APPLICATIONS OF A HYBRID METHOD TO A PLATE WITH SIMPLY SUPPORTED BOUNDARY CONDITIONS

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ABSTRACT

The EN12354 building acoustics prediction standards are based on the statistical energy analysis (SEA) method. In a traditional SEA path analysis, lightweight or heavyweight materials have different principal paths that determine the sound insulation for the specified building acoustics frequency range (50Hz-5000Hz). Different building materials require different applications of the engineering method (EN12354) to determine in-situ sound insulation with flanking. A hybrid method, such as the (finite element method) FEM-SEA hybrid approach, offers an alternative theoretical framework to predict in situ sound insulation with the capacity to combine vastly different methods. In a hybrid model, different power flow contributions (due to the deterministic and direct-field energies) are naturally separated into different matrices. This work investigates the feasibility of using a hybrid model to predict sound insulation. The hybrid method is applied to three different materials. These models are compared against traditional SEA and infinite plate models.

Keywords: *sound insulation (SI), hybrid model, building acoustics.*

1. INTRODUCTION

The hybrid method is based on the notion that a complex system can be split into a series of components, which are considered as either deterministic or statistical. Deterministic components are modelled using e.g. Finite Elements, whilst statistical components are modelled using SEA. These distinct theories are combined through the so-called diffuse field reciprocity relation [1], as illustrated in [2]. Since its first development, the hybrid method has been demonstrated on various use cases, notably including air-borne and impact sound transmission. In [3] the method is applied to a simple sound transmission problem for single and double leaf partitions, assuming the partition itself can be represented as a statistical (SEA) component. In [4] a finite element (FE) model of the partition is introduced, and the hybrid model extended to compute the variance of the sound transmission due to the assumed diffuse fields (statistical components) either side. The influence of installing a partition within a common flexible frame is addressed in [5]. Simplification of the closed form variance expressions and extension for third-octave bands is discussed in [6]. In [7] the deterministic partition, previously modelled by FE, is replaced with a transfer matrix model providing an efficient sound transmission prediction through finite-sized thick and layered wall and floor systems. The related issue of sound radiation due to structure-borne excitation is also addressed in [8], where the impacted floor is modeled by FE and the receiver acoustic volume as a diffuse field SEA component.
Statistical energy analysis (SEA) is a well-known method to calculate the sound insulation of different materials. It is the basis upon which the EN12354 building acoustics prediction standards are written. This work investigates the feasibility of using an alternative theoretical framework, based on a hybrid method, to calculate coupling loss factors and predict sound insulation. In this work we compare the hybrid approach against calculations made using traditional SEA

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and infinite plate models. Three different material models are investigated.
Section 2 describes the plate specifications and the room properties. The coupling loss factors calculated using a hybrid approach are compared with a traditional SEA method. The sound insulation is also calculated using traditional SEA and infinite plate models and compared with the results from the hybrid method. These methods are described in section 3. Finally, sections 4 and 5 present the results and conclusion of the early study.

2. DESCRIPTION OF THE MODEL

2.1 Model overview

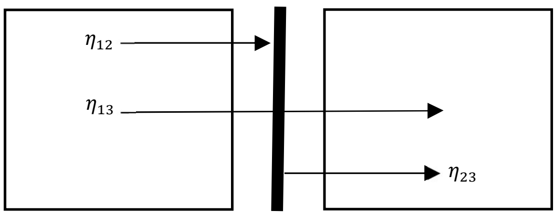

Figure 1. Two acoustic volumes separated by a flat plate [1].

The model comprises three subsystems; two acoustic volumes are separated by a flat plate (see Fig. 1). The acoustic volumes are modelled as two acoustic half spaces and the plate is modelled using both infinite plate and analytic techniques. A direct field approach is used to model the infinite plate, and a modal model with simply supported edges is used to model the deterministic plate system. These methods are combined to give the coupling loss factors and hence the sound insulation of the system.

2.2 Plate specifications

The study examples are three square plates (small to large). The properties of the plates are listed in table 1. The steel plate is also modelled in [1].

2.1 Room specifications

The rooms are assumed to be air filled with typical gas constants for air at room temperature (see table 2). For the infinite plate models, the plate and room dimensions are not considered. In the SEA model the calculation for the cross laminated timber (CLT) was performed with typical room volumes for dwellings with large rooms (59.4 m^3 and 54.0 m^3); the steel with typical room volumes for dwellings with very small rooms (both 10.0 m^3); and the Perspex has typical room volumes for dwellings with small and large rooms (27.0 m^3 and 54.0 m^3). In the hybrid model the source and receiving rooms were assumed to be the same volume both 10.0 m^3. To simplify the comparison with the hybrid model in all of the calculations the rooms are described by a loss factor that does not vary over the frequency range; this can be calculated from the reverberation time of the rooms. The value 0.01 is typical of a reverberation time of 0.44s at 500 Hz. Note in real world applications it is usually only possible to measure the total loss factor of a room or cavity.

Table 1. Plate properties.

	CLT	Steel	Perspex
Elastic modulus (Nm^{-1})	1.58×10^9	2.10×10^{11}	5.6×10^9
Poisson ratio	0.3	0.3	0.3
Dimensions (m^2)	3.6×3.6	0.5×0.5	0.91×0.91
Surface density (kgm^{-3})	37.9	39.3	11.6
Thickness (m)	0.12	0.005	0.0098
Internal loss factor (-)	0.01	0.01	0.02

Table 2. Room properties.

	Room A	Room B
Gas Density (kgm^{-3})	1.205	1.205
Speed of sound (ms^{-1})	343	343
Loss factor (-)	0.01	0.01

3. METHOD

The loss factors were determined using the hybrid method and, for comparison, a traditional SEA method. In all models the material is assumed to be isotropic, and the plates are square with equidistant grid spacing; a 20x20 grid is used where D_d=0, and a 24x24 grid is used where a value for D_d is included.

3.1 Loss factors determined by the hybrid method

3.1.1 No deterministic system (i.e. $D_p=0$)

The coupling loss factors using the hybrid method were determined by [1, 3, 9]:

$$\eta_{jk} = \left(\frac{2}{\omega\pi n_j}\right)\sum_{r,s} \mathrm{Im}\{\mathbf{D}^{(j)}_{\mathrm{dir,rs}}\}\,(\mathbf{D}^{-1}_{\mathrm{tot}}\mathrm{Im}\{\mathbf{D}^{(k)}_{\mathrm{dir}}\}\mathbf{D}^{-1*T}_{\mathrm{tot}})_{r,s} \qquad (1)$$

Where n_j is the modal density of subsystem j, $\mathbf{D}^{(j)}_{\mathrm{dir,rs}}$ is the (r, s) term of the direct field dynamic stiffness matrix of subsystem j, and $\mathbf{D}^{(k)}_{\mathrm{dir}}$ is the direct field dynamic stiffness matrix of subsystem k, the subscripts r and s indicate the (r, s) term and $\mathbf{D}_{\mathrm{tot}}$ is given by

$$\mathbf{D}_{\mathrm{tot}} = \sum_k \mathbf{D}^{(k)}_{\mathrm{dir}} = \mathbf{D}^{(1)}_{\mathrm{dir}} + \mathbf{D}^{(2)}_{\mathrm{dir}} + \mathbf{D}^{(3)}_{\mathrm{dir}} \qquad (2)$$

$D_{\mathrm{d,rs}}= 0$ therefore:

$$\omega\eta_{d,j} = 0 \qquad (3)$$

and there is no deterministic system so:

$$\mathrm{P}^{\mathrm{ext}}_{\mathrm{in,j}} = 0 \qquad (4)$$

3.1.2 Using the driving point dynamic stiffness for the deterministic system (i.e. $D_d=D_{point}$)

The coupling loss factors are determined by Eqn. (1) but in this case [1].

$$\mathbf{D}_{\mathrm{tot}} = \mathbf{D}_{\mathrm{d}} + \sum_k \mathbf{D}^{(k)}_{\mathrm{dir}} \qquad (5)$$

In this approach the ensemble average of the deterministic dynamic stiffness matrix is given by the driving point dynamic stiffness of a plate.

$$E[\mathbf{D}_{\mathrm{d}}] = \mathbf{D}_{\mathrm{p}} \qquad (6)$$

At the edges of the plate this is:

$$D_{\mathrm{p,edge}} = i\omega\sqrt{B\rho h} \qquad (7)$$

Where ω is the angular frequency, B is the bending stiffness of the plate, ρ is the plate density, and h is the plate thickness. In the middle of a plate the driving point dynamic stiffness is:

$$D_{\mathrm{p,middle}} = 8i\omega\sqrt{B\rho h} \qquad (8)$$

There is no power directly input to the plate therefore Eqn. (4) is also applied.

3.1.3 Using a deterministic dynamic stiffness (D_d)

The coupling loss factors are determined by Eqns. (1) and (5) but in this approach the deterministic dynamic stiffness matrix is given by the matrix inverse of the sum of the modal contributions.

$$\mathbf{D}_{\mathrm{d}} = \mathbf{H}_{\boldsymbol{d}}^{-1} \qquad (9)$$

The terms of H_{d} are given by [10]:

$$H_{\mathrm{d,jk}} = \frac{4}{\rho h a^2}\sum_n\sum_m \frac{\sin^2\frac{n\pi x_j}{a}\sin^2\frac{m\pi y_k}{a}}{\omega_0^2(1+i\eta_p)-\omega^2} \qquad (10)$$

Where a is the size of the square plate, ω_0 is the angular resonance frequency, η_p is the internal loss factor of the plate and n, m are integers. Similarly, to section 3.1.2 there is no power directly input to the plate therefore Eqn. (4) is also applied.

3.1.4 The direct field dynamic stiffness matrix ($D^{k2)}_{\mathrm{dir}}$)

The direct field dynamic stiffness matrix is given by the matrix inverse of the receptance matrix [1]:

$$\mathbf{D}^{(k)}_{\mathrm{dir}} = \mathbf{H}^{-1}_{\mathbf{dir}} \qquad (11)$$

The terms of $\mathrm{H}_{\mathrm{dir,jk}}$ are given by [1]:

$$H_{\mathrm{dir,jk}} = G(r_{jk}) \qquad (12)$$

where G is the Green's function for the infinite plate and r_{jk} is the distance between grid points j and k. The Green's function is given by [1]:

$$G(r_{jk}) = (-i/8Bk^2)[H_0^{(2)}(kr_{jk}) - H_0^{(2)}(ikr_{jk})] \qquad (13)$$

Where $H_0^{(2)}$ is zeroth order the Hankel function of the second kind and k is the bending wave number.

3.1.5 The direct field dynamic stiffness matrix of the rooms ($\mathbf{D}^{(j)}_{\mathrm{dir}}$)

The direct field dynamic stiffness matrix of the rooms is given by [9]:

$$\mathbf{D}^{(j)}_{\mathrm{dir}} = \frac{i8\pi\omega\rho c k_a^2}{k_s^4}\{\mathrm{sinc}(k_a r) + if(k_a r)\} \tag{14}$$

where k_a is the acoustic wavenumber, k_s is the wavenumber corresponding to the grid spacing and

$$f(k_a r) = \frac{\cos(k_a r)}{k_a r} + \frac{1}{k_a r}\int_0^{k_s r/k_a} J_0(x)dx \tag{15}$$

and

$$r = \sqrt{(x-x_0)^2 + \left(y-y_0\right)^2} \tag{16}$$

where $J_0(x)$ is a zeroth order Bessel function of the first kind. ($\mathbf{D}_{\mathrm{dir}}$ is undefined along the diagonal and is set to zero.) Langley [9] recommends four points per half wavelength. A mesh density of two points per wavelength, is instead implemented. This less dense mesh was selected due to speed and memory constraints. Also note the condition $k_s \gg k_a$,

3.2 Loss factors determined by a traditional SEA method

The coupling loss factors were determined by using the typical equations for a three-subsystem model [11, 12]. The radiation coupling is given by:

$$\eta_{\mathrm{ij}} = \frac{\rho_0 c_0 \sigma}{\omega \rho h} \tag{17}$$

Where ρ_0 is the gas density, c_0 is the speed of sound of the gas, σ is the radiation efficiency given by Leppington et al. [13]. The plate is assumed to be simply supported and installed in an infinite baffle. The non-resonant coupling is given by:

$$\eta_{\mathrm{ij}} = \frac{c_0 S}{4\omega V_i}\tau_{NR} \tag{18}$$

where S is the surface area of the plate, V_{i} is the volume of subsystem i and τ_{NR} is the non-resonant transmission coefficient, given by Leppington et al. [14]. The modal densities of the plates are given by [11, 12]:

$$n_{\mathrm{B,p}} = \frac{\pi f S}{c_{\mathrm{B,p}}^2} \tag{19}$$

Where f is frequency and $c_{\mathrm{B,p}}$ is the bending wave phase velocity and the modal densities of the rooms are given by [11, 12]

$$n_{\mathrm{R}} = \frac{4\pi f^2 V}{c_0^3} + \frac{\pi f S_{\mathrm{T}}}{2c_0^2} + \frac{L_T}{8c_0} \tag{20}$$

where

$$S_{\mathrm{T}} = 2(L_x L_y + L_x L_z + L_y L_z) \tag{21}$$

and

$$L_{\mathrm{T}} = 4(L_x + L_y + L_z) \tag{22}$$

Where L_{x}, L_{y} and L_{z} are the dimensions of the rooms.

3.3 Consistency relationship

The coupling loss factors can also be calculated or verified in the reverse direction using the consistency relationship. This is given by [11]:

$$\frac{\eta_{\mathrm{ij}}}{n_{\mathrm{j}}} = \frac{\eta_{\mathrm{ji}}}{n_{\mathrm{i}}} \tag{23}$$

4. RESULTS

4.1 Coupling loss factors

4.1.1 No deterministic system (i.e. $D_{\mathrm{d}}=0$)

The different calculation methods to determine the coupling loss factors are compared in Figs. 2, 3 and 4. The η_{12} and η_{23} loss factors are replicated using the hybrid method, however unlike the traditional method a η_{13} loss factor is obtained over the whole frequency range (not just below the critical frequency, f_{c}). The physical significance of the loss factor, η_{13} in this frequency range ($f > f_{\mathrm{c}}$) is unclear. Further work would be required to extend the upper frequency range of the hybrid model. Further work is also required to appropriately include the deterministic dynamic stiffness ($\mathbf{D}_{\mathrm{d}}$) (see also section 4.1.2 and 4.1.3).

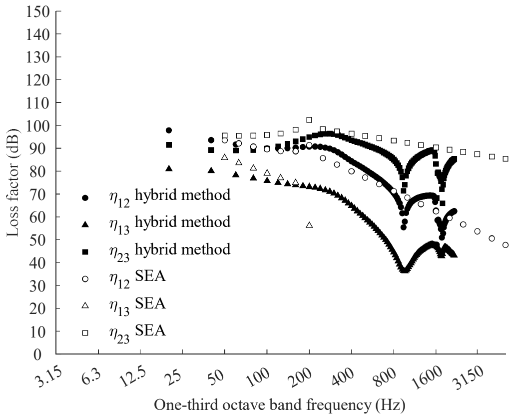

Figure 2. Coupling loss factors for the CLT plate.

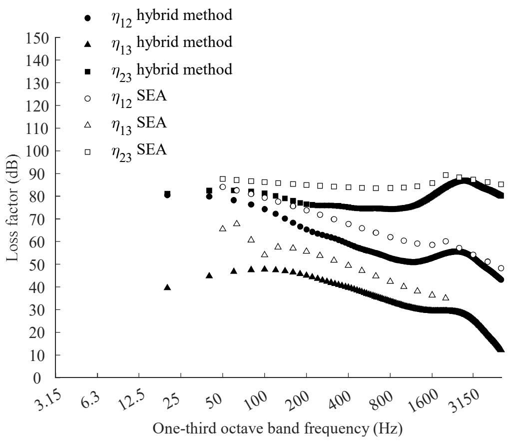

Figure 3. Coupling loss factors for the steel plate.

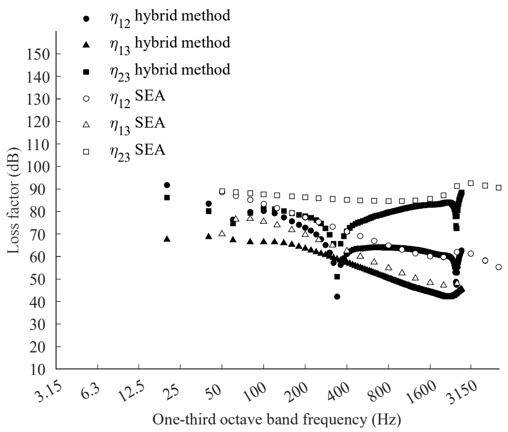

Figure 4. Coupling loss factors for the Perspex plate.

4.1.2 Using the driving point dynamic stiffness (i.e. D_p=D_{point})

Preliminary efforts to incorporate a deterministic dynamic stiffness ($\mathbf{D}_d$) are shown in Figs. 5 and 6 The coupling loss factors for the Perspex and CLT plates are smoothed; however, the agreement between the calculation methods is diminished. This is particularly true of the η_{13} loss factor for both plates, and the η_{23} loss factor for the CLT plate.

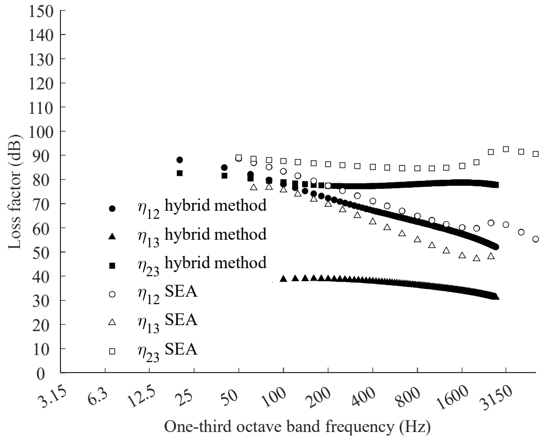

Figure 5. Coupling loss factors for the Perspex plate (D_p=D_{point}).

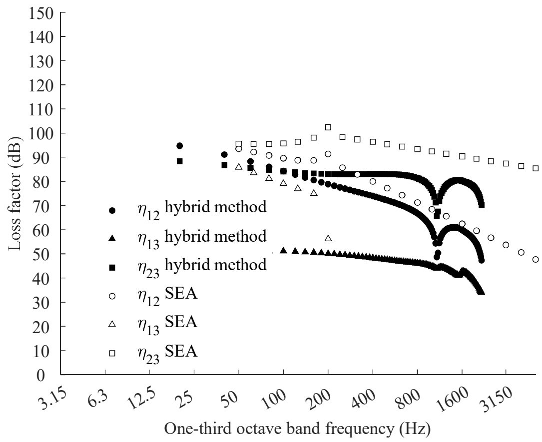

Figure 6. Coupling loss factors for the CLT plate (D_p=D_{point}).

4.1.3 Using a deterministic dynamic stiffness (D_d)

The coupling loss factors when incorporating a deterministic dynamic stiffness (D_d) are shown in Fig. 7 and 8.

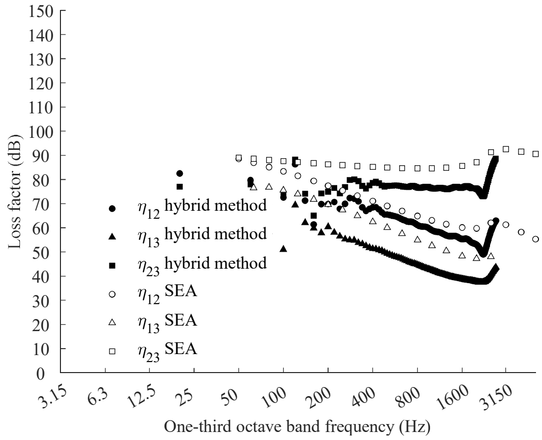

Figure 7. Coupling loss factors for the Perspex plate (including D_p).

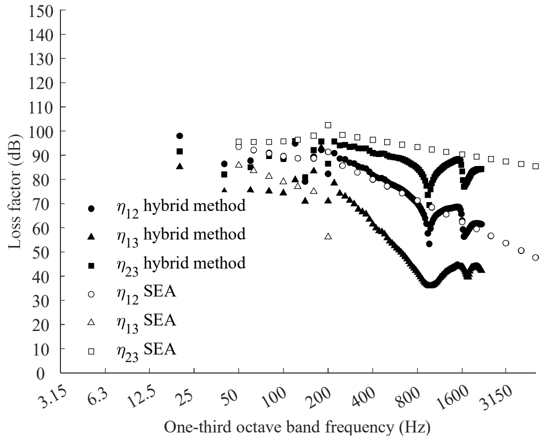

Figure 8. Coupling loss factors for the CLT plate (including D_p).

4.2 Sound insulation

The results of the calculated sound insulation are shown in Figs. 9, 10 and 11. The sound insulation for the traditional SEA and the infinite plate models are presented; further work would be required to extend the upper frequency range of the hybrid model. The critical frequencies of the CLT, steel and Perspex plates are 423 Hz, 2393 Hz and 2898 Hz respectively.

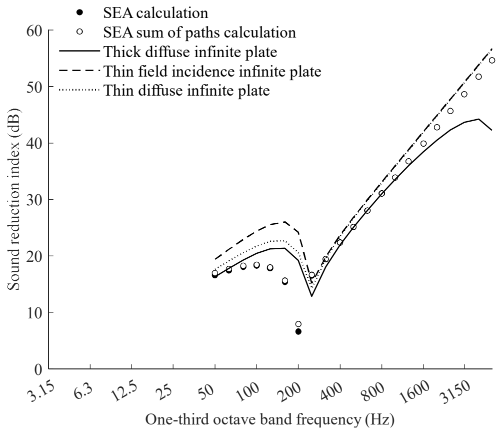

Figure 9. Sound insulation for the CLT plate.

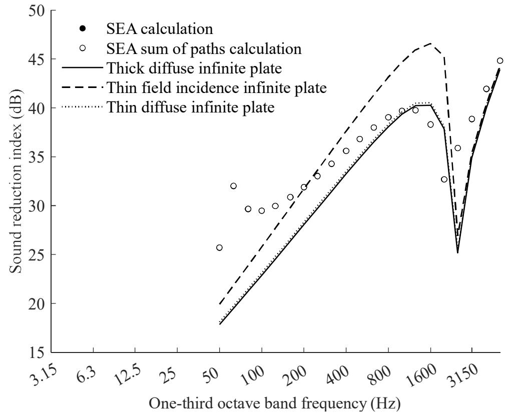

Figure 10. Sound insulation for the steel plate.

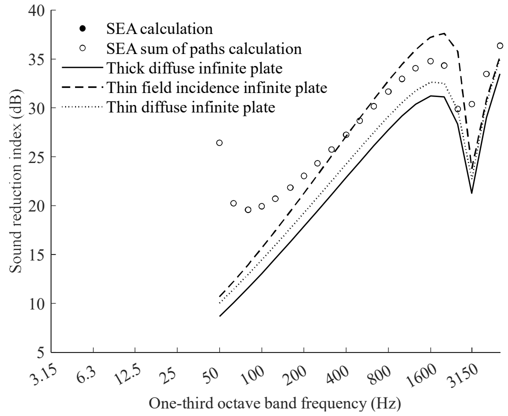

Figure 11. Sound insulation for the Perspex plate.

5. CONCLUSION

The early results are presented for our hybrid model. The η_{12} and η_{23} loss factors are replicated using the hybrid method, however unlike the traditional method a η_{13} loss factor is obtained over the whole frequency range (not just below the critical frequency, f_c). The meaning of this loss factor above the critical frequency ($f > f_c$) is unclear. Further work would be required to extend the upper frequency range of the model. Future work is also required to improve accuracy and fully include a deterministic system in the model.

6. ACKNOWLEDGMENTS

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