Financial stress relationships among Euro area countries: An R-vine Copula approach

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Abstract

One of the biggest challenges of keeping Euro area financial stability is the negative co-movement between the vulnerability of public finance, the financial sector, security markets stresses as well as economic growth, especially in peripheral economies. This paper utilises a ARMA-GARCH based R-vine copula method to explore tail dependance between the Financial Stress Indices of eleven euro area countries with an aim of understanding how financial stress are interacting with each other. We find larger economies in the Euro area tend to have closer upper tail dependence in terms of positive shocks, while smaller economies tend to have closer lower tail dependence with respect to negative shocks. The R-vine copula results underline the complex dynamics of financial stress relations existing between Euro Area economies. The estimated R-vine shows Spain, Italy, France and Belgium are the most inter-connected nodes which underlying they might be more efficient targets to treat in order to achieve a quicker stabilizing. Our results relate to the fact that Eurozone is not a unified policy making area, therefore, it needs to follow divergent policies for taming the effects of financial instability to different regions or groups of economies that are more interconnected.

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1 Introduction

The European sovereign debt crisis has been taking place since the end of 2009. Several Eurozone member states (Greece, Portugal, Ireland, Spain and Cyprus) were neither able to repay nor refinance their government debt or to bail out over-indebted banks under their national supervisory scheme without the assistance of third parties[1]. The ever-expanding sovereign risk spreads, along with the feedback effect that downgrading of the peripheral economies' bonds had on Euro Area banks, have led to a series of unprecedented reactions from the side of European policy makers. A number of financial rescue packages have been provided to the most heavily affected economies, complemented with austerity measures ultimately affecting these countries' growth recovery prospects. On the same time, the European Central Bank implemented a set of unconventional monetary policies, in an effort to ameliorate the effects of the financial instability to the Eurozone banking markets, leading to a stratospheric increase to the value of the central bank's financial balances. Despite of improving global and euro area economic and financial conditions from mid-2013, the overall outlook for financial stability has remained very challenging in the euro area. One of the biggest challenges is caused by negative co-movement between the vulnerability of public finance, the financial sector, security markets stresses and economic growth. Therefore, understanding the co-movement structure of financial stress between countries does not only provide better interpretation of single country's stress situation, but also can point out a more efficient path towards financial recovery. For this purpose, we use an ARMA-GARCH based R-vine copula method to explore the tail dependence of eleven Euro area countries's financial conditions, with an aim of understanding how the degree of financial distress in each one of them interact with each other, particularly in the case of extreme events.

The perplexed nature of the recent Global Financial Crisis, which was later transformed to a Eurozone sovereign and banking crisis clearly demonstrated that modern financial markets are highly interconnected. An adverse shock or a market's structural fragility can rapidly spread to a global scale, indicating the need for new measures and quantitative tools, able to capture the potential effect of such financial events to several financial domains. Up until recent, the early warning systems literature neglect the potential inherent weaknesses of financial systems, leading to the development of impaired tools for monitoring their conditions (Gramlich and Oet, 2011). Studies, such as Hatzius et al. (2010), Holl et al. (2012), and Oet et al. (2011), have developed an

[1]Given the significant decline in the GDP of majority of European economies, triggered by the global financial crisis 2008-2009, the annual GDP growth rate in European Union declined from 0.5% to -4% from 2008 to 2009.

array of indices by using market-based indicators in real time and high frequency associated with the banking, securities, and foreign exchange markets, often termed 'financial stress indices' (FSI). The level of the FSI indicates the interaction of financial vulnerabilities and quantifies the size of financial shocks (Grimaldi, 2010, 2011), which makes it a valid choice for representing the quantified financial stress level of a country.

However, in today's financial market, countries are closely inter-related because of the free flow of capital, existence of multinational financial institutions and funds. Investigating the dependence between counties' stress will help us better interpret the behavior of individual countries in an integrated market. Copula method as a multi-variate model can provide comprehensive information for this purpose. The construction of vine coupla of eleven euro area countries' FSIs does not only present the stress dependence of pair countries, but also be able to assess the level of the financial stability of the euro areas as a whole.

Our findings indicate certain level of historical information persistence in the FSI volatility of most Euro Area countries. Only Finland shows asymmetric feature in terms of financial stress volatility reaction to the negative shocks. Several type copula families including Gaussian, student-t , Frank, BB8 and Survival Gumbel copulas are selected to model 55 country pairs which reveals different tail dependence of FSIs. We also find that larger economies in the Euro area tend to have closer upper tail dependence, while smaller economies tend to have closer lower tail dependence. Once again, Finland is an exception and is generally tail independent with other countries in terms of financial stress in extreme situation. R-vine copula is the best choice comparing with traditional C-vine and D-vine in explaining the interdependency of these countries as a whole. The estimated R-vine copula results underline the complex dynamics of financial stress relations existing between Euro Area economies. Spain, Italy, Belgium and France manifest as central nodes to Eurozone financial interconnectedness, which underlies they might be more efficient targets to treat in order to achieve quicker recovery.

The contribution of this research is fivefold: firstly, to our knowledge, this is the first attempt to apply the extreme value and tail dependence analysis on financial stress literature; secondly, the paper identifies the persistence level as well as asymmetry of information regarding financial stress in both first and second moment; thirdly, this study exams the co-movement of financial stress of eleven countries in the European Union at the same time; fourthly, by using R-vine copulas, we do not only conquer large numbers of dimensions issues but also be able to select a wide variety of dependence, which reflect flexible upper and lower tail dependence properties; the last but not the least, the research simplifies the complex dependence structure of these euro area countries's FSI as well as provides implications for policy practice.

The remainder of the paper is as follows. Section 2 is a literature review in sovereign debt analysis and copula methods. Section 3 introduces the GARCH-based vine copula method. Section 4 explains and discuss the results we get from the analysis. And Section 5 concludes.

2 Literature Review

The recent financial crisis and subsequent recession provide a very good example of how shocks of one sector could cause wide spread financial strains in other industry or entire economy. Kliesen and Smith (2010) define that financial stress is a multidimensional problem involving a number of simultaneous or temporally proximate exogenous shocks to factors from banks and financial markets. Therefore, measures of the single country's aggregate financial stress should include sets of default probabilities for a broad categories of products or markets. Comparing with Reinhart et al. (2000) binary variables study of financial conditions[2], Grimaldi (2010, 2011) considers stress, the product across different vulnerable markets and shocks, can be determined by the interaction between various financial vulnerabilities and sizes of shocks. Chau and Deesomsak (2014) define the signs of stress including financial turmoil, exchange rates under pressure, a combination of depreciation and depletion of foreign reserves, dwindling capital inflows, withdrawals from merging economy equity and debt funds, and bank lending was scaled back.

In addition, in order to overcome a potential problem of focusing solely on one indicator and ignoring all other potential factors, some studies have combined several indicators designed to measure financial market stress into one summary variable, like an index number. Cardarelli et al. (2011) argue that FSI addresses the weakness of historical approaches to identifying episodes of financial crises, such as less attention on financial stress with little macroeconomic impact, and little attention to pure security market stresses or liquidity squeezes[3]. Louzis and Vouldis (2012) consider systemic stress equal to the amount of systemic risk which has materialized[4]. They construct a Financial Systemic Stress Index (FSSI) to exam Greek financial stress condition, and they confirm that FSSI can

[2] They use 1 for stress period and 0 for non-stress period to define the financial conditions in an economy.

[3] Kliesen and Smith (2010) construct eleven financial market variables from US (Kansas City Financial Stress Index) to measure the degree of financial stress in the markets. Hanschel and Monnin (2005) forecast Switzerland banking sector crisis by using stress index. Morales and Estrada (2010) use FSI for determining bank profitability and probability of default in Colombia. Yiu et al. (2010a) using a monthly composite financial stress index (FSI) to capture historical episodes when Hong Kong was under significant financial stress.

[4] Systemic risk, in turn, can be defined as the risk that financial instability becomes so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially.

timely identify the crisis periods as well as the level of systemic stress in the Greek financial system. Johannes et al. (2011) define financial stress as a disruption that impairs the financial markets ability to act as an efficient intermediary between lender and borrower or buyer and seller. They test how the equal-weighted financial stress index is affected by alternative historical reference periods and alternative weighting. One of their key conclusion is that the development of the index is affected to some extent over time. The differences in the way of various index development have however been few and temporary. Illing and Liu (2006) suggest stress increasing with expected financial loss, with risk or with uncertainty. They conduct an internal survey in the Bank of Canada to determine FSI by selecting which events over the past twenty-five years were most stressful for Canada's financial system. The survey-based FSI could more directly reflects the Canadian experience. Hansen (2006) states financial stress as risk, his constructed FSI from Denmark is very much related to the business cycle, macroeconomic uncertainty and monetary policy.

Copula method is widely used in finance context because it expresses the multivariates's dependence with a quartile manner which is natural in terms of extreme risk assessment. Patton (2008) finds elliptical (symmetric) copula families cannot model financial data well enough because of asymmetric responds in terms of shocks in financial markets. More extend forms of copulas have been developed since, for instance partially symmetric copula, Mix-id copula and skewed multivariate copula (McNeil and Neslehova, 2009, Joe, 1993, Joe and Hu, 1996). Nonetheless, the flexibility of dependence can be explained by these model is inadequate to explain complex financial dependencies. Joe (1997) argues that even archimedean copula families can explain a greater variety of dependance, the dimensions these families are very limited. Joe (1997) proposed the idea of vine structure of copula (Vine copula) and it has been made application by Bedford and Cooke (2002). Vine copula, on the other hand, improves the explanatory power of copula methods by allowing higher dimensions, wider range of copula families and asymmetric dependance structures. Aas et al. (2009) perform statistical inference on two particular vine structures of regular vines: C-vine and D-vine. C-vine and D-vine are two traditional vine copulas with predetermined vine structure and special cases for regular vine. Nikoloulopoulos et al. (2012) apply vine copulas in 5 dimensional stock markets Value at Risk (VaR) estimation. Dißmanna et al. (2013) suggest a 'edge' to make regular vine estimation feasible and application has been made in financial returns. A more flexible structure of vine will present more accurate information regarding the dependence. Zhang (2014) analyzes sovereign debt in crisis period with ten dimensional sample which suggests vine copula's ability of high-dimensional analysis. Therefore, our constructed vine copula with 11 dimensional countries' FSIs will provide more detailed financial stability dependence information.

3 Methodology

We exam the inter-relationship of Euro area countries's financial stress conditions by applying a multi-dimensional ARMA-GARCH based vine copula model.

3.1 Measuring financial stress

Our dataset consists of a set of financial stress indices (FSIs) for the original Euro Area economies, excluding Luxembourg. For each one of countries in this dataset, we develop four sub-indices, each one representing the prevailing financial conditions to the following financial markets: banking, bond, money and stock market. The advantage of these stress indices is the composition of a number of individual financial indicators into a single one, exhibiting important features of the markets under consideration. The relevant literature (e.g. (Cardarelli et al., 2011)) has underlined their usefulness as early warning indicators, as well as important tools for assessing financial stability and the degree of markets financial distress.

Table 1 exhibits the individual indicators employed for the FSIs construction. The dataset is formulated from existing research in this area. We confine this set of indicators to those available on a weekly basis, which are also available for all economies under investigation. We aim to include both the pre- and the post-Euro Area crisis period, by choosing to cover the period spanning from 05/01/2001 until 20/09/2013. We did not choose daily data for avoiding any potential mismatches in public holidays or trading days (Yiu et al., 2010b).

In total, the dataset consists of 21 variables, covering the four aforementioned markets (seven indicators for banking market, five for money and equity markets, four for the bond market). The objective is to capture the potential instability stemming from the different types of risks, being represented by this set of metrics. For the case of banks, we include measures exposing the degree of risk associated with their activities, together with proxies of profitability, market value and operational efficiency. For instance, bank equities realized volatility, together with bank stocks' beta and the (negatively signed) equities returns are indicators showing the level of volatility and risk perception for the bank market. Profitability and its variations is evident from the price-to-earnings ratio. Increasing value for the turnover volume indicates the investor's sentiment and uncertainty towards the developing market conditions. Dividend yields and market value are related to banking

Table 1: Variable used in Financial Stress Indices

Banking Sector	**Money Market**
Dividend Yield	TED Spread
Market Value	Inverted Term Spread
Turnover by Volume	Treasury Bill Realized Volatility
Price/Earnings ratio	Main Refinancing Rate - 2yr Government Bond Yield
Bank Equities Realized Volatility	Main Refinancing Rate - 5yr Government Bond Yield
Banking Sector Beta	
Bank Equities Returns	
Equity Market	**Bond Market**
Stock Returns	Sovereign Spread
Dividend Yield	Government Bond Realized Volatility
Price/Earnings ratio	Corporate Spread
Stocks Realized Volatility	Government Bond Duration
Market Value	

institutions fundamentals and they are a signal of the perceived level of default risk for them.

Money market indicators indicate the level of credit and counterparty risk, while interbank liquidity conditions are also represented. TED spread (the spread between the 3-month Euro-Libor from respective Treasury bills) is expected to increase in periods of worsening financial conditions. On top of that, the spreads of the main refinancing rate from the short term (2-year and 3-year) governmental bond yields is another indicator, depicting deteriorating liquidity conditions. For these particular spreads, we incorporate them with a negative sign, as negative values of these spreads exhibit higher financial stress. Finally, realized volatility measure of treasury bill of the countries under investigation is also computed, showing the degree of volatility risk in this market, while increasing value of the inverted term spread is an alarming signal of excessive default risk and financial strains in the market.

Similarly, the equity market measures exhibit the situation prevailing in the markets' uncertainty and volatility, while measures of firms' default risk are also included. Stock returns, negatively signed, is an indication of the aforementioned uncertainty, with higher price variation in period on heightening financial stress. Market value and dividend yields are exhibits of the level of default risk and credibility of the listed firms, in the same fashion as the case of the banking markets previously discussed. P/E ratio is representative of financial sustainability of firms, while the equities realized volatility indicates the degree of historical risk perception for each particular stock market in our sample.

Finally, for the case of bond markets, metrics of the level of sovereign risk, as well as private sector default risk are included to the composition of the bond sub-index. Specifically, sovereign spread is the spread between each country's 10-year government bond yield and the German 10 year government bond yield, which is considered as a safe haven investment choice. Then, the bond realized volatility proxies the perceived level of volatility risk in this market. Moreover, decreasing government bond duration is also an indicator of increasing financial stress and uncertainty. As Lee and Yau (2011) show, bond duration decreases, especially for bonds with lower ratings. Thus, countries with increasing solvency issues and deteriorating fundamentals, as in the case of Euro Area crisis, might exhibit such a behavior in their bonds' duration.

The aggregation of the variables in Table 1 is conducted by using the variance-equal approach. There are two stages in this method: first, we standardize individual variables, by demeaning them and dividing with their standard deviation. Then, an equal weight applies to each one of these variables for the calculation of the final, aggregated stress index. Finally, the country level index is based on the four sub-indices, again on an equal weight approach. In general terms, the financial stress index is calculated according to the following formula:

$$\mathrm{FSI}_t = \sum_{i=1}^{4} w_i \mathrm{fsi}_{it} \tag{1}$$

where, fsi stands for the market systemic risk index on time t and is the weight factor for the aforementioned market. By employing this method, any issues of mis-measurement or with the units of measurement are avoided. In FSIs literature, the variance-equal approach is the most heavily used, given its simplicity and the production of stress indices able to accurately capture market conditions and level of distress (Hakkio and Keeton, 2009, Magkonis and Tsopanakis, 2014, Apostolakis and Papadopoulos, 2014, MacDonald et al., 2015). This means that the equally weighted stress indices successfully represent the prevailing conditions in the financial markets. Figure 1 exhibits the FSIs for the group of countries under examination.

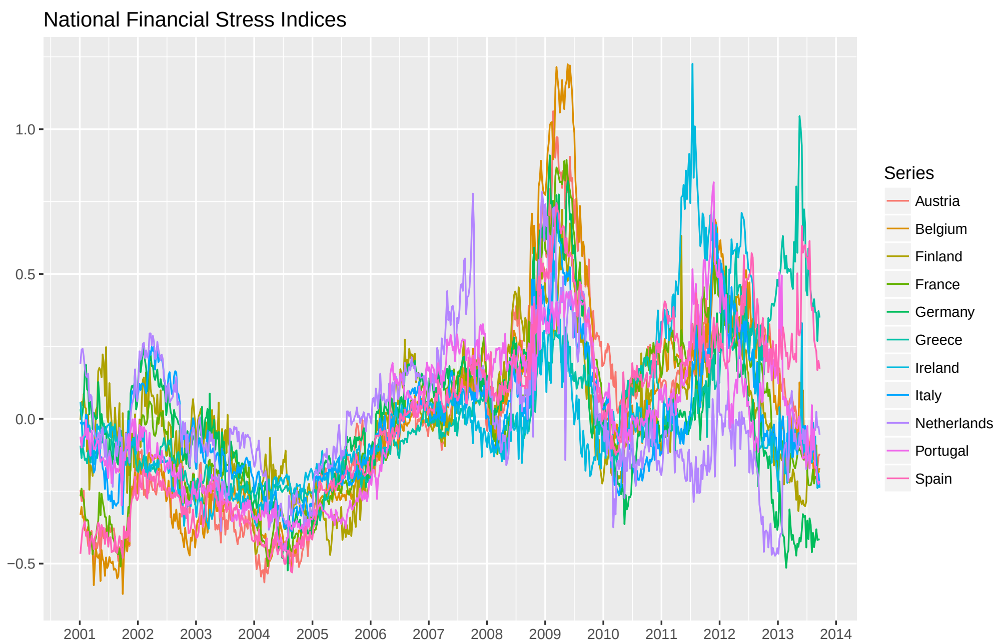


Figure 1: FSI

3.2 GARCH-vine-copula

We use three-step ARMA-GARCH based vine copula approach to model FSIs for eleven European countries. In the first step, each series of FSI will be modeled by ARMA-GARCH filters (GARCH, EGARCH, GJR-GARCH, T-GARCH are included) with student t distributed innovation and the option to include heteroscedasticity term into the mean equation. This step identifies the feature of the data from the country itself. In the second step, a bi-variate copula analysis issued to investigate the pair countries's FSIs's standardized residuals' dependence and categorize the tail dependence of each fifty-five pairs into proper copula family. In the last step, we construct vine copula structures by using copula families which are estimated in the second step. This step focuses on the dependance feature of all countries as a whole.

3.2.1 Step 1: ARMA-GARCH filter

Because of the non-stationary feature of weekly FSI, we use the first difference of the series instead[5]. It is calculated as follows:

$$R_j = -\Delta \mathrm{FSI}_j \tag{2}$$

where $j = 1, \ldots, d$, is country number, R_j will be negative the change of the index, FSI_j is country j's FSI.

In contract with other financial data, the positive change of Financial Stress Index means the more stressed the country is. In order to make explanation of results be consistent with the common sense, we look at the negative changes of FSI. Therefore, the higher the R_j means the less stressed the Country j is.

Then a standard ARMA(P, Q)-GARCH(p, q) filter is as follows

$$R_{t,j} = \mu_j + \sum_{i=1}^{P} \phi_{i,j} R_{t-i,j} + \sum_{i=1}^{Q} \theta_{i,j} \epsilon_{t-i,j} + \delta_j \sigma_{t,j}^2, \tag{3}$$

$$\epsilon_{t,j} = z_{t,j} \sigma_{t,j}, \tag{4}$$

$$\sigma_{t,j}^2 = \alpha_{0,j} + \sum_{i=1}^{q} \alpha_{i,j} \epsilon_{t-i,j}^2 + \sum_{i=1}^{p} \beta_{i,j} \sigma_{t-i,j}^2, \tag{5}$$

where $j = 1, \ldots, d$, $t = 1, \ldots, T$. $R_{t,j}$ is the negative change of FSI_j at time t. $z_t \sim T(0, 1, \nu)$, the conditions of coefficients which guarantee positive conditional volatility are $\alpha_i > 0$, $\beta_i > 0$ and $\sum \alpha_i + \sum \beta_i < 1$.

In financial market, the reaction of investors are bigger in facing of negative information comparing to the positive events are given. Therefore, we expect the asymmetric GARCH might be better fitted models. For that purpose, ARMA-GARCH will be compared with ARMA-EGARCH, ARMA-GJR-GARCH and ARMA-T-GARCH by BIC[6].

[5] Table 4 shows the unit root test results from Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979), Phillips-Perron (PP) test (Phillips and Perron, 1988) and DF-GLS test (Elliott et al., 1996) of each country's FSI. FSIs from Finland, Greece and Netherlands passes PP tests while fails ADF tests and DF-GLS tests, and all other series fails all three tests which suggests they are non-stationary with at least one unit root. Both tests results from first difference of FSIs show no more unit root. Therefore, by balancing ADF, PP and DF-GLS test results, first difference is applied to all series.

[6]

$$BIC = -2\ln L + k\ln(n)$$

where k is the number of estimated parameters, L is likelihood, n is sample size in order to find the best fitted model (Schwarz, 1978)

ARMA(P,Q)-EGARCH(p,q) is as follows:

$$R_{t,j}=\mu_j+\sum_{i=1}^{P}\phi_{i,j}R_{t-i,j}+\sum_{i=1}^{Q}\theta_{i,j}\epsilon_{t-i,j}+\delta_j\sigma_{t,j}^2, \tag{6}$$

$$\epsilon_{t,j}=z_{t,j}\sigma_{t,j}, \tag{7}$$

$$\ln\sigma_{t,j}^2=\alpha_{0,j}+\sum_{i=1}^{q}\alpha_{i,j}\left|z_{t,j}\right|-\gamma_{i,j}E\left|z_{t,j}\right|+\sum_{i=1}^{p}\beta_{i,j}\ln\sigma_{t-i,j}^2, \tag{8}$$

where $j=1,\dots,d$, $t=1,\dots,T$. $R_{t,j}$ is the negative change of FSI_j at time t. $z_t\sim T(0,1,\nu)$.

ARMA(P,Q)-GJR-GARCH(p,q) is as follows:

$$R_{t,j}=\mu_j+\sum_{i=1}^{P}\phi_{i,j}R_{t-i,j}+\sum_{i=1}^{Q}\theta_{i,j}\epsilon_{t-i,j}+\delta_j\sigma_{t,j}^2, \tag{9}$$

$$\epsilon_{t,j}=z_{t,j}\sigma_{t,j}, \tag{10}$$

$$\sigma_{t,j}^2=\alpha_{0,j}+\sum_{i=1}^{q}\alpha_{i,j}\epsilon_{t-i,j}^2+\sum_{i=1}^{q}\gamma_{i,j}\epsilon_{t-i,j}^2I_{t-i,j}+\sum_{i=1}^{p}\beta_{i,j}\sigma_{t-i,j}^2, \tag{11}$$

where $j=1,\dots,d$, $t=1,\dots,T$. $R_{t,j}$ is the negative change of FSI_j at time t. $z_t\sim T(0,1,\nu)$. $I_{t-i}=1$ if $\epsilon_{t-i}\geq 0$; $I_{t-i}=0$ if $\epsilon_{t-i}<0$.

ARMA(P,Q)-T-GARCH(p,q) is as follows:

$$R_{t,j}=\mu_j+\sum_{i=1}^{P}\phi_{i,j}R_{t-i,j}+\sum_{i=1}^{Q}\theta_{i,j}\epsilon_{t-i,j}+\delta_j\sigma_{t,j}^2, \tag{12}$$

$$\epsilon_{t,j}=z_{t,j}\sigma_{t,j}, \tag{13}$$

$$\sigma_{t,j}^2=\alpha_{0,j}+\sum_{i=1}^{q}\alpha_{i,j}\left|z_{t,j}\right|-\gamma_{i,j}z_{t,j}+\sum_{i=1}^{p}\beta_{i,j}\sigma_{t-i,j}^2, \tag{14}$$

All results are subject to Ljung-Box test on standardized residual and ARCH LM test. Passing both tests indicate there are no higher order autoregression in both first and second moments of the series. The standardized residuals will be transform to uniform distribution by applying probability integral transform (PIT) procedure. To ensure standardized residuals to be independent and identical distributed (i.i.d) and the PIT transformed series are uniform distributed, we follow Patton (2006)'s procedure. It suggests filtered series should pass density goodness of fit procedure of Diebold et al. (1998), Kolmogorov-Smirnov test (K-S test) for uniform distribution, and a joint hit test proposed by Patton (2006)(see Appendix B). Otherwise, the original filter is misspecified.

3.2.2 Step 2: Bivariate-copula analysis

A d-variate copula $C(u_1, \ldots, u_d)$ is a cumulative distribution function (cdf) with uniform marginals on the unit interval. According to the theorem of Sklar (1959), if $F_j(x_j)$ is the cdf of a univariate continuous random variable X_j, then $C(F_1(x_1), \ldots, F_d(x_d))$ is a d-variate distribution for $X = (X_1, \ldots, X_d)$ with marginal distributions F_j, $j = 1, \ldots, d$. Conversely, if F_j, $j = 1, \ldots, d$ is continuous, then a unique copula C is as follows:

$$F(x) = C(F_1(x_1), \ldots, F_d(x_d)), \forall x = (x_1, \ldots, x_d), \tag{15}$$

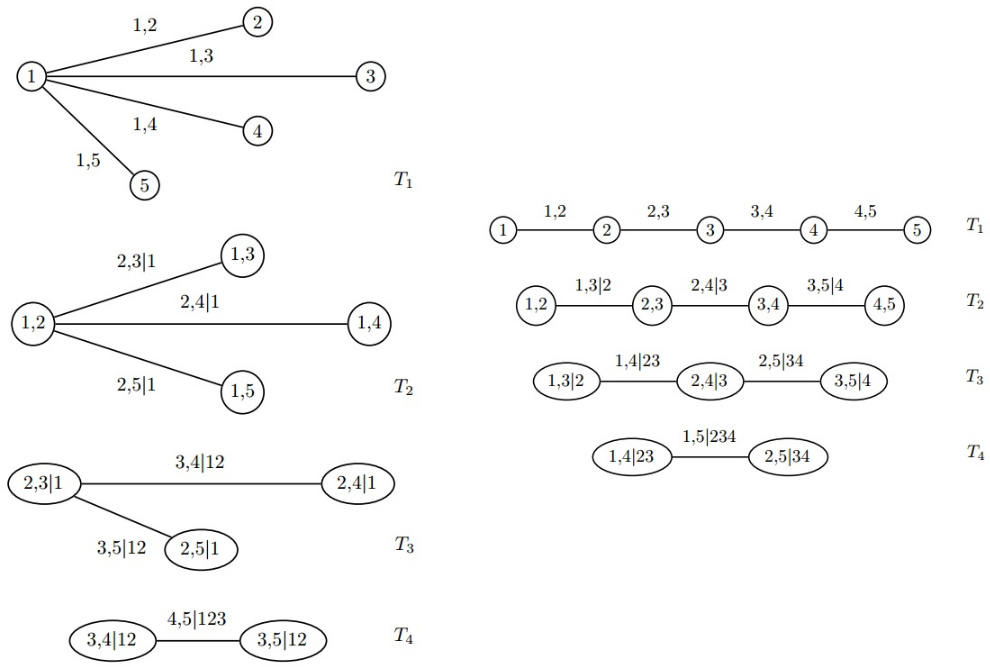
Because vine copula is based on pair-wise copula construction, bi-variate copula can be estimated for all pairs of variables. The Bivariate copula is as follows:

$$F(x) = C(F_1(x_1), F_2(x_2)), \tag{16}$$

In total, there are forty different copula families will be estimated for each pair of varibles, including independence copula, Gaussian copula, Student t copula, Clayton copula, Gumbel copula, Frank copula, Joe copula, BB1, BB7, BB8 copula and rotated forms (90°, 180°and 270°rotation). Different copula family will reveal different types of dependance and tail dependance property (see Appendix C for the properties of each copula family). For instance, Frank copulas show tail independence which is also considered as a benchmark for tail dependence, Gumbel copulas show only upper tail dependence while Clayton copulas show only lower tail dependence. Student-t copulas show reflection symmetric upper and lower tail dependence and BB families show different upper and lower tail dependence . The best fitted copula will be selected based on lowest AIC. Kendall's τ (Kendall, 1938) will also be extracted based on estimated copula parameters to show the correlation between variables (see Appendix C).

3.2.3 Step 3: Vine-copula analysis

Traditional multi-dimensional copula requires all series should have same dependance. For example, a 4-dimensional t-copula means the dependance between any two series are t copula, which is rarely the case in reality. When the dimension reaches 5 and more, the accuracy of traditional copula methods will reduce significantly. Vine copula, on the contrary, allows different types of dependance across different pairs and it also easily model with a higher dimensions, for example ten (Zhang, 2014). While a d-dimensional vine copula are built by $d(d-1)$ bivariate copulas in a $(d-1)$-level



Source: Brechmann and Schepsmeier (2012)

Figure 2: Examples of 5-dimensional C- (left) and D-vine (right)

tree form. There are several ways to construct a copula tree. C-vines and D-vines are the selected tree types in this paper. In a C-vine tree, the dependence with respect to one particular variable (as first root node) is modeled by bivariate copulas for each pair. Conditioned on this variable, pair wise dependencies with respect to a second variable are selected, which is called the second root node. In general, a root node is chosen in each tree and all pairwise dependencies with respect to this node are modeled conditioned on all previous root nodes (see Figure 2 left panel). According to Aas and Berg (2009), this gives the following decomposition of a multivariate density,

$$f(x) = \prod_{k=1}^{d} f_k(x_k) \times \prod_{i=1}^{d-1} \prod_{j=1}^{d-i} c_{i,i+j|1:(i-1)}(F(x_i|x_1,\ldots,x_{i-1}),(F(x_{i+j}|x_1,\ldots,x_{i-1})|\boldsymbol{\theta}_{i,i+j|1:(i-1)}), \tag{17}$$

where f_k,$k=1,\ldots,d$, denote the marginal densities and $c_{i,i+j|1:(i-1)}$ bivariate copula densities with parameter(s) $\boldsymbol{\theta}_{i,i+j|1:(i-1)}$ (here $i_k : i_m$ means $i_k,\ldots,i_m$). And the outer product runs over the

$d-1$ trees and root nodes i, while the inner product refers to the $d-i$ pair-copulas in each tree $i = 1, \dots, d-1$. The log-likelihood function of the C-vine copula with parameter $\boldsymbol{\theta}_{CV}$ is as follows:

$$\ell_{CV}(\boldsymbol{\theta}_{CV}|\boldsymbol{u}) = \sum_{k=1}^{N}\sum_{i=1}^{d-1}\sum_{j=1}^{d-i} \log[c_{i,i+j|1:(i-1)}(F_{i|1:(i-1)}, F_{i+j|1:(i-1)}|\boldsymbol{\theta}_{i,i+j|1:(i-1)})], \tag{18}$$

where $F_{j|i_1:i_m} := F(u_{k,j}|u_{k,i_1}, \cdots, u_{k,i_m})$ and the marginal distribution are uniform.

A D-vine provides an alternative way to choose the order of these pairs (see Figure 2 right panel). In the first level of the tree, the dependence of the first and second variable, the second and the third, the third and the fourth, and so on, are used. That means in a 5-dimensional vine copula, in the first level of the tree, pairs $(1,2), (2,3), (3,4), (4,5)$ have been modeled. While in the second level of the tree, conditional dependence of the first and third given the second variable (pair $(1,3|2)$), the second and fourth given the third (pair $(2,4|3)$), and so on. In this way it continues to construct the third level up to the $d-1$ level. According to Aas and Berg (2009) the density of a D-vine is,

$$\begin{aligned} &f(x) = \prod_{k=1}^{d} f_k(x_k) \times \\ &\prod_{i=1}^{d-1}\prod_{j=1}^{d-i} c_{j,i+i|(j+1):(j+i-1)}(F(x_j|x_{j+1}, \dots, x_{j+i-1}), (F(x_{j+i}|x_{j+1}, \dots, x_{j+i-1})|\theta_{j,j+i|x_{j+1},\dots,x_{j+i-1}}), \end{aligned} \tag{19}$$

where the outer product runs over the $d-1$ trees, while the pairs in each tree are designated by the inner product. In order to get the conditional distribution functions $F(x|\mathbf{v})$ for an m-dimensional vector $\mathbf{v}$, one can sequentially apply the following relationship,

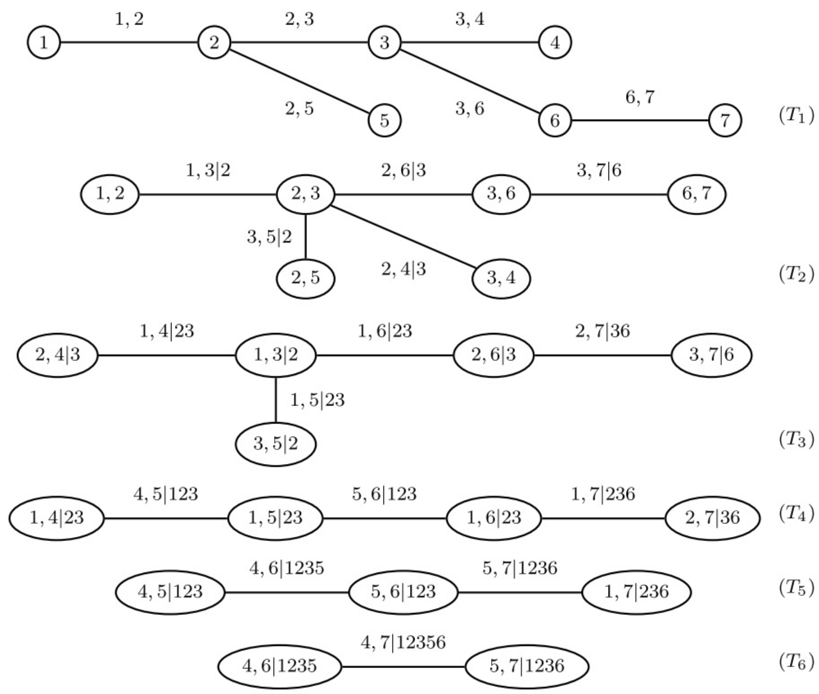
$$h(x|\mathbf{v}, \boldsymbol{\theta}) := F(x|\mathbf{v}) = \frac{\partial C_{xv_j|\mathbf{v}_{-j}}(F(x|\mathbf{v}_{-j}), F(v_j|\mathbf{v}_{-j})|\boldsymbol{\theta})}{\partial F(v_j|\mathbf{v}_{-j}}, \tag{20}$$

where v_j is an arbitrary component of $\mathbf{v}$ and $\mathbf{v}_{-j}$ denotes the $(m-1)$-dimensional vector $\mathbf{v}$ excluding v_j. Further $C_{xv_j|\mathbf{v}_{-j}}$ is a bivariate copula distribution function with parameter(s) θ specified in tree

m. The log-likelihood function of a D-vine copula with parameter $\boldsymbol{\theta}_{DV}$ is as follows:

$$\ell_{DV}(\boldsymbol{\theta}_{DV}|\boldsymbol{u}) = \sum_{k=1}^{N}\sum_{i=1}^{d-1}\sum_{j=1}^{d-i}\log[c_{j,j+i|(j+1):(j+i-1)}(F_{j|(j+1):(j+i-1)}, F_{j+i|(j+1):(j+i-1)}|\boldsymbol{\theta}_{j,j+i|(j+1):(j+i-1)})]. \tag{21}$$

A R-vine copula is more flexible than C-vine or D-vine, in terms of allowing wider range of



Source: Dißmanna et al. (2013)

Figure 3: Example of 7-dimensional R copula

structures (see Figure 3). R-vine copula introduces a new concept - edge in order to increase structure variety. Figure 3, for instance, in the first level of tree of this 7-dimensional data, pairs $(1, 2), (2, 3), (3, 4), (2, 5), (3, 6)$ and $(6, 7)$ will be estimated. While on the second level of tree, pairs $(1, 3|2), (2, 6|3), (3, 7|6), (3, 5|2), (2, 4|3)$and so on. The choice of copula in the lower level will influence the conditional copula in a higher level. Therefore, choices of copula families will influence each other. C-vine and D-vine copula will be specific cases of R-vine copula with specific edge. According to Dißmanna et al. (2013),

$$f(x) = \prod_{k=1}^{d} f_k(x_k) \times$$
$$\prod_{i=1}^{d-1} \prod_{e \in E_i} c_{C_{e,a},C_{e,b}|D_e}(F_{C_{e,a}|D_e}(x_{C_{e,a}}|x_{D_e}), (F_{C_{e,b}|D_e}(x_{C_{e,a}}|x_{D_e}), \tag{22}$$

where $e = a, b$, x_{D_e} is variable from D_e, i.e. $x_{D_e} = x_i | i \in D_e$.

The log-likelihood function of a R-vine copula with parameter $\boldsymbol{\theta}_{RV}$ and $E_1, E_2, \ldots, E_{d-1}$is as follows:

$$\ell_{RV}(\boldsymbol{\theta}_{RV}|\boldsymbol{u}) = \tag{23}$$
$$\sum_{k=1}^{N} \sum_{i=1}^{d-1} \sum_{e \in E_t} \log[c_{j(e),k(e),D(e)}(F(u_{i,j(e)}|u_{i,D(e)})|\boldsymbol{\theta}_{j(e),k(e)|D(e)})],$$

where $u_i = (u_{i,1}, \ldots, u_{i,d})' \in [0,1]^d$,$i = 1, \ldots, N$. $c_{j(e),k(e)|D(e)}$ is the bi-variate copula density with edge e and parameter $\boldsymbol{\theta}_{j(e),k(e)|D(e)}$.

4 Results and Discussion

4.1 ARMA-GARCH filter

The ARMA-GARCH filter exams the series of FSI's feature in both first and second moment. Therefore, results from ARMA-GRACH filter show the information persistence and random shock persistence in both FSI's mean and its variance.

ARMA orders are picked by comparing BIC of all order combinations from $(0, 0)$ to $(6, 6)$. Then, these orders will be used as guidance in ARMA-GARCH filter. The significance of AR (ϕ) and MA (θ) coefficients will be considered and the order with insignificant AR or MA coefficients will be removed. Ljung-Box test on standardized residuals will be conducted to make sure there are no extra auto regressive orders in the series. The first line of Table 2 shows final ARMA orders from all the countries in this sample. More than half of the sample, including Belgium, German, Greece, Ireland, Italy, Netherlands and Spain, are estimated with only one MA order. This suggests the evidence of persistence of information in FSI in their own countries is weak, however, there are strong evidence that random shocks which cannot be capture by AR process has one period (one week) persistence. AR (1) fits France the best. It reveals that, this week's FSI is significantly influenced by last week's stress in both France and Portugal. In Austria and Portugal, there are two period time persistence of information in FSI and longer random shocks (three periods) cannot be modeled by AR. Finland shows the highest order of information persistence which means information four weeks ago still significantly influence current week's FSI.

GARCH models including asymmetric GARCH are estimated in this paper. The optimal models are picked by combining BIC, coefficient significance, ARCH LM test results and i.i.d tests on PIT residuals. The second and third row of Table 2 present the best fitted GARCH model types and their orders. It is interesting to note that other than Finland and Portugal, the rest of countries' FSIs are symmetric in its own variance component. In financial data, the usual case will be asymmetric in variance component because investors generally react more when there are negative information comparing to positive news. However, because the data used in this paper is weekly frequency, it is reasonable to say that the over-reaction of negative information made by investors may be corrected in a week's time. Finland and Portugal, on the other hand, shows significant asymmetry movement of FSI across the negative and positive information. The significance of γ coefficient in Table 2 and Figure 4 is a strong sign of asymmetry. Figure 4 display when there is a negative shock in last period the deviation this period is larger than in a positive shock. δ's significance from Portugal suggests EGARCH-in-mean model fits Portugal. This shows Portugal's FSI changes can also be explained by the volatility directly of the series. Austria, Belgium, France, Germany, Greece, Ireland, and Spain are all GARCH (1,1) process which suggests information will have one period persistence in variance of FSI of these countries. In other words, in these countries, the volatilities from previous period do have strong influence to the current period of time. And this effect will be assessed by the coefficient (β) as a weight. Spain has the strongest influence ($\beta_{1,\text{spa}} = 0.951$) followed by France ($\beta_{1,\text{fra}} = 0.937$) and Italy ($\beta_{1,\text{spa}} = 0.907$). The weakest influence from last period comes from Austria ($\beta_{1,\text{spa}} = 0.825$). Netherlands' FSI is modeled as ARCH(2) model, which means it is not as volatile as other countries, however, there are certain level of random shocks cannot modeled by GARCH process in the second moment.

All countries' FSIs are modeled with a student-t distributed innovation. In finance, the data tend to be fat tails, which makes t distributed innovation a better choice compare to Gaussian. From Table 2, all coefficients are significant in the estimator 'shape' suggest t distributed innovation is a valid and better choice compare to normal distributed innovation. ARCH LM test results suggest there are no more autoregressive orders in the second moment. There is only one exception

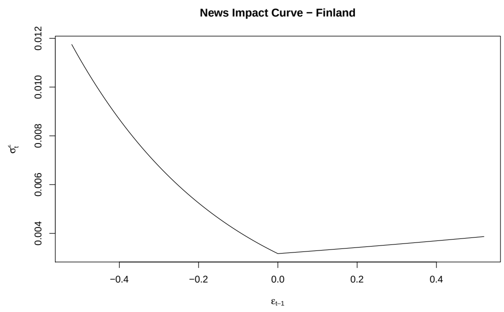
ARCH LM test, which is significant value in Netherlands model lag[5]. To address that, extra orders of ARCH and GARCH are tested, however, coefficients are all insignificant. Combined with the insignificance of lag[1] and lag[2] estimators in original ARCH LM test, it is unlikely to have extra autoregressive orders. All series past Diebold et al.(1998)'s density fit procedure in all first, second, third and fourth moments, KS test as well as Patton(2006)'s joint hit tests with 5 bins.

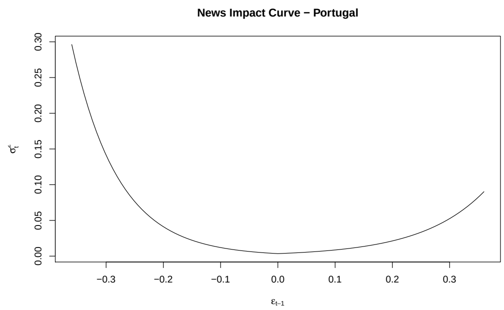
Table 2: Results of GARCH analysis

	Aus	Bel	Fin	Fra	Ger	Gre	Ire	Ita	Net	Por	Spa
ARMA order	(2,3)	(0,1)	(4,2)	(1,0)	(0,1)	(0,1)	(0,1)	(0,1)	(0,1)	(2,3)	(0,1)
GARCH type	GARCH	GARCH	EGARCH	GARCH	GARCH	GARCH	GARCH	GARCH	ARCH	EGARCH-M	GARCH
GARCH order	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(1,1)	(2,0)	(1,2)	(1,1)
μ	6.85E-04	-1.25E-03	6.85E-04	1.10E-05	1.37E-03	-8.86E-04	-3.95E-04	5.00E-04	8.19E-04	6.88E-04	-4.03E-04
ϕ_1	0.696*		1.378*	-0.069*						0.578*	
ϕ_2	-0.961*		-0.500*							-0.988*	
ϕ_3			-0.051*								
ϕ_4			-0.163*								
θ_1	-1.009*	-0.249*	-1.753*		-0.308*	-0.377*	-0.444*	-0.161*	-0.348*	-0.972*	-0.314*
θ_2	1.164*		0.983*							1.256*	
θ_3	-0.268*									-0.409*	
δ										0.010*	
α_0	1.51E-04*	2.85E-05	-1.03E-01*	1.82E-05	1.72E-04	4.26E-05	3.94E-05	3.27E-05	4.09E-03*	-6.17E-01*	1.12E-05
α_1	0.117*	0.097*	-0.062*	0.053*	0.098*	0.126*	0.115*	0.080*	0.257*	-0.072*	0.048*
α_2									0.236*		
β_1	0.825*	0.900 *	0.982*	0.937*	0.857*	0.863*	0.884*	0.907*		0.750*	0.951*
β_2										0.151*	
γ			0.084*							0.467*	
shape	8.760*	5.594*	6.118 *	5.914*	4.920*	4.797*	4.294 *	6.193*	3.087*	4.996*	5.550*
Ljung-Box Test on Standardized Residuals											
lag[]	0.005[1]	0.009[1]	1.944[1]	0.312[1]	0.021[1]	0.0003[1]	0.002[1]	0.186[1]	0.705[1]	0.112[1]	0.518[1]
lag[]	2.562[14]	0.052[2]	6.086[17]	0.604[2]	0.222[2]	0.102[2]	0.010[2]	0.217[2]	0.852[2]	3.206[14]	0.521[2]
lag[]	5.857[24]	2.251[5]	11.618[29]	1.320[5]	1.575[5]	1.191[5]	0.451[5]	0.341[5]	4.315[5]*	5.587[24]	0.797[5]
ARCH LM Test											
lag[]	0.051[3]	2.564[3]	0.564[3]	1.106[3]	0.175[3]	1.393[3]	0.330[3]	3.024[3]	1.821[3]	0.188[4]	0.0645[3]
lag[]	1.387[5]	2.629[5]	1.032[5]	1.106[5]	0.448[5]	2.924[5]	1.961[5]	3.103[5]	2.912[5]	0.759[6]	0.619[5]
lag[]	1.827[7]	3.028[7]	1.141[7]	1.132[7]	2.273[7]	3.336[7]	2.657[7]	3.361[7]	5.458[7]	1.335[8]	0.831[7]
P value of i.i.d Test on PIT Residuals											
1st moment LM test	0.998	0.947	1.000	1.000	0.728	0.990	0.880	0.961	0.983	0.999	0.946
2nd moment LM test	1.000	0.858	0.991	0.762	1.000	0.753	0.998	0.989	0.956	0.498	0.994
3rd moment LM test	1.000	0.981	1.000	1.000	0.959	0.994	0.774	0.972	0.877	1.000	0.909
4th moment LM test	1.000	0.736	0.985	0.798	1.000	0.893	0.997	0.945	0.712	0.507	0.989
K-S test	1	1	1	1	1	1	1	1	1	1	1
Joint hit test	0.629	0.984	0.956	0.977	0.982	0.999	0.583	0.948	0.925	0.999	0.856
AIC	-3.208	-3.333	-2.675	-3.671	-2.998	-3.744	-3.173	-3.490	-2.631	-3.413	-3.529
BIC	-3.140	-3.292	-2.594	-3.630	-2.958	-3.703	-3.132	-3.450	-2.590	-3.318	-3.488

Note:* is significant in the 95% confidence interval. Joint hit tests are tested with 5 bins

Figure 4: News Impact Curve





4.2 Bi-variate copula family

Transformed standardized residuals extract from ARMA-GARCH filter will be used to model the tail dependance of the group of eleven countries financial stress. Tail dependence reveals movement relationship in extreme events. We firstly use a bi-variate copula selection to understand the relationship between each pair of countries. Table 3 presents the the best fit copula families and Kendall's τ for each country pairs. All fifty-five countries pairs fall into five different copula families out of forty. They are Gaussian copula, Frank copula, student t copula, BB8 copula and survival Gumbel copula (180°rotated Gumbel copula).

Table 3: Bi-variate copula selection

country pair	family	τ	country pair	family	τ	country pair	family	τ
Aus.Bel	14	0.23159	Fin.Fra	1	0.158521	Ger.Por	10	0.193805
Aus.Fin	5	0.170072	Fin.Ger	5	0.213946	Ger.Spa	10	0.274349
Aus.Fra	5	0.333136	Fin.Gre	10	0.127411	Gre.Ire	10	0.136534
Aus.Ger	2	0.197265	Fin.Ire	5	0.099923	Gre.Ita	5	0.187348
Aus.Gre	1	0.13953	Fin.Ita	5	0.185124	Gre.Net	10	0.164461
Aus.Ire	10	0.109282	Fin.Net	10	0.198807	Gre.Por	10	0.118107
Aus.Ita	5	0.282826	Fin.Por	14	0.135049	Gre.Spa	10	0.18599
Aus.Net	2	0.255439	Fin.Spa	5	0.219798	Ire.Ita	14	0.179725
Aus.Por	1	0.140272	Fra.Ger	2	0.267848	Ire.Net	5	0.112278
Aus.Spa	5	0.246055	Fra.Gre	10	0.11686	Ire.Por	14	0.137246
Bel.Fin	5	0.18384	Fra.Ire	1	0.129468	Ire.Spa	14	0.192823
Bel.Fra	5	0.365614	Fra.Ita	1	0.335646	Ita.Net	5	0.305323
Bel.Ger	10	0.317592	Fra.Net	2	0.281529	Ita.Por	14	0.240986
Bel.Gre	1	0.136374	Fra.Por	14	0.157651	Ita.Spa	2	0.383454
Bel.Ire	1	0.20032	Fra.Spa	2	0.312687	Net.Por	14	0.195235
Bel.Ita	14	0.321766	Ger.Gre	10	0.142322	Net.Spa	10	0.302959
Bel.Net	2	0.290348	Ger.Ire	10	0.179219	Por.Spa	2	0.218047
Bel.Por	1	0.206181	Ger.Ita	5	0.293654			
Bel.Spa	5	0.329039	Ger.Net	2	0.257315			

Note: All the following family are tested, and the number shows the best fit. 0 = independence copula; 1 = Gaussian copula; 2 = Student t copula (t-copula); 3 = Clayton copula; 4 = Gumbel copula; 5 = Frank copula; 6 = Joe copula; 7 = BB1 copula; 8 = BB6 copula; 9 = BB7 copula; 10 = BB8 copula; 13 = rotated Clayton copula (180 degrees; survival Clayton); 14 = rotated Gumbel copula (180 degrees; survival Gumbel); 16 = rotated Joe copula (180 degrees; survival Joe); 17 = rotated BB1 copula (180 degrees; survival BB1); 18 = rotated BB6 copula (180 degrees; survival BB6); 19 = rotated BB7 copula (180 degrees; survival BB7); 20 = rotated BB8 copula (180 degrees; survival BB8); 23 = rotated Clayton copula (90 degrees); 24 = rotated Gumbel copula (90 degrees); 26 = rotated Joe copula (90 degrees); 27 = rotated BB1 copula (90 degrees); 28 = rotated BB6 copula (90 degrees); 29 = rotated BB7 copula (90 degrees); 30 = rotated BB8 copula (90 degrees); 33 = rotated Clayton copula (270 degrees); 34 = rotated Gumbel copula (270 degrees); 36 = rotated Joe copula (270 degrees); 37 = rotated BB1 copula (270 degrees); 38 = rotated BB6 copula (270 degrees); 39 = rotated BB7 copula (270 degrees); 40 = rotated BB8 copula (270 degrees).

Figure 5 displays examples of these copula families for selected pair countries, such as Aus.Bel, Aus.Ger, Aus.Ire and Bel.Gre. As it shows in Figure 5(a), survival Gumbel copula only presents lower tail dependance other than higher tail dependance. That suggests Austria and Belgium's financial stress will tend to increase together more compare while not decrease together. Bel.Ita, Fin.Por, Fran.Por, Ire.Por, Ire.Spa, Ita.Por, Net.Por pairs tail dependence relationship are similar to Aus.Bel. It is worth mentioning that Italy, Ireland and Portugal have stronger lower tail dependence compare to upper tail dependence. Therefore, as the peripheral countries in Europe, these three countries' financial stress movement has higher level of dependence in negative events compare to positive situations. According to ECB (2014), the countries that showed the largest increases in yield spreads against Germany between 2009 to 2014 were Greece, Italy and Portugal. These countries, referred to the troubled and heavily-indebted countries of Europe, have been the latest to report growing budgetary imbalances and to become subject to the excessive deficit procedure.

As for Figure 5(b), the best fit copula family for country pair like Austria and Finland is Frank Copula. The evidence of having tail dependence is limited because Frank copula is the benchmark for tail independence. This suggests Aus.Fin, Aus.Fra, Aus.Ita, Aus.Spa, Bel.Fin, Bel.Fra, Bel.Spa, Fin.Ger, Fin.Ire, Fin.Ita, Fin.Spa, Ger.Ita, Gre.Ita, Ire.Net, Ita.Net pairs financial stress indices are moving independently to each other when there are extreme events. From the results, Finland tends to be tail independent to other countries. Finland has thrived after joining the single currency in 1999. In addition, it has made a robust recovery even after being suffered in the recession of 2008-09. The unemployment has come down from a peak of 8.7% in early 2010, to 6% in early 2012. The recovery has been sustained by strong consumer spending, supported by a sturdy housing market. The financial system is in decent shape. Finland's public finances are healthy, too, certainly compared with those elsewhere in the euro area. Its government debt is only about 50% of GDP, much lower than Germany's 80%, and the public deficit is about 2% of GDP 2013, a paltry amount compared with those being racked up in Europes debtor countries. Of the six remaining AAA rated countries in the 17-strong zone, Finland is the only one not facing the risk of a downgrade, according to Moody's, a ratings agency (The Economist, 2012).

Figure 5(c) displays student t copula dependence structure using Aus.Ger country pair as an example. Student t copula shows series' symmetric upper and lower tail dependence, which suggests the co-movement in both positive and negative extreme events. Aus.Net, Bel.Net, Fra.Ger, Fra.Net, Fra.Spa, Ger.Net, Ita.Spa, Por.Spa country pairs are estimated as student t copula. The symmetric stress index movement between Netherlands and other European countries implies that Netherlands has shared the benefit in EU with robust economic growth throughout 2007, however, the negative effects of the financial crisis became more apparent since 2008. In the wake of the crisis, typical Dutch strengths, like the pensions system and its strong position in world trades, have turned out to be vulnerable along with other stressed economies like Portugal.

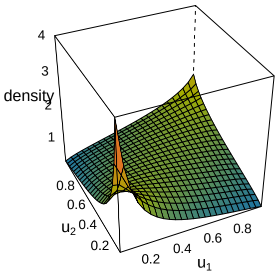
Figure 5(d) shows BB-8 copula dependence structure with Aus.Ire pair as an example. BB-8 copula family reveals an asymmetric upper and lower tail dependence, and the upper tail dependence is higher than lower tail dependence. This suggests that there is evidence of comovement between two countries' stress in both positive and negative extreme events. However, the comovement is more frequent in positive extreme events comparing in negative extreme events. Bel.Ger, Fin.Gre, Fin.Net, Fra.Gre, Ger.Gre, Ger.Ire, Ger.Por, Ger.Spa, Ger.Ire, Gre.Net, Gre.Por, Gre.Spa, Net.Por, Por.Spa are estimated as BB8 copula. Such significant positive movement between Germany and other countries (Spain, Portugal, Ireland and Greece) can be explained that Germany, the largest national economy in Europe, could be very influential for other countries' economy growth. However, the FSIs of these countries have limited impact on German FSI during stressed periods, because of its own stable domestic economy, high market liquidity position, high credit financial assets (e.g. sovereign bonds)[7]. BB-8 copula dependence structure between Greece and other high deficit and high debt countries (i.e. Spain, Portugal, France, and Ireland) implies a high dependence in both stress release and crisis time.

Figure 5(e) presents the Gaussian copula example. It also suggests a limit evidence of tail dependence. The tail dependence is stronger than Frank copula, but less significant than student t copula. Aus.Gre, Aus.Por, Bel.Gre, Bel.Ire, Bel.Por, Fin.Fra, Fra.Ire, Fra.Ita country pairs are estimated as Gaussian copula. The limited tail dependence between France and other countries might suggest that co-movement of FSIs between France and other countries are quite modest.

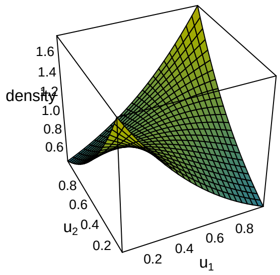
[7]From Jan 2009 to Nov 2014, there are clear diverse-movement of ten-year sovereign bond yields between German and Ireland, Portugal, Spain and Greece during peak time of European Sovereign Debt Crisis 2010-2013(ECB, 2014).

Figure 5: Sample results of Bi-variate copula analysis

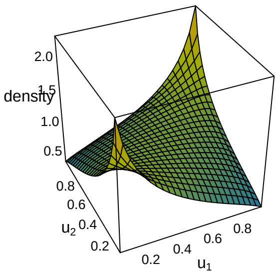
(a) Aus.Bel - 180 rotated Gumbel Copula



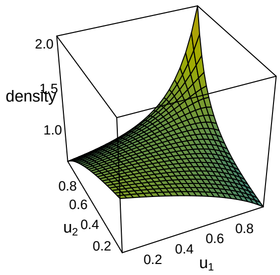
(b) Aus.Fin - Frank Copula



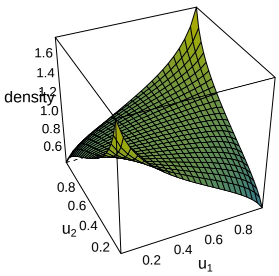
(c) Aus.Ger - t Copula



(d) Aus.Ire-BB8 Copula



(e) Bel.Gre-Guassian Copula



4.3 Vine copula

Based on the bi-variate copula family selections, vine will be structured to explain the inter dependency of these countries as a whole. We have estimated the group of countries by C-vine, D-vine and R-vine structures respectively. The first tree level of C-vine requires a node country have the most dependency to every county else. According to Kendall's τ from table 3, Italy has the highest average dependency with other countries. Therefore, Italy has been chosen as the node country for level 1 C-vine structure (see Figure 6). The D-vine, with an optimal chain dependency structure, is selected based on Travel Salesmen Problem Logarithm (Rosenkrantz et al., 1977, Belgorodski, 2010) which suggest the optimal travel route in terms of cost. The best order choice is Fin, Ger, Bel, Fra, Aus, Ita, Spa, Net, Por, Ire, Gre,(see Figure 7). With a more flexible structure, R-vine can be estimated by introducing different 'edges'. The best estimated R-copula's first level is presented in Figure 8. According to the log-likelihood calculation, R-vine is a better choice[8] . After all, C-vine and D-vine a special cases of R-vine. The first level of R-vines shows that Spain connects to four countries in terms of financial stress, which makes it the most connected country. Italy, France and Belgium are connect to other three countries respectively. Those countries represent the core of the Euro Area, in terms of potential channels of interconnection between the different economies. The R-vine structure also shows the tendency of clustering. Relatively stronger economies including Germany, Belgium, France and Austria are closely connected, while peripheral countries such as Italy, Portugal, Spain and Greece are closely connected. In practice, especially for Spain and Italy, the structural problems in their economies and their financial systems are evident and render them potential threats for Eurozone economic revival and as potential sources of triggering effects for instability in future periods of financial turmoil. This might imply the importance of the position as central nodes because of their highest level inter-dependence with other connections and it might be a more efficient route to treat these four countries' financial stress in order to achieve quicker financial recovery.

[8] The log-likelihood value for C-vine is 1017.193 from Equation 18. The log-likelihood value for D-vine is 1056.477 from Equation 21. And the log-likelihood value for R-vine is 1070.483 from Equation 23.

Figure 6: C-vine Tree level 1

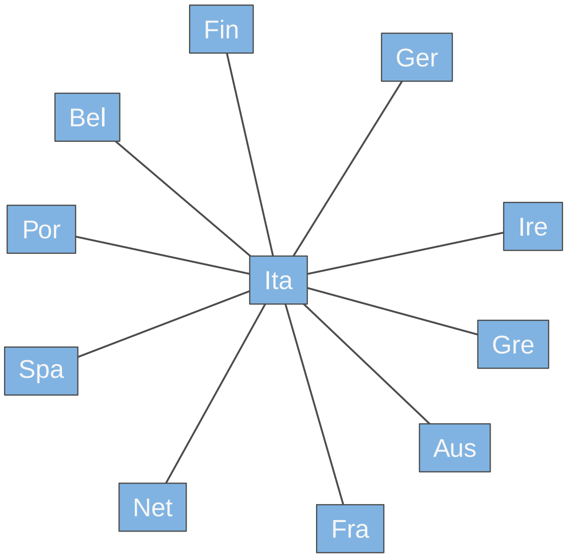


Figure 7: D-vine Tree level 1

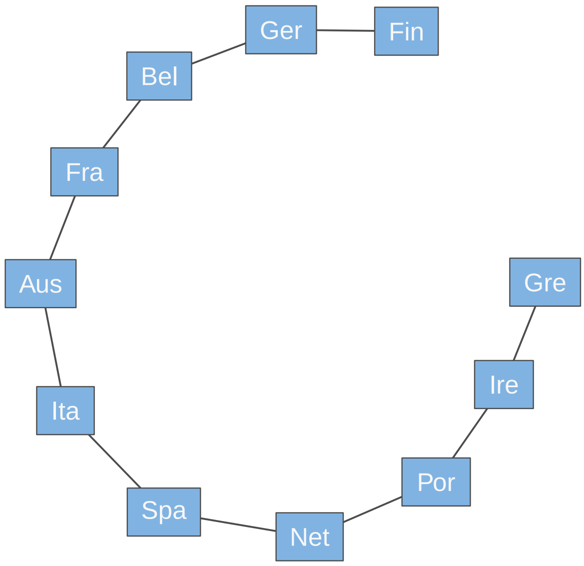
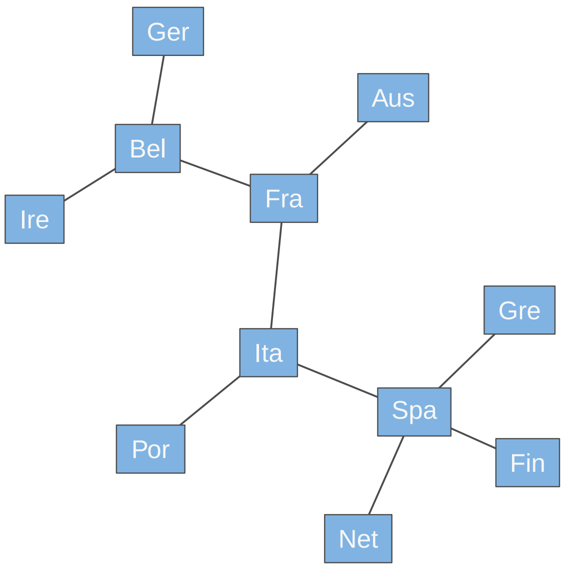


Figure 8: R-vine Tree level 1



5 Conclusion

This paper explores how Euro area countries' financial stress are co-moving with each other between 2001 and 2013. Results show certain level of historical information persistence in the volatility of FSI in most country in Euro area. Only Finland and Portugal show asymmetric feature in terms of financial stress volatility reaction to the negative shocks. We also find larger economies in the Euro area tend to have closer upper tail dependence, while smaller economies tend to have closer lower tail dependence. Finland is generally tail independent with other countries in terms financial stress in extreme situation.

R-vine copula is the best choice comparing traditional C-vine and D-vine to explain the interdependency of these countries as a whole. The estimated R-vine shows Spain, Italy, and France are the most inter-connected nodes which underlying they might be the center of financial interconnectness. We see no solid evidence that the rest of the peripheral economies can lead or affect the rest of the Euro Area. Also, Germany, as the main representer of core countries, seem to be relatively immune to direct effects from the rest of the countries.

The previously summarized results clearly demonstrate the perplexed dynamics of the highly integrated markets within the Eurozone economies. The creation of the common currency area has greatly enhanced the financial interconnectedness of these countries, which has not been followed suit by the macroeconomic conditions of these economies. As suggested by the findings, there is a certain number of countries that are strongly tied and influential to the financial stress level of the rest. Moreover, the structure of the R-vine copula suggests that countries usually characterised as the core of the Euro Area are strongly linked together, while the same holds for peripheral economies as well. A certain degree of "decoupling" is identified, which is important from a policy-making point of view. The current efforts from Eurozone policy makers to develop supranational regulatory and supervisory mechanisms for future financial crashes, such as the European Stability Mechanism (ESM), is an initiative towards the right direction. The same holds for banking supervision, which is now part of the ECBs mandate to coordinate and harmonize supervisory efforts throughout the euro area.

Nevertheless, given the "grouping" effect previously mentioned, some discretion in macroprudential and economic recovery policies can be effective for the badly hit countries to recover.

Appendix A Unit root test results

Table 4: Unit root tests on FSIs (p-values)

	Austria	Belgium	Finland	France	Germany	Greece	Ireland	Italy	Netherlands	Portugal	Spain
					FSIs						
ADF	0.83	0.51	0.34	0.65	0.62	0.21	0.54	0.41	0.21	0.71	0.09
Phillips-Perron	0.55	0.68	0.04	0.64	0.57	0.01	0.09	0.27	0.02	0.30	0.07
DF-GLS	0.91	0.89	0.41	0.84	0.74	0.87	0.69	0.39	0.57	0.67	0.99
					First difference of FSIs						
ADF	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Phillips-Perron	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
DF-GLS	0.01	0.01	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02

Note: ADF test H_0: series has one unit root. Phillips-Perron test H_0: series has one unit root. DF-GLS test H_0: series is a random walk, possibly with drift.

Appendix B Conditional density model evaluation tests

In this appendix, we follow the suggestion from Patton (2006) to test goodness-of-fit of the marginal distribution of copula models. Three tests are conducted for this purpose. Firstly, tests suggested by Diebold et al. (1998) are used to check the independence of the first four moments of transformed series. Denote d transformed series as $\{u_{j,t}\}_{t=1}^{T}$, where $u_{j,t} \equiv F_{j,t}(x_{j,t}|W_{j,t-1})$ for $t = 1, 2, \ldots, T$ and $j = 1, 2, \ldots, d$. We use the same setting as Patton (2006), regressing $(u_{j,t} - \bar{u}_j)^k$ on 10 lags of $(u_{j,t} - \bar{u}_j)^k$, for $k = 1, 2, 3, 4$. The test statistic is $(T - 20) \cdot R^2$ for each regression and is distributed under the null as χ^2_{20}. Any p-value less than 0.05 indicates a rejection of the null hypothesis that the particular model is well specified. Secondly, the Kolmogorov-Smirnov test is applied to exam the hypothesis that transformed series are $Unif(0, 1)$. Thirdly, the joint hit test suggested by Patton (2006)is used to compare the expect number and actual number of observations in each bin of an empirical histogram.

In the joint hit test, the density model in each bin will be separated as 'region' models. The settings of the test is as follows:

Let W_t be the random variable under analysis and denote the support of W_t by S. In our case, W_t will be univariate. Let $\{R_j\}_{j=0}^{K}$ be regions in S such that $R_i \cap R_j = \emptyset$ if $i \neq j$, and $\cup_{j=0}^{K} R_j = S$. Let π_{jt} be the true probability that $W_t \in R_j$ and let p_{jt} be the probability suggested by the model. Let $\Pi_t \equiv [\pi_{0t}, \pi_{1t}, \ldots, \pi_{Kt}]'$ and $P \equiv [p_{0t}, p_{0t}, \ldots, p_{Kt}]'$. The null hypothesis is the model is correctly specified, then $P_t = \Pi_t$ for $t = 1, 2, \ldots, T$. Let us define the variables to be analysed in the tests as follows:

$$H_t^j = \begin{cases} 1 & \text{if } W_t \in R_t \\ 0 & \text{otherwise,} \end{cases}$$

and $M_t \equiv \sum_{j=0}^{K} j \cdot H_t^j$. Then whether the proposed density model is correctly specified in all $K+1$ regions simultaneously can be tested by test the null hypothesis $M_t \sim i.n.i.dMultinomial(P_t)$ against $M_t \sim Multinomial(\Pi_t)$ where Π_t to be a function of P_t and variables in the time $t - 1$

information set. Π_t is specified as:

$$\pi_1(Z_t, \boldsymbol{\beta}, P_t) = \wedge\Big(\lambda_1(Z_{1t}, \beta_1) - \ln\Big[\frac{1-p_{1t}}{p_{1t}}\Big]\Big)$$

$$\pi_j(Z_t, \boldsymbol{\beta}, P_t) = \Big(1 - \sum_{i=1}^{j-1} \pi_{it}\Big) \cdot \wedge\Big(\lambda_j(Z_{jt}, \beta_j) - \ln\Big[\frac{1-\sum_{i=1}^{j} p_{jt}}{p_{jt}}\Big]\Big), \text{for } j = 2, \ldots, K.$$

$$\pi_{0t} = 1 - \sum \pi_j(Z_t, \boldsymbol{\beta}, P_t)_{j=1}^{K}$$

where $Z_t \equiv [Z_1, \ldots, Z_K]'$ and $\boldsymbol{\beta} \equiv [\beta_1, \ldots, \beta_K]'$. Denote K_β as the length of $\boldsymbol{\beta}$. The likelihood function is as follows

$$\mathcal{L}(\Pi(Z, \boldsymbol{\beta}, P)|H) = \sum_{t=1}^{T} \sum_{j=0}^{K} \ln\pi_{jt} \cdot \mathbf{1}\{M_t = j\}$$

Then the joint test will be a likelihood ratio test of $LR_{All} \equiv -2 \cdot (\mathcal{L}(P|H) - \mathcal{L}(\Pi(Z, \hat{\boldsymbol{\beta}}, P)|H)) \sim \chi^2_{K_\beta}$ with the null hypothesis that the model is correctly specified in all K regions. Our setting is following Patton (2006) which adopts 5 bins.

Appendix C Properties of the Bivariate Copula Families

C.1 Elliptical copulas

Gaussian copula function is as follows:

$$C(u_1, u_2) = \Phi_\rho(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$$

Bivariate Student-t copula is as follows:

$$C(u_1, u_2) = t_{\rho,\nu}(t^{-1}(u_1), t^{-1}(u_2))$$

C.2 Archimedean copulas

The bivariate acrchimedean copulas function is:

Table 5: Properties of the elliptical copula families

Name	Parameter range	Kendall's τ	Tail dep.(l, u)
Gaussian	$\rho \in (-1, 1)$	$\frac{2}{\pi}\arcsin(\rho)$	(0,0)
Student-t	$\rho \in (-1, 1), \nu > 2$	$\frac{2}{\pi}\arcsin(\rho)$	$\left(2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right), 2t_{\nu+1}\left(-\sqrt{\nu+1}\sqrt{\frac{1-\rho}{1+\rho}}\right)\right)$

Table 6: Properties of bivariate Archimedean copula families

Name	Function	Para. range	Kendall's τ	Tail dep.(l,u)
Clayton	$\frac{1}{\theta}(t^{-\theta} - 1)$	$\theta > 0$	$\frac{\theta}{\theta+2}$	$(2^{-\frac{1}{\theta}})$
Gumbel	$(-\mathrm{log}t)^{\theta}$	$\theta \geq 1$	$1 - \frac{1}{\theta}$	$(0, 2 - 2^{\frac{1}{\theta}})$
Frank	$-\mathrm{log}\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$	$\theta \in \Re$	$1 - \frac{4}{\theta} + \frac{4D_1(\theta)}{\theta}^{*}$	$(0, 0)$
Joe	$-\mathrm{log}(1-(1-t)^{\theta})$	$\theta > 1$	$1+\frac{4}{\theta^2}\int_0^1 t\mathrm{log}(t)(1-t)^{\frac{2(1-\theta)}{\theta}}dt$	$(0, 2 - 2^{\frac{1}{\theta}})$
BB1	$(t^{-\theta} - 1)^{-\delta}$	$\theta > 0, \delta \geq 1$	$1 - \frac{2}{\delta(\theta+2)}$	$(2^{-\frac{1}{\theta\delta}}, 2 - 2^{\frac{1}{\theta}})$
BB6	$(-\mathrm{log}(1-(1-t)^{\theta}))^{\delta}$	$\theta \geq 1, \delta \geq 1$	$1+\frac{4}{\theta\delta}\int_0^1(-\mathrm{log}(1-(1-t)^{\theta})\times$ $(1-t)(1-(1-t^{-\theta})))dt$	$(0, 2 - 2^{\frac{1}{\theta\delta}})$
BB7	$(1-(1-t)^{\theta})^{-\delta}$	$\theta \geq 1, \delta > 0$	$1+\frac{4}{\theta\delta}\int_0^1(-(1-(1-t)^{\theta})^{\delta+1}\times$ $\frac{(1-(1-t)^{\theta})^{-\delta}-1}{(1-t)^{\theta}-1})dt$	$(2^{-\frac{1}{\theta}}, 2 - 2^{\frac{1}{\theta}})$
BB8	$-\mathrm{log}\left(\frac{1-(1-\delta t)^{\theta}}{1-(1-\delta)^{\theta}}\right)$	$\theta \geq 1, \delta \in (0, 1]$	$1+\frac{4}{\theta\delta}\int_0^1(-\mathrm{log}\left(\frac{(1-t\delta)^{\theta}-1}{(1-\delta^{\theta}-1)}\right)\times$ $(1-t\delta)(1-(1-t\delta^{-\theta})))dt$	$(0, 0)^{**}$
Note:	* $D_1(\theta) = \int_0^{\theta}\frac{c/\theta}{\exp(x)-1}dx$ is the Debye function. ** For $\delta = 1$ the upper tail dependence coefficient is $2 - 2^{\frac{1}{\theta}}$.			

$$C(u_1, u_2) = \varphi^{[-1]}(\varphi(u_1) + \varphi(u_2))$$

where $\varphi : [0, 1] \to [0, \infty]$ is a continuous strictly decreasing convex such that $\varphi(1) = 0$ and $\varphi^{[-1]}$ is the pseudo-inverse as follows:

$$\varphi^{[-1]}(t) = \begin{cases} \varphi^{-1}(t), & 0 \leq t \leq \varphi(0), \\ 0, & \varphi(0) \leq t \leq \infty \end{cases}$$

C.3 Rotations of the copulas

In addition to the copula families presented in the last 2 sections, there are rotated versions of Clayton, Gumbel, Joe, BB1, BB6, BB7 and BB8 families in order to deal with more dependence structure. When the families are rotated by 180 degrees, they are also called the survival forms of the families. The copula function of these copulas will be calculated as follows:

$$
\begin{aligned}
C_{90}(u_1, u_2) &= u_2 - C(1 - u_1, u_2), \\
C_{180}(u_1, u_2) &= u_1 + u_2 - 1 + C(1 - u_1, 1 - u_2), \\
C_{270}(u_1, u_2) &= u_1 - C(u_1, 1 - u_2),
\end{aligned}
$$

where C_{90},C_{180} and C_{270} are the copula C rotated by 90,180 and 270 degree respectively.

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