions that are suitable for future research.

5.1. General progress

We have reviewed papers on maintenance optimization that appeared in the period 2001 to 2018. After describing terms that are commonly used in the modeling process, we have classified papers by the single-unit and multi-unit system dichotomy, and by the type of state space of the deterioration process modeled.

Wang (2002) provided the previous broad review on maintenance optimization. Studies on maintenance optimization before 2002 mainly considered time-based maintenance settings. Only a small stream of studies considered failure limit policies for single-unit systems, in which the failure rate of a unit is assumed to depend on some increasing state variable. Studies on multi-units systems that adopt condition monitoring are not reported at all by Wang (2002). The types of dependencies between units that have been considered back then are economic dependence and failure dependence, the latter being a specific type of stochastic dependence. Structural dependence, both from a technical and from a performance point of view, and resource dependence did not attain any attention.

Substantial developments in the field of maintenance optimization can be observed since the review by Wang (2002). Various extensions of traditional time-based maintenance policies have been considered and some studies have been devoted to the effect of uncertainties in the model specifications. Another development is that an increasing number of studies is including condition information. Especially in the most recent years of the review period, ~~a lot of~~ studies consider a condition-based maintenance setting. Both situations with continuous monitoring and with inspections are analyzed, and both periodic and dynamically scheduled inspections are considered. Another trend is to take the ordering of spare components into account, or to simultaneously optimize maintenance and production decisions. The use of condition information also becomes more prevalent in multi-unit systems, and structural dependence from a technical point of view, structural dependence from a performance point of view, and various types of resource dependence have been addressed.

The increasing number of studies on condition-based maintenance is also reflected in the citation statistics. The papers in our review with ~~the highest number of~~ citations (based on Google scholar, September 2019), are Grall et al. (2002a), Grall et al. (2002b), Marseguerra et al. (2002), Zhou et al. (2007), and Castanier et al. (2005). Except for Zhou et al. (2007), all these are early studies ~~in the review~~ ~~period~~ that consider condition-based maintenance. Grall et al. (2002a) and Grall et al. (2002b) consider aperiodic inspection scheduling for single-unit systems, and Castanier et al. (2005) do so for two-unit systems. Marseguerra et al. (2002) consider multi-unit systems with structural dependence from a performance point of view. Zhou et al. (2007) consider imperfect repairs for a single-unit system in a model without condition information.

5.2. Applications, data and model uncertainty

Only a very limited number of studies in our review report on the application of their model to a case study with real life data, and most studies do not elaborate on the prerequisites for using their models in practice. One of these prerequisites is the availability of relevant data. Studies on time-based maintenance typically assume that the lifetime distribution is known, and sufficient failure data are needed to determine an accurate estimation for this distribution. For studies on condition-based maintenance, data are needed to fit a stochastic deterioration process and to assess the failure behavior. Especially in practical situations where a conservative maintenance policy is used, limited failure data are available, implying that these assumptions are doubtful. Furthermore, other information is often implicitly assumed to be known, such as quantifications of inspection and monitoring errors, the occurrence of external shocks, and the effects of external factors.

A follow-up to the studies by Dekker (1996) and Dekker & Scarf (1998) would be useful to investigate the availability of data and the extent to which the maintenance models that have been developed are applied in practice. A hopeful observation is that, because of ongoing developments in sensor techniques and monitoring systems, equipment data are often available in modern industrial environments (Choi, Wallace & Wang, 2018). When new equipment is considered for which no data are available, a tradeoff should be sought between decisions that result in valuable data (e.g. from failures) and cost-effective decisions. Machine learning techniques could be used to facilitate the decision making in such cases.

Limited availability of data means that accurate estimations of model parameters cannot be achieved. However, only a limited number of studies take parameter or model uncertainty into account, most of them in a time-based maintenance setting. This area could benefit from additional investigations. Next to time-based settings, uncertainties are also likely to be prevalent in condition-based maintenance settings. For instance, the parameters of the deterioration process, or even its functional form could be unknown. Other possibilities are uncertainty in the failure behavior of a unit, and uncertainty in the quality of condition monitoring. The latter is one step further than for instance incorporating imperfect inspections. Future research could assume that the stochastic distribution of the inspection error is not known.

5.3. Methodologies

Various methodologies have been used to analyze the maintenance models that we have reviewed. For single-unit systems, the main methodology is numerical analysis based on renewal theory. In cases that include parameter uncertainty, simulation is used to investigate the updating process over time. Simulation is also used for more complex models (e.g. those that include production decisions) and for models with continuous deterioration levels. The framework of Markov decision processes is often used for discrete deterioration states, and stochastic dynamic programs have been used to analyze problems with a finite time horizon. For multi-unit systems, identifying and evaluating renewal cycles is only possible for very specific models or policies, or for a very small number of units. As a consequence, these studies typically resort to alternative methodologies. Commonly used methodologies are simulation to compare various (heuristic) policies, simulation-based optimization if the number of decision variables is limited, genetic algorithms, and heuristic algorithms specified by the authors.

Models for multi-unit systems and models with continuous deterioration states could mainly benefit from using other, more exact methodologies. Studies that consider a discrete state space mostly consider a small number of states. Additional insights for models with a continuous state space could be obtained by discretizing the continuous-state deterioration process into a discrete-state deterioration process with a large number of states (De Jonge, 2019). For single-unit systems the analysis could then be based on Markov decision processes, whereas one may resort to approximate dynamic programming for multi-unit systems. A final note is that structural properties are typically available only for single-unit systems. Although more challenging, investigating structural properties of policies for multi-unit systems is also of interest.

5.4. Model extensions

Imperfect condition monitoring is mostly considered in situations where inspections are required to obtain condition information. However, the ongoing advances in real-time condition monitoring results in a trend towards more continuous condition monitoring. The common approach of modeling inspection errors by using independent normally distributed errors cannot be used to model continuous noise that results from continuous monitoring. New approaches are needed for realistically modeling continuous noise.

The concept of minimal repair is often used to model quick fixes after failure in time-based maintenance models. This concept has also been extended to ‘worse-than-minimal’ repairs. Both minimal and worse-than-minimal repairs can easily be modeled in time-based maintenance settings, respectively by assuming that the failure rate remains the same, or that it increases after a repair. However, it is difficult to translate these concepts to existing condition-based maintenance models. One way of modeling minimal repairs and 'worse-than-minimal' repairs in a condition-based setting is by omitting the common, but practically often unrealistic assumption that failure occurs when a fixed failure threshold is exceeded. In practice, failure does not often occur at a fixed deterioration level and also occurs when the deterioration level has been stable for some time. Then, the failure behavior might be described by a failure intensity that depends on the deterioration level. Adopting this approach, a minimal repair would mean that the deterioration level remains the same, and a 'worse-than-minimal' repair would mean that the deterioration level increases. In both cases, however, the unit becomes functional again after the repair. Future research could incorporate such repairs.

The concept of unpunctual maintenance that has been studied in time-based maintenance settings could be extended to condition-based maintenance settings. It is not only relevant to study the effect of maintenance that is not carried out exactly when it is scheduled, but also unpunctual inspections are of interest. Also the research on maintenance interventions during warranty periods in time-based maintenance settings could be extended to condition-based maintenance settings. Another suggestion is to extend the idea of a postponed replacement after an inspection in the delay-time model to systems with more than three states. Furthermore, heuristics are mainly used when inspections have to be scheduled for a continuous deterioration process. Future research could study the exact aperiodic inspection schedule for deterioration processes with a continuous state space.

The majority of studies on multi-unit systems consider a single type of dependence, implying that ample research opportunities exist that incorporate multiple dependencies. Furthermore, existing studies mainly consider economic dependence or structural dependence from a performance point of view. The small number of studies that consider stochastic dependence either assume a model without condition information, or limit themselves to two units. Structural dependence from a technical point of view has also hardly been studied. Quite some studies on resource dependence exist. These either assume that limited time for maintenance is available in between successive missions, or that a limited number of repairmen are responsible for a set of geographically distributed assets. However, these studies typically consider time-based maintenance settings without condition information. Further extensions are encouraged that account for condition information.

The joint optimization of spare parts ordering and maintenance is mainly considered for multi-unit systems. Most of these studies assume that no condition information is available, leaving ample opportunities for future research. The joint optimization of production planning and maintenance is considered for both single-unit and multi-unit systems. All studies on multi-unit systems that also take production decisions into account consider time-based maintenance settings without condition information. Future research could be devoted to the joint optimization of job scheduling and maintenance decisions based on condition information. This topic is also suggested in the essay by Feng & Shanthikumar (2018) which describes how research in production and operations management may evolve in the era of big data.

We finish with features that have mainly been studied in single-unit settings, but that are suitable to be incorporated in multi-unit settings as well. A setting in which aperiodic inspections are scheduled dynamically is of interest; these should then be scheduled based on the conditions of all individual components. Other extensions would be to consider imperfect inspections or condition monitoring, a mixed population of weak and strong components, and uncertainty in the lifetime distribution or in the behavior of the characteristics of the deterioration process. Finally, we would like to mention that recent advances in delay-time modeling could also be extended to multi-unit systems, e.g., postponed replacements after an inspection, delayed first inspections, externally arriving shocks, and failure-inducing inspections.

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